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ABSTRACT

This paper is an abstract of an academic thesis available only in Hungarian. The organization of the abstract follows that of the thesis, which is divided into four chapters. At the beginning of each chapter there is a short summary and then the main issues discussed in the chapter are condensed into a few points. Being an abstract, all the formal propositions are presented without proofs. References are added only in unavoidable cases.

EIGENVALUES AND LABOR VALUES

Ernő Zalai

1. ALTERNATIVE DEFINITIONS AND EXISTENCE OF LABOR VALUES IN A SIMPLE MODEL OF REPRODUCTION

Chapter 1 reviews and extends the basic contributions -- especially by A. Brody, 1970 and M. Morishima, 1973 -- to the modern reformulation of the labor theory of value. We start from a rather simple model of social reproduction and examine three alternative definitions of labor values, emphasizing -- unlike the literature -- their conceptual differences. Sufficient conditions for the existence of labor values are based on the dual concepts of absence of production *per se* and impossibility of full automation. These concepts also provide new criteria for the productivity of Leontief matrices. To end the chapter we discuss some common failures in the neoclassical (mis)interpretation of labor values.

1) We assume that we know the following parameters of an economy for a given period of time:

$q = (q_j) > 0^*$ total output of commodity $j = 1,2,\ldots,n$

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*We adopt the following notational convention: (>), larger in all components, ($\geqq$) not less in any component, ($\geq$) larger in at least one component.

$Q = (q_{ij}) \geqq 0$ total (direct) input of commodity $i = 1,2,\ldots,n$ in the production of commodity $j = 1,2,\ldots,n$

$w = (w_j) \geq 0$ total (direct) labor input used in producing commodity $j = 1,2,\ldots,n$.

It is important to note that we do *not* assume a Leontief technology (no joint production and constant returns to scale) here, all we assume is the separability of inputs according to produced commodities (e.g. a Neumann - Leontief economy without constant returns to scale).

Also, we assume that the economy examined is closed (no external supply of commodities) and that for the given period as a whole the supply and demand of the various commodities are in equilibrium. And finally, in accordance with Marx's analysis, we assume a pure market economy, i.e. on average no losses in producing any of the commodities under the existing regime of equilibrium prices and wage rates.

2) Three alternative definitions of the labor value can be distilled from Marx's *Capital*: a) average labor content, b) labor requirement of the final output, c) cumulated past labor inputs. Due to their numerical identity in an input-output model the above concepts are usually taken to be equivalent, which, however, is not the case in general.

ad a) The labor content concept can best be described by its physical analogy: the total amount of labor contained in the total output of commodity j is equal to the total labor 'inflow' into its production. For simplicity and by analogy, we can refer to this procedure as the one based on a 'labor preservation rule'. Thus, if we denote the average labor content by p_j, we will have the following definitional balance identities (labor outflow = labor inflow):

$$(1) \qquad p_j q_j = \sum_{i=1}^{n} p_i q_{ij} + w_j \quad ; \qquad j = 1,2,\ldots,n \quad .$$

ad b) The second concept is based on Marx's dual treatment of value product (total labor input) and final (net) output (the latter being the use-value equivalent of the created new value.) Assume now that the economy, operating with given average input coefficients, provides one unit final output of commodity j and zero of the others. The total output ($q^{(j)}$) ensuring this final output must, thus, fulfill the following equation:

$$(2) \qquad q^{(j)} = Rq^{(j)} + e_j$$

where e_j is the j^{th} unit vector and $R = Q \langle q \rangle^{-1}$, the input coefficient matrix.

Having determined the necessary total output we can now calculate the necessary labor, which -- by definition -- is the value of commodity j:

$$(3) \qquad p_j = mq^{(j)} \quad ,$$

where $m = w \langle q \rangle^{-1}$, the vector of labor input coefficients. If not otherwise stated the product of two vectors should be understood as *scalar* product.

ad c) The third concept is based on the determination of the cumulated direct labor requirements in all phases of production giving rise to one unit output of commodity j. Here it is assumed that the production conditions have been the same in all previous phases. Thus the value of commodity j will be defined in this case as the sum of the following infinite series:

$$(4) \qquad p_j = m_j + mr^{(j)} + mRr^{(j)} + mR^2r^{(j)} + \ldots \quad ,$$

where $r^{(j)}$ is the j^{th} column of matrix R.

3) The equivalence of the above three definitions in the frame of the adopted simple model has been proved and emphasized in the literature. It has not been shown, however, that there are fundamental differences between them in the context of more general models, e.g. in the case of joint production. This fact

explains the emergence of debatable or clearly unacceptable generalizations of Marx's labor value concept. The main differences of the three value definitions are as follows. Definition a) does not require the knowledge of the 'average' input coefficients even less so the assumption of their independence from the time and scale of production. Definition b) is based on the assumption that *in the given period* of time the above coefficients do not change with the size and structure of production. Definition c) goes one step further and assumes fixed input coefficients, independent also of the time of different phases of production. (Note that such backward tracing of previous production phases will go back *ad infinitum* in time.) We will come back to the importance of these differences, when the case of joint production will be discussed.

4) In the case of the examined simple model the productivity of matrix R constitutes a crucial necessary condition for the existence of positive values. (A nonnegative quadratic matrix is called productive if its dominant eigenvalue is less than 1.) Productivity of an input coefficient matrix can be characterized and ensured in many different ways, see, for example, Nikaido, 1968. We propose here a new way, which is plausible and realistic for the Marxian analysis. We do not assume either that there is a final output produced of each commodity or that the production system is irreducible (indecomposable), as it usually is.

5) First we introduce the concept of *'production per se'*. Production *per se* is said to prevail in an economy if there is a group of industries whose commodity production does not exceed the intermediate usage of the same kind of commodities.

In other words, production *per se* implies an 'unproductive' production subsystem, which, in general, is irreconcilable with the idea of a pure market economy. Thus, in the Marxian model it is quite realistic to assume the absence of production *per se*.

The absence of production *per se* can be regarded as an assumption weakening that of irreducibility. Irreducibility implies, among other things that in order to provide final output of any commodity, every commodity is used directly or indirectly. As it will be seen, with the assumption of the absence of production *per se* one supposes only that each commodity is needed directly or indirectly for the production of at least one of the observed (actual) final outputs.

The following propositions hold.

Proposition 1: If the production of commodities exceeds their intermediate usage and every commodity is produced, i.e.

$$q \geq 1Q = Rq \quad \text{and} \quad q > 0 \quad ,$$

production *per se* exists *iff* there are commodities which are neither directly nor indirectly needed to provide the given final output $(q - 1Q)$.

Proposition 2: If the assumptions of Proposition 1 hold and there is no production *per se*, then the input coefficient matrix (R) is productive. Conversely, with productive R production *per se* is impossible.

Remark: If $q > Rq$ or R is irreducible and $q \geq Rq$, then $q > 0$ and each commodity is directly or indirectly involved in producing the final output. Therefore production *per se* is impossible (Proposition 1), and consequently (see Proposition 2) R is productive. Thus the above propositions can be viewed as generalizations of the well-known productivity criteria of Gale.

6) It can be shown that a dual counterpart of the concept of production *per se* is that of *perfect or full automation* (first introduced by Morishima - Catephores (1978) in a different context.) An economy with input coefficients R and m can be fully automated if there is $q \geq 0$ such that $q \geqq Rq$ and $mq = 0$.

The following propositions hold:

Proposition 3: An economy with a productive input-output coefficient matrix (R) can be fully automated *iff* there is a group of commodities whose production does not require labor input either directly or indirectly.

Proposition 4: If the given economy can not be fully automated, and the wage rate and all prices are positive, and, in addition, no commodity is produced at a loss, then the input-output coefficient matrix is productive.

7) In the light of the above considerations, we can prove under plausible assumptions that in a pure market economy with separable inputs the labor values of commodities are positive and uniquely determined.

Proposition 5: If the examined economy can not be fully automated (labor is indispensable) and any of the following two assumptions holds

a) the wage rate and the prices are positive and the production of no commodity (on average) creates a loss;

b) the production of commodities exceeds their intermediate usage and there is no production *per se*,

then the labor values of the commodities are positive and uniquely determined.

Remark: In this simple model-economy each of the discussed definitions of value leads to the same numerical results, as is well known. It is often asserted that Marxian values can be defined only in a fixed coefficient Leontief economy. However, all we need is the assumption of input separability. Here we give a calculation scheme that only makes use of the absolute input and output data (derived from equation (1)):

(5) $$p = w(\langle q \rangle - Q)^{-1} \quad .$$

8) It is customary in the neoclassical tradition to interpret Marx's *labor values* as *equilibrium prices* in a static (rather stationary) economy with constant returns to scale and no joint production and with labor being the only scarce resource (see the interpretation of the *non-substitution* theorem). Such an interpretation, however, does not seem to be fully justified.

First, in defining labor values Marx did not assume that the production technology would not change or that there were constant returns to scale. On the contrary, technological change was in Marx's view an important means to achieve extra profit, the driving force of the capitalist economy.

The second and related issue is that Marx treated value as an average (social) concept. He emphasized the difference between the *'social'* and the *'individual' value* produced by specific producers. Marx, in a sense, took implicitly into account the conditions constraining the reallocation of resources and the choice of technology in any given period of time, when he defined value as *socially necessary* labor.

Third, Marx's definition of value does not imply that labor is the only scarce resource. All it implies is that man-made or natural resources and their availability have nothing to do with the value of the commodities. These are only factors modifying labor values (see, for example, the "false" social values of agricultural products). Whether one accepts this value concept or not is another question. The point is that the neoclassical labor-value concept is different from that of Marx.

9) To illustrate the above point let us take an economy, where n commodities are produced, each by several no-joint-production technologies. Let the observed output in process k_j be denoted by $q_j^{k_j}$ and the total commodity input vector by $q^{(k_j)}$. (Capital is supposed to be renewed to the extent of wear and tear -- as Marx generally assumed -- thus the above inputs reflect the needs of capital renewal, too.) Let the labor input be denoted by $w_j^{k_j}$ in process k_j ($j = 1,2,\dots,n$; $k_j = 1,2,\dots,s_j$). As we noted earlier it is not important here whether there are scarce resources (capital goods, land and other natural resources) or not.

Since inputs are separable we can simply calculate the data needed for the determination of the labor values (see equation (5)):

$$q = (q_j) = \left(\sum_{k_j=1}^{s_j} q_j^{k_j} \right)$$

$$Q = (q_{ij}) = \left(\sum_{k_j=1}^{s_j} q_i^{(k_j)} \right)$$

$$w = (w_j) = \left(\sum_{k_j=1}^{s_j} w_j^{k_j} \right)$$

$$p = w(\langle q \rangle - Q)^{-1}$$

One can easily see that the values determined above will be quite different from the neoclassical 'labor values'. To determine the latter one should first of all assume that there are no scarce resources other than labor and that constant returns to scale prevails. On the basis of these assumptions one has to calculate the (marginal) equilibrium prices. As it is known, these equilibrium (shadow) prices can be determined as the optimal solution of the following linear programming problem:

Maximize

$$p^\circ c$$

subject to

$$p^\circ (\bar{Z} - \bar{Q}) \leqq \bar{w}$$

$$p^\circ \geqq 0 \quad ,$$

where $c \in R_+^n$, $\bar{Q}$ $n \times (s_1+s_2+ \dots +s_n)$ is the commodity input matrix, $\bar{w}$ the labor input vector and $\bar{Z}$ $n \times (m_1+m_2+ \dots +m_n)$ the commodity output matrix.

Note that if we evaluated the form $p(\bar{Z} - \bar{Q}) - \bar{w}$ at average labor values, in general we would get positive as well as negative (extra surplus) values. In the neoclassical case, however, there can be only negative extra surplus.

2. LABOR VALUE AND JOINT PRODUCTION

Chapter 2 is a digression into the problem of labor-value determination in a case where there is no natural way to separate inputs with respect to the produced commodities. It is shown that the three alternative definitions of value (discussed in Chapter 1) cease to lead to the same results and can not be readily applied. The question of valid generalization of Marx's value concept is discussed in the light of the attempts by Steedman (1977), Morishima (1973) and Morishima - Catephores (1978).

10) The examined model-economy in this chapter differs from the one before in two respects. We will assume that the production activities (*actually used*) can be classified only into m *basic processes*, each of them producing possibly more than one commodity. Therefore the *basic* information we have can be summarized by the following data sets:

$\bar{Z} = (z_{ij})$ the (total) amount of commodity i produced by the j^{th} basic process; $(i=1,2,\dots,n; j=1,2,\dots,m)$

$\bar{Q} = (q_{ij})$ the (total) amount of commodity i used in the j^{th} basic process;

$\bar{m} = (m_j)$ the (total) amount of labor used in the j^{th} basic process.

Note that the above economy is not assumed to operate with constant returns to scale and that capital goods are represented only to the extent of their physical depreciation (by their replacement requirement). Therefore it only resembles a von Neumann economy, which is the framework of Morishima's related investigations.

11) The analysis of the three alternative definitions of the labor values yields the following results.

The determination of the average *labor content* -- according to the labor preservation rule -- relies on the solution of the following equation system:

(6) $$p\bar{Z} = p\bar{Q} + \bar{w} \quad ,$$

(labor 'outflow' = labor 'inflow') .

Steedman (1977) has tried to generalize Marx's value concept in this way and demonstrated that it will give, in many cases, meaningless or no solution. He has failed, however, to realize that this is only a *mechanical* generalization of *one* possible definition of values.

Unlike Steedman, Morishima has tried to generalize the *'labor necessary to produce the final output'* definition. This would require first the determination of such activity levels, $x^{(j)}$ for the basic processes, which (assuming constant average input-output coefficients) would yield one unit final output of commodity j. Thus we first have to solve the following equation system:

(7) $$\bar{Z}x^{(j)} = \bar{Q}x^{(j)} + e_j \quad .$$

Having solved the above problem the value of commodity j can then be determined as

$$p_j = \bar{m}x^{(j)} \quad .$$

As Morishima has shown the meaningful solution of (7) poses similar problems as that of (6). Thus the mechanical extension of this definition can not be used either. It is also apparent that the *'cumulated labor input'* concept breaks down completely in the case of joint production (nonseparable inputs).

12) Two things should be clear in the light of the above difficulties. First, no mechanical extensions of the discussed Marxian definitions will, in general, lead to a meaningful generalization of the value concept for the case of non-separable inputs. Second, the three widely used concepts underlying the alternative definitions are equivalent only in the input-output modeling paradigm. (The first two concepts are equivalent for a wider class of models, whenever $(\bar{Q} - \bar{Z})$ is nonnegatively invertible.)

13) Morishima (1973) and later Morishima - Catephores (1978) proposed two possible (nonmechanic) generalizations of Marx's definitions. Both of them ('optimal values' and 'real values') are specific to a given bundle of commodities (nonadditive values) and resemble very much the neoclassical reinterpretation of labor values. The basic difficulties, which make it hard to accept Morishima's generalizations, can be summarized as follows.

They are based on a minimal (marginal) labor-requirement concept instead of an average one. The total value of a commodity vector is not strictly monotonically increasing. Therefore it may happen that the value of the necessary consumption is equal to the amount of labor actually used even if there is capitalist (unproductive) consumption. This would imply no exploitation, which is hardly acceptable on Marxist grounds. Conversely, it may also happen that Morishima's definition implies exploitation (surplus) even if there is no unproductive consumption. In general, the *actual* value of the observed final output (the observed labor input) is usually larger than its *optimal* or *'real'* value. (These latter two problems are closely related to the fact that Morishima does assume constant returns to scale.) Non-additivity of value also seems to pose serious interpretational problems.

14) In the light of the above shortcomings, Morishima's generalization attempts do not seem to be satisfactory either. There is no easy way to extend Marx's concept to more general cases without violating some of his basic criteria. The choice of generalization depends then to a large extent on how much importance one attaches to these different criteria. In the author's view, criteria such as the positivity and additivity of values, the equality of total labor input and the value of total final output should be met by any acceptable generalization.

In the context of the examined (still simple) joint production model one could argue that there always exists a socially-historically determined 'input-separation' rule according to which producers divide the joint inputs between joint products. On the basis of this separation rule the determination of value could then be reduced to the basic case.

Another possible generalization could be based on the extension of the 'labor-content' definition. One could, for example, calculate such nonnegative, additive values that, on the one hand minimize the difference of labor 'inflow' and 'outflow' in each of the basic processes (according to some norm), and on the other hand the total labor input of these values will be equal to the value of final output.

These are, of course, only two possibilities that can be imagined. The problem of a satisfactory generalization of Marx's value concept seems to remain an open question.

3. VALUES AND EIGENVALUES

The main theme of this chapter is the reformulation of the problem of value determination as an eigenvalue problem. We return to the basic model and assume homogeneous labor. After a brief discussion of the value of labor and the concepts of surplus labor, surplus value, and surplus product and their interrelationships, we introduce and analyze different eigenvalue problems suitable for the simultaneous determination of the value system. The existence of positive values will be proved without assuming the irreducibility of the commodity production system.

15) Labor is a *specific* commodity and the definition of its value differs from that of the *common* (non-labor) commodities. There is, thus, no symmetry in the treatment of labor and other commodities, as is often asserted. The value of one unit (say, an hour) of labor is not the total labor content of that particular commodity, but only the labor content of the common commodities needed for its reproduction (necessary consumption). Let us denote the necessary consumption per hour by a semipositive vector f and the value of one hour labor by p_0.

$$p_0 = pf = m(I - R)^{-1}f \quad .$$

The *complete* system of commodity production (labor included) can be characterized by the following *augmented coefficient matrix*.

$$\hat{R} = \begin{pmatrix} 0 & m \\ f & R \end{pmatrix}$$

From this the total labor content of one hour of labor can be determined as

$$(0,m)(I - \hat{R})^{-1} \quad ,$$

which, in general, is larger than p_0.

16) The *rate of surplus* (r) can be determined as the ratio of the surplus labor and necessary labor. The reproduction of one hour of labor necessitates p_0 amount of labor, therefore the hourly *surplus labor*: $1 - p_0$. Thus the rate of surplus:

$$r = \frac{1 - p_0}{p_0} = \frac{1 - pf}{pf} \quad .$$

Proposition 6: In an economy which cannot be fully automated the value of various commodities (including labor) and the surplus rate is positive and uniquely determined *iff* the dominant eigenvalue of the augmented input coefficient matrix is less than 1.

17) *Surplus product* can be defined and said to exist if

$$q_s = q - Rq - f(mq) \geq 0 \quad .$$

The existence of surplus product can also be characterized by the following inequality:

$$\hat{q} \geq \hat{R}\hat{q}$$

where $\hat{q} = (q_0, q)$ and q_0 is the available (reproduced) amount of labor.

The following proposition sheds light on the fundamental correspondence between surplus labor, surplus product and surplus value. *Surplus value* is defined as

$$V_s = pq - pRq - p_0 mq \quad .$$

Proposition 7: In an economy that cannot be fully automated surplus product can emerge only if there is surplus labor. If labor values exist then the surplus value is equal to the surplus labor and also to the value of surplus product. Thus the rate of surplus can be determined by any of these three terms.

Note that surplus product may exist even if labor values do not (e.g. there is production *per se*).

18) The augmented input coefficient matrix ($\hat{R}$) has been the basis of Brody's investigation. Morishima, on the other hand, used the *combined* input coefficient matrix (C), in which labor is represented through its necessary consumption:

$$C = R + fm' \quad ,$$

where fm' is the diadic product of the two vectors.

The only real difference between the two approaches is that the augmented matrix may take into account the total available (reproduced) labor, whereas the combined matrix may only take into account the labor employed in production. It can be shown that

- the dominant eigenvalue of $\hat{R}$ is less or greater than or equal to 1 *iff* the same is true for C
- the submatrix of $(I - \hat{R})^{-1}$ defined by the common commodities is, in fact, $(I - C)^{-1}$
- and if m and f are semipositive as assumed then $\hat{R}$ is irreducible *iff* C is irreducible.

19) The mutual interdependence of the values, the surplus rate, and the input coefficient can be characterized by the following generalized eigenvalue problems:

$$(8) \qquad (p_0, p) = (p_0, p)\hat{R}(r) \quad , \qquad p_0 = \frac{1}{1+r}$$

$$(9) \qquad p = pC(r) \quad , \qquad p_0 = \frac{1}{1+r}$$

where

$$\hat{R}(r) = \begin{pmatrix} 0 & (1+r)m \\ f & R \end{pmatrix}$$

$$C(r) = R + (1+r)fm' \quad .$$

It can be readily seen that if labor values exist, then the rate of surplus must be such that 1 is an eigenvalue of $\hat{R}(r)$ and $C(r)$. In addition, if the above matrices are irreducible at such value of r, then 1 must be their dominant eigenvalue.

If $r = 0$ (simple commodity production), i.e. $\hat{R}(r) = \hat{R}$, then equation (8) will be reduced to the form used by Brody. Brody's closed (eigenvalue) form of the value system in the case of positive surplus value turns out to be inappropriate.

Proposition 8: If $\hat{R}$ (or C) is irreducible then their productivity is a necessary and sufficient condition for the existence of uniquely determined and positive labor values and surplus rate.

20) The irreducibility of $\hat{R}$ or C would imply, on the one hand, that labor is indispensable in the economy, and on the other, that for the reproduction of labor, each commodity is directly or indirectly required. This latter assumption can hardly be justified since there are commodities clearly not needed for the reproduction of labor (mere luxuries or, say, warplanes). Also, from the propositions of Chapter 1 it can be felt that one does not really need such strong assumptions to ensure the uniqueness of the values, i.e. of the positive solution of the generalized eigenvalue problems (8) and (9).

This is shown with the help of three theorems, summarized here as Proposition 9. The underlying economic reasoning is, in short, the following. If labor is indispensable then the labor itself and those commodities that are directly or indirectly required for its reproduction define a set of *basic commodities* (similar to Sraffa's basic products). These basic commodities are indispensable for the production of any commodity. The *subeconomy* defined by the basic commodities is irreducible and uniquely determines both the surplus rate and the values of the basic commodities. From these, in turn, the rest of the values can be uniquely determined, if they exist at all.

Proposition 9: If the assumptions of Proposition 5 hold then the generalized eigenvalue problems (8) and (9) have a unique nonnegative solution in p_0, p, and p_0, p and 1 + r are strictly positive in this solution. *If*, in addition, there exists surplus product *or if* there is at least one profitably produced commodity with wages allowing at least the purchase of the necessary consumption at the prevailing prices, *then* the surplus rate (r) is also positive.

4. HETEROGENEOUS LABOR AND THE DETERMINATION OF VALUE

In the previous chapter we have seen that it is possible to construct a closed model that would simultaneously determine the values of the common commodities and labor as well as the rate of surplus as a function of the prevailing social-technical conditions of reproduction. It also has been seen that when labor is homogeneous this simultaneity is not a real one, since the values of the common commodities can be determined without the knowledge of the value of labor and the rate of surplus. This is not the case, however, when labor is heterogeneous and a unique rate of surplus prevails. It will also be argued that Morishima's attempt to define values for the case of heterogeneous labor is not quite appropriate. The rest of the chapter is devoted to some not clearly defined and understood Marxian concepts such as the producing force, intensity, complicatedness and productivity of labor and their relation to the size of value.

21) Morishima (1973) has suggested the following procedure for the determination of labor values in the case of heterogeneous labor. Let us first introduce the additional parameters and variables that enter the problem.

$M = (m_{kj})$ input coefficient of skilled labor of the kind $k = 1,2,\ldots,h$ in the production of commodity $j = 1,2,\ldots,n$

$m_0 = (m_{0j})$ the same for unskilled labor

$F = (f_{ik})$ the per hour necessary consumption of commodity $i = 1,2,\ldots,n$ by skilled labor kind $k = 1,2,\ldots,h$

$N = (n_{\ell k})$, $n_0 = (n_{0k})$ the skilled ($k = 1,2,\ldots,h$) and unskilled (0) labor input coefficient in the reproduction process of skilled labor kind $k = 1,2,\ldots,h$

$v = (v_k)$ the conversion rate of skilled labor $k = 1,2,\ldots,h$ into unskilled (simple) labor.

Morishima suggests the following solution scheme for the simultaneous determination of the labor values and the conversion rates:

(10) $$p = pR + vM + m_0$$

(11) $$v = pF + vN + n_0 \quad .$$

From the formal analogy it can be seen that the conversion rates can be interpreted as the labor contents of the various kinds of skilled labor. From this, however, it would be a mistake to interpret them as the values of the skilled labor commodities (see point 15 on this question). Such interpretation would treat skilled labor as other common commodities. Thus it would imply that the only source of surplus labor is unskilled (simple) labor. From a different point of view: one, in fact, had to assume that "skill production" is a capitalist enterprise, i.e. capital owners purchase various commodities, among them unskilled labor and produce skilled labor that they sell or rent out to other enterprises. Such an interpretation could perhaps be acceptable in a collective commodity production system but certainly not in a capitalist one.

22) Morishima himself carefully avoided such an interpretation. He also realized that with the above conversion rates and values one could in general expect differing rates of surplus value, i.e. exploitation for the various kinds of labor. This would then violate Marx's explicit assumption about the law of a uniform rate of exploitation. Morishima did not realize that the values and conversion rates which fulfill the above condition can be determined only if for (11) we substitute the following equation

$$(12) \qquad v = (1+r)pF + vN + n_0$$

where r is the uniform rate of surplus. Observe that if the value product of one hour simple labor is taken to be 1 unit, then its value is $\frac{1}{1+r}$. Similarly, the value of an hour of skilled labor kind k is $\frac{1}{1+r} v_k$. Thus dividing equation (12) by (1+r) and denoting by $\pi_k (k=0,1,2,...,h)$ the values of the various kinds of labor will yield:

$$(13) \qquad \pi = pF + \pi N + \pi_0 n_0$$

which is completely in agreement with the definition of the value of labor (see point 15).

Equation (10) after appropriate rearrangement yields:

$$(10') \qquad p = pR + (1+r)(\pi M + \pi_0 m_0) \quad .$$

If we assume for simplicity that the reproduction of simple (unskilled) labor requires only common commodities, in (per hour) amounts shown in vector f_0, then the value of simple labor will be given by

$$(14) \qquad \pi_0 = pf_0 \quad .$$

Combining equations (10'), (13) and (14) will yield a similar generalized eigenvalue problem as the one introduced for the case of homogeneous labor:

$$(\pi_0,\pi,p) = (\pi_0,\pi,p)\begin{pmatrix} 0 & n_0 & (1+r)m_0 \\ 0 & N & (1+r)M \\ f & F & R \end{pmatrix} , \qquad \pi_0 = \frac{1}{1+r} .$$

The derived equation is put in alternative forms and also it is further examined in the broader context of the debates on the possibility of reducing skilled to simple labor and the labor market segmentation problem.*

23) Recent debates in Hungary on the theoretical measurability of value have clearly demonstrated that some of the Marxian concepts are not defined and used unambiguously. They include such concepts as the productivity, producing force, value producing power, intensity and complicatedness of labor. An attempt is made to clearly separate these concepts from each other and reveal their mutual interdependence and also their relation to the determination of labor values.

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*For a more detailed exposition of points 21 and 22 in English see Zalai (1980).

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