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Makowski, M. and Sosnowski, J.S.

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COORDINATION OF SECTORAL PRODUCTION PLANNING USING PRICES AND QUOTAS (A CASE STUDY FOR THE POLISH AGRICULTURAL MODEL)

Marek Makowski
Janusz Sosnowski
The Systems Research Institute of the Polish Academy of Sciences

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

FOREWORD

Understanding the nature and dimensions of the world food problem and the policies available to alleviate it has been the focal point of the IIASA Food and Agriculture Program since it began in 1977.

National food systems are highly interdependent, and yet the major policy options exist at the national level. Therefore, to explore these options, it is necessary both to develop policy models for national economies and to link them together by trade and capital transfers. For greater realism the models in this scheme are being kept descriptive, rather than normative. In the end it is proposed to link models to twenty countries, which together account for nearly 80 per cent of important agricultural attributes such as area, production, population, exports, imports and so on.

As part of the development of the Polish Agricultural Model, Marek Makowski and Janusz Sosnowski have investigated the coordination of sectoral production planning in Poland. Since this work involved methodological innovations, it was carried out in joint collaboration with the Food and Agriculture Program (FAP) and the Systems and Decision Sciences Area of IIASA.

This paper presents intermediate results of research done within the framework of the elaboration of the Polish Agricultural model, which will not only be included in the system of models of the FAP, but will also be used for decision-making processes in Poland.

Kirit S. Parikh
Program Leader
Food and Agriculture Program

Andrzej Wierzbicki
Area Leader
Systems and Decision Sciences

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COORDINATION OF SECTORAL PRODUCTION PLANNING USING PRICES AND QUOTAS (A CASE STUDY FOR THE POLISH AGRICULTURAL MODEL)

1. INTRODUCTION

The agricultural model of Poland outlined by Podkaminer (in press) is composed of several submodels. One submodel, the production model, consists of m submodels, each one relevant to a specific subsector of the Polish agriculture which are either state-owned or cooperatives, or one of three types of private ownership. The coordination of the sectoral production plan has to be achieved by fixing the producers' prices for inputs and outputs so that the sectoral optimum plan is consistent with the overall optimum plan and some additional requirements, explained in Section 2, are fulfilled. Since in some cases it is not possible to meet the requirements of the sectoral plan by fixing prices, A. Wierzbicki suggested establishing quotas for specific products.

The basis for this research has been a two-stage approach outlined by L. Podkaminer. First, one starts with a model with which the full potential of the agriculture as a whole can be studied in order to determine the desired pattern and level of production for each sector. Then, with a method such as the one presented in this paper, prices can be determined—and if need be, quotas.

This paper deals mainly with the problem of developing a method for determining prices and quotas and with the problem of determining an overall plan for all agricultural sectors. This research was begun at the same time as the research presented by Podkaminer (1981) who makes a general formulation of the problem. Therefore, to avoid repetition, the problem is only briefly described in section 2. However, the method of solving the problem differs from the one proposed by Podkaminer (1981).

2. FORMULATION OF THE PROBLEM

The problem of direct versus indirect controlling of economic activities in a centrally planned economy within the context of the Polish agriculture is discussed by Podkaminer (1981), who argues that profit oriented motives should be used as the sole basic instrument for controlling the planning of the agricultural producers. A government agency would thus be able to control more efficiently both the production pattern and use of resources by setting prices and, if necessary, quotas than by any administrative measures.

According to the two-stage approach mentioned above a government agency decides on an overall plan for agriculture in such a way as to use the full potential of agriculture (including possible trade) taking current social needs into account. The method of determining such a plan plays a key role within the decision-making process, but since it is not the main topic of this paper, only a brief discussion of two possible approaches to this overall planning is presented in Appendix A. In the following we shall assume that an overall plan and the desired sectoral plans (which are a part of the overall plan) are determined prior to an attempt to set prices and, if necessary, quotas, and solve the problem of sectoral planning on the level of the producer. This assumption differs from that of Podkaminer (1981) who proposes the simultaneous determination of an overall plan and of prices.

Hence the problem can be formulated as follows:

Based on an overall plan for agriculture which could be determined, for example, as discussed in Appendix A, the aim is to find instruments for controlling the planning of producers while taking into account the following requirements:

R1 The optimal plans for all subsectors (according to each local goal function) have to be consistent with an overall plan of the Polish agriculture.

R2 Local goal functions need not be mutually consistent.

R3 The production targets are not allowed as instruments of control.

R4 Prices are expected to be the main instrument used for controlling all sectors.

R5 Prices for products and production inputs have to be the same for all sectors.

R6 If there are no prices which fulfill requirements R1 through R5, it is permissible to introduce quotas in a given sector for a given product. However, the objective is to introduce as few quotas as possible. If a quota is established, a fixed price is paid for a commodity only if the quantity sold does not exceed the quota. If there is a surplus in production, a lower price may be paid. Hence the quota is not a limit on production.

R7 Constraints on inputs can also be introduced if necessary or preferred in place of quotas.

R8 There should be almost no reason for the violation of a license, if any, for quotas and limited resources.

R9 Prices should fulfill additional requirements (that result from assuring a level of minimal and/or maximal income to be within certain bounds, to reflect changes in price over time). These are given in section 3.

The Problem of Sectoral Planning

Each sector is assumed to be composed of producers with similar technological and behavioral characteristics. We also assume that producers in each sector behave in a rational way. In other words, given prices for all products and production factors, technological constraints, and available inputs, the producers are assumed to choose for each sector a production plan which will maximize their own goal function.

Therefore, the general formulation of the problem of sectoral planning can be formulated as follows:

Find a production pattern $x_i \in R^n_+$ and use of production inputs $s_i \in R^k_+$ such as to maximize

$$cx_i - ps_i \tag{2.1}$$

subject to

$$A_i x_i + B_i s_i \le b_i \tag{2.2}$$

$$s_i \le d_i \tag{2.3}$$

$$x_i \le \bar{x}_i \tag{2.4}$$

where c and p are given vectors of prices for products and inputs respectively, A_i and B_i are matrices of fixed coefficients for technical constraints, b_i are vectors of available local inputs, d_i is a vector of common inputs and $\bar{x}_i$ is a vector of quotas.

The constraint (2.3) implies that the government agency will set a limit on inputs, whereas constraint (2.4) implies that the agency will set quotas. On first sight (2.4) may appear to be a kind of limit, but it is not. This particular formulation is used to simplify the following presentation in section 3.

A more detailed discussion of quotas is given in Appendix B.

If a quota or limit for any commodity is not introduced, the relevant constraint is disregarded.

3. THE DETERMINATION OF PRICE QUOTAS AND LIMITS

The problem boils down to determining the parameters of the goal functions (2.1), vectors c and p, the right hand side of the constraints, in other words the limits d_i and the quotas $\bar{x}_i$ of constraints (2.3) and (2.4), in such a way as to assure that the optimal solutions will be equal to a given one and additional requirements are met [see (3.12) to (3.18)].

To begin with let us start with a simple example that illustrates a case for which prices cannot be determined which would fulfill requirements R1 - R5.

We shall consider two sectors and two commodities. Let optimal solutions of an overall plan (see Figures 1a and b) be $\hat{x}_1$ and $\hat{x}_2$ respectively, and E1, E2, D1, D2 be active constraints.

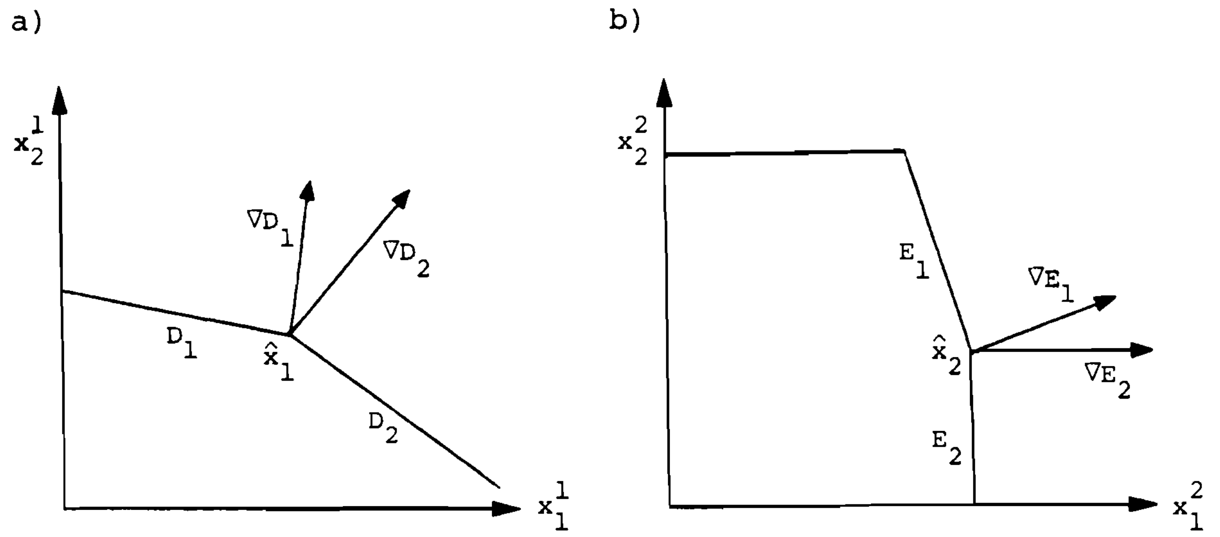

Figure 1. Example of a case in which planning can not only be controlled through prices.

It is obvious that the price vectors c_1 and c_2 have to be such that

$$c_1 = \alpha_1 \nabla D_1 + \alpha_2 \nabla D_2 \qquad \alpha_1, \alpha_2 \geq 0 \tag{3.1}$$

$$c_2 = \alpha_3 \nabla E_1 + \alpha_4 \nabla E_2 \qquad \alpha_3, \alpha_4 \geq 0 \tag{3.2}$$

where ∇D_1, ∇D_2, ∇E_1, ∇E_2 denote gradients of the active constraints and there are no α_i, $i = 1,\ldots,4$, such that $c_1 = c_2$.

The example illustrates that for some solutions one would not be able to avoid introducing a quota. Such a situation will probably occur if technologies between sectors differ considerably.

Let us consider the problem of prices from the point of view of a given sector (the index i for a given sector will be neglected in the following).

An admissible set of solutions will be defined by local constraints D1 and D2 (see Figure 2). The optimal solution determined by an overall plan is $\hat{x} = (\hat{x}_1, \hat{x}_2)$, the reason being that constraint G is due to limited global inputs. Should the constraint G be non-active,the optimal solution would be $\hat{x}^{\ell}$ (see Figure 2a). Should a limit for commonly used inputs and differences of prices between sectors be allowed, the price vector $\tilde{c}$ must be a linear combination of the gradients of the active constraints ∇G and $\nabla D2$. Should there be no limit for common inputs, the only price vector,for which the optimal solution of a given sector remains constant without further changes of constraints, will be $\nabla D2$. This in turn implies that the optimal solution will not be unique. Moreover, the producers in a given sector can choose a solution which is not an acceptable one within the framework of the global problem. Note that the price $\tilde{c}$ (see Figure 2b) can be determined by application of the Dantzig Wolfe algorithm (see Appendix A). But this price could be different in another sector. Moreover, since with this price there is no unique solution for a given sector, using the Dantzig Wolfe algorithm, a high-ranking decision-

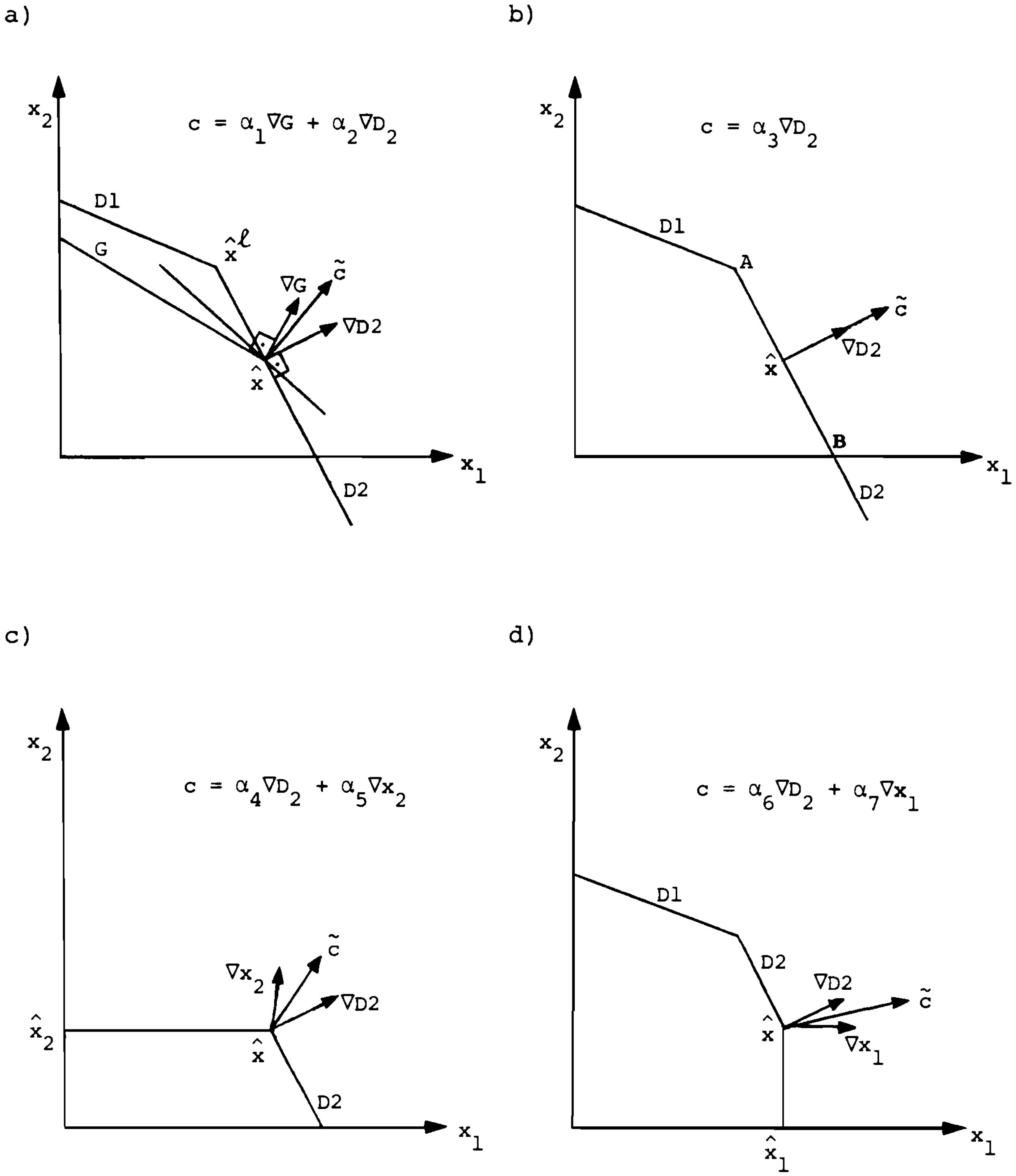

Figure 2. An illustration of different ways of price determination.

maker chooses a solution in the form of a convex combination of previously obtained sectoral solutions (in our example A and B in Figure 2b). If the Dantzig Wolfe algorithm is used, a sectoral solution must be given for a sector in addition to prices. Therefore we do not apply the Dantzig Wolfe algorithm to solve our problem.

In order to be certain that the desired solution will actually be chosen, at least one additional constraint must be introduced. The simplest would be to introduce a quota for a product, in other words a constraint of the type $x^j \leq \bar{x}^j$. For which product a quota should be introduced, depends on the final choice of the price vector $\tilde{c}$. If the price vector is a linear combination of ∇D2 and ∇x1, a quota for x_1 should be introduced, and if $\tilde{c}$ is a combination of ∇D2 and ∇x2 a quota for x_2 is established (see Figures 2c and 2d).

Let us now formulate a method of determining prices and—if needed—quotas and limits. It is possible to analyze the relationship between all the variables considered (such as prices, shadow prices, and shadow prices for quotas) using the following equations (3.3) through (3.10).

The conditions of optimality for $\hat{x}_i$ and $\hat{s}_i$ for the problems defined by (2.1 - 2.4) for each sector can be formulated as a solution to a dual system for those active constraints which are taken into consideration. Such a solution will be composed of vectors c, p, u_i, v_i and λ_i, λ_i being vectors of Lagrange multipliers of the sectoral problems (being part of the overall problem).

Let $\hat{A}_i$ and $\hat{B}_i$, i = 1,...,m, be submatrices of A_i and B_i composed of rows that have been active in a solution of an overall planning problem. The following conditions have to be fulfilled:

$$c - \hat{A}_i^T\lambda_i - u_i = 0 \qquad (3.3)$$

$$-p - \hat{B}_i^T\lambda_i - v_i = 0 \qquad (3.4)$$

$$u_i^j \le 0 \quad \text{if } \hat{x}_i^j = 0 \tag{3.5}$$

$$v_i^j \le 0 \quad \text{if } \hat{s}_i^j = 0 \tag{3.6}$$

$$u_i^j = 0 \quad \text{if } \hat{x}_i^j > 0 \text{ and no quota for } x_i^j \text{ exists} \tag{3.7}$$

$$v_i^j = 0 \quad \text{if } \hat{s}_i^j > 0 \text{ and no limit for } s_i^j \text{ exists} \tag{3.8}$$

$$u_i^j \ge 0 \quad \text{if } \hat{x}_i^j \ge 0 \text{ and quota for } x_i^j \text{ is introduced} \tag{3.9}$$

$$v_i^j \ge 0 \quad \text{if } \hat{s}_i^j \ge 0 \text{ and limit for } s_i^j \text{ is introduced} \tag{3.10}$$

where j is the index for products, i is the index for a sector, and T means a matrix transposition.

Note that the necessary condition for a problem with a linear goal function and nonlinear constraints can be formulated in a similar way, i.e. matrices $\hat{A}_i^T$ and $\hat{B}_i^T$ would be replaced by relevant gradients.

A sufficient and necessary condition for formulating the problem of prices is that the base matrix is known. This problem may be solved no matter what criterion has been adopted for choosing an overall plan, even if the resulting solution for a sector is not on a vertex.

Since the system of equations (3.3) - (3.10) does not have a unique solution, one could look for the solution which is nearest to a given one. Let $\bar{c}$ and $\bar{p}$ be vectors of reference prices (these can be world prices prices from the previous year or prices which were used in an effective plan). The following problem can then be formulated: find a vector of prices so that

$$(\|c - \bar{c}\| + \|p - \bar{p}\|) \rightarrow \min \tag{3.11}$$

subject to conditions (3.3 through 3.10) with possible extension according to the comments presented above. As a norm l_1 or l_∞ may be applied if one needs to use LP methods.

A solution to the above problem may not meet the requirements R8 and R9. If the following conditions are fulfilled then R8 and R9 would be met. R8, which states requirements for prices, can also be formulated as follows:

$$\underline{\underline{c}} \le c \le \overline{\overline{c}} \tag{3.12}$$

$$\underline{\underline{p}} \le p \le \overline{\overline{p}} \tag{3.13}$$

$$c^j = 0 \quad \text{if the product is nonmarketable} \tag{3.14}$$

$$c^j = c^k \quad \text{if } j,k \in J_k \quad \text{which is set of commodities that are the same (for example, a specific product produced by different technologies)} \tag{3.15}$$

$$\underline{\underline{R}}_i \le c\hat{x}_i - p\hat{s}_i \le \overline{\overline{R}}_i \quad \text{for } i = 1,\ldots,m \tag{3.16}$$

where $\underline{\underline{c}}$, $\overline{\overline{c}}$ and $\underline{\underline{p}}$, $\overline{\overline{p}}$ are lower and upper bounds for prices which were set in order to keep prices from varying too much from year to year. (3.16) reflects the range of income to a given sector.

Requirements, which stem from R9, will be discussed in more detail. Under a system of contracts and limits (rationing of production inputs) an exchange market for products and inputs could be developed. Should such a market develop, the government agency would loose effective control of the production pattern,and inputs and the actual production may differ considerably from that determined by the overall plan. Moreover, a grave shortage of some inputs may occur. To avoid this, one should look for instruments which would almost never cause a violation of licenses of inputs and products.

If u_i (v_i) are positive, then they can be interpreted as the marginal effectiveness of quotas (limits), and it is obvious that:

- there would be no reason to exchange licenses for the j-th product between any two sectors if

$$u_1^j = u_i^j \qquad i = 2,\ldots,m \tag{3.17}$$

- the same applies for inputs

$$v_1^j = v_i^j \qquad i = 2,\ldots,m \tag{3.18}$$

- there is no reason in the i-th sector to change from one technology used to produce a specific commodity to another one if

$$u_i^k = u_i^j \qquad j,k \in J_k \tag{3.19}$$

We shall now briefly discuss the feasibility of the price problem. Obviously, the system of equations (3.1) through (3.19) may be infeasible. In our opinion this is due to the nature of the problem and there is no alternative way of solving it. We can recommend four methods of dealing with the problem, in case it is infeasible.

1. Changing the overall plan may seem to be the simplest way to avoid this problem. But after examining the problem, we have come to the conclusion that in most cases, only a remarkable change in the overall plan would permit a feasible solution (if no other assumptions are changed). Illustratively speaking, a solution for each sector should be on a vertex that might be supported by a hyperplane common to all sectors. This may occur if the technologies of the sectors do not

differ very much or admissible sets are not "flat" in the neighborhood of optimal solutions. Therefore we would recommend the fourth approach instead of trying to change the overall plan.

2. The simplest way to find a feasible solution is to disregard some constraints which stem from R8 and/or R9.

3. If the latter proposal is not acceptable the most promising method seems to be replacing some of the hard constraints by soft constraints. Such an approach can be clearly interpreted and summarized as follows: if the income which resulted from a violation of a license is relatively small, one could then expect that no one would violate a license.

4. Another promising approach is equivalent to formulating and solving the following problem.

Find a vector of prices c, p and vectors λ_i and t (of dummy variables) which solve the following problem:

$$\min \|t\| \tag{3.19}$$

where $t_i = (t_{i1}, t_{i2}, \ldots, t_{i,n+k})$
subject to

$$\begin{bmatrix} c \\ -p \end{bmatrix} + t_i - \begin{bmatrix} \hat{A}_i^T \\ \hat{B}_i^T \end{bmatrix} \lambda_i - \begin{bmatrix} u_i \\ v_i \end{bmatrix} = 0 \qquad i = 1, \ldots, m \tag{3.20}$$

and (3.4) - (3.10) and (3.12) - (3.18).

The solution always exists and can be interpreted as follows. Let

$$\|t\| = \max_{i,j} |t_{ij}| \tag{3.21}$$

or

$$\|t\| = \max_{i,j} |\rho_j t_{ij}| \tag{3.22}$$

where $\rho_j > 0$ are weight coefficients.

By applying prices for products defined as

$$\tilde{c}_i = \hat{c} + \hat{t}_i \tag{3.23}$$

differences in prices are permissible, but sectoral solutions obtained for $\tilde{c}_i$ are consistent with an overall plan. When applying the norm (3.22) instead of (3.21) a solution can be found in which different weights are associated with prices that may differ considerably.

Note, that if

$$\varepsilon = \|\hat{t}\| \tag{3.24}$$

then differences between prices for a given commodity in any two sectors is not greater than 2ε.

Now we can examine solutions determined by each sector by applying $\hat{c}$ instead of $\tilde{c}_i$. The differences between these solutions and those obtained in an overall problem may be acceptable.

Moreover,it is reasonable to assume that each producer behaves like a homo-economicus but his choice of production pattern may slightly differ from the optimal solution obtained by computation of a model.

4. IMPLEMENTATION

4.1 Overall Production Planning

Although it is not the main topic of our research, some effort has been made to develop software which will aggregate the sectoral production models (4 agricultural ones and one which accounts for the rest of the Polish economy) into one LP model. A feasible solution (or plan) for the economy as a whole could then be determined with special emphasis on agricultural planning. The program MERGE, described by Makowski and Sosnowski (forthcoming), has been developed for this purpose.

For this overall problem one can apply a single objective which is either defined in each submodel, or introduced by defining the multi-objective problem and specifying only one goal, whichever way is easier for the user.

The alternative to defining a single objective is solving a multicriteria optimization problem (Wierzbicki, 1979). One may define several objectives, for instance, maximization of each type of product, minimization of the use of inputs. One may also define more complex objectives which are linear combinations of variables specified in the model (for example, the weighted sum of various types of meats). For each objective, a decision maker specifies a desired value (i.e. a value that she or he is willing to obtain for a corresponding objective).

As a result of solving the multicriteria overall plan, a Pareto optimal solution is chosen, such that the following selection function attains its maximum

$$w(g) = \min_i \left\{ \frac{1}{\alpha_i \bar{g}_i} (g_i - \bar{g}_i) \right\} \tag{4.1}$$

where g_i and $\bar{g}_i$ are the i-th objective and its reference point (or desired level) and α_i is a weight coefficient. In the case where the solution is such that decision makers would like to change priorities among the objectives, α_i is changed. In this case it is much easier to change α_i than a reference point.

The i-th objective is defined as

$$g_i = \sum_j \beta_{ij} x_j \tag{4.2}$$

where x_j is a variable of any submodel and β_{ij} is the corresponding weight coefficient.

The maximization of (4.1) subject to (4.2) and subject to the constraints specified in all the submodels considered, results in determining a Pareto optimal solution and the corresponding values of the objective functions. Their properties are discussed in Appendix B.

The maximization of the selection function (4.1) is performed by solving a specially generated LP problem. The approach is similar to that of Kallio, Lewandowski and Orchard-Hays (1980), but we use a slightly different selection function and a different way for providing the information needed for defining the criteria. This is much more efficient if the multicriteria problem is generated by the same program that performs the aggregation.

4.2 Determination of Prices, Quotas and Approximate Solutions

Although prices should be the major tool of controlling sectoral planning, there are situations where either quotas or an approximate solution will have to be acceptable.

Due to the size of the overall planning problem and the necessity of analyzing many scenarios with different assumptions, the only realistic way to cope is to develop a problem oriented generator. Such a generator has been made operational and generates an appropriate LP problem for finding prices and, if allowed, quotas. These are then used in sectoral planning according to the needs of the decision maker. Many alternatives may be examined. The generator can produce a MPS file corresponding to a problem reflecting the planner's requirements. Figure 3 outlines the relationship between the sectoral models, the overall planning model, and the determination of prices (and possibly, quotas).

First, an overall planning model is generated using the sectoral models. One may define which rows are to be aggregated. There is also the possibility of changing their status (for example, to make those rows neutral which are used for testing the sectoral models and to activate those rows which allow for overall balances). One may also define a new goal function or use the multiobjective optimization option. The overall planning model is then solved, and the price problem is generated.

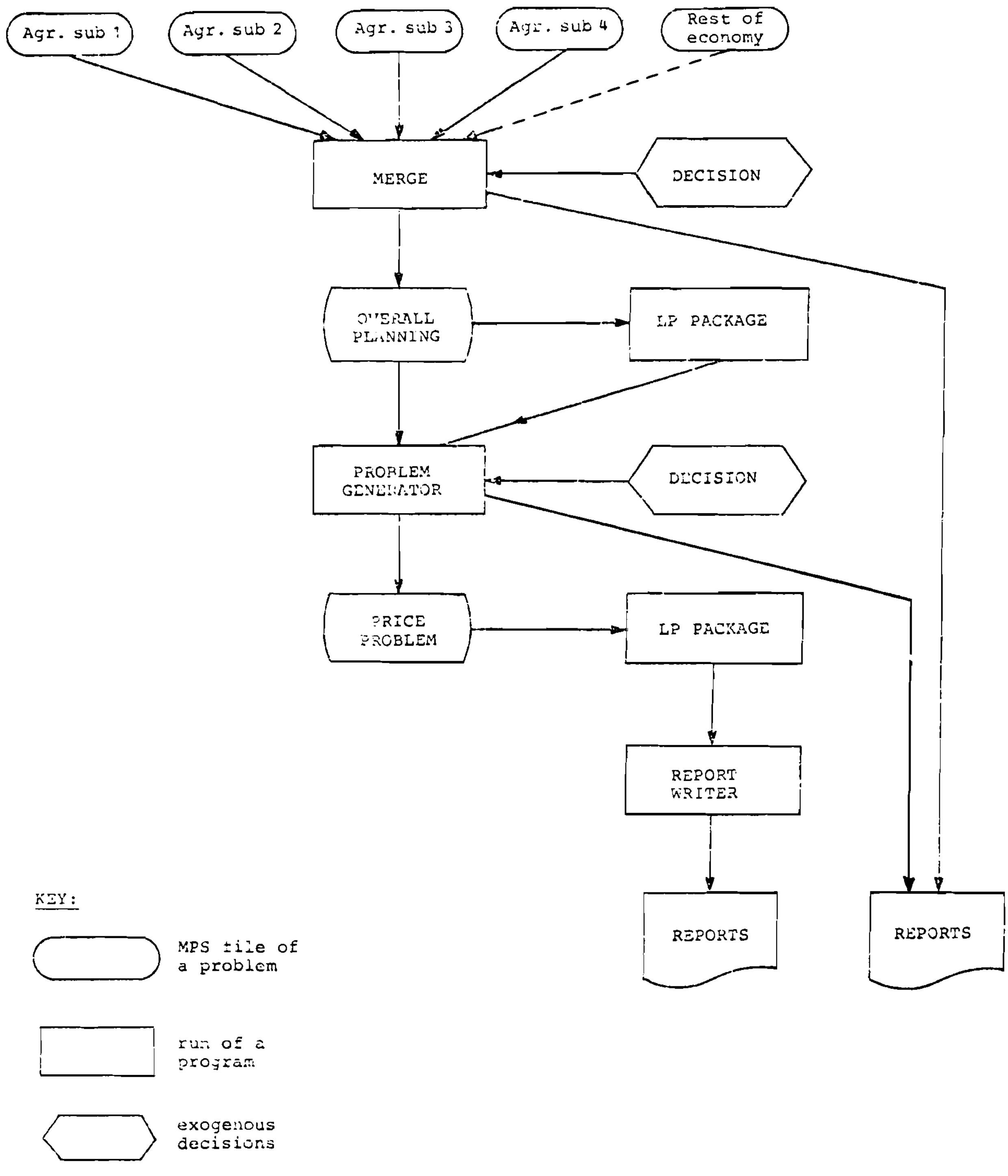

Figure 3. Flowchart of the generation of an overall planning problem and the corresponding price determination problem.

The overall model (in the form of a MPS file), its solution and the file containing assumptions (to be discussed later) are used. During the generation of the price problem, the obtained solution is briefly evaluated by the price generator, so as to assure that the models reflect the general assumptions (for example, that no constraints are imposed on the variables, since that would be equivalent, either to the case of a production goal (non-zero lower bound), or of a quota (upper bound). When this occurs, a warning is printed out and appropriate action is taken (for example, for an active upper bound, a quota is generated even if it is not allowed by the control variables which are to be discussed later on).

Finally, the price problem is solved. A writer then analyzes the results and produces a report. If an approximate solution is allowed (or a check of the price solution is desired) one can modify the sectoral models, by using a program developed by B. Lopuch. These are then solved in order to examine how much, if at all, sectoral solutions differ from the relevant part of the corresponding overall plan.

The features of the generator for the price problem will be briefly discussed. Some of the requirements mentioned in section 2 (R1, R2, R3, R4, R5) have already been included in the generator. In addition, prices can only be determined for marketable produce. (Since there is some cooperation between sectors, the group of intermediate goods is identified. Prices for those goods are generated so that the same good is an input for one sector and a product in another.

One may deal with the remaining assumptions (see section 2) by setting appropriate values for the control variables which would, or would not, according to the assumptions accepted for a certain scenario by the decision maker, allow for:

- the introduction of quotas for products
- the introduction of quotas for inputs
- shadow prices for quotas being the same for the same good in all sectors

- the formulation of a disturbed problem, i.e. one that allows for differences in prices among sectors (see section 4)
- the introduction of lower and/or upper bounds for prices
- a minimal and/or maximal income for a particular sector or sectors.

In addition, one may introduce the desired vector of prices $(\bar{c}, \bar{p}, \bar{r})$, where r stands for prices of intermediate consumption goods. The following goal function is minimized so as to allow the selection of a solution that meets the requirements set by a decision maker, as much as possible:

$$\sum_{i\varepsilon I_c} \frac{W1}{\bar{c}_i} |c_i - \bar{c}_i| + \sum_{i\varepsilon I_p} \frac{W1}{\bar{p}_i} |p - \bar{p}_i| + \sum_{i\varepsilon I_q} \frac{W1}{\bar{q}_i} |r_i - \bar{r}_i|$$

$$+ \sum_{i\varepsilon I_u} W2\, u_i + \sum_{i\varepsilon I_v} W2\, v_i + \sum_{i\varepsilon I_a} W3 |t_i|$$

where I_c, I_p, I_q, I_u, I_v, I_a are sets of indices that correspond to all types of prices (for products, inputs and intermediate consumption goods), to a set of quotas, and to variables for which the approximate solution is sought. Depending on the scenario some of the sets may be empty. The vectors c, p, r are price vectors for products, inputs and intermediate consumption goods, respectively; u, v are shadow prices of quotas for products and inputs, respectively (quotas for intermediate consumption goods are not realistic and therefore are introduced); t_i represents the difference between prices among sectors [see equation (3.20)]. However, it is advisable to formulate the desired levels of all prices since this would result in finding a solution that is even "closer" to the decision maker's expectations. One should point out that, if no reference point is given, the zero price for many goods would be determined, which is not an acceptable solution. The weight coefficients W1, W2, W3 reflect priorities assigned to the corresponding components of the goal function. It is advisable to set the value of W3 relatively high, since otherwise the solution, that allows for different prices in each sector in the first step (see equation 3.23), may be such that differences are large (see equation 3.24). Since prices have to be the same for all sectors and are defined as $\hat{c}$ in (3.23), the resulting sectoral solution may differ remarkably from the one determined by the relevant part of the overall plan. The relation between W1 and W2 depends on the preferences of a decision maker. If one prefers to have the price structure "closer" to the desired structure, at the expense of introducing more quotas or allowing shadow prices for quotas to differ among sectors, then W1 should be greater than W2. In the opposite situation the relation should be reversed.

4.3 Preliminary Results

Since the production models are still being worked on, the results presented in this paper are preliminary and are only given for the purpose of demonstrating possible ways of using the proposed approach.

In the following tables (Tables 1 and 2), results used in overall planning are presented for each approach. In the first, the goal function includes net income at world market prices, and the second uses a reference point whose components are the net production for major products (being the sum of corresponding products in all subsectors) and the use of inputs. As a reference point, the domestic prices of 1978 were used for most goods, and, as a lower bound, one half of the corresponding price. We have also tried to determine prices for the overall plan for which the goal is the maximization of production value (without taking into account the cost of inputs). Such an unrealistic approach resulted in prices whose components, for most of the inputs, were zero. This illustrates the necessity of consistency between goals for overall planning, whether it has a single or multiple objective, and the expected (or desired) price structure.

Table 1. Results of the solution to the price problem for single criterion overall planning.

	Price	Index	Ref.price
Products			
Wheat	0.9964	0.96142	0.5080
Rye	0.9669	1.24339	0.4310
Barley	0.9964	0.95373	0.5100
Oat	0.3960	0.	0.3960
Potatoes	0.1261	-0.48947	0.2470
Sugar Beet	0.0769	-0.29450	0.1090
Rapeseed	0.5200	-0.50000	1.0400
Beans	1.2150	-0.50000	2.4300
Flax Fibre	0.3000	-0.50000	0.6000
Vegetables	0.5044	-0.49560	1.0000
Fruit	0.9600	0.	0.9600
Milk	5.3200	0.	5.3200
Beef	3.2100	0.	3.2100
Pork	2.1350	-0.50000	4.2700
Lamb	4.5100	0.	4.5100
Poultry	3.3048	-0.02800	3.4000
Eggs	4.6300	0.	4.6300
Wool	0.2660	0.	0.2660
Inputs			
Melioratio	0.5500	-0.50000	1.1000
Feed	0.9964	0.54720	0.6440
Bought chi	0.0900	0.	0.0900
Exploi. co	3.6731	2.67310	1.0000
Labor (1)	6.3000	-0.50000	12.6000
Labor (2)	10.9200	-0.49908	21.8000
Labor (3)	14.2253	0.01609	14.0000
Drought fo	81.2605	0.56270	52.0000
External d	65.0000	0.	65.0000
Fertilizer	0.4922	-0.15138	0.5800
Pesticides	1.0000	0.	1.0000
Electric e	0.0010	0.	0.0010
Machinery	1.0000	0.	1.0000
Services	0.5000	-0.50000	1.0000
External m	1.0000	0.	1.0000
Coal	1.0000	0.	1.0000
Seeds	1.0000	0.	1.0000
Amortizati	4.3823	3.38230	1.0000
Veter. ser	1.0000	0.	1.0000
Taxes	2.2135	1.21350	1.0000
nksb	4.4285	no rfp	none
Cooper.			
Calves ek	6.2473	0.24946	5.0000
Heifers ek	9.3427	-0.15066	11.0000
Calves m ek	9.0000	0.	9.0000
Heif d. ek	13.0000	0.	13.0000
Calves in	7.2743	0.39890	5.2000
Heifers in	6.0000	-0.50000	12.0000
Calves m in	5.4813	-0.45187	10.0000
Heif d. in	13.6607	-0.02424	14.0000
Pig m ek	1.1000	0.	1.1000
Pig w ek	1.6000	0.	1.6000
Pig m in	1.1000	0.	1.1000
Pig w in	1.6000	0.	1.6000
Lamb w ek	2.1167	-0.18588	2.6000
Lamb m ek	2.1167	-0.18588	2.6000
Lamb w in	2.6000	0.	2.6000
Lamb m in	2.6000	0.	2.6000

Table 2. Results of the price problem for the multicriterial overall plan.

	Price	Index	Ref.price
Products			
Wheat	0.2540	-0.50000	0.5080
Rye	0.2650	-0.38515	0.4310
Barley	0.2550	-0.50000	0.5100
Oat	0.1980	-0.50000	0.3960
Potatoes	0.1235	-0.50000	0.2470
Sugar Beet	0.0545	-0.50000	0.1090
Rapeseed	0.5617	-0.45990	1.0400
Beans	1.2150	-5.00000	2.4300
Flax Fibre	0.3000	-5.00000	0.6000
Vegetables	0.5000	-5.00000	1.0000
Fruit	0.4800	-5.00000	0.9600
Milk	2.6600	-5.00000	5.3200
Beef	2.0873	-0.34975	3.2100
Pork	3.0616	-0.28300	4.2700
Lamb	2.2500	-0.50111	4.5100
Poultry	0.3246	1.74253	3.4000
Eggs	4.6300	0.	4.6300
Wool	0.2660	0.	0.2660
Inputs			
Melioratio	0.5500	-0.50000	1.1000
Feed	0.3220	-0.50000	0.6440
Bought chi	0.0900	0.	0.0900
Exploi. co	0.5000	-0.50000	1.0000
Labor (1)	12.6000	0.	12.6000
Labor (2)	10.9000	-0.50000	21.8000
Labor (3)	7.0000	-0.50000	14.0000
Drought fo	26.0000	-0.50000	52.0000
External d	131.8669	1.02872	65.0000
Fertilizer	0.2900	-0.50000	0.5800
Pesticides	0.5000	-0.50000	1.0000
Eletric e	0.2560	255.00002	0.0010
Machinery	0.5000	-0.50000	1.0000
Services	0.5000	-0.50000	1.0000
External m	0.5000	-0.50000	1.0000
Coal	1.0000	0.	1.0000
Seeds	0.5000	-0.50000	1.0000
Amortizati	0.5000	-0.50000	1.0000
Veter. ser	0.5000	-0.50000	1.0000
Taxes	0.5000	-0.50000	1.0000
nksb	0.6290	no rfp	none
Cooper.			
Calves ek	5.4181	0.08362	5.0000
Heifers ek	10.0291	-0.08826	11.0000
Calves m ek	8.8311	-0.01877	9.0000
Heif d. ek	13.0000	0.	13.0000
Calves in	5.2000	0.	5.2000
Heifers in	10.8242	-0.09798	12.0000
Calves m in	10.0000	0.	10.0000
Heif d. in	14.0394	0.00281	14.0000
Pig m ek	1.1000	0.	1.1000
Pig w ek	1.6000	0.	1.6000
Pig m in	1.1000	0.	1.1000
Pig w in	1.6000	0.	1.6000
Lamb w ek	2.6000	0.	2.6000
Lamb m ek	2.6000	0.	2.6000
Lamb w in	2.8327	0.08950	2.6000
Lamb m in	2.8327	0.08950	2.6000

Quota	Dual Price
= Wheat	0.0849
= Rye	0.0959
= Barley	0.0859
= Oat	0.0289
= Potatoes	0.1235
= Sugar Beet	0.0638
= Beans	0.7833
= Flax Fibre	0.1263
= Vegetables	0.4939
= Fruit	0.4147

Tables 1 and 2 present selected results. The following notation is used: the index is defined by

index = (price - reference price)/reference price

where reference price is the desired value of a corresponding price. Values corresponding to quotas (Table 2) are equal to a relevant shadow price associated with the introduction of a certain quota (the sign "=" means that the shadow price must be equal for all sectors, according to an assumption adopted for this run).

The solution to the price problem that corresponds to the single objective overall plan (Table 1) shows that it is sometimes possible to control sectoral planning only by setting prices. However, for some goods these prices differ markedly from the desired ones. This gap in prices for a specific commodity may be decreased by introducing quotas. We have also examined this scenario; the results will be presented in the final report.

The other solution for multicriteria overall planning (Table 2) illustrates the situation where the goals discussed in section 2 cannot be reached. First of all, prices differ in this solution much more from reference prices than in Table 1. Second, quotas for some goods have to be introduced. Finally this is an approximate solution (see section 3). Therefore, one may expect that applying these prices may result in a different solution for each sector than those required by the overall plan. The magnitude of the differences in prices may be examined by running the sectoral models using the prices that have been determined previously and additional constraints, according to the quotas. Finally, we would like to restate, that the results presented here only serve as an illustration of the method, since the model for overall planning has been generated by using submodels that have not been fully refined, and since the reference point for the overall plan has been chosen arbitrarily by the authors. Furthermore, the reference point and the lower bounds for prices (which are assumed to be equal to half of the corresponding reference price) have been chosen arbitrarily.

REFERENCES

Dantzig, G.B., and P. Wolfe (1960) Decomposition Principle for Linear Programms. In: *Operations Research*, Nr. 8.

Dantzig, G.B. (1963) *Linear Programming and Extensions.* Princenton: Princenton University Press.

Kallio, M., A. Lewandowski, and W. Orchard-Hays (1980) *An Implementation of the Reference Point Approach for Multiobjective Optimization.* WP-80-35. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Makowski, M., and J. Sosnowski (forthcoming) Implementation of an Algorithm for Scaling Matrices and Other Programs Useful in Linear Programming. CP-81-00. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Nikaido, H. (1968) *Convex Structures and Economic Theory.* Academic Press.

Podkaminer, L. (1981) *Efficient Use of Prices and Quantity Constraints for Control and Coordination of Linear Sectoral Production Models.* WP-81-110. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Podkaminer, L. (in press) System Modelling of the Polish Agriculture Development (goal, method, general outline) (in Polish). Rolnictwo i Wies.

Wierzbicki, A. (1979) *A Methodological Guide to Multiobjective Optimization.* WP-79-122. Laxenburg, Austria: International Institute for Applied Systems Analysis.

APPENDIX A: THE PROBLEM OF OVERALL AGRICULTURAL PLANNING

The agricultural production model consists of m sectors. The i-th sector produces n kinds of commodities $x_i \in R_+^n$ and uses k types of inputs $s_i \in R_i^k$. A set of admissible solutions for the i-th sector is defined by

$$A_i x_i + B_i s_i \leq b_i \tag{A.1}$$

$$s_i \leq d_i \tag{A.2}$$

$$x_i \leq \bar{x}_i \tag{A.3}$$

where A_i and B_i are matrices of technological coefficients, and b_i is a vector of inputs in sector i. A constraining value of the inputs used jointly d_i and possible quotas $\bar{x}_i$ are given. One can solve the optimization problem defined by a specified goal function and conditions (A.1) - (A.3) for each sector separately, but it is necessary to treat the allocation of some inputs as exogenous variables.

A set of admissible solutions for the overall agricultural plan is defined by

$$A_i x_i + B_i s_i \leq b_i \qquad i = 1,\ldots,m \tag{A.4}$$

$$\sum_{i=1}^{m} s_i \leq d \tag{A.5}$$

where d is a vector of inputs available for agriculture. For m = 3 a matrix, that describes the admissible set, has the structure given in Figure 4.

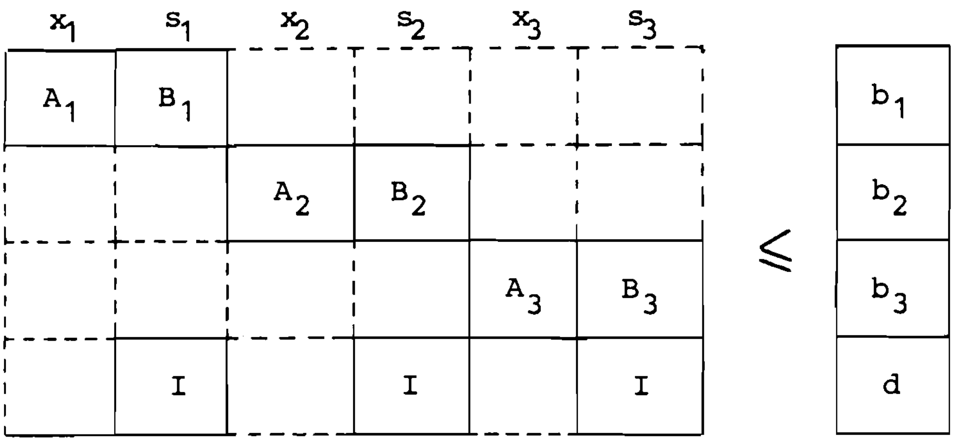

Figure 4. The structure of the constraints of the overall problem.

Let us consider the problem of a centrally planned agricultural production. Let

$$X = \sum_{i=1}^{m} x_i \qquad S = \sum_{i=1}^{m} s_i \tag{A.6}$$

where X denotes a vector of production and S denotes a vector of inputs. The overall plan may be obtained in the following two ways:

1. maximizing the agricultural production income with world market prices (or other prices acceptable to a central planner).
2. finding an efficient production (multicriteria optimization), which will be explained below.

A pair (X,S) defined by (A.6) belongs to a set of technologies T, $(X,S) \in T$, if x_i and s_i, $i = 1,\dots,m$, are such that conditions (A.4) and (A.5) are satisfied.

A vector (X,S) is more efficient than $(X',S') \in T$ if

$$\begin{pmatrix} X \\ -S \end{pmatrix} \geq \begin{pmatrix} X' \\ -S' \end{pmatrix} \quad (A.7)$$

(see Nikaido, 1968).

A vector $(X,S) \in T$ is said to be efficient (see Nikaido, 1968) if no other vector belonging to T is more efficient than (X,S). In other words, one cannot improve a production plan defined by an efficient vector because, in order to increase the production of one product, one has either to increase the use of inputs or to decrease the production of another product.

Hence, finding an efficient vector is equivalent to determining the Pareto-optimum (Wierzbicki,1979) for a criterion

$$\begin{pmatrix} X \\ -S \end{pmatrix} \rightarrow \max \quad (A.8)$$

Usually more than one efficient vector exists. So let us assume that one can define a desired production plan $(\bar{X},\bar{S})$, which does not necessarily belong to T. In this case it is possible to define a selection function of efficient production plans. The selection function may be defined as

$$w(X,S) = \min_{ij}\{\gamma_i(X^i - \overline{X}^i),\ \delta_j(S^j - \overline{S}^j)\} \qquad (A.9)$$

where $\gamma_i > 0$ and $\delta_j < 0$ are weight coefficients. An efficient production plan may be determined by

$$\max\ w(X,S) \qquad (A.10)$$

subject to constraints (A.4) - (A.6).

The problem of maximization in (A.10) can be reduced to a linear programming problem. Two examples of choosing effective points when the reference point $(\overline{X},\overline{S})$ does or does not belong to T are given in Figures 5a and 5b.

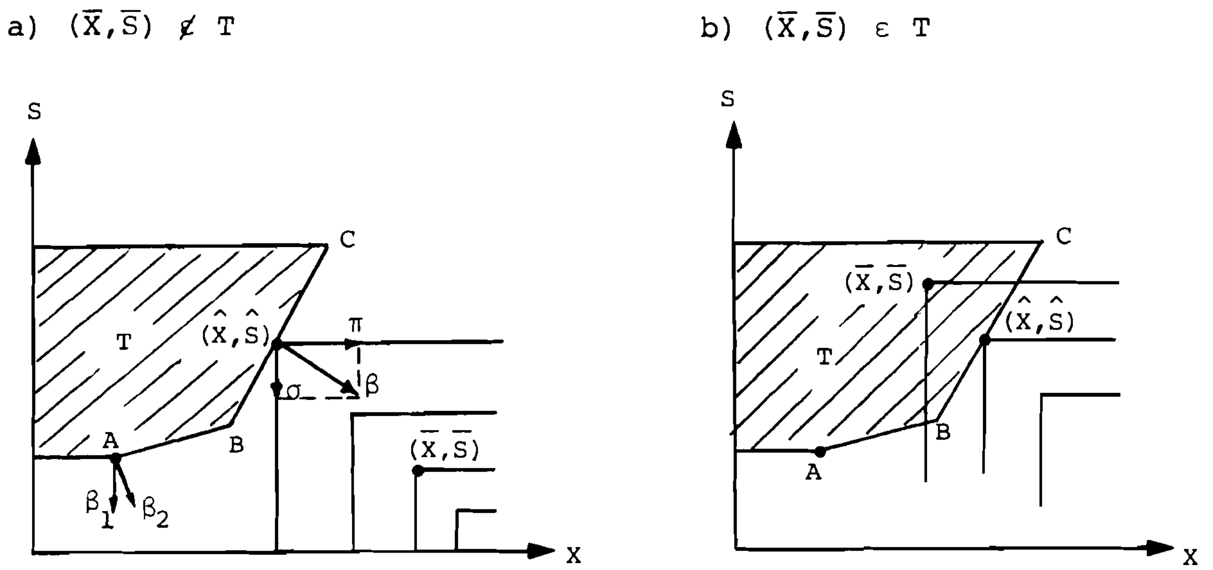

Figure 5. The determination of efficient points.

The admissible set of solutions is shaded. The set of efficient points is composed of two segments AB and BC. The efficient point nearest to the reference point $(\overline{X},\overline{S})$ is denoted by $(\hat{X},\hat{S})$.

For the reference point $(\overline{X},\overline{S}) \varepsilon T$ and the corresponding efficient point $(\hat{X},\hat{S})$ in Figure 5b the following holds:

$$\hat{X} \geq \overline{X} \quad \text{and} \quad \hat{S} \leq \overline{S} \tag{A.11}$$

The efficient point $(\hat{X},\hat{S})$ has the corresponding price vector β, with the components π (price for product X) and σ (price for the input S).

The following should be pointed out:

1. Only relative prices can be determined. In other words, any $\alpha\beta$ where α is a positive constant is also a vector of prices corresponding to the point $(\hat{X},\hat{S})$. A price vector will be called unique if the proportions between prices are fixed, unless otherwise stated. To obtain prices instead of proportions, one may either fix a price or normalize a price vector.

2. For several efficient points (in our example A,B,C) the prices are not unique. For example, at the point A any linear combination of β_1 and β_2 is a price vector.

Thus, with a selection function (A.9) and a reference point one can find an efficient production plan and a vector of prices for both products and inputs.

One could try to determine the same efficient plan by solving an LP problem, namely

$$\max \sum_{i=1}^{m} (\pi x_i + \sigma s_i) \tag{A.12}$$

subject to conditions (A.4) and (A.5). For this approach, however, one has to determine prices before solving the problem (for example, world prices can be taken). Also, at most points —the point $(\hat{X},\hat{S})$ in Figure 5—the price β applied to (A.12) results not only in the same point, but in the entire set of efficient solutions—segment AB in Figure 5. Thus, a solution of (A.12) is usually nonunique, while a solution of (A.9) is usually unique.

APPENDIX B: INTRODUCING QUOTAS

According to the requirement R9 (see section 2) formulated by A. Wierzbicki the problem of introducing a quota boils down to a change in the sectoral goal function (2.1):

$$\max \left(\sum_{j=1}^{n} \min \{c_j x_i^j,\ c_j \bar{x}_i^j + \alpha_j c_j (x_i^j - \bar{x}_i^j)\} - p s_i \right) \tag{B.1}$$

where $\bar{x}_i^j$ is a quota for the j-th product in the i-th sector and α_j $(0 \le \alpha_j < 1)$ reflects a decrease in price for any surplus in production. The function (B.1) differs from the function (2.1) only in the part which is dependent on the variables x_i^j. The function (B.1) is continuous and piecewise linear. Two examples of such a function in one (R^1) and two (R^2) dimensional space are given in Figure 6.

Let us formulate the sectoral problem with quotas in equivalent form which is more convenient for our analysis, namely

$$\max_{x_i^j \ge 0\ z_i^j \ge 0\ s_i \ge 0} \left(\sum_{j=1}^{n} (c_j x_i^j + c_j^{\alpha} z_i^j) - p s_i \right) \tag{B.2}$$

subject to

$$A_i(x_i + z_i) + B_i s_i \leq b_i \qquad (B.3)$$

$$x_i \leq \bar{x}_i \qquad (B.4)$$

where c^{α} is a vector of prices for the surplus production of a specific commodity, the surplus is denoted by z_i. If a quota is not introduced for the j-th product then $\bar{x}_i^j = +\infty$. For simplicity we have assumed that $c^{\alpha} = 0$.

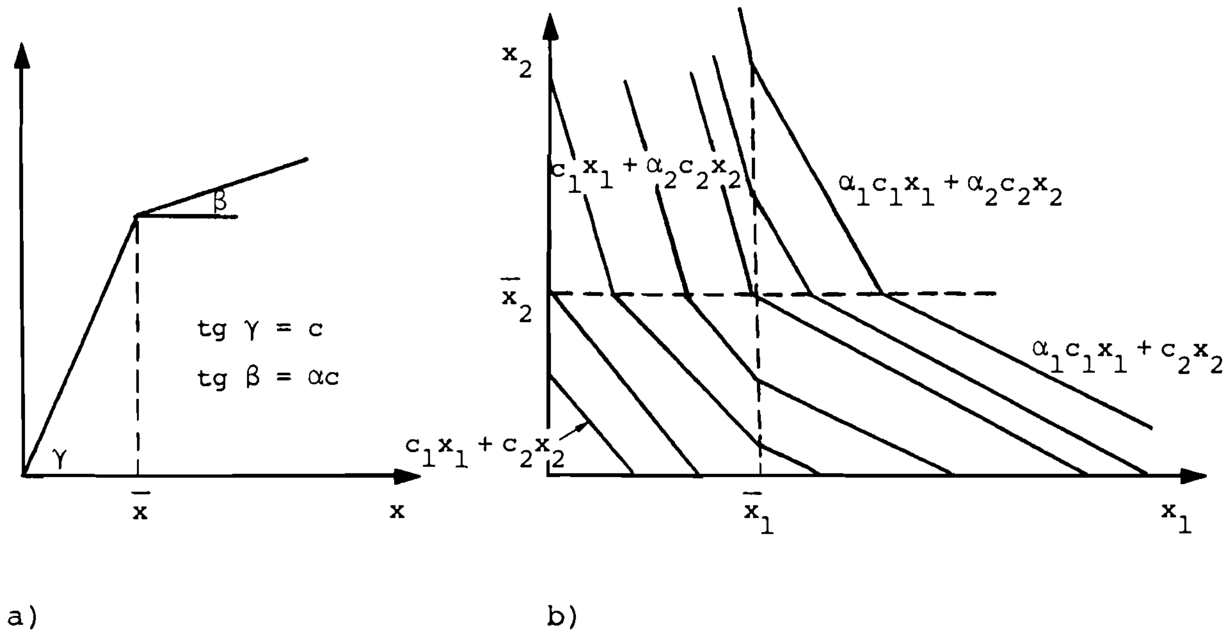

Figure 6. Examples of functions with quotas.