International Institute for
Applied Systems Analysis
IIASA www.iiasa.ac.at

Model Migration Schedules: A Simplified Formulation and an Alternative Parameter Estimation Method

Castro, L.J. and Rogers, A.

IIASA Working Paper

WP-81-063

May 1981

Working Paper

MODEL MIGRATION SCHEDULES: A SIMPLIFIED FORMULATION AND AN ALTERNATIVE PARAMETER ESTIMATION METHOD

Luis J. Castro
Andrei Rogers

May 1981
WP-81-63

International Institute for Applied Systems Analysis
A-2361 Laxenburg, Austria

MODEL MIGRATION SCHEDULES: A SIMPLIFIED FORMULATION AND AN ALTERNATIVE PARAMETER ESTIMATION METHOD

Luis J. Castro
Andrei Rogers

May 1981
WP-81-63

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

Interest in human settlement systems and policies has been a central part of urban-related work at IIASA since its inception. From 1975 through 1978 this interest was manifested in the work of the *Migration and Settlement Task*, which was formally concluded in November 1978. Since then, attention has turned to dissemination of the Task's results and to the conclusion of its comparative study, which is carrying out a comparative quantitative assessment of recent migration patterns and spatial population dynamics in all of IIASA's 17 NMO countries.

This paper is part of the Task's dissemination effort. It focuses on the mathematical description of a simplified model migration schedule and on an alternative parameter estimation method which promises to be useful in situations where access to large computers and packaged programs is limited.

Reports summarizing previous work on migration and settlement at IIASA are listed at the back of this paper. They should be consulted for further details regarding the data base that underlies this study.

Andrei Rogers
Chairman
Human Settlements
and Services Area

ACKNOWLEDGMENTS

The authors are indebted to Warren Sanderson for his useful comments on an earlier draft and to Walter Kogler for his computer programming efforts.

ABSTRACT

This paper outlines a simplified model and a new numerical parameter estimation method that may enhance the application of model migration schedules in situations where access to large computers and packaged programs may be limited.

CONTENTS

MODEL MIGRATION SCHEDULES: A SIMPLIFIED FORMULATION AND AN ALTERNATIVE PARAMETER ESTIMATION METHOD

INTRODUCTION

The model migration schedules set out in Rogers and Castro (1981a, b) were fitted to observed data by means of a rather complex nonlinear procedure based on the Levenberg-Marquardt algorithm summarized in an appendix of that paper. To enhance the application of these schedules in situations where access to large computers and packaged programs may be limited, we propose in this paper a simplified model and a simplified estimation procedure. The simplification is carried out in two steps. First, the model itself is simplified by fixing its index of labor asymmetry σ_2 to a prespecified value and by adopting a standard schedule. Second, the parameter estimation process is simplified by replacing the nonlinear estimation algorithm with a linear one. The adequacy of each simplification is assessed in turn by a comparison of the results obtained with it against those found using the original unsimplified "complete" model.

MODEL SCHEDULES AND THEIR SIMPLIFICATION

The notion of simplified model schedules appears in the literature on model nuptiality schedules and is particularly well described in a recent paper by Rodriguez and Trussell (1980). We shall adopt and then adapt their strategy and for expositional convenience will use both their simplifying assumption and their standard nuptiality function. In a subsequent paper we shall introduce our own standard migration function.

The Case of First Marriage Frequencies

The selectivity of marriage with respect to age has been extensively studied by Coale and his associates since 1971. Coale (1971) found that age patterns of first marriage follow the same basic curve and differ significantly only in the location and scaling of the age at which marriages start to occur and the proportion of the cohort eventually marrying. In a paper one year later Coale and McNeil (1972) gave an analytic expression that satisfactorily fitted many first marriage frequency distributions. The function was called a double exponential density function and was defined as

$$f(x) = \frac{\lambda}{\Gamma(\alpha/\lambda)} e^{-\alpha(x-\mu) - e^{-\lambda(x-\mu)}} \qquad (1)$$

where α, λ, and μ are the function's parameters and $\Gamma(\alpha/\lambda)$ is the gamma function value of the ratio α/λ. The assumption that the ratio α/λ was constant in different populations, allowed Coale and Trussell (1974) to formulate Equation (1) as a function of a standard schedule $f_s(x)$:

$$f(x) = Kf_s(x) \qquad (2)$$

where

$$f_s(x) = \frac{0.1946}{k} e^{-\frac{0.174}{k}(x-x_o-6.06k) - e^{-\frac{0.288}{k}(x-x_o-6.06k)}} \qquad (3)$$

x_o is the age at which a consequential number of marriages first occur, k is the number of years in the standard schedule into which one year of marriage in the observed population may be "packed," and K is the proportion of the cohort eventually marrying (i.e., a scaling parameter). This standard has a mean and variance of:

$$\bar{x} = x_o + 11.36k \tag{4}$$

and

$$S^2 = 43.34k^2 \tag{5}$$

respectively.

Rodriguez and Trussell (1980) recently proposed a method to fit model nuptiality schedules to different types of data collected by the World Fertility Survey. They proposed a modification of the standard schedule defined in Coale and Trussell (1974), keeping the assumption that α/λ is a constant.

The basic simplification adopted by Rodriguez and Trussell is the expression of the standard as a function of the mean $\bar{x}$ and the standard deviation S. To do this they derive a new standard, with zero mean and unit variance, from Equation (3) by finding the values of x_o and k that generate the desired mean and variance. The resulting new standard density function is

$$f_s(x) = \frac{1.2813}{S} e^{-\frac{1.145}{S}(x-\bar{x}+8.05S) - e^{-\frac{1.896}{S}(x-\bar{x}+8.05S)}} \tag{6}$$

The Case of Migration

Empirical studies of age-specific migration schedules have shown that the age profiles exhibited by such data have a common shape. Starting with relatively high levels during the early adolescent ages, the migration rates decrease monotonically thereafter to a low point x_ℓ, then increase until they reach a maximum high peak at age x_h, and then decrease once again to the ages of retirement. Occasionally a "post-labor force" component appears, showing either a bell-shaped curve with a peak at age x_r or an upward slope that increases monotonically to the last age included in the schedule, age w say.

Decomposing the age profile into pre-labor force, labor force, and post-labor force components, we shall restrict our attention in this paper to those profiles that only have the first two components. However, our argument is equally valid for profiles showing a post-labor force component.

In several recent papers we have shown that the observed profile of migration rates may be described by a function of the form:*

$$m(x) = m_1(x) + m_2(x) + c \tag{7}$$

where $m_1(x) = a_1 e^{-\alpha_1 x}$ for the pre-labor force component

$m_2(x) = a_2 e^{-\alpha_2 (x-\mu_2) - e^{-\lambda_2 (x-\mu_2)}}$ for the labor force component

and c is the constant term that improves the fit when migration rates at older ages are relatively high.

The area under the $m(x)$ curve is called the gross migraproduction rate (GMR), which in this paper is always assumed to be equal to unity.

An alternative way of expressing Equation (7) is as a weighted linear combination of the density functions representing the three components:

$$m(x) = \phi_1 f_1(x) + \phi_2 f_2(x) + \phi_c \left(\frac{1}{w}\right) \tag{8}$$

where w is the last age included in the schedule,

ϕ_1 and ϕ_2 are the relative shares of the pre-labor force and labor force components,

ϕ_c is the share of the constant term,

and where $f_1(x)$ and $f_2(x)$ are the density functions

---

*This assumes that the age profiles do not exhibit a post-labor force component.

$$f_1(x) = \alpha_1 e^{-\alpha_1 x} \tag{9}$$

$$f_2(x) = \frac{\lambda_2}{\Gamma(\alpha_2/\lambda_2)} e^{-\alpha_2(x-\mu_2)-e^{-\lambda_2(x-\mu_2)}} \tag{10}$$

Note that $\phi_1 + \phi_2 + \phi_c = 1$ by definition.

Equations (7) through (10) imply that

$$m_1(x) = \phi_1 f_1(x) \tag{11}$$

$$m_2(x) = \phi_2 f_2(x) \tag{12}$$

and $$c = \frac{\phi_c}{w} \tag{13}$$

The model expressed in (7) or in (8) may be called the "complete" model because it contains the parameters needed to describe the observed regularities in migration age profiles. The parameters, however, are not easily interpretable in terms of more familiar measures such as means or variances. In order to introduce such statistics into the discussion it is necessary to assume simplifications of the kind adopted in recent studies of model nuptiality schedules.

Let

$$\phi_1 f_1(x) = \frac{\phi_1}{\bar{x}_1} e^{-x/\bar{x}_1} \tag{14}$$

where $\bar{x}_1 = 1/\alpha_1$ is the mean age of the pre-labor force component. In order to perform a similar transformation in Equation (12) we have to express, as in Coale and Trussell (1974), $f_2(x)$ as a function of a standard schedule $f_s(x)$:

$$\varphi_2 f_2(x) = \varphi_2 K f_s\left(\frac{x-x_o}{k}\right) \tag{15}$$

where in our application

- x_o is the age at which a consequential number of migrations first occur in the labor force component
- k denotes the number of years in the standard schedule into which the intensity of migration in one year in the observed population may be "packed"
- K is the scaling parameter, which in our case is equal to unity since both $f_2(x)$ and $f_s(\frac{x-x_o}{k})$ are density functions.

Alternatively, following the proposal of Rodriguez and Trussel (1980), we may formulate (15) as a function of the mean age of labor force migrants $\bar{x}_2$ and the associated standard deviation S_2:

$$m_2(x) = \varphi_2 f_2(x) = \varphi_2 f_s\left(\frac{x-\bar{x}_2}{S_2}\right) \tag{15'}$$

Thus, for example, if for expositional convenience we adopt the Coale-Trussell and Rodriguez-Trussell nuptiality standard profiles, then

$$m_2(x) = \phi_2 f_2(x)$$

$$= \varphi_2 \frac{0.1946}{k} e^{-\frac{0.174}{k}(x-x_o-6.06k)-e^{-\frac{0.288}{k}(x-x_o-6.06k)}} \tag{16}$$

and

$$m_2(x) = \phi_2 f_2(x)$$

$$= \varphi_2 \frac{1.2813}{S_2} e^{-\frac{1.145}{S_2}(x-\bar{x}_2+0.805S_2)-e^{-\frac{1.896}{S_2}(x-\bar{x}_2+0.805S_2)}} \tag{16'}$$

become replacements for Equations (15) and (15'), respectively.

The substitution of (14) and (16) or (16') into Equation (8) allows us to have two simplified model migration schedules, each expressed as a function of more common statistics or measures, and each assuming that the ratio of $\sigma_2 = \lambda_2/\alpha_2$ remains constant and equal to 1.66 over all populations. Table 1 sets out the complete and the simplified models.

SIMPLIFIED AND COMPLETE MODEL MIGRATION SCHEDULES: NONLINEAR PARAMETER ESTIMATION

In this section we test whether the simplified model is a good approximation of the complete model and therefore whether the assumption that the index of labor asymmetry $\sigma_2 = \lambda_2/\alpha_2 = 1.66$ is a reasonable one.

To assess the consequences of the above assumption, we have chosen several outmigration flows that exhibit a wide range of variation of the labor asymmetry index. Table 2 sets out the parameter values for the complete model migration schedules of female flows from each of several regions (identified by numbers in parentheses) to the rest of Sweden, the rest of the United Kingdom, and the rest of Japan, respectively. This table shows, for example, that among the three countries represented, Japan's female outflow from Region 1 has the highest value of $\sigma_2 = 10.39$, whereas the corresponding highest value for Sweden is 4.95. The lowest values are also included in the same table, and they vary from a low of 1.26 for Japan to a high of 3.43 for Sweden.

In the discussion that follows we shall call the simplified model expressed as a function of x_o and k the Type A model and the one that is a function of $\bar{x}_2$ and S_2 the Type B model.

To estimate the parameters of the simplified model we have used the same Levenberg-Marquardt nonlinear algorithm that we applied in our previous paper for obtaining parameter estimates in the complete model. (See Appendix A of Rogers and Castro, 1981a.) The parameters and the corresponding derived variables for both simplified models, are presented in Tables 3 and 4.

Table 1 Complete and simplified model migration schedules.

Model		Pre-labor force component		Labor force component	Constant term
COMPLETE					
$m(x)$	$=$	$a_1 e^{-\alpha_1 x}$	$+$	$a_2 e^{-\alpha_2 (x-\mu_2) - e^{-\lambda_2 (x-\mu_2)}}$	$+\ c$
SIMPLIFIED					
TYPE A.					
$m(x)$	$\doteq$	$\frac{\phi_1}{\bar{x}_1} e^{-\frac{x}{\bar{x}_1}}$	$+$	$\phi_2 \frac{0.1946}{k} e^{-\frac{0.174}{k}(x-x_o-6.06k) - e^{-\frac{0.288}{k}(x-x_o-6.06k)}}$	$+\ \frac{\phi_c}{w}$
TYPE B.					
$m(x)$	$\doteq$	$\frac{\phi_1}{\bar{x}_1} e^{-\frac{x}{\bar{x}_1}}$	$+$	$\phi_2 \frac{1.2813}{S_2} e^{-\frac{1.145}{S_2}(x-\bar{x}_2+0.805S_2) - e^{-\frac{1.896}{S_2}(x-\bar{x}_2+0.805S_2)}}$	$+\ \frac{\phi_c}{w}$

Table 2 Parameters and variables defining the complete model migration schedule for three selected countries: Sweden, United Kingdom, and Japan.

	sweden		united kingdom		japan	
	(4)	(8)	(1)	(3)	(5)	(1)
gmr(obs)	0.838	1.237	1.197	1.040	0.783	1.414
gmr(mod)	1.000	1.000	1.000	1.000	1.000	1.000
mae%m	8.809	13.167	9.194	6.379	4.816	7.878
a1	0.025	0.019	0.015	0.012	0.022	0.012
alpha1	0.104	0.128	0.100	0.062	0.113	0.069
a2	0.080	0.094	0.076	0.054	0.083	0.042
mu2	19.832	17.620	20.392	19.324	23.021	14.772
alpha2	0.129	0.143	0.171	0.131	0.159	0.075
lambda2	0.442	0.711	0.284	0.456	0.200	0.781
c	0.003	0.003	0.005	0.005	0.005	0.004
mean age	28.139	28.990	33.477	34.032	32.047	33.258
%(0-14)	21.929	16.400	19.033	18.649	20.865	17.478
%(15-64)	70.757	74.555	66.316	67.030	66.888	69.659
%(65+)	7.314	9.044	14.651	14.321	12.247	12.864
delta1c	9.932	5.338	3.023	2.544	4.907	2.767
delta12	0.312	0.198	0.201	0.225	0.271	0.294
beta12	0.809	0.390	0.584	0.477	0.710	0.917
sigma2	3.434	4.953	1.661	3.485	1.255	10.389
x low	15.610	14.770	13.530	15.110	13.520	12.370
x high	22.580	19.850	22.080	21.960	24.000	17.650
x shift	6.970	5.080	8.550	6.850	10.480	5.280
a	27.874	28.315	27.173	28.067	28.885	32.117
b	0.038	0.052	0.027	0.026	0.025	0.025

NOTE: The numbers in the parentheses denote the regions exhibiting the lowest or highest values for σ_2.

SOURCE: Rogers and Castro (1981b)

Table 3 Parameters and variables defining the simplified model migration schedule Type A for selected regions in Sweden, United Kingdom, and Japan.

	sweden		united kingdom		japan	
	(4)	(8)	(1)	(3)	(5)	(1)
gmr(obs)	0.838	1.237	1.197	1.040	0.783	1.413
gmr(mod)	1.000	1.000	1.000	1.000	1.000	1.000
mae%m	9.779	15.632	7.363	7.321	4.130	11.764
phi1	0.215	0.122	0.148	0.305	0.199	20.428
x1	3.585	6.475	9.564	27.626	8.651	388.731
phi2	0.542	0.568	0.394	0.326	0.490	0.347
x0	15.213	14.240	14.508	15.323	14.367	11.542
k	1.052	0.849	0.981	0.938	1.212	1.059
c	0.003	0.004	0.005	0.004	0.004	-0.039
mean age	28.643	29.759	33.628	34.012	32.035	32.077
%(0-14)	21.483	16.076	18.947	18.695	20.811	18.987
%(15-64)	70.120	73.332	66.138	66.997	67.049	69.964
%(65+)	8.392	10.592	14.915	14.307	12.139	11.050
phi1/2	0.393	0.214	0.375	0.934	0.405	58.828
x low	14.520	13.250	13.760	14.390	13.880	10.240
x high	23.330	20.850	22.080	22.550	23.690	19.710
x shift	8.810	7.600	8.320	8.160	9.810	9.470
a	33.280	32.250	31.480	29.800	34.790	30.860
b	0.037	0.051	0.029	0.025	0.028	0.024

NOTE: The numbers in the parentheses denote the regions exhibiting the lowest or highest values for σ_2 in the complete model.

Table 4 Parameters and variables defining the simplified model migration schedule Type B for selected regions in Sweden, United Kingdom, and Japan.

	sweden		united kingdom		japan	
	(4)	(8)	(1)	(3)	(5)	(1)
gmr(obs)	0.838	1.237	1.197	1.040	0.783	1.413
gmr(mod)	1.000	1.000	1.000	1.000	1.000	1.000
mae%m	9.777	15.629	7.353	7.320	4.131	11.796
phi1	0.215	0.122	0.148	0.304	0.199	50.204
x1	3.587	6.475	9.579	27.604	8.650	619.517
phi2	0.541	0.567	0.394	0.326	0.490	0.346
x2	27.158	23.882	25.648	25.976	28.130	23.538
s	6.922	5.587	6.454	6.174	7.977	6.960
c	0.003	0.004	0.005	0.004	0.004	-0.067
mean age	28.643	29.759	33.628	34.012	32.035	31,962
%(0-14)	21.483	16.076	18.949	18.695	20.811	19.031
%(15-64)	70.119	73.332	66.135	66.997	67.051	70.123
%(65+)	8.393	10.592	14.916	14.308	12.138	10.847
phi1/2	0.393	0.215	0.376	0.934	0.405	145.032
x low	14.520	13.250	13.760	14.390	13.880	10.220
x high	23.330	20.850	22.080	22.550	23.680	19.690
x shift	8.810	7.600	8.320	8.160	9.800	9.470
a	33.280	32.250	31.480	29.800	34.780	30.840
b	0.037	0.051	0.029	0.025	0.028	0.024

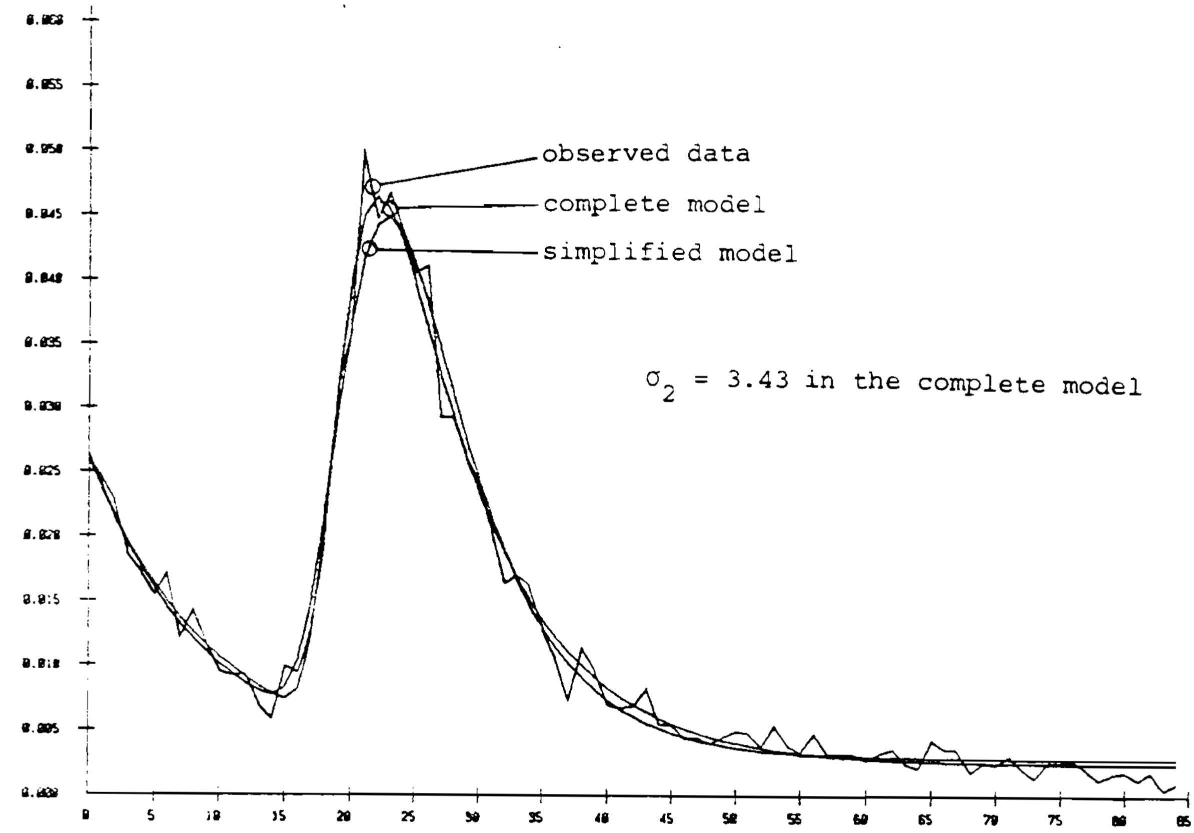
NOTE: The numbers in the parentheses denote the regions exhibiting the lowest or highest values for σ_2 in the complete model.

The two simplified models are equivalent, in the sense that given the parameters of one it is possible to estimate the parameters of the other by simple transformations of variables. Nevertheless we have estimated the parameters for each independently, thereby testing the sensitivity of the estimation procedure to two different model specifications.

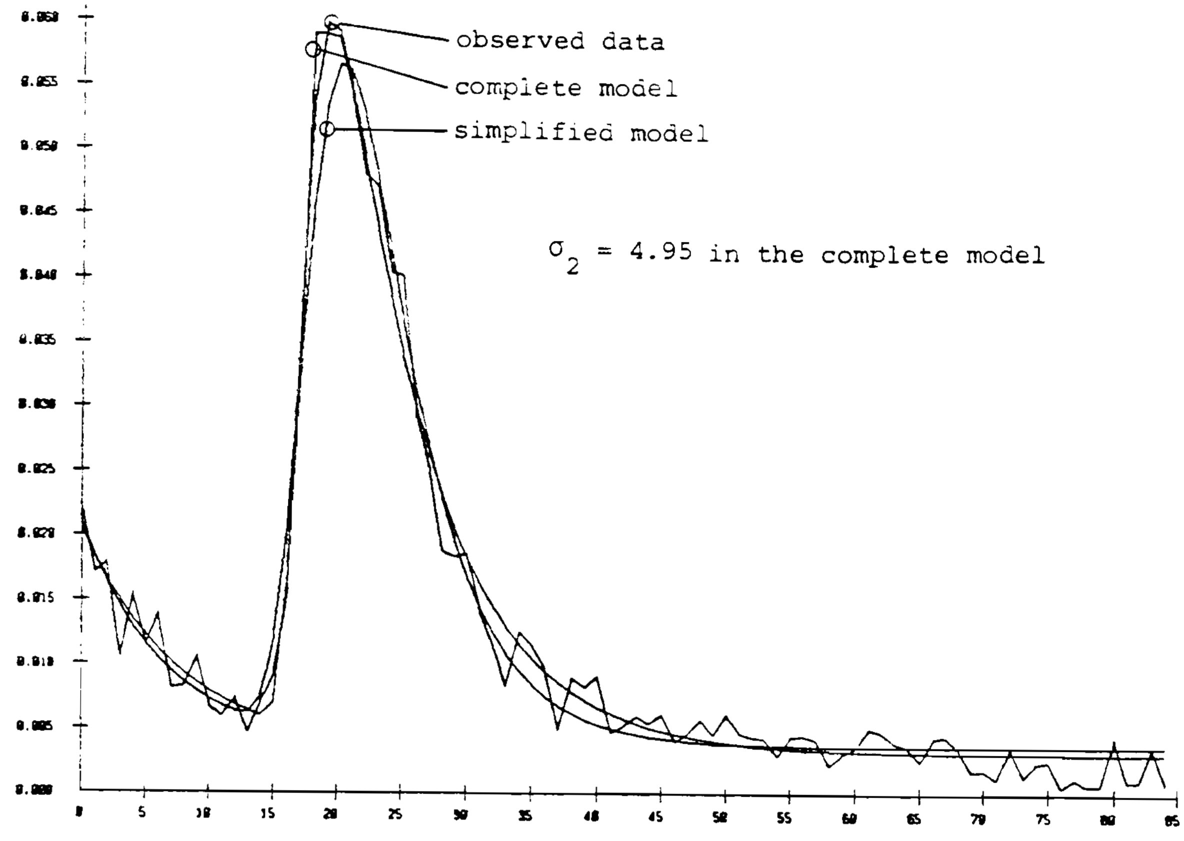
Tables 3 and 4 show that the parameters corresponding to the age profile with $\sigma_2 = 10.39$ are unrealistic, especially those for the pre-labor force component and the constant term. Region 3 of the United Kingdom also shows, in both models, a very high value for the mean age of the pre-labor force component. With the exception of these two schedules (schedules that correspond to high values of σ_2) the method yields reasonable parameter values. The problem of unrealistic values may be solved perhaps by first performing a cubic spline interpolation of the five-year age group data of the United Kingdom and Japan. The Swedish data, reported by single years of age, produce reasonable parameter values even with σ_2 indexes exceeding those observed in Region 3 of the United Kingdom. This suggests that some prior "smoothing" of the U.K. data might produce improved results.

By comparing the parameters of the simplified models in Tables 2 and 3, it is possible to observe that the particular specification does not significantly alter the parameters common to the two formulations. In Sweden, for example, the ϕ_1, ϕ_2, and c values are almost identical for both specifications.

Figures 1, 2, and 3 present the drawings of the observed data, the simplified models A and B, and the complete model migration schedules. It is interesting to observe that the simplified model age profiles of Region 3 in the U.K. and of Region 1 in Japan show a good fit to the observed data, even though the relevant parameter estimates are unrealistic. The reason for this may be that the algorithm searches for a set of parameters that produces the smallest deviation between the observed and the estimated schedules. Since the algorithm is constrained in its choice of the ratio $\sigma_2 = \lambda_2/\alpha_2$, it tries to compensate for this

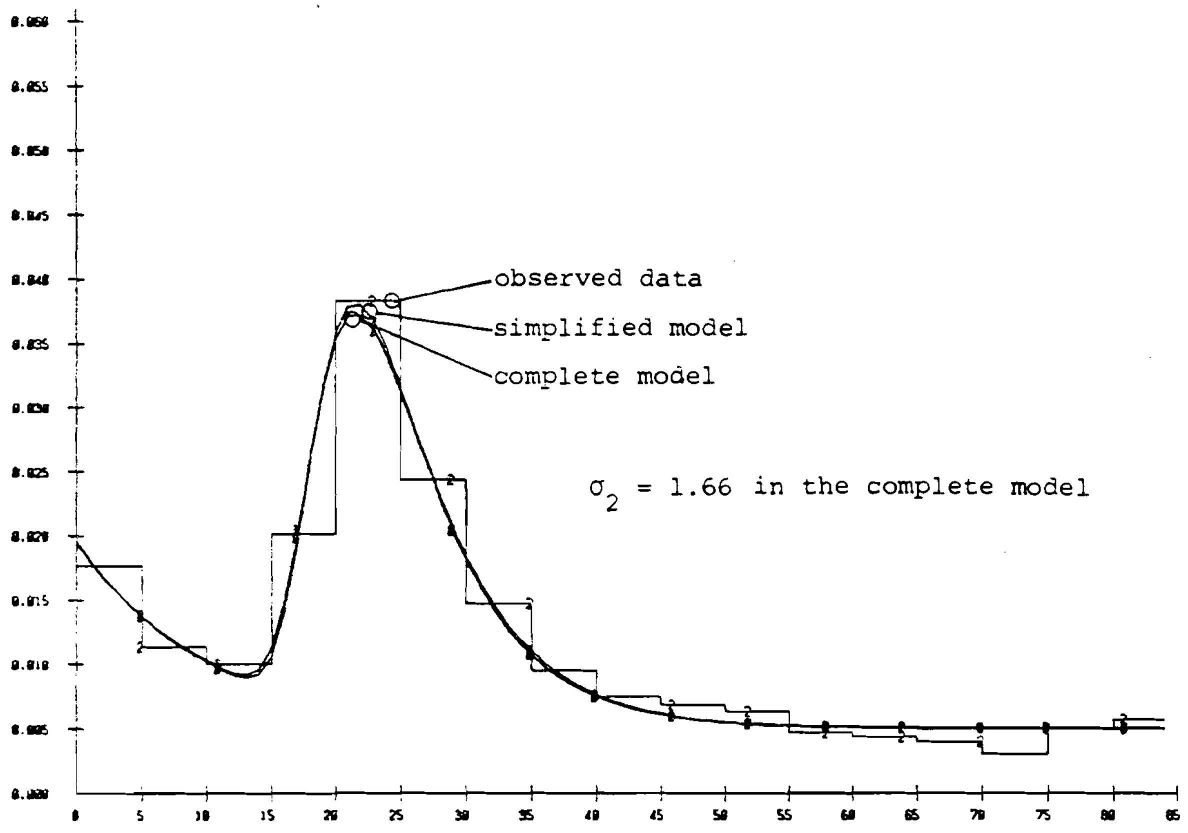


Region 4

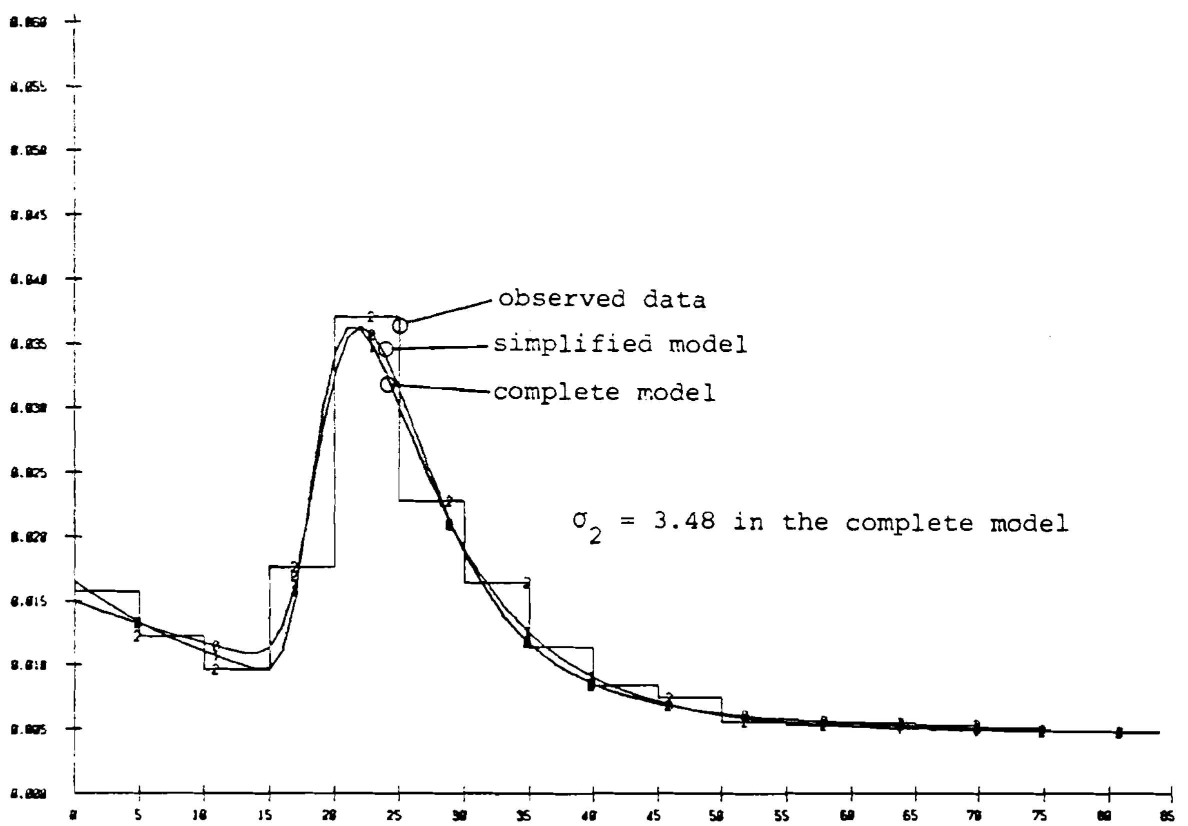


Region 8

Figure 1 Observed data and complete and simplified model migration schedules: Females, Sweden, 1974.

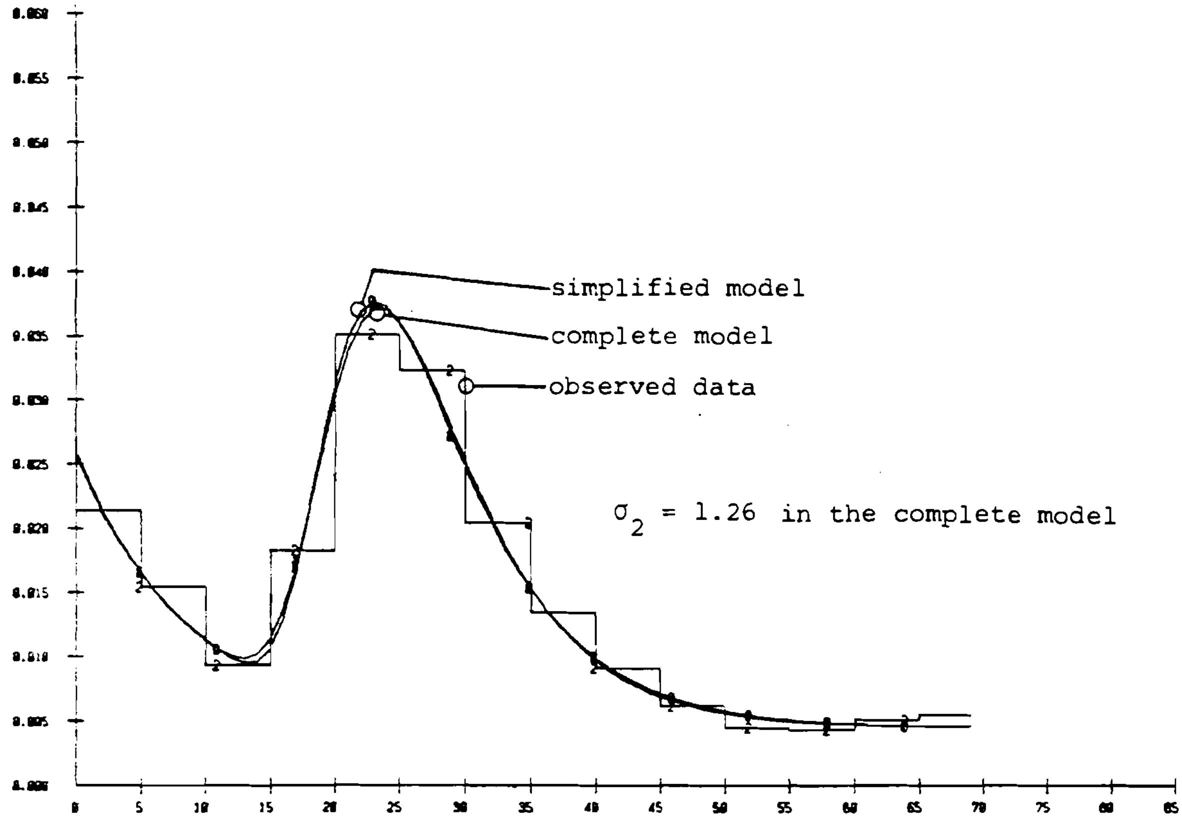


Region 1

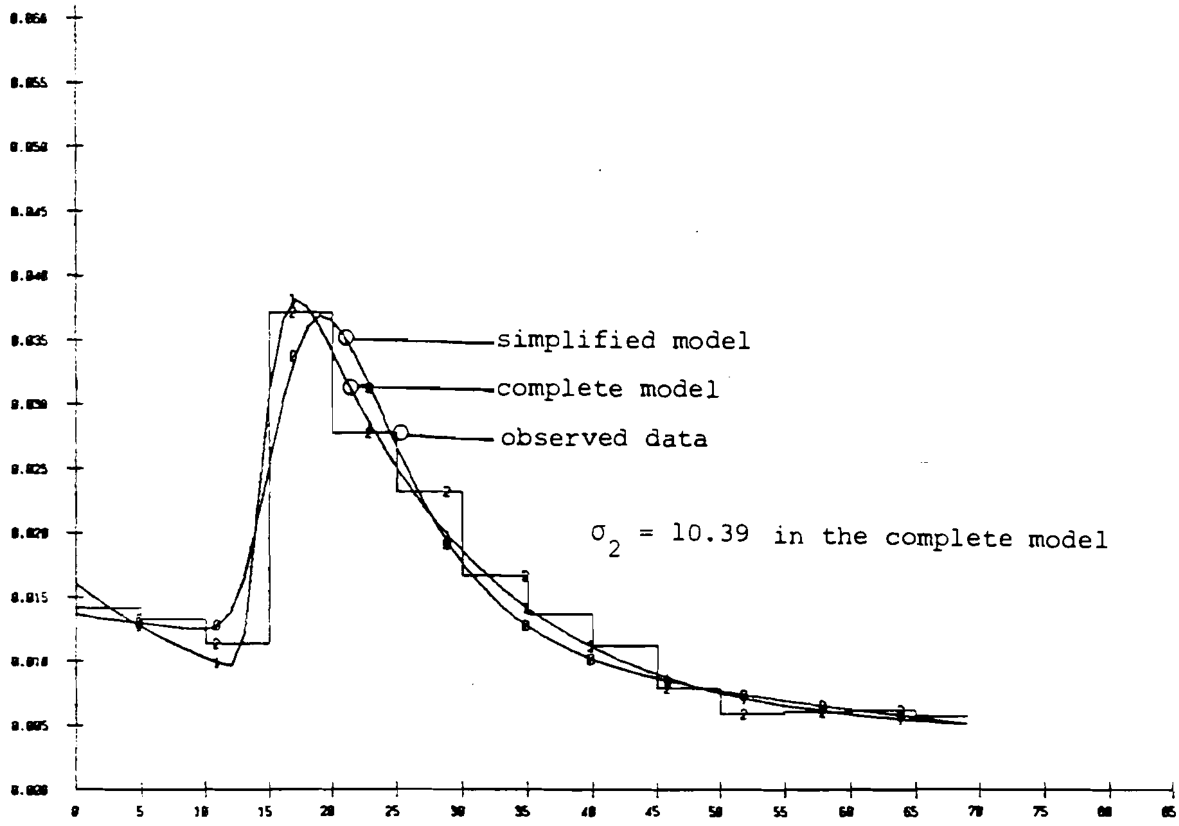


Region 3

Figure 2 Observed data and complete and simplified model migration schedules: Females, United Kingdom, 1970.



Region 5



Region 1

Figure 3 Observed data and complete and simplified model migration schedules: Females, Japan, 1970.

by changing the parameter values of the pre-labor force component and the constant term. The result therefore may be a "local" rather than a global optimum solution.

SIMPLIFIED AND COMPLETE MODEL MIGRATION SCHEDULES: LINEAR PARAMETER ESTIMATION

This section sets out a linear algorithm that may be used to estimate the parameters of the simplified or the complete model migration schedule. The calculations are simple enough to be carried out with a small electronic pocket calculator, and we illustrate them using the schedules analyzed in the previous section. The input data are taken to be the model migration schedules presented in Table 2 and not the observed data. This eliminates the need for smoothing the observed data and allows us to compare the performance of the linear estimation method with that of the nonlinear one.

Simplified Model

To estimate the parameters of the simplified model using the linear approach, we adopt the Type B specification which expresses the model schedule as a function of weights, means, and the standard deviation of the labor force component. A limitation of the linear estimation method is that it is not possible to apply it directly to the Type A model; however, since both models are equivalent formulations, x_o and k can be derived from the estimated parameter values of the Type B model.

To start the linear estimation procedure, set the parameter c equal to the average value of the fifteen oldest single-year age groups of the observed schedule m(x):

$$c = \frac{1}{15} \sum_{x=w-14}^{w} m(x) \qquad (17)$$

Given values for c and the last age group w, ϕ_c may be estimated with Equation (13):

$$\phi_c = c \cdot w \qquad (18)$$

The labor force component is estimated by first computing the function $m_2(x)$ [Equation (12)] as:

$$m_2(x) = m(x) - c \qquad \text{for } x_\ell + 1 \le x \le w - 15 \tag{19}$$

where x_ℓ is the observed (or assumed) low point. The weight ϕ_2 of the labor force component follows directly from

$$\phi_2 = \sum_{x=x_\ell+1}^{w-15} m_2(x) \tag{20}$$

and the mean age and standard deviation are defined as

$$\bar{x}_2 = \frac{1}{\phi_2}\left[\sum_{x=x_\ell+1}^{w-15} x \cdot m_2(x)\right] \tag{21}$$

and

$$S_2 = \left[\sum_{x=x_\ell+1}^{w-15} (x-\bar{x}_2)^2 \frac{m_2(x)}{\phi_2}\right]^{\frac{1}{2}} \tag{22}$$

respectively.

The parameter values of the pre-labor force component are derived in a similar fashion. First, the weight ϕ_1 is found as a residual,

$$\phi_1 = 1 - (\phi_c + \phi_2) \tag{23}$$

and the mean age $\bar{x}_1$ is defined as

$$\bar{x}_1 = \frac{1}{\phi_1}\left[\sum_{x=0}^{x_\ell} x \cdot m(x)\right] \tag{24}$$

Table 5 presents a summary of the basic steps, and Table 6 gives the parameters obtained with the linear and the nonlinear estimation methods. The observed differences between the two approaches are minor. Surprisingly, the simple linear method occasionally yields more realistic results (for example, for the schedules exhibiting the highest σ_2 values in the United Kingdom and Japan), but it always tends to underestimate the location of x_ℓ , the low point.

Complete Model

The linear procedure for estimating the constant term and the pre-labor force component of the complete model is identical to that used for the simplified model. The labor force component, however, is estimated in a different way, in order to relax the assumption that σ_2 is a constant.

Let

$$\sigma_2 = \varphi_{2b}/\varphi_{2a} \qquad (25)$$

where

$$\varphi_2 = \varphi_{2a} + \varphi_{2b} \qquad (26)$$

and

$$\phi_{2a} = \sum_{x=x_\ell+1}^{x_h} m_2(x) \qquad (27)$$

where x_h is the high point. Compute $m_2(x)$ and ϕ_2, using Equations (19) and (20), respectively. Then given σ_2 compute a_2 using the analytical expression of the migration rate at age x_h [Rogers and Castro (1981a, p. 48)]:

$$m(x_h) = a_2 \left(\frac{1}{\sigma_2}\right)^{\frac{1}{\sigma_2}} e^{-\frac{1}{\sigma_2}} \qquad (28)$$

Table 5 Linear parameter estimation: simplified model.

A. Constant term

$$c = \frac{1}{15} \sum_{x=w-14}^{w} m(x)$$

$$\phi_c = c \cdot w$$

B. Labor-force component

$$m_2(x) = m(x) - c \qquad \text{for } x_\ell+1 \leq x \leq w-15$$

$$\phi_2 = \sum_{x=x_\ell+1}^{w-15} m_2(x)$$

$$\bar{x}_2 = \frac{1}{\phi_2} \left[\sum_{x=x_\ell+1}^{w-15} x \cdot m_2(x) \right]$$

$$S_2 = \left[\sum_{x=x_\ell+1}^{w-15} (x-\bar{x}_2)^2 \frac{m_2(x)}{\phi_2} \right]^{\frac{1}{2}}$$

C. Pre-labor force component

$$\phi_1 = 1 - (\phi_c + \phi_2)$$

$$\bar{x}_1 = \frac{1}{\phi_1} \left[\sum_{x=0}^{x_\ell} x \cdot m(x) \right]$$

Table 6 Parameters and variables defining the female simplified model migration schedule for selected regions in Sweden, United Kingdom, and Japan: Linear and nonlinear parameter estimation.

	sweden				united kingdom				japan			
	region 4		region 3		region 1		region 3		region 5		region 1	
	linear	non-linear	linear	non-linear	linear	non-linear	linear	non-linear	linear	non-linear	linear	non-linear
gmr(obs)	0.839	0.839	1.237	1.237	1.197	1.197	1.040	1.040	0.783	0.783	1.414	1.413
gmr(mod)	0.988	1.000	0.985	1.000	0.976	1.000	0.976	1.000	0.980	1.000	0.978	1.000
mae%m	5.256	9.777	12.361	15.629	6.574	7.358	8.781	7.320	7.720	4.131	11.709	11.796
phi1	0.186	0.215	0.119	0.122	0.113	0.143	0.119	0.304	0.142	0.199	0.086	50.204
x1	7.176	8.587	7.630	6.475	9.005	9.579	11.118	27.604	7.527	8.650	8.986	619.517
phi2	0.597	0.541	0.613	0.567	0.447	0.394	0.451	0.326	0.492	0.490	0.527	0.346
x2	27.591	27.158	24.806	23.832	25.477	25.648	27.378	25.976	27.307	28.130	26.823	23.538
a	7.393	6.922	7.025	5.587	6.973	6.454	8.372	6.174	7.749	7.977	10.833	6.960
c	0.003	0.003	0.003	0.004	0.005	0.005	0.005	0.004	0.004	0.004	0.005	-0.067
mean age	28.340	28.643	29.103	29.759	33.576	33.628	34.337	34.012	32.131	32.035	33.501	31.962
%(0-14)	19.880	21.488	15.546	16.076	16.499	18.949	16.089	18.695	18.225	20.811	17.485	19.031
%(15-64)	72.977	70.119	75.645	73.332	69.248	66.135	69.868	65.997	69.832	67.051	69.600	70.123
%(65+)	7.243	8.393	8.810	10.592	14.253	14.916	14.043	14.308	11.943	12.138	12.915	10.847
phi1/2	0.311	0.398	0.194	0.215	0.254	0.376	0.265	0.934	0.289	0.405	0.164	145.032
x low	13.340	14.520	11.770	13.250	12.530	13.760	12.130	14.390	13.240	13.880	7.920	10.220
x high	23.240	23.330	20.960	20.850	21.640	22.080	22.730	22.550	23.040	23.680	20.770	19.690
x shift	9.900	8.810	9.190	7.600	9.060	8.320	10.600	8.160	9.800	9.800	12.850	9.470
a	29.948	33.280	30.160	32.250	29.374	31.480	32.305	29.800	30.187	34.780	35.942	30.840
b	0.076	0.037	0.042	0.051	0.031	0.029	0.025	0.025	0.030	0.028	0.022	0.024

whence

$$a_2 = m(x_h) \Big/ \left[(1/\sigma_2)^{1/\sigma_2} e^{-1/\sigma_2} \right] \qquad (29)$$

Given a_2, σ_2, and ϕ_2, estimate λ_2 by recalling Equations (10) and (12) and noting that a_2 may be expressed as

$$a_2 = \phi_2 \frac{\lambda_2}{\Gamma(\alpha_2/\lambda_2)} \qquad (30)$$

Solving for λ_2, and substituting $\sigma_2 = \lambda_2/\alpha_2$ into the expression, gives

$$\lambda_2 = \left[a_2 \Gamma(1/\sigma_2) \right] \Big/ \phi_2 \qquad (31)$$

Finally, recalling the definition of σ_2 yields α_2:

$$\alpha_2 = \lambda_2/\sigma_2 \qquad (32)$$

Table 7 outlines a summary of the basic steps of the linear parameter estimation method for the complete model. Table 8 presents both the linear and the nonlinear parameter estimations of the complete model for purposes of comparison. The differences between the two sets of estimated parameters are remarkably small.

The linear and nonlinear estimation methods also may be compared by examining the model age profiles that they generate. Figures 4 through 6 present these profiles, demonstrating that in most cases the linear estimation method gives a very adequate approximation to the complete model migration schedule that is defined by the nonlinearly estimated parameter values.

CONCLUSION

The aim of this paper has been to introduce (1) a model migration schedule expressed in terms of familiar statistical measures such as means and variances, and (2) a simple linear

Table 7 Linear parameter estimation: complete model.

A. Constant term

$$c = \frac{1}{15} \sum_{x=w-14}^{w} m(x)$$

$$\phi_c = c \cdot w$$

B. Labor force component

$$m_2(x) = m(x) - c \qquad \text{for } x_\ell+1 \le x \le w-15$$

$$\phi_2 = \sum_{x=x_\ell+1}^{w-15} m_2(x)$$

$$\phi_{2a} = \sum_{x=x_\ell+1}^{x_h} m_2(x)$$

$$\phi_{2b} = \phi_2 - \phi_{2a}$$

$$\sigma_2 = \phi_{2b}/\phi_{2a}$$

$$a_2 = m(x_h) \Big/ \left[(1/\sigma_2)^{1/\sigma_2} e^{-1/\sigma_2} \right]$$

$$\lambda_2 = \left[a_2 \Gamma(1/\sigma_2) \right] \Big/ \phi_2$$

$$\alpha_2 = \lambda_2/\sigma_2$$

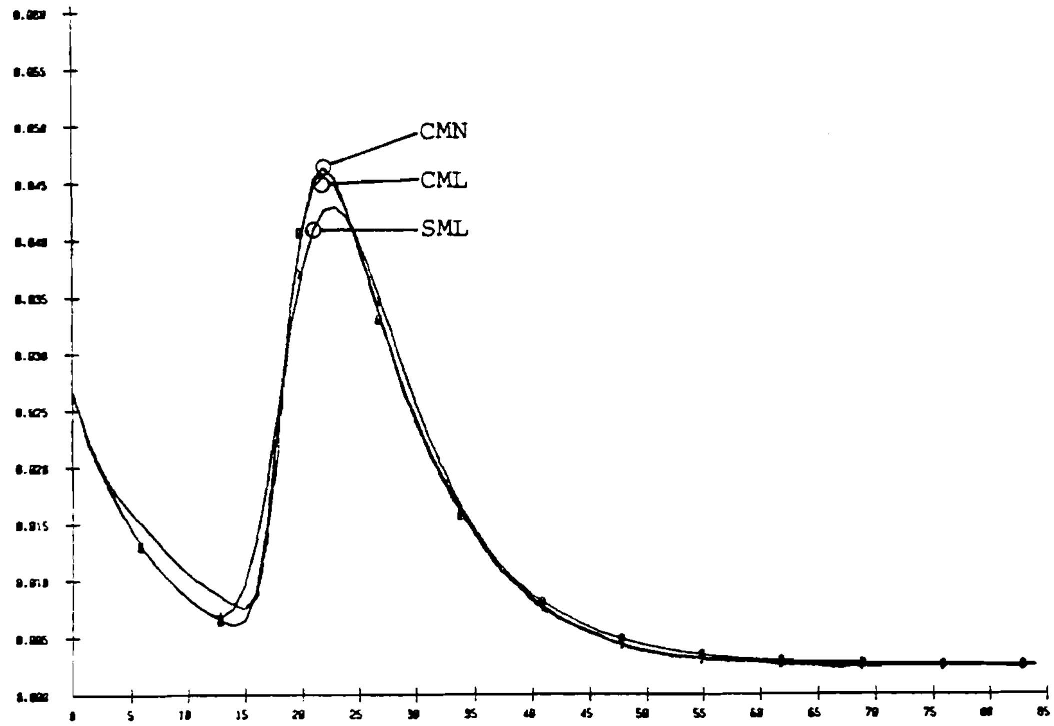
C. Pre-labor force component

$$\phi_1 = 1 - (\phi_c + \phi_2)$$

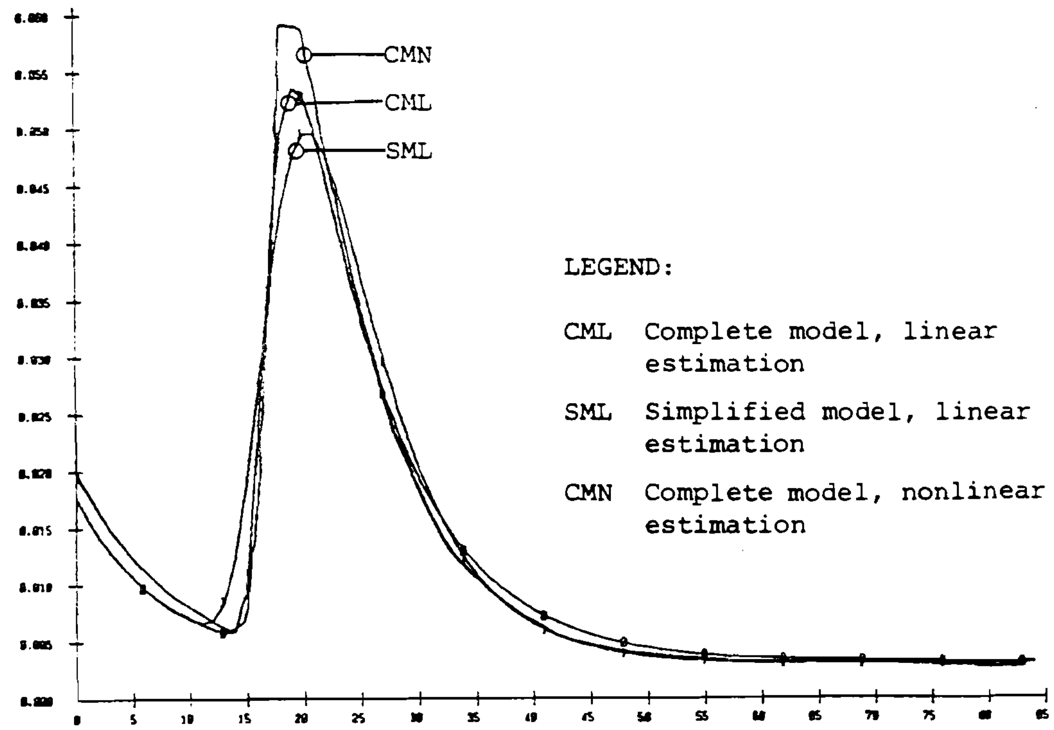
$$\bar{x}_1 = \frac{1}{\phi_1} \left[\sum_{x=0}^{x_\ell} x \cdot m(x) \right]$$

Table 8 Parameters and variables defining the female complete model migration schedule for selected regions in Sweden, United Kingdom , and Japan: Linear and nonlinear estimates.

	s w e d e n				u n i t e d k i n g d o m				j a p a n			
	region 4		region 3		region 1		region 3		region 5		region 1	
	linear	non-linear	linear	non-linear	linear	non-linear	linear	non-linear	linear	non-linear	linear	non-linear
gmr(obs)	0.838	0.838	1.237	1.237	1.197	1.197	1.040	1.040	0.783	0.783	1.414	1.414
gmr(mod)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000
mae%m	5.230	3.909	8.542	13.167	5.774	9.194	6.808	6.379	6.162	4.816	6.713	7.878
a1	0.026	0.025	0.016	0.019	0.013	0.015	0.011	0.012	0.019	0.022	0.010	0.012
alpha1	0.139	0.104	0.131	0.128	0.111	0.100	0.090	0.062	0.133	0.113	0.111	0.069
a2	0.034	0.080	0.084	0.094	0.078	0.076	0.056	0.054	0.075	0.083	0.048	0.042
mu2	17.653	19.882	17.263	17.620	19.398	20.392	18.799	19.324	21.746	23.021	13.970	14.772
alpha2	0.125	0.129	0.125	0.143	0.154	0.171	0.112	0.131	0.136	0.159	0.084	0.075
lambda2	0.385	0.442	0.615	0.711	0.293	0.284	0.414	0.456	0.227	0.200	0.467	0.781
c	0.003	0.003	0.003	0.003	0.005	0.005	0.005	0.005	0.004	0.005	0.005	0.004
mean age	28.346	28.139	29.961	28.990	34.297	33.477	35.092	34.032	33.093	32.047	33.720	33.258
%(0-14)	19.614	21.929	14.536	16.400	16.280	19.033	15.658	18.649	18.122	20.865	17.591	17.478
%(15-64)	72.693	70.757	76.124	74.555	68.900	66.316	69.606	67.030	69.372	66.888	68.665	69.659
%(65+)	7.693	7.314	9.341	9.044	14.319	14.651	14.736	14.321	12.506	12.247	13.744	12.864
delta1c	10.131	9.932	4.933	5.839	2.430	3.023	2.122	2.544	4.405	4.907	2.117	2.767
delta12	0.309	0.312	0.186	0.198	0.162	0.201	0.192	0.225	0.252	0.271	0.201	0.294
beta12	1.111	0.309	1.047	0.390	0.721	0.584	0.804	0.477	0.977	0.710	1.329	0.917
sigma2	3.066	3.434	4.911	4.953	1.900	1.661	3.698	3.485	1.668	1.255	5.583	10.389
x low	14.730	15.610	14.010	14.770	13.160	13.530	14.160	15.110	13.420	13.520	10.040	12.370
x high	22.510	22.580	19.320	19.850	22.010	22.080	21.870	21.960	23.900	24.000	17.530	17.650
x shift	7.730	6.970	5.310	5.080	8.350	8.550	7.710	6.850	10.480	10.480	7.490	5.280
a	30.003	27.874	31.213	28.315	29.949	27.173	32.606	28.067	31.593	28.885	35.190	32.117
b	0.040	0.038	0.048	0.052	0.031	0.027	0.028	0.026	0.028	0.025	0.027	0.025

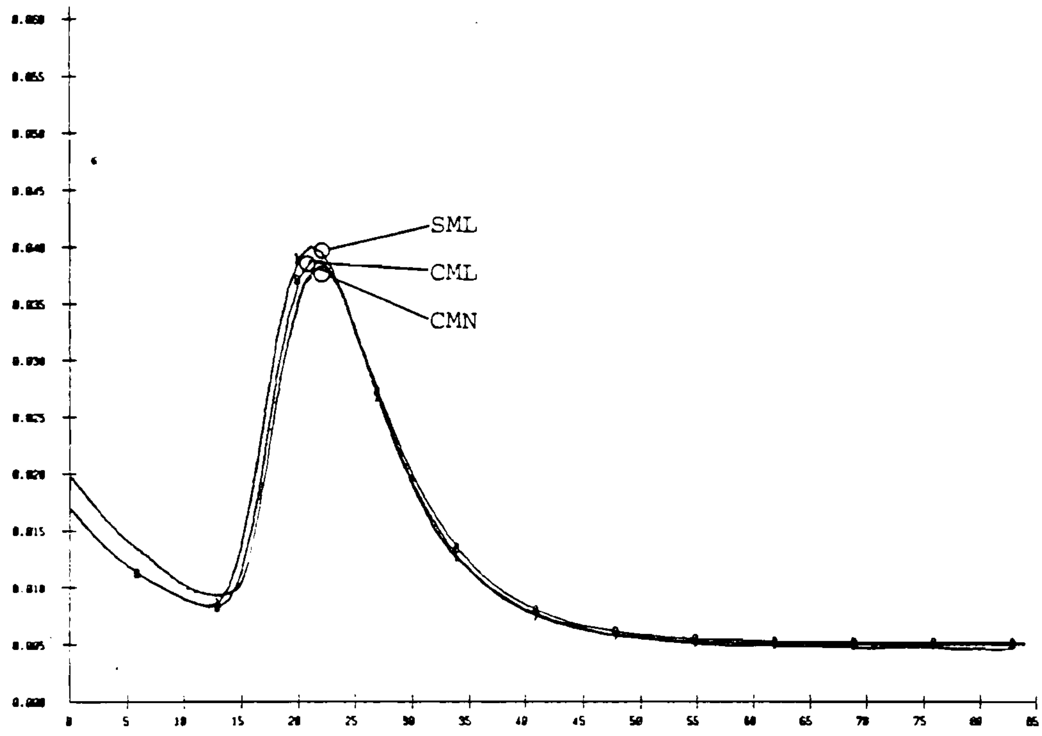


Region 4

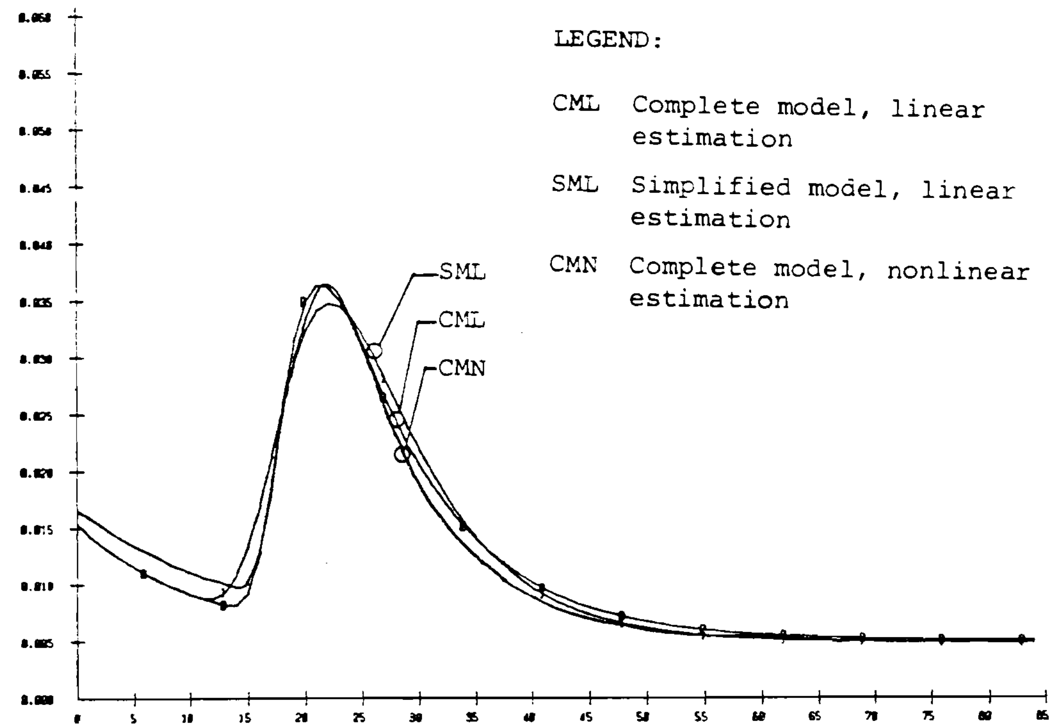


Region 8

Figure 4 Linear estimation of the complete and simplified model migration schedules compared with the nonlinear estimation of the complete model: Females, Sweden, 1970.

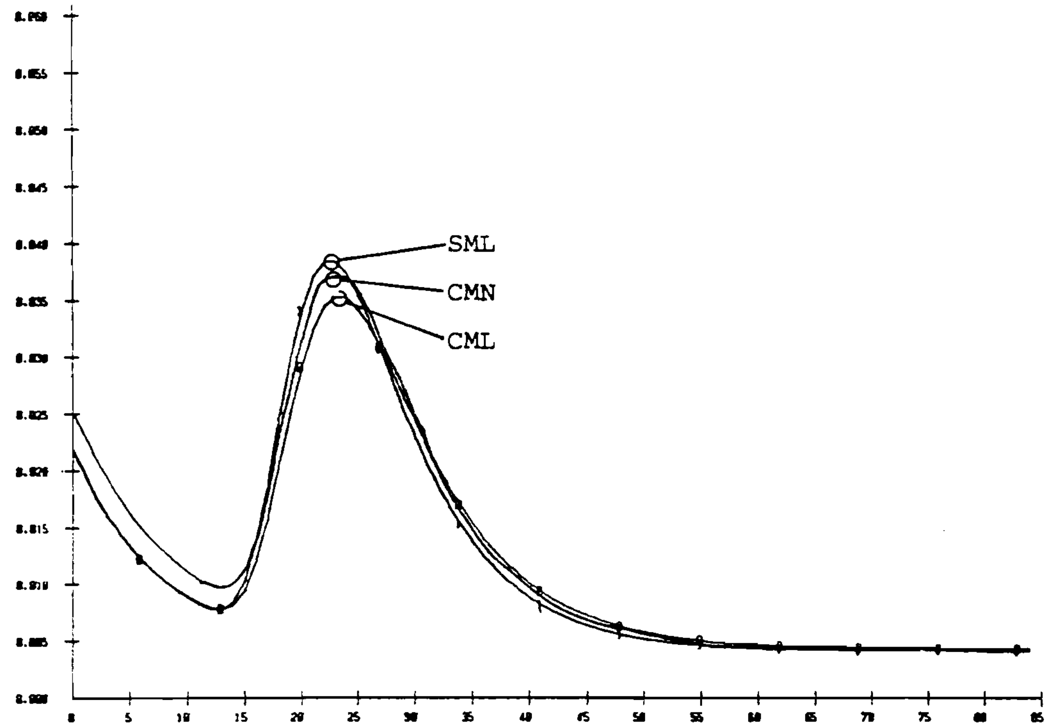


Region 1

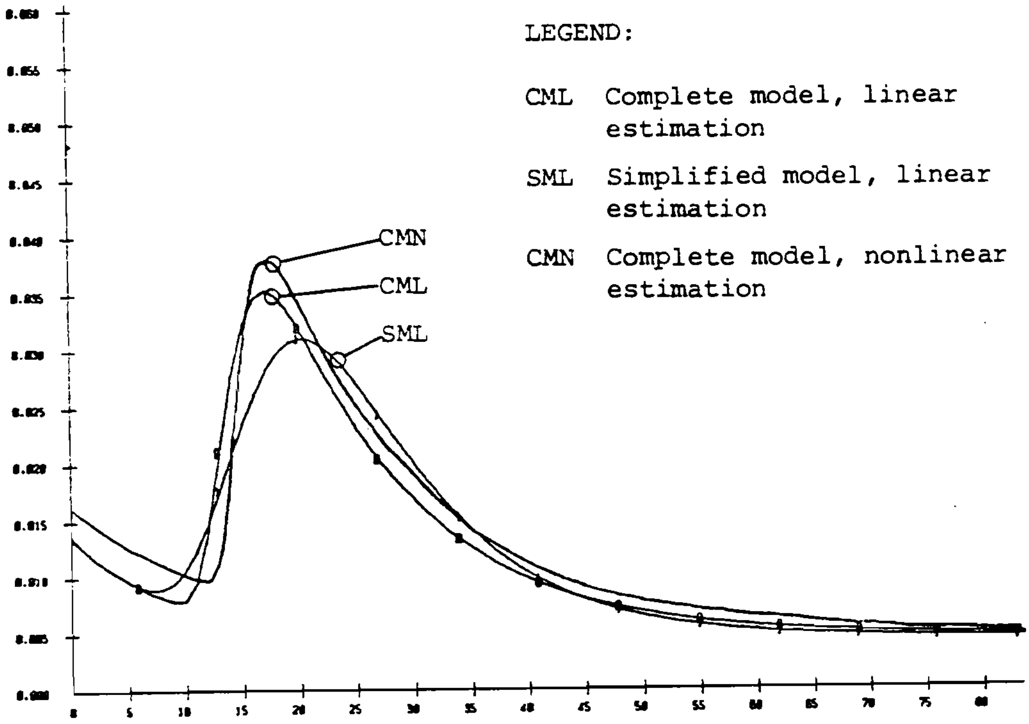


Region 3

Figure 5 Linear estimation of the complete and simplified model migration schedules compared with the nonlinear estimation of the complete model: Females, United Kingdom, 1970.



Region 5



Region 1

Figure 6 Linear estimation of the complete and simplified model migration schedules compared with the nonlinear estimation of the complete model: Females, Japan, 1970.

parameter estimation method that may be used in situations where access to large computers and packaged programs is limited.

The notion of simplified model schedules has been successfully applied in the construction of nuptiality schedules, and some of the same simplifications seem to be applicable in the case of migration. The results obtained by the simplified model are satisfactory when contrasted with those found using the original "complete" model of previous research (Rogers and Castro, 1981b). And it seems likely that the performance of the simplified model may be further improved by choosing a standard migration function that implies a more appropriate value for the labor asymmetry index σ_2. Moreover, the linear estimation method is simple enough to be implemented with small electronic pocket calculators; access to large computer facilities with their complex packaged nonlinear parameter estimation algorithms becomes unnecessary.

Finally, it appears that a promising direction for future research lies in the application of the simplified model to the analysis of family dependency relationships in migration patterns. In Castro and Rogers (1979), for example, we have shown that the age distribution of migrants $n(x)$ exhibits the same fundamental regularities found in observed age-specific migration rate schedules and that the complete model yields an adequate mathematical representation of such regularities. It is not surprising, therefore, that the simplified model, defined in this paper, also may be fitted to such observed migration age profiles. In such instances, the ratio ϕ_1/ϕ_2 of the estimated parameters defines the number of dependents per labor force migrant, and this quantity, in turn, may be a close approximation of the average family size among migrants. This aspect of the model will be explored in a forthcoming paper.

REFERENCES

Castro, L.J. and A. Rogers (1979) *Migration Age Patterns: Measurement and Analysis.* WP-79-16. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Coale, A. (1971) Age-patterns of marriage. *Population Studies* 25:193-214.

Coale, A. and D.R. McNeil (1972) The distribution by age at first marriage in a female cohort. *Journal of the American Statistical Association* 67:743-749.

Coale, A. and J. Trussell (1974) Model fertility schedules: variations in the age structure of childbearing in human populations. *Population Index* 40(2):185-206.

Rodriguez, G. and J. Trussell (1980) Maximum likelihood estimation of the parameters of Coale's model nuptiality schedule from survey data. *World Fertility Survey, Technical Bulletins* 7 Tech. 1261, Voorburg, Netherlands: International Statistical Institute.

Rogers, A. and L.J. Castro (1981a) *Model Schedules in Multistate Demographic Analysis: The Case of Migration.* WP-81-22. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Rogers, A. and L.J. Castro (1981b) *638 Model Migration Schedules: A Technical Appendix.* WP-81-23. Laxenburg, Austria: International Institute for Applied Systems Analysis.

RELATED PUBLICATIONS
OF THE MIGRATION AND SETTLEMENT TASK

THEORY AND MODELS

1. Andrei Rogers, *Migration and Settlement: Selected Essays*. RR-78-6. Reprinted from a special issue of *Environment and Planning A*.

2. Andrei Rogers and Frans Willekens, *Migration and Settlement: Measurement and Analysis*. RR-78-13.

3. Frans Willekens and Andrei Rogers, *Spatial Population Analysis: Methods and Computer Programs*. RR-78-18.

4. Andrei Rogers, *Migration Patterns and Population Redistribution*. RR-80-7. Reprinted from *Regional Science and Urban Economics*.

5. Andrei Rogers, *Essays in Multistate Demography*. RR-80-10. Reprinted from a special issue of *Environment and Planning A*.

6. Nathan Keyfitz, *Multidimensionality in Population Analysis*. RR-80-33. Reprinted from *Sociological Methodology 1980*.

NATIONAL CASE STUDIES

1. Philip Rees, *Migration and Settlement: 1. United Kingdom.* RR-79-3.

2. Kalevi Rikkinen, *Migration and Settlement: 2. Finland.* RR-79-9.

3. Åke Andersson and Ingvar Holmberg, *Migration and Settlement: 3. Sweden.* RR-80-5

4. Gerhard Mohs, *Migration and Settlement: 4. German Democratic Republic.* RR-80-6.

5. Paul Drewe, *Migration and Settlement: 5. Netherlands.* RR-80-13.

6. Marc Termote, *Migration and Settlement: 6. Canada.* RR-80-29.

7. Klára Bies and Kálmán Tekse, *Migration and Settlement: 7. Hungary.* RR-80-34.

8. Svetlana Soboleva, *Migration and Settlement: 8. Soviet Union.* RR-80-36.

9. Reinhold Koch and Hans-Peter Gätzweiler, *Migration and Settlement: 9. Federal Republic of Germany.* RR-80-37.

10. Michael Sauberer, *Migration and Settlement: 10. Austria.* Forthcoming.

11. Kazimierz Dziewoński and Piotr Korcelli, *Migration and Settlement: 11. Poland.* Forthcoming.

12. Dimiter Philipov, *Migration and Settlement: 12. Bulgaria.* Forthcoming.

13. Jacques Ledent, *Migration and Settlement: 13. France.* Forthcoming.

14. Karel Kühnl, *Migration and Settlement: 14. Czechoslovakia.* Forthcoming.

15. Zenji Nanjo, Tatsuhiko Kawashima, and Toshio Kuroda, *Migration and Settlement: 15. Japan.* Forthcoming.

16. William Frey and Larry Long, *Migration and Settlement: 16. United States.* Forthcoming.

17. Agostino LaBella, *Migration and Settlement: 17. Italy.* Forthcoming.