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International Institute for Applied Systems Analysis
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FOREWORD

The public provision of urban facilities and services often takes the form of a few central supply points serving a large number of spatially dispersed demand points: for example, hospitals, schools, libraries, and emergency services such as fire and police. A fundamental characteristic of such systems is the spatial separation between suppliers and consumers. No market signals exist to identify efficient and inefficient geographical arrangements, thus the location problem is one that arises in both East and West, in planned and in market economies.

This problem is being studied at IIASA by the Public Facility Location Task which started in 1979. The expected results of this Task are a comprehensive state-of-the-art survey of current theories and applications, an established network of international contacts among scholars and institutions in different countries, a framework for comparison, unification, and generalization of existing approaches, as well as the formulation of new problems and approaches in the field of optimal location theory.

This paper presents the first results of the application of a static location model for the analysis of the location of high schools in the town of Turin, Italy. It is a product of a collaboration between the Public Facility Location Task of the Human Settlements and Services Area at IIASA, Laxenburg, Austria, and the Institute for Economic and Social Research (IRES) Turin, Italy.

A list of publications in the Public Facility Location Series appears at the end of this paper.

Andrei Rogers
Chairman
Human Settlements
and Services

ABSTRACT

This paper reports on an analysis of the location of high schools in the town of Turin, Italy. It presents the first exploratory results of the application of a static location model and suggests some possible ways of generalizing this model into a dynamic one.

A detailed sensitivity analysis is presented, and it is shown how the results obtained lead naturally to dynamic issues. A simple Markov-chain model is used to evaluate the effects of changes in the first year admission policy, and the resulting shifts in the spatial distribution of required school capacity are shown.

Finally, a simplified version of a dynamic optimal capacity-adjustment model is proposed, and its possible extensions are outlined.

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OPTIMAL HIGH SCHOOL LOCATION: FIRST RESULTS FOR TURIN, ITALY

INTRODUCTION

The location of schools is a classic problem in the field of public service planning and provision. It can assume different features, depending on the type of geographical environment, the type of schools, and the type of demand considered. This paper deals with high schools, that is, schools of a noncompulsory type giving either professional or pre-university education. The geographical setting is the town of Turin, Italy, a highly industrialized urban area.

The noncompulsory nature of the high schools implies some interesting features, mainly in customer behavior, which make the high school location problem quite different from analogous problems for primary schools. The customer-choice process (absent in primary schools) is very rich and complex for high schools. The total demand in this case is not given, since the decision to undertake a high school course is made by customers, rather than imposed by laws. The high school student population varies widely over time and space. In the Province of Turin, for instance, it more than doubled from 1966 (about 37,000 students) to 1977 (about 76,249 students). However, this growth has not been uniform, and there is some evidence that spatial friction contributed to the generation of an uneven distribution of school attendance ratios: about 56% of the 14-18 year old population are students in the town of Turin, while this figure reduces to 44% for the rest of the province.

Choice over space is also important in high schools, and indeed the students are among the main components of commuter flows: more than 45% of the students attending a high school downtown live out of town. This spatial choice process is non-trivial, since it depends on many factors other than travel costs, such as type of vocation, neighborhood, congestion and many other externalities due both to the site and to the interactions among customers.

All of the above problems call for nonconventional models, and make the standard normative approaches unrealistic and useless. The aim of this paper is to explore some simplified versions of such models and to test some of the solution algorithms required.

A model for customer choice behavior is built first, which accounts both for demand elasticity and cross-substitutions over space. This behavioral model is then embedded into a static optimal location problem, and a computational algorithm for the resulting mathematical program is developed.

A detailed sensitivity analysis of the solution to the static problem is presented, and it is shown how the results obtained lead naturally to dynamic issues. A simple Markov-chain model is used to evaluate the effects of changes in the first year admission policy, and the resulting shifts in the spatial distribution of required school capacity are shown.

Finally, a simplified version of a dynamic optimal capacity-adjustment model is proposed, and its possible extensions are outlined.

1. THE BOUNDED-SIZE LOCATION PROBLEM

1.1 The Model

The usual formulation of a location problem for public facilities is the following:

$$\max_{S,L} F(S)$$

s.t.

$$\sum_{j \varepsilon L} S_{ij} \leq P_i$$

$$\sum_{j \varepsilon L} f_j (\Sigma_i S_{ij}) \leq B$$

where

i labels customer locations

j labels facility locations

L is a subset of facility locations, to be chosen in an optimal way

S_{ij} is the number of customers living in i and using the facility in j

P_i is the maximum number of people living in i, who can become customers of some facility belonging to the set L; from now on P_i will be called the *potential demand* living in i

$F(\cdot)$ is a function of the array $\{S_{ij}\}$, measuring the benefit accruing to the customers from each spatial arrangement of facilities; the function $F(\cdot)$ must be maximized both with respect to the $\{S_{ij}\}$ array (by the customers who are supposed to look for their own maximum aggregated benefit) and with respect to a proper choice of the subset L (by the public authority who is in charge of locating facilities). It is assumed, therefore, that the public authority agrees on measuring benefits in the same way as customers do, that is, by means of the same function $F(\cdot)$

$f_j(\cdot)$ are cost functions, measuring the cost of establishing and operating a facility of a given size in location j; the total size of a facility in j is measured by the total demand it attracts, that is $\Sigma_i S_{ij}$. The cost to be paid if j is chosen, therefore, is $f_j(\Sigma_i S_{ij})$

B is a given total budget available to establish and operate the whole set of facilities

The above formulation is quite general, yet it is not well suited for some problems often found in reality, like the school location problem. It often happens that such precise definitions of cost functions and budget are not available, either because of lack of data or because noneconomic factors determine constraints on the feasible sizes for facilities. This is indeed quite often the case with high schools, where such things as cost functions are very difficult to assess, but constraints on the feasible sizes for facilities are given from laws, regulations, rules of art, and so on. The simplest, but meaningful, case is when such constraints can be stated in the form of *bounds* on the sizes. That is, a statement of the form: *"The size of each facility should not be less than a minimum level T and more than a maximum level Z"*, can be made. Often the choice of appropriate values for T and Z is itself a subject for debate and decision, so that a sensitivity analysis on different values for T and Z is required for evaluation. The same holds true for a possible total "Budget" constraint, which in its simplest, nonmonetary form becomes a constraint on the total service capacity to be established. A tradeoff analysis on different values of total capacity is usually required for evaluation, therefore a sensitivity analysis on the "Budget" level is also required.

A general model which accounts for all the above requirements is the following one:

$$\max_{S,L} F(S)$$

s.t.

$$\sum_{j \varepsilon L} S_{ij} \leq P_i$$

$$T \leq \sum_{i} S_{ij} \leq Z \quad , \qquad j \varepsilon L$$

$$\sum_{j \varepsilon L} \sum_{i} S_{ij} \leq B$$

where

T,Z are the lower and upper bound on the facility sizes, respectively, and

B is an upper bound for the total service capacity which can be established

In order to run actual experiments, the above model requires a further specialization, obtained by introducing a suitable form for the benefit function $F(\cdot)$. A commonly used approach is to relate $F(\cdot)$ to travel cost, so that, if:

C_{ij} is the cost of a trip from i to j then

$$F(S)$$

is

$$F(S) = -\sum_{ij} S_{ij} C_{ij}$$

In this way, the problem reduces to one of finding the spatial arrangement of facilities which yields the minimum travel cost for the customers. However, this seemingly sensible decision criterion will produce strongly degenerate solutions. Unless the potential demand is forced to be *strictly* equal to the actual generated demand, an obvious optimal solution would be to do nothing, since then the total travel cost would be zero. What is wrong with this reasoning? Simply, the elasticity of demand was forgotten. Namely, the travel-cost minimizing approach neglects the fact that demand places a value on being a customer too, and not just on minimizing travel cost. In order to get the correct model of customer choice, a random utility framework can be used (see Domencich and McFadden, 1975). The only alternatives for a potential customer living in i are:

- to become a customer of a facility in j, for some $j \in L$
- not to become a customer at all

Suppose the net utility for choosing j is

$$v_{ij} + \theta$$

and the net utility for not choosing any facility is

$$w_i + \theta$$

where

v_{ij}, w_i are the deterministic (measured) components of utility while

θ is a random term accounting for possible deviations from deterministic behavior

According to the usual assuptions of random utility theory (McFadden, 1978; Ben-Akiva and Lerman, 1978; Daly, 1978), it follows that the total expected utility for a potential customer living in i is given by

$$E_i = \log(\Sigma_j e^{v_{ij}} + e^{w_i})$$

or, defining the terms

$$f_{ij} = e^{v_{ij}}$$

$$g_i = e^{w_i} \quad ,$$

$$E_i = \log(\Sigma_j f_{ij} + g_i)$$

Therefore, the total expected utility for <u>all</u> potential customers is

$$E = \Sigma_i P_i E_i = \Sigma_i P_i \log(\Sigma_j f_{ij} + g_i)$$

This function is related to the total accessibility measure discussed in Leonardi (1978, 1979a, 1979b) and analyzed in Ben-Akiva and Lerman (1978). It is also related to the consumer surplus measure defined by Neuburger (1971), and first used in an optimal location context by Coelho and Wilson (1976).

The implied demand models are obtained by taking derivatives of E with respect to the utilities v_{ij} and w_i (see Daly, 1978, for instance). They are

$$S_{ij} = \frac{\partial E}{\partial v_{ij}} = P_i \frac{f_{ij}}{\Sigma_j f_{ij} + g_i}$$, the expected number of customers living in i and using the facility in j

$$U_i = \frac{\partial E}{\partial w_i} = P_i - \Sigma_j S_{ij} = P_i \frac{g_i}{\Sigma_j f_{ij} + g_i}$$,

the expected number of potential customers who do not use any facility; U_i will also be called the *unsatisfied demand* in i

But the above two models are easily recognized to be the solution to the following *entropy maximizing* problem (Leonardi, 1980b)

$$\max_{S,U} \quad - \sum_i \left[\sum_j S_{ij} \left(\log \frac{S_{ij}}{f_{ij}} - 1 \right) + U_i \left(\log \frac{U_i}{g_i} - 1 \right) \right]$$

s.t.

$$\sum_j S_{ij} + U_i = P_i$$

Therefore, it can be concluded that maximization of the expected utility E for the customers is equivalent to maximization of the above entropy measure.

Plugging this result into the general bounded-size location model, one gets

$$\max_{S,U,L} \; -\sum_i \left[\sum_{j\varepsilon L} S_{ij} \left(\log \frac{S_{ij}}{f_{ij}} - 1 \right) + U_i \left(\log \frac{U_i}{g_i} - 1 \right) \right] \qquad (1)$$

s.t.

$$\sum_{j\varepsilon L} S_{ij} + U_i = P_i \qquad (2)$$

$$T \leq \sum_i S_{ij} \leq Z \quad , \qquad j \; \varepsilon \; L \qquad (3)$$

$$\sum_{j\varepsilon L} \sum_i S_{ij} \leq B \qquad (4)$$

Model (1)-(4) belongs to the class of problems discussed in Leonardi (1980a and 1980b). It should also be noticed that the equivalence between utility maximizing and entropy maximizing has been found and used by many authors. A recent comprehensive approach to the analysis of this equivalence and its use in urban and regional planning can be found in Brotchie, Lesse, and Roy (1979).

Before giving the details on how to solve problem (1)-(4), the calibration of the terms f_{ij} and g_i on actual data will be briefly discussed. In principle, the utilities v_{ij} and w_i can be expressed as functions of any number of relevant parameters. This would lead to what is known as a "logit" analysis (Domencich and McFadden, 1975). However, in our simplified examples, it has been assumed that

$$v_{ij} = -\beta C_{ij}$$

where C_{ij} is travel time from i to j and β is a given parameter. This amounts to reducing the demand model to a classic "gravity", or "spatial interaction", model (Wilson, 1974). As for the term w_i, it can be interpreted as a measure of the friction (other than the spatial one) encountered by people living in i to get access to the service (namely, to go to high school). In a detailed "logit" analysis it should therefore be expressed in terms of various socio-economic determinants, like family income, parents educational level, occupational sector, and so on. A disaggregation of demand in different socio-economic classes (besides place of dwelling) would also be required, in order to analyze the social equity issues related to locational decisions. Such an analysis will be the subject for future research, but is beyond the scope of this paper, whose main aim is to test the quality of the solution technique proposed for problems of the type (1)-(4). Therefore, in order to run the experiments, an averave value has been determined for the quantities g_i, which is in accordance with the observed high school attendance ratios over the past few years. That is, the "how" of g_i has been found but not the "why" behind it.

1.2 The Algorithm

Problem (1)-(4) is a nonlinear combinatorial programming problem; the combinatorial part arises from the choice of the set L, which runs over a discrete set of alternatives. An exact solution to such problems is usually very difficult to find. However, the special structure of problem (1)-(4) can be exploited to develop an efficient heuristic algorithm.

When the subset L is held constant, the resulting subproblem is a standard entropy maximizing one, which can be easily solved by means of Lagrangean duality. Let the Lagrangean function be defined

$$L(S,U,\nu,\lambda,\mu,\gamma) = -\sum_i \left[\sum_{j\varepsilon L} S_{ij} \left(\log \frac{S_{ij}}{f_{ij}} - 1 \right) + U_i \left(\log \frac{U_i}{g_i} - 1 \right) \right] +$$

$$+ \sum_i \nu_i \left(P_i - \sum_{j \varepsilon L} S_{ij} - U_i \right) +$$

$$+ \sum_{j\varepsilon L} \lambda_j \left(\sum_i S_{ij} - T \right) + \sum_{j\varepsilon L} \mu_j \left(Z - \sum_i S_{ij} \right) +$$

$$+ \gamma \left(B - \sum_{j\varepsilon L} \sum_i S_{ij} \right)$$

The above function has been built as usual, by multiplying the constraints (2)-(4) by undertermined shadow prices and adding them to the objective function (1). The following shadow prices have been introduced

ν_i is the shadow price for the constraint on total potential demand [constraint (2)]

λ_j is the shadow price for the lower bound constraint [left-hand side of (3)]

μ_j is the shadow price for the upper bound constraint [right-hand side of (3)]

γ is the shadow price for the total capacity constraint [constraint (4)]

Except for the ν_i, which can take any value, all the above shadow prices must be nonnegative, since the constraints are all inequalities.

According to duality theory for concave programs, problem (1)-(4) (with L fixed) is equivalent to the following saddle-point problem

$$\min_{\nu,\lambda,\mu,\gamma} \max_{S,U} L(S,U,\nu,\lambda,\mu,\gamma)$$

s.t.

$$\lambda,\mu,\gamma, \geq 0$$

Setting the derivatives of the Lagrangean with respect to $\{S_{ij}\}$ and $\{U_i\}$ equal to zero yields

$$- \log \frac{S_{ij}}{f_{ij}} - \nu_i + \lambda_j - \mu_j - \gamma = 0 \quad , \qquad \text{or} \quad S_{ij} = f_{ij} e^{-(\nu_i - \lambda_j + \mu_j + \gamma)} \tag{5}$$

$$- \log \frac{U_i}{g_i} - \nu_i = 0 \quad , \qquad \text{or} \quad U_i = g_i e^{-\nu_i} \tag{6}$$

Substitution of (5) and (6) in the Lagrangean gives the following *dual* function, which depends on the shadow prices only

$$D(\nu,\lambda,\mu,\gamma) = \sum_i \sum_{j\varepsilon L} f_{ij} \, e^{-(\nu_i - \lambda_j + \mu_j - \gamma)} + \sum_i g_i \, e^{-\nu_i} + \\ + \sum_i \nu_i \, P_i - T \sum_{j\varepsilon L} \lambda_j + Z \sum_{j\varepsilon L} \mu_j + \gamma B \tag{7}$$

The shadow prices ν_i can be eliminated by imposing constraint (2) on (5) and (6)

$$\sum_{j\varepsilon L} S_{ij} + U_i = e^{-\nu_i} \left(\sum_{j\varepsilon L} f_{ij} \, e^{\lambda_j - \mu_j - \gamma} + g_i \right) = P_i$$

hence (8)

$$e^{-\nu_i} = P_i \Bigg/ \left(\sum_{j\varepsilon L} f_{ij} \, e^{\lambda_j - \mu_j - \gamma} + g_i \right)$$

This result can be substituted in (5) and (6) to give a closed form for S_{ij} and U_i

$$S_{ij} = P_i \frac{f_{ij}\, e^{\lambda_j - \mu_j - \gamma}}{\sum_{j\varepsilon L} f_{ij}\, e^{\lambda_j - \mu_j - \gamma} + g_i} \tag{9}$$

$$U_i = P_i \frac{g_i}{\sum_{j\varepsilon L} f_{ij}\, e^{\lambda_j - \mu_j - \gamma} + g_i} \tag{10}$$

Substitution of (8) into (7) yields the following reduced dual function

$$V(\lambda,\mu,\gamma) = \sum_i P_i \left(\log \sum_{j\varepsilon L} f_{ij}\, e^{\lambda_j - \mu_i - \gamma} + g_i \right) - \tag{11}$$

$$- T \sum_{j\varepsilon L} \lambda_j + Z \sum_{j\varepsilon L} \mu_j - \sum_i P_i \;(\log P_i - 1) + \gamma B$$

An algorithm to minimize (11) over λ and μ for a given γ can be built, which is a special form of the usual balancing method for gravity models. Let the notation be simplified as follows

$w_j = e^{\lambda_j - \mu_j}$ is the balancing factor for the bound constraints (3)

$\alpha = e^{-\gamma}$ is the balancing factor for the total capacity constraint (4)

$\phi_i(W) = \sum_{j\varepsilon L} f_{ij}\, \alpha\, w_j$ is the accessibility measure from i

Then (9) and (10) can be rewritten as

$$S_{ij} = P_i \frac{\alpha f_{ij} w_j}{\phi_i(W) + g_i} \tag{12}$$

$$U_i = P_i \frac{g_i}{\phi_i(W) + g_i} \tag{13}$$

Note that, from (13), one can write

$$\frac{P_i}{\phi_i(W) + g_i} = \frac{U_i}{g_i}$$

and if this result is substituted in (12) one gets

$$S_{ij} = U_i \frac{\alpha f_{ij} w_j}{g_i} \tag{14}$$

Let the following function be defined

$$\rho_j(W) = \frac{\alpha \sum_i U_i f_{ij}}{g_i}$$

which can be interpreted as a demand potential for location j (ρ_j is a measure of nearness of j to unsatisfied demand U_i in all i)

Then one can write from (14)

$$\sum_i S_{ij} = w_j \rho_j(W) \tag{15}$$

which states that the total demand attracted in j is equal to the balancing factor multiplied by the demand potential.

The demand potential $\rho_j(W)$ can also be interpreted as the demand which would be attracted in j if no constraints were imposed on the size of the facility in j. The balancing factor w_j is therefore a correction term, which forces the demand to meet such constraints. An iterative procedure to compute the balancing factors, in order to satisfy constraints (3), is the following

a. start with an initial guess for w_j,

$$\text{e.g.} \quad w_j = 1 \quad , \qquad \forall\, j \in L$$

b. given the current value of the demand potentials ρ_j, update w_j for $j \in L$ according to this rule

$$\text{if } T < \rho_j < Z \quad , \qquad \text{set } w_j = 1 \tag{16}$$

$$\text{if } \rho_j \leq T \quad , \qquad \text{set } w_j = T/\rho_j \tag{17}$$

$$\text{if } \rho_j \geq Z \quad , \qquad \text{set } w_j = Z/\rho_j \tag{18}$$

Step b above is repeated over and over until convergence, which is usually quite fast. Since this is a special form of a multi-proportional adjustment problem, convergence can be assured under very general conditions (Willekens, Pór, Raquillet, 1979).

The above routine solves the subproblem of determining S_{ij} and U_i (and, consequently, the facility sizes) when the set L of open facilities is given. Now let the problem of finding the best set L be introduced. The simplest iterative algorithm will have the following structure

a. start with an initial guess on L

b. given the current L, try to improve the solution by eliminating some j from L and repeat the step

c. if b yields no improvement, try to improve the solution by adding some j (not in L) to L and go to b

b. if b and c are unsuccessful, stop

In order to avoid a combinatorial explosion, a "clever" rule to perform b and c is needed. But such a rule is naturally suggested by the output of the routine for the balancing factors. It has first to be noticed that only the locations which ended with condition (17) must be checked for possible elimination from L. These are the locations which have been forced to meet the lower bound constraint, since their unconstrained demand potential was below it. It does not make sense to eliminate a location whose demand potential is greater than the lower bound, since this would surely lead to a worse solution. In order to get the possible maximum improvement, the location ending on condition (17) with the maximum balancing factor will be tested. But, by looking at (17), it is seen that the balancing factor is inversely proportional to the demand potential. Hence, one is left with the intuitive rule that the location to be eliminated, if any, is the one with the *lowest demand potential.*

The above reasoning can be reversed for the test on locations to be added. Since the demand potential is defined (and computed) for all locations, including the ones which are not currently in L, the rule is: the location to be added, if any, is the one (not in L) with the *highest demand potential.*

The above considerations can be summarized in the following algorithm

a. start with an inital guess on L

b. suppose problem (1)-(4) has been solved for the current L, by means of the balancing factors routine; let the location $j \in L$ with lowest demand potential be dropped from L and the resulting problem (1)-(4) be solved again. If the value of the objective function (11) is higher than before, keep the reduced L and repeat the step

c. if step b has been unsuccessful, let the location $j \notin L$ with highest demand potential be added to L and the resulting problem (1)-(4) be solved. If the value of the objective function (11) is higher than before, keep the enlarged L and go to step b

d. if step c has been unsuccessful, stop

The above algorithm is specified for a given value of α, the balancing factor for the constraint on total capacity. In order to perform a sensitivity analysis on this constraint, the same algorithm can be used for different values of α. In this approach, it is better to think of α as a *tradeoff parameter*, weighing the relative importance of customer benefit, as measured by the objective function (1), and total cost, as measured by the total service capacity provided. Since α has been defined as

$$\alpha = e^{-\gamma}$$ where γ is the shadow price of the total capacity constraint

it follows that, when $\gamma = 0$ (that is, the capacity constraint is not binding) $\alpha = 1$, while when $\gamma = \infty$ (that is, the capacity constraint is very binding) $\alpha = 0$. Therefore α varies between 0 and 1, and a systematic search within this interval can be easily organized.

2. SOME RESULTS FOR THE HIGH SCHOOLS IN TURIN

As it has already been stated, the experimental setting for the applications presented in this paper has been kept as simple as possible, in order to test the efficiency of the algorithm developed in Section 1. The first simplification concerns the spatial disaggregation used. Only the town of Turin has been considered (with no hinterland) and the standard 23 districts (see, for instance, Provincia di Torino, 1978) have been assumed both as demand and possible school locations. The layout of this districting system can be seen on any of the maps given later in this paper. It is worth noting that such an aggregation has been introduced only to keep the size of the arrays reasonably small, but there is no conceptual difficulty in using a wider and more disaggregated geographical setting.

The second simplification concerns the demand. Only demand for public high schools has been considered, and possible competition between the public and the private high schools has been neglected. Moreover, the high-school demand has been aggregated over all vocations. A disaggregated analysis of demand by vocation is planned for future research; however, one should be careful in using the descriptive results of such an analysis in a normative framework, since the vocational breakdown of high schools has been changed many times in the past and is very likely to be changed in the future. Such changes will not depend on the spatial arrangement of schools, and will possibly be of an institutional nature.

As for the actual figures, the data on potential and actual demand and on existing school capacity for 1977 have been used (Provincia di Torino, 1978). There is, of course, no theoretical limitation in using more recent data, if they will become available. However, this is unlikely to affect the quality of our exercise. A few more words of explanation on the data definition are needed. The *potential demand* in each district is 14-18 year-old youths who are living in that district. The *actual* demand in each district is the high school student population living in that district (dwelling place and school location need not be the same). The existing school capacity in each district is measured in terms of the number of students each school can serve. It should be noticed that only the data on potential demand are used as an input in the optimization model, while the data on actual demand and existing school capacity are used for calibration and comparison (both of them will usually change after optimization).

The parameters for measuring the spatial and social friction of access to schools (namely the f_{ij} and g_i) have been assessed empirically, with no rigorous statistical calibration. This has been done for model-testing purposes only, and a better calibration will be produced in the near future. However, the main qualitative features of the results already obtained are unlikely to undergo big changes, since a detailed calibration will not change the order of magnitude of the empirically estimated parameters.

The friction-of-distance parameters have been set equal to

$$f_{ij} = e^{-\beta C_{ij}}$$

where a value of 0.15 has been assumed for β and the C_{ij} are the travel times, measured in minutes, between each pair of districts by public transport. (An extension of the model accounting for private transport as well could be easily developed, but this has not been done in the present version.) The value assumed for β has been suggested as reasonable from previous origin-destination surveys on school trips, carried out at IRES, Turin. As for the g_i terms, measuring the "nonspatial" friction of access to schools, an average value g has been used for all districts. This value has been assessed by trial and error, in order to approximately reproduce the total actual demand observed in year 1977. This may cause some mistakes, since the school-attendance ratio for 14-18 year-olds varies widely among districts, ranging from a minimum value of 29.81% to a maximum value of 56.66%. This is indeed an indication that social differences among the districts are nonnegligible and a motivation for further detailed analysis.

In spite of all the limitations listed above, the results obtained seem to be realistic enough to suggest that the basic mechanism has been caught, and further refinements will only improve some quantities, rather than the quality of solutions. These results are discussed in the following.

The distribution of demand and school capacity in Turin (divided in 23 districts) for the year 1977 is shown in Figure 1. The existence of very high and very low peaks and disparities suggest that the existing distribution is far from optimal. The simplest way to evaluate how far it is from optimal is to compare it with the solution of problem (1)-(4) under the assumption of no constraints on size and capacity. This comparison is shown in Figure 2. According to the unconstrained solution, the capacity should be drastically reduced in some districts, like district 1, and increased in some others, like district 11. However, this solution seems also too scattered, since some unreasonably small facilities are open, like in district 21. More realistic solutions are possibly obtained by solving (1)-(4) with a nonzero lower bound T on facility sizes. An upper bound could be introduced as well, but it is not required in this specific case, since the highest peaks obtained in the unconstrained solution are far below a reasonable upper bound.

A summary of the results obtained by systematically increasing the lower bound (with no constraint on total capacity) is shown in Figure 3 and 4. In order to understand Figure 3 some further definitions are needed. The "objective function" is the function defined by (1). By using the duality theory developed in 1.2 [namely equation (11)], it can be shown that the value of this function is the difference of two terms: an "accessibility" term, given by

$$\sum_i P_i \log (\phi_i + g_i)$$

where ϕ_i is the accessibility measure introduced in Section 1.1, and a "shadow cost" term, given by

$$T \sum_{j \epsilon L} \log w_j$$

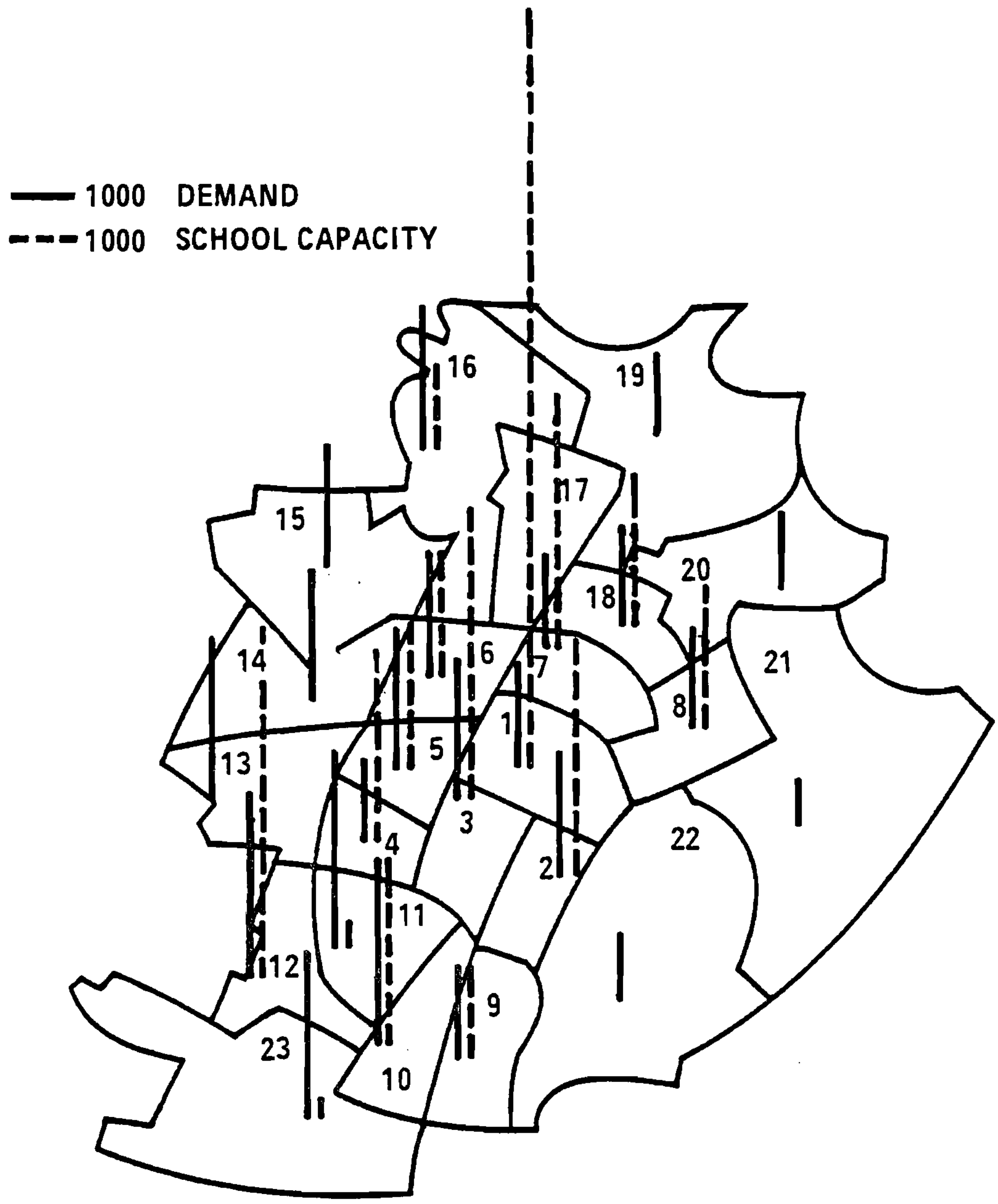


Figure 1. High school demand and capacity in the 23 districts of Turin, 1977.

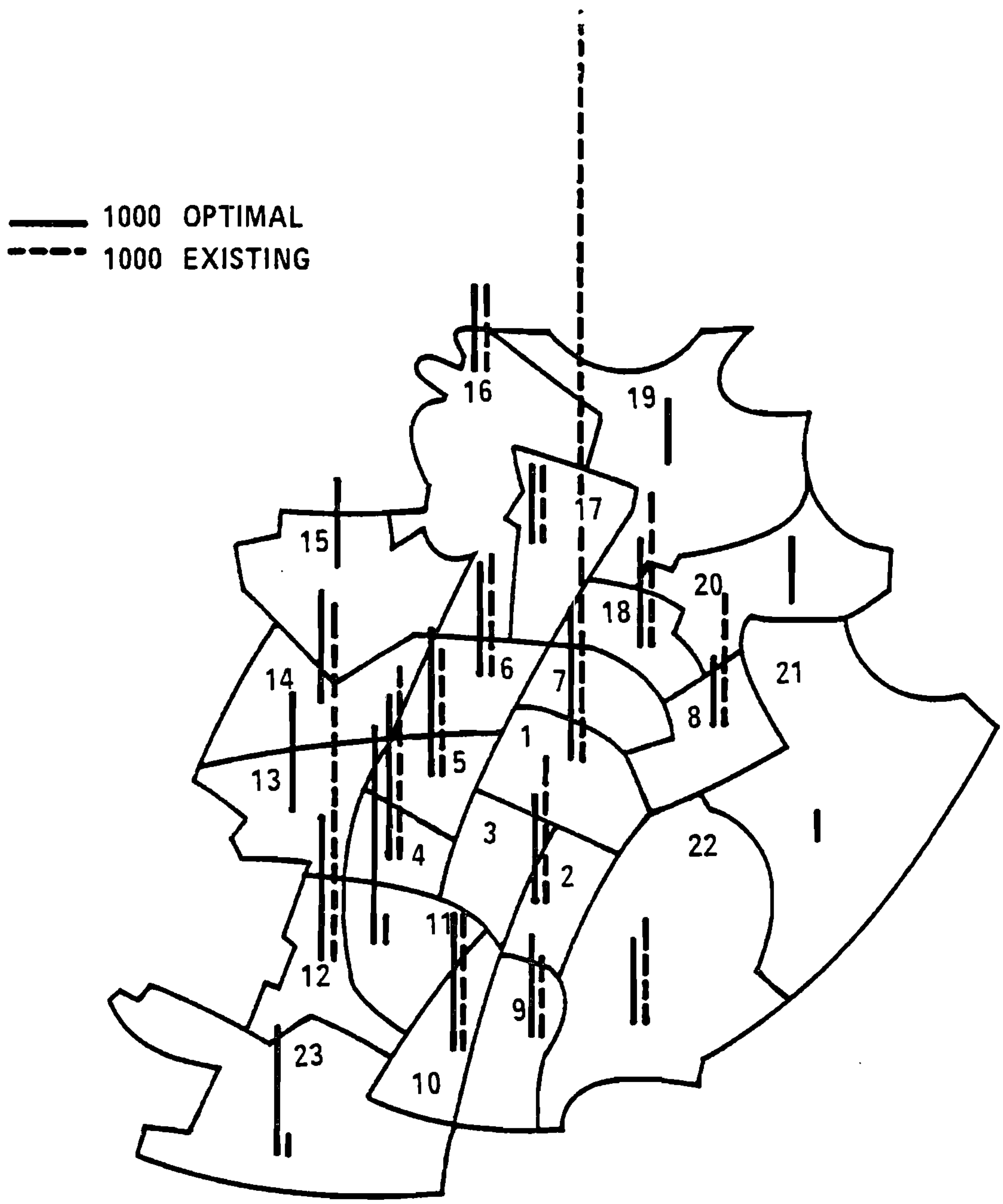
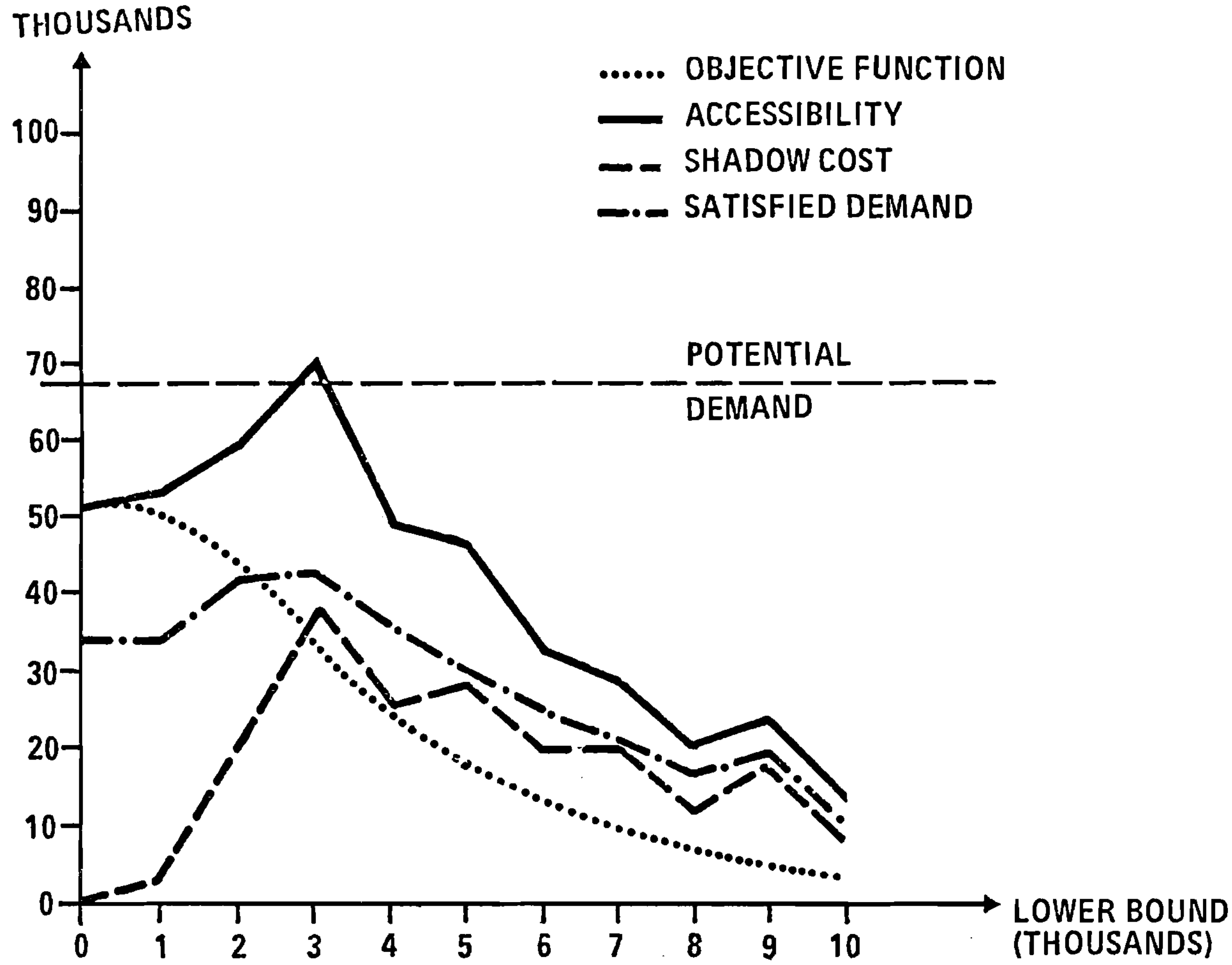


Figure 2. Comparison between existing (1977) capacity distribution and unconstrained optimal distribution.



Source: Appendix A1

Figure 3. Summary of the evaluation criteria for the sensitivity analysis on the lower bound (minimum feasible size for facilities).

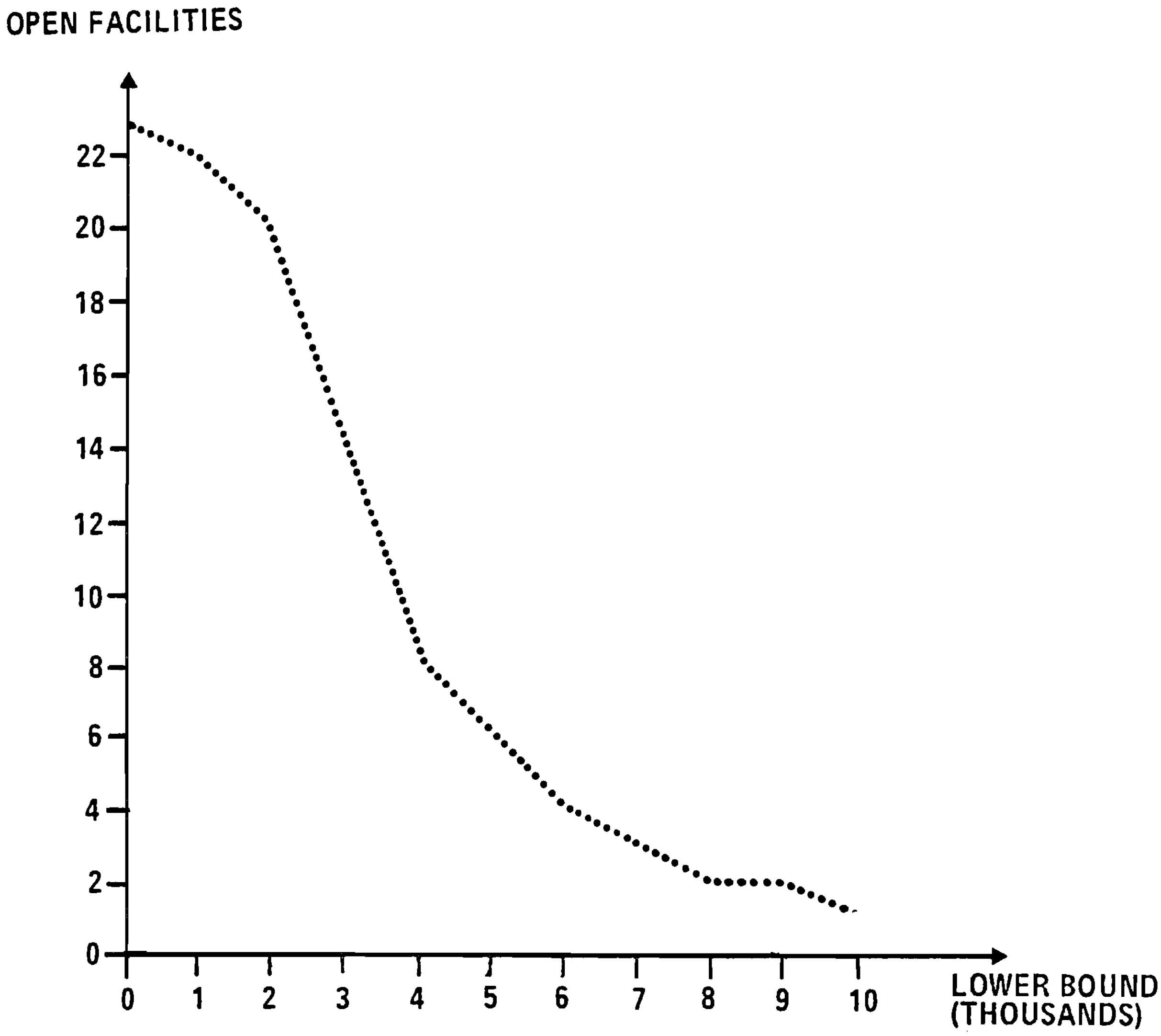


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Figure 4. Number of open facilities versus the lower bound (minimum feasible size for facilities).

where the w_j are the multipliers introduced in (12) (additive constant terms are negelected). Since contributions to the "shadow cost" come only from facilities open at the lower bound level, it is a measure of the "price" paid to keep these facilities open. (Here the term "price" is used in the sense of duality theory of mathematical programming, and is not necessarily related to money.) The satisfied demand is the fraction of the population in the 14-18 age interval attending some high school. It is given either by $\sum_{ij} S_{ij}$, where S_{ij} is defined in (12), or by $\sum_i P_i - U_i$, where U_i is defined in (13). The behavior of the above four indicators for an increasing lower bound (see Figure 3) reveals some interesting general facts. While the objective function decreases, both the accessibility and the shadow cost have a maximum for a lower bound around 2000-3000. The satisfied demand has a maximum there too, and this suggests that, if maximizing the satisfied demand is one of the goals for the optimal location, then lower bound values between 2000 and 3000 should be used. In Figure 4 the number of open facilities for each lower bound value is shown. The whole set of resulting spatial patterns is shown in Figures 5-14. While the solution of Figure 6, corresponding to a lower bound of 2000, is quite reasonable, the solution of Figure 11, corresponding to a lower bound of 7000, is unrealistic, and has been computed for sensitivity analysis only. Unrealistic as they are, the solutions obtained for very high lower bounds reveal interesting facts on a possible ranking among the districts, the highest rank being associated to the last districts which disappear from the solution. For instance, Figures 11-14 show again the importance of district 11, which in 1977 had practically no schools.

A different kind of sensitivity analysis can be performed by varying the level of total capacity or, which is the same (and easier for computation), by varying the value of the parameter α introduced in (12). As already stated in Section 1.2, this parameter can be considered as a measure of the tradeoff between the customer's benefit [the objective function (1)] and the resources to be used [constraint (4)]. When $\alpha = 1$ there is no limit to the

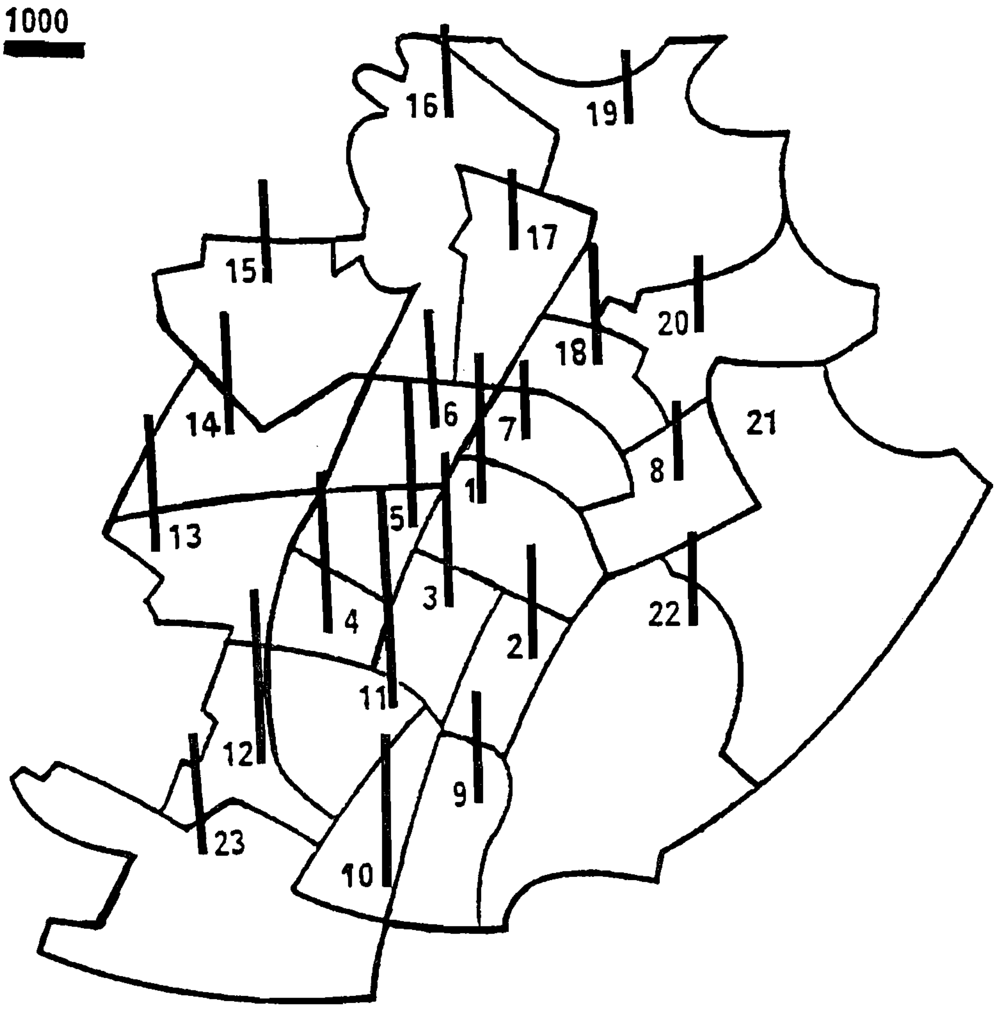


Figure 5. Optimal solution - lower bound = 1000.

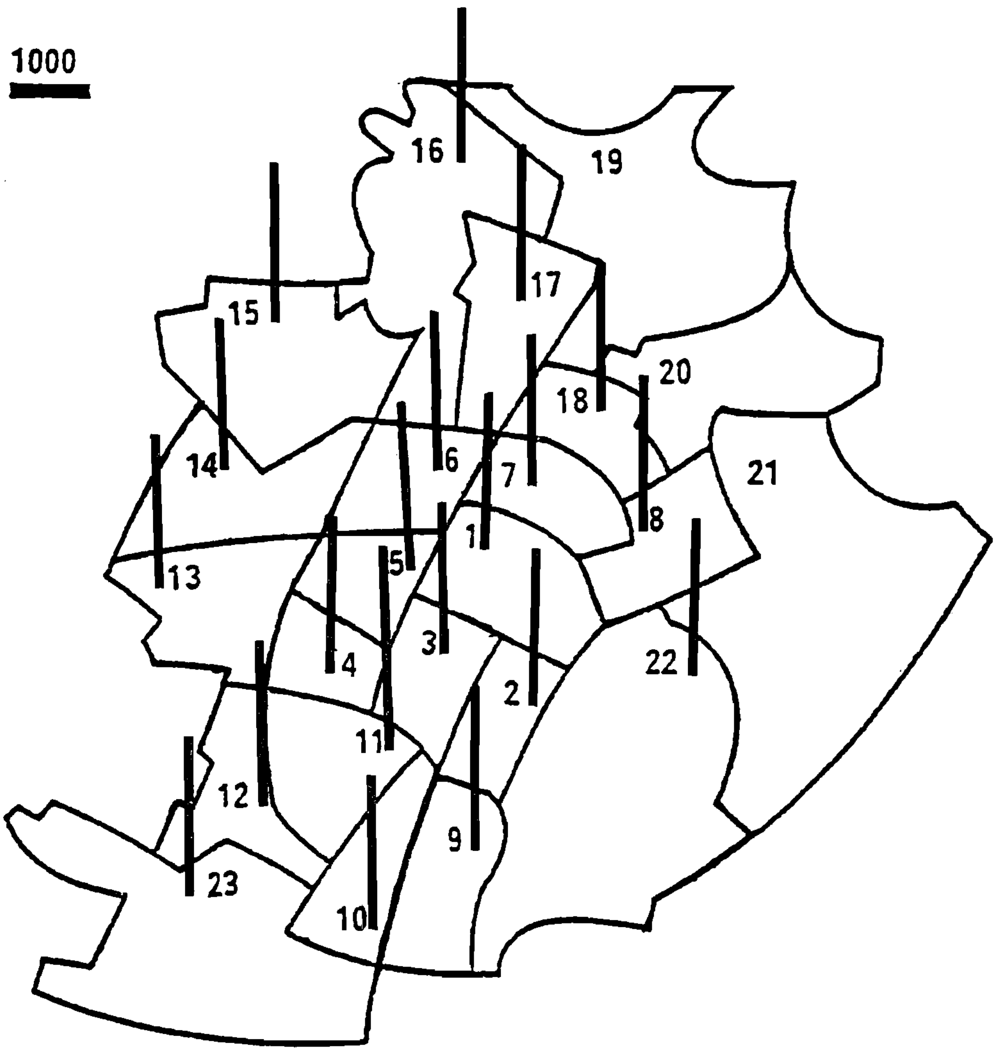


Figure 6. Optimal solution - lower bound = 2000.

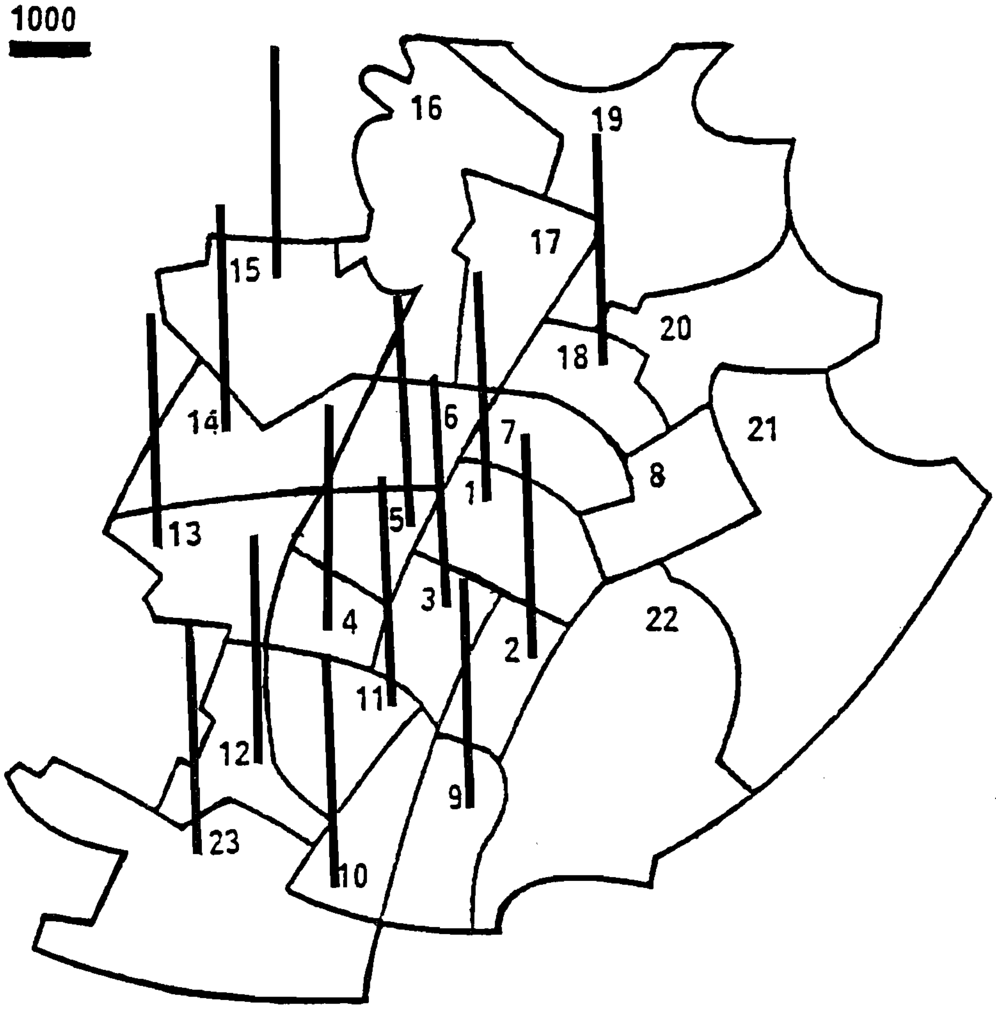


Figure 7. Optimal solution - lower bound = 3000.

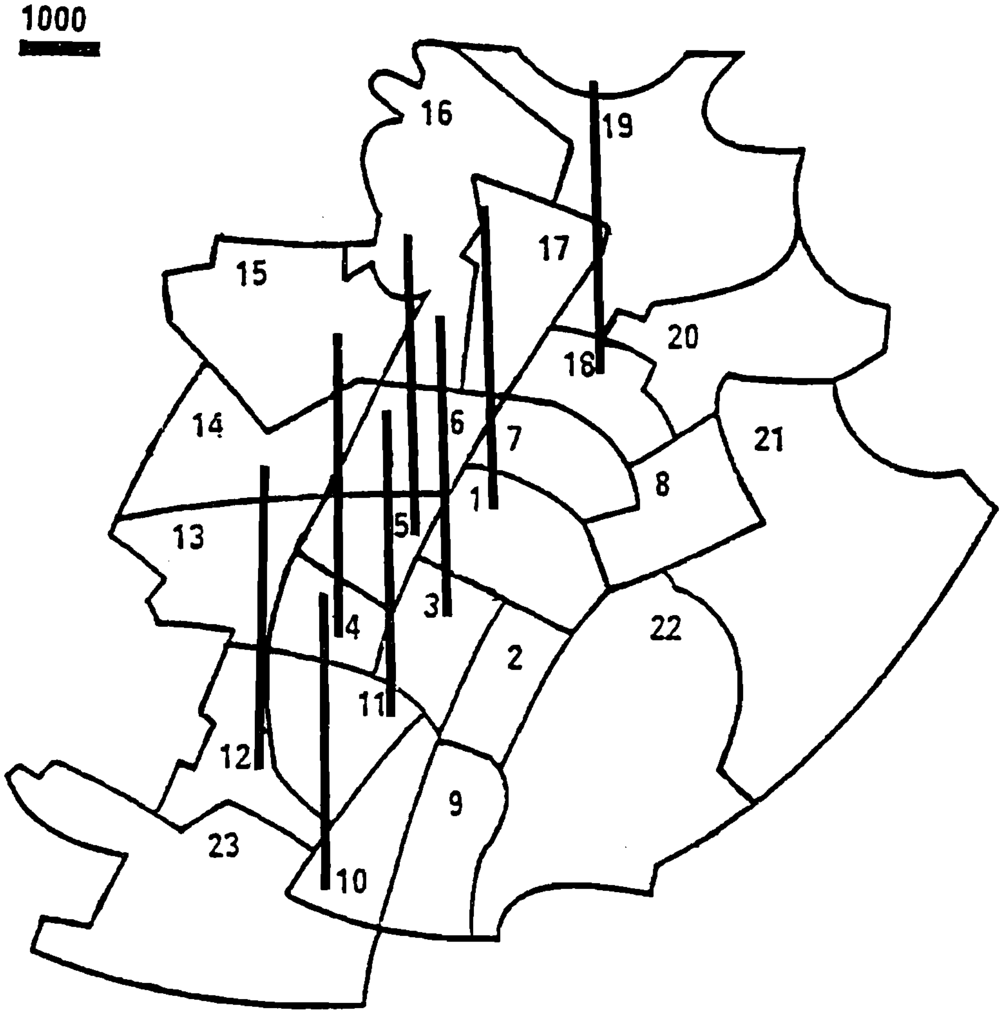


Figure 8. Optimal solution - lower bound = 4000.

Figure 9. Optimal solution - lower bound = 5000.

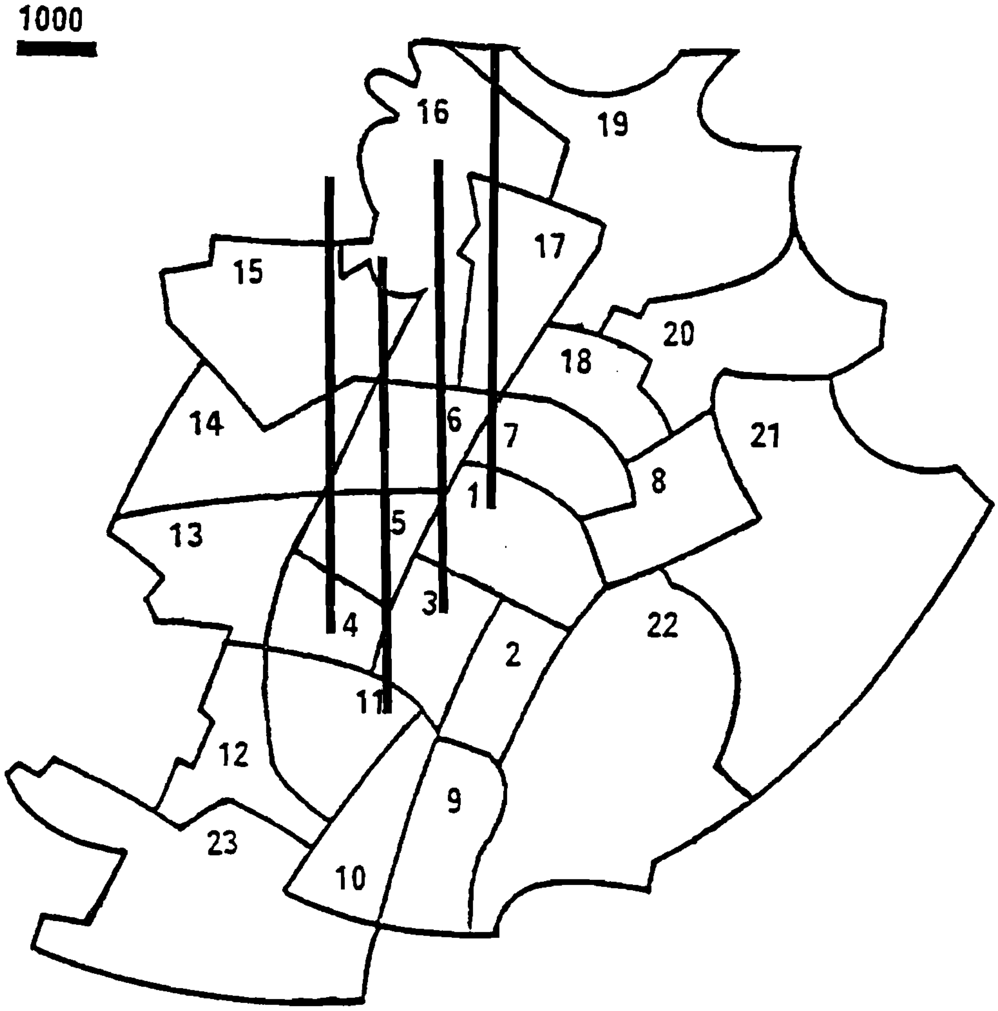


Figure 10. Optimal solution - lower bound = 6000.

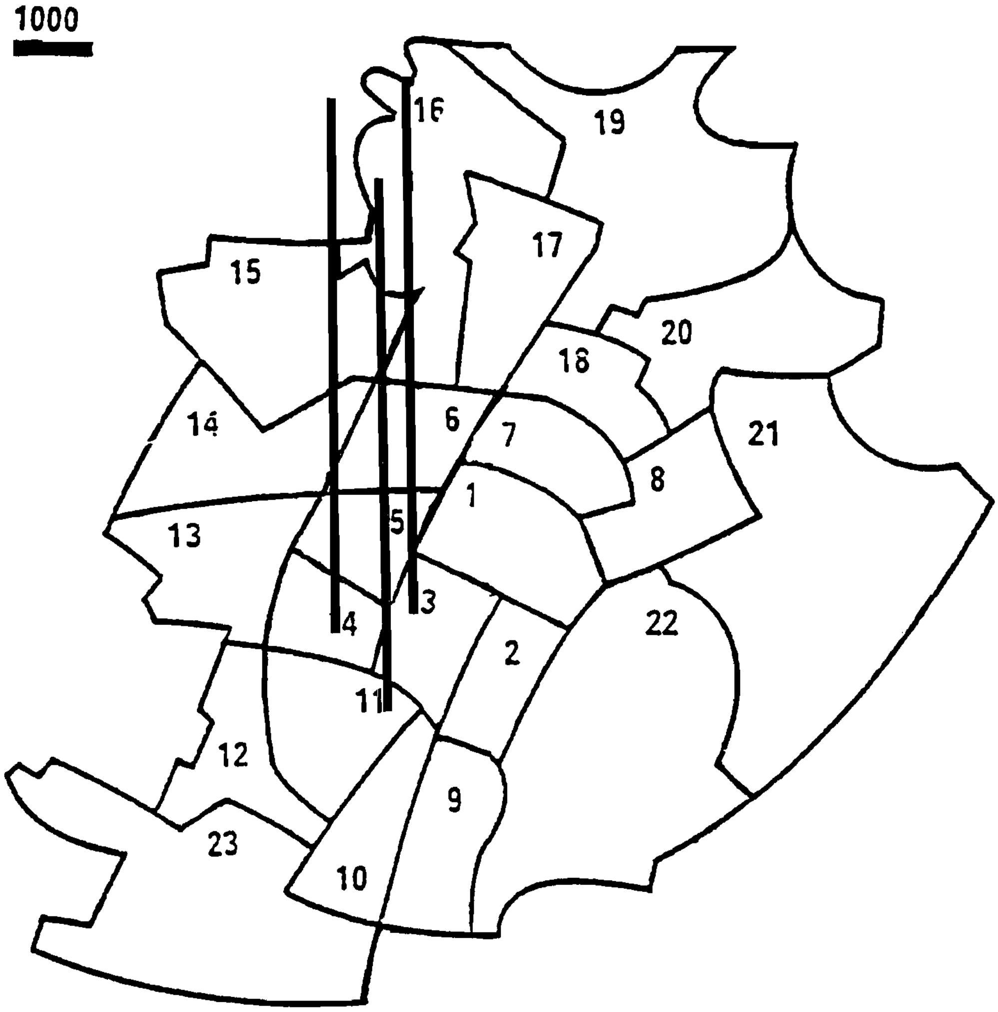


Figure 11. Optimal solution - lower bound = 7000.

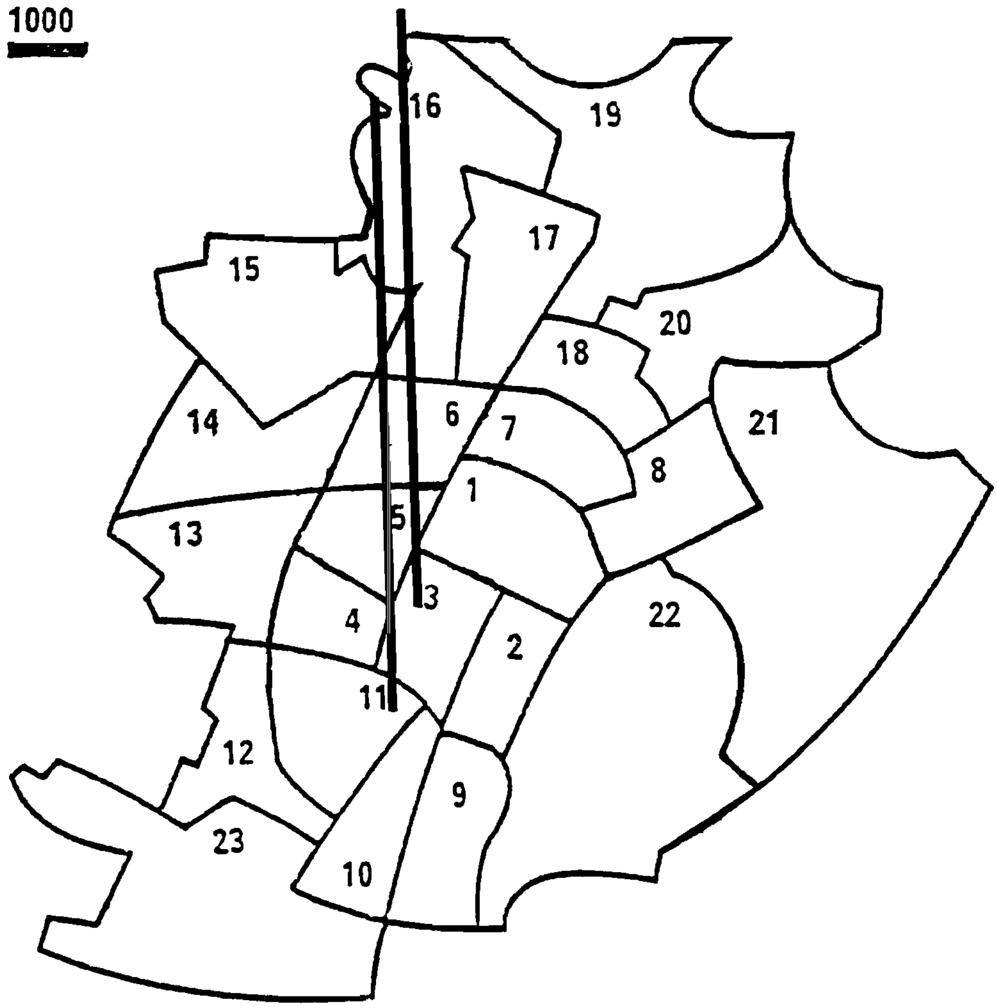


Figure 12. Optimal solution - lower bound = 8000.

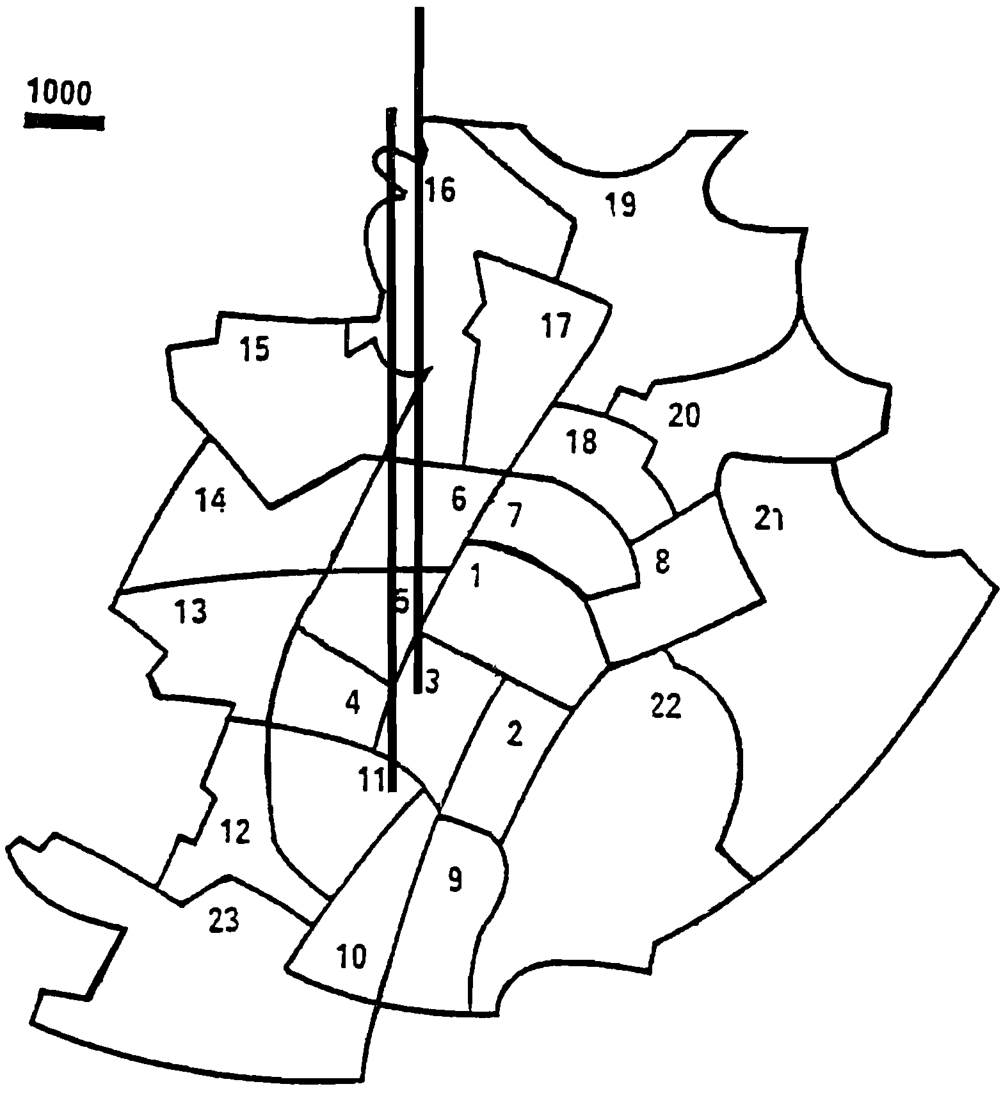


Figure 13. Optimal solution - lower bound = 9000.

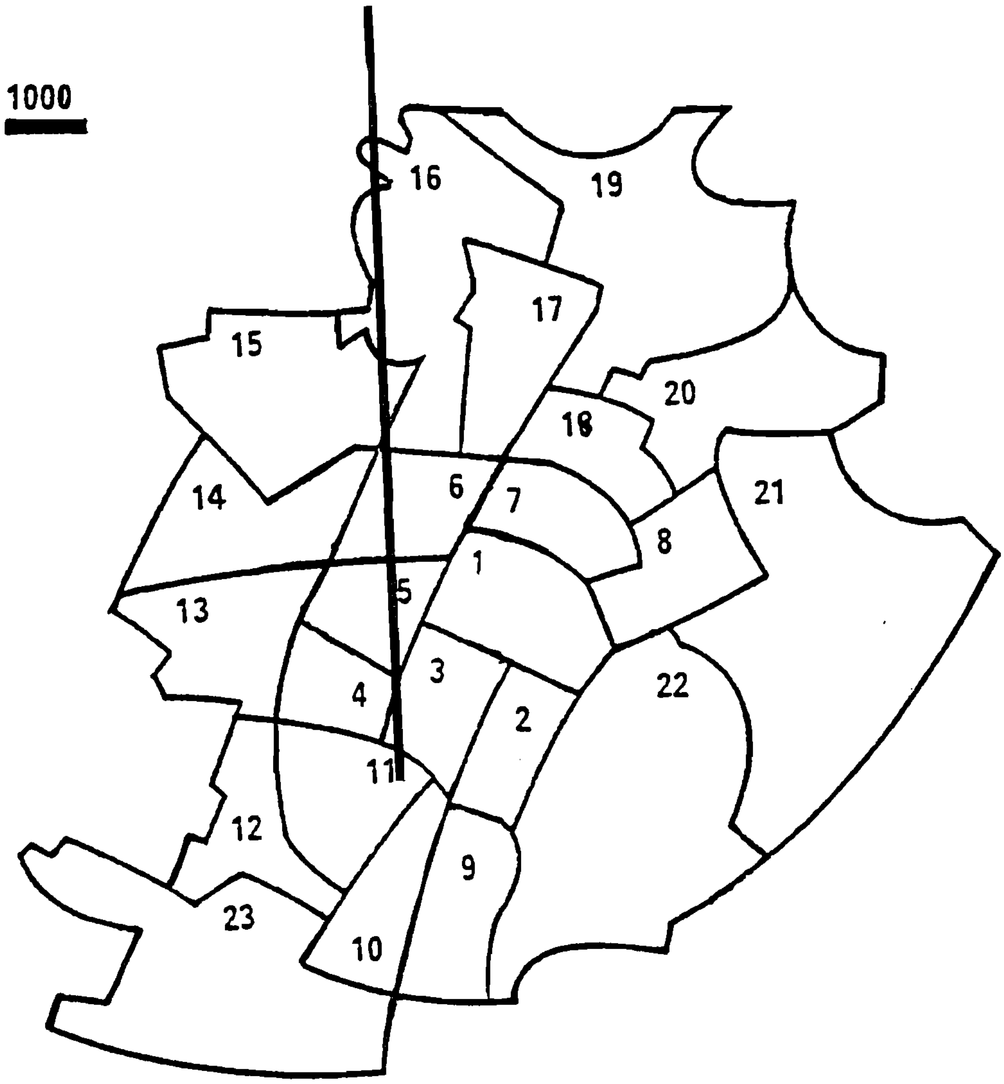
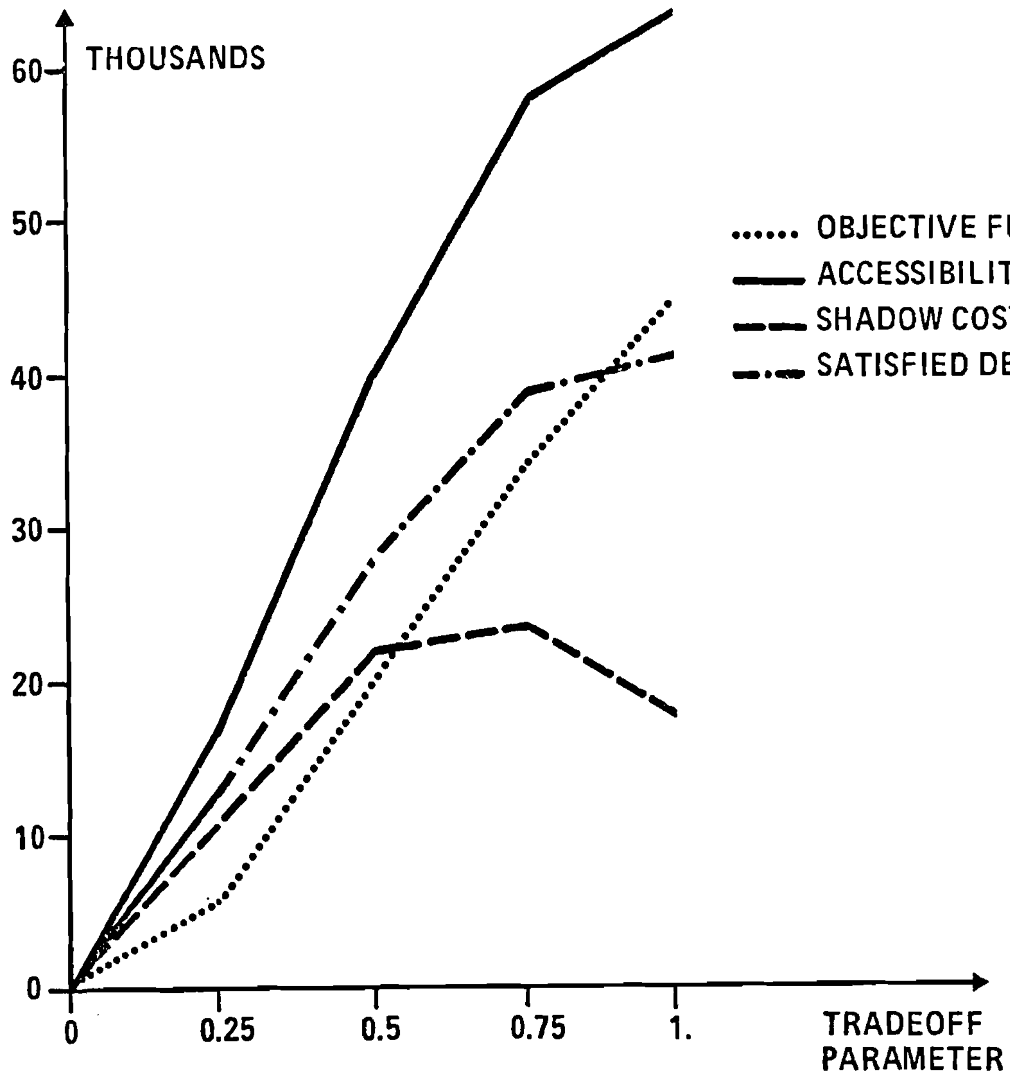


Figure 14. Optimal solution - lower bound = 10000.

available resources, while when $\alpha = 0$ no resources are available. A summary of the behavior of the solution for $0 \leq \alpha \leq 1$ is shown in Figure 15 and 16. (The lower bound--the minimum feasible size--has been kept constant and equal to 2000.) Even though the lower bound is the same, the capacity constraint reduces the number of open facilities. This is shown in Figure 17 (to be compared with Figure 6), which gives the solution for $\alpha = .25$, corresponding to a total capacity of 14000. The detailed results for the sensitivity analysis on the lower bound and on the trade-off parameter are reported in Appendix A.

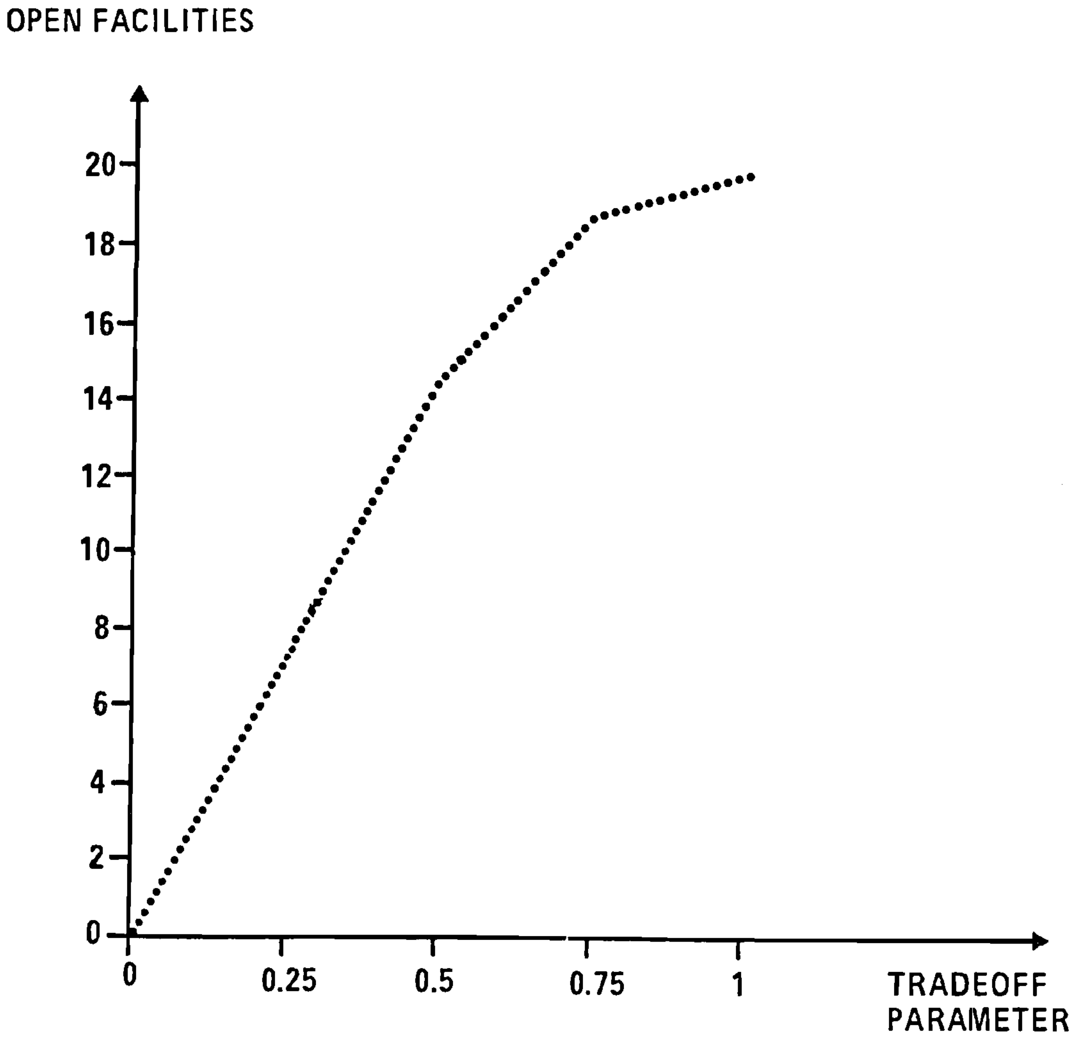
3. INTRODUCING THE GRADED STRUCTURE

A comparison of the existing (1977) distribution of high schools with the optimal unconstrained solution (Figure 2) or with the more realistic solution with a lower bound of 2000 (Figure 6) shows that big changes are needed in many districts. There are, however, many reasons why such changes should be made gradually. Even if economic reasons are neglected, the dynamic behavior of the school system itself is a sufficient reason for introducing gradual changes. Students have to stay in school many years (at least five) to complete the course, and cannot be moved arbitrarily. If the simplifying (but socially and educationally reasonable) assumption is introduced that the already enrolled students cannot be moved, then the only way to change the distribution of required school capacity is by changing the admission policy for the new students at the first grade of high school. In order to further simplfy the problem, suppose that no constraints are placed on the number of new students in all locations. This assumption may require unrealistic capacity expansions or reductions, but it is useful in order to analyze the "natural response of the system. If customers are unconstrained, then all multipliers α and w_j in equations (12) and the following are equal to 1. Let $P_i(t)$ be the potential demand for new admissions in the first grade at year t (approximately, the



Source: Appendix A2

Figure 15. Summary of the evaluation criteria for the sensitivity analysis on the tradeoff parameter, with the lower bound equal to 2000.



Source: Appendix A2

Figure 16. Number of open facilities versus the trade-off parameter.

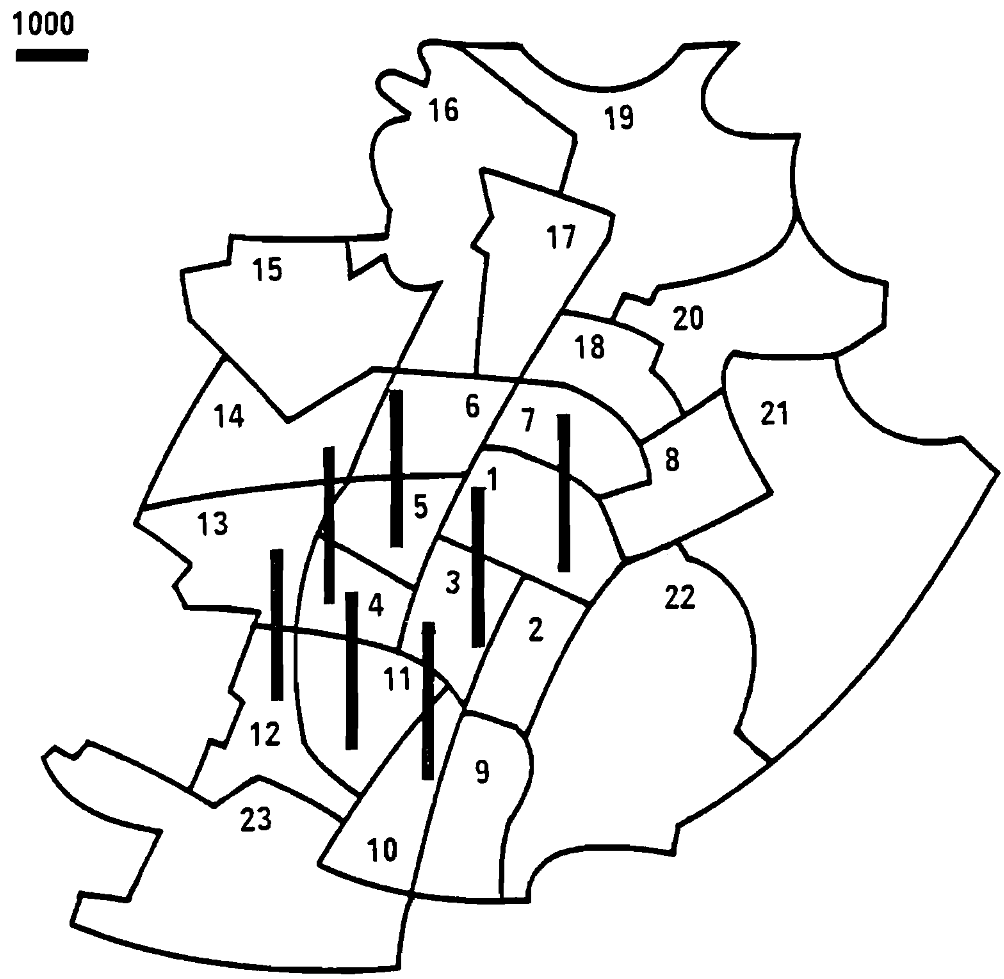


Figure 17. Optimal solution - lower bound = 2000, total capacity = 14000.

people who reached 14 years of age at year t), then (12) becomes

$$S_{ij}(t) = P_i(t) \frac{f_{ij}}{\sum_j f_{ij} + g_i}$$

and the total of new admissions to the first grade in district j at year t is

$$R_j(t) = \sum_i P_i(t) \frac{f_{ij}}{\sum_j f_{ij} + g_i} \qquad (19)$$

According to the available data for the years 1973-1976, the transitions from one grade to another during the total 5-year high school course take place according to the transition probabilities indicated in Table 1.

The resulting dynamic process is a simple open Markov-chain in discrete time. Three transitions are possible from the first four grades: stay in the same grade one more year (main diagonal of Table 1), pass to the next grade (upper diagonal of Table 1), or retire from school (column "OUT", to the right of Table 1). From the last grade (5), only two transitions are possible: stay in the same grade one more year or go out of the system, either because of retirement or (mainly) because of successful completion of the whole course.

In the absence of better forecasts (which may become available in the near future), the response of the above system has been tested with two different input functions:

a. a potential demand with a geometric growth, using the growth rate observed in the last years (approximately 5%);
b. a potential demand with a logistic growth, fitting the logistic parameters to the last 15 years data.

Table 1. Transition matrix for the 5 high school grades.

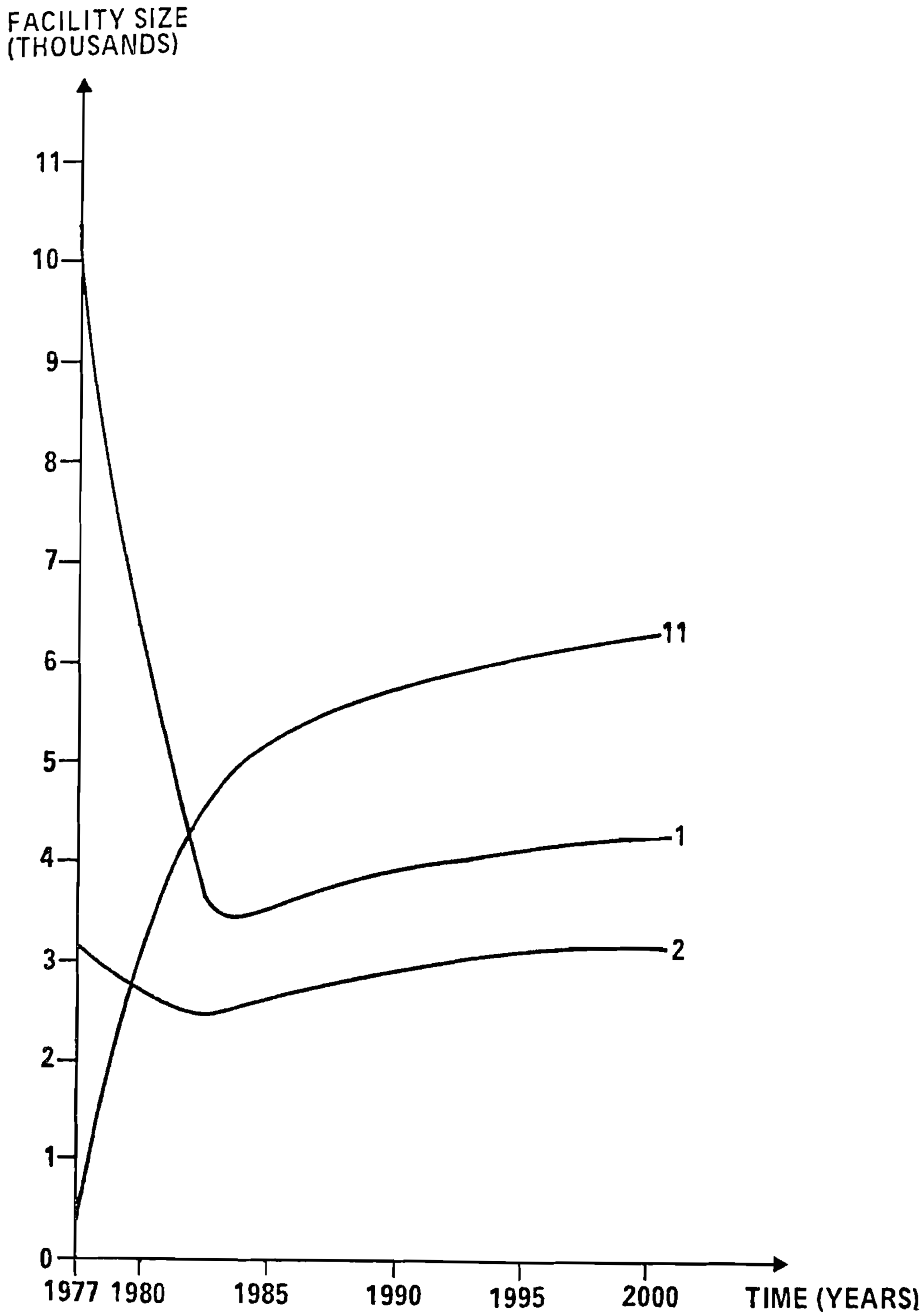
	1	2	3	4	5	OUT
1	.0712	.7381				.1907
2		.0615	.8571			.0814
3			.0530	.7720		.1750
4				.0390	.8710	.0900
5					.0369	.9631

Furthermore, in the absence of data on residential mobility (which unfortunately are unlikely to become available in the near future) the relative size of the demand in each demand location i has been kept equal to the one observed in 1977.

In spite of the above limitations, the results obtained are quite realistic and interesting. They are reported in detail in Appendix B. An example of the three typically observed behaviors is given in Figure 18, showing the response to the logistic input (the most realistic one) of districts 1, 2, and 11. District 1 decreases fast until 1983 (slightly more than 5 years from 1977, which is the minimum time required to lose the memory of the initial conditions) and then settles down slowly to its stationary value. District 2 is practically unaffected by changes. District 11 increases steadily to a value higher than the initial capacity in district 1 (and, indeed, to the highest value among all districts).

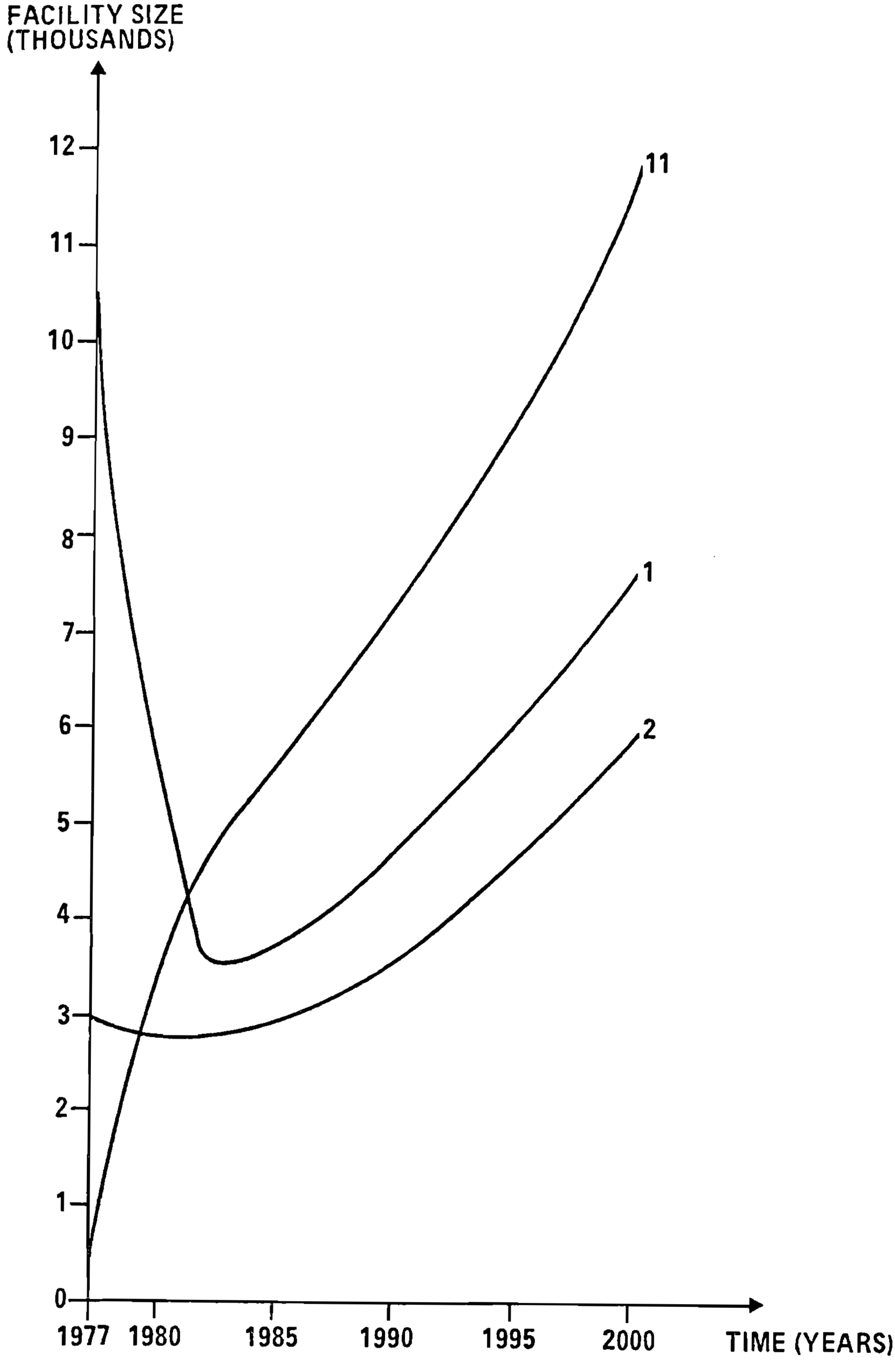
For the sake of completeness, the response of districts 1, 2, and 11 to the geometric input is shown in Figure 19, although it is clearly unrealistic.

An overall picture of the shifts in the high school capacity distribution induced by the admissions policy (19) is given in



Source: Appendix B2

Figure 18. The responses of districts 1, 2, and 11 to the logistic input.



Source: Appendix B1

Figure 19. The responses of districts 1, 2, and 11 to the geometric input.

Figure 20. The aggregation of the districts into three zones is introduced:

1. the "center" including districts from 1 to 8;
2. the "south west", including districts from 9 to 13, plus districts 22 and 23;
3. the "north west", including districts from 14 to 18.

Districts 19, 20, and 21 have no schools, both in 1977 and in most optimal solutions; therefore, they have been dropped.

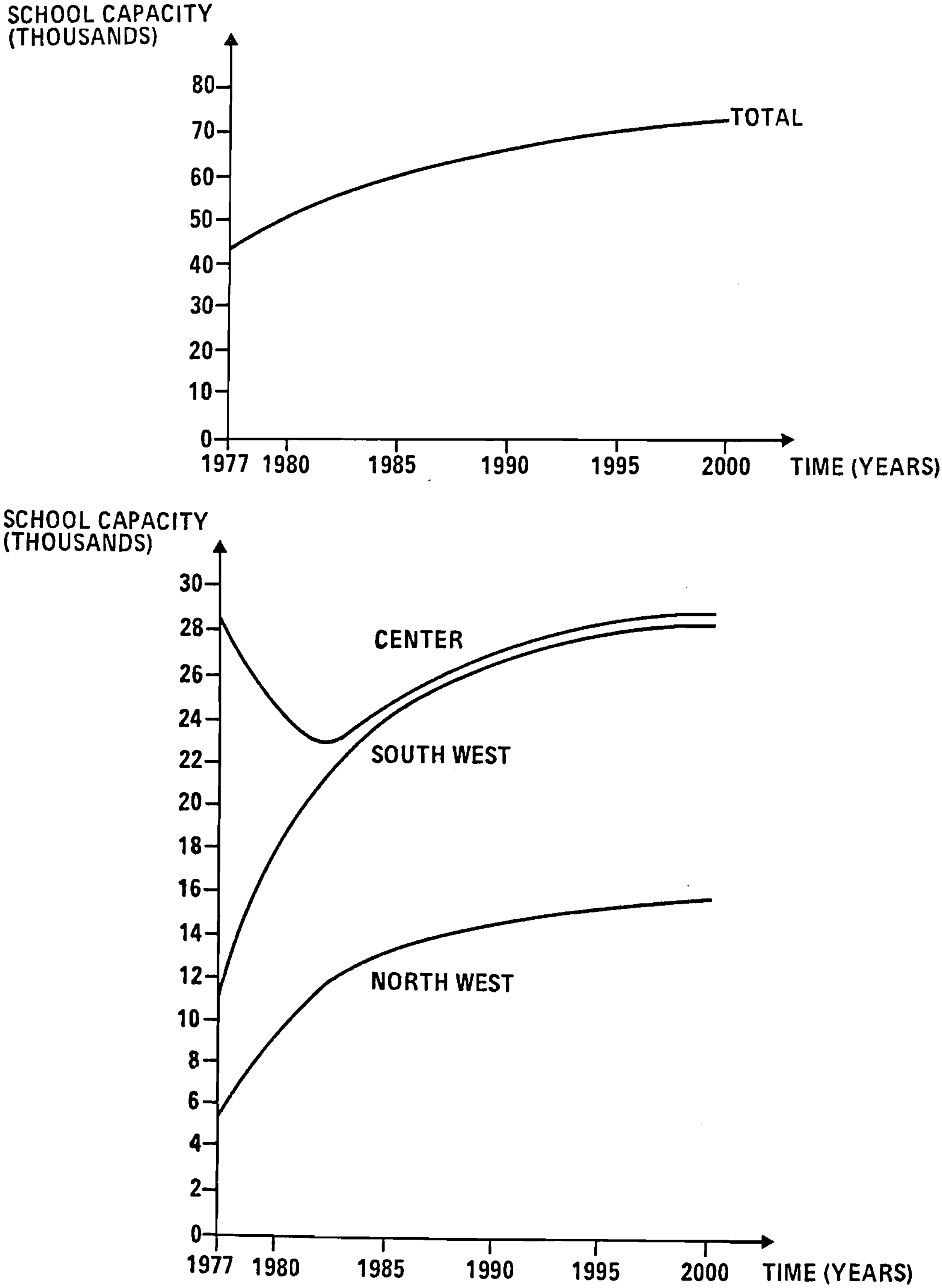
Figure 20 shows that the main effect of the "unconstrained" admissions policy is a shift of capacity toward the south west, while keeping the center approximately constant. The north west increases too, although to a lesser extent.

4. TOWARD DYNAMIC OPTIMIZATION

The simple analysis carried out in Section 3 provides a motivation for including dynamics into location models. In a first attempt to introduce optimization over time explicitly, the following simplifying assumptions are introduced:

a. demand is known deterministically in advance for each time period and each demand location;
b. demand is generally increasing, so that an already open facility will never be closed in the future;
c. the capacity of open facilities has to be fully used in all time periods;
d. time is discrete and each time period (e.g., 1 year) is numbered $t = 1,\ldots,N$;
e. the planning horizon N is finite;
f. the size of facilities can be changed only n times, with $n < N$.

Moreover, it is assumed that the system has been run long enough to have all meaningful locations open, so that only capacity expansion and reduction is possibly required. Assumption f is required to make the dynamic problem meaningful. It basically states that the size of facilities cannot be changed every day. It has to be kept constant for a while, while demand possibly undergoes fast changes.



Source: Appendix B2

Figure 20. Shift of required capacity among the three zones of Turin.

The optimization problem arising from assumptions a to f is illustrated in Figure 21, for the simple case of a single facility.

Assumption c implies that the capacity will be always less than the potential demand. Let the *unconstrained demand* be defined as the demand that would result from dropping all constraints on service capacity. The unconstrained demand may be less or more than the service capacity actually available at each time. If the total planning period N is divided in n intervals ($n = 3$ in the example of Figure 21), and the size of the facility is kept constant during each interval, then the optimal size for each interval will be some kind of "average" of the unconstrained demand. Provided the optimization subproblem for each time interval (t,τ) is solved for all $t,\tau = 1,\ldots, N-1$, $\tau > t$, and

$v_{t\tau}$ is the value of the objective function for the optimal solution of the (t,τ) interval subproblem,

then it is possible to write the following simple dynamic programming recursion for the optimal timing of an n stage process:

$$V_t(n+1) = \max_{t<\tau\leq N-n} \left[v_{t\tau} + V_\tau(n) \right] \tag{20}$$

where

$V_\tau(n)$ is the total value of an optimal n-stage process starting at time τ

Usually the value of n is not known to the decision maker; he would rather have a prospect of different solutions for different values of n. By means of (20) such a prospect can be easily obtained, for all $n = 1,\ldots,N$.

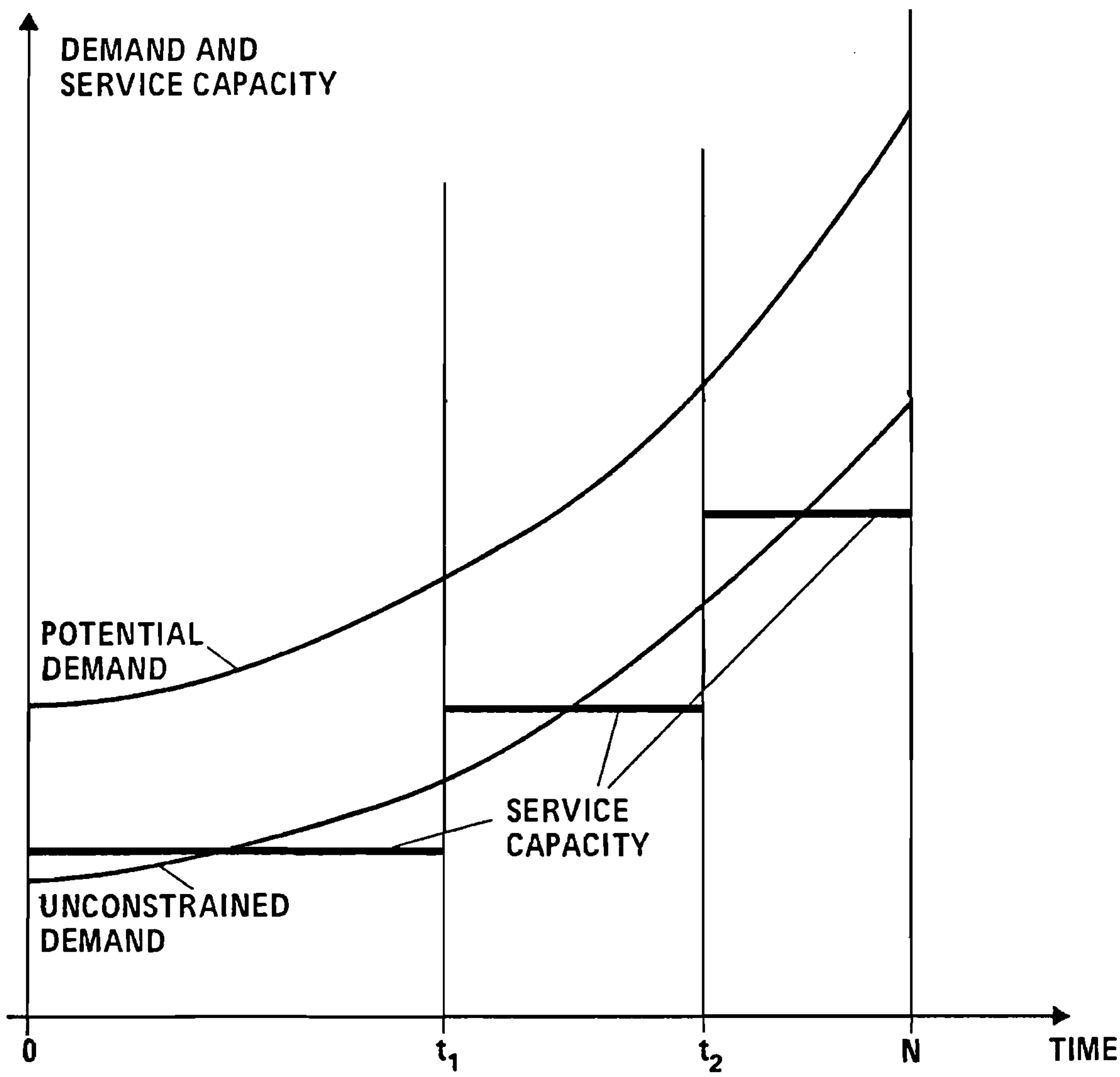


Figure 21. Dynamic size readjustment for a three-stage process.

Let now the single stage subproblem be solved, that is, the values for the $v_{t\tau}$ be determined. If the years from t to τ are renumbered from 1 to $m = j - i + 1$, the following generalization of problem (1)-(4) is obtained

$$\max_{S,Z} \quad - \sum_{k=1}^{m} \sum_{i} \left[\sum_{j} S_{ij}^{k} \left(\log \frac{S_{ij}^{k}}{f_{ij}} - 1 \right) + U_i^k \left(\log \frac{U_i^k}{g_i} - 1 \right) \right] \qquad (21)$$

s.t.

$$\sum_{j} S_{ij}^{k} + U_i^k = P_i^k \qquad (22)$$

$$\sum_{i} S_{ij}^{k} = Z_j \qquad (23)$$

where

k labels the years within the time interval (t,τ)

Z_j is the size of facility in j, to be kept constant during years $k = 1,\ldots,m$

Constraint (23) arises from assumption c above, and requires the capacity of each facility to be fully used over the whole period.

It may be shown that for the optimal Z_j the following equations must hold

$$Z_j = \prod_{k} \left(\rho_j^k \right)^{1/m} \qquad (24)$$

where

$\rho_j^k = \sum_i U_i^k \frac{f_{ij}}{g_i}$ is the potential of unsatisfied demand in j at year k

Thus (24) states the very reasonable condition that the optimal size of the facility must be equal to the geometric mean of unsatisfied demand potentials over the whole period. It can be also shown that

$$\rho_j^k = \sum_i P_i^k \frac{f_{ij}}{\phi_i^k + g_i} \qquad (25)$$

where

$\phi_i^k = \sum_i f_{ij} w_j^k$ is the accessibility measure already introduced in (12)

and

w_j^k are nonnegative multipliers with the property that $\prod_k w_j^k = 1$

A computational form for the multipliers is given by the following set of equations

$$w_j^k = \frac{\prod_k \left(\rho_j^k\right)^{1/m}}{\rho_j^k} \qquad (26)$$

which can be solved iteratively.

The algorithm to solve the dynamic optimization problem is still at its early stage of development, and its detailed description and extended application will constitute the subject of a forthcoming working paper. However, the results of the first tests seem interesting enough to deserve a brief discussion.

The algorithm has been applied to the logistic input case over the 20-year period between 1977 and 1996, and all the subdivisions in n = 1,2,...,7 stages have been generated. The overall behavior of the solution, as a function of the number of stages, is shown in Figure 22.

For graphical convenience, the sign of the objective function has been reversed, so that it must be interpreted as a "cost" to be minimized. From the diagram it is seen that this cost decreases steadily with the number of stages, but the rate of decrease is quite small after 4 stages. On the other hand, the satisfied demand over the whole time period is almost flat from 3 to 6 stages, and has a sudden decrease for more than 6 stages. This behavior has some analogies with the behavior of the solution to the bounded size location problem (see Figure 3), where the satisfied demand was found to decrease for lower bounds on the size of facilities greater than 3000. Figure 22 suggests that, if maximizing satisfied demand is of some concern as a planning goal, the split of the time horizon into stages should not be carried further than 6 stages. Moreover, there is a substantial indifference among all stage numbers between 3 and 6.

This may be useful for a further cost-benefit analysis. The increase in customers benefit (as measured by the objective function and by the satisfied demand) is very small for more than 3 stages , while the cost for changing the facility sizes may substantially increase. A 3 or 4 stage process seems, therefore, to be the most sensible solution.

The distribution over time of the required changes on total capacity is shown in Figure 23 for the 3-stage process and the 6-stage process. For the 3-stage process, the service capacity is changed in 1981 and in 1987, and the required changes are quite big (more than 6000 demand units). For the 6-stage process, the service capacity is changed in 1979, 1981, 1984, 1987, and 1991, with average small changes (slightly more than 3000 demand units).

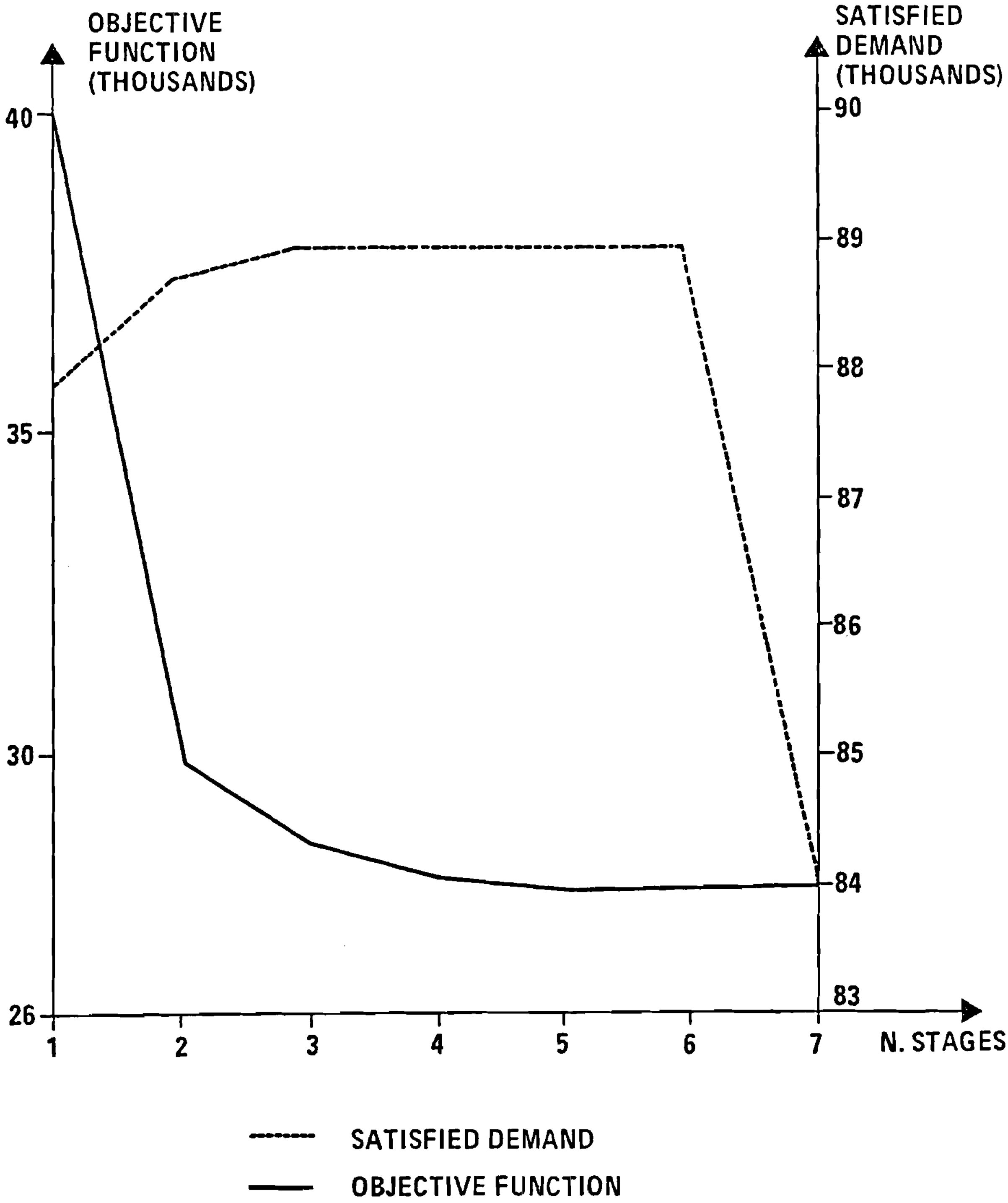


Figure 22. Dynamic capacity adjustment: a sensitivity analysis on the number of stages. (The constant $139 \cdot 10^5$ has been subtracted from the objective function.)

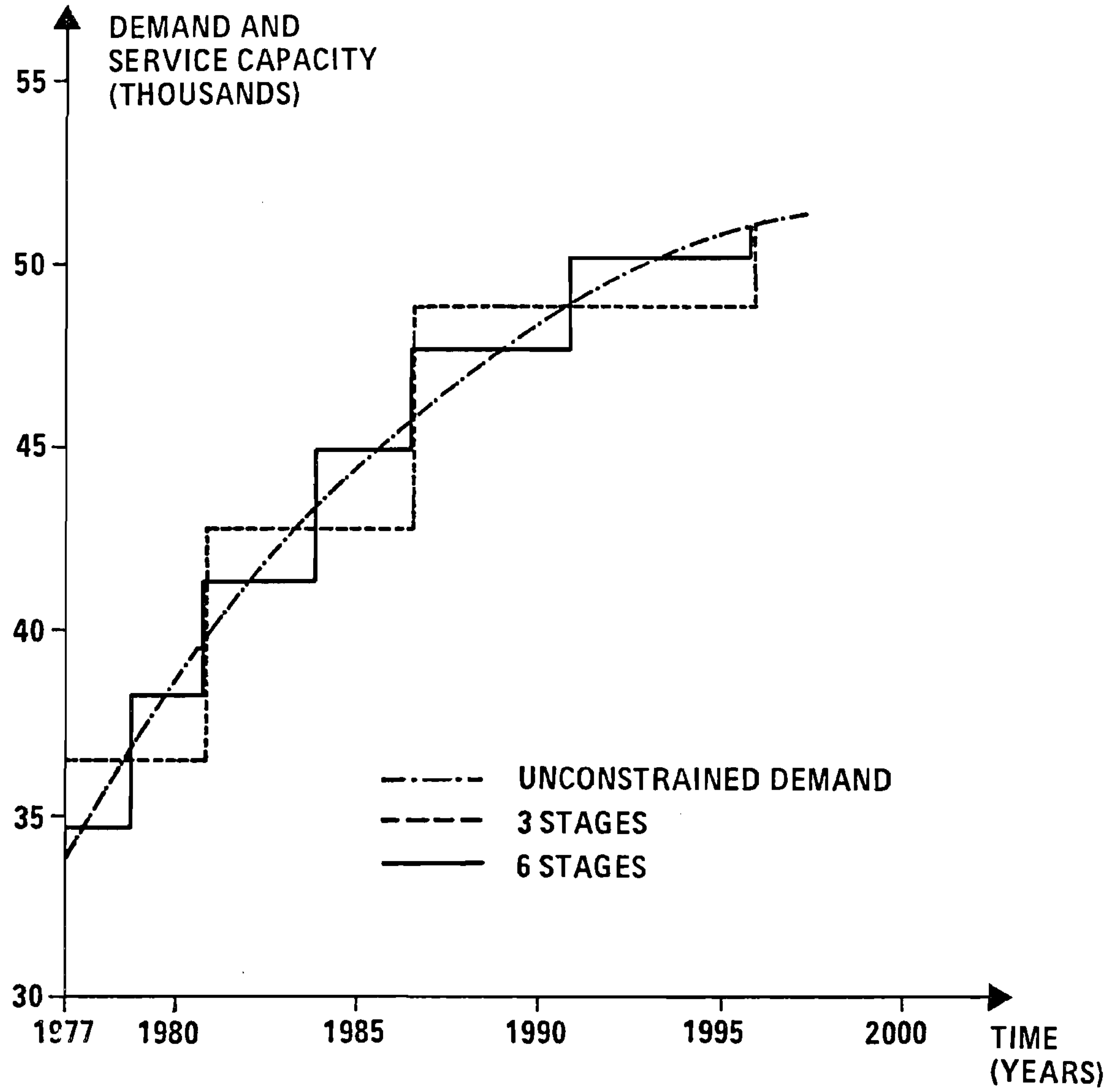


Figure 23. Optimal timing and capacity expansion. Overall results for the 3-stage and 6-stage processes.

Although the 3-stage solution requires bigger changes, the total change in capacity over the whole period is smaller than for the 6-stage solution.

Therefore, if costs are assumed to be proportional to total change in capacity, the 3-stage solution might be better than the 6-stage one. This is even more true if fixed costs have to be paid for each change.

5. CONCLUDING REMARKS

Sections 2 to 4 have shown how spatial patterns obtained by static optimization raise dynamic issues. In Section 4 a simple model for optimal timing of changes in service capacity has been proposed. In this model the set of open facilities is held fixed, and only readjustments to the sizes are introduced from time to time. If more constraints on the size of the facilities (e.g., lower and upper bounds) are introduced and/or the assumption of generally increasing demand is relaxed, then the decision to open or close some facility may become meaningful. In this case the simple structure of problem (9) is lost, and new techniques must be developed to solve the resulting combinatorial problems. If the demand is further assumed to be stochastic, then stochastic programming methods may be required. Some exploratory work has been done on stochastic versions of the static problem (Ermoliev and Leonardi, 1980; Ermoliev, Leonardi, and Vira, forthcoming), and a generalization to the dynamic case may follow the same path.

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APPENDIX A: RESULTS FOR THE STATIC BOUNDED-SIZE LOCATION MODEL

<u>Legend for Appendix A</u>

minimum size	lower bound on facility size (varies from 0 to 10,000)
maximum size	upper bound on facility size (always kept equal to 100,000)
step number	number of iterations (updating the set of chosen locations) required to reach the optimum
objective function	value of the function defined by equation (11), after dropping the constant terms
total demand	potential demand for high schools (14-18 year-old youths) living in each district*
satisfied demand	number of high school students living in each district
unsatisfied demand	difference between total demand and satisfied demand for each district
accessibility	value of the denominator in equations (12) and (13) for each district
facility size	optimal service capacity for each district

*Due to calibration and computation readjustments, there are some differences between the values in the *total demand* array and the data on 14-18 year-olds published in Provincia di Torino (1978).

used capacity	difference between facility size and minimum size for each district (meaningful only for open facilities)
unused capacity	difference between maximum size and facility size for each district
multiplier	value of the balancing factor introduced in equations (12) and (13), for each district
total	for all the arrays except *accessibility* and *multiplier*, sum over all districts; for the array *accessibility*, the first sum in the right-hand side of equation (11); for the array *multiplier*, the second sum in the right-hand side of equation (11)*

*All the remaining terms in the right-hand side of (11) have been dropped in applications, either because they are constant or because they correspond to nonactive constraints. Therefore, the sum of the two totals at the bottom of the arrays *accessibility* and *multiplier* yields the value of the *objective function*.

Appendix A. 1 Sensitivity Analysis on the Minimum Feasible Facility Size.

minimun size 0.

maximum size 100000.

tradeoff parameter 1.00000

step number 1

obj. function 50655.82031

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1584.	1220.	2.29749
2	2664.	1450.	1214.	2.19439
3	3080.	2087.	993.	3.10017
4	2306.	1472.	834.	2.76565
5	2874.	1729.	1145.	2.50932
6	2856.	1558.	1298.	2.20018
7	2398.	1194.	1204.	1.99201
8	2448.	1063.	1385.	1.76778
9	2536.	1320.	1216.	2.08532
10	4712.	2571.	2141.	2.20127
11	5230.	3259.	1971.	2.65391
12	4602.	2403.	2199.	2.09234
13	4468.	2350.	2118.	2.11000
14	3668.	1699.	1969.	1.86280
15	2992.	1182.	1810.	1.65334
16	3144.	1288.	1856.	1.69367
17	2882.	1104.	1778.	1.62127
18	2844.	1482.	1362.	2.08792
19	2100.	770.	1330.	1.57894
20	2090.	839.	1251.	1.67032
21	1032.	334.	698.	1.47746
22	1644.	848.	796.	2.06585
23	4342.	1889.	2453.	1.77012
total	69716.	35475.	34241.	50655.82031

	facility size	used capacity	unused capacity	multiplier
1	1968.	1968.	98032.	1.00000
2	1459.	1459.	98541.	1.00000
3	2183.	2183.	97817.	1.00000
4	2194.	2194.	97806.	1.00000
5	1940.	1940.	98060.	1.00000
6	1468.	1468.	98532.	1.00000
7	1164.	1164.	98836.	1.00000
8	983.	983.	99017.	1.00000
9	1448.	1448.	98552.	1.00000
10	2168.	2168.	97832.	1.00000
11	2980.	2980.	97020.	1.00000
12	2281.	2281.	97719.	1.00000
13	1797.	1797.	98203.	1.00000
14	1536.	1536.	98464.	1.00000
15	1253.	1253.	98747.	1.00000
16	1171.	1171.	98829.	1.00000
17	1047.	1047.	98953.	1.00000
18	1554.	1554.	98446.	1.00000
19	800.	800.	99200.	1.00000
20	944.	944.	99056.	1.00000
21	340.	340.	99660.	1.00000
22	1176.	1176.	98824.	1.00000
23	1621.	1621.	98379.	1.00000
total	35475.	35475.	2264526.	0.

/cont....

A. 1 (cont.)

minimun size 1000.

maximum size 100000.

tradeoff parameter 1.00000

step number 2

obj. function 50214.14063

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1586.	1218.	2.30154
2	2664.	1450.	1214.	2.19478
3	3080.	2087.	993.	3.10218
4	2306.	1472.	834.	2.76620
5	2874.	1729.	1145.	2.50992
6	2856.	1559.	1297.	2.20141
7	2398.	1201.	1197.	2.00295
8	2448.	1073.	1375.	1.77992
9	2536.	1320.	1216.	2.08544
10	4712.	2571.	2141.	2.20129
11	5230.	3259.	1971.	2.65411
12	4602.	2403.	2199.	2.09242
13	4468.	2351.	2117.	2.11022
14	3668.	1699.	1969.	1.86326
15	2992.	1183.	1809.	1.65408
16	3144.	1289.	1855.	1.69533
17	2882.	1107.	1775.	1.62357
18	2844.	1505.	1339.	2.12341
19	2100.	899.	1201.	1.74802
20	2090.	868.	1222.	1.70990
21	1032.	5.	1027.	1.00517
22	1644.	848.	796.	2.06585
23	4342.	1889.	2453.	1.77021
total	69716.	35353.	34363.	50619.76953

	facility size	used capacity	unused capacity	multiplier
1	1962.	962.	98038.	1.00000
2	1458.	458.	98542.	1.00000
3	2181.	1181.	97819.	1.00000
4	2193.	1193.	97807.	1.00000
5	1939.	939.	98061.	1.00000
6	1466.	466.	98534.	1.00000
7	1156.	156.	98844.	1.00000
8	1000.	-0.	99000.	1.02475
9	1448.	448.	98552.	1.00000
10	2168.	1168.	97832.	1.00000
11	2980.	1980.	97020.	1.00000
12	2281.	1281.	97719.	1.00000
13	1797.	797.	98203.	1.00000
14	1535.	535.	98465.	1.00000
15	1251.	251.	98749.	1.00000
16	1169.	169.	98831.	1.00000
17	1043.	43.	98957.	1.00000
18	1531.	531.	98469.	1.00000
19	1000.	-0.	99000.	1.35668
20	1000.	-0.	99000.	1.07911
21	0.	0.	100000.	2.93833
22	1175.	175.	98825.	1.00000
23	1621.	621.	98379.	1.00000
total	35353.	13353.	2264647.	-405.62891

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A. 1 (cont.)

minimun size 2000.

maximum size 100000.

tradeoff parameter 1.00000

step number 3

obj. function 45128.05859

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1834.	970.	2.89069
2	2664.	1753.	911.	2.92452
3	3080.	2278.	802.	3.83967
4	2306.	1570.	736.	3.13415
5	2874.	1883.	991.	2.89889
6	2856.	1871.	985.	2.89892
7	2398.	1502.	896.	2.67544
8	2448.	1578.	870.	2.81517
9	2536.	1610.	926.	2.73722
10	4712.	2837.	1875.	2.51247
11	5230.	3406.	1824.	2.86804
12	4602.	2475.	2127.	2.16379
13	4468.	2577.	1891.	2.36245
14	3668.	2133.	1535.	2.38987
15	2992.	1768.	1224.	2.44475
16	3144.	1934.	1210.	2.59847
17	2882.	1782.	1100.	2.62058
18	2844.	1779.	1065.	2.67012
19	2100.	314.	1786.	1.17561
20	2090.	574.	1516.	1.37868
21	1032.	11.	1021.	1.01107
22	1644.	1082.	562.	2.92507
23	4342.	2235.	2107.	2.06094
total	69716.	40786.	28930.	63773.02734

	facility size	used capacity	unused capacity	multiplier
1	2000.	-0.	98000.	1.27324
2	2000.	-0.	98000.	1.74423
3	2000.	-0.	98000.	1.10526
4	2000.	-0.	98000.	1.04516
5	2000.	-0.	98000.	1.21580
6	2000.	-0.	98000.	1.73596
7	2000.	-0.	98000.	2.16465
8	2000.	-0.	98000.	2.93251
9	2000.	-0.	98000.	1.72323
10	2000.	-0.	98000.	1.05618
11	2677.	677.	97323.	1.00000
12	2110.	110.	97890.	1.00000
13	2000.	-0.	98000.	1.26415
14	2000.	-0.	98000.	1.66798
15	2000.	-0.	98000.	2.31107
16	2000.	-0.	98000.	2.52328
17	2000.	-0.	98000.	2.91983
18	2000.	-0.	98000.	1.58956
19	0.	0.	100000.	2.72835
20	0.	0.	100000.	2.51546
21	0.	0.	100000.	5.95053
22	2000.	-0.	98000.	2.18370
23	2000.	-0.	98000.	1.43494
total	40786.	786.	2259214.	-18644.96875

/cont.....

A. 1 (cont.)

minimun size 3000.

maximum size 100000.

tradeoff parameter 1.00000

step number 7

obj. function 33579.54297

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1985.	819.	3.42338
2	2664.	1943.	721.	3.69629
3	3080.	2496.	584.	5.27154
4	2306.	1843.	463.	4.97836
5	2874.	2196.	678.	4.23755
6	2856.	1795.	1061.	2.69164
7	2398.	1032.	1366.	1.75521
8	2448.	714.	1734.	1.41199
9	2536.	1832.	704.	3.60196
10	4712.	3371.	1341.	3.51251
11	5230.	4031.	1199.	4.36099
12	4602.	3168.	1434.	3.20953
13	4468.	3245.	1223.	3.65442
14	3668.	2344.	1324.	2.77054
15	2992.	1920.	1072.	2.78985
16	3144.	1139.	2005.	1.56774
17	2882.	568.	2314.	1.24526
18	2844.	1729.	1115.	2.55156
19	2100.	309.	1791.	1.17260
20	2090.	358.	1732.	1.20636
21	1032.	4.	1028.	1.00413
22	1644.	1014.	630.	2.60808
23	4342.	2965.	1377.	3.15353
total	69716.	41999.	27717.	71765.82031

	facility size	used capacity	unused capacity	multiplier
1	3000.	-0.	97000.	2.13279
2	3000.	-0.	97000.	3.16593
3	3000.	-0.	97000.	2.10254
4	3000.	-0.	97000.	2.24485
5	3000.	-0.	97000.	2.46948
6	0.	0.	100000.	3.00160
7	0.	0.	100000.	2.98210
8	0.	0.	100000.	3.34832
9	3000.	-0.	97000.	3.32642
10	3000.	-0.	97000.	2.22010
11	3000.	-0.	97000.	1.61776
12	3000.	-0.	97000.	2.08029
13	3000.	-0.	97000.	2.77895
14	3000.	-0.	97000.	2.75946
15	3000.	-0.	97000.	3.40034
16	0.	0.	100000.	3.21430
17	0.	0.	100000.	3.11715
18	3000.	-0.	97000.	2.07613
19	0.	0.	100000.	3.99062
20	0.	0.	100000.	3.25017
21	0.	0.	100000.	8.80329
22	0.	0.	100000.	4.09146
23	3000.	-0.	97000.	3.18546
total	41999.	-1.	2258001.	-38186.27734

/cont.....

A. 1 (cont.)

minimun size 4000.

maximum size 100000.

tradeoff parameter 1.00000

step number 7

obj. function 24432.54102

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1881.	923.	3.03902
2	2664.	1242.	1422.	1.87407
3	3080.	2336.	744.	4.13949
4	2306.	1710.	596.	3.86846
5	2874.	2083.	791.	3.63535
6	2856.	1240.	1616.	1.76739
7	2398.	893.	1505.	1.59321
8	2448.	650.	1798.	1.36128
9	2536.	976.	1560.	1.62539
10	4712.	2991.	1721.	2.73806
11	5230.	3896.	1334.	3.92021
12	4602.	3129.	1473.	3.12430
13	4468.	2448.	2020.	2.21154
14	3668.	958.	2710.	1.35372
15	2992.	388.	2604.	1.14887
16	3144.	430.	2714.	1.15844
17	2882.	509.	2373.	1.21461
18	2844.	1742.	1102.	2.58158
19	2100.	338.	1762.	1.19209
20	2090.	358.	1732.	1.20676
21	1032.	3.	1029.	1.00244
22	1644.	519.	1125.	1.46188
23	4342.	1278.	3064.	1.41721
total	69716.	32000.	37716.	49553.45313

	facility size	used capacity	unused capacity	multiplier
1	4000.	-0.	96000.	2.21767
2	0.	0.	100000.	2.91507
3	4000.	-0.	96000.	2.04610
4	4000.	-0.	96000.	2.18265
5	4000.	-0.	96000.	2.43753
6	0.	0.	100000.	2.72011
7	0.	0.	100000.	3.50215
8	0.	0.	100000.	4.16930
9	0.	0.	100000.	2.84448
10	4000.	-0.	96000.	2.14583
11	4000.	-0.	96000.	1.70840
12	4000.	-0.	96000.	2.38874
13	0.	0.	100000.	2.82654
14	0.	0.	100000.	2.55914
15	0.	0.	100000.	2.98645
16	0.	0.	100000.	3.19157
17	0.	0.	100000.	3.94954
18	4000.	-0.	96000.	2.52557
19	0.	0.	100000.	5.28862
20	0.	0.	100000.	4.18308
21	0.	0.	100000.	11.66939
22	0.	0.	100000.	3.42318
23	0.	0.	100000.	2.69958
total	32000.	-0.	2268001.	-25120.91211

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A. 1 (cont.)

minimun size 5000.

maximum size 100000.

tradeoff parameter 1.00000

step number 3

obj. function 17589.42969

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1884.	920.	3.04748
2	2664.	1349.	1315.	2.02518
3	3080.	2425.	655.	4.70002
4	2306.	1767.	539.	4.27763
5	2874.	2200.	674.	4.26434
6	2856.	1349.	1507.	1.89496
7	2398.	820.	1578.	1.51957
8	2448.	410.	2038.	1.20137
9	2536.	1076.	1460.	1.73717
10	4712.	3145.	1567.	3.00795
11	5230.	3758.	1472.	3.55341
12	4602.	2526.	2076.	2.21693
13	4468.	2600.	1868.	2.39188
14	3668.	1028.	2640.	1.38931
15	2992.	263.	2729.	1.09644
16	3144.	230.	2914.	1.07881
17	2882.	149.	2733.	1.05467
18	2844.	913.	1931.	1.47312
19	2100.	56.	2044.	1.02723
20	2090.	91.	1999.	1.04537
21	1032.	3.	1029.	1.00255
22	1644.	581.	1063.	1.54638
23	4342.	1377.	2965.	1.46441
total	69716.	30000.	39716.	46849.54297

	facility size	used capacity	unused capacity	multiplier
1	5000.	-0.	95000.	2.67311
2	0.	0.	100000.	3.79417
3	5000.	-0.	95000.	2.62362
4	5000.	-0.	95000.	2.80007
5	5000.	-0.	95000.	3.24034
6	0.	0.	100000.	3.49463
7	0.	0.	100000.	4.11095
8	0.	0.	100000.	4.63888
9	0.	0.	100000.	3.71514
10	5000.	-0.	95000.	2.73629
11	5000.	-0.	95000.	1.99832
12	0.	0.	100000.	2.85832
13	0.	0.	100000.	3.65228
14	0.	0.	100000.	3.28302
15	0.	0.	100000.	3.53952
16	0.	0.	100000.	3.77409
17	0.	0.	100000.	4.33149
18	0.	0.	100000.	2.79985
19	0.	0.	100000.	5.64050
20	0.	0.	100000.	4.57013
21	0.	0.	100000.	14.54579
22	0.	0.	100000.	4.47960
23	0.	0.	100000.	3.43700
total	30000.	-0.	2270001.	-29260.11328

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A. 1 (cont.)

minimun size 6000.

maximum size 100000.

tradeoff parameter 1.00000

step number 3

obj. function 13182.77539

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1818.	986.	2.84301
2	2664.	1146.	1518.	1.75501
3	3080.	2241.	839.	3.67216
4	2306.	1503.	803.	2.87037
5	2874.	1770.	1104.	2.60293
6	2856.	991.	1865.	1.53099
7	2398.	722.	1676.	1.43051
8	2448.	350.	2098.	1.16681
9	2536.	703.	1833.	1.38327
10	4712.	1813.	2899.	1.62516
11	5230.	3207.	2023.	2.58507
12	4602.	2129.	2473.	1.86104
13	4468.	2111.	2357.	1.89582
14	3668.	746.	2922.	1.25533
15	2992.	222.	2770.	1.08013
16	3144.	184.	2960.	1.06208
17	2882.	132.	2750.	1.04804
18	2844.	870.	1974.	1.44044
19	2100.	49.	2051.	1.02388
20	2090.	75.	2015.	1.03706
21	1032.	2.	1030.	1.00193
22	1644.	367.	1277.	1.28770
23	4342.	851.	3491.	1.24378
total	69716.	24000.	45716.	33780.56641

	facility size	used capacity	unused capacity	multiplier
1	6000.	-0.	94000.	2.77964
2	0.	0.	100000.	3.76321
3	6000.	-0.	94000.	2.50834
4	6000.	-0.	94000.	2.52794
5	0.	0.	100000.	3.16749
6	0.	0.	100000.	3.59024
7	0.	0.	100000.	4.55984
8	0.	0.	100000.	5.33445
9	0.	0.	100000.	3.51128
10	0.	0.	100000.	2.55079
11	6000.	-0.	94000.	1.75702
12	0.	0.	100000.	2.73661
13	0.	0.	100000.	3.48970
14	0.	0.	100000.	3.49602
15	0.	0.	100000.	4.13061
16	0.	0.	100000.	4.40440
17	0.	0.	100000.	5.11758
18	0.	0.	100000.	3.22771
19	0.	0.	100000.	6.69189
20	0.	0.	100000.	5.34874
21	0.	0.	100000.	17.38839
22	0.	0.	100000.	4.26550
23	0.	0.	100000.	3.42669
total	24000.	-0.	2276001.	-20597.79102

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A. 1 (cont.)

minimun size 7000.

maximum size 100000.

tradeoff parameter 1.00000

step number 2

obj. function 10008.91406

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1015.	1789.	1.56710
2	2664.	815.	1849.	1.44073
3	3080.	2043.	1037.	2.97047
4	2306.	1528.	778.	2.96295
5	2874.	1732.	1142.	2.51745
6	2856.	724.	2132.	1.33968
7	2398.	380.	2018.	1.18802
8	2448.	156.	2292.	1.06795
9	2536.	685.	1851.	1.37026
10	4712.	1888.	2824.	1.66862
11	5230.	3245.	1985.	2.63492
12	4602.	2252.	2350.	1.95860
13	4468.	2170.	2298.	1.94405
14	3668.	593.	3075.	1.19300
15	2992.	119.	2873.	1.04160
16	3144.	128.	3016.	1.04228
17	2882.	64.	2818.	1.02284
18	2844.	275.	2569.	1.10689
19	2100.	21.	2079.	1.00994
20	2090.	31.	2059.	1.01498
21	1032.	1.	1031.	1.00101
22	1644.	295.	1349.	1.21859
23	4342.	840.	3502.	1.23991
total	69716.	21000.	48716.	29060.27930

	facility size	used capacity	unused capacity	multiplier
1	0.	0.	100000.	2.83696
2	0.	0.	100000.	3.87704
3	7000.	-0.	93000.	2.59191
4	7000.	-0.	93000.	2.86745
5	0.	0.	100000.	3.44099
6	0.	0.	100000.	3.78808
7	0.	0.	100000.	4.57366
8	0.	0.	100000.	5.56516
9	0.	0.	100000.	3.96629
10	0.	0.	100000.	2.96929
11	7000.	-0.	93000.	2.04583
12	0.	0.	100000.	3.23503
13	0.	0.	100000.	4.02572
14	0.	0.	100000.	3.80855
15	0.	0.	100000.	4.58878
16	0.	0.	100000.	4.95959
17	0.	0.	100000.	5.62085
18	0.	0.	100000.	3.22466
19	0.	0.	100000.	7.31913
20	0.	0.	100000.	5.72348
21	0.	0.	100000.	20.16337
22	0.	0.	100000.	4.70950
23	0.	0.	100000.	3.97039
total	21000.	-0.	2279000.	-19051.36523

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A. 1 (cont.)

minimun size 8000.

maximum size 100000.

tradeoff parameter 1.00000

step number 2

obj. function 7631.11621

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	834.	1970.	1.42364
2	2664.	699.	1965.	1.35576
3	3080.	1844.	1236.	2.49252
4	2306.	852.	1454.	1.58646
5	2874.	1102.	1772.	1.62234
6	2856.	451.	2405.	1.18744
7	2398.	324.	2074.	1.15636
8	2448.	129.	2319.	1.05552
9	2536.	488.	2048.	1.23843
10	4712.	1744.	2968.	1.58742
11	5230.	2904.	2326.	2.24893
12	4602.	1867.	2735.	1.68249
13	4468.	1009.	3459.	1.29180
14	3668.	392.	3276.	1.11952
15	2992.	67.	2925.	1.02295
16	3144.	99.	3045.	1.03256
17	2882.	47.	2835.	1.01672
18	2844.	209.	2635.	1.07938
19	2100.	14.	2086.	1.00670
20	2090.	25.	2065.	1.01221
21	1032.	1.	1031.	1.00082
22	1644.	220.	1424.	1.15435
23	4342.	677.	3665.	1.18462
total	69716.	16000.	53716.	20396.71289

	facility size	used capacity	unused capacity	multiplier
1	0.	0.	100000.	2.92374
2	0.	0.	100000.	4.04209
3	8000.	-0.	92000.	2.45737
4	0.	0.	100000.	2.70893
5	0.	0.	100000.	3.02650
6	0.	0.	100000.	3.84145
7	0.	0.	100000.	4.94357
8	0.	0.	100000.	6.17032
9	0.	0.	100000.	4.13573
10	0.	0.	100000.	3.10867
11	8000.	-0.	92000.	2.00693
12	0.	0.	100000.	3.13759
13	0.	0.	100000.	3.43661
14	0.	0.	100000.	3.97780
15	0.	0.	100000.	5.08962
16	0.	0.	100000.	5.50924
17	0.	0.	100000.	6.32005
18	0.	0.	100000.	3.55130
19	0.	0.	100000.	8.25965
20	0.	0.	100000.	6.42520
21	0.	0.	100000.	22.98557
22	0.	0.	100000.	4.97800
23	0.	0.	100000.	4.26929
total	16000.	-0.	2284000.	-12765.59668

/cont.....

A. 1 (cont.)

minimun size 9000.

maximum size 100000.

tradeoff parameter 1.00000

step number 1

obj. function 5840.48291

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	949.	1855.	1.51126
2	2664.	800.	1864.	1.42920
3	3080.	1981.	1099.	2.80255
4	2306.	958.	1348.	1.71119
5	2874.	1232.	1642.	1.75040
6	2856.	527.	2329.	1.22602
7	2398.	380.	2018.	1.18845
8	2448.	154.	2294.	1.06693
9	2536.	569.	1967.	1.28915
10	4712.	1967.	2745.	1.71676
11	5230.	3155.	2075.	2.52095
12	4602.	2091.	2511.	1.83273
13	4468.	1166.	3302.	1.35307
14	3668.	462.	3206.	1.14411
15	2992.	81.	2911.	1.02768
16	3144.	119.	3025.	1.03936
17	2882.	57.	2825.	1.02016
18	2844.	249.	2595.	1.09577
19	2100.	17.	2083.	1.00808
20	2090.	30.	2060.	1.01471
21	1032.	1.	1031.	1.00099
22	1644.	259.	1385.	1.18696
23	4342.	797.	3545.	1.22473
total	69716.	18000.	51716.	23676.05469

	facility size	used capacity	unused capacity	multiplier
1	0.	0.	100000.	3.45106
2	0.	0.	100000.	4.78361
3	9000.	-0.	91000.	2.95756
4	0.	0.	100000.	3.23570
5	0.	0.	100000.	3.60679
6	0.	0.	100000.	4.47498
7	0.	0.	100000.	5.73541
8	0.	0.	100000.	7.06475
9	0.	0.	100000.	4.87749
10	0.	0.	100000.	3.73045
11	9000.	-0.	91000.	2.45313
12	0.	0.	100000.	3.79707
13	0.	0.	100000.	4.06598
14	0.	0.	100000.	4.59464
15	0.	0.	100000.	5.78590
16	0.	0.	100000.	6.26420
17	0.	0.	100000.	7.16789
18	0.	0.	100000.	4.07449
19	0.	0.	100000.	9.35195
20	0.	0.	100000.	7.31351
21	0.	0.	100000.	25.89695
22	0.	0.	100000.	5.86678
23	0.	0.	100000.	4.99891
total	18000.	-0.	2282000.	-17835.57227

/cont.....

A. 1 (cont.)

minimun size 10000.

maximum size 100000.

tradeoff parameter 1.00000

step number 2

obj. function 4231.01611

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	216.	2588.	1.08343
2	2664.	155.	2509.	1.06181
3	3080.	838.	2242.	1.37391
4	2306.	561.	1745.	1.32183
5	2874.	221.	2653.	1.08343
6	2856.	70.	2786.	1.02513
7	2398.	38.	2360.	1.01602
8	2448.	16.	2432.	1.00651
9	2536.	293.	2243.	1.13084
10	4712.	1742.	2970.	1.58641
11	5230.	2702.	2528.	2.06850
12	4602.	1865.	2737.	1.68131
13	4468.	452.	4016.	1.11262
14	3668.	58.	3610.	1.01602
15	2992.	9.	2983.	1.00308
16	3144.	32.	3112.	1.01022
17	2882.	6.	2876.	1.00196
18	2844.	39.	2805.	1.01379
19	2100.	1.	2099.	1.00069
20	2090.	3.	2087.	1.00125
21	1032.	0.	1032.	1.00008
22	1644.	110.	1534.	1.07181
23	4342.	573.	3769.	1.15202
total	69716.	10000.	59716.	12393.57520

	facility size	used capacity	unused capacity	multiplier
1	0.	0.	100000.	2.98981
2	0.	0.	100000.	4.18795
3	0.	0.	100000.	2.68482
4	0.	0.	100000.	2.84243
5	0.	0.	100000.	3.02426
6	0.	0.	100000.	4.10539
7	0.	0.	100000.	5.35828
8	0.	0.	100000.	7.10862
9	0.	0.	100000.	4.65881
10	0.	0.	100000.	3.65994
11	10000.	-0.	90000.	2.26201
12	0.	0.	100000.	3.62211
13	0.	0.	100000.	3.69246
14	0.	0.	100000.	4.41924
15	0.	0.	100000.	6.07359
16	0.	0.	100000.	6.59300
17	0.	0.	100000.	7.60526
18	0.	0.	100000.	4.05968
19	0.	0.	100000.	10.02678
20	0.	0.	100000.	7.59793
21	0.	0.	100000.	28.53905
22	0.	0.	100000.	5.54580
23	0.	0.	100000.	5.09291
total	10000.	-0.	2290000.	-8162.55908

Appendix A. 2 Sensitivity Analysis on the Tradeoff Parameter.

minimun size 2000.

maximum size 100000.

tradeoff parameter 0.25000

step number 11

obj. function 5389.10840

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	881.	1923.	1.45849
2	2664.	494.	2170.	1.22766
3	3080.	1390.	1690.	1.82203
4	2306.	992.	1314.	1.75494
5	2874.	1181.	1693.	1.69715
6	2856.	465.	2391.	1.19469
7	2398.	248.	2150.	1.11554
8	2448.	105.	2343.	1.04476
9	2536.	363.	2173.	1.16728
10	4712.	1507.	3205.	1.47006
11	5230.	2269.	2961.	1.76659
12	4602.	1628.	2974.	1.54729
13	4468.	1083.	3385.	1.31977
14	3668.	303.	3365.	1.09014
15	2992.	63.	2929.	1.02166
16	3144.	73.	3071.	1.02372
17	2882.	35.	2847.	1.01223
18	2844.	273.	2571.	1.10629
19	2100.	13.	2087.	1.00609
20	2090.	21.	2069.	1.01010
21	1032.	1.	1031.	1.00057
22	1644.	180.	1464.	1.12316
23	4342.	431.	3911.	1.11033
total	69716.	14000.	55716.	17035.91406

	facility size	used capacity	unused capacity	multiplier
1	2000.	-0.	98000.	2.44062
2	0.	0.	100000.	3.35944
3	2000.	-0.	98000.	2.24765
4	2000.	-0.	98000.	2.32614
5	2000.	-0.	98000.	2.58669
6	0.	0.	100000.	3.31665
7	0.	0.	100000.	4.19545
8	0.	0.	100000.	5.12912
9	0.	0.	100000.	3.43083
10	2000.	-0.	98000.	2.39411
11	2000.	-0.	98000.	1.77857
12	2000.	-0.	98000.	2.40573
13	0.	0.	100000.	2.97034
14	0.	0.	100000.	3.25881
15	0.	0.	100000.	4.20901
16	0.	0.	100000.	4.48446
17	0.	0.	100000.	5.10936
18	0.	0.	100000.	3.11278
19	0.	0.	100000.	6.73397
20	0.	0.	100000.	5.36809
21	0.	0.	100000.	17.64164
22	0.	0.	100000.	4.15188
23	0.	0.	100000.	3.34616
total	14000.	-0.	2286000.	-11646.80566

/cont

A. 2 (cont.)

minimun size 2000.

maximum size 100000.

tradeoff parameter 0.50000

step number 3

obj. function 20247.03125

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1470.	1334.	2.10273
2	2664.	1395.	1269.	2.09870
3	3080.	1987.	1093.	2.81878
4	2306.	1418.	888.	2.59711
5	2874.	1679.	1195.	2.40540
6	2856.	1668.	1188.	2.40374
7	2398.	625.	1773.	1.35266
8	2448.	405.	2043.	1.19823
9	2536.	1271.	1265.	2.00463
10	4712.	2299.	2413.	1.95258
11	5230.	2948.	2282.	2.29223
12	4602.	2102.	2500.	1.84050
13	4468.	2295.	2173.	2.05608
14	3668.	1896.	1772.	2.06978
15	2992.	1371.	1621.	1.84616
16	3144.	665.	2479.	1.26849
17	2882.	325.	2557.	1.12693
18	2844.	1219.	1625.	1.74975
19	2100.	167.	1933.	1.08666
20	2090.	191.	1899.	1.10048
21	1032.	2.	1030.	1.00186
22	1644.	636.	1008.	1.63015
23	4342.	1965.	2377.	1.82667
total	69716.	29999.	39717.	41910.95703

	facility size	used capacity	unused capacity	multiplier
1	2000.	-0.	98000.	1.81303
2	2000.	-0.	98000.	2.49731
3	2000.	-0.	98000.	1.67376
4	2000.	-0.	98000.	1.70466
5	2000.	-0.	98000.	1.94747
6	2000.	-0.	98000.	2.75746
7	0.	0.	100000.	2.82018
8	0.	0.	100000.	3.32155
9	2000.	-0.	98000.	2.53522
10	2000.	-0.	98000.	1.64814
11	2000.	-0.	98000.	1.19904
12	2000.	-0.	98000.	1.55601
13	2000.	-0.	98000.	2.11592
14	2000.	-0.	98000.	2.76571
15	2000.	-0.	98000.	3.18129
16	0.	0.	100000.	3.11107
17	0.	0.	100000.	3.22914
18	2000.	-0.	98000.	2.06333
19	0.	0.	100000.	4.18040
20	0.	0.	100000.	3.35240
21	0.	0.	100000.	9.81403
22	0.	0.	100000.	3.16325
23	2000.	-0.	98000.	2.43633
total	29999.	-1.	2270001.	-21663.92578

/cont.....

A. 2 (cont.)

minimun size 2000.

maximum size 100000.

tradeoff parameter 0.75000

step number 3

obj. function 33709.96484

	total demand	satisfied demand	unsatisfied demand	accessibility
1	2804.	1737.	1067.	2.62797
2	2664.	1692.	972.	2.74143
3	3080.	2207.	873.	3.52797
4	2306.	1526.	780.	2.95779
5	2874.	1826.	1048.	2.74225
6	2856.	1819.	1037.	2.75485
7	2398.	1386.	1012.	2.37049
8	2448.	680.	1768.	1.38450
9	2536.	1564.	972.	2.61026
10	4712.	2713.	1999.	2.35730
11	5230.	3242.	1988.	2.63074
12	4602.	2307.	2295.	2.00523
13	4468.	2493.	1975.	2.26179
14	3668.	2078.	1590.	2.30626
15	2992.	1728.	1264.	2.36738
16	3144.	1895.	1249.	2.51679
17	2882.	1722.	1160.	2.48404
18	2844.	1615.	1229.	2.31400
19	2100.	261.	1839.	1.14176
20	2090.	459.	1631.	1.28150
21	1032.	4.	1028.	1.00346
22	1644.	1052.	592.	2.77545
23	4342.	2154.	2188.	1.98453
total	69716.	38160.	31556.	57828.40625

	facility size	used capacity	unused capacity	multiplier
1	2000.	-0.	98000.	1.53326
2	2000.	-0.	98000.	2.15304
3	2000.	-0.	98000.	1.36080
4	2000.	-0.	98000.	1.30854
5	2000.	-0.	98000.	1.51790
6	2000.	-0.	98000.	2.17303
7	2000.	-0.	98000.	2.51269
8	0.	0.	100000.	2.77735
9	2000.	-0.	98000.	2.16381
10	2000.	-0.	98000.	1.31455
11	2161.	161.	97839.	1.00000
12	2000.	-0.	98000.	1.17213
13	2000.	-0.	98000.	1.59589
14	2000.	-0.	98000.	2.12128
15	2000.	-0.	98000.	2.94759
16	2000.	-0.	98000.	3.22677
17	2000.	-0.	98000.	2.62035
18	2000.	-0.	98000.	1.83514
19	0.	0.	100000.	3.26888
20	0.	0.	100000.	2.90312
21	0.	0.	100000.	7.20037
22	2000.	-0.	98000.	2.72578
23	2000.	-0.	98000.	1.82321
total	38160.	160.	2261841.	-24118.44141

APPENDIX B: RESULTS FOR THE DYNAMIC 5-YEAR HIGH SCHOOL SYSTEM WITH UNCONSTRAINED ADMISSIONS POLICY

Legend for Appendix B

starting input	population reaching age 14 within year 1977
growth rate	in Appendix B1 (geometric input) the rate of constant geometric growth; in Appendix B2 (logistic input) the parameter r in the logistic differential equation $$\dot{y} = yr(1 - y/k)$$
initial size	existing school capacity at year 1977 in each district
recruitment	the new admissions to the first grade, for each district, divided by the total 14 year-old population. (These ratios have been kept constant over time)

Appendix B. 1 Response to the Geometric Input

starting input 19701.
growth rate 0.05000

	initial size	recruitment
1	10432.	0.03796
2	3137.	0.02777
3	3886.	0.04094
4	2598.	0.04088
5	1651.	0.03599
6	1553.	0.02836
7	3454.	0.02515
8	1812.	0.02104
9	1051.	0.02774
10	2240.	0.04090
11	300.	0.05572
12	5179.	0.04280
13	0.	0.03393
14	620.	0.03025
15	0.	0.02638
16	1138.	0.02453
17	1110.	0.02263
18	2184.	0.03522
19	0.	0.
20	0.	0.
21	0.	0.
22	1471.	0.02244
23	192.	0.03246
total	44008.	0.65309

total

	1	2	3	4	5	total
1977	13308.	9858.	8340.	6782.	5721.	44008.
1978	14457.	10429.	8891.	6703.	6118.	46598.
1979	15215.	11312.	9410.	7125.	6064.	49126.
1980	15978.	11926.	10195.	7542.	6430.	52071.
1981	16777.	12527.	10762.	8164.	6807.	55037.
1982	17616.	13154.	11307.	8627.	7362.	58066.
1983	18497.	13811.	11873.	9066.	7785.	61032.
1984	19422.	14502.	12467.	9520.	8183.	64093.
1985	20393.	15227.	13090.	9996.	8594.	67299.
1986	21412.	15988.	13745.	10496.	9023.	70664.
1987	22483.	16788.	14432.	11020.	9475.	74198.
1988	23607.	17627.	15154.	11571.	9948.	77907.
1989	24787.	18508.	15911.	12150.	10446.	81803.
1990	26027.	19434.	16707.	12757.	10968.	85893.
1991	27328.	20406.	17542.	13395.	11516.	90188.
1992	28695.	21426.	18419.	14065.	12092.	94697.
1993	30129.	22497.	19340.	14768.	12697.	99432.
1994	31636.	23622.	20307.	15507.	13332.	104403.
1995	33217.	24803.	21323.	16282.	13998.	109623.
1996	34878.	26043.	22389.	17096.	14698.	115105.
1997	36622.	27345.	23508.	17951.	15433.	120860.
1998	38453.	28713.	24684.	18848.	16205.	126903.
1999	40376.	30148.	25918.	19791.	17015.	133248.
2000	42395.	31656.	27214.	20780.	17866.	139910.

/cont....

B. 1 (cont.)

center

	1	2	3	4	5	total
1977	8625.	6389.	5405.	4395.	3708.	28523.
1978	5953.	6759.	5763.	4344.	3965.	26784.
1979	6030.	4810.	6099.	4618.	3930.	25486.
1980	6315.	4746.	4445.	4888.	4167.	24563.
1981	6630.	4953.	4304.	3623.	4412.	23921.
1982	6961.	5198.	4473.	3464.	3318.	23415.
1983	7309.	5458.	4692.	3589.	3139.	24188.
1984	7675.	5731.	4927.	3763.	3242.	25336.
1985	8059.	6017.	5173.	3950.	3397.	26596.
1986	8462.	6318.	5432.	4148.	3566.	27925.
1987	8885.	6634.	5703.	4355.	3744.	29321.
1988	9329.	6966.	5988.	4573.	3931.	30787.
1989	9795.	7314.	6288.	4801.	4128.	32326.
1990	10285.	7680.	6602.	5041.	4334.	33942.
1991	10799.	8064.	6932.	5293.	4551.	35640.
1992	11339.	8467.	7279.	5558.	4778.	37422.
1993	11906.	8890.	7643.	5836.	5017.	39293.
1994	12502.	9335.	8025.	6128.	5268.	41257.
1995	13127.	9801.	8426.	6434.	5532.	43320.
1996	13783.	10292.	8847.	6756.	5808.	45486.
1997	14472.	10806.	9290.	7094.	6099.	47760.
1998	15196.	11346.	9754.	7448.	6404.	50148.
1999	15955.	11914.	10242.	7821.	6724.	52656.
2000	16753.	12509.	10754.	8212.	7060.	55289.

south west

	1	2	3	4	5	total
1977	3155.	2337.	1977.	1608.	1356.	10433.
1978	5520.	2472.	2108.	1589.	1450.	13140.
1979	5953.	4226.	2231.	1689.	1438.	15537.
1980	6262.	4654.	3741.	1788.	1524.	17969.
1981	6576.	4908.	4187.	2958.	1614.	20243.
1982	6905.	5156.	4429.	3348.	2636.	22473.
1983	7250.	5414.	4654.	3550.	3013.	23880.
1984	7613.	5684.	4887.	3731.	3203.	25118.
1985	7993.	5969.	5131.	3918.	3368.	26379.
1986	8393.	6267.	5388.	4114.	3537.	27698.
1987	8813.	6580.	5657.	4320.	3714.	29083.
1988	9253.	6909.	5940.	4536.	3899.	30537.
1989	9716.	7255.	6237.	4762.	4094.	32064.
1990	10202.	7617.	6549.	5001.	4299.	33667.
1991	10712.	7998.	6876.	5251.	4514.	35351.
1992	11247.	8398.	7220.	5513.	4740.	37118.
1993	11810.	8818.	7581.	5789.	4977.	38974.
1994	12400.	9259.	7960.	6078.	5226.	40923.
1995	13020.	9722.	8358.	6382.	5487.	42969.
1996	13671.	10208.	8776.	6701.	5761.	45118.
1997	14355.	10719.	9215.	7036.	6049.	47373.
1998	15073.	11254.	9675.	7388.	6352.	49742.
1999	15826.	11817.	10159.	7757.	6669.	52229.
2000	16618.	12408.	10667.	8145.	7003.	54841.

/cont....

B. 1 (cont.)

north west

	1	2	3	4	5	total
1977	1528.	1132.	957.	779.	657.	5052.
1978	2984.	1197.	1021.	769.	702.	6674.
1979	3232.	2276.	1080.	818.	696.	8103.
1980	3401.	2526.	2008.	866.	738.	9539.
1981	3571.	2665.	2271.	1584.	781.	10873.
1982	3750.	2800.	2405.	1815.	1409.	12178.
1983	3937.	2940.	2527.	1927.	1633.	12964.
1984	4134.	3087.	2654.	2026.	1739.	13640.
1985	4341.	3241.	2786.	2128.	1829.	14325.
1986	4558.	3403.	2926.	2234.	1921.	15041.
1987	4786.	3573.	3072.	2346.	2017.	15794.
1988	5025.	3752.	3226.	2463.	2118.	16583.
1989	5276.	3940.	3387.	2586.	2223.	17412.
1990	5540.	4137.	3556.	2716.	2335.	18283.
1991	5817.	4343.	3734.	2851.	2451.	19197.
1992	6108.	4561.	3921.	2994.	2574.	20157.
1993	6413.	4789.	4117.	3144.	2703.	21165.
1994	6734.	5028.	4323.	3301.	2838.	22223.
1995	7071.	5280.	4539.	3466.	2980.	23334.
1996	7424.	5544.	4766.	3639.	3129.	24501.
1997	7795.	5821.	5004.	3821.	3285.	25726.
1998	8185.	6112.	5254.	4012.	3449.	27012.
1999	8594.	6417.	5517.	4213.	3622.	28363.
2000	9024.	6738.	5793.	4423.	3803.	29781.

location 1

	1	2	3	4	5	total
1977	3155.	2337.	1977.	1608.	1356.	10432.
1978	1010.	2472.	2108.	1589.	1450.	8629.
1979	896.	897.	2231.	1689.	1437.	7151.
1980	929.	717.	887.	1788.	1524.	5846.
1981	975.	730.	661.	755.	1613.	4735.
1982	1024.	765.	661.	540.	717.	3706.
1983	1075.	803.	690.	531.	497.	3596.
1984	1129.	843.	725.	554.	481.	3731.
1985	1185.	885.	761.	581.	500.	3912.
1986	1244.	929.	799.	610.	524.	4107.
1987	1307.	976.	839.	640.	551.	4312.
1988	1372.	1024.	881.	673.	578.	4528.
1989	1441.	1076.	925.	706.	607.	4754.
1990	1513.	1129.	971.	741.	637.	4992.
1991	1588.	1186.	1020.	779.	669.	5242.
1992	1668.	1245.	1070.	817.	703.	5504.
1993	1751.	1307.	1124.	858.	738.	5779.
1994	1839.	1373.	1180.	901.	775.	6068.
1995	1931.	1442.	1239.	946.	814.	6371.
1996	2027.	1514.	1301.	994.	854.	6690.
1997	2128.	1589.	1366.	1043.	897.	7024.
1998	2235.	1669.	1435.	1095.	942.	7375.
1999	2347.	1752.	1506.	1150.	989.	7744.
2000	2464.	1840.	1582.	1208.	1038.	8131.

/cont.....

B. 1 (cont.)

location 2

	1	2	3	4	5	total
1977	949.	703.	594.	483.	408.	3137.
1978	642.	743.	634.	478.	436.	2933.
1979	649.	520.	671.	508.	432.	2779.
1980	680.	511.	481.	538.	458.	2667.
1981	713.	533.	463.	392.	485.	2587.
1982	749.	559.	481.	373.	360.	2522.
1983	786.	587.	505.	386.	338.	2603.
1984	826.	617.	530.	405.	349.	2726.
1985	867.	647.	557.	425.	365.	2862.
1986	910.	680.	584.	446.	384.	3005.
1987	956.	714.	614.	469.	403.	3155.
1988	1004.	750.	644.	492.	423.	3313.
1989	1054.	787.	677.	517.	444.	3478.
1990	1107.	826.	710.	542.	466.	3652.
1991	1162.	868.	746.	570.	490.	3835.
1992	1220.	911.	783.	598.	514.	4026.
1993	1281.	957.	822.	628.	540.	4228.
1994	1345.	1004.	863.	659.	567.	4439.
1995	1412.	1055.	907.	692.	595.	4661.
1996	1483.	1107.	952.	727.	625.	4894.
1997	1557.	1163.	1000.	763.	656.	5139.
1998	1635.	1221.	1050.	801.	689.	5396.
1999	1717.	1282.	1102.	842.	723.	5666.
2000	1803.	1346.	1157.	884.	760.	5949.

location 3

	1	2	3	4	5	total
1977	1175.	870.	736.	599.	505.	3886.
1978	931.	921.	785.	592.	540.	3769.
1979	956.	744.	831.	629.	535.	3695.
1980	1002.	751.	681.	666.	568.	3668.
1981	1052.	786.	680.	552.	601.	3670.
1982	1104.	825.	709.	546.	503.	3688.
1983	1160.	866.	744.	569.	494.	3833.
1984	1218.	909.	782.	597.	514.	4019.
1985	1278.	955.	821.	627.	539.	4219.
1986	1342.	1002.	862.	658.	566.	4430.
1987	1409.	1052.	905.	691.	594.	4651.
1988	1480.	1105.	950.	725.	624.	4884.
1989	1554.	1160.	997.	762.	655.	5128.
1990	1632.	1218.	1047.	800.	688.	5385.
1991	1713.	1279.	1100.	840.	722.	5654.
1992	1799.	1343.	1155.	882.	758.	5936.
1993	1889.	1410.	1212.	926.	796.	6233.
1994	1983.	1481.	1273.	972.	836.	6545.
1995	2082.	1555.	1337.	1021.	878.	6872.
1996	2186.	1633.	1404.	1072.	921.	7216.
1997	2296.	1714.	1474.	1125.	967.	7577.
1998	2411.	1800.	1547.	1182.	1016.	7955.
1999	2531.	1890.	1625.	1241.	1067.	8353.
2000	2658.	1984.	1706.	1303.	1120.	8771.

/cont....

B. 1 (cont.)

location 4

	1	2	3	4	5	total
1977	786.	582.	492.	400.	338.	2598.
1978	902.	616.	525.	396.	361.	2799.
1979	952.	703.	556.	421.	358.	2989.
1980	1000.	746.	632.	445.	380.	3203.
1981	1050.	784.	673.	505.	402.	3414.
1982	1103.	823.	708.	539.	455.	3628.
1983	1158.	864.	743.	567.	486.	3819.
1984	1216.	908.	780.	596.	512.	4012.
1985	1276.	953.	819.	626.	538.	4212.
1986	1340.	1001.	860.	657.	565.	4423.
1987	1407.	1051.	903.	690.	593.	4644.
1988	1478.	1103.	948.	724.	623.	4876.
1989	1551.	1158.	996.	760.	654.	5120.
1990	1629.	1216.	1046.	798.	686.	5376.
1991	1710.	1277.	1098.	838.	721.	5645.
1992	1796.	1341.	1153.	880.	757.	5927.
1993	1886.	1408.	1211.	924.	795.	6224.
1994	1980.	1479.	1271.	971.	834.	6535.
1995	2079.	1552.	1335.	1019.	876.	6861.
1996	2183.	1630.	1401.	1070.	920.	7205.
1997	2292.	1712.	1471.	1124.	966.	7565.
1998	2407.	1797.	1545.	1180.	1014.	7943.
1999	2527.	1887.	1622.	1239.	1065.	8340.
2000	2654.	1981.	1703.	1301.	1118.	8757.

location 5

	1	2	3	4	5	total
1977	499.	370.	313.	254.	215.	1651.
1978	780.	391.	334.	251.	230.	1986.
1979	837.	600.	353.	267.	227.	2285.
1980	880.	655.	533.	283.	241.	2592.
1981	924.	690.	589.	422.	255.	2882.
1982	971.	725.	623.	472.	377.	3167.
1983	1019.	761.	654.	499.	425.	3358.
1984	1070.	799.	687.	525.	450.	3531.
1985	1124.	839.	721.	551.	473.	3708.
1986	1180.	881.	757.	578.	497.	3894.
1987	1239.	925.	795.	607.	522.	4089.
1988	1301.	971.	835.	638.	548.	4293.
1989	1366.	1020.	877.	670.	576.	4508.
1990	1434.	1071.	921.	703.	604.	4733.
1991	1506.	1124.	967.	738.	635.	4970.
1992	1581.	1181.	1015.	775.	666.	5218.
1993	1660.	1240.	1066.	814.	700.	5479.
1994	1743.	1302.	1119.	854.	735.	5753.
1995	1830.	1367.	1175.	897.	771.	6041.
1996	1922.	1435.	1234.	942.	810.	6343.
1997	2018.	1507.	1295.	989.	850.	6660.
1998	2119.	1582.	1360.	1039.	893.	6993.
1999	2225.	1661.	1428.	1091.	938.	7343.
2000	2336.	1744.	1500.	1145.	984.	7710.

/cont....

B. 1 (cont.)

location 6

	1	2	3	4	5	total
1977	470.	348.	294.	239.	202.	1553.
1978	620.	368.	314.	237.	216.	1754.
1979	660.	480.	332.	251.	214.	1938.
1980	694.	517.	429.	266.	227.	2133.
1981	729.	544.	466.	342.	240.	2320.
1982	765.	571.	491.	373.	307.	2506.
1983	803.	600.	516.	393.	336.	2648.
1984	843.	630.	541.	413.	355.	2783.
1985	886.	661.	568.	434.	373.	2923.
1986	930.	694.	597.	456.	392.	3069.
1987	976.	729.	627.	479.	411.	3222.
1988	1025.	765.	658.	502.	432.	3383.
1989	1076.	804.	691.	528.	454.	3552.
1990	1130.	844.	726.	554.	476.	3730.
1991	1187.	886.	762.	582.	500.	3916.
1992	1246.	930.	800.	611.	525.	4112.
1993	1308.	977.	840.	641.	551.	4318.
1994	1374.	1026.	882.	673.	579.	4534.
1995	1443.	1077.	926.	707.	608.	4761.
1996	1515.	1131.	972.	742.	638.	4999.
1997	1590.	1188.	1021.	780.	670.	5248.
1998	1670.	1247.	1072.	819.	704.	5511.
1999	1753.	1309.	1126.	859.	739.	5786.
2000	1841.	1375.	1182.	902.	776.	6076.

location 7

	1	2	3	4	5	total
1977	1044.	774.	655.	532.	449.	3454.
1978	595.	819.	698.	526.	480.	3117.
1979	589.	489.	739.	559.	476.	2851.
1980	615.	464.	458.	592.	505.	2635.
1981	646.	483.	422.	377.	534.	2462.
1982	678.	507.	436.	341.	348.	2310.
1983	712.	532.	457.	350.	310.	2361.
1984	748.	558.	480.	367.	316.	2469.
1985	785.	586.	504.	385.	331.	2591.
1986	824.	616.	529.	404.	347.	2721.
1987	866.	646.	556.	424.	365.	2857.
1988	909.	679.	583.	446.	383.	3000.
1989	954.	713.	613.	468.	402.	3150.
1990	1002.	748.	643.	491.	422.	3307.
1991	1052.	786.	675.	516.	443.	3473.
1992	1105.	825.	709.	542.	466.	3646.
1993	1160.	866.	745.	569.	489.	3829.
1994	1218.	910.	782.	597.	513.	4020.
1995	1279.	955.	821.	627.	539.	4221.
1996	1343.	1003.	862.	658.	566.	4432.
1997	1410.	1053.	905.	691.	594.	4654.
1998	1481.	1106.	950.	726.	624.	4886.
1999	1555.	1161.	998.	762.	655.	5131.
2000	1632.	1219.	1048.	800.	688.	5387.

/cont.....

B. 1 (cont.)

location 8

	1	2	3	4	5	total
1977	548.	406.	343.	279.	236.	1812.
1978	474.	429.	366.	276.	252.	1798.
1979	491.	376.	387.	293.	250.	1798.
1980	515.	385.	343.	311.	265.	1819.
1981	541.	404.	349.	277.	280.	1850.
1982	568.	424.	364.	280.	252.	1887.
1983	596.	445.	383.	292.	253.	1969.
1984	626.	467.	402.	307.	264.	2065.
1985	657.	491.	422.	322.	277.	2168.
1986	690.	515.	443.	338.	291.	2277.
1987	724.	541.	465.	355.	305.	2391.
1988	761.	568.	488.	373.	321.	2510.
1989	799.	596.	513.	391.	337.	2636.
1990	839.	626.	538.	411.	353.	2767.
1991	880.	657.	565.	432.	371.	2906.
1992	925.	690.	593.	453.	390.	3051.
1993	971.	725.	623.	476.	409.	3204.
1994	1019.	761.	654.	500.	430.	3364.
1995	1070.	799.	687.	525.	451.	3532.
1996	1124.	839.	721.	551.	474.	3709.
1997	1180.	881.	757.	578.	497.	3894.
1998	1239.	925.	795.	607.	522.	4089.
1999	1301.	971.	835.	638.	548.	4293.
2000	1366.	1020.	877.	670.	576.	4508.

location 9

	1	2	3	4	5	total
1977	318.	235.	199.	162.	137.	1051.
1978	596.	249.	212.	160.	146.	1364.
1979	645.	456.	225.	170.	145.	1640.
1980	679.	504.	402.	180.	154.	1919.
1981	713.	532.	453.	318.	163.	2178.
1982	748.	559.	480.	362.	283.	2432.
1983	786.	587.	504.	385.	326.	2587.
1984	825.	616.	530.	404.	347.	2722.
1985	866.	647.	556.	425.	365.	2859.
1986	909.	679.	584.	446.	383.	3001.
1987	955.	713.	613.	468.	402.	3152.
1988	1003.	749.	644.	491.	423.	3309.
1989	1053.	786.	676.	516.	444.	3475.
1990	1106.	825.	710.	542.	466.	3648.
1991	1161.	867.	745.	569.	489.	3831.
1992	1219.	910.	782.	597.	514.	4022.
1993	1280.	956.	821.	627.	539.	4223.
1994	1344.	1003.	863.	659.	566.	4435.
1995	1411.	1054.	906.	692.	595.	4656.
1996	1481.	1106.	951.	726.	624.	4889.
1997	1556.	1162.	999.	762.	656.	5134.
1998	1633.	1220.	1048.	801.	688.	5390.
1999	1715.	1281.	1101.	841.	723.	5660.
2000	1801.	1345.	1156.	883.	759.	5943.

/cont....

B. 1 (cont.)

location 10

	1	2	3	4	5	total
1977	677.	502.	424.	345.	291.	2240.
1978	894.	531.	453.	341.	311.	2530.
1979	952.	693.	479.	363.	309.	2795.
1980	1001.	745.	619.	384.	327	3076.
1981	1051.	784.	672.	493.	346.	3346.
1982	1103.	824.	708.	538.	442.	3614.
1983	1158.	865.	743.	567.	485.	3819.
1984	1216.	908.	781.	596.	512.	4013.
1985	1277.	954.	820.	626.	538.	4214.
1986	1341.	1001.	861.	657.	565.	4425.
1987	1408.	1051.	904.	690.	593.	4646.
1988	1478.	1104.	949.	725.	623.	4879.
1989	1552.	1159.	996.	761.	654.	5123.
1990	1630.	1217.	1046.	799.	687.	5379.
1991	1711.	1278.	1099.	839.	721.	5648.
1992	1797.	1342.	1153.	881.	757.	5930.
1993	1887.	1409.	1211.	925.	795.	6227.
1994	1981.	1479.	1272.	971.	835.	6538.
1995	2080.	1553.	1335.	1020.	877.	6865.
1996	2184.	1631.	1402.	1071.	920.	7208.
1997	2293.	1712.	1472.	1124.	966.	7568.
1998	2408.	1798.	1546.	1180.	1015.	7947.
1999	2528.	1888.	1623.	1239.	1066.	8344.
2000	2655.	1982.	1704.	1301.	1119.	8761.

location 11

	1	2	3	4	5	total
1977	91.	67.	57.	46.	39.	300.
1978	1159.	71.	61.	46.	42.	1378.
1979	1293.	860.	64.	49.	41.	2307.
1980	1363.	1007.	740.	51.	44.	3206.
1981	1431.	1068.	902.	574.	46.	4022.
1982	1503.	1122.	963.	719.	501.	4809.
1983	1578.	1178.	1013.	772.	645.	5186.
1984	1657.	1237.	1064.	812.	696.	5466.
1985	1740.	1299.	1117.	853.	733.	5742.
1986	1827.	1364.	1173.	895.	770.	6029.
1987	1918.	1432.	1231.	940.	808.	6331.
1988	2014.	1504.	1293.	987.	849.	6647.
1989	2115.	1579.	1358.	1037.	891.	6979.
1990	2221.	1658.	1425.	1088.	936.	7328.
1991	2332.	1741.	1497.	1143.	983.	7695.
1992	2448.	1828.	1572.	1200.	1032.	8080.
1993	2571.	1919.	1650.	1260.	1083.	8484.
1994	2699.	2015.	1733.	1323.	1137.	8908.
1995	2834.	2116.	1819.	1389.	1194.	9353.
1996	2976.	2222.	1910.	1459.	1254.	9821.
1997	3125.	2333.	2006.	1532.	1317.	10312.
1998	3281.	2450.	2106.	1608.	1383.	10827.
1999	3445.	2572.	2211.	1689.	1452.	11369.
2000	3617.	2701.	2322.	1773.	1524.	11937.

/cont.....

B. 1 (cont.)

location 12

	1	2	3	4	5	total
1977	1566.	1160.	981.	798.	673.	5179.
1978	997.	1227.	1046.	789.	720.	4779.
1979	1001.	811.	1107.	839.	714.	4471.
1980	1047.	788.	754.	888.	757.	4234.
1981	1100.	822.	716.	617.	801.	4055.
1982	1154.	862.	742.	577.	567.	3902.
1983	1212.	905.	778.	595.	523.	4014.
1984	1273.	950.	817.	624.	538.	4202.
1985	1336.	998.	858.	655.	563.	4411.
1986	1403.	1048.	901.	688.	591.	4631.
1987	1473.	1100.	946.	722.	621.	4863.
1988	1547.	1155.	993.	758.	652.	5106.
1989	1624.	1213.	1043.	796.	685.	5361.
1990	1706.	1274.	1095.	836.	719.	5629.
1991	1791.	1337.	1150.	878.	755.	5911.
1992	1881.	1404.	1207.	922.	792.	6206.
1993	1975.	1474.	1267.	968.	832.	6516.
1994	2073.	1548.	1331.	1016.	874.	6842.
1995	2177.	1625.	1397.	1067.	917.	7184.
1996	2286.	1707.	1467.	1120.	963.	7543.
1997	2400.	1792.	1541.	1176.	1011.	7921.
1998	2520.	1882.	1618.	1235.	1062.	8317.
1999	2646.	1976.	1699.	1297.	1115.	8732.
2000	2778.	2075.	1783.	1362.	1171.	9169.

location 13

	1	2	3	4	5	total
1977	0.	0.	0.	0.	0.	0.
1978	702.	0.	0.	0.	0.	702.
1979	787.	518.	0.	0.	0.	1305.
1980	830.	613.	444.	0.	0.	1887.
1981	872.	650.	549.	343.	0.	2414.
1982	915.	683.	586.	437.	299.	2921.
1983	961.	718.	617.	470.	392.	3157.
1984	1009.	754.	648.	495.	424.	3329.
1985	1060.	791.	680.	519.	446.	3497.
1986	1113.	831.	714.	545.	469.	3672.
1987	1168.	872.	750.	573.	492.	3855.
1988	1227.	916.	787.	601.	517.	4048.
1989	1288.	962.	827.	631.	543.	4250.
1990	1352.	1010.	868.	663.	570.	4463.
1991	1420.	1060.	911.	696.	598.	4686.
1992	1491.	1113.	957.	731.	628.	4920.
1993	1565.	1169.	1005.	767.	660.	5166.
1994	1644.	1227.	1055.	806.	693.	5425.
1995	1726.	1289.	1108.	846.	727.	5696.
1996	1812.	1353.	1163.	888.	764.	5981.
1997	1903.	1421.	1221.	933.	802.	6280.
1998	1998.	1492.	1283.	979.	842.	6594.
1999	2098.	1566.	1347.	1028.	884.	6923.
2000	2203.	1645.	1414.	1080.	928.	7270.

/cont....

B. 1 (cont.)

location 14

	1	2	3	4	5	total
1977	187.	139.	117.	96.	81.	620.
1978	639.	147.	125.	94.	86.	1092.
1979	703.	481.	133.	100.	85.	1502.
1980	740.	548.	419.	106.	91.	1904.
1981	777.	580.	492.	328.	96.	2273.
1982	816.	609.	523.	393.	289.	2630.
1983	857.	640.	550.	419.	353.	2818.
1984	900.	672.	578.	441.	378.	2968.
1985	945.	705.	606.	463.	398.	3117.
1986	992.	741.	637.	486.	418.	3273.
1987	1041.	778.	669.	511.	439.	3437.
1988	1094.	817.	702.	536.	461.	3609.
1989	1148.	857.	737.	563.	484.	3789.
1990	1206.	900.	774.	591.	508.	3979.
1991	1266.	945.	813.	621.	533.	4178.
1992	1329.	993.	853.	652.	560.	4387.
1993	1396.	1042.	896.	684.	588.	4606.
1994	1465.	1094.	941.	718.	618.	4836.
1995	1539.	1149.	988.	754.	648.	5078.
1996	1616.	1206.	1037.	792.	681.	5332.
1997	1696.	1267.	1089.	832.	715.	5599.
1998	1781.	1330.	1143.	873.	751.	5879.
1999	1870.	1397.	1201.	917.	788.	6173.
2000	1964.	1466.	1261.	963.	828.	6481.

location 15

	1	2	3	4	5	total
1977	0.	0.	0.	0.	0.	0.
1978	546.	0.	0.	0.	0.	546.
1979	612.	403.	0.	0.	0.	1014.
1980	645.	476.	345.	0.	0.	1467.
1981	678.	505.	427.	266.	0.	1876.
1982	711.	531.	456.	340.	232.	2270.
1983	747.	558.	479.	365.	304.	2454.
1984	784.	586.	503.	384.	329.	2587.
1985	824.	615.	529.	404.	347.	2718.
1986	865.	646.	555.	424.	364.	2854.
1987	908.	678.	583.	445.	383.	2997.
1988	953.	712.	612.	467.	402.	3146.
1989	1001.	747.	643.	491.	422.	3304.
1990	1051.	785.	675.	515.	443.	3469.
1991	1104.	824.	708.	541.	465.	3642.
1992	1159.	865.	744.	568.	488.	3824.
1993	1217.	909.	781.	596.	513.	4016.
1994	1278.	954.	820.	626.	538.	4216.
1995	1342.	1002.	861.	658.	565.	4427.
1996	1409.	1052.	904.	690.	594.	4649.
1997	1479.	1104.	949.	725.	623.	4881.
1998	1553.	1160.	997.	761.	654.	5125.
1999	1631.	1218.	1047.	799.	687.	5381.
2000	1712.	1278.	1099.	839.	722.	5651.

/cont.....

B. 1 (cont.)

location 16

	1	2	3	4	5	total
1977	344.	255.	216.	175.	148.	1138.
1978	532.	270.	230.	173.	158.	1363.
1979	571.	409.	243.	184.	157.	1564.
1980	600.	446.	364.	195.	166.	1771.
1981	630.	470.	402.	288.	176.	1967.
1982	662.	494.	424.	321.	258.	2159.
1983	695.	519.	446.	340.	290.	2289.
1984	729.	545.	468.	358.	307.	2407.
1985	766.	572.	492.	375.	323.	2528.
1986	804.	601.	516.	394.	339.	2654.
1987	844.	631.	542.	414.	356.	2787.
1988	887.	662.	569.	435.	374.	2926.
1989	931.	695.	598.	456.	392.	3072.
1990	978.	730.	628.	479.	412.	3226.
1991	1026.	766.	659.	503.	433.	3387.
1992	1078.	805.	692.	528.	454.	3557.
1993	1132.	845.	726.	555.	477.	3735.
1994	1188.	887.	763.	582.	501.	3921.
1995	1248.	932.	801.	612.	526.	4117.
1996	1310.	978.	841.	642.	552.	4323.
1997	1376.	1027.	883.	674.	580.	4539.
1998	1444.	1078.	927.	708.	609.	4766.
1999	1517.	1132.	973.	743.	639.	5005.
2000	1592.	1189.	1022.	781.	671.	5255.

location 17

	1	2	3	4	5	total
1977	336.	249.	210.	171.	144.	1110.
1978	492.	263.	224.	169.	154.	1303.
1979	527.	379.	237.	180.	153.	1476.
1980	554.	412.	338.	190.	162.	1656.
1981	581.	434.	371.	268.	172.	1826.
1982	610.	456.	392.	297.	240.	1995.
1983	641.	479.	411.	314.	267.	2112.
1984	673.	503.	432.	330.	283.	2221.
1985	707.	528.	454.	346.	298.	2332.
1986	742.	554.	476.	364.	313.	2449.
1987	779.	582.	500.	382.	328.	2571.
1988	818.	611.	525.	401.	345.	2700.
1989	859.	641.	551.	421.	362.	2835.
1990	902.	673.	579.	442.	380.	2977.
1991	947.	707.	608.	464.	399.	3125.
1992	994.	743.	638.	487.	419.	3282.
1993	1044.	780.	670.	512.	440.	3446.
1994	1096.	819.	704.	537.	462.	3618.
1995	1151.	860.	739.	564.	485.	3799.
1996	1209.	903.	776.	592.	509.	3989.
1997	1269.	948.	815.	622.	535.	4188.
1998	1333.	995.	855.	653.	562.	4398.
1999	1399.	1045.	898.	686.	590.	4618.
2000	1469.	1097.	943.	720.	619.	4849.

/cont.....

B. 1 (cont.)

location 18

	1	2	3	4	5	total
1977	660.	489.	414.	337.	284.	2184.
1978	776.	518.	441.	333.	304.	2371.
1979	820.	604.	467.	354.	301.	2546.
1980	862.	643.	543.	374.	319.	2741.
1981	905.	676.	580.	434.	338.	2931.
1982	950.	709.	610.	464.	390.	3124.
1983	998.	745.	640.	489.	419.	3291.
1984	1047.	782.	672.	513.	441.	3457.
1985	1100.	821.	706.	539.	463.	3630.
1986	1155.	862.	741.	566.	487.	3811.
1987	1213.	905.	778.	594.	511.	4002.
1988	1273.	951.	817.	624.	537.	4202.
1989	1337.	998.	858.	655.	563.	4412.
1990	1404.	1048.	901.	688.	592.	4633.
1991	1474.	1101.	946.	722.	621.	4864.
1992	1548.	1156.	993.	759.	652.	5107.
1993	1625.	1213.	1043.	797.	685.	5363.
1994	1706.	1274.	1095.	836.	719.	5631.
1995	1792.	1338.	1150.	878.	755.	5912.
1996	1881.	1405.	1208.	922.	793.	6208.
1997	1975.	1475.	1268.	968.	832.	6518.
1998	2074.	1549.	1331.	1017.	874.	6844.
1999	2178.	1626.	1398.	1067.	918.	7187.
2000	2287.	1707.	1468.	1121.	964.	7546.

location 22

	1	2	3	4	5	total
1977	445.	330.	279.	227.	191.	1471.
1978	496.	349.	297.	224.	204.	1570.
1979	523.	387.	315.	238.	203.	1665.
1980	549.	410.	349.	252.	215.	1774.
1981	576.	430.	370.	279.	228.	1883.
1982	605.	452.	388.	296.	251.	1993.
1983	636.	475.	408.	311.	267.	2097.
1984	667.	498.	428.	327.	281.	2202.
1985	701.	523.	450.	343.	295.	2312.
1986	736.	549.	472.	361.	310.	2428.
1987	772.	577.	496.	379.	326.	2549.
1988	811.	606.	521.	398.	342.	2677.
1989	852.	636.	547.	417.	359.	2811.
1990	894.	668.	574.	438.	377.	2951.
1991	939.	701.	603.	460.	396.	3099.
1992	986.	736.	633.	483.	415.	3254.
1993	1035.	773.	664.	507.	436.	3416.
1994	1087.	812.	698.	533.	458.	3587.
1995	1141.	852.	733.	559.	481.	3766.
1996	1198.	895.	769.	587.	505.	3955.
1997	1258.	940.	808.	617.	530.	4152.
1998	1321.	986.	848.	648.	557.	4360.
1999	1387.	1036.	890.	680.	585.	4578.
2000	1457.	1088.	935.	714.	614.	4807.

/cont....

B. 1 (cont.)

location 23

	1	2	3	4	5	total
1977	58.	43.	36.	30.	25.	192.
1978	676.	45.	39.	29.	27.	816.
1979	753.	501.	41.	31.	26.	1353.
1980	794.	587.	432.	33.	28.	1874.
1981	834.	622.	526.	335.	30.	2346.
1982	876.	654.	561.	419.	293.	2802.
1983	919.	686.	590.	449.	376.	3021.
1984	965.	721.	620.	473.	405.	3184.
1985	1014.	757.	651.	497.	427.	3345.
1986	1064.	795.	683.	522.	448.	3512.
1987	1117.	834.	717.	548.	471.	3688.
1988	1173.	876.	753.	575.	494.	3872.
1989	1232.	920.	791.	604.	519.	4066.
1990	1294.	966.	830.	634.	545.	4269.
1991	1358.	1014.	872.	666.	572.	4482.
1992	1426.	1065.	915.	699.	601.	4706.
1993	1497.	1118.	961.	734.	631.	4942.
1994	1572.	1174.	1009.	771.	663.	5189.
1995	1651.	1233.	1060.	809.	696.	5448.
1996	1733.	1294.	1113.	850.	730.	5721.
1997	1820.	1359.	1168.	892.	767.	6007.
1998	1911.	1427.	1227.	937.	805.	6307.
1999	2007.	1498.	1288.	984.	846.	6622.
2000	2107.	1573.	1353.	1033.	888.	6954.

Appendix B. 2 Response to the Logistic Input

starting input 19701.
growth rate 0.13000

	initial size	recruitment
1	10432.	0.03796
2	3137.	0.02777
3	3886.	0.04094
4	2598.	0.04088
5	1651.	0.03599
6	1553.	0.02836
7	3454.	0.02515
8	1812.	0.02104
9	1051.	0.02774
10	2240.	0.04090
11	300.	0.05572
12	5179.	0.04280
13	0.	0.03393
14	620.	0.03025
15	0.	0.02638
16	1138.	0.02453
17	1110.	0.02263
18	2184.	0.03522
19	0.	0.
20	0.	0.
21	0.	0.
22	1471.	0.02244
23	192.	0.03246
total	44008.	0.65309

total

	1	2	3	4	5	total
1977	13308.	9858.	8340.	6782.	5721.	44008.
1978	14389.	10429.	8891.	6703.	6118.	46530.
1979	15016.	11262.	9410.	7125.	6064.	48877.
1980	15582.	11776.	10152.	7542.	6430.	51482.
1981	16113.	12225.	10631.	8131.	6807.	53908.
1982	16609.	12645.	11042.	8525.	7333.	56154.
1983	17070.	13037.	11423.	8857.	7695.	58082.
1984	17495.	13401.	11779.	9164.	7998.	59838.
1985	17885.	13737.	12110.	9451.	8277.	61461.
1986	18241.	14046.	12416.	9718.	8537.	62957.
1987	18563.	14327.	12697.	9964.	8779.	64331.
1988	18855.	14583.	12953.	10190.	9003.	65584.
1989	19117.	14814.	13185.	10397.	9208.	66722.
1990	19353.	15022.	13396.	10585.	9396.	67750.
1991	19563.	15208.	13585.	10754.	9566.	68676.
1992	19750.	15375.	13755.	10907.	9720.	69506.
1993	19916.	15523.	13906.	11044.	9859.	70248.
1994	20063.	15655.	14042.	11167.	9983.	70909.
1995	20193.	15771.	14162.	11276.	10094.	71496.
1996	20308.	15874.	14268.	11373.	10194.	72016.
1997	20409.	15965.	14362.	11458.	10282.	72476.
1998	20498.	16046.	14445.	11534.	10360.	72883.
1999	20576.	16116.	14518.	11601.	10429.	73240.
2000	20644.	16178.	14583.	11661.	10490.	73555.

/cont.....

B. 2 (cont.)

center

	1	2	3	4	5	total
1977	8625.	6389.	5405.	4395.	3708.	28523.
1978	5926.	6759.	5763.	4344.	3965.	26757.
1979	5951.	4790.	6099.	4618.	3930.	25388.
1980	6159.	4687.	4428.	4888.	4167.	24330.
1981	6367.	4834.	4252.	3609.	4412.	23475.
1982	6563.	4997.	4369.	3423.	3307.	22659.
1983	6746.	5152.	4515.	3506.	3104.	23022.
1984	6914.	5296.	4655.	3622.	3168.	23654.
1985	7068.	5429.	4786.	3735.	3272.	24288.
1986	7208.	5550.	4906.	3840.	3374.	24879.
1987	7336.	5662.	5017.	3938.	3469.	25422.
1988	7451.	5763.	5119.	4027.	3558.	25917.
1989	7555.	5854.	5210.	4109.	3639.	26366.
1990	7648.	5936.	5294.	4183.	3713.	26773.
1991	7731.	6010.	5368.	4250.	3780.	27139.
1992	7805.	6076.	5435.	4310.	3841.	27467.
1993	7870.	6134.	5495.	4364.	3896.	27760.
1994	7928.	6186.	5549.	4413.	3945.	28021.
1995	7980.	6232.	5596.	4456.	3989.	28253.
1996	8025.	6273.	5638.	4494.	4028.	28459.
1997	8065.	6309.	5676.	4528.	4063.	28641.
1998	8100.	6341.	5708.	4558.	4094.	28801.
1999	8131.	6369.	5737.	4585.	4121.	28942.
2000	8158.	6393.	5763.	4608.	4145.	29067.

south west

	1	2	3	4	5	total
1977	3155.	2337.	1977.	1608.	1356.	10433.
1978	5493.	2472.	2108.	1589.	1450.	13113.
1979	5875.	4207.	2231.	1689.	1438.	15440.
1980	6107.	4595.	3724.	1788.	1524.	17739.
1981	6316.	4790.	4136.	2945.	1614.	19800.
1982	6510.	4956.	4325.	3308.	2624.	21724.
1983	6691.	5110.	4477.	3468.	2978.	22724.
1984	6858.	5253.	4617.	3592.	3130.	23449.
1985	7010.	5385.	4747.	3704.	3244.	24090.
1986	7150.	5505.	4867.	3809.	3346.	24677.
1987	7276.	5616.	4977.	3906.	3441.	25216.
1988	7391.	5716.	5077.	3994.	3529.	25707.
1989	7493.	5807.	5168.	4075.	3609.	26153.
1990	7586.	5888.	5251.	4149.	3683.	26556.
1991	7668.	5961.	5325.	4215.	3750.	26919.
1992	7741.	6026.	5391.	4275.	3810.	27244.
1993	7806.	6084.	5451.	4329.	3864.	27535.
1994	7864.	6136.	5504.	4377.	3913.	27794.
1995	7915.	6182.	5551.	4420.	3957.	28024.
1996	7960.	6222.	5593.	4458.	3996.	28228.
1997	8000.	6258.	5630.	4491.	4030.	28409.
1998	8034.	6289.	5662.	4521.	4061.	28568.
1999	8065.	6317.	5691.	4547.	4088.	28708.
2000	8092.	6341.	5716.	4571.	4112.	28831.

/cont.....

B. 2 (cont.)

north west

	1	2	3	4	5	total
1977	1528.	1132.	957.	779.	657.	5052.
1978	2970.	1197.	1021.	769.	702.	6660.
1979	3190.	2266.	1080.	818.	696.	8050.
1980	3316.	2494.	1999.	866.	738.	9413.
1981	3430.	2601.	2243.	1577.	781.	10633.
1982	3535.	2691.	2348.	1793.	1403.	11771.
1983	3633.	2775.	2431.	1883.	1614.	12336.
1984	3724.	2853.	2507.	1950.	1700.	12734.
1985	3807.	2924.	2578.	2012.	1762.	13082.
1986	3883.	2990.	2643.	2069.	1817.	13401.
1987	3951.	3050.	2703.	2121.	1869.	13693.
1988	4013.	3104.	2757.	2169.	1916.	13960.
1989	4069.	3153.	2807.	2213.	1960.	14202.
1990	4119.	3197.	2851.	2253.	2000.	14421.
1991	4164.	3237.	2892.	2289.	2036.	14618.
1992	4204.	3273.	2928.	2322.	2069.	14795.
1993	4239.	3304.	2960.	2351.	2099.	14953.
1994	4271.	3332.	2989.	2377.	2125.	15094.
1995	4298.	3357.	3014.	2400.	2149.	15219.
1996	4323.	3379.	3037.	2421.	2170.	15329.
1997	4344.	3398.	3057.	2439.	2189.	15427.
1998	4363.	3415.	3075.	2455.	2205.	15514.
1999	4380.	3430.	3090.	2469.	2220.	15590.
2000	4394.	3444.	3104.	2482.	2233.	15657.

location 1

	1	2	3	4	5	total
1977	3155.	2337.	1977.	1608.	1356.	10432.
1978	1006.	2472.	2108.	1589.	1450.	8625.
1979	885.	894.	2231.	1689.	1437.	7136.
1980	906.	708.	885.	1788.	1524.	5811.
1981	937.	713.	654.	753.	1613.	4669.
1982	965.	735.	645.	534.	715.	3595.
1983	992.	758.	664.	519.	492.	3425.
1984	1017.	779.	685.	533.	470.	3484.
1985	1039.	798.	704.	549.	482.	3573.
1986	1060.	816.	722.	565.	496.	3659.
1987	1079.	833.	738.	579.	510.	3739.
1988	1096.	848.	753.	592.	523.	3812.
1989	1111.	861.	766.	604.	535.	3878.
1990	1125.	873.	779.	615.	546.	3938.
1991	1137.	884.	790.	625.	556.	3991.
1992	1148.	894.	799.	634.	565.	4040.
1993	1157.	902.	808.	642.	573.	4083.
1994	1166.	910.	816.	649.	580.	4121.
1995	1174.	917.	823.	655.	587.	4155.
1996	1180.	923.	829.	661.	592.	4185.
1997	1186.	928.	835.	666.	598.	4212.
1998	1191.	933.	840.	670.	602.	4236.
1999	1196.	937.	844.	674.	606.	4257.
2000	1200.	940.	848.	678.	610.	4275.

/cont.....

B. 2 (cont.)

location 2

	1	2	3	4	5	total
1977	949.	703.	594.	483.	408.	3137.
1978	639.	743.	634.	478.	436.	2930.
1979	640.	517.	671.	508.	432.	2769.
1980	663.	505.	479.	538.	458.	2642.
1981	685.	520.	458.	391.	485.	2539.
1982	706.	538.	470.	369.	358.	2441.
1983	726.	554.	486.	377.	334.	2478.
1984	744.	570.	501.	390.	341.	2545.
1985	760.	584.	515.	402.	352.	2613.
1986	776.	597.	528.	413.	363.	2677.
1987	789.	609.	540.	424.	373.	2735.
1988	802.	620.	551.	433.	383.	2789.
1989	813.	630.	561.	442.	392.	2837.
1990	823.	639.	570.	450.	399.	2881.
1991	832.	647.	578.	457.	407.	2920.
1992	840.	654.	585.	464.	413.	2955.
1993	847.	660.	591.	470.	419.	2987.
1994	853.	666.	597.	475.	424.	3015.
1995	859.	671.	602.	479.	429	3040.
1996	863.	675.	607.	484.	433	3062.
1997	868.	679.	611.	487.	437	3082.
1998	872.	682.	614.	490.	440.	3099.
1999	875.	685.	617.	493.	443.	3114.
2000	878.	688.	620.	496.	446.	3128.

location 3

	1	2	3	4	5	total
1977	1175.	870.	736.	599.	505.	3886.
1978	926.	921.	785.	592.	540.	3764.
1979	943.	740.	831.	629.	535.	3679.
1980	977.	742.	679.	666.	568.	3631.
1981	1010.	767.	672.	550.	601.	3599.
1982	1041.	793.	693.	540.	501.	3568.
1983	1070.	817.	716.	556.	489.	3648.
1984	1097.	840.	738.	575.	502.	3752.
1985	1121.	861.	759.	592.	519.	3853.
1986	1143.	881.	778.	609.	535.	3947.
1987	1164.	898.	796.	625.	550.	4033.
1988	1182.	914.	812.	639.	564.	4111.
1989	1198.	929.	827.	652.	577.	4183.
1990	1213.	942.	840.	664.	589.	4247.
1991	1226.	953.	852.	674.	600.	4305.
1992	1238.	964.	862.	684.	609.	4357.
1993	1248.	973.	872.	692.	618.	4404.
1994	1258.	981.	880.	700.	626.	4445.
1995	1266.	989.	888.	707.	633.	4482.
1996	1273.	995.	894.	713.	639.	4515.
1997	1279.	1001.	900.	718.	645.	4543.
1998	1285.	1006.	906.	723.	649.	4569.
1999	1290.	1010.	910.	727.	654.	4591.
2000	1294.	1014.	914.	731.	658.	4611.

/cont....

B. 2 (cont.)

location 4

	1	2	3	4	5	total
1977	786.	582.	492.	400.	338.	2598.
1978	897.	616.	525.	396.	361.	2795.
1979	940.	700.	556.	421.	358.	2974.
1980	975.	737.	630.	445.	380.	3166.
1981	1009.	765.	665.	503.	402.	3344.
1982	1040.	791.	691.	533.	453.	3508.
1983	1068.	816.	715.	554.	481.	3634.
1984	1095.	839.	737.	574.	501.	3745.
1985	1119.	860.	758.	592.	518.	3847.
1986	1142.	879.	777.	608.	534.	3941.
1987	1162.	897.	795.	624.	549.	4027.
1988	1180.	913.	811.	638.	563.	4105.
1989	1197.	927.	825.	651.	576.	4176.
1990	1211.	940.	838.	662.	588.	4241.
1991	1224.	952.	850.	673.	599.	4298.
1992	1236.	962.	861.	683.	608.	4350.
1993	1247.	972.	870.	691.	617.	4397.
1994	1256.	980.	879.	699.	625.	4438.
1995	1264.	987.	886.	706.	632.	4475.
1996	1271.	994.	893.	712.	638.	4508.
1997	1277.	999.	899.	717.	644.	4536.
1998	1283.	1004.	904.	722.	648.	4562.
1999	1288.	1009.	909.	726.	653.	4584.
2000	1292.	1013.	913.	730.	657.	4604.

location 5

	1	2	3	4	5	total
1977	499.	370.	313.	254.	215.	1651.
1978	776.	391.	334.	251.	230.	1982.
1979	826.	597.	353.	267.	227.	2271.
1980	859.	647.	530.	283.	241.	2560.
1981	888.	673.	582.	421.	255.	2820.
1982	915.	697.	608.	466.	376.	3062.
1983	941.	718.	629.	488.	420.	3196.
1984	964.	738.	649.	505.	440.	3297.
1985	986.	757.	667.	521.	456.	3387.
1986	1005.	774.	684.	535.	470.	3469.
1987	1023.	789.	700.	549.	484.	3545.
1988	1039.	804.	714.	562.	496.	3614.
1989	1053.	816.	727.	573.	507.	3677.
1990	1066.	828.	738.	583.	518.	3733.
1991	1078.	838.	749.	593.	527.	3784.
1992	1088.	847.	758.	601.	536.	3830.
1993	1097.	855.	766.	609.	543.	3871.
1994	1106.	863.	774.	615.	550.	3907.
1995	1113.	869.	780.	621.	556.	3940.
1996	1119.	875.	786.	627.	562.	3968.
1997	1125.	880.	791.	631.	567.	3994.
1998	1130.	884.	796.	636.	571.	4016.
1999	1134.	888.	800.	639.	575.	4036.
2000	1138.	891.	804.	643.	578.	4053.

/cont.....

B. 2 (cont.)

location 6

	1	2	3	4	5	total
1977	470.	348.	294.	239.	202.	1553.
1978	617.	368.	314.	237.	216.	1751.
1979	652.	478.	332.	251.	214.	1927.
1980	677.	510.	427.	266.	227.	2107.
1981	700.	531.	460.	340.	240.	2271.
1982	721.	549.	479.	368.	305.	2423.
1983	741.	566.	496.	384.	332.	2520.
1984	760.	582.	512.	398.	347.	2598.
1985	777.	597.	526.	410.	359.	2669.
1986	792.	610.	539.	422.	371.	2734.
1987	806.	622.	551.	433.	381.	2794.
1988	819.	633.	562.	443.	391.	2848.
1989	830.	643.	573.	451.	400.	2897.
1990	840.	652.	582.	460.	408.	2942.
1991	850.	660.	590.	467.	415.	2982.
1992	858.	668.	597.	474.	422.	3018.
1993	865.	674.	604.	480.	428.	3051.
1994	871.	680.	610.	485.	434.	3079.
1995	877.	685.	615.	490.	438.	3105.
1996	882.	689.	620.	494.	443.	3127.
1997	886.	693.	624.	498.	446.	3147.
1998	890.	697.	627.	501.	450.	3165.
1999	894.	700.	630.	504.	453.	3181.
2000	896.	703.	633.	506.	456.	3194.

location 7

	1	2	3	4	5	total
1977	1044.	774.	655.	532.	449.	3454.
1978	592.	819.	698.	526.	480.	3115.
1979	581.	487.	739.	559.	476.	2842.
1980	600.	459.	457.	592.	505.	2612.
1981	620.	471.	417.	376.	534.	2419.
1982	640.	487.	426.	337.	347.	2236.
1983	657.	502.	440.	342.	306.	2247.
1984	674.	516.	454.	353.	309.	2305.
1985	689.	529.	466.	364.	319.	2367.
1986	702.	541.	478.	374.	329.	2424.
1987	715.	552.	489.	384.	338.	2477.
1988	726.	561.	499.	392.	347.	2525.
1989	736.	570.	508.	400.	355.	2569.
1990	745.	578.	516.	408.	362.	2609.
1991	753.	586.	523.	414.	368.	2644.
1992	760.	592.	530.	420.	374.	2676.
1993	767.	598.	535.	425.	380.	2705.
1994	773.	603.	541.	430.	384.	2730.
1995	778.	607.	545.	434.	389.	2753.
1996	782.	611.	549.	438.	392.	2773.
1997	786.	615.	553.	441.	396.	2791.
1998	789.	618.	556.	444.	399.	2806.
1999	792.	621.	559.	447.	402.	2820.
2000	795.	623.	561.	449.	404.	2832.

/cont.....

B. 2 (cont.)

location 8

	1	2	3	4	5	total
1977	548.	406.	343.	279.	236.	1812.
1978	472.	429.	366.	276.	252.	1795.
1979	484.	375.	387.	293.	250.	1790.
1980	502.	381.	342.	311.	265.	1800.
1981	519.	394.	344.	276.	280.	1814.
1982	535.	407.	356.	277.	251.	1826.
1983	550.	420.	368.	286.	250.	1874.
1984	564.	432.	380.	295.	258.	1928.
1985	576.	443.	390.	305.	267.	1980.
1986	588.	453.	400.	313.	275.	2028.
1987	598.	462.	409.	321.	283.	2073.
1988	607.	470.	417.	328.	290.	2113.
1989	616.	477.	425.	335.	297.	2150.
1990	624.	484.	432.	341.	303.	2183.
1991	630.	490.	438.	346.	308.	2213.
1992	636.	495.	443.	351.	313.	2239.
1993	642.	500.	448.	356.	318.	2263.
1994	646.	504.	452.	360.	322.	2285.
1995	651.	508.	456.	363.	325.	2304.
1996	654.	511.	460.	366.	328.	2320.
1997	658.	514.	463.	369.	331.	2335.
1998	660.	517.	465.	372.	334.	2348.
1999	663.	519.	468.	374.	336.	2360.
2000	665.	521.	470.	376.	338.	2370.

location 9

	1	2	3	4	5	total
1977	318.	235.	199.	162.	137.	1051.
1978	594.	249.	212.	160.	146.	1361.
1979	637.	453.	225.	170.	145.	1630.
1980	662.	498.	401.	180.	154.	1894.
1981	684.	519.	448.	316.	163.	2130.
1982	705.	537.	469.	358.	281.	2351.
1983	725.	554.	485.	376.	322.	2462.
1984	743.	569.	500.	389.	339.	2541.
1985	760.	583.	514.	401.	352.	2611.
1986	775.	597.	527.	413.	363.	2674.
1987	788.	609.	539.	423.	373.	2732.
1988	801.	619.	550.	433.	382.	2786.
1989	812.	629.	560.	442.	391.	2834.
1990	822.	638.	569.	450.	399.	2878.
1991	831.	646.	577.	457.	406.	2917.
1992	839.	653.	584.	463.	413.	2952.
1993	846.	659.	591.	469.	419.	2984.
1994	852.	665.	596.	474.	424.	3012.
1995	858.	670.	602.	479.	429.	3037.
1996	863.	674.	606.	483.	433.	3059.
1997	867.	678.	610.	487.	437.	3078.
1998	871.	682.	614.	490.	440.	3096.
1999	874.	685.	617.	493.	443.	3111.
2000	877.	687.	619.	495.	446.	3124.

/cont.....

B. 2 (cont.)

location 10

	1	2	3	4	5	total
1977	677.	502.	424.	345.	291.	2240.
1978	890.	531.	453.	341.	311.	2526.
1979	940.	690.	479.	363.	309.	2779.
1980	976.	736.	616.	384.	327.	3039.
1981	1009.	765.	663.	491.	346.	3275.
1982	1040.	792.	691.	531.	440.	3495.
1983	1069.	816.	715.	554.	479.	3634.
1984	1096.	839.	738.	574.	501.	3747.
1985	1120.	860.	758.	592.	518.	3849.
1986	1142.	880.	778.	609.	535.	3943.
1987	1162.	897.	795.	624.	550.	4028.
1988	1181.	913.	811.	638.	564.	4107.
1989	1197.	928.	826.	651.	577.	4178.
1990	1212.	941.	839.	663.	588.	4243.
1991	1225.	952.	851.	673.	599.	4301.
1992	1237.	963.	861.	683.	609.	4353.
1993	1247.	972.	871.	692.	617.	4399.
1994	1256.	980.	879.	699.	625.	4440.
1995	1265.	988.	887.	706.	632.	4477.
1996	1272.	994.	893.	712.	638.	4510.
1997	1278.	1000.	899.	718.	644.	4539.
1998	1284.	1005.	905.	722.	649.	4564.
1999	1288.	1009.	909.	727.	653.	4586.
2000	1293.	1013.	913.	730.	657.	4606.

location 11

	1	2	3	4	5	total
1977	91.	67.	57.	46.	39.	300.
1978	1153.	71.	61.	46.	42.	1372.
1979	1276.	856.	64.	49.	41.	2286.
1980	1329.	994.	737.	51.	44.	3155.
1981	1375.	1042.	891.	571.	46.	3925.
1982	1417.	1079.	940.	710.	499.	4646.
1983	1456.	1112.	974.	754.	637.	4934.
1984	1493.	1143.	1005.	782.	680.	5103.
1985	1526.	1172.	1033.	806.	706.	5244.
1986	1556.	1198.	1059.	829.	728.	5372.
1987	1584.	1222.	1083.	850.	749.	5489.
1988	1609.	1244.	1105.	869.	768.	5596.
1989	1631.	1264.	1125.	887.	786.	5693.
1990	1651.	1282.	1143.	903.	802.	5780.
1991	1669.	1298.	1159.	918.	816.	5859.
1992	1685.	1312.	1174.	931.	829.	5930.
1993	1699.	1324.	1187.	942.	841.	5994.
1994	1712.	1336.	1198.	953.	852.	6050.
1995	1723.	1346.	1208.	962.	861.	6100.
1996	1733.	1354.	1217.	970.	870.	6144.
1997	1741.	1362.	1225.	978.	877.	6184.
1998	1749.	1369.	1232.	984.	884.	6218.
1999	1756.	1375.	1239.	990.	890.	6249.
2000	1761.	1380.	1244.	995.	895.	6276.

/cont.....

B. 2 (cont.)

location 12

	1	2	3	4	5	total
1977	1566.	1160.	981.	798.	673.	5179.
1978	992.	1227.	1046.	789.	720.	4775.
1979	988.	808.	1107.	839.	714.	4455.
1980	1021.	779.	751.	888.	757.	4196.
1981	1056.	802.	707.	615.	801.	3981.
1982	1088.	829.	725.	570.	565.	3777.
1983	1119.	854.	749.	582.	517.	3821.
1984	1147.	878.	772.	601.	526.	3923.
1985	1172.	900.	794.	619.	543.	4028.
1986	1195.	920.	814.	637.	560.	4126.
1987	1217.	939.	832.	653.	575.	4216.
1988	1236.	956.	849.	668.	590.	4298.
1989	1253.	971.	864.	681.	603.	4373.
1990	1268.	984.	878.	694.	616.	4440.
1991	1282.	997.	890.	705.	627.	4501.
1992	1294.	1008.	901.	715.	637.	4555.
1993	1305.	1017.	911.	724.	646.	4604.
1994	1315.	1026.	920.	732.	654.	4647.
1995	1323.	1034.	928.	739.	662.	4686.
1996	1331.	1040.	935.	745.	668.	4720.
1997	1338.	1046.	941.	751.	674.	4750.
1998	1343.	1052.	947.	756.	679.	4776.
1999	1348.	1056.	951.	760.	683.	4800.
2000	1353.	1060.	956.	764.	687.	4820.

location 13

	1	2	3	4	5	total
1977	0.	0.	0.	0.	0.	0.
1978	698.	0.	0.	0.	0.	698.
1979	777.	516.	0.	0.	0.	1292.
1980	809.	605.	442.	0.	0.	1856.
1981	837.	635.	542.	341.	0.	2355.
1982	863.	657.	573.	432.	297.	2821.
1983	887.	677.	593.	459.	387.	3004.
1984	909.	696.	612.	476.	414.	3107.
1985	929.	714.	629.	491.	430.	3193.
1986	948.	730.	645.	505.	444.	3271.
1987	965.	744.	660.	518.	456.	3343.
1988	980.	758.	673.	529.	468.	3408.
1989	993.	770.	685.	540.	478.	3467.
1990	1006.	781.	696.	550.	488.	3520.
1991	1016.	790.	706.	559.	497.	3568.
1992	1026.	799.	715.	567.	505.	3611.
1993	1035.	807.	723.	574.	512.	3650.
1994	1042.	813.	730.	580.	519.	3684.
1995	1049.	819.	736.	586.	525.	3715.
1996	1055.	825.	741.	591.	530.	3742.
1997	1060.	830.	746.	595.	534.	3766.
1998	1065.	834.	751.	599.	538.	3787.
1999	1069.	837.	754.	603.	542.	3806.
2000	1073.	841.	758.	606.	545.	3822.

/cont.....

B. 2 (cont.)

location 14

	1	2	3	4	5	total
1977	187.	139.	117.	96.	81.	620.
1978	636.	147.	125.	94.	86.	1089.
1979	693.	478.	133.	100.	85.	1490.
1980	722.	541.	417.	106.	91.	1877.
1981	746.	566.	486.	326.	96.	2220.
1982	769.	586.	511.	388.	288.	2542.
1983	791.	604.	529.	409.	348.	2682.
1984	810.	621.	546.	424.	370.	2771.
1985	828.	636.	561.	438.	383.	2847.
1986	845.	651.	575.	450.	395.	2916.
1987	860.	664.	588.	462.	407.	2980.
1988	873.	676.	600.	472.	417.	3038.
1989	886.	686.	611.	482.	427.	3091.
1990	896.	696.	621.	490.	435.	3138.
1991	906.	704.	629.	498.	443.	3181.
1992	915.	712.	637.	505.	450.	3220.
1993	923.	719.	644.	512.	457.	3254.
1994	929.	725.	650.	517.	462.	3285.
1995	935.	731.	656.	522.	468.	3312.
1996	941.	735.	661.	527.	472.	3336.
1997	945.	740.	665.	531.	476.	3357.
1998	950.	743.	669.	534.	480.	3376.
1999	953.	747.	673.	537.	483.	3393.
2000	956.	749.	676.	540.	486.	3407.

location 15

	1	2	3	4	5	total
1977	0.	0.	0.	0.	0.	0.
1978	543.	0.	0.	0.	0.	543.
1979	604.	401.	0.	0.	0.	1004.
1980	629.	470.	343.	0.	0.	1443.
1981	651.	493.	421.	265.	0.	1830.
1982	671.	511.	445.	336.	231.	2193.
1983	689.	527.	461.	357.	301.	2335.
1984	707.	541.	476.	370.	322.	2415.
1985	722.	555.	489.	382.	334.	2482.
1986	737.	567.	501.	392.	345.	2543.
1987	750.	579.	513.	402.	355.	2598.
1988	761.	589.	523.	412.	364.	2649.
1989	772.	598.	533.	420.	372.	2695.
1990	782.	607.	541.	427.	379.	2736.
1991	790.	614.	549.	434.	386.	2774.
1992	798.	621.	556.	440.	393.	2807.
1993	804.	627.	562.	446.	398.	2837.
1994	810.	632.	567.	451.	403.	2864.
1995	816.	637.	572.	455.	408.	2887.
1996	820.	641.	576.	459.	412.	2908.
1997	824.	645.	580.	463.	415.	2927.
1998	828.	648.	583.	466.	418.	2943.
1999	831.	651.	586.	469.	421.	2958.
2000	834.	653.	589.	471.	424.	2971.

/cont.....

B. 2 (cont.)

location 16

	1	2	3	4	5	total
1977	344.	255.	216.	175.	148.	1138.
1978	529.	270.	230.	173.	158.	1360.
1979	563.	407.	243.	184.	157.	1555.
1980	585.	441.	362.	195.	166.	1749.
1981	605.	459.	397.	287.	176.	1924.
1982	624.	475.	414.	318.	257.	2087.
1983	641.	490.	429.	332.	286.	2178.
1984	657.	503.	442.	344.	300.	2247.
1985	672.	516.	455.	355.	311.	2308.
1986	685.	528.	466.	365.	321.	2365.
1987	697.	538.	477.	374.	330.	2416.
1988	708.	548.	486.	383.	338.	2463.
1989	718.	556.	495.	391.	346.	2506.
1990	727.	564.	503.	398.	353.	2545.
1991	735.	571.	510.	404.	359.	2579.
1992	742.	577.	517.	410.	365.	2611.
1993	748.	583.	522.	415.	370.	2638.
1994	754.	588.	527.	419.	375.	2663.
1995	758.	592.	532.	424.	379.	2685.
1996	763.	596.	536.	427.	383.	2705.
1997	767.	600.	539.	430.	386.	2722.
1998	770.	603.	543.	433.	389.	2737.
1999	773.	605.	545.	436.	392.	2751.
2000	775.	608.	548.	438.	394.	2763.

location 17

	1	2	3	4	5	total
1977	336.	249.	210.	171.	144.	1110.
1978	490.	263.	224.	169.	154.	1300.
1979	520.	378.	237.	180.	153.	1467.
1980	540.	407.	336.	190.	162.	1635.
1981	558.	424.	367.	267.	172.	1787.
1982	576.	438.	382.	293.	239.	1929.
1983	592.	452.	396.	307.	264.	2010.
1984	606.	464.	408.	318.	277.	2073.
1985	620.	476.	420.	328.	287.	2130.
1986	632.	487.	430.	337.	296.	2182.
1987	643.	497.	440.	345.	304.	2229.
1988	653.	505.	449.	353.	312.	2273.
1989	663.	513.	457.	360.	319.	2312.
1990	671.	521.	464.	367.	326.	2348.
1991	678.	527.	471.	373.	332.	2380.
1992	684.	533.	477.	378.	337.	2409.
1993	690.	538.	482.	383.	342.	2434.
1994	695.	542.	487.	387.	346.	2457.
1995	700.	547.	491.	391.	350.	2478.
1996	704.	550.	494.	394.	353.	2496.
1997	707.	553.	498.	397.	356.	2512.
1998	710.	556.	501.	400.	359.	2526.
1999	713.	558.	503.	402.	361.	2538.
2000	715.	561.	505.	404.	364.	2549.

/cont.....

B. 2 (cont.)

location 18

	1	2	3	4	5	total
1977	660.	489.	414.	337.	284.	2184.
1978	772.	518.	441.	333.	304.	2367.
1979	810.	602.	467.	354.	301.	2533.
1980	840.	635.	540.	374.	319.	2709.
1981	869.	659.	573.	432.	338.	2870.
1982	896.	682.	595.	459.	389.	3021.
1983	921.	703.	616.	478.	414.	3131.
1984	944.	723.	635.	494.	431.	3227.
1985	965.	741.	653.	510.	446.	3315.
1986	984.	758.	670.	524.	460.	3396.
1987	1001.	773.	685.	537.	473.	3470.
1988	1017.	787.	699.	550.	486.	3537.
1989	1031.	799.	711.	561.	497.	3599.
1990	1044.	810.	722.	571.	507.	3654.
1991	1055.	820.	733.	580.	516.	3704.
1992	1065.	829.	742.	588.	524.	3749.
1993	1074.	837.	750.	596.	532.	3789.
1994	1082.	844.	757.	602.	538.	3824.
1995	1089.	851.	764.	608.	544.	3856.
1996	1095.	856.	770.	613.	550.	3884.
1997	1101.	861.	775.	618.	555.	3909.
1998	1106.	865.	779.	622.	559.	3931.
1999	1110.	869.	783.	626.	562.	3950.
2000	1113.	873.	786.	629.	566.	3967.

location 22

	1	2	3	4	5	total
1977	445.	330.	279.	227.	191.	1471.
1978	494.	349.	297.	224.	204.	1568.
1979	516.	386.	315.	238.	203.	1657.
1980	535.	404.	347.	252.	215.	1754.
1981	554.	420.	365.	278.	228.	1844.
1982	571.	434.	379.	293.	250.	1928.
1983	586.	448.	392.	304.	264.	1995.
1984	601.	460.	405.	315.	275.	2056.
1985	614.	472.	416.	325.	284.	2112.
1986	627.	483.	427.	334.	293.	2163.
1987	638.	492.	436.	342.	302.	2210.
1988	648.	501.	445.	350.	309.	2253.
1989	657.	509.	453.	357.	316.	2292.
1990	665.	516.	460.	364.	323.	2328.
1991	672.	523.	467.	369.	329.	2360.
1992	679.	528.	473.	375.	334.	2388.
1993	684.	533.	478.	379.	339.	2414.
1994	689.	538.	482.	384.	343.	2436.
1995	694.	542.	487.	387.	347.	2456.
1996	698.	545.	490.	391.	350.	2474.
1997	701.	549.	493.	394.	353.	2490.
1998	704.	551.	496.	396.	356.	2504.
1999	707.	554.	499.	399.	358.	2516.
2000	709.	556.	501.	401.	360.	2527.

/cont.....

B. 2 (cont.)

location 23

	1	2	3	4	5	total
1977	58.	43.	36.	30.	25.	192.
1978	672.	45.	39.	29.	27.	812.
1979	743.	499.	41.	31.	26.	1341.
1980	774.	579.	430.	33.	28.	1844.
1981	801.	607.	519.	333.	30.	2290.
1982	825.	628.	548.	414.	291.	2707.
1983	848.	648.	568.	439.	371.	2874.
1984	869.	666.	585.	455.	396.	2972.
1985	889.	683.	602.	470.	411.	3054.
1986	907.	698.	617.	483.	424.	3129.
1987	923.	712.	631.	495.	436.	3197.
1988	937.	725.	644.	506.	447.	3259.
1989	950.	736.	655.	517.	458.	3316.
1990	962.	747.	666.	526.	467.	3367.
1991	972.	756.	675.	534.	475.	3413.
1992	982.	764.	684.	542.	483.	3454.
1993	990.	771.	691.	549.	490.	3491.
1994	997.	778.	698.	555.	496.	3524.
1995	1004.	784.	704.	560.	502.	3553.
1996	1009.	789.	709.	565.	507.	3579.
1997	1014.	793.	714.	569.	511.	3602.
1998	1019.	797.	718.	573.	515.	3622.
1999	1023.	801.	722.	577.	518.	3640.
2000	1026.	804.	725.	580.	521.	3656.

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