International Institute for Applied Systems Analysis
IIASA
www.iiasa.ac.at

Parliamentary Representation and the Amalgam Method

Balinski, M.L. and Young, H.P.

IIASA Collaborative Paper
November 1981

PARLIAMENTARY REPRESENTATION AND THE AMALGAM METHOD

M.L. Balinski and H.P. Young

November 1981
CP-81-36

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

AUTHORS

M.L. Balinski, Institut Auguste Comte, 21 Rue Descartes, 75005 Paris, France

H.P. Young, International Institute for Applied Systems Analysis, 2361 Laxenburg, Austria

ABSTRACT

This paper explains in direct operational terms the method presently established by law to apportion the seats of the Canadian Parliament among the provinces, the so-called "amalgam" method.

Canadian political history and common sense suggest fundamental principles of equity that should apply to any method of apportioning representation in the parliament. Unfortunately the amalgam method satisfies *none* of these principles.

There is exactly one method which does satisfy all of them: it is much simpler than the amalgam method, and is basically the one originally embodied in the British North America Act of 1867.

PARLIAMENTARY REPRESENTATION AND THE AMALGAM METHOD*

M.L. Balinski and H.P. Young

INTRODUCTION

The amended Article 51 of the British North America Act that came into force on December 31, 1974 specifies a rule for apportioning representation in the House of Commons known as the "amalgam method". The name is well-chosen. According to the Oxford English Dictionary the original meaning of amalgam is "a soft mass formed by chemical manipulation". The change of a single word in this definition well conveys the essence of the method.

Its prime motivation seems to have been to guarantee that no province would ever lose seats thus incidentally guaranteeing that no incumbent would lose his. (By a quirk of fate, the previous method applied in 1971 would have resulted in gains for *only* two provinces: Ontario and British Columbia.) It is a subtle scheme that appears to favor the small provinces more than it actually does, and tends to work against the rapidly growing intermediate-sized provinces of Alberta and British Columbia. At the same time the method is so opaque and intricately contrived that it raises doubts about its equity while at the same time hiding the nature of the inequities and blunting potential criticism.

*The authors are grateful to Professor Duff Spafford for constructive comments on an earlier draft of this paper.

Members of Parliament expressed their frustration on this point in the 1974 debates that preceded its adoption: "I have said that the language of this bill is terribly complicated, convoluted and contorted, and the minister's smile suggests that he found it that way too. I still think it should be possible to find someone else to draft these bills rather than lawyers."[1] The reader who enjoys puzzles is invited to read the actual definition of the method in the Appendix, and to try his hand at computing the apportionment based on the population data given in Table 2 below.

Equitable representation is at the heart of democracy. The provinces of Canada have had, and are having, disputes over the share of political and economic power. A necessary step in dealing with these problems is to adopt a method of apportioning the seats in the House of Commons that is easily understood and fair, instead of one that is "almost as complicated as the Einstein theory."[2]

This article has several objectives: first, to explain the amalgam method in operational terms and to show by example why it is so fundamentally bad; second, to identify principles of equity embedded in Canadian precedent and in common sense that should govern the allocation of seats to provinces; third, to explain that there is a simple and natural method that satisfies all of the principles -- in fact, there is only one such method. The principles themselves determine the method.

Brief History

Before trying to unravel the amalgam method, it is helpful to take a brief look at its antecedents in Canadian history.[3] The original Article 51 of the British North America Act specified a very simple and natural method of a type still used in many countries today. The idea is to select a target number of constituents per representative, or *divisor* x, and then to divide the population of every province by x to obtain its *quotient*, a whole number plus a fraction. If the fraction is less than one-half the province's quotient is rounded down, if more than one-half it is rounded up, and the result is the number of seats for the province. The Act further specified what x had to be, namely, the population of Quebec divided by 65. In other words, Quebec got 65 representatives, and determined the ideal constituency size to be applied to the other provinces. There was also a special proviso that no province could lose seats unless its population had decreased at least 5% from the preceding census. This was amended in 1915 to guarantee that no province should ever receive fewer seats than the number of its senators.

This approach, without the special way of determining x was first proposed in 1832 by Senator Daniel Webster of Massachusetts. It is the most often used method in United States history (under the name "method of major fractions"), and a slight variant of it is employed in Scandinavia where it is known as the method of "odd numbers".[4]

The result of applying the method to the 1971 populations, with Quebec assigned the 75 seats it now has (instead of 65) and floors equal to the present number of senators, is shown in Table 1.

Province	1971 Population	Floor	Quotient	Webster App't
Ontario	7 703 106	24	95.845	96
Quebec	6 027 764	24	75.000	75
British Columbia	2 184 621	6	27.182	27
Alberta	1 627 874	6	20.255	20
Manitoba	988 247	6	12.296	12
Saskatchewan	926 242	6	11.525	12
Nova Scotia	788 960	10	9.817	10
New Brunswick	634 557	10	7.895	10
Newfoundland	522 104	6	6.496	6
P.E.I.	111 641	4	1.389	4
Total	21 515 116	102		272

TABLE 1. Webster Apportionment with Divisor 80 370.187

(Quebec's Population Divided by 75)

A related approach is to divide the populations by a common divisor x and then simply drop the fractions, i.e., give each province the whole number in its quotient. This method was first proposed by Thomas Jefferson in 1792 and is used in many proportional representation systems under the name "d'Hondt's method". Still another variation on this theme, due to John Quincy Adams, is to give an extra representative for every fraction; in other words to round every quotient up to the next largest whole number.

These three methods are all examples of *divisor methods*, because they are all based on a common divisor x together with a rule that tells when to round a quotient up or down.[5] The choice of a divisor x determines the total number of representatives or *house size*: the smaller x is, the larger will be the house. However, it is also always possible to first fix the house size and then find *some* choice of x that results in an apportionment of precisely this number of seats.

The next event in Canadian apportionment history occurred in 1946, when a different approach was adopted. The total number of seats to be shared by the ten provinces was set at 254, and each province's *fair share* was calculated by multiplying 254 by the fraction which the province's population represented of the whole.[6] Seats were allotted by first giving each province the whole number in its fair share, and then distributing whatever seats remained, one each, to the provinces having largest remainders. If by this process some provinces received fewer seats than their number of senators, these were allotted, by exception, numbers of seats equal to their number of senators. In the latter event, the method was reapplied to the other provinces to distribute the total of 254 seats less those allotted by exception.

This method (without the complicated way of handling minimum requirements) had been proposed in the United States as early as 1792 by Alexander Hamilton and was used for a period in the nineteenth century until it was discovered to suffer from the peculiar anomaly known as the "Alabama paradox". By Hamilton's method a province's representation can actually *decrease* when the number of members in the house is *increased* and all populations remain unchanged. This bizarre behavior of the method outraged U.S. representatives and led to its abandonment by the Congress.

In 1952 came a further change in the Canadian law. 261 seats replaced 254, and a proviso was inserted that after a redistribution no province could have its representation in the House of Commons reduced by more than fifteen percent (this to shore up Saskatchewan). However this clause brought with it the need for the further proviso that no larger province could have fewer seats than a smaller one. The very need to append this extra clause should have been a sufficient signal that something was fundamentally wrong with the method.

The Amalgam Method

In introducing the amalgam method on December 2, 1974 the President of the Privy Council stated, "the purpose of this bill is to provide for a new and equitable method of redistributing seats in the House of Commons among the provinces of Canada. It deals with a matter that touches the very fabric of our democracy, that is of deep concern to every Canadian. A just system of the representation of the people is the very life-blood of our democratic process."[7] In fact the method (see Appendix) is so encumbered with loose-ends, codicils, and ad hoc categories that it cannot but beget a sense of doubt about its fairness in the mind of any citizen who tries to read it.

Ignoring all the embellishments, the method is essentially Jefferson's, but instead of one divisor there are three.

The provinces -- other than Quebec -- are divided into three categories: *large* (those with population over 2,500,000), *intermediate* (those with population of at least 1,500,000 but not more than 2,500,000) and *small* (those with population less than 1,500,000). Quebec is in a category by itself.

Quebec is to be assigned 75 seats in the reapportionment based on the 1971 census, and an additional 4 seats in every subsequent reapportionment. The divisor x_Q is defined to be Quebec's population divided by its assigned number of seats. In 1971 x_Q = 80 370.19.

In allocating seats within each category the following three exceptions *always apply:*

(1) No province may receive fewer seats than it had at the previous apportionment;

(2) No province may receive fewer seats than the whole number contained in its population divided by x_Q; and

(3) A larger province may never receive fewer seats than a smaller province.

The number of seats for each *large* province is found by applying Jefferson's method with divisor x_Q, that is, by dividing x_Q into its population and dropping the fraction. Thus for Ontario divide 7 703 106 by $x_Q \doteq$ 80 370.19 to obtain 95.845, which implies 95 seats.

The divisor for the small provinces, x_S, is defined to be the average constituency size of the small states at the *previous* apportionment, that is, the sum of the populations of the provinces that were small at the previous census, divided by the number of seats they received at the apportionment after that census. The number of seats for each small province is found by applying Jefferson's method to the populations of this census using the divisor x_S.

The 1961 data (see Table 2) yields the value x_S = 65 925.14. For example, Newfoundland has 1971 population 522 104, which divided by x_S gives 7.92, so it receives 7 seats. New Brunswick has 1971 population 634 557, which divided by x_S gives 9.63, so it receives 9 seats; but 9 is less than it had before, so it must receive 10.

The divisor for the intermediate provinces, x_I, is defined to be the total population of the current small provinces, divided by their total allotment as found above. In other words, x_I is the average constituency size of the small provinces resulting from the new apportionment. To find the number of seats for each intermediate province, divide its population by x_I, add the resulting quotient to the province's previous number of seats, divide the sum by 2, and drop the fraction.

This rule has an alternative description. To begin, compute the province's population divided by x_I, drop the fraction and subtract from it the number of seats the province had at the previous apportionment. This difference is the increase in representation due the province *if* Jefferson's method were used with divisor x_I. To determine the province's apportionment take one-half of this difference, drop the fraction, and add the result to the previous number of seats. Therefore, if the "increase due" is an even number the province gains half that number (e.g., if 2 it gains 1); if it is odd it gains less than half (e.g., if 1 it gains 0).

The six small provinces in 1971 have (from above) electoral quotient x_I = 66 195.85, Alberta has population 1 627 874 in 1971 and had before 19 seats. Its population divided by x_I is 24.59, which would imply 24 seats. Alternatively, the "increase due" is 24 minus 19 or 5, so its apportionment is increased by 2 seats. British Columbia's 28 seats is computed similarly.

This completes the basic description of the amalgam method, though there are other exceptions and special clauses. One is that if a province is not large and declined in population from the preceding decennial census, then it receives the same number of seats that it had before. A higher order exception states that if several exceptions arise together then the one most advantageous to a province dominates.

Finally, it may happen that there are no small provinces at some census. In this case, to find the number of seats due an intermediate province by the amalgam method its current population is divided by the average constituency size of the intermediate provinces at the previous census, and the remainder discarded.

	1961		1971		
	Population	Representation	Population	Senators	Representation
Ontario	6 236 092	88	7 703 106	24	95
Quebec	5 259 211	74	6 027 764	24	75
British Columbia	1 629 082	23	2 184 621	6	28
Alberta	1 331 944	19	1 627 874	6	21
Manitoba	921 686	13	988 247	6	14
Saskatchewan	925 181	13	926 242	6	14
Nova Scotia	737 007	11	788 960	10	11
New Brunswick	597 936	10	634 557	10	10
Newfoundland	457 853	7	522 104	6	7
Prince Edward Island	104 629	4	111 641	4	4
Total	18 200 621	262	21 515 116	102	279
Total population of small		5 076 236		3 971 751	
Total representatives of small		77		60	
Electoral quotient of small		65925.14		66195.85	

Table 2. 1961 and 1971 Populations and Apportionments

Principles and Paradoxes

What is wrong with the amalgam method? That it is a crazy-quilt of ad hoc recipes, loose-ends, and exceptions, is evident. More fundamentally, it conceals grave defects that run counter to essential provisions of the British North America Act and violates the common sense of fair division.

The amalgam method does not conform to the constitutional ideal of meting out representation in proportion to population. Proportionality as the ideal is clearly stated in the original Article 52 of the British North America Act which has never been altered:

> Article 52. The Number of Members of the House of Commons may be from Time to Time increased by the Parliament of Canada, provided the proportionate Representation of the Provinces prescribed by this Act is not thereby disturbed.

Proportionality means that what really matters is the *relative* -- not the absolute -- sizes of the provinces. The amalgam method's classification of the provinces by fixed numerical thresholds (1 500 000 and 2 500 000) defeats this. For if the population of every province were to double, then the small provinces would be just as small relative to Ontario and Quebec as they were before. However, Nova Scotia, New Brunswick, Manitoba and Saskatchewan would now be "intermediate" provinces, while Alberta and British Columbia would join the ranks of the "large". The result would be a decline in the share of representation enjoyed by these provinces even though nothing had changed in their sizes relative to the others. A reasonable method would award the same shares to the provinces whenever their relative sizes and the size of the house remained unchanged.[8] Hamilton's method has this property. So do divisor methods, for if all populations increase by the same percentage then increasing the common divisor by this percentage will result in exactly the same quotients and so the same apportionment of the house as before.

The amalgam method gives inconsistent results when all populations change at the same rate. It gives even more bizarre results when they change at different rates. Clause 5(2)(a) states that a larger province can never receive fewer seats than a smaller province. It is disturbing enough that such a clause is needed since a reasonable method would automatically be expected to have this property. But, as is often the case with patchwork, no sooner is one hole plugged than a new and larger one opens. The clause concerns comparisons between provinces in any one apportionment. What happens as the provinces change in size? *The amalgam method can actually take seats from a province that is growing in population and give them to one that is shrinking.*

For example, suppose it were discovered that British Columbia's 1971 population had been undercounted by 1000 persons while Alberta's had been overcounted by 168 000. The amalgam method would actually take one seat from British Columbia (whose numbers had increased by 1 000) and give it to Alberta (whose numbers had decreased by 168 000)! In other words, Alberta could have deliberately underreported its population, or encouraged emigration, and thereby gained a seat at the expense of British Columbia.

The reason for this peculiar behavior is that with a decline of 168 000 Alberta would have only 1 459 874 persons and therefore would be classified as a small state. Applying the 1961 small-province divisor 65 925.14 results in a quotient of 22.144 and hence 22 seats for Alberta. This increases the 1971 average constituency of the small provinces to 66 239.33, which is applied to the intermediate provinces. The resulting quotient for British Columbia is 32.996, which, averaged with its previous number of seats (23) and rounded down results in only 27 seats.

The phenomenon of a growing province giving up seats to a shrinking province is called the *population paradox.* It can occur with the method of Hamilton[9] as well as with the amalgam method.

This paradox cannot occur with a divisor method because the only way a growing state can lose seats is if its quotient decreases, i.e., a larger divisor is applied. But then the quotient of any state that is shrinking or constant in population must also decrease, so it surely cannot gain seats. It can be shown mathematically that the *only* methods that avoid the population paradox and satisfy certain regularity conditions are the divisor methods.[10]

An equally serious violation of the ideal of proportionality is connected with the minimum guarantees in the number of seats to be given each province. Beginning in 1915 Article 51A of the British North America Act mandated such floors:

> Article 51A. Notwithstanding anything in this Act a province shall always be entitled to a number in the House of Commons not less than the number of senators representing such province.

Proportionality with minimum guarantees to the small was reiterated as the goal in 1974 by the President of the Privy Council (the Honorable Mitchell Sharp) when introducing the amalgam method:

> "Representation by population remains a treasured goal and constitutes an integral aspect of this method of redistribution... Within the group of the smallest provinces, representation by population will prevail except to the extent that the floor provisions may apply. Continued population growth in the small provinces would reduce their reliance on these floor provisions and lead to a greater degree of representation by population."[11]

Fixed floor provisions are a reasonable way of protecting regional interests and are found in many federal systems. But the analgam method has "moving floors", since no province is ever permitted to lose a representative. Rather than protecting regional interests this might better be called a provision to protect incumbents' interests. If populations keep changing relative to each other and representation is maintained proportional to population, the floors will keep rising and the House will increase indefinitely in size. Since, in addition, Quebec's representation is to increase at each subsequent reapportionment, the amalgam method gives *no* control on the total size of the House (e.g., reapportionment on the basis of January 1980 estimated populations would give some 306 members, an increase of 27). This is untenable. If on the other hand a ceiling is placed on the total membership together with floors that rise at each reapportionment, then gradually over the years proportionate representation will cease to exist.

The only workable solution is to adopt fixed floors and to enable Parliament to choose whatever fixed house size it regards as reasonable. By imposing floor limitations, however, the ideal of giving each province its "fair share" of the fixed total number of seats must be modified. This is easily done. The fair shares are adjusted so that no province's share is less than its floor amount and the shares of those provinces which are above their floors are in the same proportion as their populations, the sum of all shares being the total number of seats.[12]

Table 3 shows the adjusted fair shares for the 1971 data (floors equal to the number of senators and 279 seats) and compares this with the amalgam method solution. Clearly the amalgam method does not meet the ideal of proportionality as nearly as it might -- even allowing for the advantage given to the small provinces by imposing floors.

The reason is that the amalgam method treats the provinces in three different categories "like the Three Bears",[13] each with its own divisor. The *apparent* effect of this categorization is to ensure that the intermediate provinces "fare a little better than Ontario and Quebec but not as well as the smaller provinces",[14] or so parliament believed. In reality the method may make the intermediate provinces (Alberta and British Columbia) *worse off* than Ontario and Quebec. A slight alteration of the 1971 data shows how this can happen (Table 4).

The small provinces have the same populations and distribution of seats as before, so the same divisor as before (66 195.85) is applied to the intermediate provinces. The resulting quotients for Alberta and British Columbia are 26.970 and 32.939, which when averaged with their 1961 apportionments give 22 and 27 seats respectively. Neither would get more using

Quebec's average constituency (80 370.19) as a divisor, yet both are worse off than either Quebec or Ontario in terms of average constituency size. The explanation for this apparent anomaly lies in the rule for dropping fractions. Alberta, with a population of 1 785 287 and 22 seats has an average constituency of 81 149.41 which is worse than Quebec's (80 370.19), but dividing 1 785 287 by 80 370.19 yields 22.213 and the fraction must be dropped. This observation can be used to construct other examples in which some of the small provinces are worse off than both the intermediate and the large provinces.

In dropping fractions the amalgam method, like Jefferson's, tends to favor large provinces. The reason is that dropping a fraction of say .5 represents a much greater relative loss for a province with 6 seats than one with 60 seats. Adams' method, by contrast, tends to favor small provinces because it rounds all fractions up. Webster's, on the other hand, tends on average to be even-handed. The reason is that the fraction of any province will be just as likely to be above as below .5, independent of the province's size; in the former case it is favored by Webster's method and in the latter case not, so on average the two balance out.[15]

Table 5 contrasts the three methods for the 1971 populations and a uniform minimum of 4 per province, which was taken instead of the more restrictive number of senators to better show their differences. Notice that relative to the fair shares Adams' seems to favor the small provinces while Jefferson's favors the large. The Webster solution is nearest to the fair shares in the sense that it is not possible to transfer any seat and bring both of the provinces involved nearer to their fair shares. This is not true for either the Adams or Jefferson solution. In fact Webster's is the *only* divisor method that is *always near the fair shares* in this sense.[17]

Table 6 shows the solutions and the adjusted fair shares when the floors are set at the number of senators.

Province	Floor(#Sen.s)	Fair Share	Adj.Fair Share	Amalgam Method Apportionment
Ontario	24	99.891	98.287	95
Quebec	24	78.166	76.911	75
British Columbia	6	28.329	27.875	28
Alberta	6	21.110	20.771	21
Manitoba	6	12.815	12.609	14
Saskatchewan	6	12.011	11.818	14
Nova Scotia	10	10.231	10.067	11
New Brunswick	10	8.229	10.000	10
Newfoundland	6	6.770	6.662	7
P.E.I.	4	1.448	4.000	4
Total	102	279	279	279

Table 3. Adjusted Fair Shares versus 1971 Apportionment

Province	Population	Amalgam Method App't	Av. Constituency
Ontario	7 637 167	95	80 391
Quebec	6 027 764	75	80 370
British Columbia	2 180 412	27	80 756
Alberta	1 785 287	22	81 149
Manitoba	988 247	14	70 589
Saskatchewan	926 242	14	66 160
Nova Scotia	788 960	11	71 724
New Brunswick	634 557	10	63 456
Newfoundland	522 104	7	74 586
P.E.I.	111 641	4	27 910
Total	21 602 381	279	77 428

Table 4. Example with slightly altered populations showing how the Amalgam Method may make intermediate provinces worse off than the large.

Province	Floor	Adj.Fair Share	Adams	Webster	Jefferson	Amalgam
Ontario	4	98.972	98	99	100	95
Quebec	4	77.447	77	77	78	75
B.C.	4	28.069	28	28	28	28
Alberta	4	20.916	21	21	21	21
Manitoba	4	12.697	13	13	12	14
Saskatchewan	4	11.901	12	12	12	14
Nova Scotia	4	10.137	10	10	10	11
New Brunswick	4	8.153	9	8	8	10
Newfoundland	4	6.708	7	7	6	7
P.E.I.	4	4.000	4	4	4	4
Total	40	279	279	279	279	279

Table 5. Apportionment of 279 Seats by Methods of Jefferson, Webster, and Adams. 1971 Populations, Minimum of 4 Per Province[16]

Province	Floor	Adj.Fair Share	Adams	Webster	Jefferson	Amalgam
Ontario	24	98.287	98	98	99	95
Quebec	24	76.911	76	76	78	75
B.C.	6	27.875	28	28	28	28
Alberta	6	20.771	21	21	21	21
Manitoba	6	12.609	13	13	12	14
Saskatchewan	6	11.818	12	12	11	14
Nova Scotia	10	10.067	10	10	10	11
New Brunswick	10	10.000	10	10	10	10
Newfoundland	6	6.662	7	7	6	7
P.E.I.	4	4.000	4	4	4	4
Total	102	279	279	279	279	279

Table 6. Apportionment of 279 Seats by Methods of Jefferson, Webster, and Adams: 1971 Populations Number of Senators as Minima[18]

Consequences for the Canadian Problem

Parliamentary debate, precedent, and common sense all point to certain fundamental principles that should govern the fair distribution of seats in the House of Commons.

One of these is that *the provinces' shares of representation should depend only on their relative, rather than their absolute sizes.* Another is that *a larger province should never get fewer seats than a smaller province, and in comparing different problems a growing province should never give up seats to a shrinking one.* This property must be an integral part of the method itself, not tacked on as an afterthought.

Representation in proportion to population is the constitutional ideal -- subject to floors to protect the smaller provinces. The correct standard for measuring how close a solution comes to meeting this ideal is the provinces' fair shares adjusted for their floor guarantees. In particular *it should not be possible to bring the provinces closer to their shares by a transfer of seats.* Beyond this, *a method should be even-handed in its treatment of smaller and larger provinces.* And finally, *a method should allow for the House of Commons to be fixed at any size that may be deemed desirable.*

The amalgam method satisfies *none* of these fundamental principles. On the other hand, as we already know, there is a method -- Webster's -- that satisfies *all* of the principles. It is a remarkable fact -- requiring detailed mathematical proof beyond the scope of the paper -- that Webster's is the *only* method satisfying these principles.[19]

Our conclusion is that Canada should abandon the amalgam method and return to one already firmly established in precedent: Webster's method with fixed floors and a predetermined house size.

APPENDIX. - British North America Act, Article 51.

Article 51.* (1) The number of members of the House of Commons and the representation of the provinces therein shall upon the coming into force of this subsection and thereafter on the completion of each decennial census be readjusted by such authority, in such manner, and from such time as the Parliament of Canada from time to time provides, subject and according to the following Rules:

1. There shall be assigned to Quebec seventy-five members in the readjustment following the completion of the decennial census taken in the year 1971, and thereafter four additional members in each subsequent readjustment.

2. Subject to Rules 5(2) and (3), there shall be assigned to a large province a number of members equal to the number obtained by dividing the population of the large province by the electoral quotient of Quebec.

3. Subject to Rules 5(2) and (3), there shall be assigned to a small province a number of members equal to the number obtained by dividing
(a) the sum of the populations, determined according to the results of the penultimate decennial census, of the provinces (other than Quebec) having populations of less than one and a half million, determined according to the results of that census, by the sum of the numbers of members assigned to those provinces in the readjustment following the completion of that census; and
(b) the population of the small province by the quotient obtained under paragraph (a).

4. Subject to Rules 5(1)(a),(2) and (3), there shall be assigned to an intermediate province a number of members equal to the number obtained
(a) by dividing the sum of the populations of the provinces (other than Quebec) having populations of less than one and a half million by the sum of the number of members assigned to those provinces under any of Rules 3, 5(1)b), (2) and (3);
(b) by dividing the population of the intermediate province by the quotient obtained under paragraph (a); and
(c) by adding to the number of members assigned to the intermediate province in the readjustment following the completion of the penultimate decennial census one-half of the difference resulting from the subtraction of that number from the quotient obtained under paragraph (b).

*As enacted by the British North America Act, S.C. 1974-75-76, c. 13, which came into force on December 31, 1974.

5. (1) On any readjustment,

(a) if no province (other than Quebec) has a population of less than one and a half million, Rule 4 shall not be applied and, subject to Rules 5(2) and (3), there shall be assigned to an intermediate province a number of members equal to the number obtained by dividing

(i) the sum of the populations, determined according to the results of the penultimate decennial census, of the provinces (other than Quebec) having populations of not less than one and a half million and not more than two and a half million, determined according to the results of that census, by the sum of the numbers of members assigned to those provinces in the readjustment following the completion of that census, and

(ii) the population of the intermediate province by the quotient obtained under subparagraph (i);

(b) if a province (other than Quebec) having a population of

(i) less than one and a half million, or

(ii) not less than one and a half million and not more than two and a half million

does not have a population greater than its population determined according to the results of the penultimate decennial census, it shall, subject to Rules 5(2) and (3), be assigned the number of members assigned to it in the readjustment following the completion of that census.

(2) On any readjustment,

(a) if, under any of Rules 2 to 5(1), the number of members to be assigned to a province (in this paragraph referred to as "the first province") is smaller than the number of members to be assigned to any other province not having a population greater than that of the first province, those Rules shall not be applied to the first province and it shall be assigned a number of members equal to the largest number of members to be assigned to any other province not having a population greater than that of the first province;

(b) if, under any of Rules 2 to 5(1)(a), the number of members to be assigned to a province is smaller than the number of members assigned to it in the readjustment following the completion of the penultimate decennial census, those Rules shall not be applied to it and it shall be assigned the latter number of members;

(c) if both paragraphs (a) and (b) apply to a province, it shall be assigned a number of members equal to the greater of the numbers produced under those paragraphs.

(3) On any readjustment,
(a) if the electoral quotient of a province (in this paragraph referred to as "the first province") obtained by dividing its population by the number of members to be assigned to it under any of Rules 2 to 5(2) is greater than the electoral quotient of Quebec, those Rules shall not be applied to the province and it shall be assigned a number of members equal to the number obtained by dividing its population by the electoral quotient of Quebec;

(b) if, as a result of the application of Rule 6(2)(a), the number of members assigned to a province under paragraph (a) equals the number of members to be assigned to it under any of Rules 2 to 5(2), it shall be assigned that number of members and paragraph (a) shall cease to apply to that province.

6. (1) In these Rules,
"electoral quotient" means, in respect of a province, the quotient obtained by dividing its population, determined according to the results of the then most recent decennial census, by the number of members to be assigned to it under any of Rules 1 to 5(3) in the readjustment following the completion of that census;
"intermediate province" means a province (other than Quebec) having a population greater than its population determined according to the results of the penultimate decennial census but not more than two and a half million and not less than one and a half million;
"large province" means a province (other than Quebec) having a population greater than two and a half million;
"penultimate decennial census" means the decennial census that preceded the then most recent decennial census;
"population" means, except where otherwise specified, the population determined according to the results of the then most recent decennial census;
"small province" means a province (other than Quebec) having a population greater than its population determined according to the results of the penultimate decennial census and less than one and a half million.
(2) For the purposes of these Rules,
(a) if any fraction less than one remains upon completion of the final calculation that produces the number of members to be assigned to a province, that number of members shall equal the number so produced disregarding the fraction;

(b) if more than one readjustment follows the completion of a decennial census, the most recent of those readjustments shall, upon taking effect, be deemed to be the only readjustment following the completion of that census;
(c) a readjustment shall not take effect until the termination of the then existing Parliament.

(2) The Yukon Territory as bounded and described in the schedule to chapter Y-2 of the Revised Statutes of Canada, 1970, shall be entitled to one member, and the Northwest Territories as bounded and described in section 2 of chapter N-22 of the Revised Statutes of Canada, 1970, shall be entitled to two members.

1. Commons Debates, December 2, 1974, p. 1851, from speech of Mr. Stanley Knowles.

2. Senate Debates, December 18, 1974, p. 435, from speech of Mr. Grattan O'Leary.

3. The sources are W.H. McConnell, Commentary on the British North America Act, MacMillan of Canada (1975) and Article 51 of the Act itself. McConnell's description of the amalgam method is not entirely correct; he errs in his description of the divisor to be applied to the intermediate states.

4. Subsequent discussion of the history and properties of methods draws on the authors' forthcoming book Fair Representation (Yale University Press) as well as on a series of papers by the authors in the technical literature. For a summary see M.L. Balinski and H.P. Young, "The Webster Method of Apportionment", Proceedings of the National Academy of Sciences, USA, 77(1980), pp. 1-4 (hereinafter referred to as The Webster Method).

5. The method now used to apportion the United States House of Representatives is a more complicated divisor method in which a quotient is rounded up if it is greater than the square root of the product of the two nearest whole numbers. Thus a quotient of 2.450 would be rounded up to 3 because it is greater than the square root of 2 times 3 (about 2.449).

6. The fair share is also called the quota (see The Webster Method).

7. Commons Debates, December 2, 1974, p. 1845, from speech of Mr. Mitchell Sharp.

8. This principle embodies the technical properties of homogeneity and symmetry (see The Webster Method).

9. Examples from U.S. history which show that the Hamilton method admits the population paradox may be found in Fair Representation.

10. Technically, a homogeneous and symmetric method avoids the population paradox if it is impossible for a province that has increased in population to lose seats and at the same time a province that has decreased in population to gain seats. If, in addition, the method gives exactly the fair shares whenever these shares are all whole numbers, then essentially it must be a divisor method. The proof may be found in Fair Representation.

11. Commons Debates, December 2, 1974, p. 1846, from speech of Mr. Mitchell Sharp.

12. The adjusted fair shares, also called modified quotas in Fair Representation, are defined as follows: if p_i is the population of province i, r_i is its minimum floor, and h is the number of seats or house size, find x such that $\sum_1^n \max \{p_i/x , r_i\} = h$. The numbers $q_i = \max \{p_i/x , r_i\}$ are the adjusted fair shares.

13. Commons Debates, December 2, 1974, p. 1850, from speech of Mr. Stanley Knowles.

14. Commons Debates, December 2, 1974, p. 1850, from speech of Mr. Stanley Knowles.

15. A method is biased if over many problems it tends to give some class of provinces more (or less) than its fair share. It can be shown that Webster's method is the only divisor method satisfying certain regularity conditions that is unbiased. This is done from several different points of view in Fair Representation.

16. The Adams apportionment is found with the divisor 79 000 and rounding every quotient up (except for Prince Edward Island to satisfy the floor of 4). The Webster apportionment is found with the divisor 78 100. The Jefferson apportionment is found with the divisor 77 000 and rounding every quotient down (except for Prince Edward Island).

17. A formal proof that the Webster method is characterized as the one divisor method that is "near to the quota" is given in Fair Representation.

18. Divisors that provide the solutions are as follows: for Adams, 79 400; for Webster, 79 000; for Jefferson, 77 250.

19. This fact is proved in Fair Representation.