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Modeling Dualism in Japan

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MODELING DUALISM IN JAPAN

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FOREWORD

Roughly 1.6 billion people, 40 percent of the world's population, live in urban areas today. At the beginning of the last century, the urban population of the world totaled only 25 million. According to recent United Nations estimates about 3.1 billion people, twice today's urban population, will be living in urban areas by the year 2000.

Scholars and policy makers often disagree when it comes to evaluating the desirability of current rapid rates of urban growth and urbanization in many parts of the globe. Some see this trend as fostering national processes of socioeconomic development, particularly in the poorer and rapidly urbanizing countries of the Third World; whereas others believe the consequences to be largely undesirable and argue that such urban growth should be slowed down.

This paper describes a general equilibrium model that has been constructed to analyze the demoeconomic development of Japan in this century. In order to gain deeper insights into the process of intersectoral and interregional labor mobility, special emphasis is placed on the dualistic nature of the labor market, a salient feature of Japan's development experience and of many contemporary developing economies.

A list of the papers in the Population, Resources, and Growth Series appears at the end of this report.

Andrei Rogers
Chairman
Human Settlements
and Services Area

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ABSTRACT

The history of Japan's economic development has attracted much attention because of the speed with which this country has successfully achieved a high rate of industrialization. The process of this development has been aided by traditional elements that have played an active and important role in the economy for a long time, thus allowing modern-traditional dualism to persist until well after World War II.

After briefly reviewing the pattern of "dualistic development" in Japan, this paper exposits an applied general equilibrium model, which has been built to analyze the various demographic and economic forces that have shaped the path of Japan's growth and urbanization. The time periods under study are 1905-1930 and 1953-1963.

CONTENTS

MODELING DUALISM IN JAPAN

1. INTRODUCTION

The main purpose of this paper is to describe a general equilibrium model that will be used for analyzing the economic growth and urbanization in an economy where a labor-market dualism prevails, namely, Japan in a historical setting. Two time periods were chosen for the study: 1905-1930 when economic dualism was forming in Japan and 1953-1963 when it was dissolving. This labor-market dualism can be seen in wage and productivity differentials between modern and traditional sectors. "Modern" sectors are referred to as those sectors that use largely imported (borrowed) technologies from abroad while "traditional" sectors use indigenous technologies in urban as well as agricultural areas.

In reality, however, the dichotomy is not as clear-cut as stated above. Thus, the sector division employed in the model described includes a third manufacturing sector, which could be called a hybrid sector.

This process of growth is by nature complicated with various forces interacting at various levels. Take, for example, urbanization. Throughout the periods being studied, population inflows into large cities continued. This was the result of interaction between at least three labor markets: rural, urban traditional, and urban modern. The situation of each labor market, in turn, was influenced by technologies and factors available and output demands. Through these interactions, the majority of the migrants entered traditional urban labor markets, thereby helping to form and maintain the dualism. But population inflows into cities also increased the public investment in construction of capital stock (social stock). This was one of the factors which kept the aggregate demand relatively high in the 1920s and in turn, helped the recovery of the Japanese economy from the Depression in the 1930s. Finally, urbanization did change, albeit slowly, the tastes of consumers changing the relative prices of various commodities.

The model described in this paper has been built taking into account these major demoeconomic forces and their interactions: private investments, wage determinations, internal migration, and patterns of consumption. The intended form of analysis is in three steps:

(1) The values of parameters and coefficients are estimated using conditions of the initial time period(s). Then an *ex post* simulation test is carried out, i.e., to see if the model duplicates the history of Japanese economic development with a comfortable degree of precision.

(2) Comparative statics and sensitivity analyses are carried out that give different values to exogenous variables and parameters and examine the reaction of the model to these changes.

(3) This last stage involves exploration of the impacts over time of different patterns of such dynamic features as technical progress, saving and investment, consumers' tastes, and demographic aspects of population, on the process of urbanization and dualistic development in Japan.

Why choose historical Japan for such an analysis? There is a vast amount of literature describing Japanese economic history. It is because of these detailed descriptive and statistical works, which have already been carried out, that a more holistic and systematic approach is possible.

The recent development in the field of computable general equilibrium modeling and the success of Kelley and Williamson (1974) in finding some interpretations of Japanese experiences in 1885 to 1915, which are different from conventionally accepted ones, make it more reasonable and desirable to pursue similar efforts. Kelley and Williamson use a model that has only two sectors and does not include government. Also, they study only the time period that is termed Phase I (Ohkawa 1980) or the balanced growth phase (Nakamura 1971) and exclude the period of urban dualistic development. This period is well known among development economists and economic historians as a salient feature of Japanese economic growth—and possibly a relevant feature for contemporary developing countries. The Kelley/Williamson model can be extended to include some of these various other features that could be important to the understanding of Japanese experiences. Because Japan is the only non-Western country that has successfully carried out the "modern economic growth" defined by Kuznets (1966), the dualistic development in historical Japan, we believe, is worthy of another study with this new approach: general equilibrium modeling and the use of dynamic counterfactuals.

In the next chapter we briefly review "dualistic development."* The third chapter describes the model in detail and chapter 4 discusses the possible extensions and modifications. Concluding observations are given in chapter 5.

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*This section is not intended as a full survey of the topic. For more detailed discussion see Ohkawa (e.g., 1972), Ohkawa and Rosovsky (1973), Nakamura (1971), Minami (1973), and the International Development Center of Japan reports in the references.

2. A BRIEF HISTORY OF JAPAN'S DUALISTIC DEVELOPMENT

2.1. A General Overview

Past experiences of Japan have received considerable attention because of her rapid rate of growth from the second half of the 19th century. It is not only that she grew continuously, but also that there was a noteworthy trend of accelerated growth during this period,* starting with a moderate rate of growth and gradually increasing. At the same time, long swings, or fluctuations, in the economic growth occurred. Japan experienced 3.5 long swings between the end of the Matsukata Deflation (1887) and the 1970s. Each cycle was about 20 years long. This "trend acceleration" is easily observed in Table 1 where the swings are controlled. For example, the average annual growth rates of gross national expenditure (GNE) during upswing periods are 3.21 percent (1887-97), 3.30 percent (1904-19), 4.88 percent (1930-38), and 9.56 percent (1953-69). The growth acceleration has also been accompanied by structural change in the economy.

Table 1. Long-term pattern of aggregate growth rates, with a constant price series: average annual growth rates (in percent) (U) = upswing, (D) = downswing.

Period (length in years)	GNE	Total Population	Per Capita GNE
Long-swing phases			
(U) 1887-1897 (10)	3.21	0.96	2.25
(D) 1897-1904 (7)	1.83	1.16	0.67
(U) 1904-1919 (15)	3.30	1.19	2.11
(D) 1919-1930 (11)	2.40	1.51	0.89
(U) 1930-1938 (8)	4.88	1.28	3.60
(D) 1938-1953 (15)	0.58	1.36	-0.78
(U) 1953-1969 (16)	9.56	1.03	8.53

Source: Ohkawa and Shinohara (1979) page 10.

*Ohkawa and Rosovsky (1973) call this "trend acceleration" of the Japanese economy.

The structural changes can be observed in sectoral shifts of production. The industrial share in net domestic products, for example, increased from 20 percent in 1887* to 52 percent in 1938*, while the agricultural share decreased from 42.5 percent to 18.5 percent during the same period. Within the industrial sector there was also a continuous shift from light to heavy or from traditional to modern industries.

These shifts were a result of the introduction (importation) of advanced technologies from abroad into a largely agrarian and traditional society: the fate of a late starter in industrialization. It was, therefore, only natural that there was a period of time in the history of Japan when traditional (indigenous) elements and modern (imported) elements coexisted. The period when the traditional elements played an important, albeit changing, role was fairly long. They were not to be eliminated through competition immediately after modern elements were brought into the economy. The years of Japanese economic development, therefore, can be said to fall into a number of distinct phases in accordance with the different roles traditional and modern elements played. Ohkawa (e.g., 1980)** divides them into four phases:

Phase 0	1868 ~ 1885
Phase I	1885 ~ World War I (1915-19)
Phase II	World War I ~ 1958-62
Phase III	1958-62 ~ present

Phase 0 was an initial period where the institutional changes that were required for the modern economic growth took place. The year of Meiji Restoration (1868), the date when the modern economic growth became the national objective, is considered the first year of Phase 0, although some modern elements already existed before that time.

Phase I was the period when the modernization largely depended on the traditional elements both in agriculture and nonagriculture. The dependency took the form of financial sources of growth (taxes and savings), foreign exchange (exports), and supplies of labor and food.

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*These are the center years of five-year moving averages. Figures are from Ohkawa and Shinohara (1979:35).

**The rest of this section is based on Chkawa's argument at large.

Phase II was the period when the modern sectors started to grow more independently from the traditional sectors than before. The strong dependency of the modern sectors on the traditional sectors as the source of labor supply, however, continued. This period started with the first investment spurt in the modern sectors and saw a fluctuating but continuous growth of modern elements, while the growth potential of the traditional sectors faced some constraints. The so-called "dualistic development patterns" was, therefore, firmly established toward the beginning of this phase and continued until well after the end of World War II.

Phase III started around 1958-62 and continues until now. It is a phase when modern elements dominate all sectors of the economy.

A quick glance at exports also endorses the changing but important role played by the traditional sectors. In the very initial phase of economic development after 1868 (the Meiji Restoration), raw silk, silk worm cocoons, and the green tea exports were 85 percent of the total exports. By the turn of the century more industrial goods, produced by traditional manufacturing, became the dominant exports: raw silk, other silk goods, cotton goods and other miscellaneous goods. Textile goods were the most important exports until the 1930s, consisting of 60 to 70 percent of all manufactured exports. Within the textile exports, however, there was a remarkable shift from silk to cotton products.

An increase in heavy industry's share of exports started around the 1920s and became dominant, replacing textiles, only after World War II. One well known dynamic feature of the Japanese economy is that exports of manufactured goods were preceded by periods of imports and import-substitution production. [See Akamatsu (1966) "wild-geese formation."]

The role of traditional sectors to finance Japan's high growth is thus obvious. The three primary export commodities in the early phase were products of the agricultural sector. The major exports from the turn of the century were manufactured textiles and miscellaneous goods produced by the traditional manufacturing sector without imported technology or by sectors with imported but largely modified technologies (e.g., cotton, matches).

Among the different phases of Japanese experiences, we will focus our attention on Phase II because this is the period when the "dualism" occurred and persisted, and will study the mechanism of urbanization and growth in the dualistic economy using the model presented in chapter 3 of this paper. The next section reviews some features of dualistic development in more detail.

2.2. Dualistic Development: A Closer Look

The second phase of Japanese economic development started from the time of the private investment spurt with an almost unlimited increase in export demand due to World War I. This spurt increased the size of firms in general and in particular the heavier industry firms. The average annual growth rate of the manufacturing sector during World War I was an extraordinary 9.3 percent. But the growth rates were quite uneven within the sector: machinery grew at 28.1 percent, metal grew at 10.7 percent, textiles at 4.1 percent, and miscellaneous goods at 2.2 percent (Yasuba 1966).

The distribution of income was also strikingly uneven. The real wage rate of urban workers and real agricultural income decreased until 1918, while those who received profit incomes were highly benefitting from the boom. The deteriorating income distribution during the economic boom increased dissatisfaction among the poor, paving the road to the rice riots of 1918.

The stagnant Japanese agriculture after World War I is also noteworthy. This was partially caused by the world-wide agricultural crises when supply increased as demand decreased because of deflationary pressures. These were due to the efforts of all industrialized nations for the restoration of the gold-standard to pre-war rates. Another reason for the stagnation of Japanese agriculture was a shift of government policy after the rice riots of 1918. This policy no longer protected the farmers but appeased the urban workers. Imports of cheaper rice from the colonies were promoted at the cost of accelerating the decline of domestic agriculture. To make matters worse, the increasing urbanization decelerated the growth of consumers' demand for traditional food. As a result, there was a drastic relative decline in agricultural income. The rural area, therefore, no longer had the capacity to support its population, causing an increase in labor surplus.

The "push" pressure of the rural areas increased, but the "pull" factors in urban areas were limited for two reasons. First, even during the boom, investment was largely concentrated on more capital intensive modern industries, and the gross employment effects were limited. Second, the rate of turnover in modern industry started to decline drastically, partly because of the increasing capital intensity which caused technology-specific skills to become important. With formal schooling giving only general background training, almost all the skill training was done on the job. Once skills were acquired it was in the interests of both employers and employees to have employees stay with the same firm. The wage policy of employers in modern industry, therefore, can be regarded as minimizing the sum of wages and loss of training costs through turnovers.* The prevailing wage levels in traditional sectors were much less relevant to the wage of modern sectors than any economic textbook might suggest.

The limited demand for labor in the modern industrial sector, coupled with the decline of the population-supporting-capacity of the rural areas, made the alternatives that any job seekers had quite limited. The surplus labor flowed into various urban traditional sectors. The number of gainfully employed in small-scale manufacturing, retail services, and other traditional services increased drastically. For example, the number increased by 101 percent for retail and wholesale services, and by 42 percent for other services (Nakamura 1971). The increase of labor in these sectors pushed the wage levels down thus giving birth to wage differentials between different sectors and between different sizes of firms.**

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*In addition, modern sector employers had incentive to pay higher wages in order to isolate their workers from the uprising restlessness of urban laborers and to keep the "cream of the labor force" attached to them.

**Before World War I, wage differentials were of a spatial nature, i.e., the segmented labor markets with limited spatial labor mobility and flow of information were mainly responsible for the differentials, while the dualism born after World War I was formed with a relatively high factor mobility and flow of information.

The interaction between urban labor markets and rural labor markets was, therefore, quite important. Wage levels went down with the decline of agricultural income not only for traditional sector employees but also for female workers (in mainly textile sectors). These female workers had been recruited directly from the rural areas and had a high turnover rate.

The question posed from a neoclassicist point of view, then, is why this dualistic disequilibrium phenomenon persisted for so long. It is hoped that, with the help of the model presented in the next chapter, we will gain more insights into this question. A plausible hypothesis behind the model is that this dualism occurs with a rapid technical or technological progress accompanied by an increased demand for technology-specific skills. Both the firms that own the new technology and the skilled labor that uses the technology, earn quasi-rents. This situation can last a long time if the investment that embodies new technologies stays at a high level, restricting the diffusion of the latest technologies.

Both workers and employers have a vested interest in each other, since the skills workers now have are technology-specific. It is disadvantageous for the worker to quit so long as other opportunities are no more attractive than the present job, and it is costly to the employer if the worker does quit. So, the employers pay higher wages to reduce the number of resignations, and workers stay in the same firm, possibly increasing their efficiency through learning-by-doing. It is a type of bilateral monopoly (Tan 1980).

This hypothesis also assumes the continuing existence of surplus labor in the traditional sector. Without this surplus the wage level of traditional sectors would have been pushed up to a level where workers in modern sectors would have not felt as much incentive to stay in the same firm as they actually did. This is what occurred in the 1960s in Japan when the surplus labor in the traditional sector was exhausted and the economic dualism largely disappeared (along with the diffusion of technologies and the productivity increase in agriculture).

From the observations and hypotheses presented, it is anticipated that the investments in new technologies and an elastic labor supply are quite important in forming the dualism seen in post-World War I Japan. The model described in the following chapter emphasizes these aspects.

3. THE MODEL

3.1. General Features

The model described below is built to capture aspects of Japanese economic development in this century as described in the previous section. Its specific features are as follows:

- It is a vintage model in which each capital stock in some sector is specified by the year of installation.
- It distinguishes the *ex ante* and the *ex post* production possibility sets, and it assumes putty-semiputty technologies for various urban sectors with varying elasticities of substitution.
- It assumes that the productive physical investment is carried out by investors with differing expectations, albeit myopic, on future factor and commodity prices and on demand.
- It defines the rural-urban labor migration as depending largely on urban job opportunities as well as on the difference in (expected) income differentials deflated by cost-of-living indices.
- "Trade" has three specifications (1) all exports are given the Armington specifications, (2) all raw materials and machinery imports are considered noncompetitive, and (3) all industrial finished goods are considered competitive and given the Armington specifications.

3.2. Sector Division

The manufacturing sector is divided into three subsectors in order to capture differences in technologies employed within the sector. It is assumed that the size of firms reflects the degree of indigenousness of the technology employed. As one of the criteria for determining whether an industry belongs to the modern or the traditional subsector, therefore, the proportion of small-scale firm employment to total employment in one industry is considered.*

*The pre-World War II period firms with five or less employees and the post-World War II period firms with 49 or less employees are considered as small-scale firms.

In addition, several other criteria are employed such as capital-labor ratios, and labor and capital productivities. For the pre-World War II period, the conventional modern-traditional dichotomy is applied to these criteria, but a third subsector, the textile industry, is singled out because of its important role in the development of Japanese economy in this period.

The service sector in the urban area is divided into two subsectors: capital-intensive modern services, which includes transportation, communication, electricity, and other modern services, and traditional services, which include retails, wholesales, personal services, financial services, or carpentry for residential buildings.*

Construction is considered as another sector due to its specific characteristics, namely, nontradeability and high labor-capital ratio.

For the rural region, economic activities consist of the primary (agriculture), and the rural service sectors. "Agriculture" also includes sericulture, fishery, and forestry. The rural service sector *should* include rural retails, rural traditional transportation, rural personal services, rural carpentry, etc., but due to the data limitation, only rural carpentry is considered as belonging to this sector in the pre-World War II period.

Finally, we have two additional sectors to account for rental flows in urban and rural housing.

The difference between the pre- and the post-World War II divisions is more apparent than real. What is considered modern in the pre-war period can be considered in the medium range of capital intensity in the post-war period, e.g., machinery and printing. The singling out of the textile industry for the pre-war period is only natural given the role it played as a major export sector, with modern or modified modern technology utilizing the unskilled (female) labor force from rural areas. This distinct role was not played by the textile industry or any other "single" sector in the post-World War II period.

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*The banking and other financial services will be considered as traditional, mainly because of the way statistical data were recorded in pre-World War II periods.

Table 2. Sector divisions in Japan's model.

		1905-1930	1950-1963*
Urban	Industry I	Modern industry (MM)	High capital intensity industry (HK)
		Chemical Machinery Metal Mining (Printing)[a] (Stone, clay & glass)[a]	Petroleum Steel Nonfernous Chemicals (Ceramics)[a] (Pulp & paper)[a]
	Industry II	Textile industry (TE)	Moderate capital intensity industry (MK)
			Machinery (Mining)[a] Textile Printing Rubber
	Industry III	Traditional Industry (TM)	Low capital intensity industry (LK)
		Food Lumber & wood Miscellaneous	Clothes Leather Furniture Metals Wood Food Miscellaneous
	Construction	Construction	Construction
	Urban service	Modern service (MS)	Modern Service (MS)
		Modern transport Electricity Other public services	Modern transport Electricity & other utilities Public services Banking & modern finances
		Traditional service (US)	Traditional service (US)
		All the rest of service activities Urban housing construction	Retail & wholesale services Personal services Urban housing construction
Rural	Rural service	Rural service (RS)	Rural service (RS)
		Rural housing construction	Rural housing construction (Retail in rural areas)[b]
	Agriculture	Agriculture Fishery Forestry Sericulture	
	For rental flows	Urban housing Rural housing	

Parentheses mean tentative for reasons of:

[a]being on the borderline of sector divisions
[b]data possibly not being available

*The post-World War II sector division of manufacturing depends very much on the work of Motai and Ohkawa (1978).

3.3. Production Relations

Persistent wage differentials between modern and traditional sectors in Japan have puzzled many economists and have been a source of heated arguments. It was not only the wage differentials that were persistent but also the very existence of the traditional sectors. In the stereotyped concept of modernization, traditional sectors are residual and are supposed to be phased out and replaced by more modern sectors as the country develops. This did not occur in Japan for a long time. Many authors have tried to explain this persistent (if not happy) coexistance with lasting wage differentials as discussed in the previous chapter.

There have been various attempts at explaining Japan's economic development. In this paper I will try to explain it from the economic and technological points of view, not only because these aspects are easier to quantify, but also because I believe they should be elaborated first. These aspects are less culture-specific and are therefore useful for finding any relevance to, or comparability with, the experiences of other countries.

The maintained hypotheses are:

- The *ex post* elasticity of substitution of the modern sector's technology is much smaller than that of the traditional sector.
- The physical life time of equipment in the modern sector is much longer than in the traditional sector.
- The equipment of the modern sector is, by definition, "borrowed" technology, while the technology used in the traditional sector is mainly indigenous.
- The investment of each sector is carried out at the most efficient point with a given expected factor price ratio.
- The technical progress is more rapid and labor saving in the modern sector.

Most of these can be clearly shown by the following diagram of isoquants:

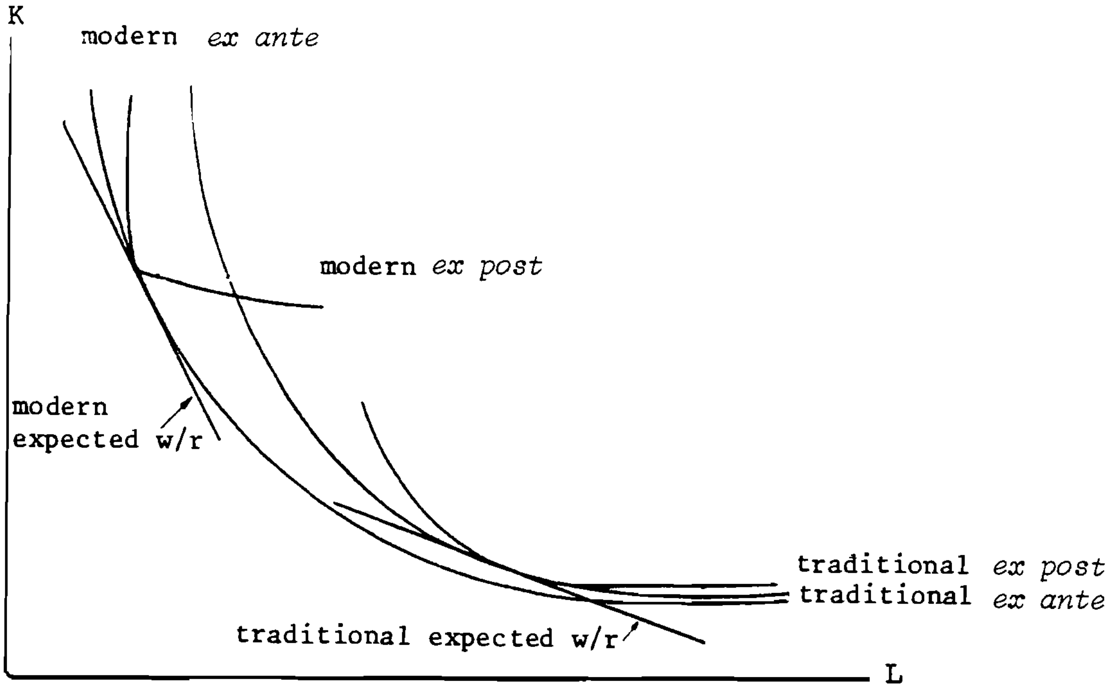


Figure 1. Different *ex ante* functions and different wage-rental (w/r) expectations.

The implicit scenario is that because of the limited substitutability in the modern sectors, available capital stock and new investment almost determine the sectors' employment of labor. The rest of the urban labor force will then have to seek employment in the nonmodern urban sectors: i.e., traditional sectors, where, due to the high elasticity of substitution, employees can be hired, but with lower wages.

Finally, where capital stock is concerned, a distinction is made between structure and equipment. Capital structure (KC) is supplied by the construction sector and has a longer lifetime than capital equipment (KE), which is supplied by the modern manufacturing sector.

Production relations are formally expressed in the following production functions. The indices are:

1 = Industry I (MM for 1905-1930; HK for 1950-1963)
2 = Industry II (TE for 1905-1930; MK for 1950-1963)
3 = Industry III (TM for 1905-1930; LK for 1950-1963)
4 = Construction

5 = Modern service
6 = Traditional urban service
7 = Rural service
8 = Agriculture and primary activities (including sericulture)
9 = Urban housing
10 = Rural housing

Production Technology

$i = 1,2,3,5$

Ex ante

$$Q_i = A_i \left\{ \hat{\phi}_i \left[\hat{\xi}_i \left(\hat{\varepsilon}_i \, \underline{S}_i^{\hat{\delta}_i} + (1 - \hat{\varepsilon}_i) \, \underline{KE}_i^{\hat{\delta}_i} \right)^{\frac{\hat{\eta}_i}{\hat{\delta}_i}} + (1 - \hat{\varepsilon}_i) \, \underline{KC}_i^{\hat{\eta}_i} \right]^{\frac{\hat{\zeta}_i}{\hat{\eta}_i}} + (1 - \hat{\phi}_i) \, \underline{L}_i^{\hat{\zeta}_i} \right\}^{\frac{1}{\hat{\zeta}_i}} \qquad (1)$$

where ^ means *ex ante* and a bar under a variable means it is measured in efficiency units.

Ex post

$$Q_{iv} = A_i \cdot A_{iv} \left\{ \phi_{iv} \left[\xi_{iv} \left(\varepsilon_{iv} \, \underline{S}_i^{\delta_{iv}} + (1 - \varepsilon_{iv}) \, \underline{KE}_{iv} \cdot e^{-\sigma_E (t-v)} \right)^{\frac{\eta_{iv}}{\delta_{iv}}} + (1 - \xi_{iv}) \underline{KC}_{iv} \, e^{-\sigma_C (t-v)} \right]^{\frac{\zeta_{iv}}{\eta_{iv}}} + (1 + \phi_{iv}) \, \underline{L}_i^{\zeta_{iv}} \right\}^{\frac{1}{\zeta_{iv}}} \qquad (2)$$

where v = vintage

$$\phi_{iv} = \frac{\hat{\phi}_i}{\hat{\phi}_i + (1 - \hat{\phi}_i)(k^*)^{\zeta_{iv} - \hat{\zeta}_i}}$$

$$\xi_{iv} = \frac{\hat{\xi}_i}{\hat{\xi}_i + (1 - \hat{\xi}_i)(h*)^{\eta_{iv} - \hat{\eta}_i}}$$

$$\varepsilon_{iv} = \frac{\hat{\varepsilon}_i}{\hat{\varepsilon}_i + (1 - \hat{\varepsilon}_i)(s*)^{\delta_{iv} - \hat{\delta}_i}}$$

where

k* = the efficient composite capital (S,KE,KC) labor ratio at given factor price ratios and a given *ex ante* production function.

h* = the efficient skill and equipment composite (S,KE) and structure capital (KC) ratio at given factor price ratios and a given *ex ante* production function.

s* = the efficient skill-equipment capital ratio at given factor price ratios and a given *ex ante* production function.

A_{iv} = a sector and vintage specific parameter (see Appendix 1).

(There is a one-to-one correspondence between production sectors and commodities.)

i = 4 (construction)

$$Q_4 = A_4 \left[\varepsilon_4 \, \underline{KE}_4^{\rho_4} + (1 - \varepsilon_4) \, \underline{L}_4^{\rho_4} \right]^{\frac{1}{\rho_4}} \qquad (3)$$

i = 6,7 (TS,RS)

$$Q_i = A_i \, (\underline{KE}_i)^{\beta_1^i} \, (\underline{KC}_i)^{\beta_2^i} \, (\underline{L}_i)^{1-\beta_1^i-\beta_2^i} \qquad (4)$$

i = 8 (A)

$$Q_8 = A_8 \left[\theta_8 \, \underline{KE}_8^{\rho_8} + (1 - \theta_8) \, \underline{KC}_8^{\rho_8} \right]^{\frac{\beta_1^8}{\rho_8}} (L_8)^{\beta_2^8} \, (\chi_8)^{1-\beta_2^8-\beta_1^8} \qquad (5)$$

$i = 9$

$$Q_9 = A_9 \ (KC_9)^{\beta^9} \ (\chi_9)^{(1-\beta^9)} \tag{6}$$

$i = 10$

$$Q_{10} = A_{10} \ KC_{10} \tag{7}$$

where

Q_i = total output of sector i

A_i = sector-specific shift parameter

A_{iv} = sector- and vintage-specific shift parameter

ϕ_i, ξ_i, ε_i, θ_i = distribution parameters where constant-elasticity-of-substitution (CES) forms are used

δ_i, η_i, ζ_i, ρ_i = substitution parameters where CES forms are used

β^i = production parameters in Cobb-Douglas forms

$S_{i(v)}$ = skilled labor of sector i (and of vintage v)

$L_{i(v)}$ = unskilled labor of sector i (and of vintage v)

$KE_{i(v)}$ = capital equipment of sector i (and of vintage v)

$KL_{i(v)}$ = capital structure of sector i (and of vintage v)

χ_i = land area of sector i

Equations (1) and (2) are for i = 1,2,3, and 5 and a putty-semiputty assumption is used. The *ex post* elasticities of substitution are supposed to be smallest for modern manufacturing, i = 1, and increase as the sectors become more traditional. The parameter values for *ex ante* and *ex post* production functions of the modern service sector will be derived from more empirical data. These three-stage nested constant-elasticity-of-substitution functions, are specified to incorporate the following assumptions:

1. Equipment and skill are most complementary.

2. The composite of these are complimentary, although to a lesser degree, to structures.

3. The unskilled labor is considered as more of a substitute for any of these capital goods.

The construction sector ($i = 4$) is assumed to use only equipment and (unskilled) labor for building roads, harbors, and factory structures, etc. This simplification seems to be permissible given the technology used and the pattern of employment in this sector. Cobb-Douglas specifications are used for traditional service sectors and the primary sector. Capital stock in traditional service sectors includes building structures (usually shared with structures for residential purposes) and some tools.

The Cobb-Douglas specification in agriculture (except for specifying a capital composite where CES is used) is based on past research. Most studies indicate that the elasticity of substitution between factors in this sector is or is close to unity. It is also considered that the disaggregation of this sector is not required in this case study, because Japan has enjoyed relatively unimodal agricultural development.

Some explanation of the specification with regard to embodied technical changes is in order here. This specification increases not only the number of production functions but also the complexities in the following sense.

1. Each vintage should have different input-output relations.

2. Each vintage should have different quality outputs, i.e., newer machines produce better goods.

3. Each vintage can have quite different rates of return on factor inputs.

In the case of point 1, when the input-output data are not available for the pre-World War II period, production functions will be specified in order to include intermediates in the functions in a Cobb-Douglas manner.* That is,

*I am grateful to Warren Sanderson for pointing out that this is possibly easier than estimating the input-output tables.

$$Q_i = (Y_i)^{\alpha^{VA}} \cdot \prod_j (A_{ji})^{\alpha_j}$$

and

$$\alpha^{VA} + \sum_j \alpha_j = 1$$

where

Q_i = total output of sector i

Y_i = total value added of sector i

A_{ji} = input of j good in the production of sector i

α^{VA} = distribution parameter for value added

α_j = distribution parameter for input j

There are problems, however. One is the method of estimation of α, and the other is that only intermediate input substitution through relative price change is accounted for and not input saving technical progress. The estimation of α, for the time being, will only be carried out by simulation—a choice for α that replicates history with a certain degree of precision. The second problem would be the topic of future research. For the post-World War II period, where the input-output data are more available, both value-added and total-output specifications could be tried; the former would use variable rather than fixed input-output relations.

In the case of point 2 (the quality differences among vintages), it will be assumed that products of the same category are completely homogeneous in the eyes of consumers. They pay the same price and get the same satisfaction from products produced by equipment of different vintages.

In point 3, rates of return on capital will be allowed to vary by vintages,* but the wage rate will be completely the same within each sector for the same kind of labor regardless of the difference in vintage of the equipment used by the laborers.

*Aggregation of capital of different vintages is impossible unless the technical progress is purely capital augmenting, which we have ruled out.

There are two elements determining the lifetime of capital stock: physical and economic. Both will be taken into account. The first is specified by the maximum number of years a certain kind of capital stock could stay in production. This will be determined by exogenously given fixed rates of depreciation, or lifetimes under the one-hoss-shay assumption.*

Economic obsolescence is another element determining the lifetime of capital stocks. This will be taken into account by the following criterion: if any capital stock has a negative net rate of return for two consecutive years, the whole vintage of that sector will be disposed of without any cost regardless of how old that stock is.

3.4. Pricing and Factor Returns

We begin this section by defining the cost of production and the value added price. The cost of production is the minimum cost required to produce the commodity, and the cost per unit of the commodity is the producer's price. This is the total cost of all inputs;

$$p_i = l_i \frac{L_{iv}}{Q_{iv}} + q_i \frac{S_{iv}}{Q_{iv}} + r_{Eiv} \frac{KE_{iv}}{Q_{iv}} + r_{Civ} \frac{KC_{iv}}{Q_{iv}} + \sum_{j=0}^{n} p_j^c \frac{A_{ji}}{Q_{iv}}$$

where

p_i = producer's price

l_i = user cost of unskilled labor

q_i = user cost of skilled labor

r_{Ei} = user cost of capital equipment (quasi-rent)

r_{Ci} = user cost of capital structure (quasi-rent)

p_j^c = purchase price of commodity j

*The production functions in equation (2) only use the fixed-rate-depreciation assumption but the one-hoss-shay assumptions will be tried in some simulation.

A_{ji} = inputs of commodity j to product commodity i
(j = 0 is imports)*

The value added price is defined as

$$P_i^{VA} = P_i - \sum_j P_j^C \frac{A_{ji}}{Q_i}$$

$$= l_i \frac{L_{iv}}{Q_{iv}} + q_i \frac{S_{iv}}{Q_{iv}} + r_{Eiv} \frac{KE_{iv}}{Q_{iv}} + r_{Civ} \frac{KC_{iv}}{Q_{iv}}$$

The *ex post* user costs of primary factors and the noncompetitively imported commodity are in turn defined as:

$$l_i = (1 + t_L) W_i^L + b_i T_i \quad \text{and} \quad \frac{\partial l_i}{\partial W_i^L} = 0 \quad \text{for } i = 1,2,5$$

$$q_i = (1 + t_S) W_i^S$$

$$r_{Eiv} = \left(\frac{1}{1 - t_r} R_{iv}^E + \sigma_i^E\right) P_1$$

$$r_{Civ} = \left(\frac{1}{1 - t_r} R_{iv}^C + \sigma_i^C\right) P_4$$

$$P_o^C = \left[1 + t_{(i)}^{FNC}\right] P_o^W \cdot \tau$$

where

t_L = tax rate on unskilled labor wage (employer's contribution)

t_S = tax rate on skilled labor wage (employer's contribution)

t_r = tax rate on capital profit income

$t_{(i)}^{FNC}$ = tariff rate on noncompetitive imports**

---

*This specification assumes only one kind of imported inputs. But it is relatively easy to disaggregate them, and I shall do so as needed.

**Average tariff rates will be used for most simulations. However, when more detailed aspects of tariff policies need to be studied, sector-specific rates $t_{(i)}^{FNC}$ or $t_{(i)}^{FC}$ will be employed.

W_i^L = wage rate for unskilled labor

W_i^S = wage rate for skilled labor

R_i^E = after-tax rate of return on capital equipment

R_i^C = after-tax rate of return on capital structure

σ_i^E = depreciation rate of capital equipment

σ_i^C = depreciation rate of capital structure

P_1 = price of capital equipment ($\equiv$ price of modern manufacturing sector goods)

P_4 = price of capital structure ($\equiv$ price of construction sector services)

P_o = user price of imported goods

P_o^W = world price of imported goods

τ = exchange rate

b_i = rate of unskilled labor turnover

$b_i = b_i(W_1, \ldots, W_8)$

T_i = sector-specific hiring and training cost of unskilled labor, but $T_i = 0$ for $i = 3,6,7,8$

The above equations are conventional and need no further explanation except for the user cost of unskilled labor. This cost for sector 1,2, and 5 (modern manufacturing, textile, and modern service) is assumed to depend on the hiring and training cost and the rate of turnover as well as wage and tax.* This specification means that even the unskilled require a certain amount of training in these sectors. This training is firm-specific and is not considered as "cost" unless those who received the training leave. This also includes the cost of recruitment—incurred by looking for appropriate employees. This cost was most explicit in the textile

*A similar specification is used for studying migration in Stiglitz (1973). The lost training cost is treated as similar to depreciation for convenience.

sector where the sector had to send recruiters of countryside girls to the rural areas.

The consumers' prices are (1) for domestically produced goods:

$$P_i^c = (1+t_i^{id})\ P_i\ (1+\psi_i)$$

where

t_i^{id} = indirect tax rate

ψ_i = markup for transport cost, $\psi_i = 0$ when consumed in the area of production

and (2) for competitive imports:

$$P_o^c = (1+t^{FC})\ P_o^W \cdot \tau$$

Finally, the neoclassical marginal principles apply with respect to factor returns,* that is,

$$l_i = P_i^{VA}\ \frac{\partial Q_{iv}}{\partial L_{iv}} \qquad \text{for all } v$$

$$q_i = P_i^{VA}\ \frac{\partial Q_{iv}}{\partial S_{iv}} \qquad \text{for all } v$$

*For traditional sectors, of course, these are just imputed shares. For the agricultural sector, I explicitly assume that the population perceive the average income as their measure of welfare, especially in making decisions to migrate. See section 3.5.3. on Migration. As to the urban traditional sectors I assume the marginal principle still holds even in rural-urban migration decisions, thinking that many migrants would not become self-employed proprietors right away. In other words, they would first be employed by a small factory or retail shops. Of course, when self-employed proprietors using family labor decide to become employed laborers, the average family income (not wage income) should then be measured against the wage rate of the possible employment. This intraurban, intersectoral movement is left for a later sensitivity analysis.

$$r_{Eiv} = P_i^{VA} \frac{\partial Q_{iv}}{\partial KE_{iv}}$$

$$r_{Civ} = P_i^{VA} \frac{\partial Q_{iv}}{\partial KC_{iv}}$$

3.5. Factor Allocation

The three kinds of capital goods (skill, equipment, and structures) and labor are allocated over various sectors for production of goods and services. The way the factors are allocated is crucial for an economy; that is, how new factors are being allocated and how the existing labor force is shifting among sectors are the major determinants of the future path the economic-demographic development of the country. The following is the description of how the allocation is determined in the model.

3.5.1. Physical Productive Private Investment

Physical productive private investment applies to both capital equipment and capital structures. Investors are supposed to be fully informed of *ex ante* technical possibilities, and they also have the sector-specific expectations on future factor prices and demand for their output. The most efficient factor employment per unit of output (or value added) could be obtained using the Shephard's Lemma (Shephard 1953) by taking the first order derivative of the unit cost function with respect to the user cost of that factor. Thus,

$$\frac{\partial \mu}{\partial r_E} = \left(\frac{KE}{Q}\right)^*$$

where

$$\mu = \frac{B}{Q}$$

B = the cost function implied by the production technology

$\left(\frac{KE}{Q}\right)^*$ = the optimum capital equipment-output ratio given the technology and the user cost of capital.

It would be assumed that the investors have the expectation on future demand in addition to the factor prices. Suppose then, that D_i^e is the expected demand per year for the sector i commodity, and S_i^A is the actual capacity of supplying this commodity less the depreciation. Then the new capacity for this product needed for the next period is:*

$$DD_i \underset{def}{=} D_i^e - S_i^A$$

The new capital stocks required are the expected investments and can be specified as

$$I^e_{E\cdot i} = P_1 \left(\frac{\partial \mu_i}{\partial \nu^e_{E\cdot i}}\right) \cdot DD_i$$

for equipment in sector i, and

$$I^e_{C\cdot i} = P_4 \left(\frac{\partial \mu_i}{\partial \nu^e_{C\cdot i}}\right) \cdot DD_i$$

*If DD_i turns out negative, an atypical case for a growing economy, it will be assumed that the gross investment in that sector is 0.

for structures in sector i, where $I^e_{E\cdot i}$, $I^e_{C\cdot i}$ are expected investments in sector i for equipment and structures, and $\nu^e_{E\cdot i}$, $\nu^e_{C\cdot i}$ are *expected* costs of the use of capital equipment and structures.

The total investment however is constrained by total available saving although this period's excess demand for investment has some effect on the next period's saving patterns of capital income earners. Thus the actual investment in sector i in capital goods k (k = E,C) is

$$I^A_{k\cdot i} = \frac{I^e_{k\cdot i}}{\sum_k \sum_j I^e_{kj}} \; SAV_p$$

where

j = capital-using sectors (1,2,3,4,5,6,7,8)

k = E and C

SAV_p = saving available for physical productive investment

Finally, the specification of how expectations are formulated is in order.* It is assumed that investors are myopic. Thus

$$D_i^{e\cdot t+1}(t) = \gamma_D \left[D_i^A(t) - D_i^{e\cdot t}(t-1) \right] + D_i^A(t)$$

$$W_i^{e\cdot t+1}(t) = \gamma_W \left[W_i^A(t) - W_i^{e\cdot t}(t-1) \right] + W_i^A(t)$$

$$r_{k\cdot i}^{e\cdot t+1}(t) = \gamma_{r_k} \left[r_{k\cdot i}^A(t) - r_{k\cdot i}^{e\cdot t}(t-1) \right] + r_{k\cdot i}^A(t) \qquad k = E,C$$

$$\nu_{k\cdot i}^{e\cdot t+1}(t) = \gamma_{\nu_k} \left[\nu_{k\cdot i}^A(t) - \nu_{k\cdot i}^{e\cdot t}(t-1) \right] + \nu_{k\cdot i}^A(t) \qquad k = E,C$$

---

*Expectations on factor costs are based on the technology used by the "best practice" firms of the sector.

$$q_i^{e\cdot t+1}(t) = \gamma_q \left[q_i^A(t) - q_i^{e\cdot t}(t-1) \right] + q_i^A(t)$$

where

$X_i^{e\cdot t}(t-1)$ = the expected value of X for time t in sector i, and the expectation is formed at t-1.

$X_i^A(t)$ = the actual value of X at time t in sector i

γ_X = an X-variable-specific positive constant

$\nu_{k\cdot i}$ = the perceived cost of capital in the future, which consists of rate of borrowing and depreciation cost

3.5.2. Public Investment

Public investment in general will be taken as exogenous in the model, mainly because of the very nature of the public investment, which is influenced by exogenous factors; for example, typhoons, earthquakes, and wars.* But a word of clarification should be added here. It is assumed that some government organizations, government-run factories, national railway, electricity generators, and so forth, are in the appropriate sectors. That is, railway and power are placed in MS sectors and public factories in the modern or textile manufacturing sectors. Therefore, the parts that are taken as exogenous in the public investment are public infrastructure construction (both agricultural and general), natural disaster reconstruction, military, religious and cultural facilities construction, furniture and fixture, and nonmilitary building construction. Only public infrastructure construction is really important, and more efforts will be given to endogenize it in the future. Nonmilitary building construction is important in a sense that a good part of it is school buildings. This will be discussed later in Section 3.10 on skill accumulation.

*During the period of 1900-1940, the proportion of the public investment in construction that went into the repair work of damages caused by typhoons and earthquakes varies from 10 to 50 percent, and during the same time period in government equipment investment the proportion of military investment varied from less than 10 percent to close to 90 percent.

3.5.3. Migration

Here only intranational migration in a narrow sense is considered. This means the population movement to Korea, Taiwan, and Manchuria would be exogenous. Also "migration" here would be understood as intersectoral flows of population. The more emphasis is, therefore, given to the shift of population from agricultural to nonagricultural sectors. From our assumption of sector division, this movement is synonymous to rural-urban migration *in the model*. But we should be aware of differences between nonagricultural and urban sectors. In any case, a very simplistic approach is taken with regard to this migration, despite a myriad of works discussing this issue.* The literature from the economic point of view, especially concerning developing countries, has not gone much beyond the Harris-Todaro specification.

I shall also resort to the Harris-Todaro concept and specify migration as a function of a ratio of the expected urban income to the average income in the rural sector. The expected urban income is a weighted average of various urban wages. These weights, however, are different from the conventional Harris-Todaro framework. Here, because labor migration in Japan was highly demand-oriented, the number of available jobs for migrants in each sector will be used as a weight. This weight will consist, therefore, of the number of new jobs created by new investment and the amount of employee turnover.

Thus the rural-urban migration (M_{ru}) is specified as follows:

$$\frac{M_{ru}}{L_r} = m \ln \frac{\tilde{W}_u}{\tilde{W}_r} \qquad (8)$$

where

L_r = rural labor force

m = parameter

---

*For a summary of more or less an economic view, see Todaro (1975) and for a demographic aspects, see Rogers (1977 and 1980).

$$\tilde{W}_u = \frac{1}{COL_u} \cdot \frac{\sum_i \sum_j d^j \theta_i^j W_i^j}{\sum_i \sum_j d^j \theta_i^j}$$

j = skilled (S)
unskilled (L) labor
i = urban sectors

$d^j = d \qquad 0 < d < 1$ if j = skilled

$d^j = 1$ if j = unskilled

$$\theta_i^S = \left[I^A_{KE \cdot i}(-1) + I^A_{KC \cdot i}(-1) \right] \cdot \left[\frac{S_i(-1)}{P_1(-1)\,KE_i(-1) + P_4(-1)\,KC_i(-1)} \right]$$

$$+ b_i^S(-1) \cdot S_i(-1)$$

$$\theta_i^L = \left[I^A_{KE \cdot i}(-1) + I^A_{KC \cdot i}(-1) \right] \cdot \left[\frac{L_i(-1)}{P_1(-1)\,KE_i(-1) + P_4(-1)\,KC_i(-1)} \right]$$

$$+ b_i^L(-1) \cdot L_i(-1)$$

$$\tilde{W}_r = \frac{1}{COL_r} \left[\rho_1 y_o + \rho_2 y_p + (1 - \rho_1 - \rho_2) y_N \right]$$

y_o = average income of owner cultivators

y_p = average income of peasants

y_N = average income of nonagricultural rural workers

ρ_1 = proportion of owner cultivators in the rural population

ρ_2 = proportion of peasants in the rural population

COL_j = cost of living index in area j = $\sum_i \omega_i^j P_i^j$

j = urban, rural

$\omega_i^r = \omega_i \ (E^r)$ $\quad E^r$ = expenditure of rural agricultural labor

$\omega_i^u = \sum_h V^h \ \omega_i \ (E^h)$ $\quad V^h = \frac{\text{population in income class h}}{\text{urban population}}$

E^h = expenditure of income class h

ω_i = budget share on commodity i

d = discount factor for skilled employment

3.6. Consumer's Demand

3.6.1. Static Consumption

For more than two decades economists have explored specifications of a complete system of demand equations that are able to extract various price and income responses from (usually) limited data, at the same time satisfying various restrictions derived from the consumer's theory.

For the analysis of developing countries, two often-used and therefore time-tested systems are the linear expenditure system (LES) and the addilog demand system. More flexible specifications include, but are not limited to, the Rotterdom (e.g., Theil 1976), the translog (Christenson et al. 1975), and the almost ideal demand system (AIDS) (Deaton and Muellbauer 1980) specifications. The latter ones often use the duality theory that the first order partial derivative of the expenditure function with respect to a price of a commodity is the (Hicksian) demand for that commodity of a utility-maximizing consumer, or the Roy's identity. The former specifications have the advantages of enabling clearer and more straightforward interpretations of various concepts such as elasticities or minimum basic needs, in addition to the relative ease of estimation of parameters, but at the same time the constraints imposed on the system *a priori* are more rigid than systems in the other group.

In this paper, the more flexible demand systems are favored, due mainly to our desire to avoid rigid assumptions on preference of LES (ELES)* and addilog, and eventually to examine implications of discarding homogeneity or Slutsky symmetry assumptions. Furthermore, among the various flexible specifications, it is proposed here that AIDS** be used first for this case study. This system has advantages in that all of the following are simultaneously satisfied:

(1) It gives an arbitrary first-order approximation to any demand system.

(2) It satisfies axioms of choice exactly.

(3) It aggregates perfectly over consumers without invoking parallel linear Engel curves.

(4) It has a functional form consistent with known household-budget data.

(5) It is simple to estimate avoiding nonlinearity in parameters.

The simplified equations to be estimated and used are

$$\omega_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \beta_i \ln \frac{E}{P_{ind}} \tag{9}$$

where

ω_i = budget share for commodity i

$\alpha_i, \beta_i, \gamma_{ij}$ are constants i,j = 1,...,n

P_j = price of commodity i

E = total consumption expenditure

*Extended linear expenditure system
**The derivation of this system is described in Appendix 2.

P_{ind} = an aggregate price index*

Constraints on parameter values are, for adding-up:

$$\sum \alpha_i = 1, \quad \sum_{i=1}^{n} \gamma_{ij} = 0, \quad \sum_{i=1}^{n} \beta_i = 0$$

for homogeneity:

$$\sum_j \gamma_{ij} = 0$$

and for symmetry:

$$\gamma_{ij} = \gamma_{ji}$$

The following consumption commodity groups will be used:

(1) consumer's durable
(produced by the modern manufacturing sector)

(2) cloths and clothing
(produced by the textile industry)

(3) processed food and miscellaneous
(produced by the traditional industry)

(4) transportation, communication, electricity, and fuels
(produced by the modern service sector)

(5) other services
(produced by traditional service sectors)

---

*Deaton and Muellbauer (1980) derived this index as

$$\ln P_{ind} = \alpha_o + \sum_k \alpha_k \ln P_k + \frac{1}{2} \sum_k \sum_j \gamma_{k_j} \ln P_k \ln P_j$$

but they propose simplification of this to

$$\ln P_{ind} = \sum \omega_k \ln P_k$$

This is subject to more examination using Japanese data before it will be employed.

(6) agricultural products (rice, vegetable, etc.)
(produced by agriculture)

(7) housing

The following form of income classes will be employed for analyses:

(1) urban unskilled

(2) urban skilled

(3) rural labor

(4) rural landlords and urban capital income earners*

Saving ratios are taken from the historical data year by year due to the difficulty of endogenizing them through intertemporal optimization. The only system that has endogenous saving ratios is the extended linear expenditure system. The saving ratio in this system is determined by the supernumerary income ratio and the marginal propensity to consume (assumed to be fixed at the ratio of the personal discount rate and the market interest rate). This is a nice feature for short-term or cross-section analyses. For the long-term simulation, however, to assume that the marginal propensity to consume and the amount of basic minimum bundles are fixed may be subject to careful examination. If these assumptions did not seem to hold, the endogenous saving could do more harm than good. This is why the ELES has not been chosen as a system for the base simulation, although the use of its modified version is not excluded from future work, especially for the dynamic study of demand.

3.6.2. Demographic Variables

It is important that the links between demographic and economic features be included in our model. One of the more important such links exists in household consumption demand, i.e., different demographic characteristics of households are highly likely to

*Landlords and capitalists are considered similar due to the fact that many landlords were absentee landlords living in urban areas, or many did become capitalists by investing in nonagricultural sectors.

affect their patterns of demand. These features include the number of family members, the number of children, and the age and sex of family heads, etc. There have been some promising attempts to incorporate demographic variables into complete demand systems such as LES or QES (Quadratic Expenditure System)(see, for example, Pollak and Wales 1978), and flexible systems.

With AIDS, however, some demographic variables could be incorporated into the demand equations directly in the following manner

$$\omega_i^h = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \beta_i \ln (E^h/k^h P_{ind})$$

where h is household index and the new variable k^h is interpreted as some measure of family size, thus E^h/k^h would be, according to Deaton and Muellbauer, the "needs-corrected-per-capita-expenditure" level.

3.6.3. Dynamic Features

There is no denying that the static consumption specification described above has a serious problem in that it ignores all dynamic aspects. For example, it would be misleading to consider the purchase of consumers' durables as the consumption of these durables. We must also consider habit formation effects, or "inventory" adjustment behavior, as well as changes in taste over a long time period. One suspects that a part of the reason many econometric studies reject homogeneity or other assumptions derived from the consumer's theory is because they ignore such dynamic features.

In the first stage of this case study, however, we shall only take into account change of taste over the long run. We specify the trend change of parameters in the following way:

$$X_{i \cdot t} = X_{i \cdot o} + X_i(t)$$

or the parameter X (any parameter) is decomposed into the base year value $X_{i \cdot o}$ and the trend part $X_i(t)$. X_i can be positive or negative.

3.7. Relations with the Rest of the World*

3.7.1. Export

There is no doubt that export has played an important and active role in Japanese economic growth.** As a resource-poor country, Japan has to import raw materials and export manufactured goods. As described in detail in chapter 2, however, the composition of the trade kept on changing; exports started from raw silk in the early Meiji period, then later other textile exports increased and finally from the 1920s on, the export of machinery and other heavier industrial products began to show a distinct rate of increase. It is also noteworthy that all the exports were preceded by imports in the import-substitution phase, i.e., the so-called "wild-geese-formation."

This brief review clearly shows that a mere vent for surplus specification is not enough for a model of Japan, much less for the exogenous export. I shall propose therefore to use the Armington specification (Armington 1969).***

$$EX_i = C_i^x \cdot (WT)_i \left(\frac{P_i^{Jex}}{\tau \cdot P_i^W} \right)^{\eta_i^W}$$

where

EX_i = export of sector i

C_i^x = constant for this sector

$(WT)_i$ = total amount of the world trade of this sector goods

---

*Taiwan and Korea came under Japanese rule in 1896 and 1910, respectively. Even under the Japanese rule they are considered as a part of the rest of the world, thereby making the trade with them a part of foreign trade (see section 3.7.4.).

**There has been a long argument as to whether Japanese economic growth was actually export-led. The argument is inconclusive, but Dohner (1979) suggests that in pre-World War II Japan, growth was export-led, and in post-World War II Japan, export was export-price-led in the sense of Cave's definition (Cave 1971).

***The accounting consistency is maintained by assuming government's trade arbitrages. Other specifications, such as the use of a hypothetical composite good of imported and domestically produced goods will be used in the future.

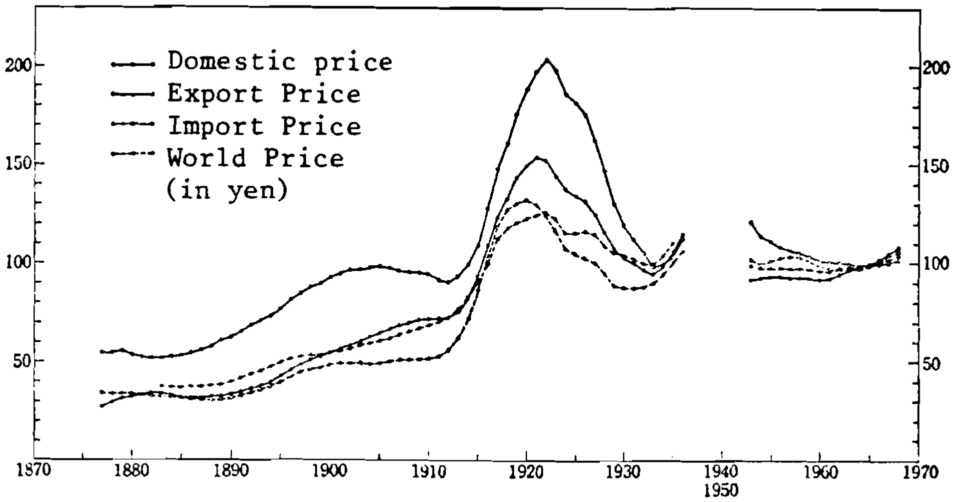
P_i^{Jex} = Japanese export price of sector i

P_i^W = world price of sector i

η_i^W = the elasticity of substitution in the i good world market*

τ = exchange rate

This is a logical specification to use because of the divergences between domestic prices, export prices, and world prices that are apparent in the pre-World War II period as shown in Figure 2. Also, if we study the movement of prices of each product, we can see that between 1900-1930, domestic prices increased (relative to average) for food products and decreased for ceramics, chemicals, metals, and textiles. These were passed on to the export prices. On the other hand, the import price increased for ceramics and machinery, and decreased for chemicals, food, and metals (see Table 3).



Source: LTES Volume 14

Figure 2. Domestic, export, import, and world trade prices for manufacturing goods (seven year moving average).

*This elasticity of substitution comes from the Armington CES specification for utilities that can be achieved by consuming products of the same kind but of different origins, e.g., clothes from the UK and clothes from Singapore.

Table 3. Percent change in prices of products between the 1900s and 1930s.

	Increase	Average	Decrease
	10% or more above average	Average ± 10%	10% or more below average
Domestic	Wood products Food products Miscellaneous	Machinery	Cermaics Chemicals Metal Textiles
Export	Wood products Food products Chemicals	Machinery Metal Ceramics	Textiles Miscellaneous
Import	Ceramics Machinery Miscellaneous	Textiles	Wood products Chemicals Food products Metal

Source: LTES Volume 14.

From this table, it is apparent that ceramics, textiles, and machinery had price changes favoring exports, while food and wood imports were bound to increase. Together with this, a calculation of the world price and the total world trade (an approximation for world demand) would determine the export of Japanese goods. It is easy to bring in more complexity by considering the formation of the yen-block with different elasticities for the block countries. But for our purpose here, it should be enough to consider only these purely economic factors.

3.7.2. Imports

Imports are a mirror image of exports in a sense that they are goods demanded by Japanese from the rest of the world. In developing countries, there are many goods that cannot be produced domestically but are needed to maintain and expand the economy. These are called noncompetitive imports. They will be treated as a function of total outputs and the Armington specification will not be used.

For competitive imports, i.e., goods that can be supplied within the country but are nevertheless imported, the Armington specification is employed. Therefore, for noncompetitive imports:

$$M^{NC} = \sum_j a_{oj} \; Q_j \; P_o^W \; \tau(1 + t^{FNC})$$

Here a_{oj} (the input-output coefficient) can vary over time and should for a long term analysis. Also, dividing a_{oj} into the sectors of origin, e.g., using a_{oij} as noncompetitive imports of good i to sector j, is possible for the post-World War II period t^{FNC} is the tariff rate on noncompetitive imports.

In general it is assumed, for the periods concerned, that raw materials and intermediate goods are noncompetitive. Competitive imports are therefore finished products (consumer goods). The specification of competitive imports is,

$$M_i^C = C_i^M \cdot D_i \cdot \left(\frac{\tau \cdot P_i^W \left[1 + t_{(i)}^{FC}\right]}{P_i} \right)^{\xi_i^W}$$

where

D_i = total domestic demand

P_i^W = world import price

P_i = domestic price

$t_{(i)}^{FC}$ = tariff rate on competitive imports (see footnote p.22)

ξ_i^W = elasticity (see footnote p. 36)

Then the trade deficit is

$$TD = \sum_i P_i^W \; \tau \; M_i^C + M^{NC} - \sum P_j^{Jex} \cdot EX_j$$

which is defined as foreign saving

$$TD \underset{def}{=} FS$$

This is assuming that the trade deficit is covered by some means of foreign capital inflow, which was, in general, the case. This aspect is briefly mentioned below.

3.7.3. Capital Movements

Capital movements are treated residually or exogenously. It is not that they were unimportant. On the contrary, despite the general belief that they were unimportant for Japan, Yamazawa and Yamamoto (1979) find that growth is accompanied by significant increases of capital inflow to finance the excess demand (see also Key 1970). But I shall take it as residually determined from the current deficit, because the sum of long- and short-term capital inflows more or less offset the current deficit every year. Other capital flows are taken as exogenous because of their one-shot nature, for example, war reparations.

3.7.4. Taiwan and Korea

Even though Taiwan and Korea are treated as among the "rest of the world" in our model, it is probably worthwhile to review the roles they played in forming the Japanese trade patterns in anticipation of future studies. Trade with these two countries increased rapidly after they came under Japanese rule, and they were given different roles from the Japanese point of view.

(1) Taiwan was regarded as a supplier of food and raw materials to Japan and therefore had always a positive surplus with her trade with Japan.

(2) Korea, in turn, was given a role of a market for Japanese products, with the exception of rice exports to Japan in 1918 to 1930.*

*This was a typical forced famine export. As described before this had different socioeconomic implications to Japan itself. That is, it stabilized the urban unrest caused by higher food prices, but at the same time it accelerated the decline of Japan's agriculture through competition by offering food at cheaper prices.

Trade with Taiwan and Korea also differed in that only yen were used as payment; there was no incremental demand/supply of foreign exchange.

3.8. Government

Many of the government activities have already been studied in sectors for production and investment, e.g., railways, power generation, government factories, etc. The rest of the government activities are considered as exogenous. They are

(1) Nonmilitary government consumption

(2) Nonmilitary government investment (general infrastructure, agricultural investment, buildings, furniture, natural disaster reconstruction)

(3) Transfers to households

(4) Subsidies to various private sectors*

(5) Transfers to the rest of the world

(6) Military expenditure**

Therefore, the government expenditure is

$$G_T^E = \sum_i P_i^c G_i + W_G + \sum_h TH_h + \sum_i O_i + \overline{MIL} + G_{PRIV}^E + \overline{FI}$$

where

G_T^E = total government expenditure

P_i^c = purchaser's price of sector i good

G_i = purchase by government from sector i for consumption or investment (not included in G_{PRIV}^E)

W_G = wage bill of the government

TH_h = transfer to household of labor type h

O_i = subsidies to sector i

---

*This includes the trade arbitrages by the government.

**There are various interpretations on economic contributions of military expenditure. It is simply assumed here, that it is nonproductive spending synonymous to consumption.

$\overline{MIL}$ = military expenditure

G^E_{PRIV} = all current and capital expenditures paid by the government for railways, ships, government factories, power stations, etc. (endogenous)

$\overline{FI}$ = foreign investment

The infrastructure and agriculture investment are assumed to accelerate the Hicks neutral technical progress.* Government revenue (G^R) has a more complicated form, but a more straightforward interpretation:

$$G^R = \sum_v \sum_i \Big[t_L \, W^L_i \, L_{i.v} + t_S \, W^S_i \, S_{i \cdot v} + t_r \frac{r_{Eiv}}{1 - t_{ri}} KE_{i \cdot v} + t_r \frac{r_{Civ}}{1 - t_{ri}} KC_{i \cdot v} \Big]$$

$$+ t_W \, W_G + \sum_i P_i \, t^{id}_i \, Q_i + \sum t^L_y \, W^L_i \, L_i + \sum t^S_y \, W^S_i \, S_i$$

$$+ t^r_\chi \, \chi_8 + t^u_\chi \, \chi_9 + \sum_i \sum_j P^W_i \, t^{FNC}_{(i)} \, a_{oij} \, Q_j \cdot \tau$$

$$+ \sum_i P^W_i \, t^{FC}_{(i)} \, M^C_i \, \tau + \overline{F}$$

---

*From the point of view of the general equilibrium framework, however, these are very difficult items for us to assign to any specific sectors, even for agricultural infrastructure, because the infrastructure investment would influence all sectors, albeit unevenly, and to say that "harbors" are capital stock for manufacturing or that rural roads are capital stock of only agriculture is likely to bring in an unwanted bias into the whole picture. I assume therefore that these infrastructure investments affect the product augmenting technological change. Therefore, for example, assuming GI_j as government investment in area j

$$A_i = A_i(GI_u, GI_r, t)$$

$A_{i \cdot 1} > A_{i \cdot 2} \geqq 0$ for i = urban sectors

$A_{i \cdot 2} > A_{i \cdot 1} \geqq 0$ for i = agriculture

where

$A_{i \cdot k}$ = partial derivative of A_i with respect to the k^{th} argument

The inside of the parentheses and t_w and W_G are what employers have to pay: social security contributions and profit tax. The next term, $\sum_i P_i t_i^{id} Q_i$ is the indirect tax where applied. The following two terms are income tax for unskilled and skilled workers (agricultural labor is considered unskilled with very little, if any, income tax). The next two terms $t_\chi^r \chi_8$ and $t_\chi^u \chi_9$ are agricultural and urban land tax. The next two are tariff income for noncompetitive and competitive imports. Finally, $\bar{F}$ is the exogenously given foreign capital inflow.*

$$GS = \text{government saving}$$
$$= G^R - G_T^E$$

3.9. Housing and Urban Land

A simple framework for studying housing demand and housing investment is presented here. The land availability constraint is imposed only on urban areas. A flat urban rent will be assumed as in Kelley and Williamson (1980). The incorporation of non-zero sloped urban rent gradients is possible, but with the help of highly unrealistic assumptions. This, therefore, would not be tried at first, and the framework of the approach is described in Appendix 3. In the first case of flat rent gradients, the size of urban areas (supply of urban land) has to be either given from the equilibrium of demand for urban-use and agricultural-use land or given exogenously.

From the rental flow accounting equation for urban housing** we have:

$$Q_9 = A_9 \; KC_9^{\beta^9} \cdot \chi_9^{(1-\beta^9)}$$

---

*The possibility of a part of $\bar{F}$ and the new debt spent as consumption is disregarded.

**This could be a misspecification because the most empirical studies seem to indicate elasticities of factor substitution in the neighborhood of 0.5 (DeLeeuw and Ekanem 1971, Muth 1968). We should be ready to change this function to a CES after more examination of the data.

the housing price is derived as

$$\text{Min } (r_{C9}\ KC_9 + r_{\chi 9}\ \chi_9)$$

Then

$$TP_9 = A_9^{-1}\ (\beta_9)^{-\beta^9}\ (1-\beta^9)^{-(1-\beta^9)}\ (r_{C9})^{\beta^9}\,(r_{\chi 9})^{(1-\beta^9)} \cdot Q_9$$

where TP_9 is the price of the total housing, setting

$$\frac{TP_9}{Q_9} = P_9 \ \&\ A_9^{-1} \cdot \beta^{9^{-\beta_9}}\ (1-\beta^9)^{-(1-\beta^9)} = \bar{\bar{A}}_9$$

$$P_9 = \bar{\bar{A}}_9\ r_{C9}^{\beta^9} \cdot r_{\chi 9}^{1-\beta^9}$$

From the AIDS demand function

$$\omega_9 = \frac{P_9\ Q_9}{E} = \alpha_9 + \sum_j \gamma_{9\cdot j}\ \ln P_j + \beta_9\ \ln (E/P_{ind})$$

Demands for structure and land are (D denotes demand)

$$\frac{\beta^9\ P_9}{r_{C9}} \cdot Q_9 = KC_9^D$$

$$\frac{(1-\beta^9)\ P_9}{r_{\chi 9}}\ Q_9 = \chi_9^D$$

Assume $r_{\chi 9}(-1)$ and $r_{C9}(-1)$ are equilibrium rates of return in the previous period, which give the housing cost at $P_9(-1)$. Then, migrants and the natural increase of the urban population of the previous period increase the demand for housing. The depreciation of the housing stock decreases the supply, in the absence of new investment, which in turn raises the housing price.

We now turn to the supply side of housing, which remains the critical unknown among urban economists. We will, however, proceed in a very straightforward manner. It will be assumed that:

(1) Housing structures are completely mallable.

(2) The urban land is given exogenously to housing investor's decisions.

(3) Urban landlords invest in housing structures that everybody rents.

(4) Landlords are more risk averse in their investment behaviors.

Landlords invest in housing structures if the rental stream will give them higher expected rates of return than the expected rate of return on other investment opportunities. This expected rate of return ($\tilde{i}$) is assumed to be lower than the average of all capital returns due to the risk-averse nature of landlords. The expectation of rents formed by landlords are also simple and myopic

$$\tilde{r} = r(-1) + \gamma\left[r(-1) - r(-2)\right]$$

where the sub- and super-scripts are suppressed. Then

$$\frac{\tilde{r}_{C9}\ K_9 + \tilde{r}_{\chi 9}\ \chi_9 - TKC_9}{\tilde{i}} \geq B_{C9}\ (P_6, KC_9) + P_\chi\ \chi_9$$

is necessary for them to invest in housing assuming that their expectation of the structures' lifetime is fairly long, where

T = maintenance cost

B_{C9} = cost of building urban housing structure

P_χ = price of land

However, because $\frac{\tilde{r}_{\chi 9}}{\tilde{i}} \approx P_\chi$, this can be reduced to:

$$\frac{\tilde{r}_{C9} - T}{\tilde{i}} \geq B_{C9}(P_6)$$

where

$$B_{C9}(P_6) = \frac{B(P_6, KC_9)}{KC_9}$$

Therefore, the construction of new housing will continue until

$$\frac{\tilde{r}_{C9} - T}{\tilde{i}} = B_{C9}\ (P_6)$$

or until all the demand is met by existing supply. Thus, at the end of every period, migrants move with the expectation that land and structure rents will be the same in the future as the last period, and landlords invest in structure in accordance with their expectations on rents and other profit opportunities. The resulting supply and demand would determine the rents for this period.

This investment in housing is assumed to be financed by the landlord's own saving or borrowing from family members in the pre-World War II periods and thus avoids the problem created by the assumption that $\tilde{i}$ is less than the market rate of interest. In the post-World War II period, however, the specification should be that the housing investment competes for funds with other investments.

The supply of new urban land on the other hand is determined by *ex post* comparisons between the agricultural rent and the urban land rent of the previous period. Thus the land rent would be determined by two derived demand functions: one for agricultural land and the other for urban land, or the three equations

$$\chi_8^D = P_8\ (1 - \beta_1^8 - \beta_2^8)\ Q_8 \cdot r_{\chi 8}^{-1}$$

$$\chi_9^D = P_9\ (1 - \beta^9)\ Q_9 \cdot r_{\chi 9}^{-1}$$

$$r_{\chi 8} = r_{\chi 9}$$

would solve the land allocation between the urban and agricultural users.

3.10. Human Capital Investment

There are three ways of investing in human capital for an economy. One is through formal schooling, which enhances the general level of human capital for the whole economy. Another way is on-the-job training where workers pay—"general training"—because they could use that training in other firms. Still a third way is the on-the-job training where the cost is covered by the employers because of the firm-specific nature of training.

In the model, it is assumed that all formal schooling is paid for by governments and all on-the-job training is firm-specific.

In the modern sectors, both unskilled labor and skilled labor receive specific training. First the firm has to pay T_i (hiring and training costs) for every unskilled laborer, which the firm gains back as increased productivity, unless the laborer quits. Hence the specification of the user cost of labor in section 3.4

$$l_i = (1 + t_L)\ W_i^L + b_i\ T_i$$

The skilled laborers are trained from the unskilled, depending on the rate of return on "skill." For new investment, the required skill is

$$\left(\frac{\partial \mu}{\partial q^e}\right)_i \cdot (DD_i)$$

where

DD_i = expected excess demand in sector i

$\left(\frac{\partial \mu}{\partial q^e}\right)_i$ = efficient skill-output ratio in sector i (from *ex ante* production function)

Next, it is assumed that "skill" is movable within a sector among various vintages. Each vintage would require the amounts of skilled and unskilled labor that would equate their cost to their marginal productivity, given the capital stocks.

$$P_i^e \frac{\partial Q_{vi}}{\partial S_{vi}} = q_i^e \quad , \quad P_i^e \frac{\partial Q_{vi}}{\partial L_{vi}} = l_i^e$$

The required amount of skill in each vintage can be obtained by solving these two equations for S_{vi} and L_{vi}. Then the total new skill requirement of the sector i is

$$D_{Si}^e = \frac{\partial \mu_i}{\partial q_i^e} DD_i + \sum_{v=1}^{N} S_{vi}^* - (1 - \delta^S) \sum_{v=0}^{N(-1)} S_{vi}^A(-1)$$

where

S_{vi}^* = the value of S from the solution of the above two equations

$S_{vi}^A(-1)$ = the total skill employed for vintage v in this sector in the previous period

N = the vintage of the oldest machine in this period

N(-1) = the vintage of the oldest machine in the last period

δ^S = the rate of retirement of the skilled laborer

The cost of skill training is assumed to be the price of the capital intensive service sector. Thus, the intended investment in skill is

$$I_S^e = P_5 \left[D_{S\cdot 1}^e + D_{S\cdot 2}^e + D_{S\cdot 5}^e \right]$$

This amount competes with the productive investment. The final actual investment in skill (I_S^A) is

$$I_S^A = \frac{I_S^e}{I_S^e + \sum_i \sum_j I_{ij}^e} SAV$$

where

$i = K, E$

$j = \text{sectors}$

SAV = savings available for skill and productive physical investment ($\equiv SAV_{Total} - I_{Housing}$)

3.11. Savings

It has been postulated that the saving propensities from various incomes are exogenously given, rather than determined by intertemporal utility maximization. Even this simple specification of exogenous propensities would have an adjustment mechanism of total saving if the saving propensity out of capital income were higher than that of wage income through the change in functional income distribution. It might make more sense, in any case, to assume a more explicit mechanism. It will be assumed therefore that the saving propensities out of capital income varies with the rate of return on capital in the previous period. Thus, saving ratio out of capital income of sector i

$$= s_{k\cdot i} = s_{k\cdot i}\left[r_{E\cdot i}(-1)\ ,\ r_{C\cdot i}(-1)\right]$$

and

$$\frac{\partial s_{k\cdot i}}{\partial r_{E\cdot i}(-1)}\ ,\ \frac{\partial s_{k\cdot i}}{\partial r_{C\cdot i}(-1)} > 0$$

4. MODIFICATIONS AND EXTENSIONS

The model described in the last chapter is already highly ambitious in richness of specification, but many of the important features of developing economies are still insufficiently developed. They include:

1. Rural-urban migration
2. Endogenous training cost
3. Labor supply
4. Size distribution of income
5. Investment-saving relationship
6. Agriculture
7. Inflation
8. Disequilibrium
9. Dynamization

The first four relate to demography, which is unfortunate for a model purporting to study demoeconomics. Among them I consider the rural-urban migration as the most important and its modification in specification is relatively easy. The fifth feature is the recurring question of why investment and saving has to be equal, especially when we exclude the inventory accumulation from the model. This also relates to distribution of income, if saving takes the form of lower real wages (forced saving). The sixth feature, agriculture, is weakly represented in the model and the seventh, money, is almost totally absent.* The eighth and ninth features we hope to pursue in the long run and are not related to the extensions of the model in the immediate future.

4.1. Rural-Urban Migration

This phenomenon is specified in the model only as a function of differentials between the expected urban wage and the average rural income (Todaro framework). Selective migration and farmer's subjective equilibria may also be incorporated in some way, although the Todaro framework has proved to be a useful and powerful way of explaining net internal migration.

*The inclusion of "interest rate" in investment decision means money is implicit in the model.

Selective migration essentially means that those who consider themselves more likely to succeed (in terms of having higher expected lifetime income) in the urban area are more apt to migrate. Then this story is just the other side of the human capital story, i.e., age, education, skill, and sex, which changes the probability of getting a higher income in the future. If, therefore, people are rational in that they seek a higher level of satisfaction and that the higher level of satisfaction is correlated with a higher level of income, a random utility model can be applied in the specification of rural-urban migration. That is, a logit* analysis can be employed with assumed utility functions of potential migrants.

One way, but certainly not the only way, to incorporate the selectivity would be to estimate earning functions in urban areas with such human attributes as age and education being independent variables. The utility a person gets is just a monotonic function of this earning, then the probability that a person migrates (P_{ru}) is

$$P_{ru} = \frac{e^{U[E^u(Age,\ Age^2,\ Ed,\ Sex)]}}{e^{U[E^u(Age,\ Age^2,\ Ed,\ Sex)]} + e^{U(E^r)}}$$

where

$U(\cdot)$ = assumed utility function

$E^i(\cdot)$ = earnings in area i, i = urban, rural

Age = age of potential migrants

Ed = education of potential migrants

Sex = sex of potential migrants

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*The use of an ordinary conditional logit is enough here for the assumed simplicity of decisions: to migrate or to stay. There are no alternative destinations once one decides to migrate; he just goes to an "urban area." If there is more than one homogeneous urban area assumed to be possible destinations for migrants, nested logit or probit should be employed to avoid the "independence of irrelevant alternatives" assumption.

If the data on distributions of age and educational background (years of education completed) are available, we can obtain the total migration flow of each year by integrating or summing up over all ages and educational backgrounds. This, of course, hides other difficult problems, especially the fact that different "kinds" of migration may be associated with different probabilities even with the same human attributes. For example, there is a distinguishable difference in attitude when a whole family is migrating, or when only one member of a family is migrating. Also there is ample reason to believe that a drastic difference exists between the probability that a first son will migrate and the probability that the remaining siblings will migrate. These have to be taken into account for a fuller specification of migration studies in Japan.*

Another way of extending the migration specification proposed by Kaneda (1979) and Ohkawa (1980) is by using subjective equilibria. Kaneda's argument employs the concept of marginal valuation of labor, which is equalized to the marginal product. Ohkawa's argument is the use of an "indifference point" between migrating (shifting to nonagricultural employment) and staying in farming. He proposes that there exists some critical level of marginal product in agriculture ($\widetilde{MP}_8$) and if the actual marginal productivity (MP_8) is below $\widetilde{MP}_8$, people would migrate, but if $MP_8 \geq \widetilde{MP}_8$, then people would rather stay in farming. He sees that $\widetilde{MP}_8$ is related to the market wage rate and that about 60 percent of the wage rate is $\widetilde{MP}_8$. According to this idea, and assuming demand determines migration, migration would be specified as follows:

$$M_{ru} \equiv \sum_i \left(\frac{\partial \hat{c}_i}{\partial w_i} \bigg/ \frac{\partial \hat{c}_i}{\partial r_i} \right) \cdot I_i^a - g \cdot L_u$$

if

$$W_8 \overset{=}{\text{def}} P_8 \cdot \frac{\partial Q_8}{\partial L_8} < \Theta \cdot \frac{\partial Q_u}{\partial L_u} \cdot P_u \overset{=}{\text{def}} \Theta W_u$$

*The arguments of using random utility theory presupposes the existence of individual sample data or sufficient general data.

otherwise

$$M_{ru} \equiv 0$$

The labor-capital ratio of the best-practice firms (^ means *ex ante*) is within the parentheses, and

W_8, W_u = agricultural and urban wage rates

Q_u, L_u, P_u = some hypothetical (for exposition) urban output, urban labor force, and price of urban output. (In the model proposed here, this actually means some weighted average of various urban sectors' value for marginal products.)

g = the natural growth rate of urban labor force

θ = the rate at which farmers are indifferent to staying or migrating.

Actually this can be studied with a very close specification to what has been proposed above (equation 8), if we assume away the instantaneous adjustment—i.e., all newly created jobs will be filled instantaneously by migrants—and assume some speed of adjustment, $\bar{m}$. Then,

$$\frac{M_{ru}}{L_8} = \bar{m}\left(\frac{\widetilde{MP}_8 - MP_8}{MP_8}\right)$$

$$= \bar{m}\left(\frac{\theta W_u - W_8}{W_8}\right)$$

But this is just the first order approximation to

$$\bar{m} \ln\left(\frac{\theta W_u}{W_8}\right)$$

$$= \bar{m} \ln\left(\frac{W_u}{W_8}\right) + \bar{m} \ln \theta$$

Now W_8 is the marginal product (MP) rather than the average product (AP) as proposed in the specification above, but from the Cobb-Douglas production function

$$\frac{MP_8}{AP_8} = \beta_2^8$$

So the above specification becomes

$$\bar{m} \ln \left(\frac{W_8}{\beta_2^8 \cdot AP_8} \right) + \bar{m} \ln \theta$$

$$= \bar{m} \ln \left(\frac{W_u}{AP_8} \right) + \bar{m} \left(\ln \theta - \ln \beta_2^8 \right)$$

which is different from my specification only by the constant term. Of course W_u is a weighted average of various urban wages, which may or may not be the same as my definition of $\tilde{W}_u$. (I will examine both specifications with empirical data.) As to Kaneda's proposal, I shall not explicitly use it now because of the difficulty involved in modeling a truely subjective evaluation of labor.

4.2. Endogenous Training Cost

The allocation of unskilled labor in this model is strongly influenced by the exogenous sector-specific training and hiring costs. They determine almost the entire wage structure of urban sectors. This might be more rigidity than we need. In addition, the underlying assumption that these training and hiring costs are actually technology-specific makes it imperative that we endogenize these costs. That is, it may be assumed that the higher the capital-labor ratio, the higher the amount of training needed.

This is a plausible assumption, and a closer study of post-World War II data will be carried out in order to find numerical evidence for obtaining explicit specification of this training cost function.

4.3. Labor Supply

Labor supply is implicitly assumed in the model as a fixed proportion of the total population. This, however, should be modified to account for the sensitive response of the population to various signals of the market with regard to the decision of whether to work or not. The rate of wage is usually considered as having a positive correlation with the labor supply especially in a developing country.* The lower wage through depression, for example, is known to have such effects as discouraged workers and added workers. We should also take into account the general tendency of females to participate in the labor force more as time goes one, and the tendency of young workers to decrease as the demand for education increases.

4.4. The Size Distribution of Income

The discussion of how well off people are depends on their family income or size distribution of income. The model, however, is capable of explaining only the functional distribution of income, i.e., how much is earned by labor and how much is the return to capital in each sector. But we cannot ignore the political economy of size distribution of income, especially in the settings of contemporary developing countries. Some tools should be employed in order to map the functional distribution into the size distribution. In the past, Adelman and Robinson (1978) assumed occupational-group-specific log normal distributions of income, while Lysy and Taylor (1978) disaggregated labor into 130 different types of labor, thereby getting an approximate size distribution directly from the functional distribution. Reyes-Heroles (1980) studied size and functional distribution for the base year, and used their derived mapping method for the whole simulation. Eckaus et al. (1979) used ingeneous estimation work for locating six income classes in Egypt: bottom 60 percent, middle 30 percent, and top 10 percent, for both urban and rural, from value-added share data. Furthermore, Tulpulé (1976) studied size distribution of income using more demographic data: marriage and

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*The backward-bending part of a labor supply curve is ignored here.

family formation, the number of persons working in the family, etc. This Japanese model as described in the last chapter, can be extended to include the size distribution of income in various directions, especially for the post-World War II period, for which there is a relative abundance of household income data (e.g., Mizoguchi 1975). Because the purpose of this model is the analyses of demoeconomics, it would be more interesting to extend the mapping method developed by Tulpulé (1976).

For the pre-World War II period, however, the data on size distribution of income are at best meager. A less ambitious method should be employed. See, for example, Otsuki and Takamatsu (1979) in this regard.

4.5. Investment-Saving Relationship

There is no reason to believe the *ex ante* saving desire is equal to the *ex ante* investment desire. These decisions are made by different individuals. *Ex post* saving and investment, however, are equal by the Walras' Law. How, then, is this equality realized? Most neoclassical models, starting with the Solow-Swan-Meade model, assume that saving is the constraint on investment or that saving behavior determines investment. But the other story is as plausible. The desire to invest forces saving to equal the desired investment. This is most typically done by forced saving through lowering the real wages of workers, with, for example, an inflation with a nominally fixed wage rate. This presupposes a higher saving propensity from capital income. Similarly, the increase in the capital income earners' consumption demand shifts the income share more favorably to capitalists: the Widow's cruse. In Japan the real wage increase always lagged behind the productivity increase during economic upswings therefore making the scenario of forced saving highly likely. (The model will be modified to incorporate this.) For discussion of inflation see section 4.7.

4.6. Agriculture

The model is essentially a model of urbanization and industrial growth. It has been intentional to treat agriculture as simply as possible. Fortunately, agriculture in Japan was and is relatively unimodal in a sense that there has not been a drastic dichotomy between modern and traditional and large-scale and small-scale farming. This makes it more convenient to treat it as one sector.

A more disaggregated treatment of farming can be done depending on the future direction of the study. Especially in view of the situation in Japan, the division of farmers into peasants, owner cultivators, and farmers with family members employed in nonagricultural activities, may turn out to be useful for studies of income distribution, labor supply, and migration.

4.7. Inflation and Money

In such a general equilibrium model as the one discussed in the previous chapter, the money is usually a veil, i.e., the whole model is homogeneous of degree zero with respect to all prices (commodities and factors), therefore, all that the model produces is a set of relative prices. An inflation of several hundred percent does not change the picture at all if the relative prices are constant. It has been agreed by many, however, that the monetary phenomena do have real effects, especially in the short-to-medium run.

If our interests are completely with the medium- to long-run range structural changes in the economy, it may be that we can leave our model as it is. If, however, we become more interested in the short-to-medium run, including increased investment through forced saving caused by a general inflation with a fixed nominal wage rate, then it may be better to include the monetary sector, with the real part of the model not having the homogeneous character. Adelman and Robinson (1978) do just that; they assume: (1) changing prices changes the structure of the demand for transaction balances; (2) total investment is fixed in nominal terms; (3) interest payment is specified in nominal terms; and (4) the exchange rate is specified in nominal terms, thus making the real part of their model nonhomogeneous.

We could also (1) specify the wage rate in nominal terms, and (2) elect to use nonhomogeneous consumers' demand functions. The first of the two enables one, again, to tell a forced saving story. The second depends on the empirical estimation of the system of consumers' demand. Results of so many empirical studies say that the homogeneity hypothesis is to be rejected.* This may either be due to the insufficient cares given to the dynamic aspects of consumers' demand by researchers, or due to consumers not behaving as the utility theory suggests. If we maintain the second hypothesis, then it would be interesting to see the consequences of discarding homogeneity assumptions with regard to the model behavior when inflationary pressures exist.

4.8. Disequilibrium

All the arguments so far have been made assuming that the economy is in some form of equilibrium, i.e., all markets clear, period by period, and there exists no unemployment of factors. The disequilibrium features are incorporated by: (1) fixing the rural factor endowments which causes rural-urban income differentials, thereby making migration intrinsic to the model; (2) using unbalanced technical progress; (3) employing the sector-specific training cost hypothesis which causes wage rates within the urban sectors to differ; and (4) assuming all investment decisions take place using expectations of both prices and quantities.

The equilibrium obtained in this way, therefore, is not fully Walrasian, but we are still in the Walrasian-Tâtonnement framework in a sense that each agent (household and firm) is small enough to take prices parametrically—the "auctioneer" is the only one who can change prices. If, however, developing economies are inherently in disequilibrium, there may be a good reason to pursue in the direction of establishing disequilibrium models. The recent development in macroeconomic disequilibrium or non-Walrasian equilibrium theory** seems to offer some possibilities of approaching toward this purpose.

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*See Barten (1977) for a summary of empirical tests on theoretical constraints.

**For a review, see Drazen (1980).

In this school of thinking, each agent perceives limited possibilities of trade and reformulates the trade pattern accordingly. In the earlier works (Patinkin 1965, Clower 1965, Barro and Grossman 1976, Benassy 1975, and Drèze 1975) prices are fixed even with non-zero excess demands, and quantities are supposed to adjust through some kind of rationing schemes. Recently, efforts have been made in order to endogenize prices. Benassy (1976) attempts to study a monopolistic equilibrium where prices are fixed for an intraperiod but flexible for an interperiod. Hahn (1978) does not rule out a Walrasian equilibrium but asks if any other equilibrium exists, assuming that agents make conjectures, based on the quantity signals and prevailing market prices on offer prices at which they think they can trade beyond the quantity constraints.

It seems that all these ideas are relevant for studying developing economies in general where supply constraints prevail in many commodity markets and demand constraints in factor markets. It appears, however, that these ideas are not still directly applicable in the kind of models described above. First of all, their mathematical rigor and abstractness make it difficult to apply them in *numerically* computable models. Second, the very fact that these disequilibrium studies have been motivated in order to analyze the unemployment of developed countries has been a hindrance to the initiation of their application in developing economies. Third, "(the) work is in its infancy" (Drazen 1980:304), and being developed rapidly. It may not be wise, therefore, to judge its applicability with the present state of the art.

Despite these three problems and the ever-existing question of data, the relevance that disequilibrium theories have to urbanization and economic growth, especially in a dualistic framework, seems to be sufficiently great that we should continue pursuing the possibility of incorporating disequilibrium theories in our modeling work in the future.

At least, at the present state, we should seek possibilities for studying the effects of quantity constraints and spillovers with *probably* fixed prices in certain markets. For example, it would be interesting to study the effects of the price control and rationing scheme for rice, which was in effect in Japan until well after World War II.

4.9. Dynamization

Developing economies with rapidly shifting structures are inherently dynamic as well as in disequilibria. Our model however, is that of a period-by-period static equilibrium. Between periods, dynamic changes occur in the model endogenously (migration and investment) or parametrically (technical progress, total population growth, etc.). Although this is a useful device, it may be more appropriate to use a continuous time framework. Some such work has been started (e.g., Wierzbicki 1979). It might very well be, however, that the model presented in this paper is too rich in specifications to dynamize completely.

It would be of interest to simplify the model and study implications of dynamization. This could be a topic for future research after the completion of the simulation and counterfactual analyses with the present model.

5. CONCLUSIONS AND COMMENTS ON FEASIBILITY

The purpose of this paper is to exposit an applied general equilibrium model for analyzing Japanese historical experiences of urbanization and economic growth in a dualistic framework. The model is rather large and is complex in that it has a vintage structure of production, five production factors (capital equipment, capital structure, unskilled labor, skilled labor, and land), the expectation-determined investment, and sector (technology)-specific training costs determining wage structure. The complete estimation and simulation of the model as it stands now would be a large enough challenge to a model builder. However, there are certain aspects which are important in studies of developing economies and are insufficiently developed or are missing from the model presented in chapter 3.

Among the modification and extension possibilities stated in chapter 4, the more urgently needed changes should be

(1) Demographic aspects, especially more elaboration on the specification of rural-urban migration

(2) Use of the concept of forced saving

(3) Endogenizing training costs

(4) Study on size distribution of income

Future efforts will be made in order to solve and operationalize the full model and to incorporate some extensions mentioned above.

Finally, a remark should be made on the feasibility of the full model. It is obvious that the model and suggested extensions will make a heavy demand on data. The very fact that the model structure is so complex makes its fully-simultaneous estimation of parameters impossible: the model is definitely under-identified. We will have to depend on partial estimates or other research results (also usually partial analyses) for many of the parameter estimates. A set of parameter values will be selected that will replicate the history with the highest degree of precision. This replaces the simultaneous estimation procedure. It is regrettable that, until now, this is the only possible way of estimation for this kind of modeling.

Once, however, this estimation method is accepted, then the feasibility of the full operationalization would not be so difficult, especially for the post-World War II period where there is a relative abundance of data.

For the pre-World War II period, however, a certain simplification might be needed. In general, the existence of the LTES* series offers a good opportunity for accurate historical analyses with this kind of model. Even the LTES, however, is not complete for our purposes. For example, it does not contain data on sector-specific capital stock or input-output relationships.

*Long Term Economic Statistics of Japan, edited by Ohkawa, Shinohara and Umemura. See references for the topic of each volume.

There is a problem of demand matching, i.e., the data on production have different sector divisions than the household demand data's commodity divisions. This becomes a problem in our model because it basically postulates a one-to-one correspondence between sectors and commodities. Many variables and parameters, therefore, have to be "guesstimated," although some can be obtained by going back to the raw data (census, etc.) or by looking at some relationships between available data and missing data. [Napier (1969) enhances the capital stock data greatly by estimating sector and firm size-specific capital stock from horsepower data.]

Where do all these statements lead us? It may be said that for the operationalization of the full model for the pre-World War II period, we are less than optimistic but are not completely pessimistic either. Some simplifications, e.g., merging equipment and structure capital into a "homogeneous capital stock," may be needed, but the calibration of the relatively complete model should be possible, and that possibility will be sought. For the post-World War II period, a more or less full specification of the model will be employed for analyses.

At the time of writing, a simplified version of the model* described in chapter 3 was solved using 1960 data. Some sensitivity analyses and comparative statics have also been carried out. The calibration of the full model will be attempted after more data collection.

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*Simplified to five sectors and two factors of production without government.

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APPENDIX 1: MATHEMATICAL STATEMENT OF THE MODEL

Indices

i j (= 0,...,10)	Sectors* (i=0 is foreign sector)
v (= 1,...,V_i)	Vintages
k (= KE, KC, and S)	different capital stocks
h (= capitalists and landlords, skilled unskilled, rural)	household types

Endogenous Variables

Q_i output

Y_i value added

A_i shift parameters of production**

A_{iv} vintage-specific shift parameters

S_i skilled labor

S total skill (S = new skilled labor)

L_i unskilled labor

L total labor

*There is a one-to-one correspondence between production sectors and commodities.

**These are, in simplified simulations, exogenous.

KE_i	equipments and machineries
KE	total equipments and machineries
KC_i	structures and buildings
KC	total structures and buildings
χ_8	agricultural land
χ_9	urban residential land
A_{ji}	intermediate inputs, input of j good in production of sector i
l_i	user cost of unskilled labor
q_i	user cost of skilled labor
r_{Eiv}, r_{Civ}	(*ex post*) user cost of KE and KC from quasi-rents
$\nu_{E \cdot i}, \nu_{C \cdot i}$	(*ex ante*) expectation of user cost of KE and KC
W_i^L, W_i^S	wage rates on L (unskilled) and S (skilled) labor
R_{iv}^E, R_{iv}^C	after-tax rates of return on KE and KC
D_i	domestic demands
P_i	producers' prices
P_i^C	purchasers' prices
P_i^{VA}	value-added price (primary factor cost of production)
P_i^{Jex}	Japan's export prices
r_{C9}	rent on urban housing structure
$r_{\chi 9}$	rent on urban residential land
r_{10}	rent on rural housing structure
$r_{\chi 8}$	rent on agricultural land
b_i	quit-rates
$I_{k \cdot i}^e$	desired investment
$I_{k \cdot i}^e$	actual investment

SAV	total saving - housing investment
SAV_p	saving available for physical productive investment
DD_i	expected demand minus present supply capacity net of depreciation
$X_i^{e \cdot t+1}(t)$	value of X_i expected for $(t+1)^{th}$ time period and expectation made at t^{th} time period. X^e expected value of X X^A actual value of X
M_{ru}	rural-urban migration
L_r	rural labor force (or population)
L_u	urban labor force
$\tilde{W}_u$	expected wage income in urban area
$\tilde{W}_r$	expected income in rural area
θ_i^L, θ_i^S	expected new employment in urban sector i for unskilled and skilled labor
COL_u, COL_r	cost-of-living indices for urban and rural areas
ω_i	overall budget share of household on good i
ω_i^h	budget shares of expenditure on good i by household type h
E^h	household expenditure by household type h
E	total household expenditure
P_{ind}	price index
Y^h	total household income
G^R	government revenue
G_{PRIV}^E	endogenous part of government expenditure
G_T^E	total government expenditure
M^{NC}	noncompetitive imports
M_i^C	competitive imports
FS	foreign saving (= Trade deficit TD)
GS	government saving

s_i^k saving propensities of capitalists and landlords

B_i cost function of sector i

μ_i unit cost function of sector i

EX_i exports

CD_i consumer's demand

V^h share of population in urban household type h in total urban population

Exogenous Variables and Parameters

t time

N total population

χ total arable and urban land area

G_i government spending

W_G government wage bills

TH_h transfers (to households)

O_i subsidies (to firms)

$\overline{MIL}$ military expenditure

t_L employers' social security contribution on W^L

t_S employers' social security contribution on W^S

t_w tax rate on government employees

t_r tax rate on capital income

t_χ^u, t_χ^r land tax on urban and rural land holdings

t_i^{id} indirect tax rates

t_y^S, t_y^L income tax on skilled and unskilled labor income

$t_{(i)}^{FNC}$ tariff rate on noncompetitive imports

$t_{(i)}^{FC}$ tariff rate on competitive imports

τ exchange rate $\left(= \frac{\text{yen}}{\text{foreign currency}}\right)$

$\bar{F}$ foreign capital inflow

$\overline{FI}$ foreign investment

P_i^W world prices

P_{oi}^W import prices

P_o^C purchaser's price of competitive imports

$\phi_i, \xi_i, \varepsilon_i, \theta_i$ distribution parameters in production in CES forms

$\delta_i, \eta_i, \zeta_i, \rho_i$ substitution parameters in production in CES forms

β^i production elasticity parameter (or distribution parameter) in production in Cobb-Douglas form

a_{ji} input-output coefficient (from sector j to sector i)*

σ_i^k depreciation rates

$\left.\begin{matrix}\gamma_D, \gamma_W \\ \gamma_{rE}, \gamma_{rC} \\ \gamma_q, \gamma_\nu\end{matrix}\right\}$ expectation adjustment parameters on D, W, r_E, r_C, q, and ν

ψ_i transportation mark-up rates

d^S discount factor on skilled labor wage used by potential migrants in migration decisions

s_i^h saving propensities, h = household types other than captialists and landlords

$\alpha_i, \gamma_{ij}, \beta_i$ consumption function parameters, i,j = commodities

$\tilde{i}$ subjective discount rate of urban landlords

T maintenance cost of urban housing

η_i^W, ξ_i^W elasticities of trade specifications

T_i sector-specific training and hiring costs

*These are, in some simulations, endogenous.

C_i^x, C_i^m constants in trade functions (exports and competitive imports)

WT_i world trade

g_u, g_r growth rates of urban labor, rural labor, and land area

λ_f rate of factor (f = KE, KC, L, and S) augmenting technical progress

λ_i rate of product augmenting technical progress

Sector Divisions

1. Manufacturing I
2. Manufacturing II
3. Manufacturing III
4. Construction
5. Modern Service
6. Traditional Urban Service
7. Rural Service
8. Agriculture
9. Urban Housing
10. Rural Housing

Production Relations

i = 1,2,3, and 5

Ex-ante Technology

$$Q_i = A_i \left\{ \hat{\phi}_i \left[\hat{\xi}_i \left(\hat{\varepsilon}_i \; \underline{S}_i^{\;\hat{\delta}_i} + (1-\hat{\varepsilon}_i) \; \underline{KE}_i^{\;\hat{\delta}_i} \right)^{\frac{\hat{\eta}_i}{\hat{\delta}_i}} + (1-\hat{\varepsilon}_i) \; \underline{KC}_i^{\;\hat{\eta}_i} \right]^{\frac{\hat{\zeta}_i}{\hat{\eta}_i}} + (1-\hat{\phi}_i) \; \underline{L}_i^{\;\hat{\zeta}_i} \right\}^{\frac{1}{\hat{\zeta}_i}}$$

where $\hat{X}$ means X is an *ex ante* parameter and $\underline{Y}$ means that the variable Y is measured in efficiency units.

Ex post technology

$$Q_{iv} = A_i \cdot A_{iv} \left\{ \phi_{iv} \left[\xi_{iv} \left(\varepsilon_{iv} \, \underline{S}_i^{\,\delta_{iv}} + (1-\varepsilon_{iv}) \, \underline{KE}_{iv} \cdot e^{-\sigma_E (t-v)} \right)^{\frac{\eta_{iv}}{\delta_{iv}}} + (1-\xi_{iv}) \underline{KC}_{iv} \, e^{-\sigma_C (t-v)} \right]^{\frac{\zeta_{iv}}{\eta_{iv}}} + (1-\phi_{iv}) \, \underline{L}_i^{\,\zeta_{iv}} \right\}^{\frac{1}{\zeta_{iv}}}$$

where v = vintage

$$\phi_{iv} = \frac{\hat{\phi}_i}{\hat{\phi}_i + (1-\hat{\phi}_i)(k^*)^{\zeta_{iv}-\hat{\zeta}_i}}$$

$$\xi_{iv} = \frac{\hat{\xi}_i}{\hat{\xi}_i + (1-\hat{\xi}_i)(h^*)^{\eta_{iv}-\hat{\eta}_i}}$$

$$\varepsilon_{iv} = \frac{\hat{\varepsilon}_i}{\hat{\varepsilon}_i + (1-\hat{\varepsilon}_i)(s^*)^{\delta_{iv}-\hat{\delta}_i}}$$

where

k* = the efficient composite capital (S,KE,KC) labor ratio at given factor price ratios and a given *ex ante* production function.

h* = the efficient skill and equipment composite (S,KE) and structure capital (KC) ratio at given factor price ratios and a given *ex ante* production function.

s* = the efficient skill-equipment capital ratio at given factor price ratios and a given *ex ante* production function.

$$A_{iv} = \left[\frac{\hat{\phi}_i \; TKR^{\frac{\hat{\zeta}_i - \zeta_{iv}}{\eta_{iv}}} + (1 - \hat{\phi}_i) \; \underline{\tilde{L}}^{\hat{\zeta}_i - \zeta_{iv}}}{\hat{\phi}_i \; TKR^{\frac{\hat{\zeta}_i}{\eta_{iv}}} + (1 - \hat{\phi}_i) \; \underline{\tilde{L}}^{\hat{\zeta}_i}} \right]^{\frac{1}{\zeta_{iv}}}$$

where

$$TKR = \frac{\hat{\xi}_i \; SKR^{\frac{\hat{\eta}_i}{\delta_{iv}}} + (1 - \hat{\xi}_i) \; \underline{\widetilde{KC}}^{\hat{\eta}_i}}{\hat{\xi}_i \; SKR^{\frac{\hat{\eta}_i - \eta_{iv}}{\delta_{iv}}} + (1 - \hat{\xi}_i) \; \underline{\widetilde{KC}}^{\hat{\eta}_i - \eta_{iv}}}$$

and

$$SKR = \frac{\hat{\varepsilon}_i \; \underline{\tilde{S}}^{\hat{\delta}_i} + (1 - \hat{\varepsilon}_i) \; \underline{\widetilde{KE}}^{\hat{\delta}_i}}{\hat{\varepsilon}_i \; \underline{\tilde{S}}^{\hat{\delta}_i - \delta_{iv}} + (1 - \hat{\varepsilon}_i) \; \underline{\widetilde{KE}}^{\hat{\delta}_i - \delta_{iv}}}$$

All of the factor inputs ($\underline{\tilde{S}}_i$, $\underline{\widetilde{KE}}_i$, $\underline{\widetilde{KC}}_i$, and $\underline{\tilde{L}}_i$) are efficient unit output inputs of the *ex ante* production function of sector i at the time of installation.

i = 4 (construction)

$$Q_4 = A_4 \left[\varepsilon_4 \; \underline{KE}_4^{\rho_4} + (1 - \varepsilon_4) \; \underline{L}^{\rho_4} \right]^{\frac{1}{\rho_4}}$$

i = 6,7 (TS,RS)

$$Q_i = A_i \; (\underline{KE}_i)^{\beta_1^i} \; (\underline{KC}_i)^{\beta_2^i} \; (\underline{L}_i)^{1-\beta_1^i-\beta_2^i}$$

i = 8 (A)

$$Q_8 = A_8 \left[\theta_8 \; \underline{KE}_8^{\rho_8} + (1 - \theta_8) \; \underline{KC}_8^{\rho_8} \right]^{\frac{\beta_1^8}{\rho_8}} (L_8)^{\beta_2^8} \; (X_8)^{1-\beta_2^8-\beta_1^8}$$

i = 9

$$Q_9 = A_9 \; (KC_9)^{\beta^9} \; (X_9)^{1-\beta^9}$$

i = 10

$$Q_{10} = A_{10} \; KC_{10}$$

Commodity Pricing

i = 1,2,3, and 5

$$P_i = l_i \frac{L_{iv}}{Q_{iv}} + q_i \frac{S_{iv}}{Q_{iv}} + r_{Eiv} \frac{KE_{iv}}{Q_{iv}} + r_{Civ} \frac{KC_{iv}}{Q_{iv}} + \sum_{j=0}^{n} P_j^c \frac{A_{ji}}{Q_{iv}}$$

i = 4

$$P_4 = l_4 \frac{L_4}{Q_4} + r_{E4} \frac{KE_4}{Q_4} + \sum_{j=0}^{n} P_j^c \frac{A_{j4}}{Q_4}$$

$i = 6,7$

$$P_i = l_i \frac{L_i}{Q_i} + r_{Ei} \frac{KE_i}{Q_i} + r_{ci} \frac{KC_i}{Q_i} + \sum_{j=0}^{n} P_j^c \frac{A_{ji}}{Q_i}$$

$i = 8$

$$P_8 = l_8 \frac{L_8}{Q_8} + r_{E8} \frac{KE_8}{Q_8} + r_{c8} \frac{KC_8}{Q_8} + r_{\chi 8} \frac{\chi_8}{Q_8} + \sum_{j=0}^{n} P_j^c \frac{A_{j8}}{Q_8}$$

$i = 9$

$$P_9 = r_{C9} \; KC_9 + r_{\chi 9} \; \chi_9$$

$i = 10$

$$P_{10} = r_{C10} \; KC_{10}$$

where

$$l_i = (1 + t_L) \; W_i^L + b_i \; T_i \quad \text{and} \quad \frac{\partial l_i}{\partial W_i^L} = 0 \quad \text{for } i = 1,2,5$$

$$q_i = (1 + t_S) \; W_i^S$$

$$r_{Eiv} = \left(\frac{1}{1 - t_r} R_{iv}^E + \sigma_i^E \right) P_1$$

$$r_{Civ} = \left(\frac{1}{1 - t_r} R_{iv}^C + \sigma_i^C \right) P_4$$

$$P_o^c = (1 + t^{FNC}) \; P_o^W \cdot \tau \quad \text{if noncompetitive imports}$$

$$P_o^C = (1 + t^{FC})\ P_o^W \cdot \tau \qquad \text{if competitive imports}$$

$$P_i^C = (1 + t_i^{id})\ P_i\ (1 + \psi_i) \qquad i \neq 0$$

$$l_i = P_i^{VA} \frac{\partial Q_{iv}}{\partial L_{iv}}$$

$$q_i = P_i^{VA} \frac{\partial Q_{iv}}{\partial S_{iv}}$$

$$r_{Eiv} = P_i^{VA} \frac{\partial Q_{iv}}{\partial KE_{iv}}$$

$$r_{Civ} = P^{VA} \frac{\partial Q_{iv}}{\partial KC_{iv}}$$

$$A_{ji} = a_{ji}\ Q_i$$

Investment Allocation

$$I_{E \cdot i}^e = P_1 \left(\frac{\partial\ \mu_i}{\partial\ \nu_{E \cdot i}^e} \right) \cdot DD_i$$

$$I_{C \cdot i}^e = P_4 \left(\frac{\partial\ \mu_i}{\partial\ \nu_{C \cdot i}^e} \right) \cdot DD_i$$

$$I^e_{S\cdot i} = P_5 \quad \frac{\partial\mu_i}{\partial q^e_i} \; DD_i \; + \sum_{v=1}^{N} S^*_{vi} - \sum_{v=0}^{N(-1)} S^A_{vi}(-1)$$

$$I^A_{k\cdot i} = \frac{I^e_{k\cdot i}}{\sum_h \sum_j I^e_{hj}} \; SAV$$

i,j = capital using sectors

k,h = S,KE,KC

To get S^*_{vi}, solve for each v and i

$$p^e_i \frac{\partial Q_{vi}}{\partial S_{vi}} = q^e_i$$

$$p^e_i \frac{\partial Q_{vi}}{\partial L_{vi}} = l^e_i$$

with given $KE = \overline{KE_{v\cdot i}}$, $KC = \overline{KC_{v\cdot i}}$

Expectations

$$D^{e\cdot t+1}_i(t) = \gamma_D \left[D^A_i(t) - D^{e\cdot t}_i(t-1)\right] + D^A_i(t)$$

$$W^{e\cdot t+1}_i(t) = \gamma_W \left[W^A_i(t) - W^{e\cdot t}_i(t-1)\right] + W^A_i(t)$$

$$r^{e\cdot t+1}_{k\cdot i}(t) = \gamma_{r_k} \left[r^A_{k\cdot i}(t) - r^{e\cdot t}_{k\cdot i}(t-1)\right] + r^A_{k\cdot i}(t) \qquad k = E,C$$

$$\nu_{k\cdot i}^{e\cdot t+1}(t) = \gamma_{\nu_k}\left[\nu_{k\cdot i}^{A}(t) - \nu_{k\cdot i}^{e\cdot t}(t-1)\right] + \nu_{k\cdot i}^{A}(t) \qquad k = E,C$$

$$q_i^{e\cdot t+1}(t) = \gamma_q\left[q_i^A(t) - q_i^{e\cdot t}(t-1)\right] + q_i^A(t)$$

Migration

$$\frac{M_{ru}}{L_r} = m \ln \frac{\tilde{W}_u}{\tilde{W}_r}$$

$$\tilde{W}_u = \frac{1}{COL_u} \cdot \frac{\sum_i \sum_j d^j \theta_i^j W_i^j}{\sum_i \sum_j d^j \theta_i^j} \qquad \begin{array}{l} j = \text{skilled, unskilled} \\ i = \text{urban sectors} \end{array}$$

$$d^j = d \qquad 0 < d < 1 \qquad \text{if } j = \text{skilled}$$

$$d^j = 1 \qquad \text{if } j = \text{unskilled}$$

$$\theta_i^S = \left[I_{KE\cdot i}^A(-1) + I_{KC\cdot i}^A(-1)\right] \cdot \left[\frac{S_i(-1)}{P_1(-1)\, KE_i(-1) + P_4(-1)\, KC_i(-1)}\right]$$

$$+ b_i^S(-1) \cdot S_i(-1)$$

$$\theta_i^L = \left[I_{KE\cdot i}^A(-1) + I_{KC\cdot i}^A(-1)\right] \cdot \left[\frac{L_i(-1)}{P_1(-1)\, KE_i(-1) + P_4(-1)\, KC_i(-1)}\right]$$

$$+ b_i^L(-1) \cdot L_i(-1)$$

$$\tilde{W}_r = \frac{1}{COL_R}\left[\rho_1\, y_o + \rho_2\, y_p + (1 - \rho_1 - \rho_2)\, y_N\right]$$

where

y_o = average income of owner cultivators

y_p = average income of peasants

y_N = average income of nonagricultural rural workers

ρ_1 = proportion of owner cultivators in the rural population

ρ_2 = proportion of peasants in the rural population

$COL_j = \sum_i \omega_i^j\, P_i^j$ $\qquad$ j = urban, rural

$\omega_i^r = \omega_i\ (E^r)$ $\qquad$ E^r = expenditure of rural agricultural labor

$\omega_i^u = \sum_h V^h\, \omega_i (E^h)$ $\qquad$ $V^h = \frac{\text{population in income class h}}{\text{urban population}}$

E^h = expenditure of income class h

ω_i = consumption expenditure share on commodity i

d = discount factor for skill employment

Household Consumption Demand (CD)

$$\omega_i^h = \frac{P_i^c\, CD_i^h}{E^h} = \alpha_i + \sum_j \gamma_{ij}\ \ln\ P_j^c + \beta_i\ \ln\ \frac{E^h}{P_{ind}}$$

i = commodities (consumption durables, cloth and clothing, processed foods and misscellaneous, transportation and electricity, etc., other services, agricultural products, and housing)

h = income classes (urban unskilled, skilled, rural labor, and landlords and capitalists)

$$E^h = (1 - s^h)\ Y^h$$

s_i^k = savings ratio for capitalists and landlords of sector i

$$= s_i^k \left[P_i \frac{\partial Q_i}{\partial KE_i} (-1)\ ,\ P_i \frac{\partial Q_i}{\partial KC_i} (-1) \right]$$

Housing Market

Demand

$$Q_i = \frac{E \cdot \omega_i}{P_i^c} \qquad \begin{array}{l} i = 9 \text{ for urban} \\ i = 10 \text{ for rural} \end{array}$$

where Q_i's are total demand.

For urban areas

$$KC_9^D = \frac{\beta^9\ P_9}{r_{C9}}\ Q_9$$

$$\chi_9^D = \frac{(1-\beta^9)\ P_9}{r_{\chi 9}}\ Q_9$$

For rural areas

$$KC_{10}^D = \frac{Q_{10}}{A_{10}}$$

Housing investment

$$r_{C9}^e = r_{C9}(-1) + \gamma \left[r_{C9}(-1) - r_{C9}(-2) \right]$$

then investment in urban housing is made such that

$$\frac{r^e_{C9} - T}{\tilde{i}} = \mu_{C9}\ (P_6)$$

$$\tilde{i} = d^9 \cdot i \qquad\qquad 0 < d^9 < 1$$

Rural housing investment

$$I^A_{10} = \text{Min}\left[\sigma_{10}\ KC_{10}(-1) + \frac{\Delta E^r \cdot \omega_{10}(-1)}{P_{10}(-1)},\ s^r \cdot Y^r\right]$$

where

ΔE^r = the increase in rural consumption

Y^r = rural income

Land Market (superscript D denotes demand)

$$\chi^D_8 = P_8\ (1 - \beta^8_1 - \beta^8_2)\ Q_8\ r^{-1}_{\chi 8}$$

$$\chi^D_9 = P_9\ (1 - \beta^9)\ Q_9 \cdot r^{-1}_{\chi 9}$$

$$r_{\chi 8} = r_{\chi 9}$$

Government

$$G^E_T = \sum_i P_i G_i + W_G + \sum_h TH_h + \sum_i O_i + \overline{MIL} + G^E_{PRIV} + \overline{FI}$$

$$G^R = \sum_v \sum_i \Big[t_L W_i^L L_{i.v} + t_S W_i^S S_{i \cdot v}$$

$$+ t_r \frac{r_{Eiv}}{1 - t_{ri}} KE_{i \cdot v} + t_r \frac{r_{Civ}}{1 - t_{ri}} KC_{i \cdot v} \Big]$$

$$+ t_W W_G + \sum_i P_i t_i^{id} Q_i + \sum t_y^L W_i^L L_i + \sum t_y^S W_i^S S_i$$

$$+ t_\chi^r \chi_8 + t_\chi^u \chi_u + \sum_i \sum_j P_i^W t_{(i)}^{FNC} a_{oij} Q_j \cdot \tau$$

$$+ \sum_i P_i^W t_{(i)}^{FC} M_i^C \tau + \overline{F}$$

$$GS \equiv G^R - G_T^E$$

Foreign Sectors

Export

$$EX_i = C_i^x \cdot (WT)_i \left(\frac{P_i^{Jex}}{e \cdot P_i^W} \right)^{\eta_i^W}$$

Noncompetitive imports

$$M^{NC} = \sum_i \sum_j a_{oij} Q_j P_{oi}^W \cdot \tau \left[1 + t_{(i)}^{FNC} \right]$$

or simply

$$M^{NC} = \sum_j a_{oj} Q_j P_o^W \cdot \tau (1 + t^{FNC})$$

Competitive imports

$$M_i^C = C_i^M \cdot D_i \cdot \left(\frac{\tau \cdot P_i^W \left[1 + t_{(i)}^{FC} \right]}{P_i} \right)^{\xi_i^W}$$

$$TD = \sum_i P_i^W \tau M_i^C + M^{NC} - \sum P_j^{Jex} \cdot EX_j$$

Market Clearing

$$\sum_v \sum_i L_{vi} = L$$

$$\sum_{i=8,9} \chi_i = \chi$$

$$\sum_v \sum_i S_{vi} = S$$

$$\sum_v \sum_i KE_{vi} = KE$$

$$\sum_v \sum_i KC_{vi} = KC$$

$$Q_1 = \sum_h \frac{\omega_1^h \cdot E^h}{P_1^{C \cdot k}} + \sum_{i=1}^{8} \frac{1}{P_1} \cdot I_{KE \cdot i}^A + G_1 + EX_1 - (M_1^{NC} + M_1^C) + \sum_j A_{1 \cdot j}$$

$$Q_2 = \sum_h \frac{\omega_2^h \cdot E^h}{P_2^{C \cdot k}} + G_2 + EX_2 - M_1^C + \sum_j A_{2j}$$

$$Q_3 = \sum_h \frac{\omega_3^h \cdot E^h}{P_3^C} + G_3 + EX_3 - M_1^C + \sum_j A_{3j}$$

$$Q_4 = G_4 + \sum_{\substack{i=1 \\ i\neq 4}}^{8} \frac{1}{P_4} \cdot I^A_{KC \cdot i}$$

$$Q_5 = \sum_h \frac{\omega_5^h \cdot E^h}{P_5^C} + G_5 + \sum_{\substack{i=1 \\ i\neq 4}}^{5} \frac{1}{P_5} I^A_{S \cdot i} + \sum_i \psi_i \; P_i \cdot Q_i^O + \sum_j A_{5 \cdot j}$$

$$Q_6 = \sum_h \frac{\omega_6^h \cdot E^h}{P_6} + I_9^A \qquad h = \text{urban households}$$

$$Q_7 = \frac{\omega_7 \cdot E^R}{P_7} + I_{10}^A$$

$$Q_8 = \sum_h \frac{\omega_8^h \cdot E^h}{P_8^C} + G_8 + EX_8 - M_8^{NC} + \sum_j A_{8 \cdot j}$$

$$Q_9 = D_9$$

$$Q_{10} = D_{10}$$

Factor Growth

$$\chi = (1 + g_\chi)\ \chi(-1)$$

$$L_u = g_u L_u + L_u(-1) + M_{ru} - \Delta S$$

$$L_r = g_r L_r + L_r(-1) - M_{ru}$$

$$S = \Delta S + S(-1) - \sigma^S(-1)$$

Technical Progress

$$\underline{KE}_i = \underline{KE}_i(-1) \cdot e^{\lambda_{KEi}}$$

$$\underline{KC}_i = \underline{KC}_i(-1) \cdot e^{\lambda_{KCi}}$$

$$\underline{S}_i = \underline{S}_i(-1) \cdot e^{\lambda_{Si}}$$

$$\underline{L}_i = \underline{L}_i(-1) \cdot e^{\lambda_{Li}}$$

$$A_i = A_i \left[G(-1), t\right] \qquad i = 1,\ldots,8$$

where

$$G(-1) = \left[G_1(-1),\ldots,G_8(-1)\right]$$

$$A_i = A_i(-1) \cdot e^{\lambda_i} \qquad i = 9,10$$

APPENDIX 2: DERIVATION OF AIDS* FUNCTION

One type of PIGLOG** class preferences is denoted by the following expenditure (cost) function

$$\log C(U,P) = (1 - u) \log \left[a(p)\right] + u \log \left[b(p)\right]$$

where

u ($0 \leqq u \leqq 1$) is the utility

$u = 1$ is the utility of "bliss"

$u = 0$ is the utility of "subsistence"

p is the price vector

$a(p), b(p)$ are cost functions of subsistence and bliss, respectively

$$\log a(p) = a_o + \sum_k \alpha_k \log P_k + \frac{1}{2} \sum_k \sum_j \tilde{\gamma}_{kj} \log P_k \log P_j$$

$$\log b(p) = \log a(p) + \beta_o \prod_k P_k^{\beta_k}$$

*An abbreviation of "almost ideal demand systems." See Deaton and Muellbauer (1980).

**Muellbauer (1974, 1975).

From the duality theory (see, for example, Diewert 1974, or Shephard 1953)

$$\frac{\partial C}{\partial P_i} = q_i$$

Therefore

$$\frac{\partial \log C(u,p)}{\partial \log p_i} = \frac{p_i \; q_i}{C} = \omega_i$$

where

p_i = price of good i

q_i = quantity of good i

ω_i = budget share on good i

Then

$$\omega_i = \alpha_i + \sum \gamma_{ij} \log p_j + \beta_i \, u \, \beta_o \, \Pi \, P_k^{\beta_k}$$

$$\gamma_{ij} = \frac{1}{2} (\tilde{\gamma}_{ij} + \tilde{\gamma}_{ji})$$

By inverting C(u,p), we get u as a function of p and total expenditure. Then, by substitution,

$$\omega_i = \alpha_i + \sum_j \gamma_{ij} \log P_j + \beta_j \log \frac{E}{P_{ind}}$$

where E is the total expenditure (= C) and

$$\log P_{ind} = \alpha_o + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \log P_k \log P_j$$

which is the equation in the text.

APPENDIX 3: A WAY OF INCORPORATING URBAN RENT GRADIENTS

In this appendix we examine how we could incorporate a non-zero urban rent gradient. This would increase the complexity very much, and needs more unrealistic assumptions. Trade-offs between the potential benefits and costs should be evaluated completely. A brief description of what could be done follows:

Assumptions

1. Same utility function for each urbanite. (For exposition a Stone-Geary utility function is used.)
2. Monocentric city (one CBD - central business district).
3. Homogeneous and plain topography.
4. Community cost to CBD known and constant.
5. All employed in CBD.

Then, an urbanite maximizes his or her utility function (u) subject to the budget constraint:

$$\text{Max } U = m_o \ln d + m_1 \ln Q_{HU} + \sum_{i=2}^{n+2} m_i \ln C_i$$

$$\text{s.t.} \quad y_j = k \cdot d + P_{HU} Q_{HU} + \sum_{i=2}^{n+2} P_i C_i$$

where the variables are

Q_{HU} = quantity of urban housing (HU)

d = distance from CBD

k = cost of commuting to CBD per unit of distance

m_i $(i=0,\ldots,n+2)$ = constants

C_i = consumption of non-housing good i

From the production function of urban housing service,

$$Q_{HU} = \alpha_{HU} K_{HU}^{\beta^H} \chi_{HU}^{(1-\beta^H)}$$

and

$$P_{HU} = \frac{r_{HK} K_{HU} + r_{H\chi} \chi_{HU}}{Q_{HU}}$$

it is

$$\text{Max } U = m_o \ln d + m_1 \left[\ln \alpha_{HU} + \beta^H \ln K_{HU} + (1 - \beta^H) \ln \chi_{HU}\right] + \sum_{i=2}^{n+2} m_i \ln C_i$$

$$\text{s.t.} \quad y_j = k \cdot d + r_{HK} K_{HU} + r_{H\chi} \chi_{HU} + \sum_{i=2}^{n+2} P_i C_i$$

If we assume r_{HK} is not related with the distance from CBD and that $r_{H\chi}$ picks up all the price difference related with the community cost:

FOC (λ = Lagrangean multiplier)

$$\frac{m_o}{d} - \lambda k - \lambda \chi_{HU} \cdot r'_{H\chi}(d) = 0 \quad *$$

$$\frac{m_1(1-\beta^H)}{\chi_{HU}} - \lambda r_{H\chi} = 0$$

$$\frac{m_1 \beta^H}{K_{HU}} - \lambda r_{HK} = 0$$

$$\frac{m_i}{C_i} - \lambda P_i = 0$$

From the budget constraint

$$\lambda = \frac{1}{y}\left(\sum_{i=1}^{n+2} m_i + m_o \frac{k}{k + \chi_{HU} \cdot r'_{H\chi}}\right) = \frac{\sum m_i}{y - kd} \quad **$$

---

* ' (prime) means partial derivative.

**From,

$$k + \chi_{HU} \cdot r'_{H\chi} = \frac{m_o}{\lambda d} \quad ,$$

$$\lambda = \frac{\sum m_i + m_o \frac{k}{y + \chi r'}}{y}$$

$$= \frac{\sum m_i + k \lambda d}{y}$$

which gives the solution for λ:

$$\lambda = \frac{\sum m_i}{y - kd}$$

$$R_{HU} = \frac{m_1 (1 - \beta^H) y}{r_{HR} \cdot \left(\sum_{i=1}^{n+2} m_i + \frac{m_o k}{k + R_{HU} \cdot r'_{H\chi}} \right)} = \frac{m (1 - \beta^H)(y - kd)}{r_{H\chi} \cdot \left(\sum_{i=1}^{n+2} m_i \right)}$$

$$r'_{HK} = (\text{rent gradient})^*$$

$$= \frac{\left[- r_{HR} m_o + \frac{m_1 (1 - \beta^H)}{\chi_{HU}} y - (\sum m_i) r_{H\chi} \right] k}{- m_1 (1 - \beta^H) y + (\sum m_i) \chi_{HU} \cdot r_{H\chi}}$$

Demand for other goods

$$K_{HU} = \frac{m_1 \beta^H (y - kd)}{r_{HK} \left(\sum_{i=1}^{n+2} m_i \right)}$$

$$C_j = \frac{m_j (y - kd)}{P_j \left(\sum_{i=1}^{n+2} m_i \right)} \qquad j = 2, \ldots, n+2$$

---

*

$$\frac{\partial r'_{HK}}{\partial y} = \frac{- (1 - \beta^H) m_o m_1 k}{\left[m_1 (1 - \beta^H) y - \sum m_i \chi_{HU} r_{HK} \right]^2} > 0 \qquad (\because m_o < 0)$$

Therefore, in this specification of utility, the richer have to live further away from the CBD than the poor people, because the richer have a flatter rent gradient than the poorer (recall $r'_{HK} < 0$). This may be a drawback of the model.

Total Urban Demand (TUD)

$$X_{HU}^{TUD} = 2\Pi \int_{0}^{\kappa} d \; \frac{m_1 \;(1 - \beta^H)(y - kd)}{r_{HX} \sum_{i=1}^{n+2} m_i} \, dd \Bigg|_{y = y_u}$$

$$+ \; 2\Pi \int_{\kappa}^{b} d \; \frac{m_1 \;(1 - \beta^H)(y - kd)}{r_{HX} \sum_{i=1}^{n+2} m_i} \, dd \Bigg|_{y = y_S}$$

$$K_{HU}^{TUD} = 2\Pi \int_{0}^{\kappa} d \; \frac{m_1 \; \beta^H \; (y - kd)}{r_{HK} \sum_{i=1}^{n+2} m_i} \, dd \Bigg|_{y = y_u}$$

$$+ \; 2\Pi \int_{\kappa}^{b} d \; \frac{m_1 \; \beta^H \; (y - kd)}{r_{HK} \sum_{i=1}^{n+2} m_i} \, dd \Bigg|_{y = y_S}$$

$$C_j^{TUD} = 2\Pi \int_{0}^{\kappa} d \; \frac{m_j}{P_j} \; \frac{(y - kd)}{\left(\sum_{i=1}^{n+2} m_i \right)} \, dd + 2\Pi \int_{\kappa}^{b} d \; \frac{m_j \;(y - kd)}{P_j \left(\sum_{i=1}^{n+2} m_i \right)} \, dd$$

where κ is the distance from CBD, where

$$r_{H\chi}\Big|_{y=y_u} = r_{H\chi}\Big|_{y=ys}$$

and b is the distance from CBD to the city border. Income of unskilled and skilled urban labor are y_u and y_S, respectively.

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