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Cost Allocation in Water Resources -- Three Gaming Experiments with Young Scientists at IIASA

Stahl, I.

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COST ALLOCATION IN WATER RESOURCES-- THREE GAMING EXPERIMENTS WITH YOUNG SCIENTISTS AT IIASA

Ingolf Ståhl

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

This paper is one in a series of reports on experiments with a game concerning cost allocation in water resources. The ultimate purpose of the game is to be an aid in finding better methods for allocating joint costs in projects when several parties, e.g., municipalities, join together to save costs by building a larger facility instead of several smaller ones.

Gaming, i.e., the actual playing of games, can be seen as a complement to other, more deductive, methods, for example, game theory. Since the idea is that the planners involved shall really want to use the allocation scheme, it is important that the scheme is congruent with the planner's own thinking. Gaming can first of all be seen as an "acid test" of the proposed game theoretic suggestion. If some theory is not appealing in an experimental setting, it is most likely not so in real application either. Furthermore, gaming can be seen as a direct way of finding out what ideas of distribution are really held by planners: How do intelligent decision makers, with a reasonable time for thinking through the problem, arrive at a compromise between different concepts, such as efficiency and equity, in negotiations of this type?

ABSTRACT

This paper reviews three gaming experiments with a game on cost allocation in water resources, carried out with young scientists from eight countries at IIASA. The game is aimed at testing some different methods of cost allocation.

In earlier experiments the game had mainly been used to test the predictive ability of these methods. In the three experiments reported on here the emphasis was on testing the normative relevance of these methods. All the players were initially given an overview of the methods. Furthermore, in two of the games each of the players obtained a "consultants report" on a method for which he should argue.

The results did not appear to be affected by the introduction of this normative influence, except in one respect: while in the earlier games the solution had not been in the core, a core solution was now obtained in all three games.

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COST ALLOCATION IN WATER RESOURCES--
THREE GAMING EXPERIMENTS WITH
YOUNG SCIENTISTS AT IIASA

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INTRODUCTION

IIASA working papers WP-80-38 and WP-80-82 contain descriptions of a game on cost allocation in water management as well as a report on the actual playing of this game; first in November 1979 with water planners in Sweden, and secondly in April with regional planners in Tuscany, Italy. The reader is recommended to read either one of these two working papers prior to proceeding to the present one.

Furthermore, WP-80-134 reports on two gaming experiments carried out with Swedish doctoral students. It is not necessary to read this paper but some references will be made to these experiments.

Although the focus of the IIASA gaming project is to involve, as far as possible, real decision makers, it is also of importance to obtain many game runs to be able to study the structure of the game more generally. When an opportunity was given to run this water game three times with scientists participating in the IIASA summer program for young scientists the opportunity was, utilized.

It should be mentioned that one should not take the connotation "young" too literally. The vast majority of these scientists have been involved in research, after their primary degree, for several years. Although material is lacking for an exact comparison, it is the author's strong impression that both as regards age, degrees and numbers of years in research or other work the IIASA young scientists group is more advanced than the group of Swedish doctoral students mentioned above. It should be added that in particular there appeared to be a considerably greater experience among the members of the IIASA young scientists program in quantitative methods.

It should also be mentioned that two IIASA scientists, not belonging to the young scientists program, also participated. We had planned for 3 game groups with 6 players in each group, i.e., a total of 18 players, but at the last moment, contrary to our expectations, two members of the young scientists group did not turn up for the game. In order to be able to still run the three experimental groups we, therefore, used these two IIASA scientists.

THEORETICAL BACK-GROUND

The main aim of the preceding experiments were to provide a test for some different methods of cost allocation presented in IIASA WP-79-77. These methods are presented also in the earlier mentioned WP-80-38 and WP-80-82, and also partly in appendix B below.

Here, we shall briefly mention that the methods can be characterized as:

1. Simple proportional methods, such as allocating costs in proportion to population and demand.

2. A method used in practice, called the SCRB-method.

3. A method based on a certain type of game theoretic reasoning, involving the step by step build up of larger and larger coalitions, called the Shapley value.

4. Three methods based on the game theoretic concept of the core. The core is the set of all solutions which fulfil the following three principles:

    1) Individual Rationality:
    No municipality shall pay a higher cost than it would have to pay, if it were to fulfill its water needs completely on its own.

    2) The "Full Cost" Principle:
    Total costs should be covered, leaving no surplus and no loss to any third party.

    3) "Group Rationality", refers to subcoalitions, i.e., coalitions smaller than the grand coalition and implies that the sum of payments made by the members of every such subcoalition should not be larger than the cost that this subcoalition incurs if it is working on its own.

There are, however, a great many solutions in the core. One way to obtain a unique solution in the core is to assume that one gives subsidies to the various subcoalitions so that one obtains a unique core.

In the WP-79-70 three schemes for such subsidies were discussed.

1. Each coalition obtains the same subsidy. The solution then obtained is called the Nucleolus, or as in this paper[1] the Ordinary Nucleolus.

2. Each coalition obtains a subsidy in direct proportion to the number of members in this coalition, i.e. the subsidy obtained per member is the same for each coalition. This method is called the Weak Least Core, or as in this paper, the Weak Nucleolus[1].

3. Each coalition obtains a subsidy that is directly proportional to its costs, i.e., the subsidy percentage is the same for each coalition. This method is called the Proportional Least Core.

The subsidy rate is made just big enough to make the solution unique.

Choosing between these three methods, WP-79-70 focussed on the so called monotonicity principle: If costs go up, no party shall pay less and if costs go down, no party shall pay more.

Since the Weak Least Core was the only one of the three core concepts which fulfilled this principle in all games, this was in WP-79-70 preferred from a normative point of view.

Later theoretical development by P. Young has involved another core solution, which here shall be called the Proportional Nucleolus. It is similar to the method above called the Proportional Least Core, but the subsidies are now no longer proportional to costs, but instead to cost savings. The savings are calculated as the difference between the cost of the coalition and the sum of all the costs that the members of the coalitions would have incurred, if each one of them had been completely on his own.

[1]We have preferred to use the different names: Nucleolus - Ordinary Nucleolus, and, Weak Least Core - Weak Nucleolus reserving the first name in the pair for the solution in WP-79-70 (as well as in WP-80-38 and WP-80-82 etc.) and the second pair in the name for the solution presented in RR-80-32. There is, due to a difference in the computer algorithm, a slight difference, as regards the values for M and T.

This Proportional Nucleolus has (according to P. Young, 1980) the advantage of always fulfilling the monotonicity principle. It has, furthermore, an advantage over the Weak Least Core (Weak Nucleolus) in that it fulfills the following principle:

> A player who never contributes to any cost savings when joining with other parties or coalitions, shall not realize any cost savings above his go alone costs.

Because of this, the Proportional Nucleolus is the principle suggested from a normative point of view in an IIASA Research Report (RR-80-32) published after the playing of this game.[1]

THE EXPERIMENTAL DESIGN

Different cost levels for test of monotonicity principle.

The experimental design that we wanted to try out was influenced by the outcome of the playing in the five earlier games.

In all five games, the game theoretic concept that had been the least successful from a predictive point of view, was the concept which from a normative point of view in IIASA WP-79-77 had been regarded to have the most desirable properties, namely the Weak Least Core. As mentioned above, of the game theoretic concepts discussed in the mentioned WP, this concept was the only core concept that fulfilled the monotonicity requirement in every game.

The two earliest experiments in Sweden and in Italy had, however, in no way given the participants any reasons for reflecting upon the monotonicity principle, since they had not had to think about the effect of total costs going up or down.

In the game with the Swedish Doctoral Students we, therefore, tested with one group the effect of focussing the attention of the participants explicitly on the monotonicity principle. This was done by introducing two levels of costs of the grand coalition, i.e., when all six municipalities join together: one of 83.82 mkr (millions of Swedish crowns); one of 87.82 mkr. The 83.82 figure is the one used in the previous experiments. The reason for putting the second level 4 mkr above, is that this is the figure used by Young *et alia* in WP-79-77. For this pair of figures, it is shown that the Nucleolus, which in the Swedish and Italian experiments was the most successful of the three studied core concepts, violated the monotonicity principle: When total costs are 83.82, party K pays 5.00; when costs increase to 87.82, K pays only 4.51.

The players in this group got the instruction that the grand coalition must register two payment distributions, one when costs are 87.82 and the other when costs are 83.82 mkr.

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[1] Young, Okada and Hashimoto (1980)

The play of this group indicated that the introduction of two specific cost levels did not matter in this case. Obviously no conclusions can be drawn on the basis of only one play run; indeed a great many are required. Since the experiment with the doctoral students had been critized on the grounds that they had not been paid money in proportion to their cost savings, but a prize had been given to the best player, it was especially desirable to replicate the experiment with two cost levels, now paying all players in proportion to cost savings.

Test of normative aspects of methods

Due to the negative result of introducing different cost levels in the mentioned student experiments, it was, however, also found suitable to test more explicitly the normative value of certain of the methods.

Such a normative test must, however, be carefully designed in order to have validity. In particular, it is important to avoid so called "authority effects". These might accrue, e.g. in situations when every player is supplied with the same suggestion for division of costs. If the game does not concern great amounts of money the players might follow this proposal, just in order to show that they have understood the proposal or in order to please the experimenter. A more reasonable approach to the normative testing of the model is to supply different players with different advice on how to make the division. Such advice could also involve specific arguments for the proposed cost division proposal. One can then possibly see, if the arguments for one division procedure are "stronger" than those for others in the sense that the other players will abandon their division procedures in favor of this one. (Stahl 1980a).

Hence we decided to set up an experiment, where in the game, each of the six players (municipalities) had a sheet of paper from a consultant with information regarding a method that he believes that the player should argue for. The player was to be quite free to use or disregard the arguments in this paper. Having six players we decided on supplying papers arguing for six different methods:

We selected the demand proportional method as the representative of the simple proportional methods, partly because this had fared better than the population proportional method in the earlier tests.

The Shapley value and the SCRB - method were natural candidates being different and discussed in all earlier reports on the experiments. Of the four core methods discussed above, we left out the method above called the Proportional Weak Core (with subsidies proportional to cost) since the method above called the Proportional Nucleolus, (with subsidies proportional to cost savings) was regarded as better, never violating the monotonicity principle.

For these six methods the solution was computed on the basis of table 1 showing the costs of every possible coalition. Table 2 was then obtained.

Table 1. Total cost of each possible coalition.

A	21.95	AHK	40.74	AHKL	48.95
H	17.08	AHL	43.22	AHKM	60.25
K	10.91	AHM	55.50	AHKT	62.72
L	15.88	AHT	56.67	AHLM	64.03
M	20.81	AKL	48.74	AHLT	65.20
T	21.98	AKM	53.40	AHMT	74.10
		AKT	54.85	AKLM	63.96
AH	34.69	ALM	53.05	AKLT	70.72
AK	32.86	ALT	59.81	ALMT	73.41
AL	37.83	AMT	61.36	HKLM	48.07
AM	42.76	HKL	27.26	HKLT	49.24
AT	43.93	HKM	42.55	HKMT	59.35
HK	22.96	HKT	44.94	HLMT	64.41
HL	25.00	HLM	45.81	KLMT	56.61
HM	37.89	HLT	46.98	AKMT	72.27
HT	39.06	HMT	56.49	AHKLM	69.76
KL	26.79	KLM	42.01	AHKMT	77.42
KM	31.45	KLT	48.77	AHLMT	83.00
KT	32.89	KMT	50.32	AHKLT	70.93
LM	31.10	LMT	51.46	AKLMT	73.97
LT	37.86			HKLMT	66.46
MT	39.41				
				AHKLMT	83.82

Table 2: Allocations in Millions of Swedish crowns

Method	A	H	K	L	M	T
Shapley Value	20.01	10.71	6.61	10.37	16.94	19.18
Ordinary Nucleolus	20.35	12.06	5.00	8.61	18.32	19.49
Proportional Nucleolus	20.36	12.46	3.52	8.67	18.82	19.99
Weak Nucleolus	20.03	12.52	3.94	9.07	18.54	19.71
S.C.R.B.	19.54	13.28	5.62	10.90	16.66	17.82
Demand Proportional	13.33	16.32	7.43	7.00	29.04	10.69

From table 2 we see that the shapley value is party H's best method, the Proportional Nucleolus K's best and the SCRB's M's best method. Hence we assign these methods to these players. For the remaining players the Demand Proportional method is the best method. The relative advantage of this method, compared to the second best, is highest as regards T. For party L the Ordinary Nucleolus, not assigned to any other player, is the second best method. Finally, for A, SCRB is the second best method. Since this is already assigned as is the Shapley Value, while the Weak Nucleolus is unassigned, we assign the Weak Nucleolus to A. Hence we obtain the following scheme of allocation of methods to the players.

A : Weak Nucleolus

H : Shapley Value

K : Proportional Nucleolus

L : Ordinary Nucleolus

M : SCRB

T : Demand Proportional Method

Having assigned each method to one of the players we proceeded to write a "consultant's paper" on each of these six methods. It appeared suitable to limit the length of each paper to one page. It did not seem reasonable to give each method the same length of presentation; e.g., the Demand Proportional method requires little further specification, while for each core method we need to explain both the core concept and the specific subsidy scheme of the particular core method.

The six "consultant's reports" are presented in Appendix B.

Test of effect of prize structure

As mentioned briefly above, the experiments carried out on the doctoral students had been criticized on the ground that we had not given money to every player in proportion to his cost savings in the game, but instead given a prize only to that player who by some less precise standard could be regarded as the best player[1]. The gist of the critique was that if some players believed they were not doing so very well, and hence had a smaller chance of being the winner they would lose interest completely and then play in a somewhat less serious fashion.

We therefore, wanted a design in which the difference between two groups as regards the experimental design would only concern the prize structure.

[1]The "prize will be given to that player who according to the judgement of the game leader acts as the most skillful representative for his municipality" among the players taking part in the game.

Experimental set up

In order to incorporate the three factors discussed above, we set up an experimental design of the following type for the three groups. (Table 3.)

Provided one, of course, made a great many experimental runs for each of these groups one should be able to ascribe a possible difference in behavior to whether the players got a "consultant's report" or had two cost levels or to the differences in prize structure. Making at this time only one run for each group, one obviously has to be very careful about one's conclusions.

THE PLAYING OF THE GAME

Prior to the actual playing of the game the participants in the game were given a lecture of approximately one hour's duration about the game's background and the six methods discussed above. The idea was that everyone would in principle have some general familiarity with all the six methods, but would possibly have more detailed knowledge about only one of the methods through the "consultant's report".

	Group 1	Group 2	Group 3
Focus on normative	Consultant's reports	Consultant's reports	Two cost levels of the grand coalition
Prize structure	Proportional to cost savings	Prize only to winner	Proportional to cost savings

Table 3. Experimental design

After the lecture, the three groups of players were seated around separate tables located in the same room, but some meters apart. There appeared to be little possibility of either group observing what the other group did, but it was possible for the game leader to be in contact with all groups.

The seating around each table was intentionally kept as close as possible to that in the earlier games.[1]

The participants were allotted, by random choice, to a specific group and a specific municipality role.[2]

Next the participants obtained the gaming instructions presented in appendix A. We have indicated, by the use of square brackets and figures, which parts were available to different groups.

There was time for questions, but there were no questions that could not be given an answer by directly referring to the gaming instructions.

As regards the playing of the game, we shall report on each group separately.

<u>Group 1</u>

Group 1 played for money prizes in direct proportion to cost savings and each player had a special "consultant's report" arguing for one method of cost allocation.

H and L started immediately to try to form a coalition pointing at the high cost savings to be made here, 32.96 - 25.00 = 7.96. H wanted to split these savings evenly, but L insisted on splitting the costs more in proportion to demand, which favors L. While H and L bargained with each other, K argued for a three party coalition HKL.

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[1]The same municipalities sat next to each other as in the earlier games, but due to the size of the tables two of the municipalities were seated at the end of the table in contrast to earlier games.

[2]An exception to the random allotment was made for one of the two IIASA scientists participating (see p.2). Being very well versed in game theory, he was assigned to role H in group 1, since role H (together with role K) can be regarded as the most interesting from a game theoretical point of view. As discussed later the outcome as well as the bargaining procedure in group 1 did <u>not</u>, diverge significantly from that of the other groups.

Before these three parties could agree on a coalition, M and T, after 20 minutes (counted from the start of the actual game), formed a coalition with a division 19.41 to M and 20.00 to T. This involves a split of savings, which is more favorable to T than basing the cost split on the one party coalition costs or than splitting it evenly. Then after 23 minutes H, K and L formed a three party coalition. H and L first gave K cost savings of 4 and then split the remaining savings approximately 50:50. This led to the distribution: H: 10.78, K: 6.90, and L: 9.58.

The negotiation then proceeded, focussing on forming the grand coalition. It was then generally understood that a temporary coalition was formed by HKLMT, but this coalition was not registered, as the savings involved appeared too small. The main problem in the formation of the grand coalition was how much A should get. A here wanted to regard the bargaining as a two-party game, stressing that he was the only single player able to block the formation of the grand coalition, thus regarding the other parties as committed to a five party coalition.

After long negotiations, an agreement was reached (after 79 minutes counted from the start) on the following distribution:

A: 18.65;	H: 10.38;	K: 6.60;
L: 9.18;	M: 19.21;	T: 19.80

This distribution implied that A obtained a cost saving of 3.3, while HKLMT together obtained a total cost saving of 1.5, compared to the costs earlier registered, distributed in pieces of 0.2 - 0.4 to the five parties.

The grand coalition then remained stable for 15 minutes and came into force.

Group 2

In Group 2 only the "winner" would get a prize. Everyone got a "consultant's report".

HKL here immediately set out to try to form a coalition and after 26 minutes from the start an agreement was reached on H: 10.61; K: 6.78 and L: 9.87. This implies a division where the savings over the costs when parties go alone were distributed roughly in proportion to these go alone costs.

At 33 minutes from the start, M and T agreed on a coalition with M: 18.76 and T: 20.65. This division was determined on more ad hoc grounds.

At 46 minutes from the start the five party coalition HKLMT was formed with H: 10.57; K: 6.75; L: 9.83; M: 18.71 and T: 20.61. The slight savings over the coalitions HKL and MT were spread out evenly.

Finally after 59 minutes from the start the grand coalition was formed with A: 21.02; H: 9.85; K: 6.29; L: 9.15; M: 17.83 and T: 19.68. The relevant savings were then regarded to be the difference between the cost of the five party coalition and the sum of the costs of each party going alone. These savings of 4.59 were then distributed in accordance with the share that the go alone cost of the party constituted of the sum of all go alone costs. In this step the parties thus did not take into consideration the payments that had been obtained in the earlier steps. Party A lost most by this procedure. This explains why A in this group paid more than in other games. (See table 4 on p. 12.) This grand coalition then remained to come into force.

Group 3

In this group the parties were to obtain money prizes proportional to savings. There were no consultant's reports, but instead two possible levels of costs for the grand coalition.

After 7 minutes H, K and L formed a coalition with 11.54 to H; 5.37 to K and 10.34 to L. Here the total savings of 16.61 were distributed in three even parts. This is obviously more favorable for K than distributing cost savings in proportion to the go alone costs and this explains why municipality K was relatively better off in this game than in other games (see table 4).

Next after 22 minutes from the start M and T formed a coalition with 19.12 to M; 20.29 to T. This implied that cost savings were split evenly.

After this the grand coalition was considered, with A driving hard to get into a coalition.

A temporary, but not registered coalition HKLMT was however formed, implying that the small savings of 0.2 compared to HKL and MT are spread out evenly (0.04 to each).

Then after long discussions, 77 minutes after the start, the grand coalition was formed. The parties first reached an agreement on the higher cost level of 87.82. Here the savings of 0.6 (0.59) were distributed evenly, each getting a reduction of 0.1; A with reference to his go alone costs and the other with reference to the temporary coalition HKLMT. Then as regards the lower cost level of 83.82, the additional savings were distributed so that A got a considerably larger portion, namely 1.2, while the others shared the remaining 2.8 in equal portions of 0.56. This implied a distribution of the 83.82 as follows:- A: 20.65; H: 10.84; L: 4.67; L: 9.65; M: 18.42 and T: 19.59. This grand coalition also remained to come into force.

COMPARISONS WITH METHODS AND EARLIER GAMES

We sum up the comparison between the outcome of the present three experiments, the four earlier experiments as well as the discussed six methods in table 4.

Table 4: Summary of outcomes of experiments and methods

GAME	A	H	K	L	M	T
1	18.65	10.38	6.60	9.18	19.21	19.80
2	21.02	9.85	6.29	9.15	17.83	19.68
3	20.65	10.84	4.67	9.65	18.42	19.59
SWEDISH GAME	21.15	9.70	6.00	9.10	18.37	19.50
ITALIAN GAME	20.81	9.55	6.10	8.88	18.72	18.75
STUDENT GAME A	18.15	12.77	8.10	13.25	12.90	18.65
STUDENT GAME B	18.56	13.79	6.75	8.00	17.66	19.05
SHAPLEY VALUE	20.01	10.71	6.61	10.37	16.94	19.18
ORDINARY NUCLEOLUS	20.35	12.06	5.00	8.61	18.32	19.49
PROPORTIONAL NUCLEOLUS	20.36	12.46	3.52	8.67	18.82	19.99
WEAK NUCLEOLUS	20.03	12.52	3.94	9.07	18.54	19.71
S.C.R.B.	19.54	13.28	5.62	10.90	16.66	17.82
DEMAND PROPORTIONAL	13.33	16.32	7.43	7.00	29.04	10.69

In order to see how well the theoretical allocations fit these experimental values, we have used three measures of difference:

1) The sum of absolute differences. With T as the theoretical value and E as the experimental value the measure is:

$$\sum_{i=1}^{6} | T_i - E_i |$$

2) The sum of the squared differences, i.e.,

$$\sum_{i=1}^{6} (T_i - E_i)^2.$$

Compared to measure 1, this gives a higher relative weight to large discrepancies.

3) The sum of the relative squared differences, i.e., of the squared differences after dividing each difference by the theoretical value, i.e.

$$\sum_{i=1}^{6} (T_i - E_i)^2 / T_i.$$

The idea behind this measure is that a difference is more important if it is relatively large in comparison with the "expected" value.

As an additional method of forecasting the outcome of each game we also included the result of the original game played in Skane with Swedish water planners. We then obtained the results presented in tables 5, 6 and 7 below.

From these tables we see:

1. The outcome of the Swedish game, i.e., with the water planners in Skane, is by far the best predictor. (Best in two games, second in one game.)

2. The allocation according to population was the worst and the one according to demand the second worse in all three games by large margins. This is a conclusion which also holds for all other games played.

3. The SCRB was placed third from the bottom in everyone of the three games. The SCRB fared better in earlier games.

4. The difference in the outcome of the four game theoretic method is not very great. The Shapley Value is, however, the best predictor as it was best in two games. Although it did not rank so well in the third game the Shapley Value still led to very low difference values. As regards the three Core methods, the Ordinary Nucleolus, which in this game violates monotonicity, fared better than the Weak Nucleolus and the Proportional Nucleolus, which do not violate monotonicity. It is interesting to note that this

Table 5: Game 1

		difference measure 1	2	3
1	Swedish game	5.00	7.87	0.45
2.	Shapley Value	5.78	8.91	0.56
3.	Ordinary Nucleolus	6.75	9.49	0.97
4.	Weak Nucleolus	7.05	14.03	2.28
5.	Proportional Nucleolus	7.96	17.19	3.23
6.	SCRB	11.02	23.54	1.73
7.	Demand	31.21	214.19	13.16
8.	Population	51.25	585.28	45.80

Table 6: Game 2

		difference measure 1	2	3
1.	Swedish game	1.34	0.45	0.03
2.	Shapley Value	4.80	4.39	0.34
3.	Ordinary Nucleolus	5.39	7.56	0.81
4.	Weak Nucleolus	6.83	14.14	2.05
5.	Proportional Nucleolus	7.82	16.23	2.83
6.	SCRB	10.36	22.30	1.64
7.	Demand	35.65	278.55	16.87
8.	Population	55.07	681.03	51.08

Table 7: Game 3

		difference measure 1	2	3
1.	Ordinary Nucleolus	3.09	2.79	0.28
2.	Swedish game	3.66	3.63	0.47
3.	Weak Nucleolus	3.85	4.10	0.42
4.	Proportional Nucleolus	4.84	5.31	0.72
5.	Shapley Value	5.32	7.07	0.78
6.	SCRB	9.28	15.88	1.18
7.	Demand	35.73	256.43	16.36
8.	Population	51.91	624.52	46.24

happened in spite of the fact that we, in these three games, informed the players of the monotonicity principle: in games 1 and 2 by instructing players A and K to argue on the basis of this principle and in game 3 by having the parties agree on distributions for two cost levels.

5. The results of the three games appear from tables 5 - 7 to be fairly similar. Although no conclusions can be drawn on the basis of only three games, this is a first indication that the variation in the experimental variables (prize structure and ways of focus on normative aspect) did not play a very strong role. A more formal way of looking at the possible difference due to the experimental design would be to use the result in one game as a predictor of the outcome in another game and then compute the above mentioned difference measures. If we use the result in game 6 as a predictor of the outcome in game 5, we obtain the following values on the difference measures: 1: 4.74; 2: 7.91; 3: 0.42;[1]. If we use the result in game 7 as a predictor of the outcome in game 6 we obtain the following measures: 1: 4.16; 2: 4.35; 3: 0.70. Looking in tables 5 - 7, we see that these values are quite low, roughly equivalent in size to those of the Swedish game.

[1] Using game 5 as a predictor of the outcome of game 6 gives the same values for measures 1 and 2 and a slightly different value (0.44) for measure 3.

These indications that the changes in experimental variables did not matter were supported by observing the playing of the game and by talking afterwards with the players. Although the amounts of money involved in the three games here were higher than in the Swedish and Italian games[1], the prize structure did not appear to have influenced the players.

Furthermore, the way the players in game 3 approached the agreement on two cost levels in no way gave them special reasons to favor the Weak or Proportional Nucleoli. Although the parties did not violate the monotonicity principle when going from one cost level to the other, this consistency would have had an effect in the discussed regard only if the agreement regarding the first cost level had been exactly on the distribution of the Ordinary Nucleolus.

It furthermore appeared that the consultant's report did not appear to matter either. Each player read his report briefly at the start of the game but during the actual negotiations, as far as we could observe, the players neither refered to or reread these reports.[2]

THE CORE CONCEPT AND NORMATIVE THEORY

An interesting result of the game, which however cannot be seen from tables 4 and 5, is that in none of the three games does the agreement violate the core. In games 1 and 3 the solution lies inside the core; in game 2 it lies exactly on the boundary. H obtained 9.85 and hence A, K, L, M and T together obtained 83.82-9.85= 73.97. As can be seen from table 1 they could have obtained this just as well by forming the five party coalition AKLMT.

In this respect the outcome of these three games differs from the four games played earlier. The core was violated in all of these four games. One can wonder to what extent the pre-information regarding the core concept influenced the parties in these three games.

Here, it is of particular interest to make a comparison with the Swedish and Italian games, since the results in these two games corresponded closely to the results of the three games

1 Ten Austrian schillings corresponded in games 1 and 3 to one million Swedish crowns in the original game, implying that a total of approximately AS250 were at stake in these games. In the Swedish game 1 crown (roughly 3 schillings) corresponded to one million crowns. In the Italian game the corresponding amount was only 100 lire.

2 It should be noted that in group 1, involving a game theoretician (see footnote 2, P.9) knowing all the arguments for the Proportional Nucleolus, the Shapley Value was on top of the game theoretical methods, while the Proportional Nucleolus fared worse.

presented here.[1] In the Swedish and Italian games the violation of the core consisted in party H paying less than his "individual marginal cost" 9.85 (see table 4). As noted above, if H pays less than 9.85 in the grand coalition, AKLMT have reasons to throw out H and instead form a five person coalition.

While checking whether a particular solution is in the core in general is rather cumbersome, checking that a party shall pay his "individual marginal cost" is easy to compute. In particular, since the core idea in the introductory presentation started by presenting the idea of individual marginal cost coverage, it is possible that the introductory lecture could account for the difference between these three games on the one hand and the Swedish and Italian games on the other hand as regards the violation of the core principle.

In earlier papers we have noted that the idea of individual rationality is a powerful one[2] even from a predictive point of view. It appears after these three gaming experiments reasonable to regard the idea of individual marginal cost coverage as a powerful idea from the normative point of view, in the sense that one can easily influence players to behave accordingly.

On the basis of our experience from these games we would like to suggest the following hypothesis. It will be easier to get players to follow the ideas of individual marginal cost coverage than the more general idea of marginal cost coverage extended to groups of users (which in connection with the full cost principle is equivalent to the idea of group rationality). Further experiments would, however, be required for the confirmation of this hypothesis.

In line with this hypothesis we would finally like to suggest the following very simple aid for cost allocation situations of this type, namely to supply all parties with information regarding what should be their highest respectively lowest payments on the basis of the ideas of individual rationality and individual marginal cost coverage.

For this game a table of the following type is suggested:

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[1] As regards the Swedish game the difference measures are shown in tables 5 - 7. The average difference measure values for the Italian game, when used as predicting method for the three games, are 3.45; 3.81; 0.33 respectively, that is quite low. The corresponding measures for the games with Swedish Doctoral Students are 17.17; 64.56; 5.12 and 8.67; 20.20; 1.58, i.e., significantly higher.

[2] See WP-80-134, p.12

Table 8: Highest and lowest costs of each party

Party	Remaining 5 player coalition	Cost of grand coalition	-Cost of remaining 5 players coalition	=Lowest Payment	Highest payment =go alone costs
A	HKLMT	83.82	-66.46	= 17.36	21.95
H	AKLMT	83.82	-73.97	= 9.85	17.08
K	AHLMT	83.82	-83.00	= 0.82	10.91
L	AHKMT	83.82	-77.42	= 6.40	15.88
M	AHKLT	83.82	-70.93	= 12.89	20.81
T	AHKLM	83.82	-69.76	= 14.06	21.98

APPENDIX A: GAME INSTRUCTIONS

You have been invited to participate in a simple game. The game concerns the allocation of costs in a water project. This project aims at bringing water to six municipalities. You will represent one of these. On this occasion, as the sole representative of this municipality, you will represent both the producer and the consumer side.

You will participate in this project either completely on your own, or in cooperation with one or several of the other participants in the game, who are acting as representatives for other municipalities.

All in all, representatives of six municipalities, called A, H, K, L, M, and T, participate in the game. All participants (= municipalities) must in some way take part in the water project, but their costs will depend on how they form coalitions with other participants.

Should a municipality not enter into a coalition with any other municipality, it will pay that sum in Table 1 which represents what each municipality would be obligated to pay if acting alone.

Each player can, however, by acting skillfully both during the formation of coalitions and during the allocation of the total costs within the coalition, get away with a lower payment, in some cases, a considerably lower one.

The details of the game are as follows: By lottery, each player is assigned the role of the representative of one of the six municipalities.

1,3 [Next, each player obtains (in the form of an IOU) the aforementioned sum of money corresponding to the maximum amount that he might have to pay, should he participate in the water project completely on his own]

After this, the players sit down around the table and the coalition-formation negotiations can begin.

The players then must try to form coalitions and reach agreement on how much each of the participants in the formed coalition shall pay of the total cost to the whole coalition. This total cost of each possible coalition is seen in the attached table. (Table 1).

3 [As noted in Table 1, the cost of the grand coalition AHKLMT, will either be 838.2 or 878.2 depending on whether the government will give a special subsidy of 40 for the plant of this coalition. No government subsidy will be given to any coalition other than the grand coalition AHKLMT. Whether the subsidy really will be paid or not will not be determined until after the game is finished, with either event being equally likely. Therefore, every grand coalition must register two payment distributions, one for the case when costs are 878.2 and the other for the case when costs are 838.2.]

As soon as the first coalition has been formed and agreement has been reached as to the allocation of the total costs of this coalition among its members, they register the coalition with the game director. He will then record the names of the coalition participants, as well as the payment each of them would make toward the total costs of the coalition. Once a coalition has been registered, its content, i.e., the participants and the cost allocation, is announced to all participants of the game.

A coalition does not come into force, however, until 15 minutes have elapsed since its registration, and then only provided that none of its members has been registered in another coalition during this period. Hence a player can leave one coalition and join another in order to decrease the amount of his payment. Furthermore, a coalition dissolves by registering a new coalition with additional members. For new coalitions, the rule still applies that it does not come into force until it has been registered unchanged for 15 minutes.

If the players of a coalition before these 15 minutes have elapsed still want to remain in the game, they can do so by once more registering the same coalition as previously. The players will then remain in the game for at least another 15 minutes.

Once a coalition has come into force, each of its members confirms with the game leader that his municipality is willing to pay the amount agreed upon at the time of the registration. These participants then cease to take an active part in the game, but may remain at the table if they wish to do so.

The game continues in this way until all participants are members of a coalition which has come into force (with the possible exception of a single "leftover" participant). Should the game continue more than 90 minutes from the time of its start, it will be brought to an end and those coalitions registered (but not broken) at the time will come into force.

1,3 [At the end of the game, each player pays the game leader the amount he is registered for, or if he is not a member of any coalition, his costs when going alone. He can then use his initially obtained IOU and can then keep the surplus.]

2 [A money prize will be given to that player who according to the judgement of the game leader acts as the most skillful representative for his municipality among the six players taking part in the game.]

Finally, it should be stressed that the aim of the game is to shed light on how municipalities negotiate in the field of water resources and what would be reasonable cost allocation models in this area. Hence it is important that you try as much as possible to act as one could expect a representative for a municipality to act during such negotiations where the economic interest of the municipality are at stake.

You have also been supplied by a consultant with information regarding a method that he believes that you should argue for. You are quite free to use or disregard the arguments. It should be mentioned that the other players have received similar information concerning other methods.

Table 1. Total Cost of Each Possible Coalition (in Austrian shillings).

A	219.5	AHK	407.4	AHKL	489.5
H	170.8	AHL	432.2	AHKM	602.5
K	109.1	AHM	555.0	AHKT	627.2
L	158.8	AHT	566.7	AHLM	640.3
M	208.1	AKL	487.4	AHLT	652.0
T	219.8	AKM	534.0	AHMT	741.0
		AKT	548.5	AKLM	639.6
AH	346.9	ALM	530.5	AKLT	707.2
AK	328.6	ALT	598.1	ALMT	734.1
AL	378.3	AMT	613.6	HKLM	480.7
AM	427.6	HKL	272.6	HKLT	492.4
AT	439.3	HKM	425.5	HKMT	593.5
HK	229.6	HKT	449.4	HLMT	644.1
HL	250.0	HLM	458.1	KLMT	566.1
HM	378.9	HLT	469.8	AKMT	722.7
HT	390.6	HMT	564.9	AHKLM	697.6
KL	267.9	KLM	420.1	AHKMT	774.2
KM	314.5	KLT	487.7	AHLMT	830.0
KT	328.9	KMT	503.2	AHKLT	709.3
LM	311.0	LMT	514.6	AKLMT	739.7
LT	378.6			HKLMT	664.6
MT	394.1				
				AHKLMT	838.2 or 878.2

DATA ON WATER DEMAND

	A	H	K	L	M	T
Water Demand: (Mm^3/yr)	6.72	8.23	3.75	3.53	14.64	5.39

APPENDIX B:

INFORMATION FOR PLAYER A

As seen from Table 2 on allocations according to different methods, the Weak Nucleolus appears to be the one most favorable to you (the Shapley value gives you slightly smaller costs but this might not have much chance since it does not lie in the core; see below).

The main idea behind the Weak Nucleolus is as follows: First of all, it lies within the Core. The Core is based on the following three principles:

1) Individual Rationality:
No municipality shall pay a higher cost than it would have to pay, if it were to fulfill its water needs completely on its own.

2) The "Full Cost" Principle:
Total costs should be covered, leaving no surplus and no loss to any third party.

3) "Group Rationality", implying that the sum of payments made by the members of every coalition which is smaller than the grand coalition, should not be larger than the cost that this coalition incurs if it is working on its own.

Demand proportional allocations violate the principle of individual rationality, e.g., for M, who on his own can get away with paying 20.81.

The SCRB and the Shapley value violate the principle of group rationality. According to the SCRB procedure, HKL shall together pay 29.80 and according to the Shapley value 27.69. Should they not join the grand coalition, but remain satisfied with the three-party coalition HKL, they would only have to pay the cost of this coalition, 27.26.

There are, however, a great many solutions in the core. One way to obtain a unique solution in the core is to assume that one gives subsidies to the various subcoalitions so that one obtains a unique core.

One simple system of such subsidies is the Weak Nucleolus. Then each player obtains the same subsidy in every coalition.

The Weak Nucleolus is better than the Ordinary Nucleolus since the Ordinary Nucleolus does not obey the following principle: If costs go up, no one shall pay less; if costs go down, no one shall pay more.

INFORMATION FOR PLAYER H

As seen from Table 2 on allocations according to different methods, the Shapley value is the one giving you the lowest costs. The main idea behind this value is as follows:

The grand coalition is formed step by step; first one party joins together with another part to form a three-party coalition, and then another party is added to form a four-party coalition, etc., until finally the grand coalition is formed. There are in this 6 player game, 720 ways or orders in which such a procedure can take place, depending on which party "signs up" first, and which party "signs up" next. For each order, a party joining a coalition is thought only to pay the incremental costs (i.e., the difference between the cost of the new coalition and the cost of the one he joins). The Shapley value for each party is then the party's <u>average</u> payments, computed over all 720 coalition formation orders.

The Shapley value in the table has been computed by a computer program along this line of reasoning.

INFORMATION FOR PLAYER K

As seen from Table 2 on allocations according to different methods, the Proportional Nucleolus is the one giving you the lowest costs.

The main idea behind the Proportional Nucleolus is as follows: First of all, it lies within the Core. The Core is based on the following three principles:

1) Individual Rationality:
   No muncipality shall pay a higher cost than it would have to pay, if it were to fulfill its water needs completely on its own.

2) The "Full Cost" Principle:
   Total costs should be covered, leaving no surplus and no loss to any third party.

3) "Group Rationality", implying that the sum of payments made by the members of every coalition which is smaller than the grand coalition, should not be larger than the cost that this coalition incurs if it is working on its own.

Demand proportional allocations violate the principle of individual rationality, e.g., for M, who on his own can get away with paying 20.81.

The SCRB and the Shapley value violate the principle of group rationality. According to the SCRB procedure HKL shall together pay 29.80 and according to the Shapley value 27.69. Should they not join the grand coalition, but remain satisfied with the three-party coalition HKL, they would only have to pay the cost of this coalition, 27.26.

There are, however, a great many solutions in the core. One way to obtain a unique solution in the core is to assume that one gives subsidies to the various subcoalitions so that one obtains a unique core.

One system of such subsidies is the Proportional Nucleolus. Then each subcoalition obtains a subsidy that is proportional to its costs.

The Proportional Nucleolus is better than the Ordinary Nucleolus since the Ordinary Nucleolus does not obey the following principle: If costs go up, no one shall pay less; if costs go down, no one shall pay more.

The Proportional Nucleolus also is better than the Weak Nucleolus, since the Weak Nucleolus does not obey the following principle: A player who never contributes to any cost savings when joining with other parties or coalitions, shall not realize any cost savings above his go alone costs.

INFORMATION FOR PLAYER L

As seen from Table 2 on allocations according to different methods, the Ordinary Nucleolus is the one giving you the lowest costs (besides the Demand Proportional allocation method but this might not have much chance since it does not lie in the core; see below).

The main idea behind the Ordinary Nucleolus is as follows: First of all, it lies within the Core. The Core is based on the following three principles:

1) Individual Rationality:
No municipality shall pay a higher cost than it would have to pay, if it were to fulfill its water needs completely on its own.

2) The "Full Cost" Principle:
Total costs should be covered, leaving no surplus and no loss to any third party.

3) "Group Rationality", implying that the sum of payments made by the members of every coalition which is smaller than the grand coalition, should not be larger than the cost that this coalition incurs if it is working on its own.

Demand proportional allocations violate the principle of individual rationality, e.g., for M, who on his own can get away with paying 20.81.

The SCRB and the Shapley Value violate the principle of group rationality. According to the SCRB procedure, HKL shall together pay 29.80 and according to the Shapley Value 27.69. Should they not join the grand coalition, but remain satisfied with the three-party coalition HKL, they would only have to pay the cost of this coalition, 27.26.

There are, however, a great many solutions in the core. One way to obtain a unique solution in the core is to assume that one gives subsidies to the various subcoalitions so that one obtains a unique core.

The best known system of such subsidies is the Ordinary Nucleolus. Then each subcoalition obtains the same subsidy.

INFORMATION FOR PLAYER M

As seen from Table 2 on allocations according to different methods, the SCRB method is the one giving you the lowest costs. This method has been developed specifically for practical use in water resources planning.

We define the marginal cost for a party as the marginal cost of being the last to join the grand coalition. The idea behind the SCRB method is then that each player should first of all pay his marginal cost. Then certain unallocated costs would remain. These would then be shared in relation to the remaining benefit of each player.

The "remaining benefit" is defined as the difference between the cost if the municipality goes alone and its marginal costs. The payment made by a party is then computed as the marginal cost plus its share of the non-allocated costs where the share is the party's share of remaining benefits.

For example, your marginal cost is the difference 12.89 between the costs of AHKLMT = 83.82 and the cost of AHKLT = 70.93. The total of the marginal costs is 61.39 and hence, 83.82 - 61.39 = 24.43 remain unallocated.

Your remaining benefit is 20.81 - 12.89 = 7.92. The total amount of remaining benefits can be computed to be 47.23. Hence, your share of the remaining benefits is (7.92/47.23) = 0.169. Hence, you pay 12.89 + 0.169 x 22.43 = 16.66.

INFORMATION FOR PLAYER T

As seen from Table 2 on allocations according to different methods, the Demand Proportional method is the one giving you the lowest costs.

This allocation method is the only one in the table which reflects the user side. Allocations according to this principle will imply that the cost of water per cubic meter will be the same in all municipalities. It is hence the only distribution that takes into account some principle of equity on the consumer side. This could be considered to be a factor that should weigh more heavily than the purely "municipal egoistic" reasoning behind the other allocation principles.

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