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Beaumont, J.R.

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TOWARDS A COMPREHENSIVE FRAMEWORK FOR LOCATION-ALLOCATION MODELS

John R. Beaumont
Department of Geography,
Keele University, U.K.

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

THE AUTHOR

John R. Beaumont is currently a Lecturer in Geography at Keele University, England, and was formerly a member of an urban and regional systems group at Leeds University. In addition to his research interests in public facility location problems, at present he is also examing a number of energy issues for future cities and the dynamics of spatial structure.

FOREWORD

The public provision of urban facilities and services often takes the form of a few central supply points serving a large number of spatially dispersed demand points: for example, hospitals, schools, libraries, and emergency services such as fire and police. A fundamental characteristic of such systems is the spatial separation between suppliers and consumers. No market signals exist to identify efficient and inefficient geographical arrangements, thus the location problem is one that arises in both East and West, in planned and in market economies.

This problem is being studied at IIASA by the Public Facility Location Task, which started in 1979. The expected results of this Task are a comprehensive state-of-the-art survey of current theories and applications, an established network of international contacts among scholars and institutions in different countries, a framework for comparison, unification, and generalization of existing approaches, as well as the formulation of new problems and approaches in the field of optimal location theory.

This paper draws on the the abstract work of location theorists to develop a suitable perspective for decision makers faced with public facility location problems. The elasticity of consumer demand, particularly with respect to accessibility, and the influence of fellow consumers on demand are the two principal focal points addressed by the author.

Related publications in the Public Facility Location Task are listed at the end of this report.

Andrei Rogers
Chairman
Human Settlements
and Services Area

ABSTRACT

The multi-facility generalisation of the Weber problem is used as a point of departure from which to examine a number of conceptual issues of location-allocation models. The restrictive assumption that consumers are allocated to their nearest facility is relaxed; an alternative formulation incorporating a spatial interaction model is outlined. Furthermore, it is argued that perfectly inelastic demand formulations should also be extended, because effected consumer utilisation is directly dependent on the pattern of facilities, particularly the distance consumers have to travel to reach the facilities. The final characteristic of static models to be considered is multi-level formulations, and it is suggested that this could provide a means to develop dynamic models.

CONTENTS

TOWARDS A COMPREHENSIVE FRAMEWORK FOR LOCATION-ALLOCATION MODELS

1. INTRODUCTION

Alfred Weber's (1909) abstract location theory for the individual firm, originating from the work of Fermat, can be regarded as the foundation of a wide range of location-allocation models that have been formulated to examine public facility location problems. In this paper, the aim is to employ the original, simple Weber model as a point of departure towards a development of a generalised and integrated framework for location-allocation problems. It provides both a background to discuss existing models and an opportunity to formulate new models.

Location-allocation problems, jointly optimising the location of facilities and the allocation of consumers to them (by minimising or maximising a specified objective function), can assist decision makers evaluate and compare the combinational ramifications of alternative locational goals.

At the outset, it is important to distinguish between services for which consumers must travel to a facility, such as schools, health clinics, libraries, and recreational facilities, and services which are delivered to consumers, particularly emergency (health and fire) services; particular attention in this paper will be given to the consumer attracting, rather than delivery, systems. This does not mean that the location-allocation framework should not be employed to study locational issues relating to emergency services, although, because of the nature of the problem, user behaviour does not have to be modelled directly. In addition to the need for explicit consideration of the characteristics of the services being provided, it is necessary to recognise the existence of different decision makers; in contrast to the original Weber problem, the components of location and allocation are usually undertaken by different decision makers. For instance, location is usually decided by the organisation providing the service, whereas spatial allocation is usually dependent on individual consumer's choice (excepting, say, the patient referral system in health service provision).

Weber's model is formally stated in Section 2 to highlight the problematic characteristics in relation to its analytical structure and its behavioural context. It is important to note that, in many respects, the Weber problem is a special case of the generalisations considered below. Section 3 is the major section of this paper. The brief reference in Section 2 to recent incorporations of a spatial interaction model into location-allocation models is followed by a long discussion on elasticity of consumer demand, specifically the direct influence of consumers' accessibility to facilities. Another related behavioural aspect, the influence of other consumers on an individual consumer's effective demand, is considered in association with the idea of supply-led demand in public service. These issues are extended in the examination of hierarchical and dynamic models, important theoretical issues which are frequently neglected. The general context of the entire paper is the development of a structure appropriate for decision makers involved in locating public facilities; this is especially illustrated in the exploratory examination of hierarchical and dynamic models. The concluding comments continue this theme by making some general suggestions about the role of location-allocation models in the decision-making process.

2. MULTIFACILITY GENERALISATION OF THE WEBER PROBLEM: DIRECTIONS TOWARDS A COHERENT FRAMEWORK

The multifacility generalisation of the Weber model is the elementary and fundamental form of the location-allocation problem dealing with spatial efficiency. It is a so-called "median" problem, involving the optimum location of supply facilities which minimise the total weighted travel distance (or transport costs) contracted in satisfying demand. The problem of optimally locating a set of p (uncapacitated) supply facilities, $\{\underline{s}\}$, to minimise the aggregate weighted distance between them and the n exogenously given demand points is represented as a continuous space location-allocation problem

$$\min_{\{\underline{s},\underline{\lambda}\}} TC = \sum_{i=j}^{n} \sum_{j=1}^{p} O_i \lambda_{ij} c_{ij} \qquad (2.1)$$

subject to

$$\sum_{j=1}^{p} \lambda_{ij} = 1 \qquad i = 1,\ldots,n \qquad (2.2)$$

$$\lambda_{ij} = \begin{cases} 1 \\ 0 \end{cases} \qquad \begin{array}{l} i = 1,\ldots,n \\ j = 1,\ldots,p \end{array} \qquad (2.3)$$

where O_i is the quantity demanded at location x_i, y_i, and the distance (or generalised cost) between the demand point i and the supply point j is represented by c_{ij} ($i = 1,\ldots,n$; $j = 1,\ldots,p$) which is usually assumed to be the Cartesian or Euchdean distance,

$$c_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{\frac{1}{2}} \qquad (2.4)$$

λ_{ij} is a binary variable, possessing a unitary value if the demand point i is allocated to supply facility j, and zero otherwise. It is noted that both exact and heuristic solution procedures have been developed for this optimisation problem [see, for example, Beaumont (1981) for more details].

A number of characteristics of this problem are listed below to provide the foundation of the remaining sections of this paper. All the models examined in this paper are concerned with locating facilities on a plane rather than on a specified network. Whereas it would, obviously, be interesting to compare the effects of these alternative spatial structures, it is argued that a plane is appropriate when the transport network is well developed and when the facilities are not restricted to a small set of possible locations.

Another characteristic shared by all the models that are discussed in this paper is the employment of the same distance metric [see equation (2.4)]. It is only a special case of the general distance metric.

$$c_{ij} = \left[(x_i - x_j)^p + (y_i - y_j)^p \right]^{\frac{1}{p}} \tag{2.5}$$

and it is noted that a number of formulations of location-allocation models have applied rectilinear distance

$$c_{ij} = |x_i - x_j| + |y_i - y_j| \tag{2.6}$$

In common with the vast majority of location-allocation models, the allocation of consumers to facilities in the Weber problem is based on the nearest centre hypothesis. Constraint (2.2) ensures the "all-or-nothing" allocation procedure, and no consumers have to travel to further centres because of the inclusion of capacity constraints. To relax this restrictive assumption, extensions, incorporating a spatial interaction model to reflect consumers' location choice, have been recently postulated (see, for instance, Beaumont 1980, Coehlo and Wilson 1976, Hodgson 1978, and Leonardi 1978) and, in fact, it has been demonstrated that the Weber problem is a special limiting case (Beaumont 1980). This behavioural aspect is developed below, particularly as aggregate distance (or transport cost) is no longer an appropriate criterion of user benefit when consumer choice is included. Indeed, it should be recalled that Weber's theoretical justifications for determining the least (transport) cost location was that, under the assumptions of perfect competition, uniform non-transportation costs, and perfectly inelastic demand, this criterion is also the optimum profit-maximising location.

In relation to the demand side of location-allocation models, the assumption of exogenously known, perfectly (price) inelastic consumer demand (that is, the level of demand is independent of the distance to the facilities, and, therefore, independent of their location pattern) can be relaxed. Firstly, the majority of models are deterministic; the magnitude and location of demand is known. However, given the obvious difficulties of accurately forecasting future situations, the Weber problem has been examined under conditions of uncertainty (see, for example, Cooper 1978), and more general, stochastic location-allocation models have been recently proposed (Ermoliev and Leonardi 1980). Stochastic formulations are not considered in this paper. Secondly, and arguably more fundamental, the assumption of perfectly inelastic demand is clearly a special limiting case of a whole continuum of possible situations. In the proposed extensions and reformulations of the Weber problem, it is assumed that the level of effective demand is directly dependent on the consumers' accessibility to facilities. At this stage, it is sufficient to note that the introduction of demand elasticity means that, in general, an objective function to minimise aggregate transport costs no longer satisfies Weber's original aim of profit maximisation; it is necessary to explicitly maximise producers' profit (or producers' surplus).

If consumers' demand is assumed to be solely dependent on accessibility (simply stated, it is dependent on the distance to and the relative attractiveness of all facilities), a self-perpetuating, positive feedback situation would result if the problem is uncapacitated. An extension of a facility's capacity would produce, by definition, an improvement in its accessibility, which, in turn, would increase consumers' demand because of the particular (elastic demand) model formulation. As Figure 1 illustrates, these stages provide a mechanism for continual growth. The introduction of an upper size limit, or capacity constraint,

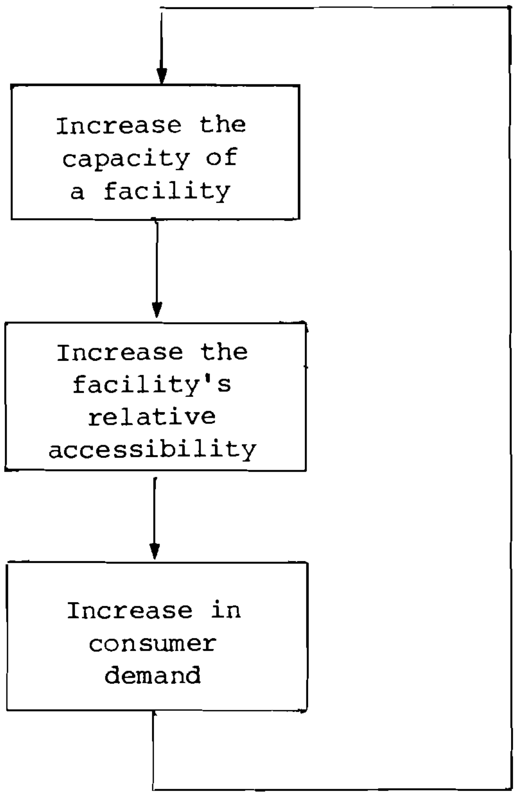

Figure 1. Positive feedback in an uncapacitated location-allocation model.

is advocated, and this can have important spatial effects, such as a modification to the pattern of allocation of consumers to facilities and a change in overall consumer demand.

Two further, basic theoretical, rather than descriptive, extensions towards a coherent structure for location-allocation models are multi-level, hierarchical models and dynamic models. The vast majority of models, such as the Weber problem, are single-level, static formulations. Given the hierarchical organization of various public facilities, multi-level representations are required to analyse the coordinated spatial provision of related services. Similarly, static formulations, concerned with one particular time, also usually fail to capture the essence of the location problem. A static formulation is sufficient if neither the magnitude nor location of demand changes over time and if the costs of relocation are negligible.

The problematic characteristics of many location-allocation models have been highlighted, and this provides a foundation to develop a more coherent modeling structure. Given the particular interest in consumer-attracting service systems, attention is given to the interplay between demand-side mechanisms (including price elasticity of consumer demand with consumers' spatial choice behaviour) and a supply-side mechanism (based on a facility's supply capacity). In addition to an incorporation of more realistic assumptions about consumers' behaviour, some attention will be given to underlying welfare theory and organizational features of supply.

3. BASIC CHARACTERISTICS OF A COHERENT FRAMEWORK: DEMAND ELASTICITY, CAPACITY CONSTRAINTS, HIERARCHICAL STRUCTURE, AND DYNAMICS

In transportation planning, trip generation and distribution models have played significant roles in travel forecasting. These component-of-travel demand models should be incorporated into location-allocation problems to adequately represent consumers' behaviour. Simply stated, it is assumed that trip generation (or demand) is a function of consumers' accessibility to opportunities and that gravity-type spatial interaction models reflect consumers' trip distribution.

In this section, a discussion of these aspects of demand is related to capacity constraints, and, in the last two subsections, these characteristics are extended in hierarchical and dynamic models.

3.1. Demand Elasticity

Even if *in situ* potential demand is uniform, unequal consumption results from consumers' different relative locations to facilities; indeed, it is the nature of geographical space that creates impurity in public goods. This has been a major impetus in the recent study of the conflict between spatial efficiency and equity criteria in the location of public facilities. In general, concentration in a few, large facilities would be more efficient than dispersion of numerous, small facilities which would be relatively equitably distributed. Often, however, the analyses are based on models that assume perfect (price) inelasticity of consumer demand, and, consequently, it is impossible to assess the effect on overall patterns of utilisation [a criterion that was deemed important by Teitz (1968) in his seminal exposition on public facility location problems). The level of utilisation is not wholly a characteristic of particular demand locations, independent of the overall spatial context, but is a function of supply characteristics, such as the distance to facilities and the size of facilities.

A variety of demand elasticity functions based on accessibility have been proposed (see Beaumont 1981, Leonardi 1980a, and Wilson 1973), although there remains a need to extend the definition of this fundamental concept to cover multi-purpose trips as well as single-purpose trips. However, the concept of accessibility provides an operational measure of supply opportunities available to consumers, and, for illustrative purposes, particular attention is given to Hansen's (1959) frequently cited definition of accessibility.

In order to represent the observed trip distribution patterns of consumers, it is necessary to relax the restrictive allocation of consumers to their nearest facilities. Benefits can be derived by patronising a more distant outlet, and this can be represented by a gravity-type spatial interaction model. Formally, these aspects of trip generation and distribution are displayed in the following model

$$T_{ij} = O_i \, e^{\alpha \log \sum_{j=1}^{p} W_j e^{-\beta C_{ij}}} \frac{W_j e^{-\beta C_{ij}}}{\sum_{j=1}^{p} W_j e^{-\beta C_{ij}}} \qquad (3.1)$$

where T_{ij} is the actual demand from a consumer at location i from a facility at location j. O_i scales the level of demand with regard to consumers' accessibility of facilities from zone i, which arguably is itself a function of the level of supply. In the provision of many public services, for example, such as health care, supply creates the demand level rather than the demand determining the level of supply. The term

$$e^{\alpha \log \sum_{j=1}^{p} W_j e^{-\beta C_{ij}}} \qquad (3.2)$$

relates to elasticity of demand; specifically, trip generation is a function of accessibility, Hansen-type (1959) accessibility. W_j is defined as normalised facility size (and is an endogenous relative measure of attractiveness), β is a distance deterrent parameter reflecting the significance of transportation costs in trip behaviour, and α is a scaling parameter relating to demand elasticity. (Perfect inelasticity is when α equals zero, and when it is less than one, it represents a decreasing marginal effect of accessibility on effective demand changes.) The final term is part of a conventional, production-constrained spatial interaction model (Wilson 1971), which represents the probability of a consumer located at i patronising a supply facility at j. This permits an extension of the common, behavioural assumption that consumers travel to their nearest facility, which is, in fact, a special, limiting case when β is infinite.

In relation to this discussion about demand, it is appropriate to allude to the conventional concepts of neoclassical economics: the substitution and income effects. For example, *ceteris paribus*, if, for residents in one zone i, the distance to all the facilities increased in identical proportions, the pattern of trip distribution would remain the same but the level of effective demand would decline—the income effect. At another extreme, if a large, new facility was located in zone i, residents in zone i would obviously alter their patterns of trips because of its relative attractiveness in comparison to its competing facilities—the substitution effect. In practice, when assessing overall changes in the pattern of travel demand, it is usual to explicitly distinguish both these effects.

Furthermore, it is possible to give (Hansen-type) accessibility a stronger theoretical basis by the employment of the Marshallian microeconomic concept of consumer surplus, which, it has been demonstrated, is basically an accessibility index. For instance, Leonardi (1978) discusses a so-called log-accessibility formulation, and, in defining a suitable criterion for measuring consumer benefit, Beaumont (1980) examines a location-allocation model that maximises location surplus. In both models, however, it is assumed that demand is perfectly inelastic, which it has already been argued is only a special case, and that trips are single- rather than multi-purpose in nature.

One possibility is to extend the conventional spatial interaction model by incorporating origin-specific β paramters, β_i. Following the argument adopted in an earlier paper (Beaumont 1981) the location-allocation problem would be written as

$$\underset{\{\underline{s}\}}{\text{Max}}\ LS = \sum_{i=1}^{n} \frac{O_i}{\beta_i} \log \sum_{j=1}^{p} W_j\, e^{-\beta_i C_{ij}} \qquad (3.3)$$

One advantage of the model represented by equation (3.1) is that it is possible to formulate an associated locational surplus objective function that can be maximised. The derivation here is based on the alteration in benefit relating to the changes in transportation costs for a different pattern of supply. [As Neuberger (1971) and Williams (1976) demonstrate, it is possible to derive surplus functions that reflect not only changes in transportation costs, but also modifications in the magnitude and distribution of facilities and consumer demand; it is indicated later that these extensions are especially important.] Such an evaluation, based on transport cost changes, is especially pertinent given the contemporary situation regarding the real price and continued availability of petroleum, and it also provides a useful illustration in the overall argument. The change in benefit, ΔLS is given by the path integral

$$\Delta LS = -\sum_{i=1}^{n} \sum_{j=1}^{p} \int_{\rho} d\underline{c}\ T_{ij}(\underline{c}) \qquad (3.4)$$

where ρ is some path in cost-space between the initial and final configuration of supply points, $\underline{s}^0$ and $\underline{s}^1$, respectively. Using equation (3.1) as the travel demand model, the locational surplus objective function is

$$\underset{\{\underline{s}\}}{\text{Max}}\ LS = \frac{1}{\alpha\beta} \sum_{i=1}^{n} O_i\, e^{\alpha \log \sum_{j=1}^{p} W_j\, e^{-\beta C_{ij}}} \qquad (3.5)$$

and a heuristic solution procedure can be derived.

At this stage, it should be noted that, although the summation of individual components to determine aggregate consumer surplus is a very useful property, such welfare functions should take account of distributional issues. Arguably, it might be more appropriate to apply an objective function concerned with the equalisation of consumers' accessibility to facilities. With regard to this spatial equity issue, attention is drawn to the family of so-called centre problems, which minimise the maximum of the weighted distances between supply and demand points. It should also be remembered that models formulated to maximise aggregate utilisation usually fail to take account of the distributional dimension.

3.2. Capacity Effects

To date, the proposed extensions have been primarily concerned with the demand side of location-allocation models. Clearly, the very essence of such problems relates to the complex interactions between supply and demand. This interaction is modelled in conjunction with the concept of a "pure public good," which is concerned with whether an individual consumer's effective utilisation is affected by the number of other consumers who want to share the same facility. Complete purity is unlikely, and a simple and common way of incorporating some indicator of supply-side capabilities is through the use of capacity constraints. However, conceptually, it is more satisfactory to represent this feature as a modification to the existing attractiveness measure W_j, because it is a component in consumers' travel decision-making process (and it would remain possible to also include capacity constraints). Indeed, Leonardi (1980b) combines both these facets together in an accessibility- and congestion-sensitive demand location model. By modifying the attractiveness function, it is possible to represent how additional users of a public facility reduce its attractiveness to others. A simple definition of an alternative attractiveness measure, suggested by Leonardi (1980b), is unused capacity; as stated earlier, in the provision of public services, supply often leads to demand. Assuming that, for a particular type of facility, the capacity constraint for each facility is identical, say W, by definition, the unused capacity of a particular facility located at j, U_j, is

$$U_j = W - W_j \qquad (3.6)$$

where W_j is a simple function of the magnitude of flows to the particular facility

$$W_j = f\left(\sum_{i=1}^{n} T_{ij}\right) \qquad (3.7)$$

Such a simple measure of attractiveness provides an endogenous mechanism to overcome the positive-feedback type, snowballing growth effect, which was mentioned earlier. The associated elastic travel demand model is

$$T_{ij} = O_i \, e^{\alpha \log \sum_{j=1}^{p} U_j e^{-\beta C_{ij}}} \frac{U_j e^{-\beta C_{ij}}}{\sum_{k=1}^{p} U_k e^{-\beta C_{ik}}} \qquad (3.8)$$

and the location-allocation problem to maximise locational surplus is

$$\underset{\{\underline{s}\}}{\text{Max}} \; LS = \frac{1}{\alpha\beta} \sum_i O_i \, e^{\alpha \log \sum_{j=1}^{p} U_j e^{-\beta C_{ij}}} \qquad (3.9)$$

which does not, however, exhibit a conventional neoclassical structure, because the demand function has additional terms to equation (3.8) arising from the functional interrelationship represented by equation (3.7).

3.3. Hierarchical Structures

It is somewhat surprising that the vast majority of location-allocation models fail to take account of hierarchical, multi-level structures, which are of fundamental importance for a co-ordinated provision of related services. Attention usually focuses on the interactions between consumers and facilities at one level, disregarding interactions between facilities. In the provision of health care facilities, for instance, a hierarchical structure is usually present, relating to the different types of treatment available. Whilst the spatial distribution of these different treatments would probably exhibit different characteristics (such as specialist treatments being provided in only a few locations, and more general, everyday services being provided in a relatively large number of locations), it is important to be able to reflect the patient referral system by explicitly taking account of interactions between service levels.

A number of alternative representations of this structural characteristic can be suggested. Simple disaggregation of a spatial interaction model by type of service would not incorporate the functional interdependence between different levels unless some kind of joint (multi-level) attractiveness function was employed. Such a formulation has been employed in the numerical simulations of the evolution of spatial structure undertaken by Beaumont, *et al.* (1982).

A second approach would be to extend the multi-facility, generalisation of the Weber problem to include both the weighted distances between demand and supply points and between supply points of different orders (assuming facilities of the same level provide identical services). Assuming that there are N levels (k = 1,...,N), the problem to minimise aggregate weighted distance can be formally stated as

$$\min_{\{\underline{s}(k)\}} AWC = \sum_{k=1}^{N} \sum_{i=1}^{n} \sum_{j(k)=1}^{p(k)} O_i(k)\ P_{ij(k)}\ C_{ij(k)} + \sum_{k=1}^{N-1} \sum_{j(k)=1}^{p(k)} \sum_{l(k+1)=1}^{p(k+1)} V_{j(k)l(k+1)}\ C_{j(k)l(k+1)} \tag{3.10}$$

where there are n demand points and p(k) facilities in order k. The spatial allocation component between consumers and facilities, $P_{ij(k)}$ could be defined as a conventional spatial interaction model; V_{kl} allocated each facility of order k to the nearest facility of a different order (l). Whilst this model *per se* is not explored in any further detail, here its characteristics are incorporated in the models described below.

A third way of generating a hierarchical structure is to build the hierarchy downwards as an incremental type of location-allocation model. As an illustration, an N-level, successively inclusive hierarchy [that is, a facility of order x provides services of order x and also the services of all lower orders (x-1,x-2,...,1)] is described. Although this structure is obviously a special case, which is only opposite for particular types of services, this assumption is easily relaxed. The first stage can be thought of as a usual, single-level location-allocation model concerned with optimally locating (according to some specified criterion) a set of P(N) supply facilities of order N, the highest order, in relation to the spatial distribution of n (where n > P(N) demand points). Once this problem has been solved, it provides the information for the first incremental problem. Specifically, given the optimum pattern of facilities of order N (and the original distribution of demand points), the next stage is to optimally locate a set of P(N-1) [where P(N-1) > P(N)] supply facilities of order N-1. As order N-1 services are provided by the highest order facilities (at least, under the assumption of a successively inclusive hierarchy), it is necessary to optimally locate only P(N-1) - P(N) facilities of order N-1, given the location of the facilities of order N. As Figure 2 shows schematically, this procedure is repeated until the complete spatial hierarchy is determined.

A general extension to this kind of derivation of a hierarchy would be to take account of distances between facilities of different orders (specifically facilities of a higher-order when a successively inclusive hierarchy exists). In such a situation, it would be necessary to successively re-examine the location of existing orders when locating an additional order to ensure system optimality.

Within this proposed hierarchiacal framework, it is possible to employ various extensions to the Weber problem that have already been considered. An additional framework would be to view hierarchical development as a temporal process, incorporating it into dynamic models (which are briefly considered in the next section).

3.4. Dynamic Models

Arguably, the essence of the public facility location problem is dynamics, because of the large capital investment involved. In general, however, location-allocation models are concerned with producing optimal, static spatial patterns. This is only appropriate if the costs of facility relocation are negligible and if neither the magnitude nor distribution of demand changes over time. In relation to the spatial dimension, it has already been suggested that large facilities are attractive to consumers and probably enable suppliers to accrue scale economies but that a concentration in a few facilities increases transport costs and may reduce levels of utilisation. In relation to the temporal dimension, it would be argued that a large number of small facilities would provide flexibility for future investment, although capacity problems may be unnecessarily numerous in the early stages. Clearly, the phasing of spatial development against a continually changing environment is a very difficult, but a very important, facility location problem.

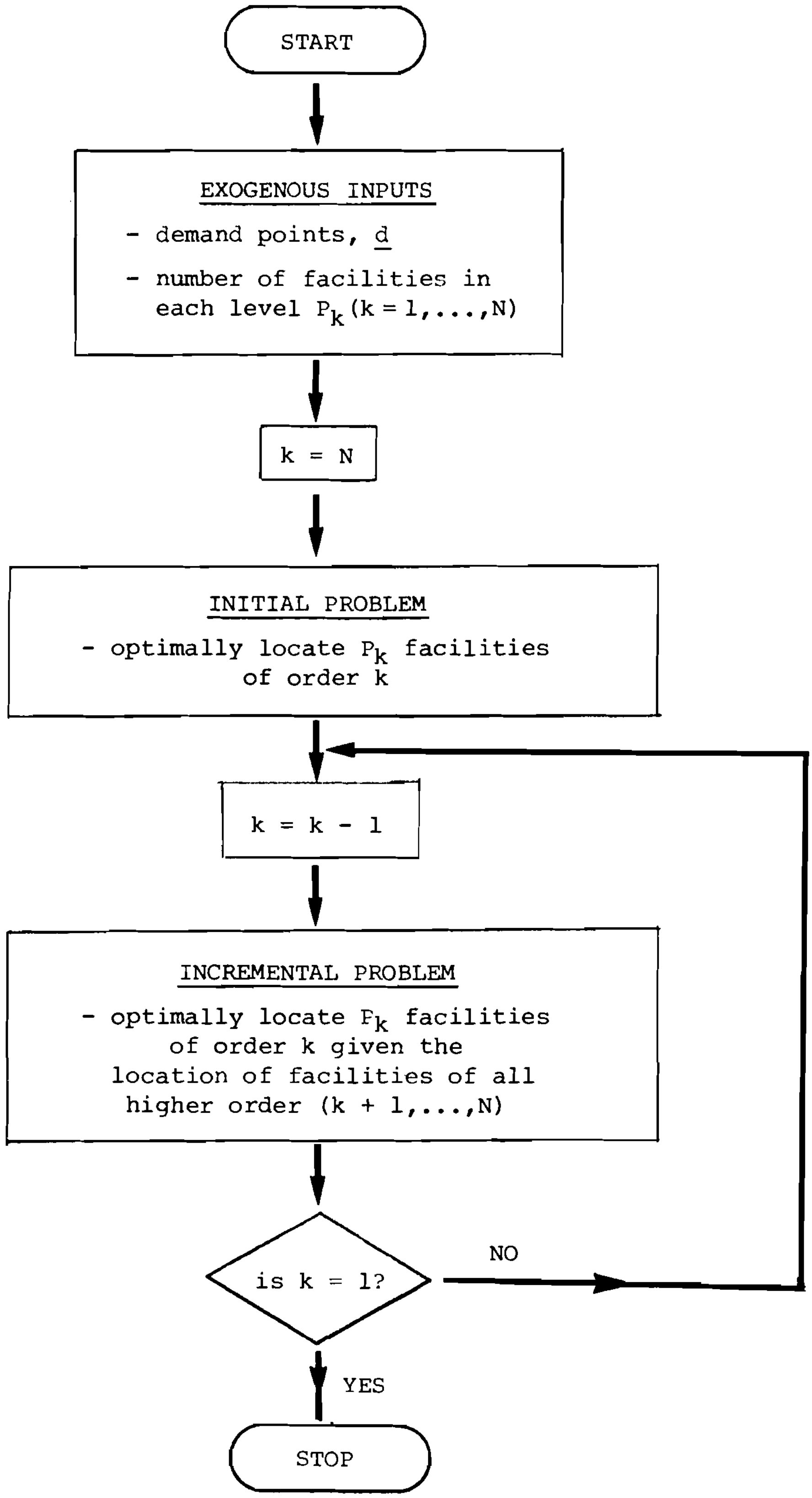

Figure 2. A hierarchical location-alloation model: a successively inclusive hierarchy.

In this section, a brief discussion of a dynamic, hierarchical framework is presented after a description of a dynamic extension of the Weber problem. For further details, see Scott (1971, 1975) and Sheppard (1974) who offer general structural frameworks for the marriage of the spatial and temporal dimensions in facility location problems.

Scott (1975) extended the Weber problem to minimise total weighted distance (or transport costs) over T time periods, during which time the number (n_t), level (O_{it}) and location (x_{it}, y_{it}) of demand points alter. The problem was to locate one facility in the first time period which takes into account these future variations. Formally, this was represented as

$$\underset{\{x_{it}, y_{it}\}}{\text{Min}} \quad z = \sum_{t=1}^{T} \sum_{i=1}^{n_t} O_{it} \, C_{ijt} \tag{3.11}$$

where

$$C_{ijt} = \left[(x_{it} - x_{jt})^2 + (y_{it} - y_{jt})^2 \right]^{\frac{1}{2}} \tag{3.12}$$

and the subscript t represents the time period (and discounted costs could be employed).

Some of the extensions considered in previous sectors suggest avenues to pursue. For instance, hierarchical development could be related to aspects of facility relocation and expansion (or contraction), which could be combined with an endogenous, dynamic attractiveness function dependent on changing capacity levels (specifically, unused capacity) and demand characteristics. For comparability, a framework for a dynamic (successively inclusive) hierarchical location-allocation model is outlined. The development process is assumed to be primarily incremental in nature, taking account of existing infrastructure. Whilst no specific optimising criterion is suggested, it is noted that, if a consumers' surplus objective function was used, it would be necessary to include changes in benefits related to changes in origin and destination characteristics as well as to changes in transport costs. For example, extra trips may be beneficial because of the increased attractiveness accessibility of some facilities, but this should not be considered solely in terms of extra transport costs.

Figure 3 portrays a dynamic, incremental (successively inclusive) hierarchical location-allocation model. Conceptually, given the long-term nature of planning for public facilities and the need to explicitly take account of existing stock, a beneficial property of the framework is its explicit, combined representation of present and future distribution of facilities of different orders. Indeed, it is the present configuration of facilities which is probably the most important factor determining the potential and the specific strategy required for a more efficient and equitable provision of services. Future development is, therefore, realistically mirrored as a part of an ongoing process. Decisions relating to facility extension or facility construction must take account of not only the aspects related to consumer behaviour, such as the amount of unused capacity and level of realised demand, but also the aspects related to supplier behaviour, such as operating costs [for example, in relation to scale (dis)economies] and capital costs (which are important determinants of the number of facilities). (Indeed, in general, insufficient attention has been given to providing a sound economic basis for cost functions; this is in stark contrast to, say, the analyses of consumers' spatial choice behaviour.) At the beginning of each time period, for example, if an aggregate budget constraint was known, it would be possible to simply determine the maximum number of new facilities that could be constructed (and examine the opportunities for the expansion of existing facilities). Assuming an elastic demand formulation, it would be possible to consider trade-offs between facility attractiveness-capacity problems and distance effects on the overall level of utilisation by successively analysing problems with different numbers of facilities. Particular attention should be given to whether there is a common core of optimum locations in these various analyses.

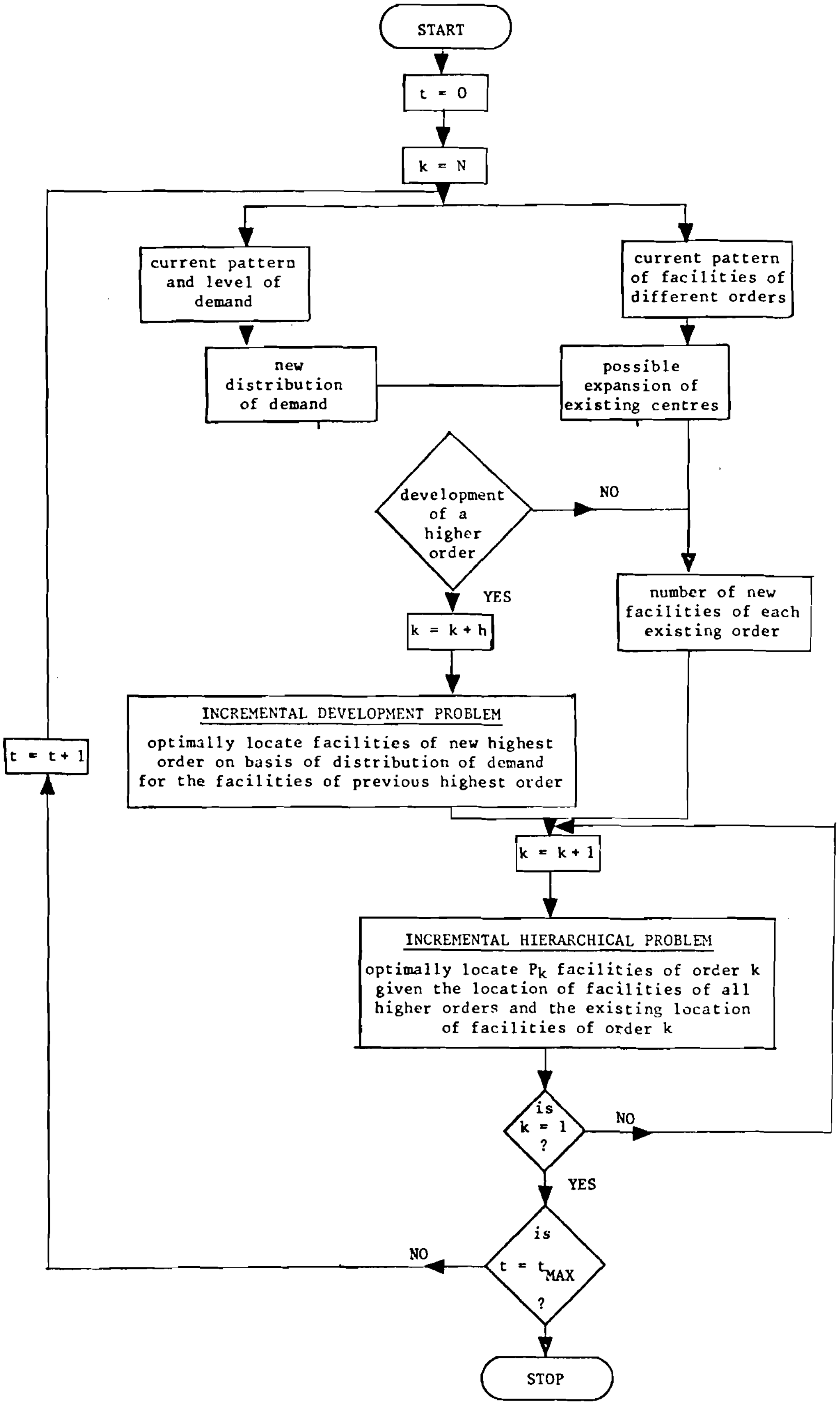

Figure 3. A dynamic (successively inclusive) hierarchical location-allocation model.

Such a framework is, therefore, very flexible, and whilst it is necessary to specify clearly the actual models required in particular situations, it should be easily operationalised. Its position in relation to the planning process is briefly discussed in the concluding section.

4. CONCLUDING COMMENTS

A detailed re-examination of the properties of Weber's original problem provided a suitable foundation from which to attempt to develop a comprehensive framework for location-allocation models. Most of the fundamental issues are raised, although the resulting discussion is necessarily selective and exploratory, rather than a definitive statement on public facility location problems. Theoretical issues relating to dynamic and hierarchical models obviously require and merit further attention.

Sufficient different formulations, however, have been outlined to suggest that the prescriptive analytical rigour will assist decision makers to evaluate and compare the implications of alternative locational goals relating to spatial efficiency and equity. In fact, the planning process, applying location-allocation models, could be seen simply as an on-going decision-making process. The idea of a continual planner-computer interactive decision-making process has been alluded to elsewhere (Beaumont 1981), and the framework for the dynamic, hierarchical model, for instance, was developed to be sufficiently flexible to permit this important characteristic. Moreover, whilst there was no discussion about possible solution algorithms, it has been suggested that heuristic procedures would be especially attractive, because they not only conceptually separate the processes of location and allocation, which usually involve different decision makers, but also facilitate an active involvement of planners.

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