International Institute for
Applied Systems Analysis
IIASA www.iiasa.ac.at

Multiregional Population Projection: An Analytic Approach

Liaw, K.-L.

IIASA Working Paper

WP-81-081

June 1981

MULTIREGIONAL POPULATION PROJECTION: AN ANALYTIC APPROACH

Kao-Lee Liaw

June 1981
WP-81-81

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

FOREWORD

The evolution of human populations over time and space has been a central concern of many scholars in the Human Settlements and Services Area at IIASA during the past several years. From 1975 through 1978 some of this interest was manifested in the work of the Migration and Settlement Task, which was formally concluded in November 1978. Since then, attention has turned to disseminating the Task's results, to concluding its comparative study, and to exploring possible future activities that might apply the mathematical methodology to other research topics.

This paper is part of the Task's dissemination effort. It is a draft of a chapter that is to appear in a volume entitled *Migration and Settlement: A Comparative Study*. Other selected publications summarizing the work of the Migration and Settlement Task are listed at the back.

Andrei Rogers
Chairman
Human Settlements
and Services Area

ABSTRACT

This paper studies the dynamic properties of age-by-region population systems which are projected into the future by the Rogers model with fixed age- and region-specific rates of birth, death, and interregional migration. Insights are obtained through the analytic solution of the mathematical model.

To make the analytic solution of the Rogers model understandable this paper considers the Rogers model as a multiregional generalization of the classical Leslie model and then makes a useful synthesis of the early findings about the dynamic properties of single-region population systems and the well-known facts about a pure migration matrix.

The Rogers model is applied to three real-world systems: the Swedish female population observed in 1974, the Soviet population of both sexes observed in 1974, and the female population of Great Britain observed in 1970. Common as well as distinct dynamic properties of the three systems are found.

ACKNOWLEDGMENTS

I am grateful to Andrei Rogers for providing me with a wonderful research environment at IIASA and a demographic model that is substantively meaningful and analytically and computationally tractable. I am also indebted to all individuals who contributed to the construction of the data base that is used in this report. Comments from Philip Rees on the first draft are highly appreciated. Thanks to Susanne Stock for typing the paper carefully and to Maria Rogers for editing my work into a more readable form. The remaining flaws are all mine.

CONTENTS

MULTIREGIONAL POPULATION PROJECTION--
AN ANALYTIC APPROACH

1. INTRODUCTION

The basic question to be answered in this paper is: How would the characteristics of a population, such as the size, spatial distribution, and regional age profiles, evolve through time if an observed set of age- and region-specific rates of birth, death, and *interregional migration* were to remain constant? Before attempting to answer this question, it is necessary to know what *useful* knowledge can be obtained from the answer.

First, to the extent that certain attributes of the basic demographic processes will remain relatively stable in the foreseeable future, the answer may serve well as a prediction. Consider the Canadian interregional population system for example. It is reasonable to assume that interregional outmigration rates will remain through the 1980's to be much higher for the destinations of Alberta and British Columbia than for the destination of Quebec. Thus, with respect to the relative shares of the national population by these provinces, the multiregional population projection based on the constant rates of the 1970's (Liaw 1980b) can serve as a prediction with a small margin of error. Similarly, because of the very stable nature of age-specific mortality rates and the implausibility of a sharp rise in fertility level, the increase in the mean age of the Canadian population that is projected with the 1971-76 fixed growth matrix (Liaw 1980b) is likely to be close to what will actually happen in the next two decades. Essentially, the projection with fixed rates reveals whether the characteristics of the *status* of a population system is compatible with those of its basic demographic *processes*. If they are highly incompatible, then either the status or the processes or both must experience significant changes. It is through the incompatibility, or the lack of it, that the projection with fixed rates helps us speculate what will happen in the future.

Second, population projection with fixed rates can reveal, without excessive complications, the major causal relationships which are inherent in a human population. An advantage of the knowledge of these relationships is that it can prevent us from misinterpreting demographic data. For example, a multiregional population projection using a fixed growth matrix can show clearly that within individual regions, there is a strong causal relationship running from a high fertility level to a steep age profile, and then to a low crude death rate (Liaw 1980a). The knowledge of this relationship enables us to avoid inferring incorrectly from the fact that the crude death rate is lower in the northern territories of Canada than in the rest of the country that somehow the harsh environment of Arctic region is particularly conducive to longevity.

Third, the transmission of population waves, which is of particular importance to the providers of age-specific goods and services (e.g., baby food producers, daycare centers, schools, and universities) but is usually ignored, becomes a prominent feature that can be studied systematically when a population projection with age-specific fixed rates is carried out.

The causal relationships of particular interest in this paper are those concerning the transmission of population waves, the change in regional age profiles, and the spatial redistribution of the population. We will use the discrete-time age-by-region *Rogers model* and its *analytic solution* to carry out the investigation. Besides showing the dynamic properties that are common among industrialized countries, our results will also highlight the distinct features of each population system. From a theoretical point of view, our results also clarify how the dynamic properties of the non-spatial Lotka or Leslie model of population growth can be synthesized with well-known properties of an irreducible stochastic matrix to facilitate the study of age-by-region population systems.

The three age-by-region population systems to be analyzed in this paper are: the Swedish female population, 1974; the Soviet population of both sexes, 1974; and the female population of Great Britain, 1970. Each system is disaggregated into eighteen age groups (0-4, 5-9,..., 85+) and eight regions (see Figures 1.1 through 1.3). For the Soviet Union, the first seven regions contain only urban areas, whereas the eighth "region" contains the rural areas of the whole country. The single-year data base for each nation available at IIASA has been transformed into a five-year growth matrix by IIASA's standard procedure.

Section 2 introduces the Rogers model and its analytic solution. Section 3 shows how indepth understanding of the Rogers model can be obtained by considering it as a generalization of the Leslie model. Section 4 provides a detailed analysis of the Swedish population system and demonstrates the plausibility of the line of thought advocated in section 3. Sections 5 and 6 report the findings of applying the Rogers model directly to the Soviet and British population systems. Finally, section 7 summarizes the main points and concludes the paper.

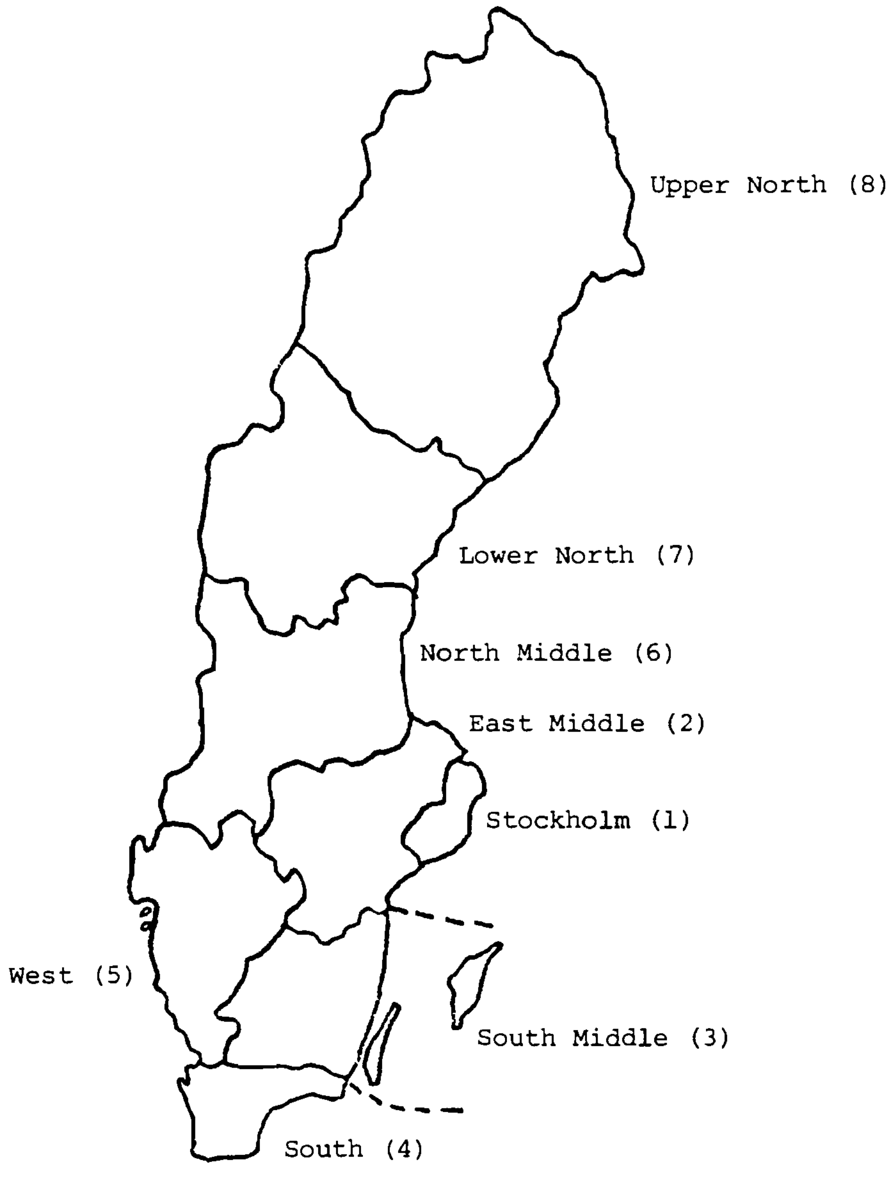

Figure 1.1 The regions of the Swedish system. (Source: Andersson and Holmberg, 1980.)

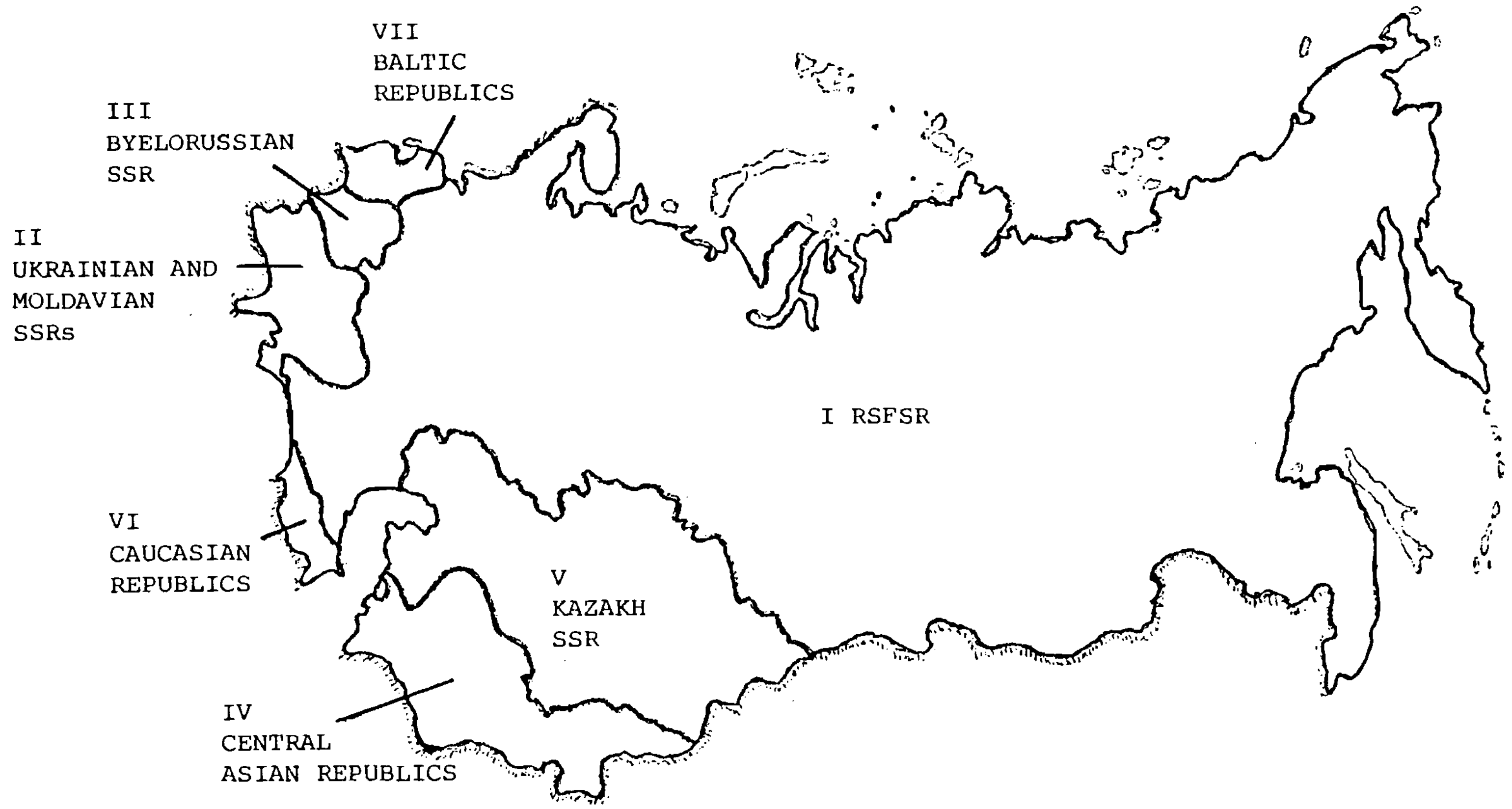

Figure 1.2 The regions of the Soviet system. (Source: Soboleva, 1980.)

Figure 1.3 The regions of the British system. (Source: Rees, 1979.)

2. THE ROGERS MODEL AND ITS ANALYTIC SOLUTION

Consider a population system with r regions and w five-year age groups (0-4, 5-9,...). Using five years as the unit time interval, the (discrete) *Rogers model* (Rogers, 1975:117-129) is of the form

$$\begin{bmatrix} \underset{\sim}{K}_1(t+1) \\ \underset{\sim}{K}_2(t+1) \\ \underset{\sim}{K}_3(t+1) \\ \vdots \\ \underset{\sim}{K}_f(t+1) \\ \hline \underset{\sim}{K}_{f+1}(t+1) \\ \vdots \\ \underset{\sim}{K}_w(t+1) \end{bmatrix} = \left[\begin{array}{ccccc|ccccc} \underset{\sim}{B}_1 & \underset{\sim}{B}_2 & \cdots & \underset{\sim}{B}_{f-1} & \underset{\sim}{B}_f & & & & & \\ \underset{\sim}{S}_1 & \underset{\sim}{0} & \cdots & \underset{\sim}{0} & \underset{\sim}{0} & & & & & \\ \underset{\sim}{0} & \underset{\sim}{S}_2 & & \underset{\sim}{0} & \underset{\sim}{0} & & & \underset{\sim}{0} & & \\ & \cdots & \cdots & \cdots & & & & & & \\ \underset{\sim}{0} & \underset{\sim}{0} & \cdots & \underset{\sim}{S}_{f-1} & \underset{\sim}{0} & & & & & \\ \hline \underset{\sim}{0} & \underset{\sim}{0} & \cdots & \underset{\sim}{0} & \underset{\sim}{S}_f & \underset{\sim}{0} & \underset{\sim}{0} & \cdots & \underset{\sim}{0} & \underset{\sim}{0} \\ & & & & & \underset{\sim}{S}_{f+1} & \underset{\sim}{0} & \cdots & \underset{\sim}{0} & \underset{\sim}{0} \\ & & \underset{\sim}{0} & & & & \cdots & \cdots & \cdots & \\ & & & & & \underset{\sim}{0} & \underset{\sim}{0} & & \underset{\sim}{S}_{w-1} & \underset{\sim}{0} \end{array}\right] \begin{bmatrix} \underset{\sim}{K}_1(t) \\ \underset{\sim}{K}_2(t) \\ \underset{\sim}{K}_3(t) \\ \vdots \\ \underset{\sim}{K}_f(t) \\ \hline \underset{\sim}{K}_{f+1}(t) \\ \vdots \\ \underset{\sim}{K}_w(t) \end{bmatrix} \qquad (2.1)$$

for $t = 0,1,2,\ldots$

where $\underset{\sim}{K}_a(t)$ for $a = 1,2,\ldots,w$ represents the interregional population distribution of the a-th group at time t; the $\underset{\sim}{B}$ submatrices show how babies are born, survive, and migrate among regions; and the $\underset{\sim}{S}$ submatrices indicate how existing people in individual age groups survive and migrate within a unit time interval.* The order of $\underset{\sim}{K}_a(t)$ is $r \times 1$, and those of $\underset{\sim}{B}_a$ and $\underset{\sim}{S}_a$ are $r \times r$. We may write the model more compactly as

$$\underset{\sim}{K}(t+1) = \underset{\sim}{G}\underset{\sim}{K}(t) \qquad (2.2)$$

---

*If foreign migration is important, the model can be modified to accommodate it (see Liaw 1978b and 1979).

where the *growth matrix* $\underset{\sim}{G}$ has the order of wr × wr, and the population vectors $\underset{\sim}{K}(t+1)$ and $\underset{\sim}{K}(t)$ the order of wr × 1.

Using the empirically valid assumption that the non-zero eigenvalues of $\underset{\sim}{G}$ are distinct, it was shown in Liaw (1978a) that the *analytic solution* of equation (2.2) is

$$\underset{\sim}{K}(t) = \sum_i \lambda_i^t \underset{\sim}{Q}_i \underset{\sim}{P}_i' \underset{\sim}{K}(0) + \text{"Residual"} \qquad (2.3)$$

where λ_i is a non-zero *eigenvalue* of $\underset{\sim}{G}$, and $\underset{\sim}{Q}_i$ and $\underset{\sim}{P}_i'$ are respectively the normalized *right* and *left eigenvectors* of $\underset{\sim}{G}$ associated with λ_i. In other words, λ_i, $\underset{\sim}{Q}_i$, and $\underset{\sim}{P}_i'$ are computed from the conditions

$$\underset{\sim}{G}\underset{\sim}{Q}_i = \lambda_i \underset{\sim}{Q}_i$$

$$\underset{\sim}{P}_i'\underset{\sim}{G} = \lambda_i \underset{\sim}{P}_i'$$

$$\underset{\sim}{P}_i'\underset{\sim}{Q}_i = 1$$

and

$$\lambda_i \neq 0$$

The "residual" is a vector filled with zeros up to the last reproductive age group (f) and will become a zero vector for $t \geq w - f$. By letting $c_i = \underset{\sim}{P}_i'\underset{\sim}{K}(0)$, the analytic solution can also be written as

$$\underset{\sim}{K}(t) = \sum_i \lambda_i^t c_i \underset{\sim}{Q}_i + \text{"Residual"} \qquad (2.4)$$

Ignoring the residual, we see that the age-by-region population at time t is simply a linear combination of the right eigenvectors of $\underset{\sim}{G}$, with the weights being $\lambda_i^t c_i$.* The relative importance of each term at t = 0 depends on the magnitude of the scalar c_i. However, since c_i remains constant through time, the relative importance of each term for large t depends increasingly on the magnitude of λ_i. As t becomes very large, only the *dominant component* (i.e., the term associated with the largest eigenvalue λ_1) remains important. In other words, we have

$$\lim_{t\to\infty} \underset{\sim}{K}(t)/\lambda_1^t = c_1 \underset{\sim}{Q}_1$$

where, according to the Frobenius theorem (Gantmacher, Vol. 2, 1969:53-54), both c_1 and $\underset{\sim}{Q}_1$ are real and non-negative. Clearly, the system will approach a fixed *long-run age-by-region population distribution*, which is represented by the dominant right eigenvector $\underset{\sim}{Q}_1$. The annual discrete *long-run growth rate* of every subpopulation is $\lambda^{0.2} - 1$. The scalar c_1 is the *stable equivalent population*. The *momentum* of the population system is

$$m = [(c_1 - k)/k] \cdot 100\%$$

where k is the total population size at t = 0. The momentum represents the percentage amplification (or deduction, if m is negative) of the "ultimate" population size due to the difference between the initial and the long-run age-by-region distributions. By "ultimate" we mean the time when the dominant component begins to overwhelm the remaining components in the analytic solution. It is obvious that the dominant component, as well as other components associated with positive real eigenvalues, will behave *monotonically* through time.

---

*Note that the right eigenvectors can be arbitrarily scaled as long as $\underset{\sim}{P}_i' \underset{\sim}{Q}_i = 1$ is satisfied. For interpretational and computational convenience, the sum of the magnitudes of the elements in each real right eigenvector is set to unity, and the sum of the squares of the elements of each complex right eigenvector is also set to unity.

Most of the eigenvalues of the growth matrix are complex numbers, which occur necessarily in conjugate pairs due to the fact that $\underset{\sim}{G}$ is real. For substantive interpretations, we must transform all complex terms into real form. Let $\alpha_i + i\beta_i$ and $\alpha_i - i\beta_i$ be a pair of complex eigenvalues of $\underset{\sim}{G}$, with $\underset{\sim}{U}_i' + i\underset{\sim}{V}_i'$, $\underset{\sim}{U}_i' - i\underset{\sim}{V}_i'$, $\underset{\sim}{X}_i + i\underset{\sim}{Y}_i$, and $\underset{\sim}{X}_i - i\underset{\sim}{Y}_i$ as the associated left and right eigenvectors. Then the analytic components associated with these two eigenvalues can be changed into the real form:

$$2d_i\sigma_i^t[\cos(t\theta_i + e_i)\underset{\sim}{X}_i - \sin(t\theta_i + e_i)\underset{\sim}{Y}_i] \qquad (2.5)$$

where

$\sigma_i = (\alpha_i^2 + \beta_i^2)^{\frac{1}{2}}$ is the *magnitude* of the eigenvalue

$\theta_i = \tan^{-1}(\beta_i/\alpha_i)$ is the *amplitude* of the eigenvalue

$d_i = \{[\underset{\sim}{U}_i'\underset{\sim}{K}(0)]^2 + [\underset{\sim}{V}_i'\underset{\sim}{K}(0)]^2\}^{\frac{1}{2}}$

and $e_i = \tan^{-1}[\underset{\sim}{V}_i'\underset{\sim}{K}(0)/\underset{\sim}{U}_i'\underset{\sim}{K}(0)]$

The element in the combined analytic component (2.5) corresponding to the a-th age group and the j-th region can be further simplified into

$$2d_i\gamma_{iaj}\sigma_i^t \cos(t\theta_i + e_i + \phi_{iaj}) \qquad (2.6)$$

where

$$\gamma_{iaj} = (X_{iaj}^2 + Y_{iaj}^2)^{\frac{1}{2}} \qquad (2.7)$$

and

$$\phi_{iaj} = \tan^{-1}(Y_{iaj}/X_{iaj}) \qquad (2.8)$$

It is to be understood that X_{iaj} and Y_{iaj} are respectively the elements of $\underset{\sim}{X}_i$ and $\underset{\sim}{Y}_i$ corresponding to the a-th age group and j-th region. Equation (2.6) shows that each pair of complex components in the analytic solution is actually a *cyclical component* with θ_i as the *frequency* (in radians/five years). The *period* (in years) of the component is

$$p_i = 5(2\pi/\theta_i) \tag{2.9}$$

Since the cosine function assumes a value between +1 and -1 for all t, the elements in the cyclical component fluctuate below the smooth exponential *upperbounds* $2d_i\gamma_{iaj}\sigma_i^t$ as t increases. If the phase angles $e_i + \phi_{iaj}$ are zero, then all elements start at t = 0 from the upperbounds; otherwise, the starting points are within $\pm 2d_i\gamma_{iaj}$. The number of years which is required to reduce the upperbounds by half of their original sizes is called the *half-life*, which is

$$t_i = 5(-\ln 2/\ln \sigma_i) \tag{2.10}$$

Now, let us see what determines what in a cyclical component. Equations (2.9) and (2.10) show that the period and half-life of a cyclical component are completely determined by a complex eigenvalue of the growth matrix $\underset{\sim}{G}$. Equations (2.7) and (2.8) show that the *inter-age-group* and *interregional* contrasts in upperbounds and phase angles of a cyclical component are completely determined by a complex right eigenvector of $\underset{\sim}{G}$. The initial age-by-region population distribution $\underset{\sim}{K}(0)$, through its inner product with a complex left eigenvector, affects only the overall level d_i and the overall phase direction e_i of a cyclial component [see equation (2.5)].

The cyclical component with the longest half-life is called the *dominant cyclical component*, and its corresponding eigenvalue the *dominant complex eigenvalue*. For simplicity, we will treat negative real eigenvalues as a special kind of complex eigenvalue whose amplitude is π radians per five years.

Separating the cyclical from the monotonic components, the analytic solution for the projected population size in age group a, region j, and at time t is

$$K_{aj}(t) = \sum_{i=1}^{n_1} c_i Q_{iaj} \lambda_i^t + \sum_{i=1}^{n_2} 2d_i \gamma_{iaj} \sigma_i^t \cos(t\theta_i + e_i + \phi_{iaj}) + \text{"Residual"} \qquad (2.11)$$

where $K_{aj}(t)$ and Q_{iaj} are respectively the elements in $\underset{\sim}{K}(t)$ and $\underset{\sim}{Q}_i$ corresponding to the a-th age group and j-th region; and n_1 and n_2 represent respectively the number of monotonic and cyclical terms. It is a common empirical observation that the number of monotonic components (n_1) turns out to be equal to the number of regions (r) in the system. For each age- and region-specific subpopulation, the relative importance of the analytic components at t = 0 depends on the initial *loadings* $c_i Q_{iaj}$ and $2d_i \gamma_{iaj} \cos(e_i + \phi_{iaj})$.

For simplicity, we adopt the following notational convention. The subscript "a" always refers to an age group; whereas the subscript "j" is used to identify a region. All components are arranged in descending order of the magnitudes of the corresponding eigenvalues (i.e., $\lambda_1 > \lambda_2 > \lambda_{n_1}$ and $\sigma_1 > \sigma_2 > \ldots > \sigma_{n_2}$).

3. THE ROGERS MODEL AS A GENERALIZATION OF THE LESLIE MODEL

Our strategy to obtain indepth understanding of the dynamic properties of the Rogers model is to consider it as a generalization of the Leslie model by splitting the population system into r regions. In this section, we will (1) describe the Leslie model and the mathematical properties of its analytic solution, and (2) study the properties of a *factorizable Rogers model* by synthesizing the early findings in classical mathematical demography and the well-known properties of an irreducible, column-stochastic matrix. Although an empirically-constructed Rogers model is in general not factorizable, the factorized version serves to clarify certain important features which recur in empirical applications.

3.1 The Leslie Model

When there is only one region, the Rogers model is reduced to the Leslie model:

$$\hat{K}(t+1) = \hat{\underset{\sim}{G}}\hat{\underset{\sim}{K}}(t) \tag{3.1}$$

where $\hat{\underset{\sim}{K}}(t)$, $\hat{\underset{\sim}{K}}(t+1)$, and $\hat{\underset{\sim}{G}}$ correspond to $\underset{\sim}{K}(t)$, $\underset{\sim}{K}(t+1)$, and $\underset{\sim}{G}$ in equation (2.2). The w × w matrix $\hat{\underset{\sim}{G}}$ is the same as $\underset{\sim}{G}$, except that the $\underset{\sim}{B}_a$ and $\underset{\sim}{S}_a$ submatrices are now reduced to scalars B_a and S_a. It can be shown that, with realistic assumptions, the growth matrix $\hat{\underset{\sim}{G}}$ has a unique dominant eigenvalue which is real and positive (Keyfitz, chapter 3, 1968). All the remaining non-zero eigenvalues are either complex or real negative. Therefore, for the a-th age group, the analytic solution of the Leslie model is

$$\hat{K}_a(t) = \hat{c}_1\hat{Q}_{1a}\hat{\lambda}_1^t + \sum_{i=1}^{n_2} 2\hat{d}_i\hat{\gamma}_{ia}\hat{\sigma}_i^t \cos(t\hat{\theta}_i + \hat{e}_i + \hat{\phi}_{ia}) + \text{"Residual"} \tag{3.2}$$

where all quantities are analogous to those in equation (2.11). Ignoring the residual, the dominant component is the only monotonic term; the remaining ones are all cyclical.

The Leslie model and its continuous-time counterpart, the Lotka model, have been studied both mathematically and empirically for many decades. The major results are summarized in Keyfitz (1968) and Coale (1972). The main points about the analytic solution of the Leslie model are as follows.

1. The dominant eigenvalue of the growth matrix $\hat{\underset{\sim}{G}}$ and the corresponding right and left eigenvectors are relatively insensitive to alternative ways of discretizing the fertility and mortality schedules. In other words, the dominant component can actually reflect the intrinsic nature of the population system.
2. The dominant eigenvalue is positively related to the area under the net fertility function (i.e., the net reproduction rate) and negatively related to the mean age of this function.*
3. The period of the dominant cyclical component is almost equal to the length of a generation and is positively and strongly related to the mean age of the net fertility function.
4. The half-life of the dominant cyclical component is negatively and strongly related to the dispersion of the net fertility function. In other words, a highly

*With discrete age groups, the net fertility function F_a is defined as

$$F_a = B_a S_{a-1}, \ldots, S_1 \qquad \text{for } a = 2,3,\ldots$$

where the right-hand-side quantities are taken from equation (2.1), with matrices being replaced by the corresponding scalars. The net reproduction rate is the sum of F_a across all age groups. We will call the sum of all B_a as the gross reproduction rate (GRR) for simplicity, although B_a are affected by infant mortality.

concentrated fertility schedule results in a small rate of attenuation for population waves that are transmitted through successive generations.

5. Higher frequency eigenvalues depend on peculiarities in the fertility and mortality schedules in a way that is difficult to describe and, hence, difficult to understand intuitively. Furthermore, these eigenvalues are also very sensitive to alternative ways of approximating the fertility and mortality schedules. There is not much insight about a population that can be learned from the components associated with these eigenvalues, except that they guarantee the "material balance" of equation (3.2) for any initial population.

6. The dominant cyclical component is damped, relative to the dominant component, by about 2.5% to 5% annually. In other words, the ratio of $\hat{\sigma}_1/\hat{\lambda}_1$ is usually between 0.88 and 0.77. Cyclical components with higher frequencies are attenuated at least twice as rapidly as the dominant cyclical component.

7. If the initial population is heavily concentrated in the age interval 0-24, the stable equivalent population size ($\hat{c}_1$) will be much larger than the initial total population size (i.e., there will be a large momentum for the population to continue growing even if the fertility level is already below replacement level). This is due to the fact that the elements of the dominant left eigenvector remain at a high level in the age interval 0-24 and drop sharply afterwards toward zero at the end of the reproductive age interval. The typical age pattern of the dominant left eigenvector is determined by the fertility and survival schedules of the Leslie model, according to the formula

$$\hat{P}_{1,a+1} = (\hat{\lambda}_1/S_a)\hat{P}_{1,a} - (B_a/S_a)\hat{P}_{1,1} \qquad (3.3)$$

for $1 < a < f$

For the young age groups, $(\hat{\lambda}_1/S_a)$ is usually close to one, and (B_a/S_a) is zero or nearly so. This explains why $\hat{P}_{1,a}$ remains more or less at the same level for small a. But, the large values of B_a around the peak of the fertility pull $\hat{P}_{1,a+1}$ down sharply. For $a > f$, $\hat{P}_{i,a} = 0$ for any i.

8. The initial upperbound of a cyclial component $(2\hat{d}_i\hat{\gamma}_{ia})$ tends to be small, when the initial population is not highly concentrated in the youngest age group. The higher the frequency, the stronger this tendency. This relationship is due to the fact that the elements of the left eigenvector associated with a high frequency eigenvalue decline in magnitude very rapidly as age increases. The age pattern of a complex left eigenvector can be explained by a formula like equation (3.3).*

9. For most empirical data, the time-path of the 0-4 age group of the Leslie model can be quite accurately approximated by the sum of the dominant and dominant cyclical components. That is,

$$\hat{K}_a(t) \doteq \hat{c}_1\hat{Q}_{1a}\hat{\lambda}_1^t + 2\hat{d}_1\hat{\gamma}_{1a}\hat{\sigma}_1^t \cos(t\hat{\theta}_1 + \hat{e}_1 + \hat{\phi}_{1a}) \qquad (3.4)$$

Thus, for the youngest age group, the time-path of the projected population size can be visualized as a damped wave with a period of about 25-30 years, which is superimposed on a smooth exponential trend.

*The complex left eigenvectors of the Leslie model are discrete analogues of $Q_i(a)$ in equation (3.20) in Coale (1972), page 68.

3.2 The Factorizable Rogers Model

Now, let us disaggregate the population system into r regions as well as w age groups. In order to obtain an easily understandable analytic result, we assume in this subsection that fertility and mortality schedules do not vary among regions, and that the level and pattern of migration are constant across all age groups. In other words, it is assumed that the non-zero submatrices in equation (2.1) can be factorized in the following manner:

$$\mathbf{B}_a = B_a \hat{\hat{\mathbf{G}}}$$

and

$$\mathbf{S}_a = S_a \hat{\hat{\mathbf{G}}}$$

where $\hat{\hat{\mathbf{G}}}$ is a fixed r × r column-stochastic matrix, and B_a and S_a are respectively the birth and survival rates of the growth matrix $\hat{\mathbf{G}}$ of the corresponding Leslie model. For simplicity, we will call $\hat{\hat{\mathbf{G}}}$ and $\hat{\mathbf{G}}$ a *pure migration matrix* and a *Leslie matrix*, respectively.

Using the Kronecker's product*, we find that our assumptions about the age-by-region population system lead to the *factorizable Rogers model*:

---

*Let $\hat{\mathbf{G}} = \begin{bmatrix} B_1 & B_2 & B_3 \\ S_1 & 0 & 0 \\ 0 & S_2 & 0 \end{bmatrix}$ and $\hat{\hat{\mathbf{G}}} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$. Then the Kronecker's product of $\hat{\mathbf{G}}$ and $\hat{\hat{\mathbf{G}}}$ is

$$\underset{\sim}{K}(t+1) = (\hat{\underset{\sim}{G}} \otimes \hat{\hat{\underset{\sim}{G}}})\underset{\sim}{K}(t) \tag{3.5}$$

Its analytic solution then becomes

$$\begin{aligned} \underset{\sim}{K}(t) &= (\hat{\underset{\sim}{G}} \otimes \hat{\hat{\underset{\sim}{G}}})^t \underset{\sim}{K}(0) = (\hat{\underset{\sim}{G}}^t \otimes \hat{\hat{\underset{\sim}{G}}}^t)\underset{\sim}{K}(0) \\ &= (\sum_k \hat{\lambda}_k^t \hat{\underset{\sim}{Q}}_k \hat{\underset{\sim}{P}}'_k) \otimes (\sum_\ell \hat{\hat{\lambda}}_\ell^t \hat{\hat{\underset{\sim}{Q}}}_\ell \hat{\hat{\underset{\sim}{P}}}'_\ell)\ \underset{\sim}{K}(0) + \text{"Residual"} \qquad (3.6) \\ &= \sum_k \sum_\ell (\hat{\lambda}_k \hat{\hat{\lambda}}_\ell)^t (\hat{\underset{\sim}{Q}}_k \otimes \hat{\hat{\underset{\sim}{Q}}}_\ell)(\hat{\underset{\sim}{P}}_k \otimes \hat{\hat{\underset{\sim}{P}}}_\ell)' \underset{\sim}{K}(0) + \text{"Residual"} \end{aligned}$$

where $\hat{\lambda}_k$ is a non-zero eigenvalue of the Leslie matrix $\hat{\underset{\sim}{G}}$, and $\hat{\underset{\sim}{P}}'_k$ and $\hat{\underset{\sim}{Q}}_k$ are the left and right eigenvectors of $\hat{\underset{\sim}{G}}$ associated with $\hat{\lambda}_k$; and $\hat{\hat{\lambda}}_\ell$ is a non-zero eigenvalue of the pure migration matrix $\hat{\hat{\underset{\sim}{G}}}$, and $\hat{\hat{\underset{\sim}{P}}}'_\ell$ and $\hat{\hat{\underset{\sim}{Q}}}_\ell$ are the left and right eigenvectors of $\hat{\hat{\underset{\sim}{G}}}$ associated with $\hat{\hat{\lambda}}_\ell$.

Comparing equations (3.6) and (2.3), we see that for a factorizable Rogers model,

---

(footnote continued from previous page)

$$\hat{\underset{\sim}{G}} \otimes \hat{\hat{\underset{\sim}{G}}} = \left[\begin{array}{cc|cc|cc} B_1m_{11} & B_1m_{12} & B_2m_{11} & B_2m_{12} & B_3m_{11} & B_3m_{12} \\ B_1m_{21} & B_1m_{22} & B_2m_{21} & B_2m_{22} & B_3m_{21} & B_3m_{22} \\ \hline S_1m_{11} & S_1m_{12} & 0 & 0 & 0 & 0 \\ S_1m_{21} & S_1m_{22} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & S_2m_{11} & S_2m_{12} & 0 & 0 \\ 0 & 0 & S_2m_{21} & S_2m_{22} & 0 & 0 \end{array}\right]$$

For a brief but useful introduction, see Theil (1971:303-308).

$$\lambda_i = \hat{\lambda}_k \hat{\hat{\lambda}}_\ell \tag{3.7}$$

$$\mathbf{Q}_i = \hat{\mathbf{Q}}_k \otimes \hat{\hat{\mathbf{Q}}}_\ell \tag{3.8}$$

and

$$\mathbf{P}_i = \hat{\mathbf{P}}_k \otimes \hat{\hat{\mathbf{P}}}_\ell \tag{3.9}$$

In other words, each non-zero eigenvalue of $\mathbf{G}$ is a non-zero eigenvalue of $\hat{\mathbf{G}}$ times an eigenvalue of $\hat{\hat{\mathbf{G}}}$; and each left (right) eigenvector of $\mathbf{G}$ is a Kronecker's product of a left (right) eigenvector of $\hat{\mathbf{G}}$ and a left (right) eigenvector of $\hat{\hat{\mathbf{G}}}$. Since we have examined the eigenvalues and eigenvectors of $\hat{\mathbf{G}}$ in section 3.1, we will now focus on $\hat{\hat{\mathbf{G}}}$.

Because the pure migration matrix $\hat{\hat{\mathbf{G}}}$ is an irreducible, non-negative matrix, we know from the Frobenius theorem (Gantmacher, Vol. 2, 1959:53-54) that its dominant eigenvalue and dominant right and left eigenvectors are positive. Furthermore, the fact that $\hat{\hat{\mathbf{G}}}$ is column-stochastic guarantees that the dominant eigenvalue equals unity, and that the sum of the elements in the right eigenvector associated with any of the subdominant eigenvalues $(\hat{\hat{\lambda}}_2,\ldots,\hat{\hat{\lambda}}_r)$ is zero. In other words,

$$\hat{\hat{\lambda}}_1 = 1 \tag{3.10}$$

and

$$\sum_{j=1}^{r} \hat{\hat{Q}}_{\ell,j} = 0 \qquad \text{for} \quad \ell = 2,\ldots,r \tag{3.11}$$

Equation (3.10) is intuitively clear, because the pure migration matrix can never cause an increase or a decrease in total population size. Equation (3.11) shows that every subdominant right eigenvector $\hat{\hat{Q}}_\ell$ ($\ell > 1$) represents an *interregional zero-sum game*.* Finally, every subdominant left eigenvector $\hat{\hat{P}}_\ell$ ($\ell >$ 1) must have at least one *positive* and one *negative* element, because its inner product with the dominant right eigenvector $\hat{\hat{Q}}_1$ must be zero.

There are two additional properties of $\hat{\hat{G}}$ that can be easily proved in a biregional case and have not been violated in empirically-constructed migration models. First, due to the fact that migration rates are always much smaller than stayer rates (i.e., $\hat{\hat{G}}$ has a dominant diagonal), all eigenvalues of $\hat{\hat{G}}$ are real and positive. Second, as the level of migration is increased, the subdominant eigenvalues become smaller.

Now, we are ready to synthesize. We begin by considering the eigenvalues of the factorizable Rogers model. Because $\lambda_i = \hat{\lambda}_k \hat{\hat{\lambda}}_\ell$, we see that corresponding to each eigenvalue of the Leslie model, there is a *cluster* of exactly r eigenvalues in the factorizable Rogers model. Within each cluster, all the eigenvalues have the same frequency (because all $\hat{\hat{\lambda}}_\ell$ are real and positive), and the eigenvalue with the largest magnitude is exactly equal to an eigenvalue of the Leslie model(because $\hat{\hat{\lambda}}_1 = 1$ and $\hat{\hat{\lambda}}_\ell < 1$ for $\ell > 1$). Furthermore, the difference in magnitude among the eigenvalues in each cluster increases as the level of migration is raised, because the subdominant eigenvalues of $\hat{\hat{G}}$ are negatively affected by the level of migration.

The components associated with the subdominant positive real eigenvalues (λ_2 to λ_r) are called *spatial components*, because they convey the information about spatial redistribution. Each spatial component of the factorizable Rogers model represents

*This zero-sum property was not realized by Feeney when he proved incorrectly that there is a one-to-one correspondence between the eigenvalues of the Leslie model and those of the corresponding nearly factorizable Rogers model (Feeney 1970).

an interregional zero-sum game for every age group, because equation (3.11) implies $\sum_{j=1}^{r} Q_{iaj} = 0$ for all spatial components. The relative importance of a spatial component at t = 0 depends on the relative size of $c_i = (\hat{\hat{P}}_1 \otimes \hat{\hat{P}}_i)K(0)$. Since $\hat{\hat{P}}_1$ is strictly positive, and $\hat{\hat{P}}_i$ (i > 1) has both positive and negative (i.e., mutually cancelling) elements, it is usually the case that c_i (i > 1) are smaller in magnitude than c_1. In other words, the spatial components are usually less important than the dominant component, even at the initial stage. The sum of all spatial components is called the *superimposed spatial component*, which is found to have very smooth time-paths in all known empirical applications, although this is not a necessary implication of the fact that the individual spatial components all have smooth exponential time-paths.

The r cyclical components associated with the most durable cluster of complex eigenvalues are called *major cyclical components*, because with the common period of one generation, they determine jointly the major features of the transmission of population waves from one generation to another. Within this group there is an important distinction between the dominant cyclical component and the remaining ones. The former represent population waves that move *synchronically* across all regions for each age group (because the dominant right eigenvector $\hat{\hat{Q}}_1$ of $\hat{\hat{G}}$ is real and positive), whereas each of the latter represents population waves that move in *opposite* directions simultaneously in two non-empty subsets of regions (because every subdominant right eigenvector of $\hat{\hat{G}}$ has the aforementioned zero-sum nature). We will call these two contrasting types of waves as synchronical and opposite waves, respectively. Because each subdominant left eigenvector $\hat{\hat{P}}_\ell$ (ℓ > 1) of $\hat{\hat{G}}$ has both positive and negative elements, and the dominant left eigenvector $\hat{\hat{P}}_1$ is strictly positive, the dominant cyclical component is usually more important than the other major cyclical components, even at the initial stage. In other words, we would anticipate the actual population waves to be more synchronical than opposite.

4. THE DYNAMIC PROPERTIES OF THE SWEDISH FEMALE POPULATION SYSTEM

4.1 The Leslie Model of Swedish Female Population

The initial age distribution and the non-zero elements of the growth matrix of the Leslie model of Swedish female population are shown in Table 4.1. The fertility schedule has a high mean age of 29.76 years and a small standard deviation of 5.97 years. Since more than 95% of the population survive from the first to the tenth age group, the net reproduction rate (0.90) is very close to the gross reproduction rate (0.91). Both mortality and fertility levels are low, with the latter being significantly below the replacement level.

Table 4.1 The initial age distribution and the non-zero elements of the growth matrix of the Leslie model of Swedish female population, 1974.

Age group	Initial population	Birth rates (B_i)	Survival rates (S_i)
0-4	300,755	-	0.9949989
5-9	289,672	-	0.9988349
10-14	264,778	0.0000276	0.9986243
15-19	261,624	0.0289772	0.9979913
20-24	282,670	0.1672100	0.9978095
25-29	328,133	0.2963153	0.9972473
30-34	274,276	0.2490735	0.9959975
35-39	226,512	0.1223440	0.9939728
40-44	221,743	0.0378757	0.9903277
45-49	239,077	0.0068123	0.9850863
50-54	269,042	0.0003645	0.9771738
55-59	246,333	-	0.9657391
60-64	248,353	-	0.9435229
65-69	225,571	-	0.9011995
70-74	185,246	-	0.8281277
75-79	133,952	-	0.7138729
80-84	81,641	-	0.8100655
85+	49,668	-	-

Note: The data are consolidated from the Rogers model, using the initial age-by-region population distribution as the weighting scheme.

The eigenvalues of the growth matrix of the Swedish Leslie model are shown in Table 4.2 and Figure 4.1. We see that as the frequency ($\hat{\theta}_i$) increases, the magnitude ($\hat{\sigma}_i$) tends to decrease. The dominant eigenvalue is $\hat{\lambda}_1$ = 0.9833, which implies a long-run growth rate of -0.3% per year and a half-life of 205 years. The smallness of the dominant eigenvalue reflects mainly the low level of the net reproduction rate and partly the unusual length of the mean age of the net fertility function (29.74 years).

The age patterns of the dominant left and right eigenvectors of the Swedish Leslie model, together with the initial and projected national age profiles are shown in Figure 4.2. First, we see that the elements of the dominant left eigenvector indeed remain at a high level in the first five age groups and then drop very sharply and become zero beyond the end of the reproductive range. Since there is a high concentration of the initial population in the first six age groups, it is not surprising that the stable equivalent (c_1 = 4,879,000) is substantially greater than the initial total population size (4,129,000). The corresponding population momentum is 18%. This means that by the time the system achieves the stable growth pattern, the total population size projected with the observed distribution will be 18% greater than the total population size projected with initial population which was rearranged in advance according to the stable age profile. Because of the positive growth momentum, the Swedish female population continues to expand from 4.13 million in 1974 until the early 1990's when a maximum population size of 4.23 million is reached.

The dominant right eigenvector in Figure 4.2 shows that the long-run age profile starts at about 5.6% in the 0-4 age group, rises smoothly and gently toward a maximum of about 6.2% in the 50-54 age group, and then declines sharply to a minimum of about 2.7% in the 85+ age group. The shape is typical of the industrialized countries whose net reproduction rates are below the replacement level. The *dissimilarity index* between the 1974 and the long-run age profiles is 7.9%,

Table 4.2 The non-zero eigenvalues of the Leslie model of Swedish female population, 1974.

Real Part ($\hat{\alpha}_i$)	Imaginary Part ($\hat{\beta}_i$)	Magnitude ($\hat{\sigma}_i$)	Half-life ($\hat{t}_i$)	Frequency ($\hat{\theta}_i$)	Period ($\hat{P}_i$)	Annual Rate of Change (%)
0.98325	0.00000	0.98325	205.17	0.00	∞	-0.337
0.50654	0.72482	0.88427	28.18	55.05	32.70	-2.430
-0.18255	0.59307	0.62053	7.26	107.11	16.81	-9.102
-0.15590	0.36628	0.39807	3.76	113.06	15.92	-16.825
-0.39484	0.21429	0.44925	4.33	151.51	11.88	-14.789
-0.44793	0.00000	0.44793	4.31	180.00	10.00	-14.839
-0.08181	0.00000	0.08181	1.38	180.00	10.00	-39.387

Note: Each complex conjugate pair is represented by the eigenvalue with positive imaginary part. Half-lives and periods are in years, whereas frequencies are measured in degrees per five years. The second and third pair of complex eigenvalues have periods of 17 and 16 years instead of 21 and 13 years found in most national populations.

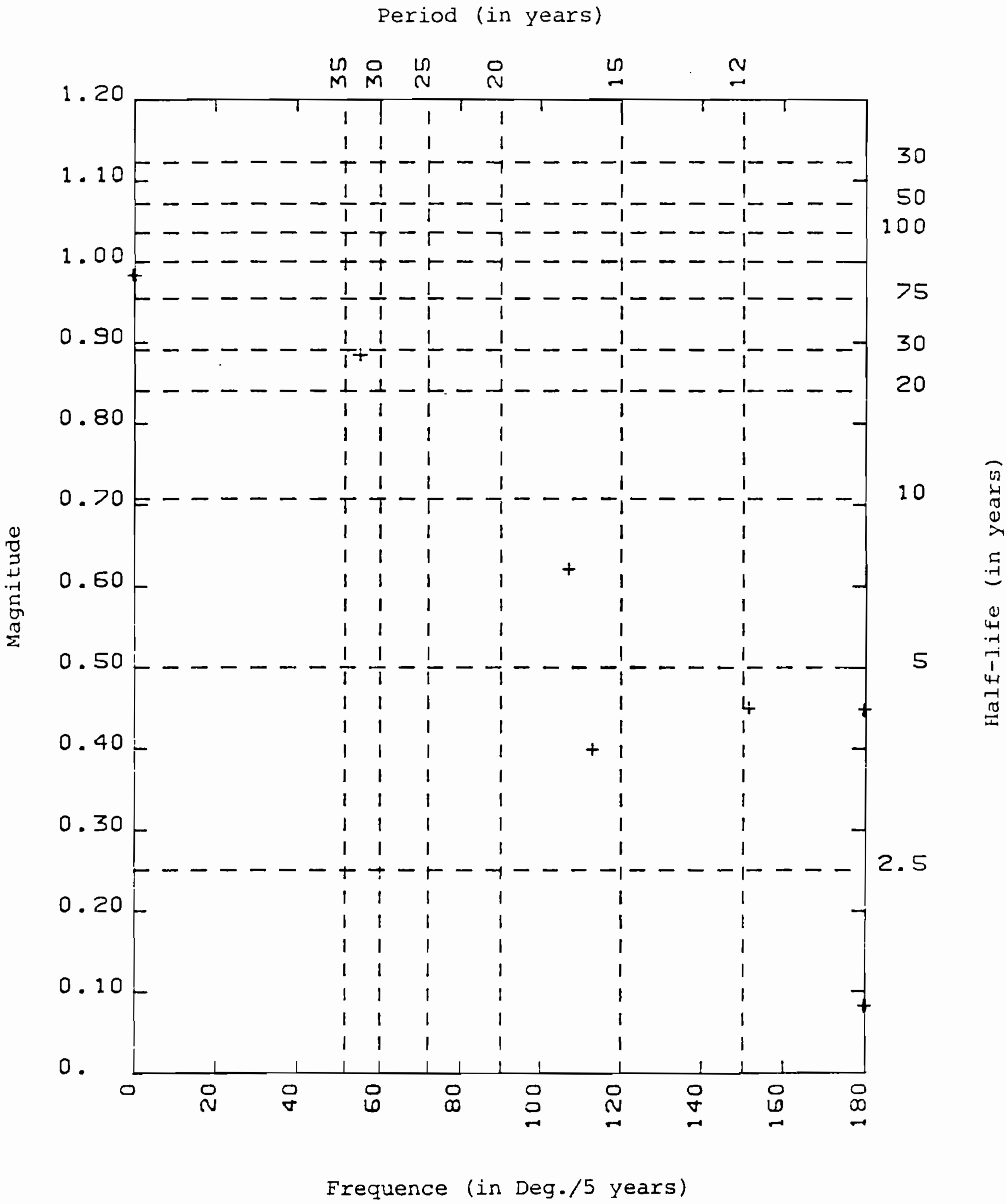

Figure 4.1 The pattern of non-zero eigenvalues of the growth matrix of the Leslie model of the 1974 Swedish female population. For complex eigenvalues, only those with positive imaginary part are shown.

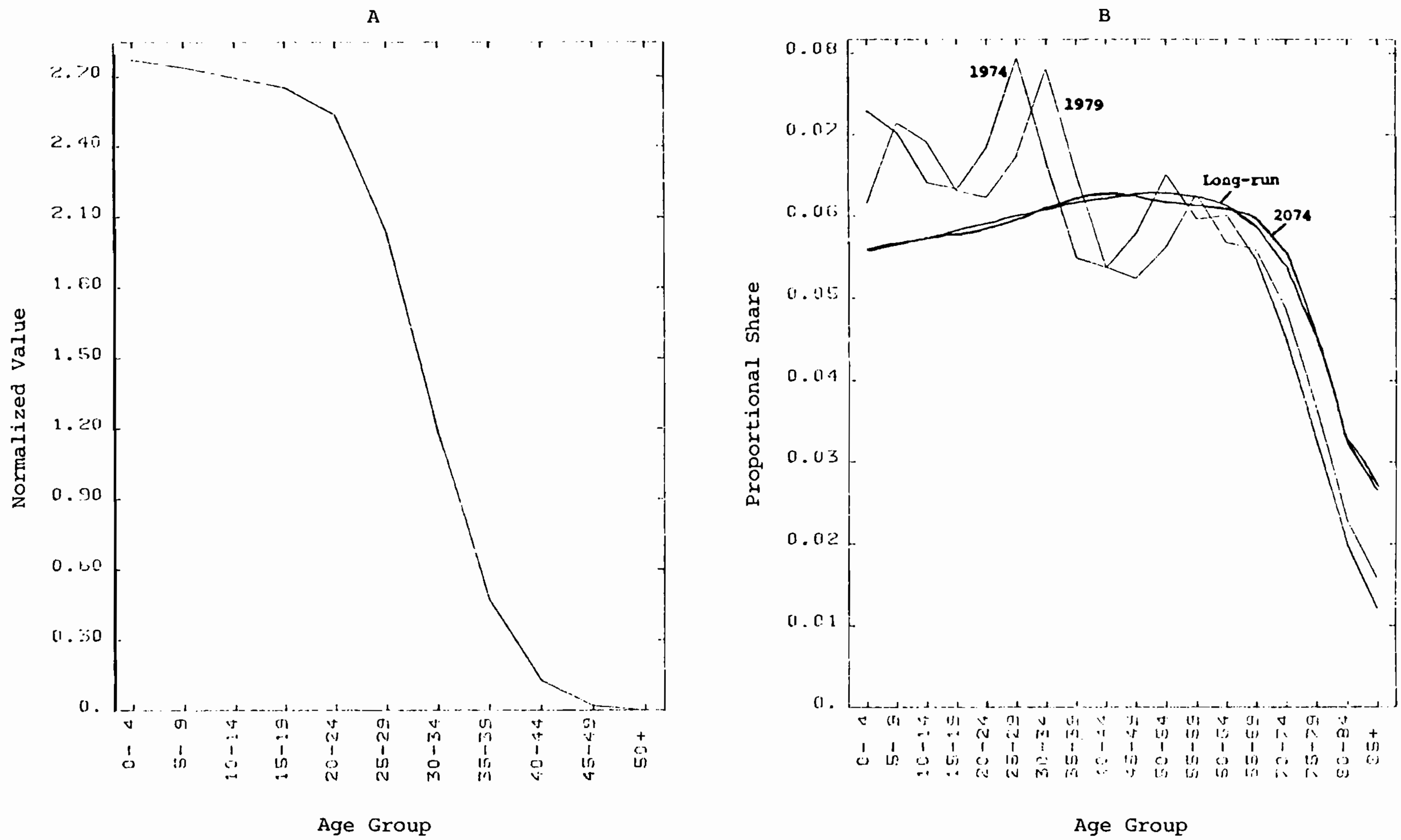

Figure 4.2 The normalized dominant left eigenvector (in A) and the initial, projected, and long-run age profiles (in B) of the Leslie model of the 1974 Swedish female population.

which means that 7.9% of the population has to be redistributed in order to make the two age profiles identical. If we use a dissimilarity index of 2% to indicate the "completion" of the convergence toward the long-run distribution, then the convergence of the age profile of the Swedish Leslie model is completed in less than 60 years.

The period of the dominant cyclical component of the Swedish model (p_1 = 32.70 years) is perhaps the longest of all recorded national populations. This is, of course, due to the very high mean age of the fertility schedule (29.76 years). The corresponding half-life is also exceptionally long (28.18 years), thanks to the strong concentration of the fertility schedule (standard deviation = 5.97 years), which is the result of very deliberate and effective birth control. This half-life implies a damping rate, relative to the dominant component, of 2.1% per year, which is smaller than those of the 47 populations examined by Coale (1972). Therefore, although the overall shape of the long-run age profile is nearly achieved in about 60 years, population waves passing through individual age groups are still visible even in 100 years (see Figures 4.2b and 4.3).

We see in Figure 4.3a that for the 0-4 age group, the time-path of the sum of the dominant and the dominant cyclical components provides a good approximation to that of the actual projected population. By 1984, the difference between the two time-paths becomes negligible. The population wave has the first trough around 1989 (due to the deficit in potential mothers who were born in the late 1950's and early 1960's), which is followed by a peak around 2004 (due to a "surplus" of mothers who were born in the late 1960's and early 1970's).

Figure 4.3b is quite instructive in demonstrating the tendency that, for relatively old age-groups, the sum of the dominant and the dominant cyclical components becomes a poor approximation in the short run for the projected population. The lack of fit is mainly due to the past changes in fertility level (a babybust in the 1930's, a babyboom in the 1940's, etc.).

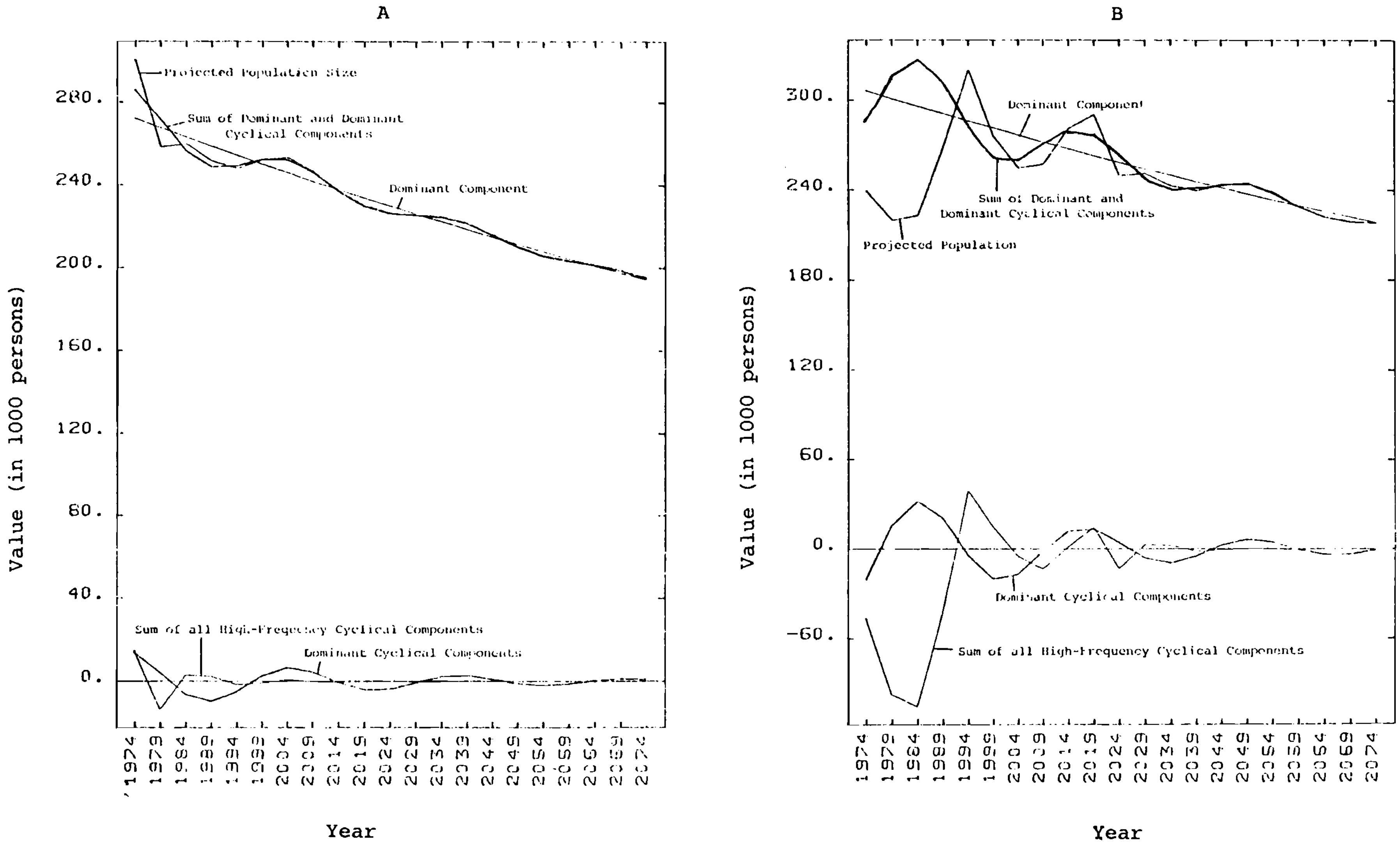

Figure 4.3 Decomposition of projected populations of the 0-4 (A) and 45-49 (B) age groups into the analytic components. Leslie model of the 1974 Swedish female population.

In a pure mathematical sense, this lack of fit is completely accounted for by the high frequency components. However, it does not really make *substantive* sense to say that the fluctuations in the fertility level in the previous decades can be *explained* by the high-frequency eigenvalues and eigenvectors of the 1974 growth matrix. With very short half-lives (between 7.3 and 1.4 years), all high frequency components are damped at least three times faster than the dominant cyclical component.

In summary, the Leslie model of the 1974 Swedish female population system has (1) a low long-run growth rate which is compensated for in the near future by a large growth momentum, (2) new population waves with an exceptionally small damping rate and long period, and (3) a smooth long-run age profile which has a weak maximum in the 50-54 age group.

4.2 The Factorizable Rogers Model of the Swedish Female Population

The pure migration matrix $\hat{\hat{\underset{\sim}{G}}}$ of the Swedish female population is shown in Table 4.3. It has been aggregated from the growth matrix of the Rogers model, using the 1974 age-by-region population distribution as the weighting scheme. The origin-destination migration rates range between 0.16% (South to Lower North) and 3.03% (Stockholm to East Middle). The region with the highest overall outmigration rate is East Middle (9.34%), whereas the lowest is West (5.41%). The eigenvalues of $\hat{\hat{\underset{\sim}{G}}}$ are all positive, lying between $\hat{\hat{\lambda}}_1 = 1$ and $\hat{\hat{\lambda}}_8 = 0.8776$.

The pattern of eigenvalues of the factorizable Rogers model of the Swedish female population is shown in Figure 4.4. It is indeed true that (1) corresponding to each eigenvalue of the Leslie model, there is a *cluster* of exactly r = 8 eigenvalues in the factorizable Rogers model; (2) within each cluster, all eigenvalues have the same frequency; and (3) the eigenvalue with the largest magnitude in each cluster is equal to an eigenvalue of the Leslie model. It is interesting to note that the

Table 4.3 The pure migration matrix of the 1974 Swedish female population system.

	1. Stockholm	2. E.Middle	3. S.Middle	4. South	5. West	6. N.Middle	7. Lower N	8. Upper N
1	0.9100260	0.0291638	0.0144563	0.0115044	0.0100683	0.0203012	0.0269362	0.0247430
2	0.0302616	0.9065597	0.0183964	0.0085602	0.0104117	0.0252818	0.0165243	0.0178878
3	0.0082713	0.0099692	0.9120762	0.0126780	0.0086577	0.0040046	0.0033789	0.0040753
4	0.0106424	0.0098901	0.0227336	0.9458716	0.0112034	0.0044099	0.0054205	0.0049233
5	0.0107466	0.0153346	0.0232770	0.0140280	0.9467833	0.0158879	0.0102412	0.0096963
6	0.0147180	0.0170390	0.0048151	0.0036324	0.0072849	0.9193038	0.0119886	0.0082651
7	0.0080689	0.0054704	0.0015785	0.0015529	0.0025006	0.0059100	0.9115434	0.0117927
8	0.0072653	0.0065733	0.0026669	0.0021724	0.0030900	0.0049010	0.0139669	0.9186165
Total Migration	0.0899740	0.0934403	0.0879238	0.0541284	0.0532167	0.0806962	0.0884566	0.0813835

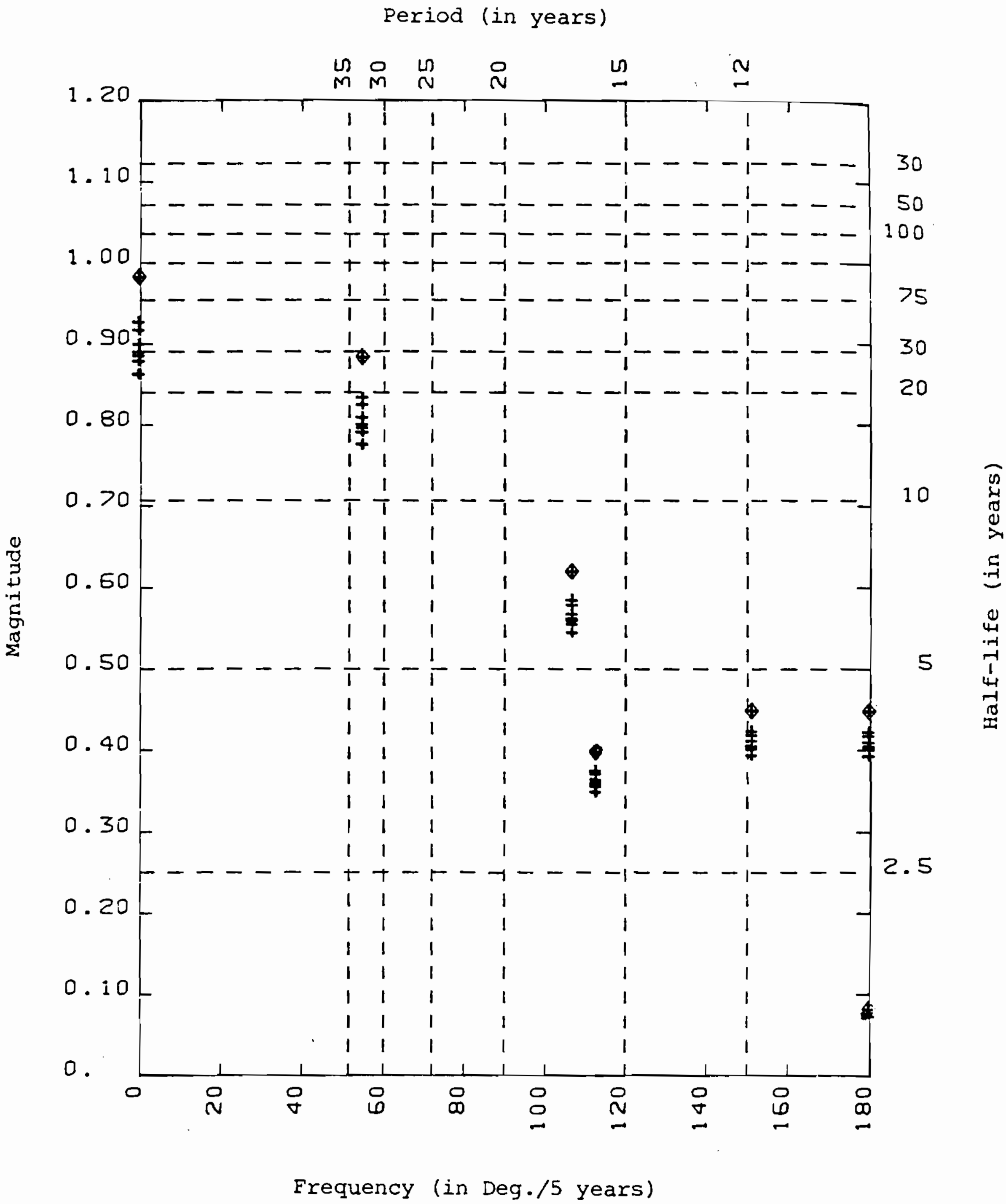

Figure 4.4 The pattern of non-zero eigenvalues of the growth matrix of the factorizable Rogers model of the 1974 Swedish female population. The eigenvalues of the corresponding Leslie model are shown as diamonds.

difference in magnitude among the major complex eigenvalues is due to the moderate level of interregional migration and has nothing to do with the interregional difference in the shapes of regional fertility schedules.

The differential importance of the dominant and spatial components are indicated in Table 4.4. First, the fact that the initial regional loadings of the dominant are much greater in magnitude than those of the spatial components suggests that the initial interregional population distribution is rather compatible with the current basic demographic processes. The dissimilarity index between the initial and the long-run interregional population distribution is only 4.15%. Second, the fact that the spatial components have large *positive* loadings on Stockholm, East Middle, and South Middle on the one hand, and large *negative* loadings on West and South on the other indicates that the general trend of geographical redistribution is *from* Stockholm, East Middle, and South Middle *to* West and South. Third, the fact that the spatial component with the shortest half-life (23.50 years) has the largest positive and negative loadings on Stockholm and East Middle, respectively, is due to the fact that the interregional migration rates between these two regions are at the highest level. The annual relative damping rate is 2.57% for this component and 1.16% for the most durable spatial component.

The eight major cyclical components have a common period of 32.70 years and have annual relative damping rates within the range of 2.09% and 4.61%. Since this range overlaps with the range of the damping rates of the spatial components, it might happen that very large loadings are associated with λ_8 and σ_1 and hence, spatial convergence proceeds more rapidly than the convergence in regional age profiles. However, we see in Tables 4.4 and 4.5 that relatively high loadings are associated with λ_2, λ_5, λ_7, σ_1, σ_7, and σ_8. Therefore, the Swedish interregional population system, just as the Canadian system (Liaw 1980a), shows the usual tendency that convergence in the regional age profiles proceeds at a faster rate than spatial convergence.

Table 4.4 The half-lives, initial levels, and initial regional loadings on the 0-4 age group of the dominant and spatial components of the factorizable Rogers model of the 1974 Swedish female population.*

Half-life (t_i)	Initial Level (c_i)	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Lower N	Upper N
		Initial Regional Loadings ($c_i Q_{iaj}$)							
205.17	4,879,430	44,933	43,095	23,719	43,829	57,739	29,539	13,663	15,755
45.89	246,088	1,599	1,527	-862	-4,376	-1,620	1,634	1,005	1,100
40.30	60,405	276	184	76	869	-1,685	52	104	124
32.90	25,799	50	210	52	-29	-196	408	-149	-346
29.84	267,198	1,210	1,636	4,256	-2,376	-1,235	-2,993	-850	353
28.71	20,874	-88	30	-75	51	-6	188	-413	313
27.03	198,608	2,798	1,904	-2,115	156	683	-2,277	-903	-245
23.50	34,081	724	-873	67	-19	46	102	-59	13

*The unit of half life is measured in years, and c_i and $c_i Q_{iaj}$ are in persons.

Table 4.5 The half-lives, periods, and regional information about the 0-4 age group of the major cyclical components of the factorizable Rogers model of the 1974 Swedish female population.

Half-life (t_i)	Period (P_i)	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Lower N	Upper N
				Initial Regional Upperbounds ($2d_i\gamma_{iaj}$)					
28.18	32.70	2,349	2,253	1,240	2,291	3,018	1,544	714	824
19.08	32.70	113	108	61	308	115	115	71	77
18.04	32.70	141	94	39	442	857	26	53	63
16.39	32.70	72	299	73	41	279	581	213	493
15.59	32.70	162	219	569	318	165	401	114	47
15.28	32.70	103	36	87	60	7	219	483	366
14.79	32.70	2,442	1,662	1,845	136	596	1,987	788	214
13.67	32.70	2,069	2,495	191	54	132	290	169	37
				Initial Regional Phase Angles ($e_i + \phi_{iaj}$)					
28.18	32.70	15.59	15.59	15.59	15.59	15.59	15.59	15.59	15.59
19.08	32.70	-161.59	-161.59	18.41	18.41	18.41	-161.59	-161.59	-161.59
18.04	32.70	-107.67	-107.67	-107.67	-107.67	72.33	-107.67	-107.67	-107.67
16.39	32.70	-62.58	-62.58	-62.58	117.42	117.42	-62.58	117.42	117.42
15.59	32.70	-1.29	-1.29	-1.29	178.71	178.71	178.71	178.71	-1.29
15.28	32.70	-60.29	119.71	-60.29	119.71	-60.29	119.71	-60.29	119.71
14.79	32.70	-64.16	-64.16	115.84	-64.16	-64.16	115.84	115.84	115.84
13.67	32.70	-84.38	95.62	-84.38	95.62	-84.83	-84.83	95.62	-84.38
				Initial Regional Loadings [$2d_i\gamma_{iaj}\cos(e_i + \phi_{iaj})$]					
28.18	32.70	2,262	2,170	1,194	2,207	2,907	1,487	688	793
19.08	32.70	-107	-102	58	292	109	-109	-67	-73
18.04	32.70	-43	-28	-12	-134	260	-8	-16	-19
16.39	32.70	33	138	34	-19	-128	268	-98	-227
15.59	32.70	162	219	569	-318	-165	-401	-114	47
15.28	32.70	51	-18	43	-30	4	-109	239	-181
14.79	32.70	1,064	724	-804	59	260	-866	-344	-93
13.67	32.70	203	-244	19	-5	13	28	-17	4
SUM	-	3,626	2,858	1,101	2,052	3,258	291	272	250

The major cyclical components are characterized in Table 4.5. Since the initial regional phase angles of the dominant cyclical component for the 0-4 age group are all equal to 15.59 degrees, the *synchronical* population waves start near their maximums at t = 0. The regional phase angles of each of the remaining major cyclical components are arranged in two exactly opposite directions (i.e., they differ by 180 degrees). Thus, each of the components represents *opposite* population waves that involve a zero-sum type of trade-off among the regions. Since the regional upperbounds of fluctuations are in general greater for the dominant cyclical components than for the remaining major cyclical components, we see in Figure 4.5 that the regional time-paths of the superimposed major cyclical component for the 0-4 age group appear more synchronical than opposite. The effect of the subdominant major cyclical components is shown to be the advancement of the first trough in South Middle, North Middle, Lower North, and Upper North by five years, and the delay of the first trough in Stockholm by the same length of time. Since the dominant cyclical component is much more durable than the remaining ones, the tendency of increasing synchronization is clearly seen in Figure 4.5.

As a consequence of the fact that $\underset{\sim}{Q}_1 = \hat{\underset{\sim}{Q}}_1 \otimes \hat{\underset{\sim}{Q}}_1$, the factorizable Rogers model has the unattractive property that all regional long-run age profiles must be identical to the smooth long-run profile of the corresponding Leslie model. This is, of course, the price we pay for analytic clarity. However, the factorizable Rogers model is much better than a set of single-region Leslie models as an approximation to the general Rogers model.

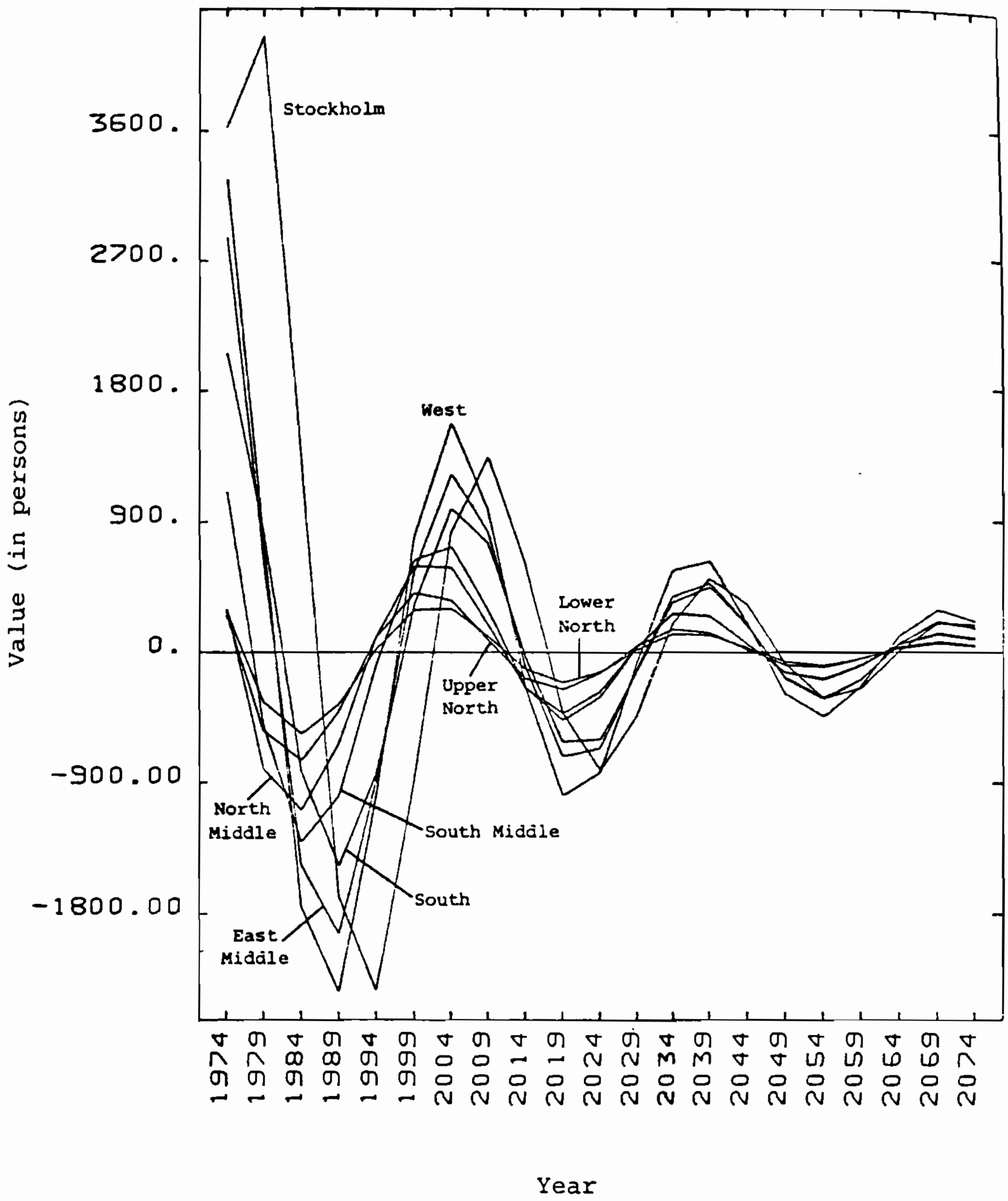

Figure 4.5 The regional time-paths of the superimposed major cyclical component for the 0-4 age group of the factorizable Rogers model of the 1974 Swedish female population.

4.3 The (Non-factorizable) Rogers Model of the Swedish Female Population

The non-zero submatrices of the growth matrix of the Rogers model of the 1974 Swedish female population is shown in Appendix A. The regional fertility, survival, and interregional outmigration schedules are characterized by gross rates, mean ages, and standard deviations in Table 4.6. The regional survival schedules have almost identical mean ages (about 41 years) and standard deviations (about 24 years) but have gross rates that are somewhat higher in the highly urbanized regions (Stockholm, South, and West) than in the rest of the country. Similarly, regional fertility schedules are almost identical with respect to mean age (30 years) and standard deviation (6 years), but differ somewhat in gross rate, ranging from 0.83 in Stockholm to 0.96 in Upper North. In contrast, interregional migration schedules differ more across regions in all three characteristics: the gross rates range from 1.29 in East Middle to 0.74 in West; the mean ages from 25.3 in Lower North to 28.6 in Stockholm; and the standard deviations from 16.8 in Lower North to 19.9 in Stockholm.

The eigenvalues of the growth matrix are shown in Figure 4.6. The overall negative relationship between the durability (half-life) and frequency of the analytic components of the Rogers model is similar to that of the corresponding Leslie model. Comparing Figures 4.4 and 4.6, we find that the dispersion in the two most durable clusters of eigenvalues of the Rogers model is similar to that of the corresponding factorizable model.

Table 4.7 shows how the dominant and spatial components convey the information about interregional redistribution process. First, the initial loadings of the dominant component are much larger in magnitude than those of the spatial components. This implies that the initial and long-run spatial distributions are very similar. For the whole system, the spatial dissimilarity index between the two distributions is only 4.14%. Second, the spatial components have relatively large *positive* initial loadings on Stockholm, East Middle, and South Middle, and relatively large

Table 4.6 The gross rates, mean ages, and standard deviations of the regional fertility, survival, and migration schedules of the (non-factorizable) Rogers model of the Swedish female population.

	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Lower N	Upper N
				Fertility Schedules				
Gross Rate	0.8326	0.9310	0.9571	0.9152	0.9262	0.9064	0.9228	0.9623
Mean Age	30.05	29.43	29.92	29.75	29.87	29.37	29.72	29.94
Stand. Dev.	5.99	5.91	6.00	5.89	5.98	6.01	5.94	5.98
				Survival Schedules				
Gross Rate	16.1606	16.0535	16.0780	16.1583	16.1168	15.9728	16.0286	16.0201
Mean Age	41.03	40.75	40.80	40.98	40.88	40.56	40.73	40.69
Stand. Dev.	24.07	23.92	23.93	24.02	23.97	23.82	23.93	23.90
Gross Death Rate	1.8394	1.9465	1.9220	1.8417	1.8832	2.0272	1.9714	1.9799
				Interregional Migration Schedules				
Gross Rate	1.2510	1.2908	1.2402	0.7535	0.7367	1.1708	1.2821	1.1029
Mean Age	28.63	26.59	26.05	26.16	26.47	25.91	25.30	26.08
Stand. Dev.	19.85	17.81	17.35	17.03	17.69	17.43	16.82	17.21

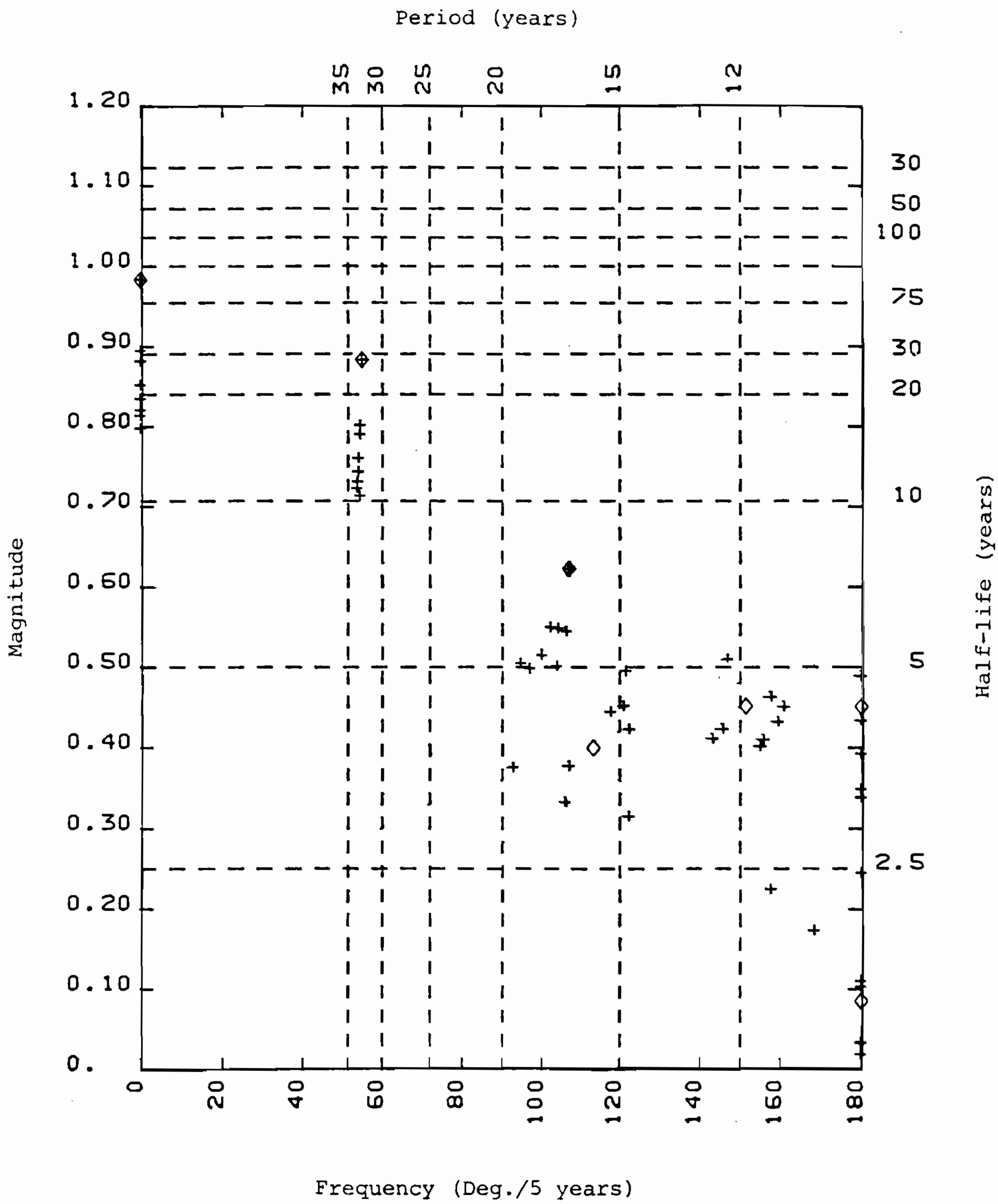

Figure 4.6 The pattern of non-zero eigenvalues of the (non-factorizable) Rogers model of the 1974 Swedish female population. The eigenvalues of the corresponding Leslie model are shown as diamonds.

Table 4.7 The half-lives, initial levels, and initial regional loadings on the 0-4 age group of the dominant and spatial components of the (non-factorizable) Rogers model of the 1974 Swedish female population.

Half-life (t_i)	Initial Level (C_i)	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Lower N	Upper N
		$C_i Q_{i,1j}$							
209.39	4,860,345	44,359	44,571	23,984	41,785	60,979	26,198	12,236	17,735
31.42	311,545	1,865	1,726	-871	-4,103	-1,904	1,322	826	1,357
27.75	164,942	596	414	222	1,874	-3,505	68	140	274
21.72	191,276	541	1,420	440	-401	-993	1,377	-286	-2,001
19.23	158,708	57	351	2,163	-1,003	-528	-1,154	-250	299
17.58	64,679	423	-114	-67	-35	98	-532	597	-341
16.84	154,426	1,107	1,056	-665	14	269	-1,107	-695	69
15.36	97,537	1,193	-1,261	88	-25	57	177	-192	19

negative initial loadings on South and West. This implies that the spatial redistribution process, though rather unimportant, is mainly from Stockholm, East Middle, and South Middle to West and South. Third, the most rapidly damped spatial component has loadings of relatively large magnitudes on Stockholm and East Middle. This reflects the high mobility level between these two regions. All these three main points were seen to be true for the factorizable Rogers model, although the right eigenvectors associated with subdominant positive eigenvalues of the non-factorizable model are no longer guaranteed to have the zero-sum nature. Of particular interest is the fact that, although the individual spatial components have very different relative annual damping rates (1.85% for λ_2 and 4.08% for λ_8), the regional time-paths of the superimposed spatial component remain very smooth not only for a whole region but also for individual age groups (see Figure 4.7).

Table 4.8 and Figure 4.8 show how the transmission of population waves through the 0-4 age group is accounted for by the major cyclical components. First, the regional upperbounds of fluctuations ($2d_i\gamma_{iaj}$) are significantly larger for the dominant cyclical component than for the remaining cyclical components. This suggests that regional population waves will look more like synchronical rather than opposite waves. Note that the nearly synchronical nature of the dominant cyclical component is indicated by the fact that its regional phase angles are bunched narrowly in only one direction. The regional time-paths of the superimposed major cyclical component in Figure 4.8 suggests that the nearly synchronical population waves will cause the youngest age group to have a major trough in 1989 and a major peak in 2004. Second, the relative damping rates of the major cyclical components range between 2.09% per year for σ_1 and 6.20% per year for σ_8. These damping rates are, on the average, greater than those of the spatial components. This explains why convergence in the age profiles tends to proceed faster than spatial convergence. Third, the fact that the damping rate of the dominant cyclical component is exceptionally small (due to the high concentration of the regional fertility

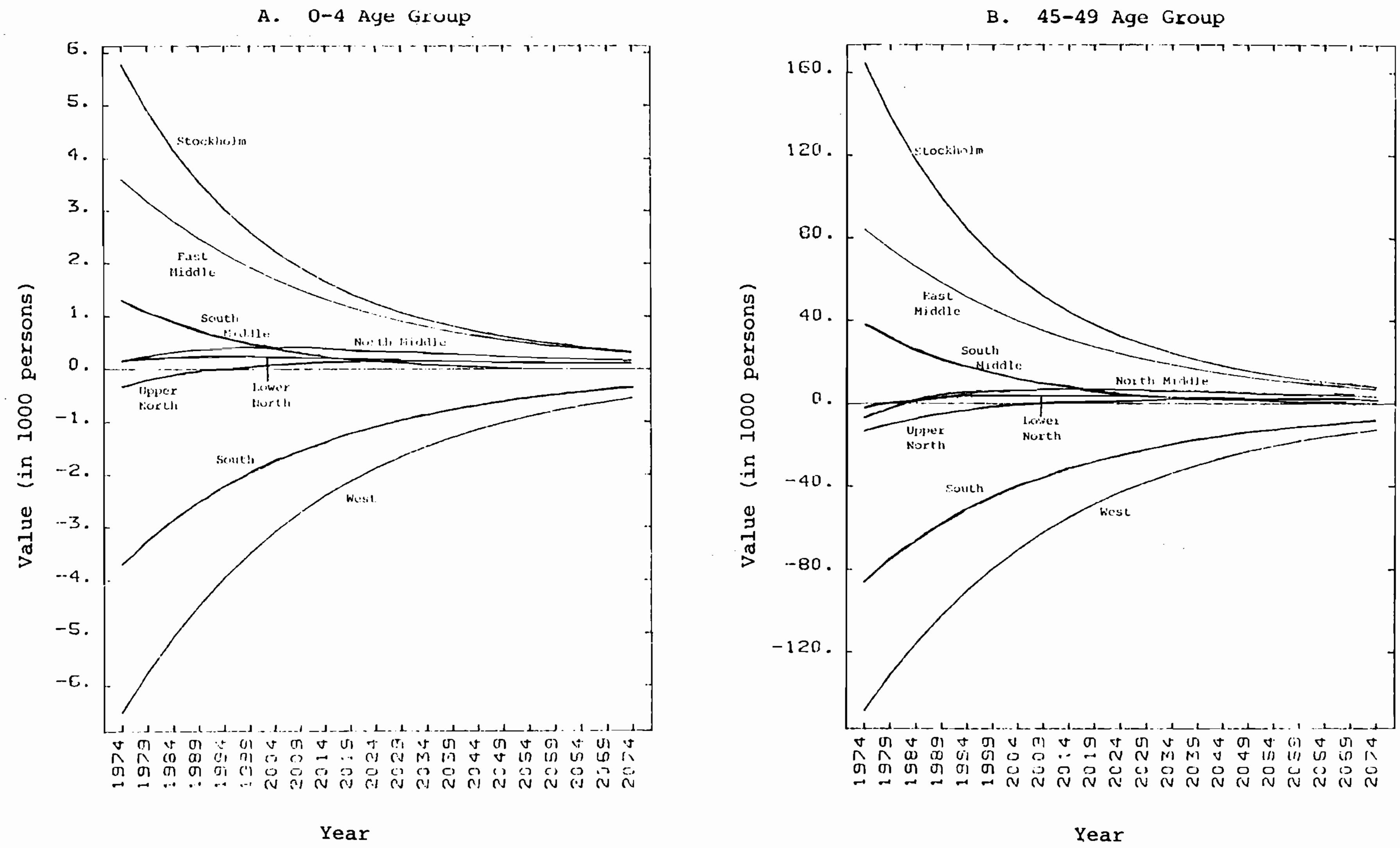

Figure 4.7 The regional time-paths of the superimposed spatial component for the 0-4 age group (A) and all ages (B) of the (non-factorizable) Rogers model of the 1974 Swedish female population.

Table 4.8 The half-lives, periods, and regional information about the 0-4 age group of the major cyclical components of the non-factorizable model of the 1974 Swedish female population.

Half-life (t_i)	Period (P_i)	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Lower N	Upper N
				Initial Regional Upperbounds ($2d_i\gamma_{iaj}$)					
28.26	32.96	2,383	2,308	1,240	2,202	3,190	1,284	627	933
15.70	32.93	182	156	74	378	158	110	75	126
14.73	32.97	124	80	35	337	652	21	30	60
12.67	33.20	314	481	177	163	331	384	93	643
11.70	33.19	56	25	406	169	96	198	58	83
11.08	33.28	286	148	51	27	62	302	296	165
10.69	33.42	1,216	499	322	22	175	586	524	75
10.26	32.96	502	917	160	23	8	355	93	23
				Initial Regional Phase Angles ($e_i + \phi_{iaj}$)					
28.26	32.96	12.22	21.73	13.91	15.86	16.09	10.83	10.83	9.22
15.70	32.93	-158.04	-142.35	25.49	32.41	10.65	-146.50	-161.83	-167.03
14.73	32.97	-77.20	-53.99	-95.97	-87.91	100.23	-26.23	-70.94	-79.69
12.67	33.20	-31.37	17.14	8.13	172.08	-175.97	14.63	-140.48	-173.03
11.70	33.19	174.01	88.65	54.73	-122.21	-131.61	-104.46	-151.77	43.81
11.08	33.28	23.16	178.76	-85.80	174.79	4.31	-174.31	14.90	-149.01
10.69	33.42	-18.03	77.87	-167.79	132.07	-18.39	174.81	174.64	-35.76
10.26	32.96	156.37	-58.26	114.65	20.28	-102.93	103.86	26.08	115.60
				Initial Regional Loadings [$2d_i\gamma_{iaj}\cos(e_i + \phi_{iaj})$]					
28.26	32.96	2,329	2,144	1,203	2,119	3,065	1,220	616	921
15.70	32.93	-169	-124	67	319	155	-92	-71	-123
14.73	32.97	28	47	-4	12	-116	19	10	11
12.67	33.20	268	460	175	-161	-330	371	-72	-638
11.70	33.19	174	89	55	-122	-132	-104	-152	44
11.08	33.28	263	-148	4	-27	62	-301	286	-142
10.69	33.42	1,156	105	-315	-15	166	-584	-522	61
10.26	32.96	-460	482	-67	22	-2	-85	84	-10
SUM	-	3,361	2,967	1,299	2,179	2,936	500	279	140

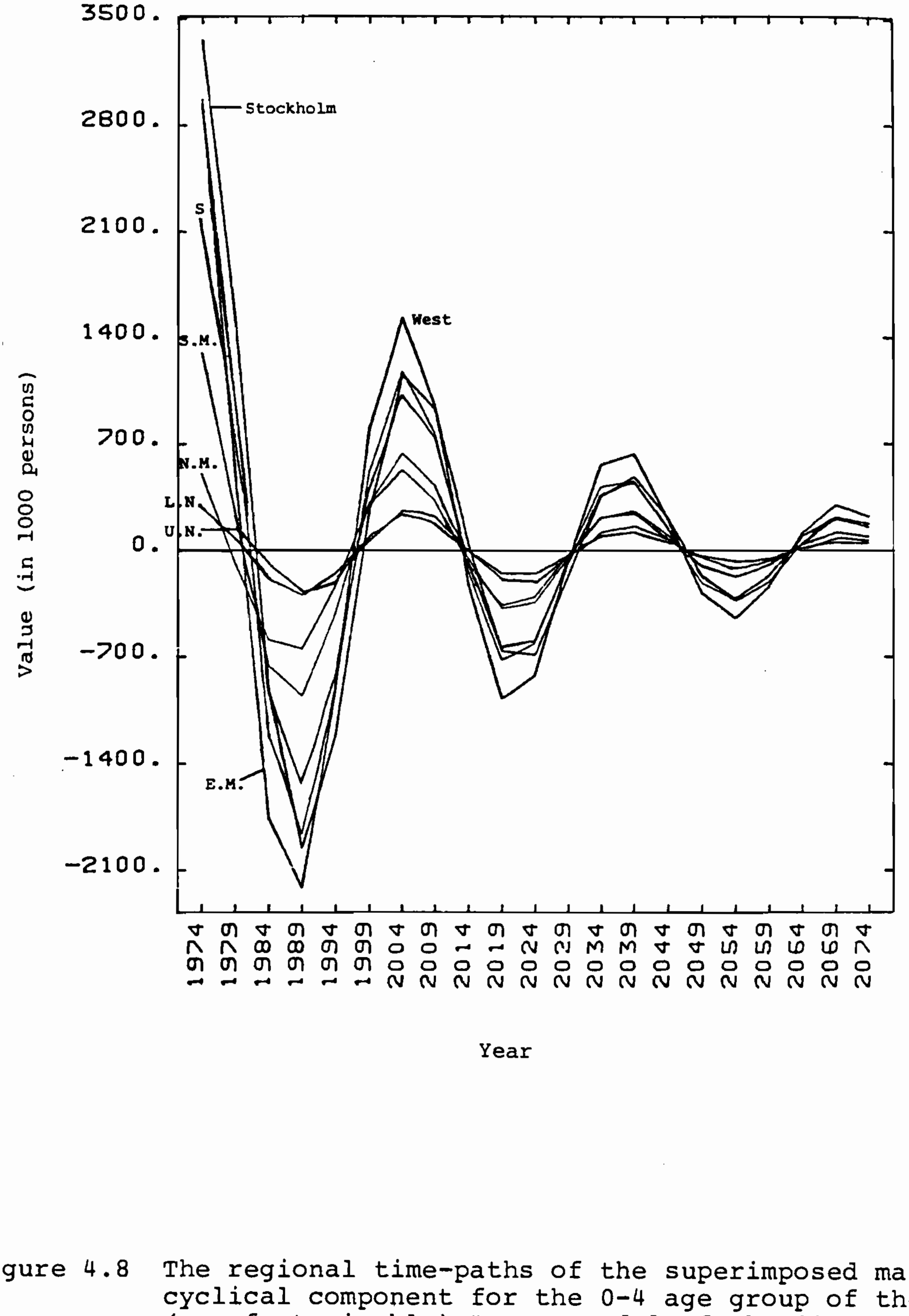

Figure 4.8 The regional time-paths of the superimposed major cyclical component for the 0-4 age group of the (non-factorizable) Rogers model of the 1974 Swedish female population.

schedules around the mean ages) implies that population waves may remain visible even after the overall pattern of the long-run regional age profiles has been closely approached. Again, these points were observed from the factorizable model. Finally, comparing Figures 4.4 and 4.6, we can infer that the dispersion of major complex eigenvalues depends more on the level of migration than on the interregional difference in the shapes of regional fertility schedules.*

Because not only the propensity to move but also the choice of destination of the Swedish population are highly selective with respect to age, the dominant right eigenvector of the growth matrix of the Rogers model implies a set of long-run regional age profiles in Figure 4.9 that are very different from the smooth long-run national age profile of the corresponding Leslie matrix. In Upper North, Lower North, North Middle, and South Middle, the long-run regional age profiles have a surplus in the youngest three or four age groups and a deficit in the 20-34 age groups, whereas Stockholm's long-run age profile displays the opposite pattern. This is, of course, the result of two contrasting migration pattern: *capital outflow* versus *capital inflow* (Rogers 1979). The smoothest long-run regional age profiles are found in East Middle, West, and South, reflecting the fact that age-selectivity in migration does not always cause irregularities in age profiles, even when the total in- and out-flows are not balanced in individual regions. Finally, the difference in the gross reproduction rate between Stockholm (0.83) and Upper North (0.96) are reflected in the difference in *slope* between their long-run age profiles. The point is that a large difference in the overall slope of long-run regional age profiles is usually the result of interregional fertility differences.

---

*This point escaped the author's attention in an earlier paper (Liaw 1979).

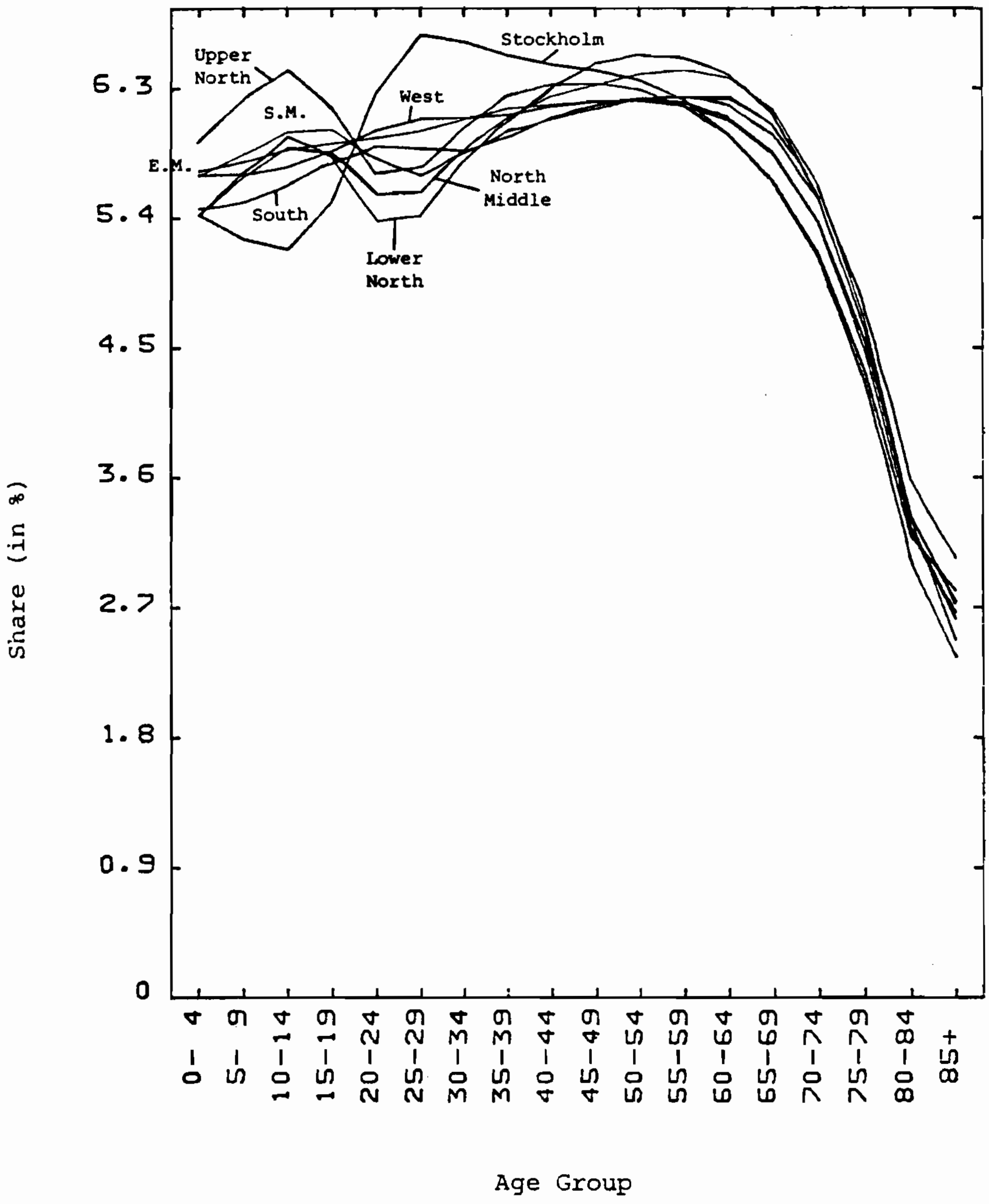

Figure 4.9 Regional long-run age profiles of the (non-factorizable) Rogers model of the 1974 Swedish female population.

Now let us look at the *actual projection result* in the light of the analytic components. Figure 4.10 shows (1) that the initial and the long-run spatial distributions are very similar; (2) that the spatial redistribution process is non-cyclical and proceeds at a slow speed; and (3) that the major gaining regions are indeed West and South, and the major losing regions are Stockholm, East Middle, and South Middle. These are consistent to what we have learned from the dominant and spatial components of the analytic solution.

The projected time-paths of the regional populations in the youngest age group are shown, with those of the meaningful analytic components, in Figure 4.11. We see in this figure (1) that each actual projected regional population fluctuates around a trend which is the sum of the dominant and superimposed spatial components; (2) that the fluctuations around the trend have a period of about 33 years and are closely approximated by the time-path of the superimposed major cyclical component; (3) that the spatial components are much less important than the dominant component in determining the trend, particularly in the two northern regions; and (4) that the damped population waves, with a major trough around 1989 and a major peak around 2004, are still quite visible in 60 years. Therefore, at least for the youngest age group, the analytic solution appears to be a useful scheme of decomposition.

Despite the fact that the long-run growth rate is negative 0.3% per year, the average annual growth rate of the whole population in the first 20 years (1974-1994) is projected to be positive 0.1%. This positive growth is consistent with the positive growth momentum of 18%. However, the system switches to negative growth before the turn of the century.

Lacking moving pictures, Figure 4.12 is used to indicate the convergence toward various long-run regional age profiles. All initial regional age profiles reflect, to various extents, past fertility changes from the low level in the 1925-39 period to a sharp rebound in the 1940's, to another low level in the 1950's, and then to a weak recovery in the 1960's just before the on-set

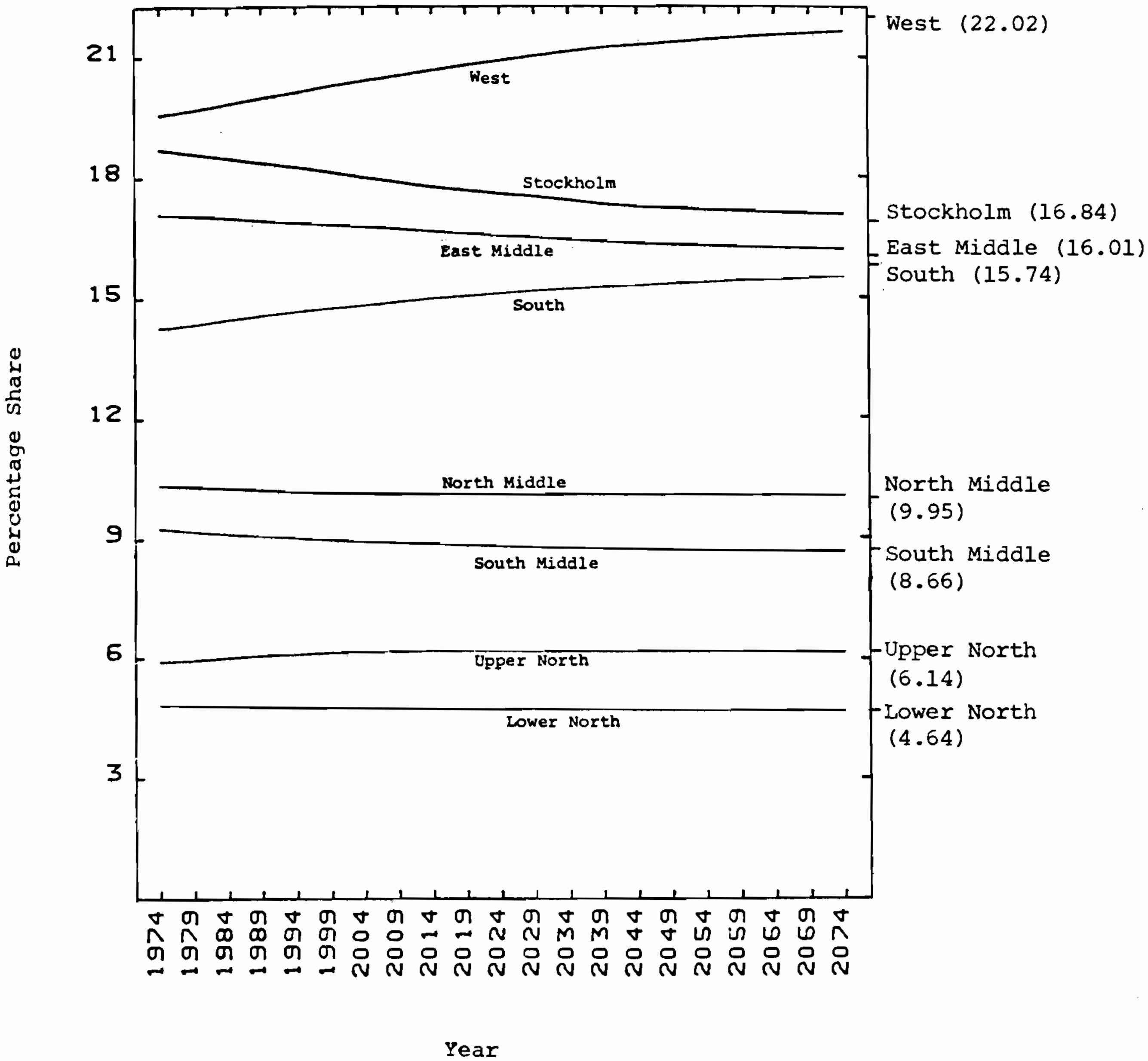

Figure 4.10 The projected regional shares of the Rogers model of the 1974 Swedish female population. The long-run shares are marked on the right axis.

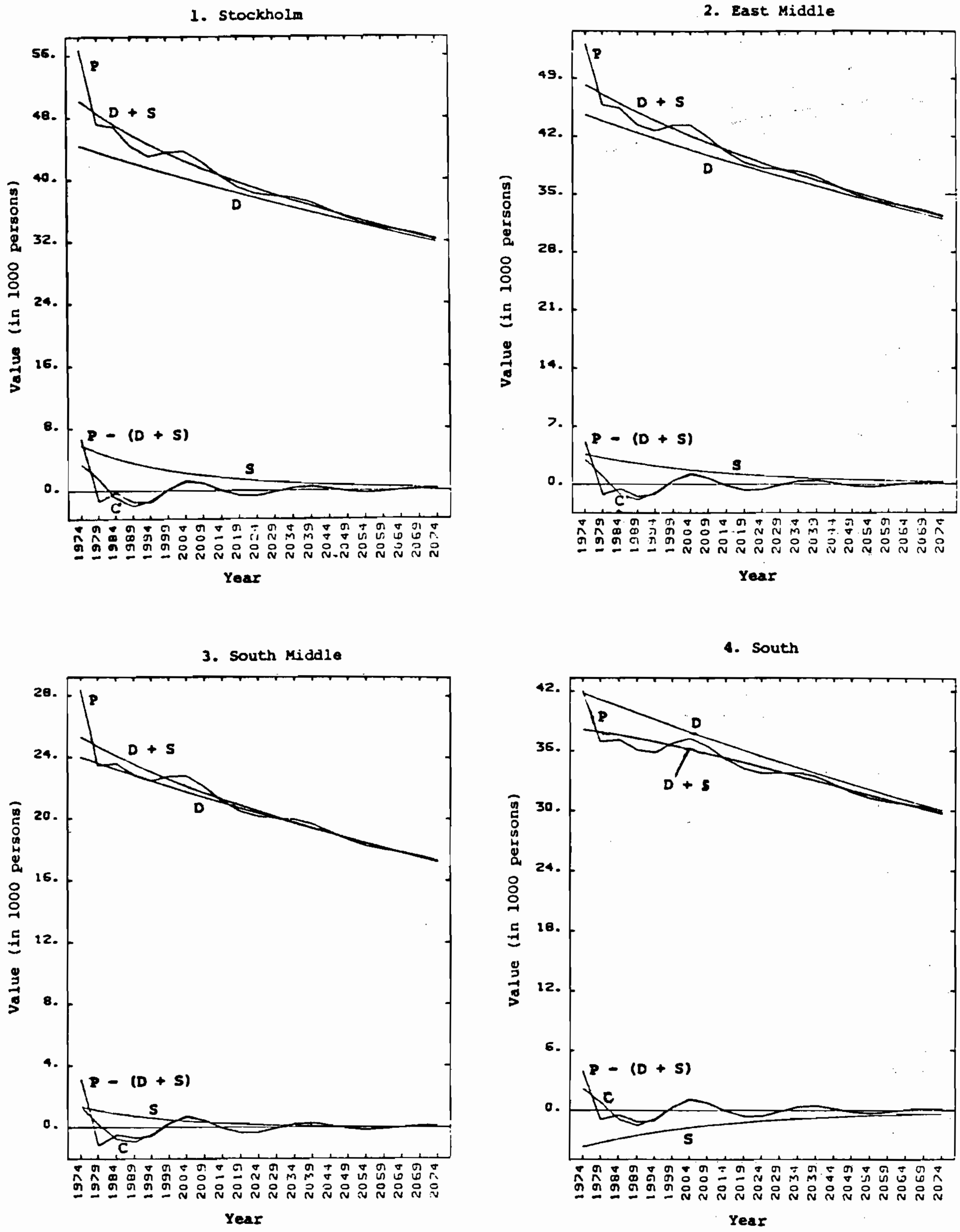

Figure 4.11 The analytic decomposition of the projected regional population sizes in the 0-4 age group of the (non-factorizable) Rogers model of the 1974 Swedish female population. P = projected population size; D = dominant component; S = superimposed spatial component; C = superimposed major cyclical component.

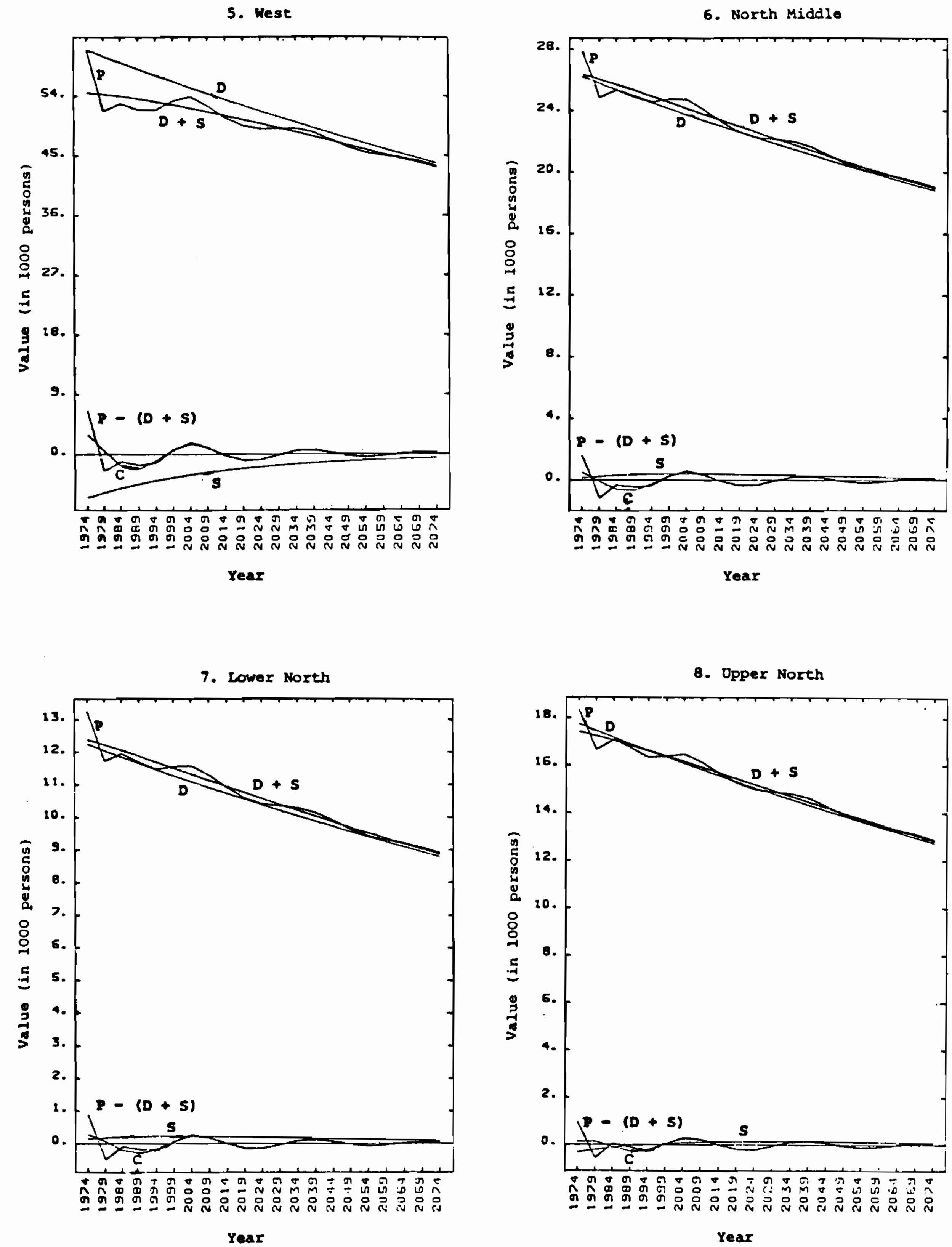

Figure 4.11 Continued.

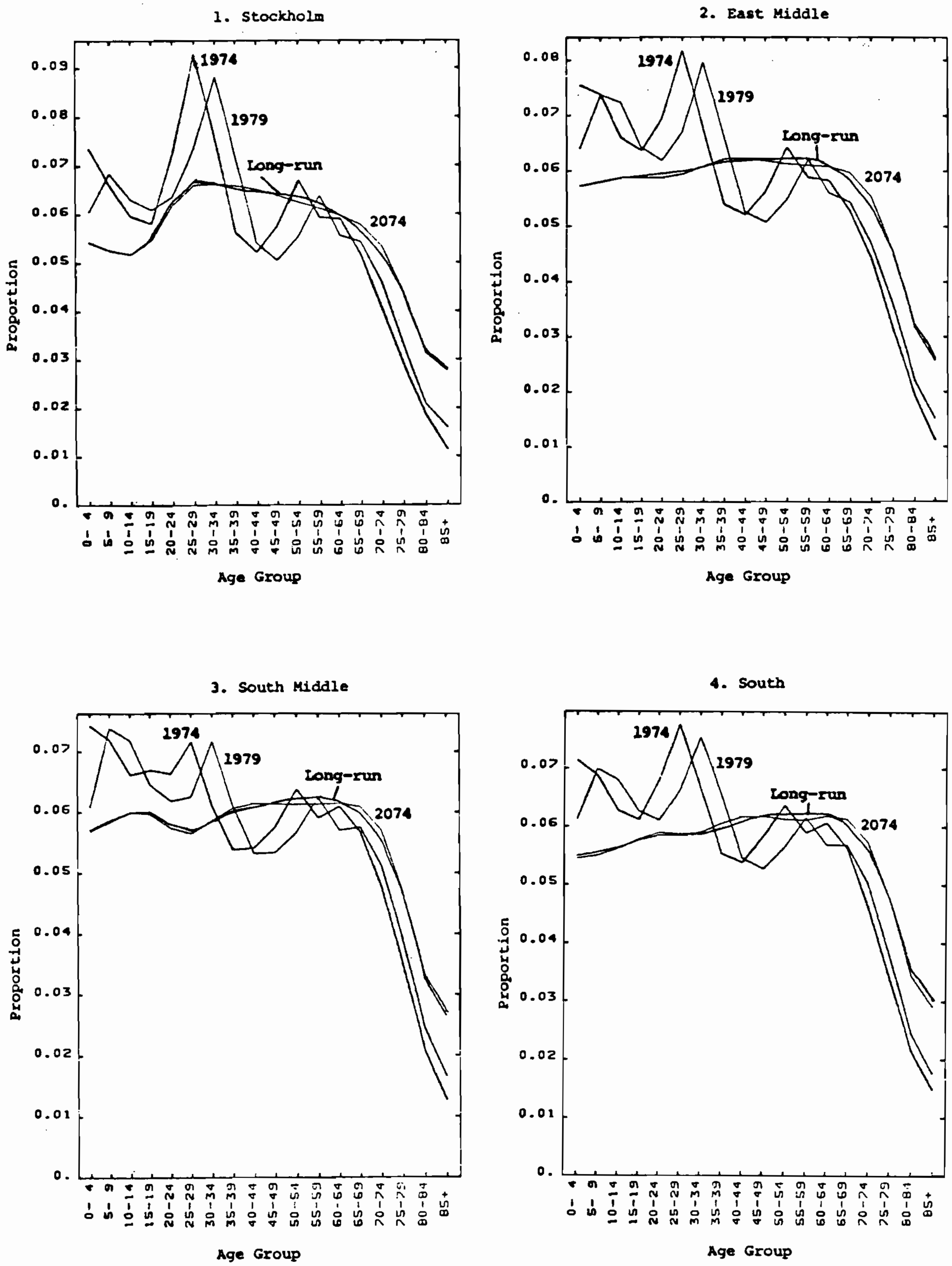

Figure 4.12 The initial, projected, and long-run regional age profiles of the (non-factorizable) Rogers model of the 1974 Swedish female population.

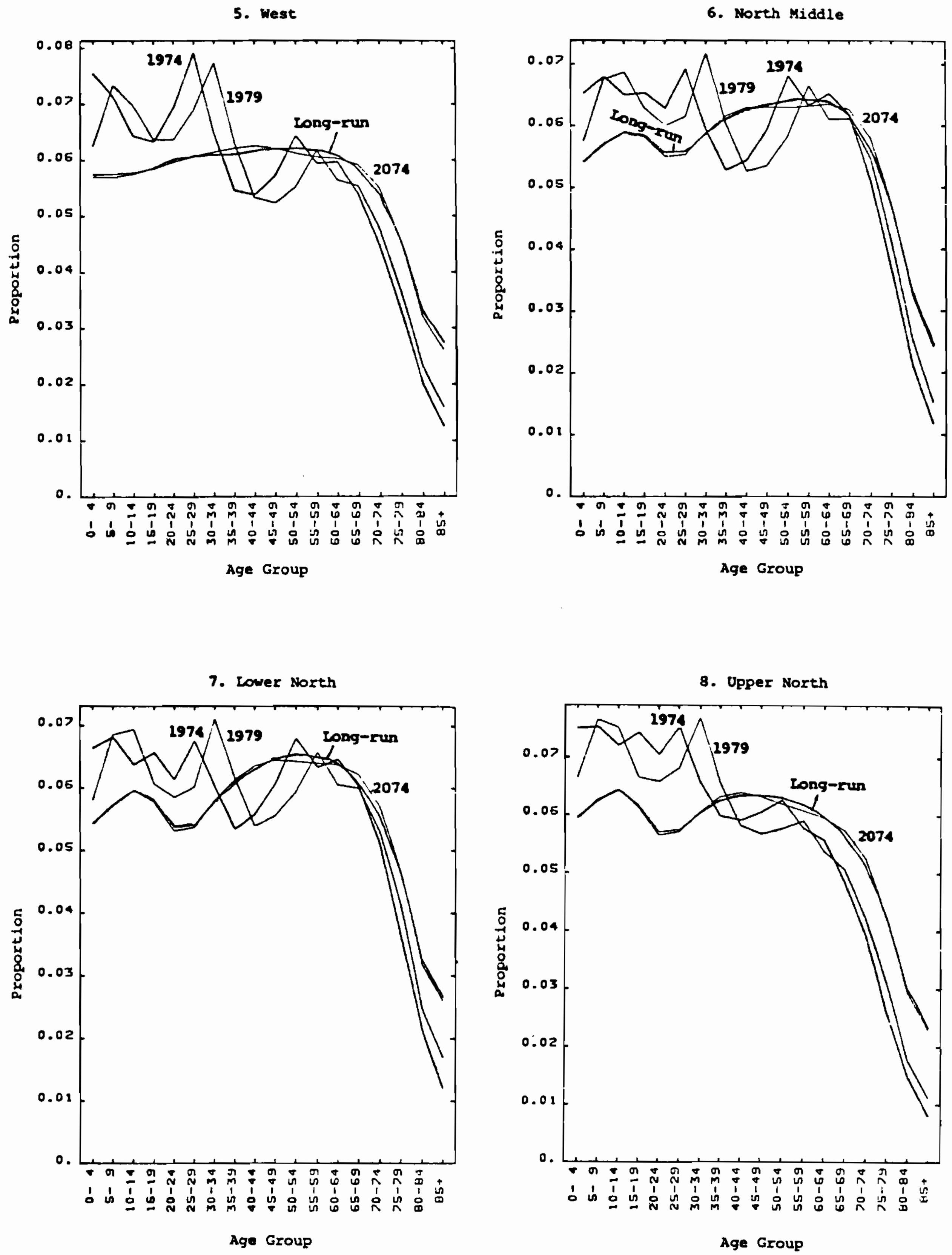

Figure 4.12 Continued.

of a deeper decline in the 1970's. The selectivity in the migration process in the recent past reinforces the concentration of Stockholm's population in the 25-29 age group on the 1974 age profile. The dissimilarity index between the initial and long-run age profiles ranges from 6.13% in Lower North to 9.22% in Stockholm. No matter whether the regional long-run age profiles are smooth or irregular, the convergence in age profile is completed (i.e., within 2%) for all regions in 55 years. Figure 4.12 shows the closeness of the 2074 and the long-run regional age profiles, although systematic deviations, due to the dominant cyclical component's small rate of attenuation, remain visible. In other words, the major outline of a regional long-run age profile is achieved long before the population waves become invisible.

Table 4.9 summarizes the evolution of regional age profiles in terms of mean age, percentage older than 64, and dissimilarity index. The trend in every region is, as a reflection of the very low fertility level, toward an older population. In terms of all three indices, the projected regional age profiles are already quite similar to their long-run counterparts by the year 2014. For government officials in Canada and the United States who are concerned with the problems of ageing population, it would be instructive to watch how the Swedes deal with the increasingly high proportion of senior citizens, which will remain much higher than in North America for many more decades.

Table 4.9 Changing characteristics of regional age profiles of the 1974 Swedish female population system projected by the non-factorizable Rogers model.

Year	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Lower N	Upper N	Sweden
					Mean Age				
1974	37.9	37.8	38.6	38.9	38.2	39.7	39.7	36.5	38.3
1984	39.6	39.3	40.1	40.1	39.4	40.8	40.8	38.1	39.7
1994	40.7	40.2	40.8	40.9	40.0	40.2	40.2	39.1	40.5
2004	41.1	40.6	41.0	41.2	40.2	41.2	41.3	39.6	40.8
2014	41.6	41.2	41.5	41.7	40.8	41.7	41.8	40.3	41.3
Long-run	42.4	42.1	42.4	43.1	42.3	42.8	42.8	41.3	42.4
					Percentage Older Than 64				
1974	15.1	15.9	17.3	17.3	16.4	18.2	18.1	13.6	16.4
1984	18.0	18.2	19.6	19.4	18.6	20.4	20.1	16.2	18.7
1994	18.9	19.0	20.2	20.0	18.9	20.6	20.3	17.1	19.3
2004	17.4	17.6	18.9	18.7	17.4	18.7	18.6	16.6	17.9
2014	20.0	20.1	20.7	20.9	19.3	20.4	20.4	18.6	20.1
Long-run	21.1	21.5	22.2	23.0	21.8	22.2	22.2	20.3	21.8
					Dissimilarity Index w.r.t. Long-run Regional Age Profile				
1974	9.2	8.6	7.0	7.7	7.7	6.2	6.1	8.5	7.9
1984	6.7	6.6	5.2	6.5	6.4	4.9	4.9	6.8	6.2
1994	5.2	5.2	4.2	5.4	5.6	4.4	4.4	5.3	5.0
2004	4.2	4.3	3.8	4.6	4.9	4.0	4.0	4.1	4.3
2014	3.3	3.2	3.0	3.5	3.7	3.1	3.0	2.9	3.3
2024	2.3	2.2	2.2	2.3	2.3	2.0	2.0	1.7	2.2
2034	1.7	1.7	1.7	1.8	1.7	1.5	1.5	1.2	1.6

5. THE DYNAMIC PROPERTIES OF THE SOVIET AGE-BY-REGION POPULATION SYSTEM

The growth matrix of the Soviet age-by-region population system is shown in Appendix B. The regional schedules of fertility, survival, and interregional migration are characterized by the gross rate, mean age, and standard deviation in Table 5.1. It is interesting to note the sharp interregional contrast in fertility schedules between the northern urban regions on the one hand and the rural and southern urban regions on the other.

The dominant eigenvalue of the growth matrix is $\lambda_1 = 1.0297$, which implies an annual long-run growth rate of 0.59% and a long-run doubling time of 118 years. The fact that this dominant eigenvalue is much larger than that of the Swedish system is mainly due to the higher gross reproduction rate (1.28) and partly due to the lower mean age (24.5 years) of the national fertility schedule of the Soviet Union.

Figure 5.1 shows that the general pattern of eigenvalues of the Rogers model follows that of the corresponding Leslie model. But, because of the higher level of migration, the dispersion of the eigenvalues in each cluster is greater than what we saw in the Swedish model.

Table 5.2 shows the relative importance of the dominant and spatial components. The stable equivalent population is 269,969,000, which is greater than the initial size of 250,869,000. Thus, the momentum of growth is 7.5%. This momentum and the upswing of the population waves cause the average annual growth rate for 1974-1979 (0.98%) to be much higher than the corresponding long-run growth rate (0.59%). The values of c_i for the spatial components are much smaller than the value of c_1, with the major exception of c_8 which clearly signifies the rapid and sweeping process of urbanization. The half-life of this "urbanization component" is only 10.26 years, which corresponds to a relatively damping rate of 7.1% per year. Other spatial components that are worth mentioning are the one with a half-life of 17.36 years (representing the spillover effect from RSFSR to Ukraine and

Table 5.1 The gross rates, mean ages, and standard deviations of the regional fertility, survival, and migration schedules of the Rogers model of the 1974 Soviet Union population.

	RSFSR	Uk & Mo	Byelo	Cent. A	Kazakh	Caucasian	Baltic	Rural
				Fertility Schedules				
Gross Rate	1.0186	1.0518	1.0938	1.7807	1.2339	1.4179	1.0001	1.6605
Mean Age	23.50	23.53	23.99	25.90	24.57	24.62	24.07	25.26
Stand. Dev.	5.85	5.99	5.99	7.06	6.49	6.23	6.14	6.95
				Survival Schedules				
Gross Rate	15.8054	15.8373	16.1009	16.0455	15.8687	16.0153	15.8414	15.8316
Mean Age	40.71	40.67	41.23	41.37	40.94	41.10	40.68	40.84
Stand. Dev.	24.06	24.04	24.30	24.36	24.19	24.22	24.05	24.11
Gross Death Rate	2.1946	2.1627	1.8991	1.9545	2.1313	1.9847	2.1586	2.1684
				Interregional Migration Schedules				
Gross Rate	1.1641	1.5392	2.0572	1.8961	2.7684	0.9889	1.4111	2.7102
Mean Age	31.55	30.89	34.51	34.63	35.20	32.64	29.92	29.10
Stand. Dev.	20.93	20.34	22.25	21.31	22.13	20.81	19.89	17.55

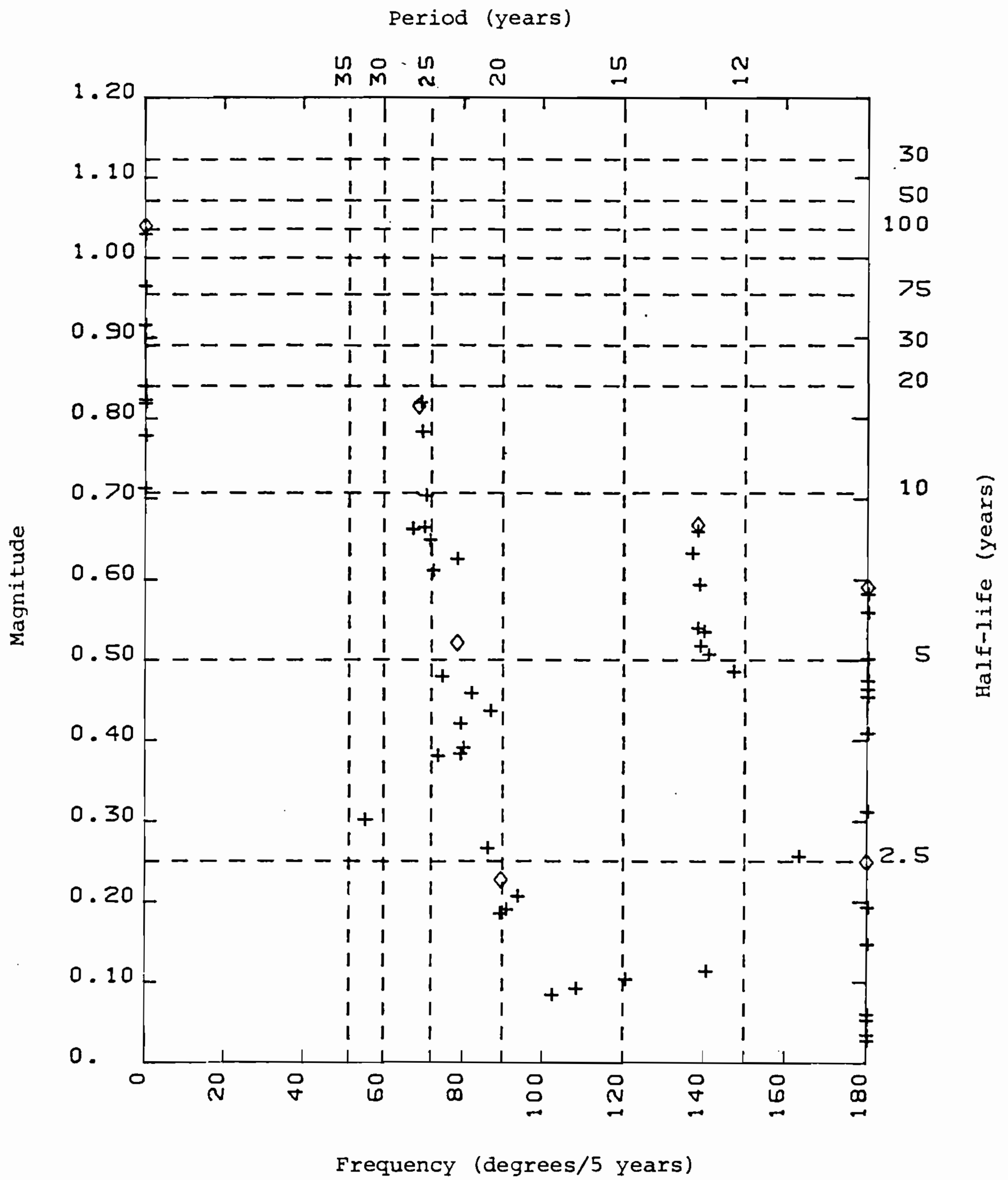

Figure 5.1 The pattern of non-zero eigenvalues of the growth matrix of the Rogers model of the 1974 Soviet population. The eigenvalues of the corresponding Leslie model are shown as diamonds.

Table 5.2 The doubling time, half-lives, initial levels, and initial regional loadings on the 0-4 age group of the dominant and spatial components of the Rogers model of the 1974 Soviet Union population.

DT & HL* (t_i)	Initial Level (C_i)	RSFSR	Uk & Mo	Byelo	Cent. A	Kazakh	Caucasian	Baltic	Rural
		Initial Regional Loadings ($C_i Q_{iaj}$)							
118.40	269,696,224	8,508,172	3,037,482	624,017	2,419,982	949,821	1,169,465	468,186	5,998,505
95.69	7,401,454	123,967	48,029	11,897	63,160	17,415	-275,751	9,064	89,699
39.58	37,565,192	753,137	429,198	94,499	-1,141,819	8,676	-64,469	83,066	397,112
20.00	3,479,886	38,199	44,575	-9,768	-5,139	5,956	-521	-69,579	-3,392
17.77	3,100,184	8,852	-45,012	68,586	-67	534	347	-28,745	-5,780
17.36	17,777,740	385,372	-357,403	-59,876	-33,562	53,302	-1,946	3,089	26,180
13.83	8,173	203	-14	-2	17	-186	-1	-5	-3
10.26	195,885,584	-2,433,099	-657,837	-264,928	-81,090	-244,888	-48,740	-109,692	3,587,041

*Note: Doubling time refers only to the dominant component.

Moldavia), and the ones with half-lives of 39.58 and 95.58 years (reflecting the higher natural growth in the Central Asian and Caucasian Republics due to their higher levels of fertility). The relative annual rates of damping for these three components are 4.5%, 2.3%, and 1.3%, respectively. The effects of urbanization, spillover, and interregional difference in fertility upon the individual regions are combined into the smooth and non-cyclical regional time-paths of the superimposed spatial component, which are illustrated in Figure 5.2 for the 0-4 age group. The time-paths of the actual projected regional shares in Figure 5.3 are indeed smooth and non-cyclical and show all urban regions as gainers. The rural share of the national population drops rapidly from 40.4% in 1974 to 23.8% in 2004, a level which is already quite close to the long-run share of 20.2%. Reflecting mainly the rapidity of urbanization, the spatial dissimilarity index drops from 20.2% in 1974 to 4.9% in 2004, and then declines to a negligible 0.7% in 2074. As far as the spatial pattern of the urban subpopulation is concerned, Table 5.3 shows that the redistributional potential is very weak, with the dissimilarity index between the 1974 and the long-run spatial distributions being less than 5%. Thus, urbanization is seen to have little effect on the spatial pattern of the urban population at the regional level.

The dominant cyclical component has a half-life of 17.48 years (which implies a relative annual damping rate of 3.3%) and a period of 26.04 years. These values are rather similar to those of the Canadian system of the late 1960's (Liaw 1980a) but are significantly different from those of the contemporary Swedish system. The difference is related to the mean ages and standard deviations of the fertility schedules. Table 5.4 shows the initial regional upperbounds, phases, and loadings of the major cyclical components for the 0-4 age group. The table indicates that nearly synchronical waves (which are represented by the dominant cyclical component) are more important than opposite waves (which are represented by the remaining major cyclical components). The regional time-paths of the superimposed major cyclical component for the 0-4 age

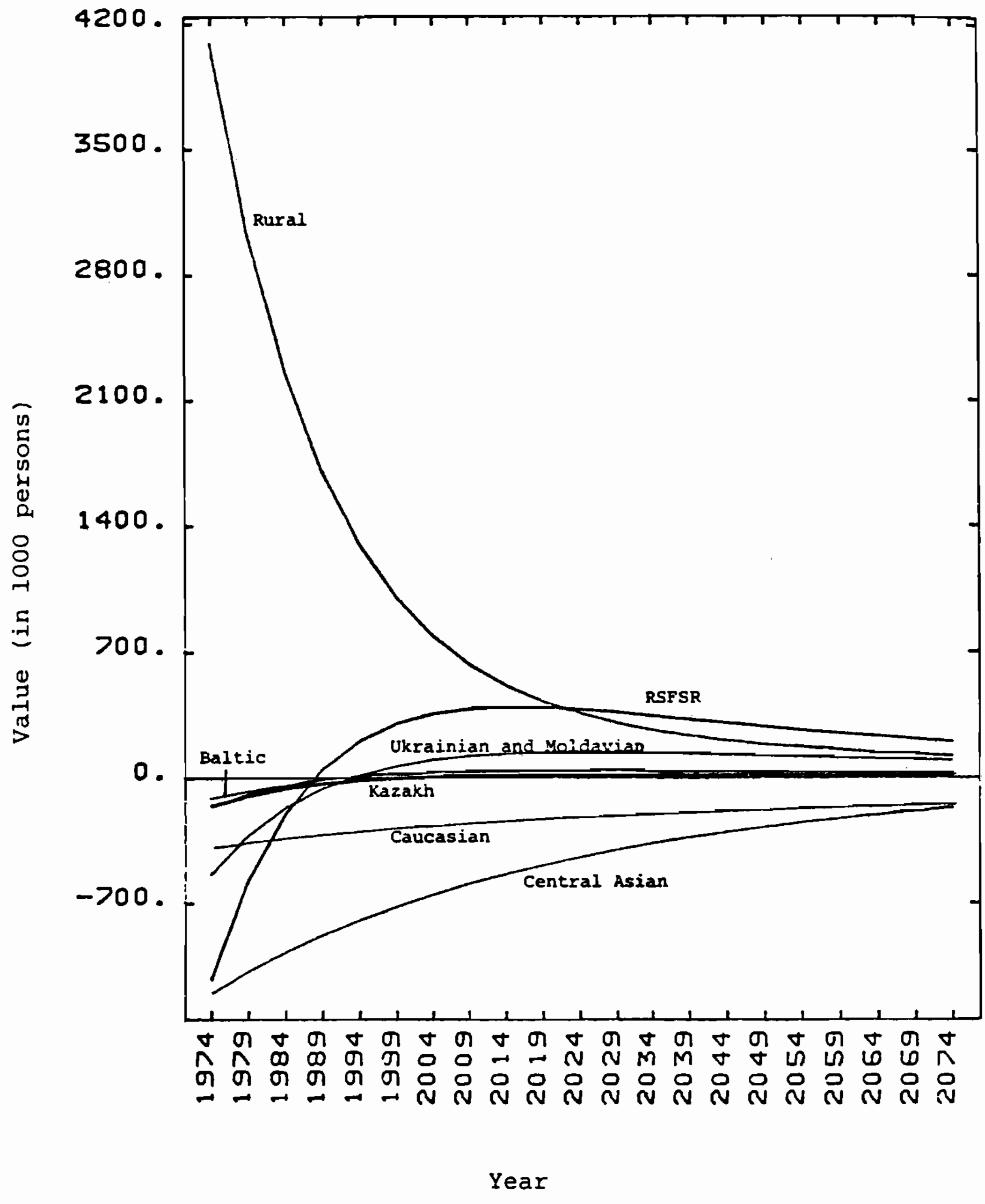

Figure 5.2 The regional time-paths of the superimposed spatial component for the 0-4 age group of the Rogers model of the 1974 Soviet population.

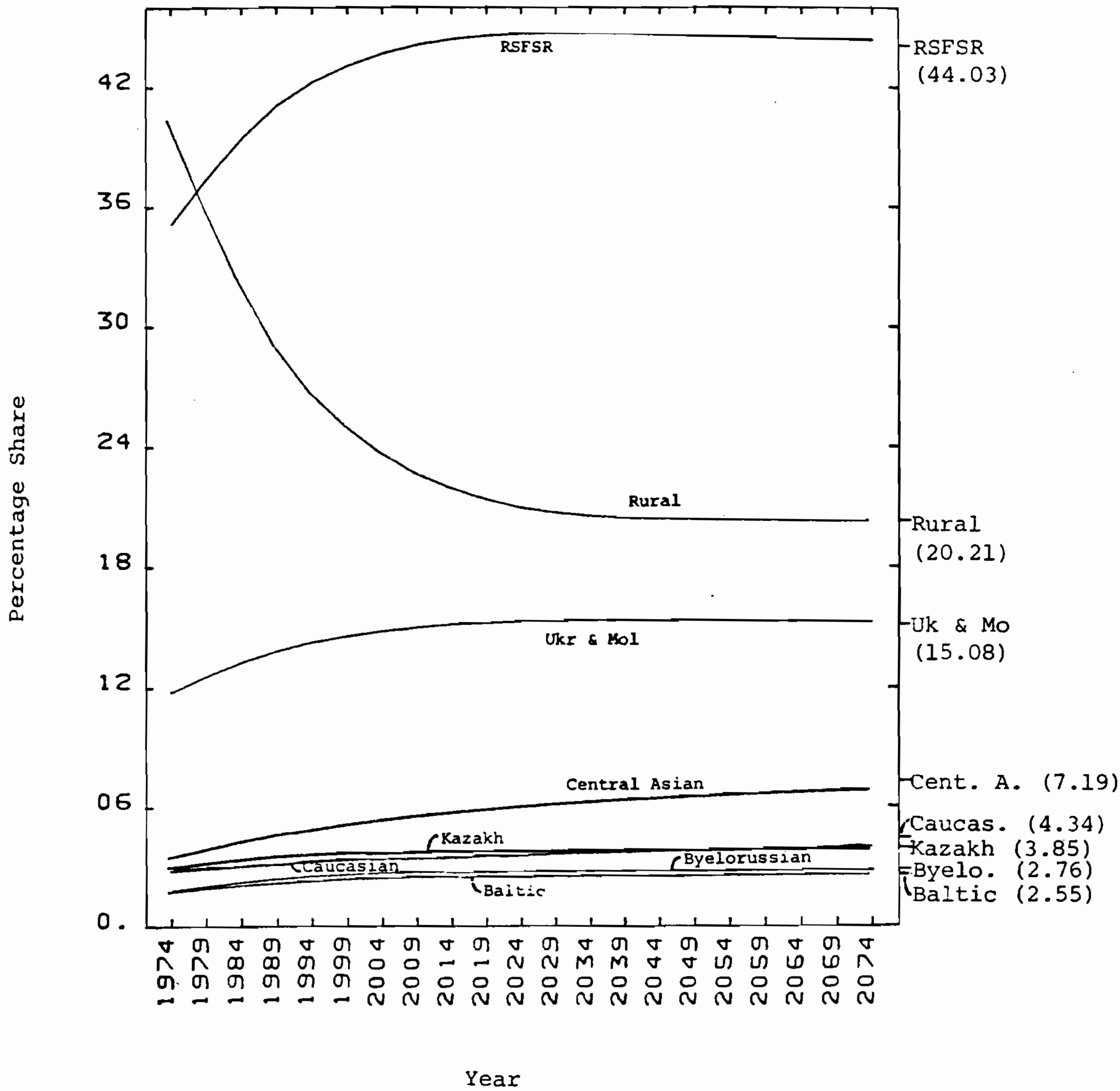

Figure 5.3 The projected regional shares of the Rogers model of the 1974 Soviet population. The long-run regional shares are marked on the right axis.

Table 5.3 The spatial redistribution of the urban subpopulation of the Soviet regional population projected by the Rogers model.

Time	RSFSR	Uk & Mo	Byelo	Cent. A	Kazakh	Caucas.	Baltic	Urban*
			Percentage of the Total Urban Population					
1974	58.95	19.74	3.04	5.80	4.91	4.62	2.90	59.63
2074	55.67	19.14	3.51	8.59	4.83	5.01	3.24	79.68
Long-run	55.18	18.90	3.45	9.02	4.82	5.44	3.19	79.79
			Difference from the Long-run Distribution					
1974	3.80	0.84	-0.41	-3.22	0.09	-0.82	-0.29	4.73
2074	0.49	0.24	0.06	-0.43	0.01	-0.43	-0.05	0.85

*In this column, the first three numbers are the urban shares in percentage of the total population, and the last two numbers are spatial dissimilarity indices of the urban subpopulation.

Table 5.4 The half-lives, periods, and regional information about the 0-4 age group of the major cyclical components of the Rogers model of the 1974 Soviet Union population.

Half-life (t_i)	Period (P_i)	RSFSR	Uk & Mo	Byelo	Cent. A	Kazakh	Caucas.	Baltic	Rural
				Initial Regional Upperbounds ($2d_i\gamma_{iaj}$)					
17.48	26.04	1,209,827	362,857	85,631	189,927	102,450	236,554	54,081	465,037
14.20	25.93	153,831	47,410	12,608	27,048	13,789	130,870	7,768	55,967
9.88	25.55	47,499	25,238	7,819	68,248	2,082	1,024	4,011	8,614
8.50	25.74	4,190	1,472	5,275	617	320	50	828	173
8.44	26.84	17,627	5,885	3,567	2,259	1,647	258	29,908	4,194
8.02	25.20	53,177	55,465	1,112	3,664	3,366	18	281	2,173
7.39	22.97	298,329	86,463	27,196	10,311	24,901	6,181	8,496	449,843
7.04	24.91	22,523	797	164	3,038	25,998	45	247	1,608
				Initial Regional Phase Angles ($e_i + \phi_{iaj}$)					
17.48	26.04	173.53	175.10	170.21	154.03	165.79	165.06	150.41	-174.24
14.20	25.93	168.23	170.28	164.11	148.40	160.20	-9.31	140.64	-175.00
9.88	25.55	98.38	100.70	78.61	-80.03	-94.73	-85.22	35.05	132.88
8.50	25.74	-157.26	-158.87	38.57	-22.32	-152.67	3.34	-53.52	-150.48
8.44	26.84	155.29	162.76	106.21	-126.87	165.43	-110.59	-21.76	69.85
8.02	25.20	-179.71	-0.99	-145.47	-19.76	179.98	137.76	86.70	51.37
7.39	22.97	-69.63	-65.68	-56.49	-86.74	-47.51	-92.61	-67.74	97.13
7.04	24.91	63.57	59.35	119.62	32.13	-121.92	-167.67	-95.49	-42.09
				Initial Regional Loadings [$2d_i\gamma_{iaj}\cos(e_i + \phi_{iaj})$]					
17.48	26.04	-1,202,131	-361,532	-84,385	-170,744	-99,317	-228,557	-47,030	-462,725
14.20	25.93	-150,595	-46,729	-12,126	-23,037	-12,974	129,145	-6,006	-55,754
9.88	25.55	-6,920	-4,686	1,544	11,814	-172	85	3,284	-5,862
8.50	25.74	-3,865	-1,373	4,124	570	-285	50	492	-150
8.44	26.84	-16,013	-5,620	-996	-1,355	-1,594	-91	27,777	1,445
8.02	25.20	-53,176	55,457	-916	3,448	-3,366	-13	16	1,357
7.39	22.97	103,822	35,605	15,013	586	16,819	-281	3,219	-55,854
7.04	24.91	10,026	407	-81	2,573	-13,745	-44	-24	1,193

group in Figure 5.4 suggest that the regional population waves moving through the youngest age group will have a major peak in 1989 and a major trough in 1999. Note that both Table 5.4 and Figure 5.4 show that the rural region is significantly affected by a cyclical component whose period and half-life are respectively only 23 and 7.4 years. Since the mean age of the rural fertility schedule is over 25 years, the short period cannot be directly related to the fertility schedule. It appears that a very high level of outmigration from a region can shorten the period as well as the half-life of the region's population wave.

Of particular interest is the rapid growth of the youngest population in the first fifteen years in all urban regions, with the rates of increase ranging from 60% in the Baltic and Caucasian Republics to 106% in Byelorussia. Table 5.4 shows that this is the result of the joint impact of intrinsic growth (dominant component), urbanization (spatial components), and population waves (cyclical components). In every region, the contribution of population waves is more important than that of intrinsic growth. With the exception of the Caucasian region, urbanization is also more important than intrinsic growth, and exceeds the importance of population waves in two regions (Baltic and Central Asian). During the initial fifteen years, the youngest population in the rural region is *reduced* by 9%, which is the balance of the gains through intrinsic growth (+6%) and population waves (+10%) and the loss through urbanization (-25%). A conclusive message from Table 5.5 is that no matter how the fertility level changes in the Soviet Union, there will definitely be a substantial increase (decrease) of young children in the urban (rural) areas.

Figure 5.5 shows the decomposition of the projected regional time-paths of the 0-4 age group into those of the analytic components. Every region's time-path fluctuates around a smooth trend which is the sum of the dominant and superimposed spatial components. The fluctuations are well "explained" by the superimposed cyclical component and are damped faster than those we

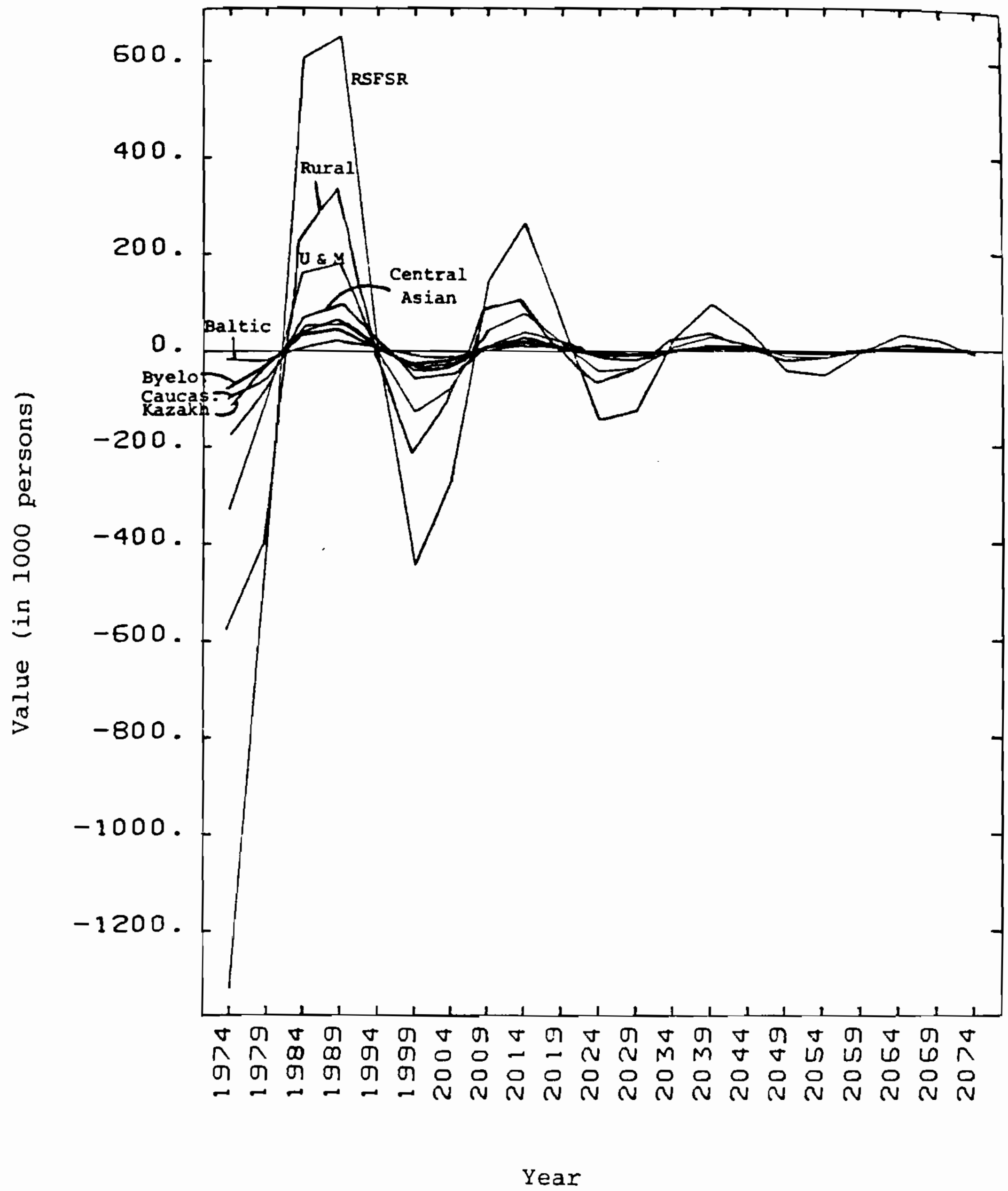

Figure 5.4 The regional time-paths of the superimposed major cyclical component for the 0-4 age group of the Rogers model of the 1974 Soviet population.

saw in the Swedish model. Note that the population waves in the Soviet Union are in opposite phases to those in Sweden; this is a reflection of the fact that after the common babyboom in the late 1940's, the fertility levels change differently in the two countries (Soboleva 1980:8-15; and Andersson and Holmberg 1980:15-18).

Table 5.5 Decomposition of projected Soviet population change in the 0-4 age group between 1974 and 1989.

Region	Projected		Dominant	Spatial	Cyclical*	
	Absolute Change (in 1000 persons)					
RSFSR	4,482		781	1,770	2,531	(1,972)
Uk & Mo	1,430		279	478	673	(509)
Byelo	337		57	146	163	(123)
Cent. A	859		222	326	310	(275)
Kazakh	431		87	126	219	(171)
Caucas.	382		107	75	200	(166)
Baltic	191		13	28	18	(13)
Rural	-885		+551	-2,394	+958	(+914)
Soviet	7,258		2,127	17	5,114	(4,170)
	Percentage Change with Respect to the 1974 Observed Size					
RSFSR	82	[4.1]	14	22	46	(36)
Uk & Mo	71	[3.6]	14	24	33	(25)
Byelo	106	[4.9]	17	42	47	(36)
Cent. A	85	[4.2]	22	32	31	(27)
Kazakh	69	[3.6]	14	20	35	(27)
Caucas.	60	[3.2]	17	12	31	(26)
Baltic	60	[3.2]	14	28	18	(13)
Rural	-9	[-0.7]	+6	-25	+10	(+10)
Soviet	37	[2.1]	11	0	26	(21)

*Numbers in parentheses show the growth attributed to the superimposed cyclical component. The values in square brackets are average annual percentage changes.

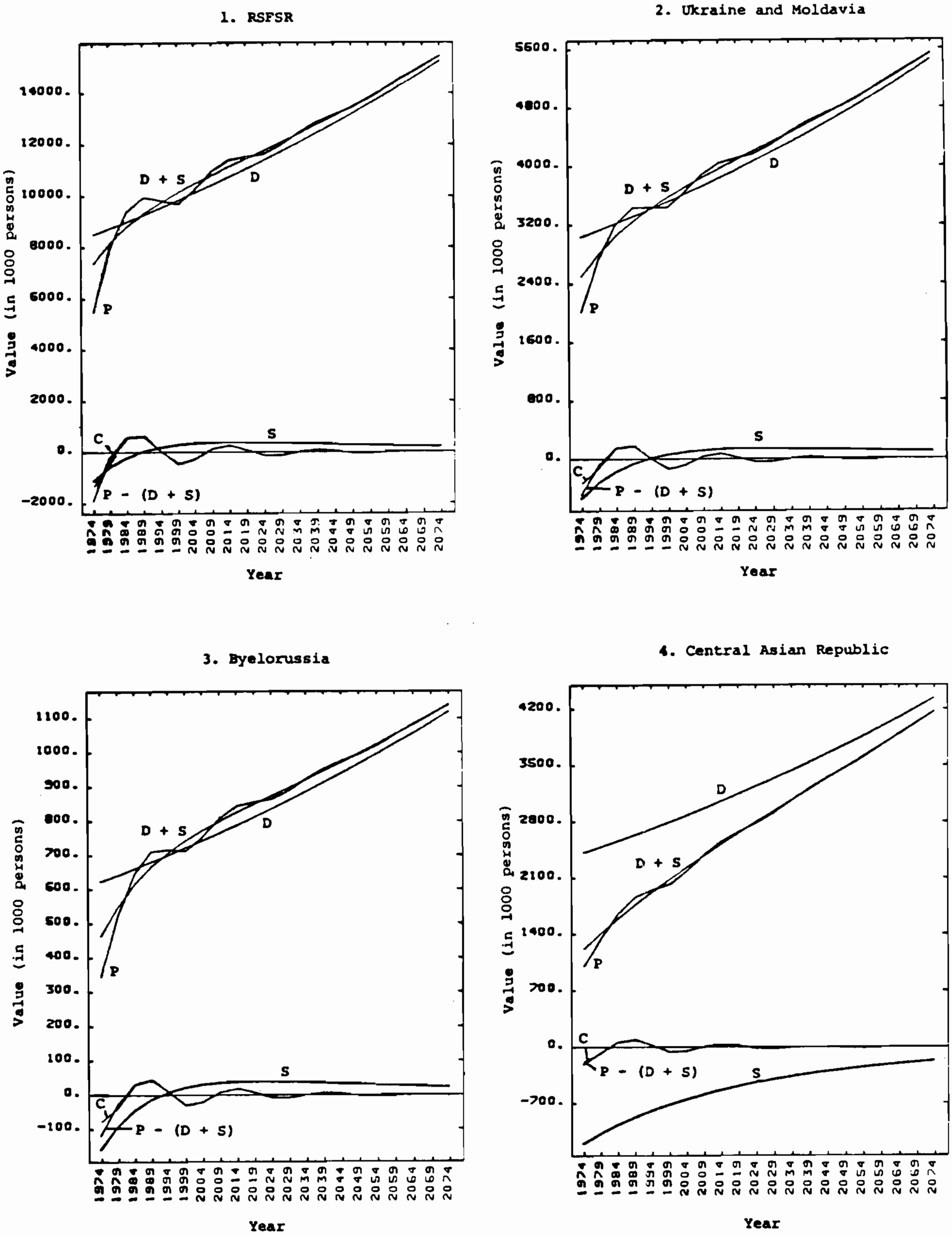

Figure 5.5 The analytic decomposition of the projected regional population sizes in the 0-4 age group of the Rogers model of the 1974 Soviet population. P = projected population size; D = dominant component; S = superimposed spatial composition; C = superimposed major cyclical component.

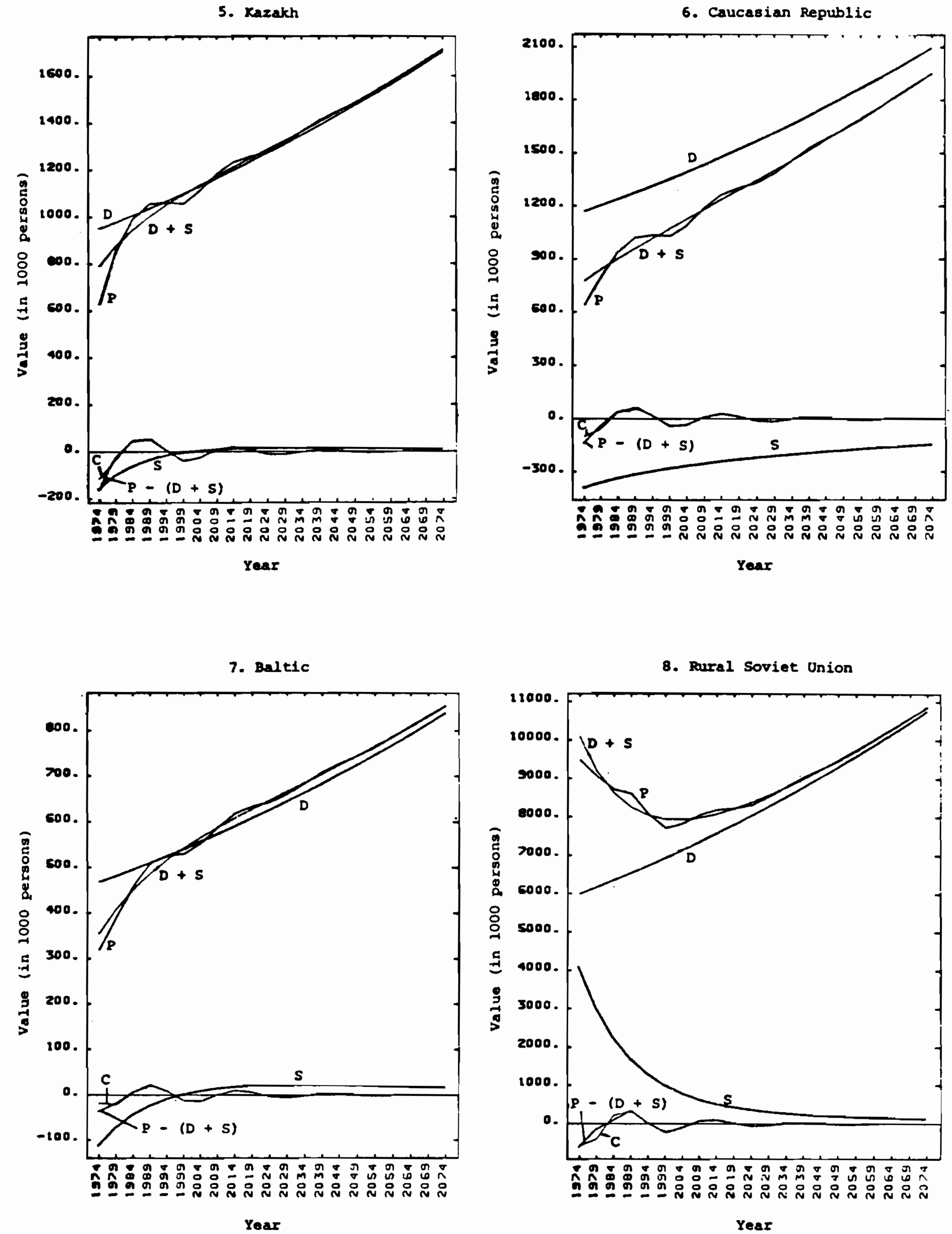

Figure 5.5 Continued.

Despite heavy rural-urban migration and the difference in fertility development between the essentially European and the essentially Asian regions, most of the 1974 regional age profiles in Figure 5.6 exhibit clearly the scars of the double-blow of World War II in the 50-54 and 25-29 age group and the subsequent second generation effect in the 0-4 age group. However, the similarly irregular initial age profiles converge to three different types of relatively smooth long-run profiles: (1) age profiles with a significant surplus in the young adult age group (found in northern urban regions including RSFSR, Ukrainian and Moldavian Republics, Byelorussia, and the Baltic Republics); (2) relatively smooth and steep age profiles without a marked effect of migration selectivity (found in southern urban regions including Kazakh and Central Asian and Caucasian Republics); and (3) a steep age profile with a significant deficit in the young and middle labor force age groups (found in the rural Soviet Union). The interregional contrast in steepness of the long-run age profiles can be seen as a clear reflection of the large interregional difference in the fertility level. However, the fact that the long-run age profiles of the southern urban regions do not show a significant surplus in the young adult age groups cannot be easily inferred by comparing the age patterns of the interregional migration schedules.

Table 5.6 shows that the trend toward a moderate increase in mean age and percent older than 64 years is for some regions, particularly the rural part, less significant than the fluctuations. For the whole nation, the long-run mean age is 35.1 years, which is only 2.2 years greater than the 1974 figure, whereas the percentage of population older than 64 is 13% in the long run, compared with 10% in 1974. The dissimilarity indices in Table 5.6 show that the convergence toward long-run regional age profiles is almost completed in 60 years.

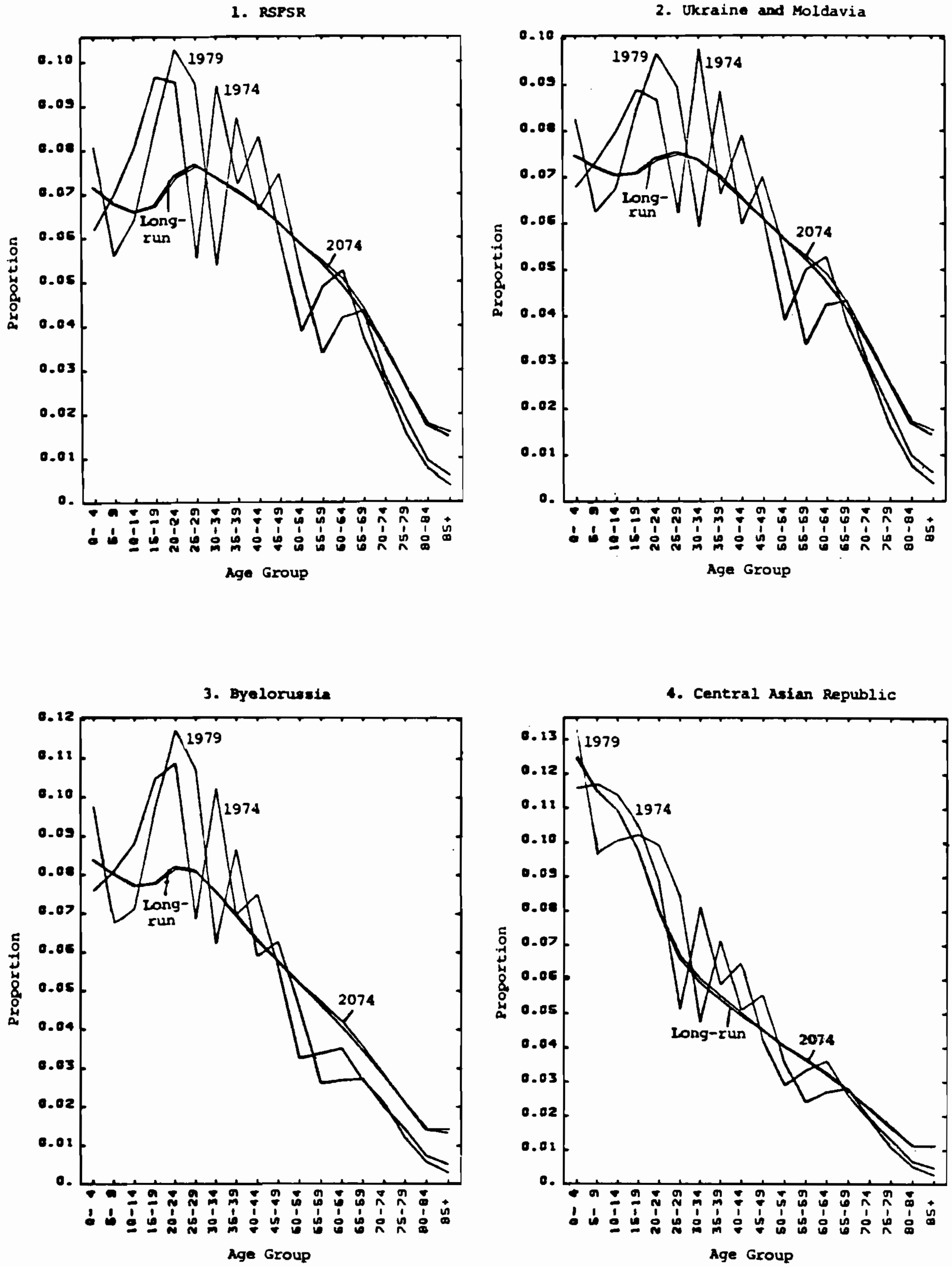

Figure 5.6 The initial, projected, and long-run regional age profiles of the Rogers model of the 1974 Soviet population.

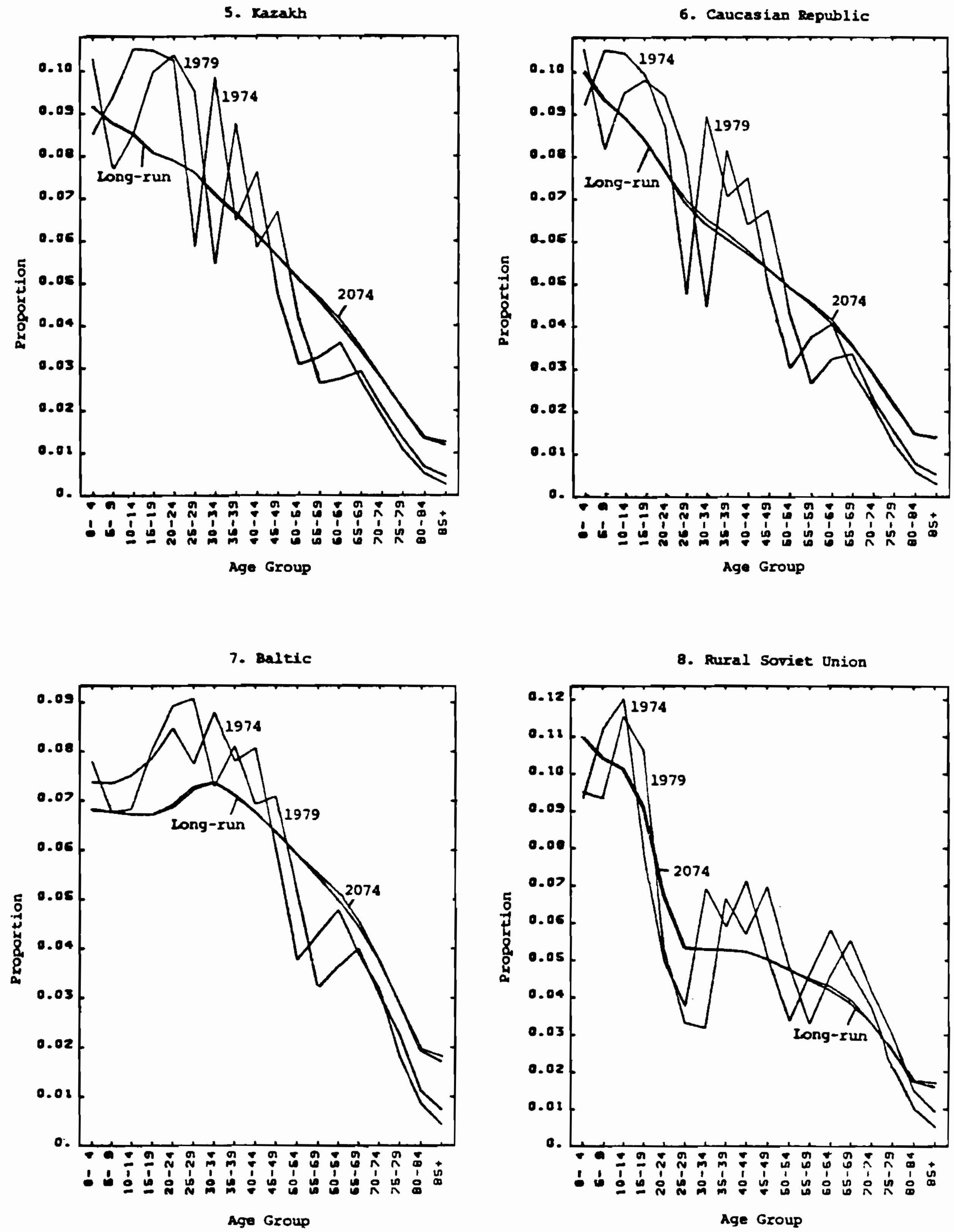

Figure 5.6 Continued.

Table 5.6 Changing characteristics of regional age profiles of the 1974 Soviet population system projected by the Rogers model.

Year	RSFSR	Uk & Mo	Byelo	Cent. A	Kazakh	Caucas.	Baltic	Rural	Soviet Union
					Mean Age				
1974	33.8	34.0	30.7	27.7	29.5	29.9	34.0	32.8	32.9
1984	34.3	34.0	30.5	27.7	30.4	30.8	34.3	34.9	33.9
1994	34.8	34.2	30.9	27.6	31.1	31.2	34.7	35.1	34.1
2004	35.6	34.9	32.0	28.1	32.0	31.9	35.6	34.5	34.5
2014	36.2	35.5	32.9	28.4	32.6	32.3	36.5	33.8	34.8
Long-run	37.0	36.3	34.1	29.0	33.2	33.0	37.7	32.7	35.1
					Percentage Older than 64				
1974	9.0	9.4	6.8	6.3	6.4	7.1	10.2	12.0	10.0
1984	10.9	11.0	7.1	7.0	7.5	8.6	10.9	16.3	12.2
1994	10.0	10.1	6.8	6.1	7.2	8.0	10.0	15.5	11.1
2004	11.5	10.9	8.0	6.9	8.8	9.8	11.5	16.7	12.2
2014	11.6	11.2	8.6	7.0	9.2	9.8	12.2	15.4	11.9
Long-run	13.8	13.4	11.2	8.9	10.8	11.6	14.8	13.1	13.0
					Dissimilarity Index w.r.t. Long-run Regional Age Profile				
1974	10.9	8.4	10.4	6.8	11.7	10.5	8.6	10.0	8.3
1984	9.9	8.7	11.3	9.0	10.0	9.1	8.6	8.9	6.6
1994	7.5	6.9	8.4	7.1	7.1	7.1	7.2	7.7	5.6
2004	5.3	5.1	5.7	4.1	4.7	4.7	5.5	6.3	4.4
2014	3.8	3.7	3.8	3.7	3.4	3.9	3.8	3.8	3.4
2024	2.8	2.7	2.6	2.2	2.3	2.5	2.6	2.6	2.5
2034	2.0	1.9	1.8	1.7	1.7	1.9	1.8	1.8	1.8

6. THE DYNAMIC PROPERTIES OF THE GREAT BRITAIN AGE-BY-REGION POPULATION SYSTEM

The growth matrix of the British age-by-region population system is shown in Appendix C. The regional schedules of the basic demographic processes are characterized in Table 6.1. As usual, the migration schedules show greater interregional differences than the fertility and survival schedules do.

The dominant eigenvalue of the system's growth matrix is $\lambda_1 = 1.0235$, which implies an annual long-run growth rate of 0.47% and a long-run doubling time of 149 years. The magnitude of λ_1 reflects the moderately high gross reproduction rate of 1.15 and a young mean age of 24.2 years of the system's fertility schedule.

Figure 6.1 shows that the overall pattern of the eigenvalues of the system's growth matrix is similar to that of the corresponding Leslie matrix, which in turn exhibits the commonly observed pattern that the three longest periods of cyclical components are about 25, 21, and 13 years and that the component with the 21-year period has a slightly smaller half-life than the component with the 13-year period (Coale 1972:70-79). The dispersion of eigenvalues in each cluster is significantly less than that of the corresponding cluster in the Soviet model because of Great Britain's relatively low level of migration.

The relative importance of the dominant and spatial components is shown in Table 6.2. The stable equivalent population of 27,442,000, being slightly *smaller* than the 1970 observed population size of 27,859,000, implies a growth momentum of -1.5%. This rarely-observed negative momentum of growth is a reflection of the interesting fact that the 1970 fertility level in Great Britain was higher than not only those of many contemporary European countries but also its own average level in the five immediately preceding decades (Rees 1979:14). As a consequence of the negative momentum, the average annual growth rate in 1970-1990 (0.36%) is found to be smaller than the corresponding long-run value (0.47%).

Table 6.1 The gross rates, mean ages, and standard deviations of the regional fertility, survival, and migration schedules of the Rogers model of the 1970 female population of Great Britain.

	North	E.Midland	W.Midland	E.Anglia	S.E.	S.W.	Wales	Scotland
				Fertility Schedule				
Gross Rate	1.1978	1.1751	1.1977	1.1196	1.0875	1.1261	1.1519	1.1991
Mean Age	24.04	23.98	24.36	23.88	24.42	24.00	23.95	24.51
Stand. Dev.	6.37	6.25	6.47	6.14	6.28	6.18	6.29	6.36
				Survival Schedule				
Gross Rate	15.7100	15.8448	15.7949	16.0570	15.9916	15.9209	15.7328	15.6334
Mean Age	40.10	40.39	40.27	40.87	40.72	40.54	40.15	39.96
Stand. Dev.	23.67	23.80	23.73	24.03	23.96	23.87	23.69	23.62
Gross Death Rate	2.2900	2.1552	2.2051	1.9430	2.0084	2.0791	2.2672	2.3666
				Interregional Migration Schedule				
Gross Rate	0.7844	1.3763	1.0517	1.5825	0.9358	1.5740	1.0707	0.7447
Mean Age	30.94	30.48	30.19	30.25	33.55	31.17	32.14	27.55
Stand. Dev.	21.51	21.51	20.28	21.36	22.44	21.56	22.70	19.29

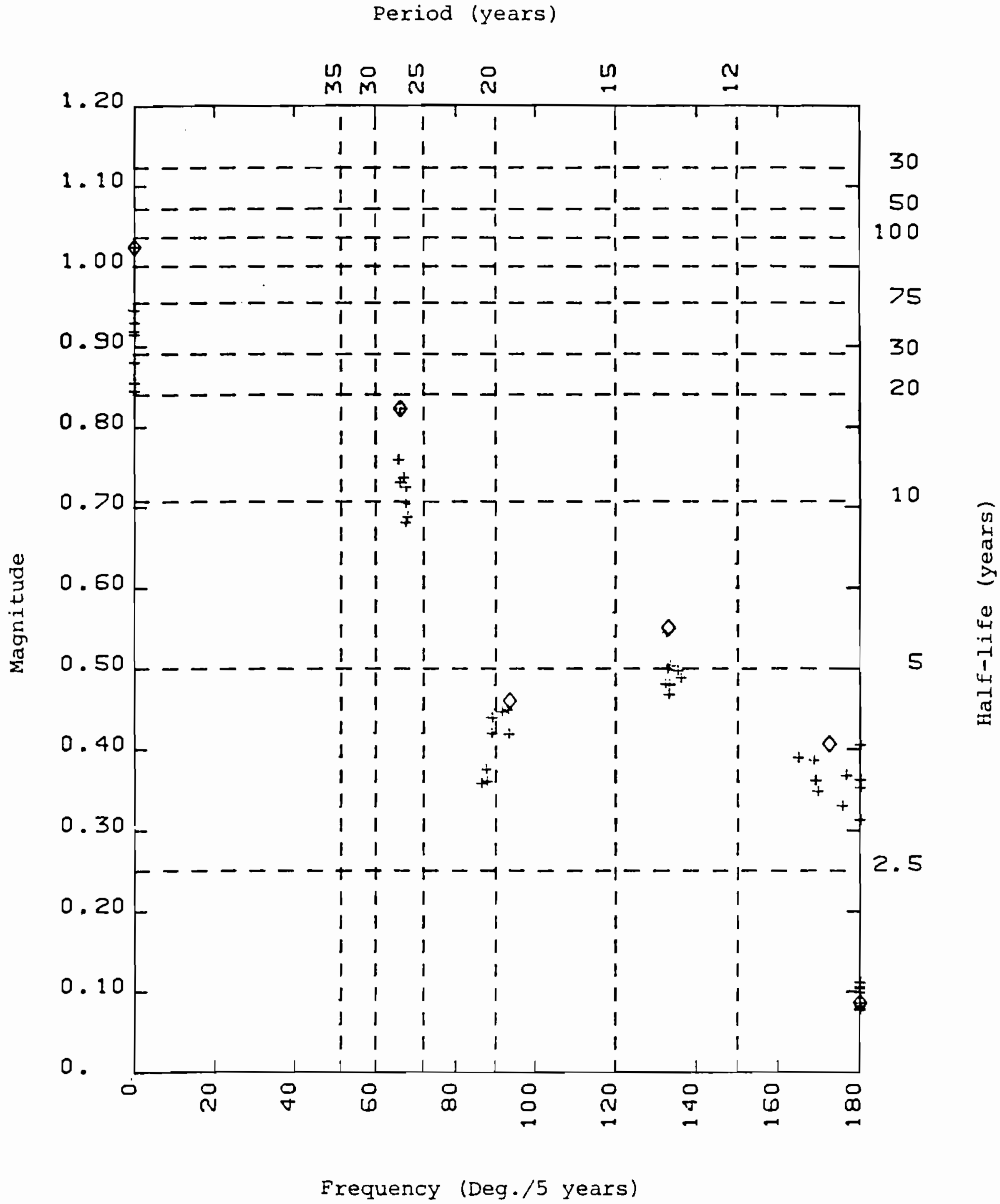

Figure 6.1 The pattern of non-zero eigenvalues of the growth matrix of the Rogers model of the 1970 British female population. The eigenvalues of the corresponding Leslie model are shown as diamonds.

Table 6.2 The doubling time, half-lives, initial levels, and initial regional loadings on the 0-4 age group of the dominant components of the Rogers model of the 1970 British female population.

DT & HL* (t_i)	Initial Level (C_i)	North	E.Midland	W.Midland	E.Anglia	S.E.	S.W.	Wales	Scotland
		Initial Regional Loadings ($C_i Q_{i,j}$)							
148.96	27,442,036	561,588	163,795	219,910	91,130	650,670	174,715	99,114	175,172
61.28	1,176,560	-10,821	-3,375	-9,304	-2,022	-9,163	-3,554	-5,293	41,024
47.61	1,222,139	38,789	-2,188	-10,450	-2,840	-21,792	-6,029	-1,170	1,324
41.04	535,297	-549	5	10,970	-2,222	-13,644	-2,237	3,458	2,286
39.08	292,035	828	-2,144	-5,243	-696	-1,881	110	8,863	223
27.28	819,192	8,248	-21,092	4,808	-3,147	10,405	2,220	-1,161	283
22.10	104,677	147	-792	257	2,442	85	-2,255	103	33
20.56	1,469,073	638	1,488	2,109	-19,533	36,588	-21,230	514	302

*The doubling time refers only to the dominant component.

The fact that the initial levels (c_i) of the spatial components are much smaller than the value of c_1 indicates that the initial and the long-run interregional distributions are quite similar. In other words, similar to the situation in Sweden, the initial spatial distribution in Great Britain is rather compatible with the basic demographic processes. From the pattern of initial loadings in Table 6.2, we see clearly that East Midland, East Anglia, and South West will be the gaining regions (because of the large negative loadings), whereas Scotland will be a losing region (because of the positive loadings). Being loaded positively by spatial components of short half-lives and negatively by spatial components of long half-lives, the South East will have its share decreased first and then increased. These inferences about spatial redistribution are supported by the time-paths of the superimposed spatial component in Figure 6.2 and the projected time-paths of regional shares in Figure 6.3.

The major cyclical components have periods of about 27 years and half-lives of between 17.79 and 9.04 years. The relative rates of damping of these components range between 4.3% and 7.9% per year. This suggests that population waves will be damped more rapidly in Great Britain than in Sweden and the Soviet Union. Since it is shown in Table 6.3 that the regional upperbounds of the dominant cyclical component are in general greater than those of the other major cyclical components, the regional population waves are likely to be more synchronical than opposite. The superimposed major cyclical component in Figure 6.4 further suggests that the more or less synchronical population waves moving through the youngest age group in every region will have a trough between 1975 and 1980 and a peak between 1990 and 1995. However, since loadings of the major cyclical components are much smaller than those of the dominant component, the new population waves that will be transmitted through the first age group to the older ones are of little importance in most regions. The decomposition of the projected regional time-paths of the 0-4 age group in Figure 6.5 shows how population waves fluctuate around the smooth trend which

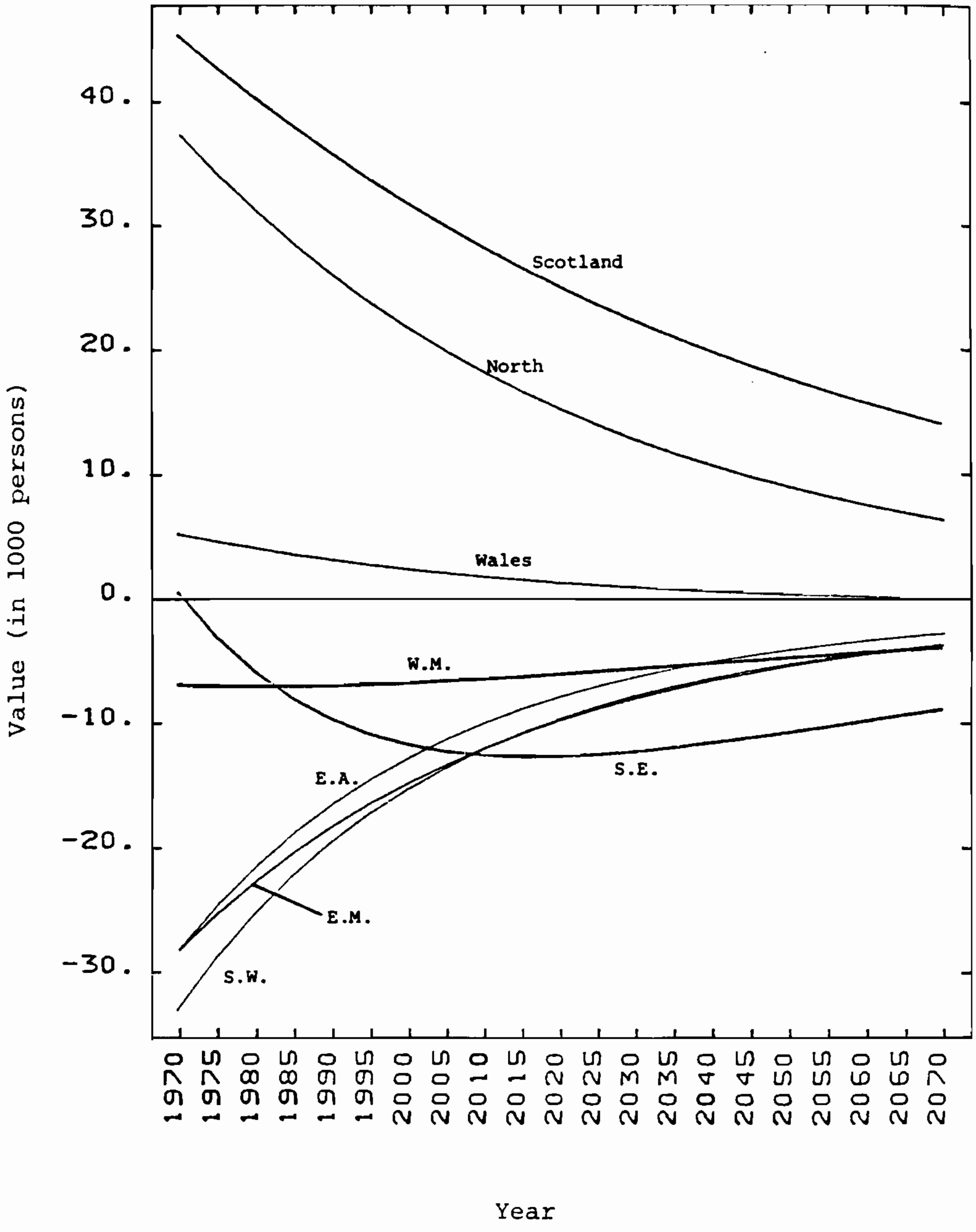

Figure 6.2 The regional time-paths of the superimposed spatial component for the 0-4 age group of the Rogers model of the 1970 British female population.

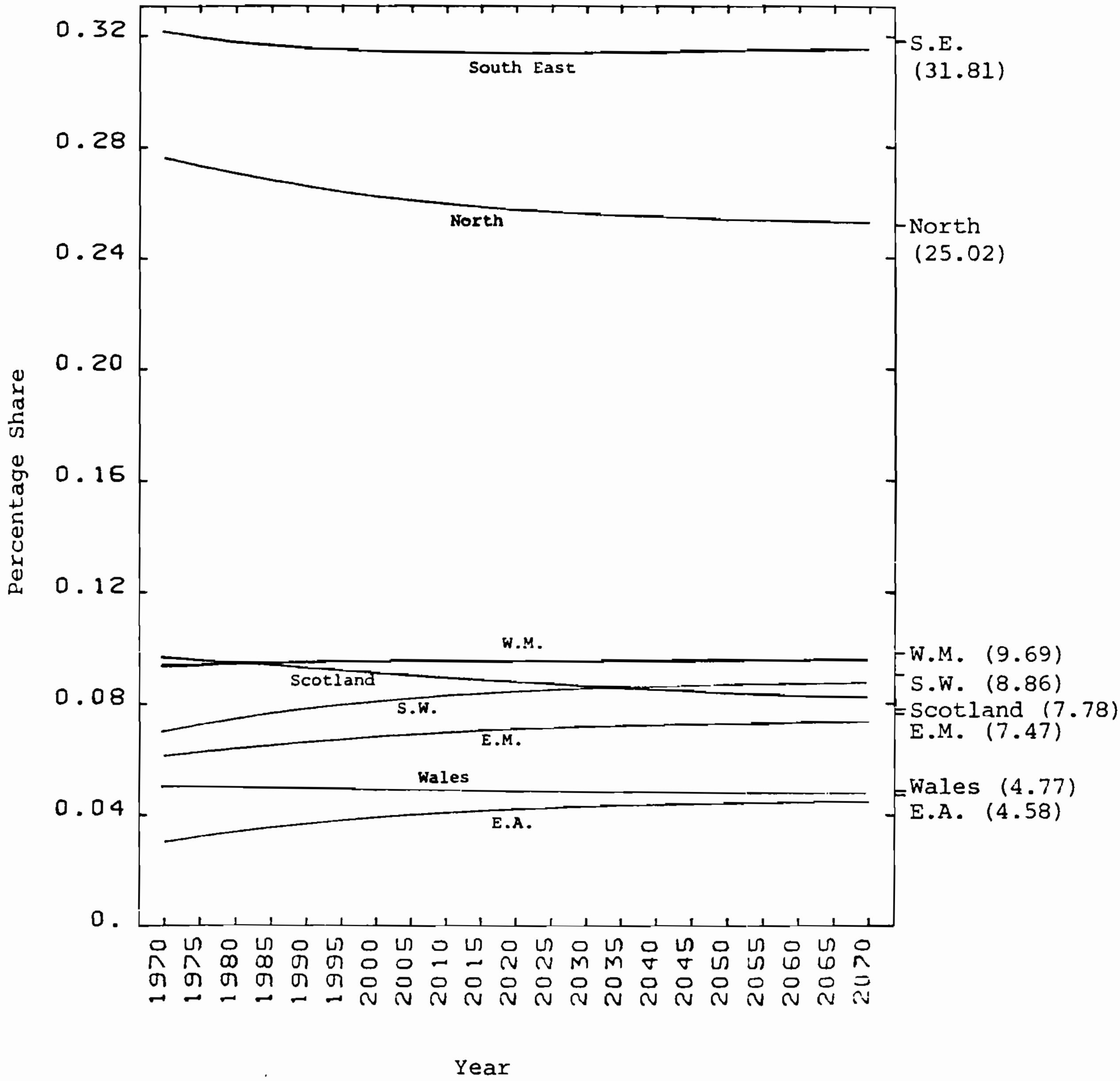

Figure 6.3 The projected regional shares of the Rogers model of the 1970 British female population. The long-run regional shares are marked on the right axis.

Table 6.3 The half-lives, periods, and regional information about the 0-4 age group of the major cyclical components of the Rogers model of the 1970 female population of Great Britain.

Half-life (t_i)	Period (P_i)	North	E.Midland	W.Midland	E.Anglia	S.E.	S.W.	Wales	Scotland
				Initial Regional Upperbounds ($2d_i\gamma_{iaj}$)					
17.79	27.25	16,727	6,136	7,229	3,708	27,810	6,925	3,201	6,647
12.56	27.42	1,386	966	1,673	780	4,690	1,248	754	9,632
11.33	26.90	8,881	782	1,943	710	5,684	1,325	892	337
11.05	27.29	903	370	2,334	269	2,064	246	256	154
10.77	26.69	957	636	572	369	980	256	2,665	74
9.89	26.71	985	1,748	432	380	818	246	186	20
9.27	26.53	163	720	162	1,919	503	1,138	68	14
9.04	26.71	129	226	222	954	2,299	1,862	88	39
				Initial Regional Phase Angles ($e_i + \phi_{iaj}$)					
17.79	27.25	67.66	65.50	53.84	67.70	49.86	64.42	66.91	47.41
12.56	27.42	-37.74	-46.04	-58.87	-41.10	-72.49	-47.40	-26.26	129.78
11.33	26.90	82.00	-1.34	-55.20	-76.08	-113.34	-76.47	48.77	-138.52
11.05	27.29	21.06	-72.95	-101.94	129.97	99.59	143.45	-26.61	-84.90
10.77	26.69	-103.29	-18.20	14.92	-11.88	-37.82	96.29	140.51	-170.04
9.89	26.71	8.90	-158.96	41.84	-145.54	32.02	93.74	-137.54	26.16
9.27	26.53	-98.71	73.76	-87.73	-110.68	109.25	58.31	-123.96	-52.01
9.04	26.71	-59.71	-105.20	0.34	149.25	-1.20	171.16	-52.05	-42.92
				Initial Regional Loadings [$2d_i\gamma_{iaj}\cos(e_i + \phi_{iaj})$]					
17.79	27.25	6,357	2,545	4,265	1,407	17,927	2,991	1,255	4,498
12.56	27.42	1,096	671	865	588	1,411	845	676	-6,163
11.33	26.90	1,237	782	-1,248	-696	-7,009	-1,320	1,065	-371
11.05	27.29	842	108	-483	-173	-344	-198	229	14
10.77	26.69	-220	604	553	361	774	-28	-2,057	-72
9.89	26.71	973	-1,631	322	-313	694	-16	-137	18
9.27	26.53	-25	201	6	-678	-166	598	-38	8
9.04	26.71	65	-59	223	-820	2,299	-1,840	54	28

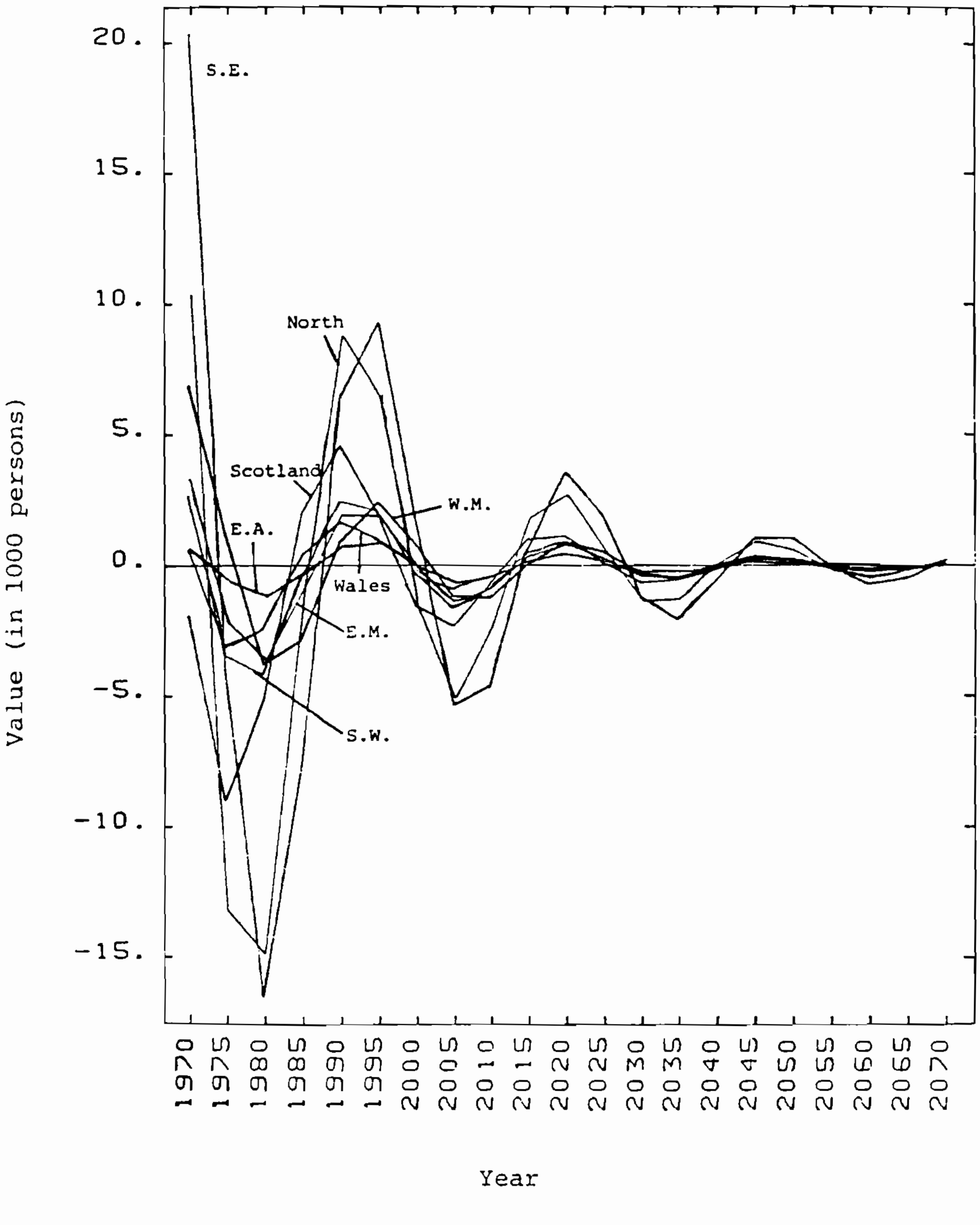

Figure 6.4 The regional time-paths of the superimposed major cyclical component for the 0-4 age group of the Rogers model of the 1970 British female population.

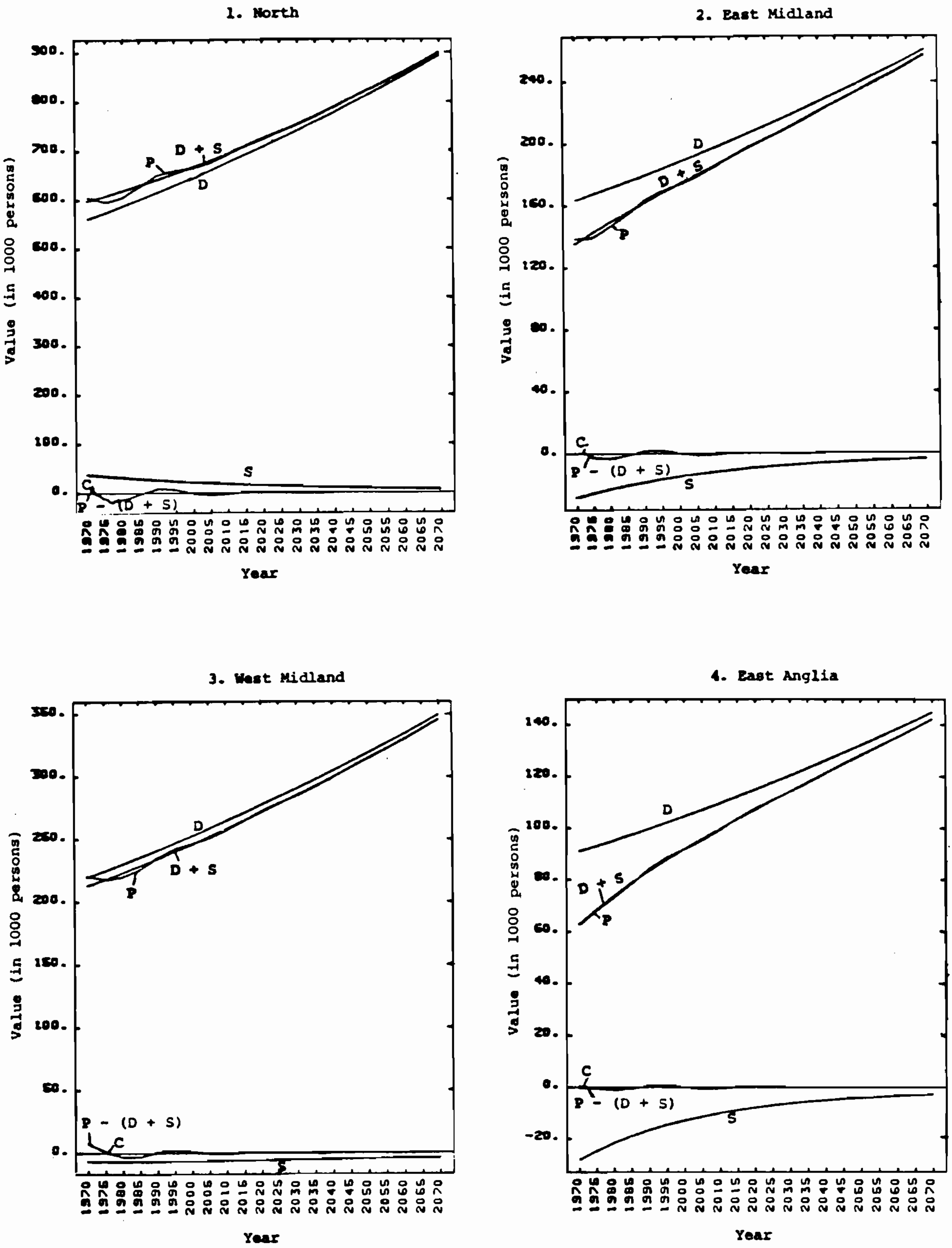

Figure 6.5 The analytic decomposition of the projected regional population sizes in the 0-4 age group of the Rogers model of the 1970 British female population. P = projected population size; D = dominant component; S = superimposed spatial component; C = superimposed major cyclical component.

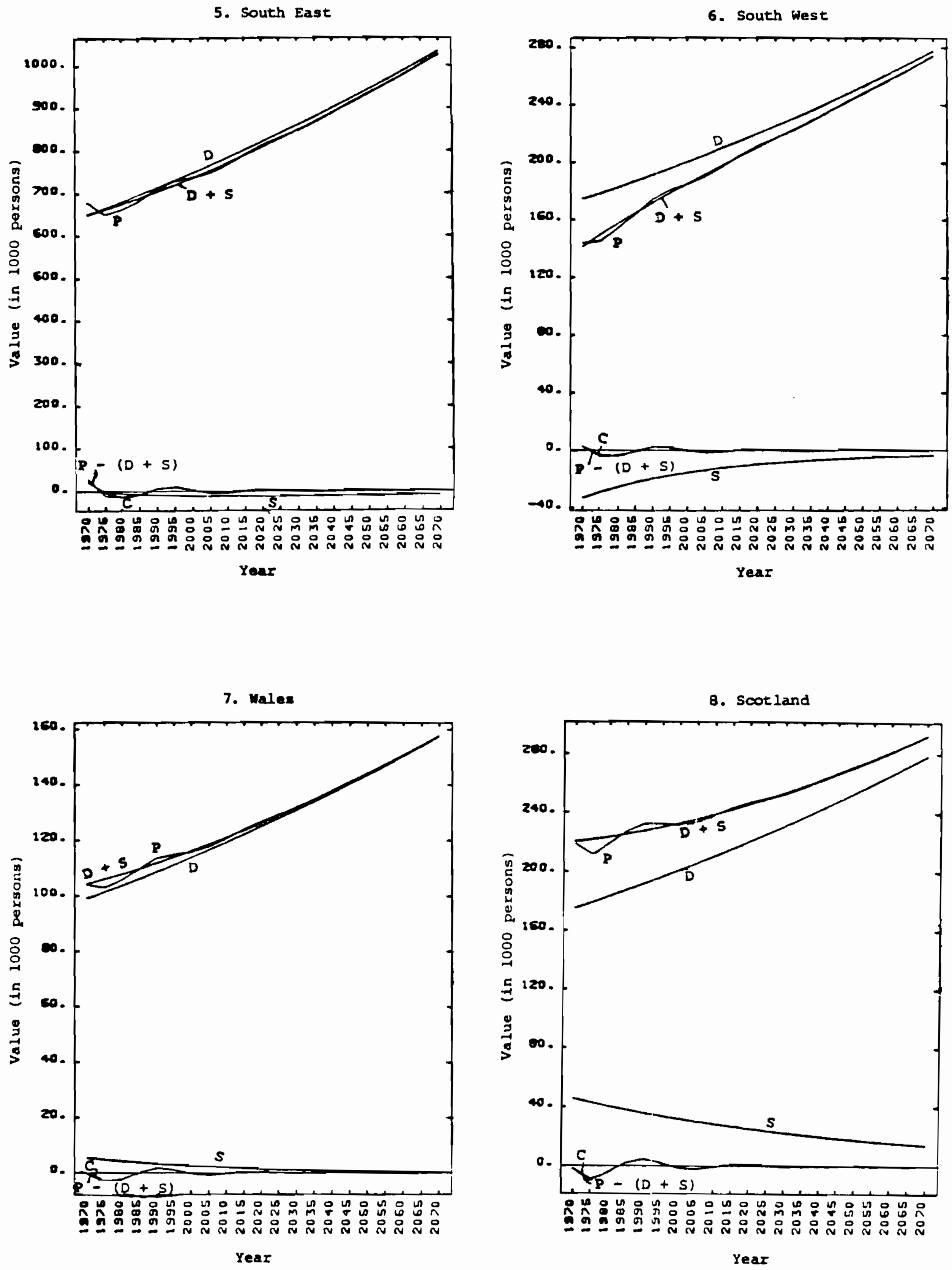

Figure 6.5 Continued.

is the sum of the dominant and the spatial components. The actual fluctuations are very well approximated by the superimposed major cyclical components.

Figure 6.6 shows that the initial and the long-run regional age profiles are quite similar in the overall slopes. Therefore, the convergence toward the long-run regional age profiles involves mainly the smoothing-out of the initial irregularities, which for practical purposes is achieved in 50 years. The regional mean ages and percentages older than 64 in Table 6.4 show that the clearly ageing trends we saw in the Swedish and Soviet populations do not exist in the British population. The reduction in the dissimilarity indices reflects mainly the quick disappearance of the major irregularities in the initial regional age profiles.

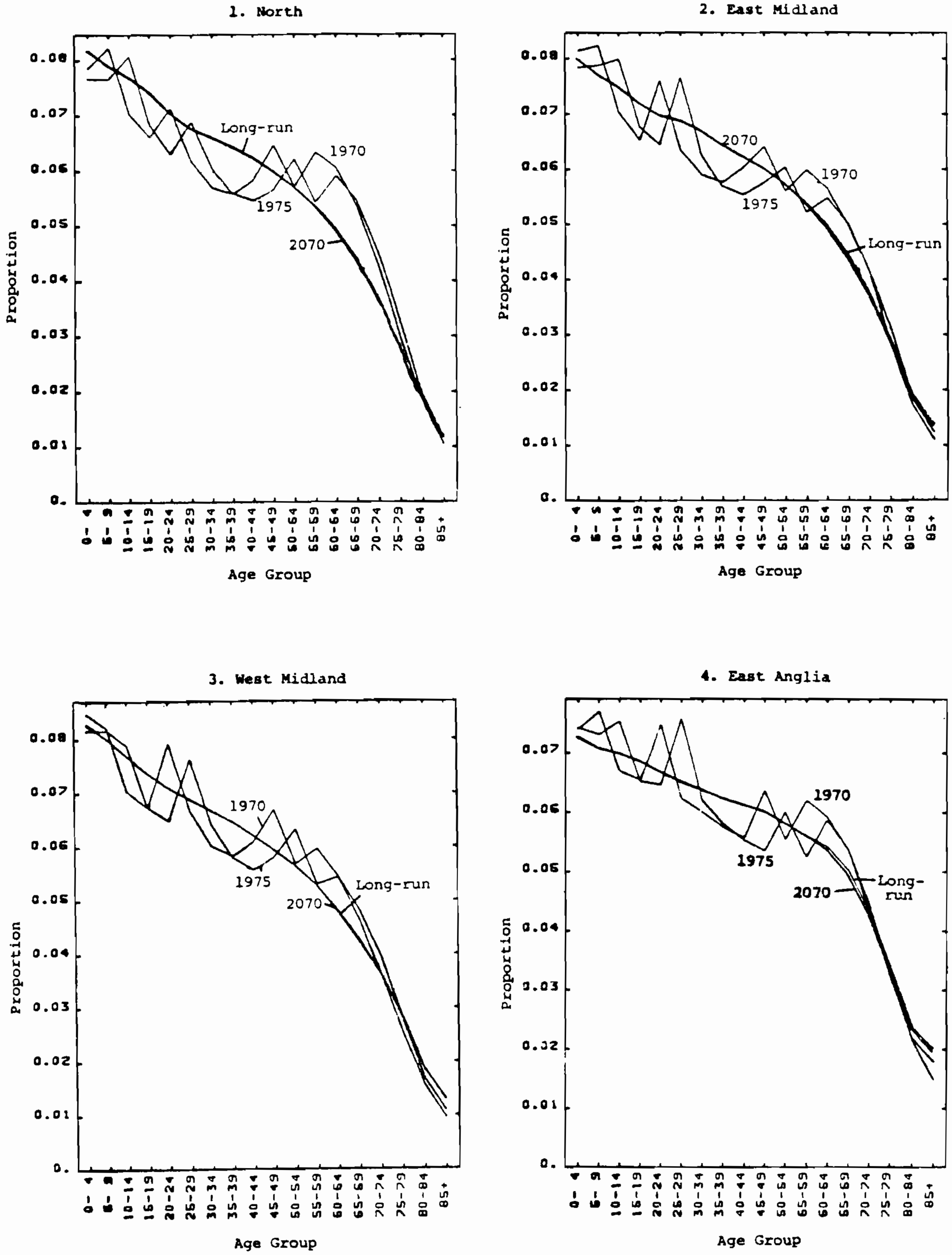

Figure 6.6 The initial, projected, and long-run regional age profiles of the Rogers model of the 1970 British female population.

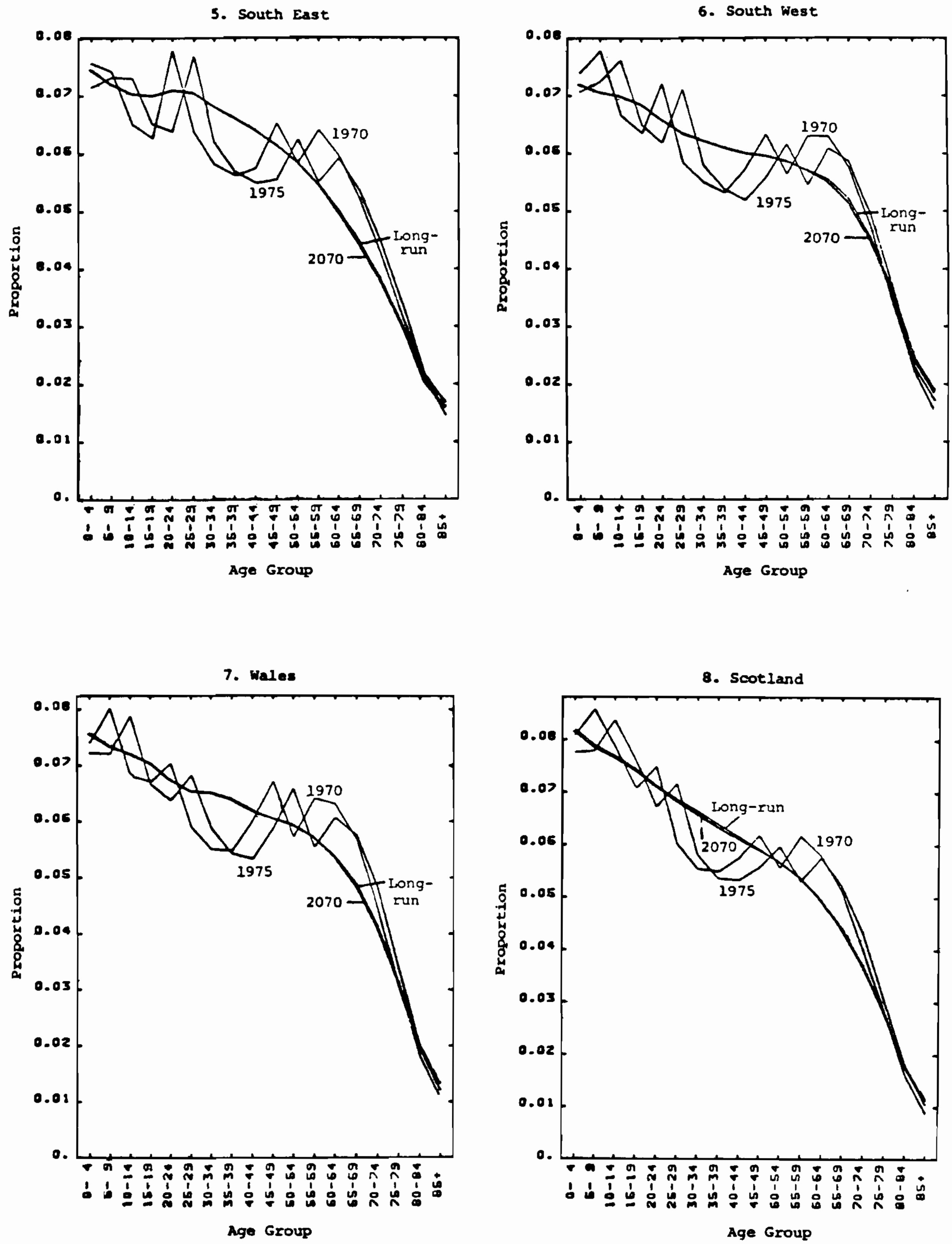

Figure 6.6 Continued.

Table 6.4 Changing characteristics of regional age profiles of the 1970 British female population projected by the Rogers model.

Year	North	E.Midland	W.Midland	E.Anglia	S.E.	S.W.	Wales	Scotland	G.B.
				Mean Age					
1970	37.4	36.8	36.1	38.2	38.2	38.8	38.0	36.4	37.6
1980	37.4	36.7	36.4	37.9	38.2	38.9	38.3	36.7	37.6
1990	36.7	36.2	36.3	37.5	37.6	38.5	37.9	36.3	37.1
2000	36.1	35.8	35.8	37.3	37.0	38.0	37.3	35.8	36.6
2010	35.9	35.8	35.7	37.4	36.9	38.0	37.2	35.8	36.5
Long-run	36.0	36.5	35.9	38.5	37.2	38.8	37.4	35.9	36.8
				Percentage Older than 64					
1970	15.4	14.8	13.3	16.6	16.1	17.6	16.0	14.4	15.5
1980	16.8	15.7	15.1	17.4	17.5	18.9	17.6	16.1	16.9
1990	16.0	15.0	15.3	16.8	16.8	18.4	17.2	15.5	16.4
2000	14.3	13.6	14.0	15.3	14.9	16.7	15.6	14.0	14.7
2010	13.3	12.8	13.2	14.8	13.9	15.9	14.5	13.0	13.8
Long-run	13.8	14.4	13.8	17.2	14.9	17.7	15.4	13.8	14.8
				Dissimilarity Index					
1970	4.7	3.9	3.8	3.4	4.8	4.1	4.4	4.2	4.2
1980	4.4	3.1	3.2	3.0	4.3	3.3	3.9	3.8	3.7
1990	3.1	2.9	2.6	3.1	3.4	3.1	3.0	2.9	3.0
2000	2.1	2.6	2.1	3.4	2.5	2.9	2.6	2.1	2.3
2010	1.5	2.2	1.6	2.9	1.9	2.4	1.8	1.6	1.8
2020	1.1	1.6	1.2	2.0	1.4	1.8	1.3	1.1	1.3

7. SUMMARY AND CONCLUSION

We started with the belief (1) that it is useful to project into future a population which is subject to a set of time-invariant age- and region-specific rates of birth, death, and interregional migration, and (2) that the dynamic behavior of the projected age-by-region population can be better understood by studying the analytic solution of the Rogers model, which involves the eigenvalues and eigenvectors of the system's growth matrix as well as the initial age-by-region population distribution.

We then used a Leslie model and a factorizable Rogers model to illuminate some salient features of the analytic solution of the general (non-factorizable) Rogers model. Based on the Swedish data, we have shown that it is analytically useful to realize that an actual Rogers model is close to being factorizable. This realization made it easy to understand the characteristics of the dominant, spatial, and major cyclical components. After showing that the factorizable Rogers model is a simple generalization of the Leslie model, most of the findings in the literature about the dynamics of age-disaggregated single-region populations were synthesized with the well-known properties of an irreducible stochastic matrix for studying the dynamics of empirical age-by-region population systems. For example, we now understand why the sum, across all regions, of the elements of a spatial component is always close to zero, and why the dominant cyclical component represents nearly synchronical population waves, whereas the remaining major cyclical components represent opposite population waves.

The three empirical population systems turned out to be similar in several respects. First, the Swedish and British female populations as well as the *urban* part of the Soviet population have initial spatial distributions that are quite compatible with the current basic demographic processes. In other words, the spatial redistribution potential is rather weak in all three cases. This is in sharp contrast to the contemporary Canadian age-by-region population system which has a strong westward shifting potential. Second, the projected age profiles of the capital regions in all three countries approach the long-run profiles which have a significant surplus of young adults. This is the long-run implication of the different migration schedules that have previously been classified into the capital-inflow and capital-outflow types by Andrei Rogers. However, we hasten to add that it is usually impossible to predict the shapes of the long-run age profiles by comparing a set of regional migration schedules, because the *spatial* pattern of migration flows are generally very complicated, unless there are only two or three regions in the system. Third, all regional population waves moving through the 0-4 age group are nearly synchronical *within each country*, because of the importance of the dominant cyclical component in representing the population waves. This importance derives from the fact that the impacts on the observed age profiles of major changes in fertility and mortality in the preceding decades were truly nationwide.

Each population system also has its distinct characteristics. Being most "mature" in terms of the theory of demographic transitions, the Swedish female population system, which already had very old regional age profiles, continues to age in such a way that every region would eventually have more than 20% of the population older than 64 years. American and Canadian officials who have been increasingly concerned with economic and social problems of ageing would probably benefit a lot by watching Sweden carefully. The projected population waves moving through the youngest age group in the Swedish regions are exceptional both in their long periods and in their small rates of damping. With the rural part of the whole nation considered as a region, the Soviet population system shows the unusual fact that spatial convergence proceeds faster than the convergence in regional age profiles. It is possible that by the end of the century, the Soviet Union could be as urbanized as the present industrialized countries in the West. The rich varieties of long-run age profiles of the Soviet regions reveal that the Soviet population is not only very heterogeneous but also lagging far behind the Swedish and British populations in the evolution of the basic demographic processes. The joint effect of intrinsic growth, urbanization, and population waves is expected to result in an extremely high growth rate in the 0-4 age group of every urban region in the Soviet Union during the fifteen-year period between 1979 and 1984. A unique feature of the British system is that due to relatively high levels of regional fertility schedules, there is a lack of the trend toward older regional age profiles. Since the 1970 high fertility levels are not likely to last, this feature is only of passing interest. A more interesting characteristic of the British system is that the projected regional population waves moving through the youngest age group are very small and will be damped rapidly.

Finally, let us face the question: Could equally informative results about regional population dynamics be obtained by more simplistic models such as a set of single-region Leslie models or a spatial interaction model without age-disaggregation? The answer, I would say, is no. The major shortcomings of single-region Leslie models are (1) that these models usually lead to the *relative* regional growth which is contrary to the observed demographic developments in most industrialized countries, because relative regional growth in these countries has been determined mainly by the spatial imbalance in the migration process but has little to do with the interregional difference in fertility and mortality levels; and (2) that these models do not allow regional age profiles to approach undulated long-run profiles which reflect the age-selectivity of migration. A spatial interaction model without age-disaggregation ignores, among other things, the important demographic feature of population waves which are undermining the financial foundation of higher education and scholarly research in countries like Canada and the United States.

APPENDIX A: INPUT DATA OF THE ROGERS MODEL OF THE 1974 SWEDISH FEMALE POPULATION

A.1 Initial Population (in persons)

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
0-4	56749.	53199.	28376.	42021.	60930.	27911.	13246.	18343.
5-9	51015.	52027.	27485.	40517.	57595.	29025.	13603.	18405.
10-14	46053.	46538.	25303.	36922.	51924.	27749.	12710.	17609.
15-19	44810.	44881.	25619.	36014.	51108.	27911.	13100.	18181.
20-24	56064.	48960.	25363.	40003.	55998.	26831.	12236.	17215.
25-29	71893.	57725.	27387.	45715.	63932.	29568.	13487.	18426.
30-34	58211.	47702.	23321.	39232.	52383.	25359.	12016.	16052.
35-39	43329.	38044.	20600.	32585.	44102.	22573.	10659.	14620.
40-44	40308.	36706.	20723.	31697.	43472.	23255.	11132.	14450.
45-49	44404.	39758.	22015.	34344.	46273.	25381.	12094.	14808.
50-54	51817.	45343.	24412.	37490.	51997.	29107.	13562.	15314.
55-59	45810.	41435.	22586.	34757.	47987.	27023.	12629.	14106.
60-64	45551.	41100.	23322.	35763.	48249.	27854.	12895.	13619.
65-59	39868.	37156.	21695.	33171.	43514.	26333.	12018.	11816.
70-74	31139.	31181.	18173.	27203.	36060.	21756.	10161.	9573.
75-79	22509.	22152.	13361.	20049.	26633.	15688.	7222.	6338.
80-84	14353.	13588.	7962.	12656.	16217.	8974.	4264.	3627.
85+	8797.	7851.	4843.	8734.	10060.	5028.	2389.	1966.

A.2 The B-submatrices

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
3	0.0000966	0.0000024	0.0000011	0.0000013	0.0000009	0.0000012	0.0000015	0.0000009
3	0.0000025	0.0000001	0.0000000	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000006	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000013	0.0000006	0.0000012	0.0000617	0.0000007	0.0000002	0.0000003	0.0000002
3	0.0000008	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000012	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000007	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000006	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0204392	0.0013669	0.0006921	0.0004367	0.0004139	0.0011209	0.0013610	0.0013842
4	0.0014769	0.0279601	0.0009767	0.0004377	0.0005483	0.0016647	0.0009745	0.0009875
4	0.0003651	0.0005142	0.0231259	0.0006208	0.0004207	0.0002511	0.0002097	0.0002300
4	0.0004920	0.0005231	0.0011250	0.0262194	0.0005075	0.0002672	0.0003122	0.0002588
4	0.0004978	0.0007957	0.0011627	0.0006191	0.0253205	0.0009363	0.0006919	0.0004760
4	0.0007928	0.0011197	0.0002634	0.0002053	0.0004320	0.0309108	0.0006886	0.0004711
4	0.0003930	0.0002714	0.0000800	0.0000712	0.0001088	0.0003354	0.0216723	0.0004199
4	0.0003218	0.0003424	0.0001402	0.0000940	0.0001615	0.0002943	0.0006504	0.0221984

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
5	0.1133603	0.0105488	0.0057331	0.0039022	0.0034992	0.0094143	0.0127436	0.0116883
5	0.0099943	0.1457162	0.0084873	0.0037317	0.0044353	0.0123167	0.0086600	0.0082348
5	0.0026793	0.0039762	0.1323162	0.0051381	0.0034850	0.0017907	0.0016408	0.0017378
5	0.0031698	0.0037948	0.0094360	0.1445870	0.0042194	0.0020505	0.0027053	0.0022870
5	0.0036848	0.0064205	0.0101506	0.0057752	0.1446851	0.0077325	0.0053456	0.0044946
5	0.0048812	0.0060947	0.0021938	0.0015668	0.0029477	0.1446248	0.0051988	0.0035729
5	0.0027558	0.0022079	0.0009181	0.0006316	0.0009938	0.0026666	0.1322414	0.0042175
5	0.0030200	0.0026939	0.0012522	0.0009313	0.0013821	0.0023563	0.0063553	0.1360769
6	0.2167756	0.0152854	0.0081646	0.0066469	0.0056374	0.0114057	0.0154385	0.0116533
6	0.0151389	0.2580289	0.0116136	0.0057250	0.0066356	0.0153261	0.0112650	0.0104585
6	0.0039514	0.0060442	0.2579397	0.0079249	0.0053971	0.0026231	0.0021808	0.0025929
6	0.0046620	0.0056872	0.0130298	0.2638573	0.0063866	0.0026428	0.0037104	0.0030197
6	0.0056666	0.0098493	0.0156486	0.0092930	0.2706211	0.0106893	0.0062571	0.0063187
6	0.0073359	0.0095751	0.0030740	0.0024450	0.0042627	0.2484203	0.0070625	0.0051493
6	0.0040764	0.0034198	0.0013545	0.0009283	0.0015666	0.0037275	0.2510986	0.0064368
6	0.0046486	0.0045319	0.0019999	0.0016077	0.0022611	0.0037620	0.0101033	0.2691848
7	0.2074633	0.0080950	0.0042813	0.0039393	0.0032724	0.0044973	0.0057533	0.0042693
7	0.0097816	0.2182447	0.0058616	0.0030420	0.0037123	0.0068885	0.0049523	0.0050880
7	0.0024808	0.0033361	0.2393828	0.0044230	0.0030418	0.0014464	0.0010542	0.0016367
7	0.0030772	0.0031144	0.0063367	0.2353026	0.0034904	0.0013049	0.0017744	0.0013772

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
7	0.0033486	0.0050343	0.0080279	0.0347481	0.2308583	0.0045219	0.0027261	0.0028463
7	0.0047895	0.0057143	0.0016541	0.0014752	0.0025369	0.2091400	0.0035493	0.0030979
7	0.0026871	0.0020840	0.0006781	0.0005554	0.0009539	0.0020647	0.2166800	0.0033781
7	0.0025836	0.0026696	0.0011659	0.0009600	0.0013887	0.0022785	0.0055867	0.2418923
8	0.1107420	0.0026938	0.0014564	0.0013260	0.0011355	0.0015014	0.0020093	0.0014246
8	0.0039821	0.1038670	0.0020581	0.0010741	0.0013466	0.0024991	0.0018726	0.0017971
8	0.0009635	0.0011545	0.1172759	0.0015551	0.0011041	0.0005154	0.0004091	0.0006527
8	0.0012363	0.0011323	0.0022927	0.1183723	0.0012648	0.0004738	0.0006769	0.0014663
8	0.0013065	0.0018193	0.0029074	0.0016984	0.1173244	0.0016761	0.0010636	0.0010278
8	0.0019559	0.0021600	0.0006448	0.0005430	0.0009495	0.1068144	0.0014970	0.0012198
8	0.0011227	0.0007761	0.0002571	0.0002050	0.0003769	0.0008325	0.1156535	0.0012384
8	0.0010298	0.0009697	0.0004461	0.0003093	0.0005214	0.0009190	0.0020466	0.1252627
9	0.0335143	0.0006706	0.0004001	0.0003186	0.0003039	0.0003963	0.0005495	0.0003940
9	0.0010633	0.0309822	0.0005679	0.0002810	0.0003765	0.0006976	0.0005400	0.0005206
9	0.0002762	0.0003176	0.0375650	0.0004184	0.0003260	0.0001612	0.0001351	0.0002029
9	0.0003023	0.0002837	0.0006011	0.0350364	0.0003251	0.0001162	0.0001895	0.0001251
9	0.0003339	0.0004614	0.0008163	0.0004175	0.0384571	0.0004569	0.0003053	0.0002904
9	0.0005069	0.0005529	0.0001696	0.0001345	0.0002622	0.0338825	0.0004045	0.0003501
9	0.0002881	0.0001876	0.0000713	0.0000443	0.0000924	0.0002110	0.0388498	0.0003316
9	0.0002579	0.0002493	0.0001112	0.0000703	0.0001388	0.0002307	0.0005565	0.0417442

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
10	0.0059017	0.0001153	0.0000955	0.0000407	0.0000524	0.0000543	0.0000662	0.0000616
10	0.0001721	0.0060029	0.0001402	0.0000368	0.0000664	0.0000977	0.0000657	0.0000829
10	0.0000389	0.0000507	0.0101177	0.0000519	0.0000525	0.0000200	0.0000131	0.0000310
10	0.0000458	0.0000472	0.0001391	0.0050413	0.0000551	0.0000152	0.0000229	0.0000193
10	0.0000523	0.0000800	0.0001992	0.0000538	0.0073604	0.0000629	0.0000371	0.0000464
10	0.0000609	0.0000948	0.0000408	0.0000176	0.0000451	0.0051523	0.0000463	0.0000551
10	0.0000444	0.0000320	0.0000182	0.0000053	0.0000160	0.0000281	0.0050530	0.0000492
10	0.0000407	0.0000428	0.0000271	0.0000096	0.0000246	0.0000323	0.0000680	0.0072160
11	0.0003642	0.0000075	0.0000029	0.0000011	0.0000043	0.0000009	0.	0.0000039
11	0.0000102	0.0004080	0.0000043	0.0000010	0.0000054	0.0000017	0.	0.0000053
11	0.0000023	0.0000033	0.0003186	0.0000014	0.0000043	0.0000003	0.	0.0000020
11	0.0000028	0.0000031	0.0000043	0.0001393	0.0000046	0.0000003	0.	0.0000013
11	0.0000030	0.0000051	0.0000061	0.0000014	0.0006198	0.0000011	0.	0.0000030
11	0.0000049	0.0000064	0.0000013	0.0000005	0.0000038	0.0000928	0.	0.0000037
11	0.0000027	0.0000022	0.0000006	0.0000002	0.0000013	0.0000005	0.	0.0000033
11	0.0000024	0.0000028	0.0000008	0.0000002	0.0000020	0.0000006	0.	0.0004822

A.3 The S-submatrices

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
1	0.86462456	0.03123900	0.01503998	0.01250798	0.01169449	0.01602869	0.02138704	0.01333628
1	0.04745152	0.87428534	0.02337867	0.01101816	0.01442395	0.03081500	0.02061964	0.01853055
1	0.01142278	0.01366660	0.86858318	0.01793507	0.01171895	0.00608477	0.00369524	0.00614059
1	0.01254992	0.01260139	0.02364333	0.92764475	0.01312439	0.00472194	0.00666299	0.00430892
1	0.01362664	0.01910409	0.03919014	0.01688767	0.92441130	0.01889841	0.01062661	0.01026992
1	0.02239566	0.02430711	0.00700099	0.00544723	0.01020214	0.90140903	0.01509106	0.01114955
1	0.01233829	0.00857618	0.00234872	0.04180303	0.00407398	0.00784313	0.89350122	0.01271265
1	0.01096325	0.01108593	0.00371656	0.00281232	0.00526240	0.00931928	0.02167331	0.91919696
2	0.90339613	0.02361430	0.01118439	0.00810108	0.00848455	0.01157654	0.01441403	0.00871437
2	0.03312900	0.91504550	0.01546983	0.00729058	0.00958897	0.02318759	0.01352430	0.01242668
2	0.00914204	0.00947350	0.92569667	0.01268501	0.00803396	0.00416985	0.00322754	0.00355116
2	0.01003235	0.00943294	0.01847940	0.95271307	0.01025665	0.00376855	0.00418873	0.00349531
2	0.01053397	0.01261692	0.01981175	0.01119354	0.94938850	0.01364399	0.00968097	0.00699607
2	0.01607476	0.01609829	0.00477791	0.00326633	0.00695871	0.93316430	0.01130622	0.00548676
2	0.03963471	0.00527840	0.00083800	0.00148657	0.00309782	0.00452640	0.93044913	0.01048943
2	0.00703972	0.00724453	0.00252687	0.00198866	0.00319798	0.00473980	0.01269293	0.94733095
3	0.90760171	0.03746138	0.02035214	0.00997397	0.01311530	0.03515407	0.04755421	0.05277738
3	0.02834058	0.89509356	0.02041322	0.00764844	0.00989359	0.03462692	0.02068534	0.02252538
3	0.00934043	0.01051427	0.89459211	0.01372399	0.00913371	0.00453878	0.00525494	0.00458356
3	0.01167345	0.01061051	0.02892498	0.94821286	0.01132485	0.00573629	0.00715195	0.00653620
3	0.01134244	0.01584702	0.02621270	0.01286761	0.94602585	0.02099273	0.01942109	0.01168076
3	0.01423160	0.01913439	0.00423509	0.00295677	0.00769615	0.88812393	0.01326777	0.00779257
3	0.00923595	0.00443184	0.00113289	0.00155156	0.00192860	0.00617728	0.87074667	0.01019889
3	0.00679400	0.00544217	0.00267373	0.00158473	0.00284432	0.00353932	0.01400395	0.88230246
4	0.85158692	0.07260177	0.04005737	0.02498956	0.02227733	0.07244349	0.10175949	0.09723971
4	0.04680333	0.80102052	0.04515101	0.01783073	0.02104707	0.06609709	0.04572725	0.04434098
4	0.01358214	0.01952001	0.78056240	0.02667128	0.01759639	0.00784699	0.00818532	0.00730514
4	0.01656054	0.01309619	0.05666092	0.86310581	0.02318271	0.01087056	0.01446456	0.01316810
4	0.01931505	0.03327436	0.05609743	0.03169253	0.89020389	0.04432510	0.03166787	0.02579544
4	0.02249095	0.03147580	0.01028826	0.00667553	0.01386592	0.77505338	0.02608047	0.01588483
4	0.01258519	0.00912035	0.00398226	0.00290592	0.00429325	0.01204485	0.73829085	0.02174009
4	0.01533612	0.01196927	0.00543240	0.00417129	0.00585890	0.00915733	0.03123130	0.77241051

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
5	0.81722954	0.07700025	0.04475637	0.03295306	0.02676915	0.06420286	0.05923180	0.06839865
5	0.05903687	0.70760393	0.05180371	0.02397584	0.02678486	0.06997393	0.05023199	0.04794686
5	0.01509291	0.02333999	0.75431097	0.03225692	0.02109581	0.00921614	0.00825326	0.00793008
5	0.01846502	0.02259533	0.06033084	0.84757298	0.02757518	0.01095571	0.01523917	0.01361903
5	0.02267246	0.03979086	0.06555065	0.04037943	0.86393982	0.04759331	0.02665165	0.02749472
5	0.02999424	0.03813632	0.01248581	0.00965837	0.01770815	0.76608671	0.03179541	0.02010072
5	0.01631069	0.01322210	0.00529228	0.00390637	0.00626451	0.01546849	0.73544711	0.03108467
5	0.01904464	0.01618094	0.00724026	0.00686414	0.00797019	0.01282553	0.04147710	0.78211635
6	0.85455906	0.04709636	0.02326746	0.02414646	0.01833047	0.02731580	0.03456793	0.02594455
6	0.04813085	0.84397072	0.03109697	0.01589224	0.01879860	0.03761522	0.02549978	0.02845485
6	0.01210948	0.01623630	0.85531735	0.02236785	0.01456684	0.00666562	0.00496152	0.00619885
6	0.01594152	0.01519619	0.03367628	0.89275092	0.01867724	0.00708757	0.00824909	0.00753988
6	0.01749646	0.02524540	0.03874270	0.02656790	0.90338099	0.02368537	0.01361228	0.01454372
6	0.02348749	0.02765210	0.00745967	0.00732651	0.01280331	0.87375391	0.01659013	0.01466311
6	0.01283419	0.01023551	0.00241803	0.00321258	0.00478163	0.01044077	0.86318272	0.02000936
6	0.01225193	0.01196639	0.00525339	0.00528603	0.00614097	0.01025207	0.02944977	0.87946308
7	0.89762101	0.02718067	0.01269597	0.01416157	0.01112710	0.01492105	0.01906568	0.01371054
7	0.03333296	0.90174690	0.01742468	0.00877445	0.01146972	0.02190392	0.01467308	0.01514895
7	0.00884833	0.01024524	0.91024262	0.01311379	0.00975168	0.00380731	0.00391269	0.00413660
7	0.01252743	0.01046738	0.02276810	0.93650876	0.01282723	0.00511374	0.00484859	0.00411102
7	0.01158891	0.01570455	0.02306776	0.01566994	0.93639529	0.01407352	0.00861233	0.00752030
7	0.01479760	0.01777429	0.00537321	0.00409065	0.00779880	0.92150766	0.01483641	0.00870608
7	0.00822613	0.00619246	0.00111297	0.00216358	0.00340721	0.00711780	0.91494888	0.01254005
7	0.00773641	0.00684704	0.00362747	0.00240275	0.00363829	0.00725329	0.01507553	0.93018015
8	0.92566246	0.01795041	0.00955577	0.00848186	0.00688789	0.01039544	0.01004479	0.00944795
8	0.02166461	0.92768729	0.01020488	0.00666799	0.00647742	0.01567398	0.00802029	0.01012676
8	0.00645945	0.00635519	0.93617690	0.00719742	0.00579902	0.00349899	0.00386535	0.00271294
8	0.00924320	0.00876714	0.01838685	0.95971039	0.00809859	0.00351531	0.00264493	0.00275747
8	0.00810764	0.01063625	0.01464908	0.00965460	0.95875829	0.01045827	0.00660559	0.00462630
8	0.00965509	0.01313661	0.00335311	0.00225454	0.00538345	0.94169492	0.01008334	0.00622817
8	0.00691126	0.00416249	0.00055239	0.00131729	0.00164963	0.00504093	0.94327164	0.00911867
8	0.00463858	0.00511597	0.00168698	0.00077972	0.00174296	0.00334511	0.00741190	0.94931239
9	0.94998166	0.01404544	0.00716241	0.00514595	0.00521968	0.00859998	0.00757735	0.00810256
9	0.01655596	0.94201112	0.00849552	0.00390693	0.00418745	0.01315069	0.00419575	0.00777960
9	0.00544182	0.00495411	0.94860446	0.00588514	0.00382159	0.00297401	0.00173407	0.00217898
9	0.00632418	0.00644736	0.01295395	0.96696991	0.00549049	0.00286967	0.00170437	0.00191609
9	0.00608186	0.00824330	0.01048330	0.00546861	0.96577477	0.00727079	0.00561299	0.00307029
9	0.00744608	0.00908837	0.00266158	0.00159885	0.00389493	0.95060307	0.00604361	0.00367694
9	0.00471342	0.00260199	0.00056484	0.00091712	0.00109323	0.00299125	0.95863181	0.00709950
9	[illegible]	0.00297824	0.00112224	0.00081966	0.00087491	0.00170397	0.00472119	0.95795876

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
10	0.94397789	0.01138092	0.00515151	0.00375096	0.00413197	0.00675276	0.00630622	0.00595452
10	0.01460722	0.94932849	0.00642486	0.00292929	0.00403481	0.00979896	0.00509305	0.00631345
10	0.00416362	0.00416897	0.95647091	0.00531024	0.00320673	0.00158960	0.00133527	0.00210988
10	0.00520139	0.00492149	0.00846242	0.96399033	0.00444174	0.00237325	0.00209675	0.00146529
10	0.00482439	0.00590548	0.00692446	0.00420254	0.96529198	0.00516651	0.00468853	0.00337239
10	0.00715133	0.00698036	0.00223978	0.00136989	0.00310627	0.95525712	0.00631315	0.00433535
10	0.00336644	0.00221776	0.00067509	0.00054918	0.00077979	0.00290704	0.95280510	0.00527639
10	0.00142564	0.00160201	0.00117154	0.00074049	0.00075641	0.00127476	0.00505908	0.95529401
11	0.93906558	0.00848452	0.00364420	0.00369102	0.00253576	0.00507750	0.00758551	0.00424375
11	0.01263325	0.94754994	0.00457794	0.00241050	0.00336427	0.00772880	0.00459449	0.00642383
11	0.00313731	0.00353021	0.95401728	0.00456377	0.00283257	0.00130738	0.00093741	0.00196519
11	0.00665378	0.00414420	0.00686406	0.96185750	0.00406693	0.00141327	0.00169696	0.00134062
11	0.00465951	0.00456555	0.00615792	0.00388310	0.96206534	0.00395745	0.00295869	0.00278334
11	0.00613957	0.00666380	0.00155428	0.00114726	0.00226686	0.95398037	0.00530118	0.00391323
11	0.00276662	0.00202999	0.00021594	0.00047165	0.00048985	0.00237983	0.94777560	0.00436679
11	0.00127963	0.00132742	0.00051224	0.00053424	0.00058154	0.00120707	0.00344466	0.94883138
12	0.92300856	0.00833621	0.00288230	0.00245115	0.00277671	0.00394658	0.00821892	0.00456343
12	0.01364761	0.93455976	0.00488705	0.00223346	0.00287951	0.00662042	0.00462975	0.00647240
12	0.00395465	0.00321560	0.94287648	0.00352416	0.00261537	0.00157352	0.00040756	0.00172550
12	0.00816542	0.00362875	0.00739704	0.95719320	0.00431052	0.00106816	0.00257597	0.00090193
12	0.00461869	0.00496251	0.00714996	0.00353953	0.95177639	0.00415516	0.00248131	0.00241217
12	0.00648334	0.00662724	0.00135735	0.00076685	0.00193919	0.94425511	0.00499447	0.00210400
12	0.00285026	0.00148085	0.00021706	0.00034537	0.00040790	0.00180770	0.93796083	0.00522781
12	0.00124016	0.00152601	0.00021204	0.00021307	0.00044752	0.00106716	0.00238312	0.93845701
13	0.93622473	0.00839440	0.00358255	0.00235376	0.00276630	0.00465610	0.00806552	0.00687193
13	0.01243939	0.91503694	0.00521968	0.00222047	0.00248499	0.00710377	0.00500460	0.00505556
13	0.00429961	0.00330093	0.92258415	0.00309194	0.00267779	0.00130140	0.00058312	0.00128586
13	0.00590955	0.00273818	0.00689175	0.93430108	0.00424595	0.00130675	0.00207087	0.00074627
13	0.00409866	0.00372417	0.00565936	0.00324270	0.93198758	0.00418545	0.00132466	0.00197084
13	0.00756638	0.00532630	0.00145535	0.00082091	0.00222643	0.91910875	0.00317180	0.00093513
13	0.00243933	0.00117515	0.00031089	0.00029441	0.00025736	0.00169191	0.91484016	0.00499010
13	0.00145371	0.00152364	0.00011013	0.00007149	0.00025519	0.00034688	0.00257159	0.91658837
14	0.87692320	0.00700416	0.00274589	0.00245355	0.00194242	0.00397302	0.00527260	0.00584803
14	0.00821651	0.87932634	0.00327950	0.00190907	0.00188314	0.00641090	0.00433696	0.00261681
14	0.00322409	0.00274954	0.88468295	0.00254367	0.00209786	0.00092111	0.00100163	0.00081868
14	0.00386076	0.00213920	0.00556622	0.89521313	0.00312114	0.00101942	0.00082445	0.00086343
14	0.00299621	0.00233300	0.00296126	0.00263597	0.89448506	0.00272434	0.00061113	0.00129236
14	0.00549032	0.00338083	0.00110533	0.00072785	0.00174546	0.87789208	0.00180149	0.00126649
14	0.00153570	0.00471080	0.00044348	0.00022592	0.00027822	0.00116977	0.87563789	0.00238883
14	0.00132969	0.00076146	0.00000228	0.00007182	0.00010871	0.00046133	0.00175862	0.88302416

Age Group	Stockholm	E.Middle	S.Middle	South	West	N.Middle	Upper N.	Lower N.
15	0.81590420	0.00533004	0.00205170	0.00173772	0.00154843	0.00270557	0.00206505	0.00270639
15	0.00641242	0.80784470	0.00190020	0.00161241	0.00127721	0.00353927	0.00315198	0.00131316
15	0.00217372	0.00162820	0.81259429	0.00160056	0.00133555	0.00075128	0.00065625	0.00051670
15	0.00241994	0.00143771	0.00453358	0.82940704	0.00218605	0.00054810	0.00138182	0.00098968
15	0.00239169	0.00204262	0.00196062	0.00225272	0.82687026	0.00126439	0.00069597	0.00145150
15	0.00279835	0.00197485	0.00075641	0.00033800	0.00083224	0.80900151	0.00201057	0.00092373
15	0.00149785	0.00059021	0.00051216	0.00016439	0.00019280	0.00097441	0.80714995	0.00074370
15	0.00066557	0.00054250	0.00013546	0.00090195	0.00006958	0.00041655	0.00116789	0.81007910
16	0.70953542	0.00414121	0.00197304	0.00219444	0.00168343	0.00281650	0.00249190	0.00214353
16	0.00603467	0.69625169	0.00233417	0.00095155	0.00098858	0.00367023	0.00243553	0.00174566
16	0.00166876	0.00115905	0.69526052	0.00118610	0.00097704	0.00039244	0.00032812	0.00032346
16	0.00213024	0.00098523	0.00357390	0.72731088	0.00159190	0.00043036	0.00058128	0.00112415
16	0.00142167	0.00154314	0.00209294	0.00169379	0.71396482	0.00114885	0.00061614	0.00110586
16	0.00224977	0.00205827	0.00030665	0.00021896	0.00055249	0.68441117	0.00151359	0.00076641
16	0.00128382	0.00018386	0.00029850	0.00000187	0.00016320	0.00108694	0.68345839	0.00107088
16	0.00046949	0.00056006	0.00015188	0.00000081	0.00007645	0.00028364	0.00083899	0.69232810
17	0.85544497	0.00284844	0.00173613	0.00218443	0.00187524	0.00246455	0.00324181	0.00228669
17	0.00463554	0.78272742	0.00257121	0.00040439	0.00078899	0.00484766	0.00240056	0.00141108
17	0.00127260	0.00096018	0.79271007	0.00103088	0.00080452	0.	0.00061195	0.
17	0.00163971	0.00057734	0.00394118	0.83027780	0.00128999	0.00029139	0.	0.00144195
17	0.00090555	0.00095654	0.00130594	0.00082150	0.80544376	0.00115867	0.00060964	0.00143341
17	0.00204660	0.00252213	0.	0.00019342	0.00030189	0.72438624	0.00172226	0.00134982
17	0.00093931	0.	0.	0.	0.00016627	0.00090140	0.79586974	0.00148684
17	0.00035212	0.00018597	0.	0.	0.	0.00028159	0.	0.76833934

APPENDIX B: INPUT DATA OF THE ROGERS MODEL OF THE 1974 SOVIET POPULATION

B.1 Initial Population (in persons)

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
0-4	5473041.	2008964.	345809.	1006117.	626439.	639035.	319269.	9479226.
5-9	6187733.	2165421.	368119.	1016642.	689083.	727290.	318804.	11361861.
10-14	7146911.	2360301.	401585.	987207.	772609.	722002.	325808.	12187578.
15-19	8531010.	2625942.	477164.	912056.	769605.	687247.	340960.	7881302.
20-24	8417263.	2558641.	494904.	768913.	751482.	600058.	367230.	5019938.
25-29	4891112.	1833893.	311167.	444311.	430065.	329356.	335266.	3815153.
30-34	8360389.	2890042.	465269.	706462.	724320.	620381.	381429.	7027913.
35-39	6369820.	1956664.	317094.	507751.	475335.	489091.	338017.	5973726.
40-44	7336668.	2338082.	340802.	565143.	561348.	520286.	349940.	7228710.
45-49	5232353.	1823433.	254861.	364483.	348579.	337782.	260618.	5120336.
50-54	3412439.	1155164.	148175.	251560.	226350.	209286.	163198.	3413558.
55-59	4322378.	1478028.	154102.	290098.	240006.	260769.	185977.	4718742.
60-64	4643167.	1559734.	159789.	314080.	263927.	282086.	207363.	5903079.
65-69	3262766.	1126475.	122192.	224941.	197945.	202353.	168345.	4729684.
70-74	2351788.	833919.	95216.	163022.	137392.	147467.	137658.	3757960.
75-79	1345139.	476972.	54460.	93243.	78583.	84346.	78736.	2149420.
80-84	632406.	224244.	25604.	43837.	36945.	39654.	37017.	1010531.
85+	313881.	111299.	12708.	21758.	18337.	19682.	18373.	501556.

B.2 The B-submatrices

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
3	0.0356050	0.0034385	0.0034524	0.0028826	0.0049592	0.0019206	0.0025617	0.0062023
3	0.0013797	0.0366395	0.0012665	0.0005967	0.0012553	0.0006032	0.0008309	0.0023117
3	0.0001826	0.0001856	0.0270801	0.0000768	0.0001611	0.0000571	0.0002634	0.0004837
3	0.0003568	0.0001810	0.0001595	0.0367131	0.0008644	0.0001674	0.0001206	0.0005081
3	0.0004277	0.0002553	0.0002195	0.0005983	0.0304943	0.0001132	0.0001270	0.0005547
3	0.0000926	0.0000316	0.0000559	0.0000767	0.0000719	0.0358573	0.0000533	0.0002097
3	0.0001458	0.0001241	0.0004365	0.0000544	0.0000828	0.0000453	0.0275101	0.0003141
3	0.0033790	0.0034917	0.0033477	0.0028501	0.0036813	0.0011958	0.0038711	0.0398672
4	0.1866549	0.0325911	0.0324691	0.0308835	0.0457482	0.0201118	0.0256677	0.0630583
4	0.0121826	0.1832697	0.0114712	0.0067628	0.0117306	0.0061687	0.0081760	0.0226403
4	0.0019816	0.0020543	0.1594940	0.0011038	0.0018777	0.0007526	0.0028415	0.0055356
4	0.0047073	0.0027579	0.0024647	0.2339568	0.0106292	0.0026687	0.0020209	0.0075179
4	0.0041885	0.0027716	0.0024440	0.0065130	0.1525243	0.0014145	0.0016571	0.0062531
4	0.0012707	0.0011700	0.0008407	0.0011754	0.0010723	0.2349557	0.0008253	0.0030105
4	0.0013787	0.0012317	0.0037654	0.0007096	0.0009326	0.0005333	0.1522850	0.0031940
4	0.0355781	0.0373016	0.0356647	0.0344607	0.0399797	0.0150670	0.0410659	0.2134588

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
5	0.3155999	0.0310005	0.0315906	0.0326217	0.0483558	0.0213843	0.0210306	0.0804776
5	0.0104875	0.3067644	0.0095134	0.0055214	0.0099099	0.0054881	0.0055537	0.0242742
5	0.0018127	0.0017934	0.3117481	0.0009320	0.0016860	0.0006770	0.0023347	0.0068005
5	0.0055596	0.0027966	0.0024571	0.4706411	0.0136492	0.0033969	0.0016435	0.0110162
5	0.0047405	0.0027636	0.0024233	0.0081867	0.3066283	0.0015233	0.0012691	0.0087477
5	0.0014388	0.0012640	0.0008239	0.0013344	0.0011503	0.4564410	0.0007247	0.0043429
5	0.0013276	0.0010949	0.0038398	0.0006341	0.0008137	0.0005078	0.2752907	0.0037780
5	0.0348319	0.0349836	0.0337886	0.0357479	0.0399278	0.0147052	0.0352065	0.4205397
6	0.2071006	0.0059235	0.0070576	0.0072897	0.0114532	0.0047308	0.0036365	0.0160470
6	0.0028117	0.1960666	0.0025512	0.0013029	0.0026321	0.0014374	0.0011234	0.0058588
6	0.0004123	0.0003538	0.2263387	0.0001720	0.0003756	0.0001394	0.0004656	0.0014857
6	0.0014240	0.0005410	0.0005199	0.3884404	0.0039876	0.0008652	0.0002545	0.0025747
6	0.0012132	0.0005649	0.0005572	0.0022611	0.2442391	0.0003470	0.0001964	0.0019476
6	0.0002491	0.0001091	0.0001159	0.0002118	0.0001815	0.3358128	0.0000881	0.0006992
6	0.0004677	0.0003282	0.0015652	0.0001820	0.0002443	0.0001631	0.2044373	0.0013575
6	0.0103103	0.0090822	0.0099410	0.0108586	0.0122358	0.0040858	0.0086750	0.3160051

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
7	0.0744597	0.0027626	0.0032034	0.0037022	0.0053258	0.0018850	0.0019312	0.0074458
7	0.0010722	0.0868060	0.0011215	0.0006630	0.0011949	0.0005580	0.0005810	0.0026638
7	0.0001483	0.0001507	0.0921142	0.0000846	0.0001621	0.0000516	0.0002256	0.0006340
7	0.0006921	0.0003102	0.0003006	0.2230380	0.0021532	0.0004040	0.0001660	0.0012723
7	0.2005756	0.0003183	0.0003081	0.0013616	0.1145810	0.0001655	0.0001264	0.0010525
7	0.0001052	0.0000035	0.0000568	0.0001023	0.0000862	0.1367565	0.0000500	0.0002863
7	0.0001433	0.0001192	0.0005511	0.0000750	0.0000908	0.0000512	0.1053533	0.0004944
7	0.0049078	0.0050227	0.0054795	0.0062312	0.0066323	0.0018920	0.0053292	0.1848688
8	0.0391693	0.0008412	0.0010136	0.0016254	0.0020721	0.0006695	0.0005036	0.0032043
8	0.0003184	0.0450036	0.0003460	0.0002721	0.0004502	0.0001940	0.0001462	0.0011354
8	0.0000486	0.0000495	0.0495072	0.0000361	0.0000661	0.0000190	0.0000644	0.0002973
8	0.0002120	0.0000930	0.0000924	0.1571183	0.0008005	0.0001321	0.0000425	0.0004615
8	0.0001658	0.0000904	0.0000900	0.0005463	0.0762533	0.0000529	0.0000297	0.0004081
8	0.0000257	0.0000232	0.0000142	0.0000332	0.0000245	0.0814342	0.0000111	0.0000904
8	0.0000493	0.0000407	0.0002007	0.0000334	0.0000371	0.0000196	0.0437738	0.0002360
8	0.0015907	0.0016642	0.0018762	0.0024809	0.0025025	0.0006503	0.0015854	0.1289549
9	0.0047143	0.0001790	0.0001274	0.0004043	0.0003309	0.0001196	0.0000758	0.0007996
9	0.0000364	0.0014465	0.0000445	0.0000687	0.0000735	0.0000355	0.0000226	0.0002904
9	0.0000055	0.0000047	0.0067964	0.0000089	0.0000107	0.0000034	0.0000099	0.0000752
9	0.0000286	0.0000113	0.0000132	0.0429604	0.0001299	0.0000228	0.0000067	0.0000958

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
9	0.0000222	0.0000105	0.0000132	0.0001370	0.0133284	0.0000101	0.0000050	0.0001010
9	0.0000332	0.0000026	0.0000019	0.0000072	0.0000038	0.0159896	0.0000017	0.0000193
9	0.0000054	0.0000038	0.0000247	0.0000081	0.0000058	0.0000034	0.0073612	0.0000586
9	0.0002146	0.0002013	0.0002690	0.0005344	0.0004035	0.0001145	0.0002495	0.0351029
10	0.0004639	0.0000076	0.0000125	0.0000918	0.0000703	0.0000200	0.0000065	0.0001666
10	0.0000036	0.0004349	0.0000044	0.0000158	0.0000159	0.0000060	0.0000020	0.0000610
10	0.0000005	0.0000005	0.0036749	0.0000020	0.0000023	0.0000006	0.0000008	0.0000157
10	0.0000082	0.0000032	0.0000037	0.0100479	0.0000419	0.0000065	0.0000017	0.0000256
10	0.0000029	0.0000014	0.0000317	0.0000310	0.0029869	0.0000018	0.0000006	0.0000211
10	0.0000005	0.0000004	0.0000003	0.0000016	0.0000008	0.0027906	0.0000002	0.0000041
10	0.0000005	0.0000004	0.0000023	0.0000018	0.0000012	0.0000006	0.0006043	0.0000122
10	0.0000407	0.0000380	0.0003512	0.0031264	0.0000964	0.0000243	0.0000433	0.0074971
11	0.0000202	0.0000003	0.0000003	0.0000261	0.0000103	0.0000032	0.0000001	0.0000336
11	0.0000001	0.0000186	0.0000001	0.0000044	0.0000023	0.0000010	0.0000000	0.0000123
11	0.0000000	0.0000000	0.0000160	0.0000006	0.0000003	0.0000001	0.0000000	0.0000032
11	0.0000000	0.0000000	0.0000000	0.0029130	0.0000021	0.0000003	0.	0.0000028
11	0.0000001	0.0000000	0.0000000	0.0000079	0.0004527	0.0000002	0.0000000	0.0000038
11	0.0000000	0.	0.	0.0000004	0.0000001	0.0004611	0.	0.0000007
11	0.0000000	0.0000000	0.0000001	0.0000005	0.0000002	0.0000001	0.0000147	0.0000025
11	0.0000003	0.0000002	0.0000002	0.0000236	0.0000066	0.0000016	0.0000002	0.0015209

B.3 The S-submatrices

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
1	0.92946631	0.02442562	0.02681884	0.01525818	0.03705462	0.01138451	0.01521396	0.03525473
1	0.01102477	0.93116790	0.00952468	0.00249060	0.00811288	0.00337597	0.00452027	0.01269662
1	0.00158221	0.00141509	0.91842556	0.00030266	0.00112412	0.00030732	0.00200100	0.00326481
1	0.00310855	0.00113785	0.00102729	0.92601174	0.00750230	0.00109436	0.00055947	0.00291754
1	0.00466435	0.00217133	0.00194887	0.00470382	0.89593565	0.00076527	0.00068514	0.00403621
1	0.00047085	0.00036471	0.00019545	0.00021638	0.00026253	0.95359361	0.00018428	0.00070094
1	0.00161128	0.00115151	0.00545848	0.00026781	0.00060261	0.00033174	0.94116640	0.00260754
1	0.02499387	0.02282715	0.02352159	0.01377844	0.02354043	0.00579812	0.02324552	0.91296828
2	0.96270043	0.01932676	0.02136248	0.01336557	0.02873648	0.00914328	0.01296796	0.02643778
2	0.00865209	0.95595968	0.00736924	0.00211771	0.00613894	0.00262527	0.00373816	0.00925996
2	0.00114626	0.00107380	0.94413525	0.00025234	0.00083403	0.00023777	0.00162915	0.00234785
2	0.00198922	0.00075184	0.00068530	0.96503776	0.00500962	0.00071445	0.00038864	0.00186774
2	0.00322259	0.00156845	0.00141412	0.00375658	0.93767333	0.00055495	0.00051892	0.00277038
2	0.00035553	0.00020843	0.00015975	0.00018775	0.00020227	0.97933125	0.00014356	0.00052619
2	0.00128846	0.00097478	0.00470327	0.00026625	0.00049461	0.00029033	0.95922756	0.00209124
2	0.01851596	0.01731993	0.01848555	0.01191599	0.01799165	0.00458542	0.01956283	0.95146726
3	0.87162534	0.07079336	0.07350881	0.06221904	0.10476645	0.04114068	0.05583540	0.13240190
3	0.02564612	0.85170197	0.02473237	0.01165038	0.02427962	0.01181301	0.01655211	0.04532641
3	0.00426214	0.00439271	0.82390076	0.00186463	0.00367433	0.00139413	0.00671392	0.01196439
3	0.00657150	0.00334887	0.00302648	0.85654199	0.01657109	0.00320466	0.00232778	0.00983016
3	0.00817375	0.00492127	0.00437118	0.01240084	0.78165597	0.00226262	0.00254598	0.01118607
3	0.00207124	0.00183027	0.00128787	0.00179127	0.00163849	0.91639596	0.00122868	0.00486789
3	0.00322475	0.00270409	0.01046879	0.00125195	0.00186491	0.00105727	0.84455448	0.00741443
3	0.05464565	0.05712818	0.05629320	0.04057503	0.06126793	0.02007842	0.06707194	0.77226114
4	0.79277724	0.15787835	0.15730655	0.15137477	0.22402062	0.09810990	0.12651385	0.31413245
4	0.05286211	0.69658965	0.05708690	0.02949239	0.05153487	0.02704012	0.03620549	0.10187909
4	0.00885174	0.00923946	0.65299046	0.00492672	0.00845467	0.00337285	0.01315558	0.02584964
4	0.01516523	0.00887755	0.00792510	0.67848724	0.03473223	0.00866895	0.00654236	0.02477416
4	0.01751277	0.01160120	0.01020303	0.02811541	0.55753422	0.00593136	0.00696331	0.02709856
4	0.00496492	0.00458005	0.00329431	0.00463832	0.00420378	0.80963403	0.00323952	0.01197326
4	0.00652038	0.00503789	0.01820864	0.00334014	0.00437591	0.00253697	0.68881840	0.01588553
4	0.09493954	0.10015299	0.09570225	0.09341813	0.10783450	0.04043744	0.11251964	0.46946326

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
5	0.76555437	0.15063021	0.15063961	0.16082773	0.23639062	0.10435717	0.10420106	0.41293669
5	0.05221387	0.68494886	0.04707502	0.02001657	0.05029347	0.02783884	0.02862378	0.15160126
5	0.00777592	0.00781537	0.64292880	0.00403572	0.00732562	0.00293653	0.01046940	0.03177965
5	0.01632205	0.00824370	0.00716557	0.63213085	0.04045131	0.01010354	0.00485334	0.03362935
5	0.21855063	0.01090334	0.00936436	0.03332215	0.50937963	0.00599495	0.00501368	0.03671714
5	0.00496813	0.00432993	0.00281559	0.00460349	0.00394125	0.78929913	0.00248390	0.01520892
5	0.00673021	0.00563724	0.01937050	0.00327319	0.00413767	0.00263380	0.71333361	0.02100517
5	0.11777087	0.11979486	0.11383191	0.12388285	0.13689454	0.05020237	0.12297831	0.30357164
6	0.86097163	0.06902955	0.07696662	0.08566695	0.13523661	0.05556162	0.03999191	0.21388558
6	0.02685649	0.84299529	0.02315418	0.01247222	0.02560554	0.01398726	0.01025316	0.06502888
6	0.00355313	0.00310020	0.80460379	0.00143818	0.00327179	0.00121589	0.00389450	0.01522743
6	0.00693737	0.00339579	0.00314106	0.76833741	0.02505549	0.00552846	0.00151181	0.01736987
6	0.01039359	0.00465290	0.00448032	0.02007106	0.70850486	0.00296628	0.00155213	0.01907569
6	0.00258818	0.00198199	0.00117834	0.00218387	0.00188559	0.88394409	0.00087112	0.00772822
6	0.00353597	0.00251505	0.01155155	0.00136570	0.00182631	0.00126376	0.87300366	0.01167538
6	0.07216012	0.06429640	0.06746680	0.07737963	0.08682983	0.02879835	0.05966419	0.63652736
7	0.92211121	0.03969602	0.04577013	0.04729361	0.07366817	0.02563120	0.02563391	0.09450668
7	0.01281075	0.96216142	0.01297672	0.00659222	0.01326978	0.00612862	0.00621104	0.02752746
7	0.00149026	0.00147743	0.87903529	0.00068659	0.00150040	0.00047322	0.00206428	0.00552335
7	0.00432289	0.00193520	0.00183854	0.87524539	0.01287877	0.00244641	0.00096935	0.00736513
7	0.30530535	0.00285513	0.00274299	0.01106656	0.83271235	0.00141141	0.00104280	0.00859562
7	0.00120420	0.00106279	0.00063757	0.00104482	0.00094366	0.94061506	0.00053566	0.00299639
7	0.00169381	0.00136519	0.00654947	0.00071402	0.00095280	0.00056270	0.91314274	0.00503923
7	0.03617810	0.03664968	0.03983118	0.04223367	0.04728388	0.01334237	0.03746699	0.82994181
8	0.92306919	0.03647505	0.04352612	0.04348467	0.07092269	0.02285573	0.02142048	0.08225662
8	0.01031331	0.90333152	0.01117735	0.00521875	0.01137384	0.00489002	0.00460552	0.02155614
8	0.00121111	0.00125074	0.87993598	0.00053404	0.00128432	0.00037156	0.00162883	0.00447327
8	0.00373389	0.00162948	0.00160969	0.87732804	0.01234289	0.00209484	0.00071129	0.00616224
8	0.03479723	0.00259551	0.00254652	0.01058269	0.83322126	0.00122845	0.00079607	0.00753459
8	0.00103486	0.00094694	0.00057192	0.00092157	0.00084005	0.94244415	0.00043287	0.00251355
8	0.00135536	0.00112713	0.00573645	0.00054705	0.00077881	0.00043722	0.91941011	0.00381559
8	0.03342523	0.03520159	0.03945013	0.04058535	0.04677302	0.01209585	0.03350221	0.84688705
9	0.93140692	0.02365668	0.03095046	0.03247135	0.05153059	0.01762076	0.01565930	0.05494741
9	0.00693014	0.92346156	0.00767597	0.00383344	0.00806658	0.00365975	0.00330932	0.01382198
9	0.00092003	0.00079299	0.90397596	0.00039492	0.00091491	0.00027812	0.00116299	0.00289919
9	0.00258715	0.00105566	0.00113550	0.89263117	0.00912267	0.00158472	0.00052048	0.00416947
9	0.00328386	0.00166878	0.00160089	0.00776073	0.86260322	0.00094877	0.00058864	0.00497036
9	0.00077342	0.00066960	0.00043143	0.00071521	0.00066337	0.94264072	0.00035407	0.00176863
9	0.00098344	0.00076676	0.00429786	0.00042872	0.00059755	0.00036245	0.92764163	0.00267878
9	0.02305002	0.02267562	0.02768291	0.03009959	0.03385043	0.00930776	0.02429819	0.87986058

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
10	0.91688162	0.02174120	0.03013467	0.03165746	0.05260611	0.01781003	0.01468969	0.05245716
10	0.00722611	0.91129684	0.00802777	0.00392317	0.00876254	0.00393182	0.00332238	0.01415323
10	0.00074236	0.00069627	0.89050108	0.00034868	0.00088783	0.00025848	0.00105678	0.00256499
10	0.00249662	0.00096766	0.00109279	0.87774187	0.00964120	0.00156883	0.00048683	0.00407067
10	0.00332796	0.00162738	0.00184816	0.00810153	0.84267831	0.00101218	0.00056337	0.00503520
10	0.00079855	0.00007630	0.00043509	0.00073799	0.00072467	0.92545468	0.00033722	0.00177846
10	0.00094660	0.00071475	0.00428282	0.00042122	0.00061219	0.00036054	0.91361791	0.00256571
10	0.02206989	0.02103120	0.02737520	0.02960661	0.03471508	0.00942430	0.02309435	0.86593401
11	0.92351788	0.01560191	0.02461199	0.02253496	0.04030541	0.01308161	0.01099635	0.03613546
11	0.00519366	0.91952425	0.00680030	0.00284913	0.00693905	0.00298637	0.00257647	0.01015874
11	0.00048442	0.00045657	0.89464277	0.00021571	0.00061547	0.00016917	0.00073509	0.00170244
11	0.00173721	0.00066683	0.00086100	0.89486212	0.00733015	0.00110663	0.00034673	0.00277301
11	0.00240807	0.00117776	0.00150944	0.00592435	0.86080265	0.00074161	0.00041485	0.00352780
11	0.00069286	0.00051742	0.00035636	0.00050997	0.00057595	0.92937267	0.00025475	0.00127897
11	0.00070740	0.00053904	0.00368840	0.00029165	0.00047310	0.00025624	0.91696477	0.00188478
11	0.01590563	0.01534307	0.02271111	0.02140673	0.02690431	0.00697834	0.01768335	0.89070332
12	0.91542244	0.01360116	0.02446583	0.02059513	0.03817460	0.01144154	0.01003992	0.02753568
12	0.00456097	0.91153133	0.00683557	0.00264650	0.00654617	0.00267549	0.00235303	0.00778980
12	0.00041788	0.00039504	0.88430578	0.00020773	0.00057645	0.00016217	0.00066196	0.00127489
12	0.00127538	0.00048983	0.00072955	0.88779692	0.00593246	0.00077680	0.00029453	0.00182158
12	0.00191039	0.00093620	0.00138411	0.00494905	0.85187870	0.00059465	0.00035515	0.00246753
12	0.00035979	0.00030610	0.00025637	0.00033051	0.00038187	0.92062068	0.00014074	0.00071130
12	0.00069327	0.00053146	0.00415797	0.00031369	0.00050501	0.00026690	0.91232264	0.00164370
12	0.01392488	0.01348094	0.02275515	0.01971662	0.02571346	0.00616742	0.01628795	0.90085477
13	0.87714273	0.01496493	0.02656743	0.02256030	0.04078465	0.01252941	0.01023977	0.02689069
13	0.00594125	0.87184989	0.00742041	0.00288879	0.00698942	0.00292522	0.00239355	0.00760579
13	0.00049433	0.00046560	0.85061812	0.00023632	0.00065597	0.00018723	0.00072632	0.00133514
13	0.00125261	0.00047562	0.00068991	0.85187888	0.00561885	0.00075942	0.00027344	0.00157103
13	0.00204564	0.00100068	0.00145288	0.00524267	0.81724072	0.00063082	0.00034861	0.00231951
13	0.00029619	0.00024710	0.00021041	0.00028028	0.00030966	0.88625604	0.00006475	0.00053769
13	0.00090875	0.00069189	0.00539285	0.00043272	0.00064227	0.00034140	0.88025761	0.00194627
13	0.01560726	0.01513505	0.02504110	0.02189399	0.02762821	0.00684742	0.01685079	0.87460351
14	0.82818776	0.01362748	0.02309257	0.02097511	0.03767731	0.01158348	0.00837300	0.02253665
14	0.00468414	0.82374316	0.00666899	0.00277230	0.00666182	0.00278750	0.00201787	0.00658394
14	0.00050568	0.00046278	0.81344765	0.00024311	0.00066052	0.00018623	0.00064027	0.00122631
14	0.02121463	0.00045187	0.00061671	0.81661975	0.00538062	0.00074806	0.00023171	0.00136900
14	0.00197385	0.00094106	0.00131387	0.00504398	0.78074640	0.00061261	0.00028717	0.00200432
14	0.00028476	0.00023300	0.00016051	0.00027801	0.00030315	0.84304280	0.00007306	0.00346306
14	0.00098426	0.00073472	0.00546298	0.00044632	0.00067648	0.00037451	0.83185053	0.00185042
14	0.01448174	0.01359565	0.02158547	0.02023342	0.02548681	0.00627030	0.01375648	0.82851934

Age Group	RSFSR	Uk & Mo	Byelo.	Cent. A.	Kazakh	Caucas.	Baltic	Rural
15	0.75489748	0.01173368	0.01939926	0.01845139	0.03381445	0.01010826	0.00681323	0.01858913
15	0.00449365	0.74973530	0.00591006	0.00256535	0.00627167	0.00255707	0.00170636	0.00571899
15	0.00247942	0.00043124	0.74912775	0.00025283	0.00063436	0.00018606	0.00056295	0.00110533
15	0.00110400	0.00040924	0.00055710	0.75955337	0.00504080	0.00068387	0.00020387	0.00117717
15	0.00176080	0.00082818	0.00116359	0.00452962	0.71808183	0.00155858	0.00024017	0.00168585
15	0.00326045	0.00021739	0.00015792	0.00030234	0.00028657	0.77321887	0.00009208	0.00040257
15	0.00095526	0.00070544	0.00516832	0.00039461	0.00065345	0.00038447	0.75487709	0.00168773
15	0.01265841	0.01169485	0.01809026	0.01781140	0.02285805	0.00545837	0.01121177	0.74959946
16	0.65161747	0.01026649	0.01710528	0.01628582	0.02973787	0.00887039	0.00595435	0.01629492
16	0.00393320	0.64670229	0.00520013	0.00226316	0.00551734	0.00225493	0.00147749	0.00501149
16	0.00442350	0.00037889	0.65796632	0.00023511	0.00055392	0.00016856	0.00048738	0.00097660
16	0.00097453	0.00035080	0.00052648	0.66931498	0.00444415	0.00060766	0.00018357	0.00104139
16	0.00154855	0.00072479	0.00104378	0.00402282	0.62465048	0.00049200	0.00020812	0.00148480
16	0.00022860	0.00019336	0.00013257	0.00025003	0.00026566	0.67598689	0.00008978	0.00035443
16	0.00083528	0.00061408	0.00452580	0.00033644	0.00056461	0.00033252	0.65118200	0.00147792
16	0.01109849	0.01024787	0.01593937	0.01574257	0.02011663	0.00478490	0.00983683	0.64863569
17	0.83643103	0.02403554	0.04290342	0.04216680	0.06972251	0.02224498	0.01434429	0.03835720
17	0.00925187	0.82232445	0.01317599	0.00606640	0.01326458	0.00569026	0.00356257	0.01181049
17	0.00108542	0.00099641	0.91624266	0.00060190	0.00145635	0.00049446	0.00130355	0.00246631
17	0.00254699	0.00099072	0.00160453	0.96022135	0.01146213	0.00155304	0.00053874	0.00273901
17	0.00365950	0.00177695	0.00273070	0.01031791	0.80241281	0.00133514	0.00051154	0.00354134
17	0.00058278	0.00048340	0.00062010	0.00081152	0.00059650	0.94352794	0.00012746	0.00090379
17	0.00199557	0.00150224	0.01139576	0.00094298	0.00145587	0.00090336	0.83266348	0.00350210
17	0.02619516	0.02416007	0.04015175	0.04068094	0.04772224	0.01232108	0.02315760	0.83842230

APPENDIX C: INPUT DATA OF THE ROGERS MODEL OF THE 1970 BRITISH FEMALE POPULATION

C.1 Initial Population (in persons)

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
0-4	604500.	133700.	220300.	62700.	673500.	144000.	103600.	219600.
5-9	633100.	140300.	213900.	65200.	665500.	151400.	112400.	231300.
10-14	540200.	117600.	183100.	56600.	583200.	129400.	95400.	212700.
15-19	508000.	111000.	175300.	55300.	562300.	123400.	94000.	191000.
20-24	543100.	129500.	206600.	63300.	698700.	140200.	98600.	202300.
25-29	473300.	102000.	173500.	52700.	571000.	113500.	82600.	162600.
30-34	437100.	100300.	156500.	50800.	521400.	106900.	77000.	149500.
35-39	429500.	98000.	151300.	48600.	504100.	103400.	76700.	148300.
40-44	443600.	102600.	153900.	47100.	515400.	111400.	84300.	155300.
45-49	497100.	109100.	174400.	53900.	535300.	123200.	94100.	167000.
50-54	430500.	95500.	147900.	46900.	523200.	109600.	80100.	150100.
55-59	467000.	101900.	155500.	52500.	574700.	122600.	89800.	166500.
60-64	455100.	96000.	142000.	59100.	534600.	122500.	88400.	156300.
65-69	409500.	84500.	121100.	45400.	466700.	111300.	79500.	138300.
70-74	327300.	70400.	94300.	36600.	381400.	92100.	62000.	108400.
75-79	227948.	43236.	66404.	27601.	281679.	66469.	42592.	75900.
80-84	136756.	29388.	40372.	18268.	187772.	43094.	25358.	43700.
85+	79896.	13626.	24624.	12531.	129949.	29037.	14850.	23600.

C.2 The $\underset{\sim}{B}$-submatrices

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
3	0.0595299	0.0033428	0.0019759	0.0017179	0.0013918	0.0016030	0.0020847	0.0017327
3	0.0008109	0.0536496	0.0011911	0.0012264	0.0007314	0.0006743	0.0004942	0.0005006
3	0.0007328	0.0014375	0.0568953	0.0008309	0.0006173	0.0012062	0.0011446	0.0005123
3	0.0003241	0.0009499	0.0002880	0.0474748	0.0008047	0.0004180	0.0002025	0.0001768
3	0.0020175	0.0037232	0.0022340	0.0052701	0.0466730	0.0054277	0.0021673	0.0020265
3	0.0006183	0.0009715	0.0010002	0.0010598	0.0013396	0.0473774	0.0012073	0.0005201
3	0.0004549	0.0003327	0.0005446	0.0003134	0.0003101	0.0005331	0.0554784	0.0001584
3	0.0004663	0.0005936	0.0002502	0.0005293	0.0004413	0.0005426	0.0003122	0.0511467
4	0.2792422	0.0153820	0.0088760	0.0035492	0.0074493	0.0078990	0.0104650	0.0081509
4	0.0041197	0.2085163	0.0055528	0.0059622	0.0037017	0.0029128	0.0024626	0.0021718
4	0.0037307	0.0062726	0.2142390	0.0038254	0.0031160	0.0055731	0.0052546	0.0024296
4	0.0016280	0.0033261	0.0011840	0.1946982	0.0036170	0.0019861	0.0011834	0.0010043
4	0.0110092	0.0140353	0.0112248	0.0256401	0.1900413	0.0273474	0.0126477	0.0103139
4	0.0030332	0.0040777	0.0043252	0.0046726	0.0064808	0.1928912	0.0059063	0.0026058
4	0.0021259	0.0014619	0.0025822	0.0014761	0.0015792	0.0025670	0.2185027	0.0008005
4	0.0024558	0.0026262	0.0012317	0.0024392	0.0022090	0.0025099	0.0015023	0.2145544

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
5	0.3493030	0.0120472	0.0102175	0.0110945	0.0090563	0.0102499	0.0123951	0.0107436
5	0.0051007	0.3229563	0.0072056	0.0080791	0.0043638	0.0034541	0.0033075	0.0031355
5	0.0043327	0.0082405	0.3293859	0.0052094	0.0040544	0.0067384	0.0063024	0.0032015
5	0.0020373	0.0044130	0.0014157	0.3031110	0.0052381	0.0024894	0.0017481	0.0011557
5	0.0134151	0.0170933	0.0133363	0.0353964	0.3082981	0.0365321	0.0151837	0.0141229
5	0.0037294	0.0051901	0.0052274	0.0063123	0.0084146	0.3048721	0.0069537	0.0032661
5	0.0026112	0.0018176	0.0031439	0.0017134	0.0018001	0.0032263	0.3258583	0.0009532
5	0.0032415	0.0035109	0.0016317	0.0037741	0.0029663	0.0037490	0.0024598	0.3395090
6	0.2530800	0.0105698	0.0057185	0.0063147	0.0048518	0.0058113	0.0066671	0.0066934
6	0.0024112	0.2504415	0.0037640	0.0043586	0.0025105	0.0018355	0.0016697	0.0017844
6	0.0024034	0.0047323	0.2575132	0.0031375	0.0021826	0.0035972	0.0030291	0.0018459
6	0.0010103	0.0025163	0.0007522	0.2291712	0.0029691	0.0011838	0.0008566	0.0005781
6	0.0050693	0.0078467	0.0061608	0.0185098	0.2562955	0.0181394	0.0058943	0.0073528
6	0.0013431	0.0029116	0.0027966	0.0033940	0.0045735	0.2385471	0.0031570	0.0017767
6	0.0014004	0.0011644	0.0016490	0.0010397	0.0008876	0.0018175	0.2441662	0.0004935
6	0.0017518	0.0021080	0.0008185	0.0022867	0.0016502	0.0022879	0.0014227	0.2763413
7	0.1313571	0.0042892	0.0024609	0.0025010	0.0019907	0.0022108	0.0028818	0.0029486
7	0.0009936	0.1173425	0.0015700	0.0017341	0.0010014	0.0007178	0.0006189	0.0007537
7	0.0010239	0.0020057	0.1393010	0.0012378	0.0009138	0.0015477	0.0012951	0.0007928

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
7	0.0004171	0.0010626	0.0003362	0.1023341	0.0012656	0.0004525	0.0003212	0.0002640
7	0.0023395	0.0030637	0.0025395	0.0063256	0.1274165	0.0069477	0.0023152	0.0030485
7	0.0007739	0.0011505	0.0013066	0.0014177	0.0018696	0.1107627	0.0012631	0.0007793
7	0.0005325	0.0004528	0.0006796	0.0004193	0.0003473	0.0006549	0.1205251	0.0001791
7	0.0007161	0.0003440	0.0003176	0.0008455	0.0006419	0.0009100	0.0005185	0.1455821
8	0.0522094	0.0013075	0.0008471	0.0007421	0.0006127	0.0006345	0.0009235	0.0009707
8	0.0003350	0.0440193	0.0005372	0.0005437	0.0003134	0.0002182	0.0002131	0.0002606
8	0.0003374	0.0005947	0.0569165	0.0003737	0.0002788	0.0004707	0.0004180	0.0002645
8	0.0001400	0.0003344	0.0001162	0.0389182	0.0004128	0.0001424	0.0001069	0.0000820
8	0.0007487	0.0009274	0.0008243	0.0020972	0.0473169	0.0021053	0.0007232	0.0009523
8	0.0002609	0.0003638	0.0004075	0.0004634	0.0005839	0.0410742	0.0004270	0.0002569
8	0.0001940	0.0001242	0.0002343	0.0001132	0.0001038	0.0001393	0.0461008	0.0000551
8	0.0002243	0.0002756	0.0001207	0.0002858	0.0001945	0.0002746	0.0001719	0.0563483
9	0.0115062	0.0002517	0.0001620	0.0001348	0.0001080	0.0001109	0.0001679	0.0001777
9	0.0000652	0.0100505	0.0001032	0.0001038	0.0000589	0.0000380	0.0000402	0.0000497
9	0.0000655	0.0001104	0.0128944	0.0000705	0.0000502	0.0000811	0.0000710	0.0000514
9	0.0000272	0.0000653	0.0000222	0.0083328	0.0000771	0.0000257	0.0000197	0.0000149
9	0.0001375	0.0001789	0.0001541	0.0003859	0.0098717	0.0003332	0.0001259	0.0001805
9	0.0000514	0.0000716	0.0000789	0.0000924	0.0001063	0.0038184	0.0000801	0.0000494

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
9	0.0000385	0.0000253	0.0000473	0.0000213	0.0000136	0.0000347	0.0097467	0.0000100
9	0.0000441	0.0000568	0.0000154	0.0000529	0.0000346	0.0000497	0.0000319	0.0120425
10	0.0005589	0.0000168	0.0000110	0.0000077	0.0000058	0.0000071	0.0000104	0.0000099
10	0.0000035	0.0006992	0.0000074	0.0000060	0.0000032	0.0000024	0.0000025	0.0000028
10	0.0000035	0.0000073	0.0009116	0.0000041	0.0000026	0.0000050	0.0000042	0.0000029
10	0.0000015	0.0000044	0.0000015	0.0005029	0.0000042	0.0000016	0.0000012	0.0000008
10	0.0000074	0.0000119	0.0000103	0.0000223	0.0005562	0.0000247	0.0000076	0.0000101
10	0.0000028	0.0000047	0.0000052	0.0000047	0.0000055	0.0005977	0.0000049	0.0000028
10	0.0000021	0.0000017	0.0000032	0.0000012	0.0000010	0.0000022	0.0006251	0.0000005
10	0.0000024	0.0000039	0.0000010	0.0000031	0.0000018	0.0000032	0.0000020	0.0006968

C.3 The S-submatrices

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
1	0.92943054	0.03802904	0.02030272	0.02390038	0.01677461	0.01963680	0.02392048	0.02260368
1	0.00399334	0.87134695	0.01385778	0.01767744	0.00924686	0.00720209	0.00544325	0.00762420
1	0.00807900	0.01700375	0.91078722	0.01142962	0.00717569	0.01256762	0.01136691	0.00597938
1	0.00417102	0.01156235	0.00350952	0.83854628	0.01221074	0.00509226	0.00263257	0.00187603
1	0.02057114	0.02839478	0.02133961	0.07113873	0.91707418	0.06599107	0.02014853	0.02430227
1	0.00709354	0.01216490	0.01106764	0.01650721	0.01802069	0.86637130	0.01273214	0.00641247
1	0.00521262	0.00371236	0.00586630	0.00323097	0.00309332	0.00551300	0.90853263	0.00131364
1	0.00574063	0.00815638	0.00260431	0.00857003	0.00572134	0.00850091	0.00487696	0.91949624
2	0.95330840	0.02646350	0.01609220	0.01640790	0.01203222	0.01525874	0.01422552	0.01501325
2	0.00663733	0.90899973	0.01095522	0.01086033	0.00664330	0.00644857	0.00292702	0.00643266
2	0.00503095	0.01274926	0.92793928	0.00661550	0.00474150	0.00859891	0.00924903	0.00354033
2	0.00330829	0.00990824	0.00355379	0.89286363	0.00847221	0.00469216	0.00121610	0.00130102
2	0.01760846	0.02322769	0.02036464	0.05216878	0.94477683	0.04940630	0.01622712	0.01797999
2	0.00522921	0.01001304	0.01022287	0.01333903	0.01491264	0.90493214	0.00958746	0.00440367
2	0.00377036	0.00245739	0.00456280	0.00176589	0.00263689	0.00369265	0.94216013	0.00107093
2	0.00380417	0.00491105	0.00304214	0.00460022	0.00460374	0.00590030	0.00316424	0.94373762
3	0.93982732	0.03161889	0.01960049	0.01454820	0.01370690	0.01521528	0.01776015	0.01540420
3	0.00790344	0.88899642	0.01176657	0.01046368	0.00742461	0.00738168	0.00416194	0.00465541
3	0.00632260	0.01365555	0.91260564	0.00739375	0.00598962	0.01237407	0.01200427	0.00455704
3	0.00320016	0.01102873	0.00324506	0.88591093	0.00835254	0.00472526	0.00140426	0.00189644
3	0.02624953	0.03469221	0.03065033	0.06269132	0.93769181	0.06850280	0.02873927	0.02507891
3	0.00633970	0.01046326	0.01136243	0.01052679	0.01672527	0.88008505	0.01376370	0.00544663
3	0.00432303	0.00309328	0.00556883	0.00308681	0.00362997	0.00569916	0.91832663	0.00183052
3	0.00436696	0.00506566	0.00313542	0.00398009	0.00501790	0.00482423	0.00221829	0.93954521
4	0.88584417	0.05352201	0.03042765	0.02624860	0.02653524	0.02641495	0.03327470	0.02487933
4	0.01516319	0.81783932	0.02050549	0.01926360	0.01392990	0.01070994	0.00804152	0.00676954
4	0.01348363	0.02235394	0.35747480	0.01165075	0.01181548	0.02135560	0.02058821	0.00819204
4	0.00563841	0.01372776	0.00413243	0.80785072	0.01346725	0.00730578	0.00366522	0.00332599
4	0.05145656	0.06430326	0.05403659	0.10675109	0.89154613	0.12203234	0.06137639	0.04778250
4	0.01083058	0.01411138	0.01637311	0.01501967	0.02582509	0.79343152	0.02316487	0.00937815
4	0.00712456	0.00469592	0.00952519	0.00512505	0.00633385	0.00953591	0.84352529	0.00315123
4	0.00347164	0.00762916	0.00506749	0.00630797	0.00340605	0.00756622	0.00399621	0.89383001

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
5	0.87303197	0.06351913	0.03436301	0.03460538	0.03237172	0.03572861	0.03874517	0.03301928
5	0.01759708	0.79957307	0.02494578	0.02361403	0.01639417	0.01121853	0.01049200	0.00947186
5	0.01562900	0.02613239	0.84402931	0.01392939	0.01354330	0.02220562	0.02183784	0.00934653
5	0.00701853	0.01343713	0.00449701	0.76703542	0.01720558	0.00854851	0.00585426	0.00391297
5	0.05301012	0.06444525	0.05334064	0.12481617	0.86953330	0.13646445	0.06232560	0.05163080
5	0.01224979	0.01621899	0.01849325	0.01858637	0.03061730	0.76089132	0.02467011	0.01038323
5	0.00875285	0.00567142	0.01159343	0.00538321	0.00719304	0.01206668	0.82621616	0.00381329
5	0.01032040	0.00379623	0.00642512	0.00967365	0.01016330	0.01073972	0.00722297	0.87582350
6	0.91603036	0.04899584	0.02586926	0.02323400	0.02267319	0.03030648	0.02736380	0.02904290
6	0.01157958	0.85116643	0.01917465	0.01304915	0.01176210	0.00913924	0.00764218	0.00753148
6	0.01004612	0.02130539	0.83837751	0.01267850	0.00940715	0.01550037	0.01371501	0.00683135
6	0.00506657	0.01138335	0.00362770	0.82109612	0.01373956	0.00594697	0.00452077	0.00284544
6	0.03125861	0.03723650	0.03276739	0.08996207	0.90255785	0.08923961	0.03033844	0.03548372
6	0.00365857	0.01390365	0.01433187	0.01396154	0.02437792	0.82716948	0.01469522	0.00772447
6	0.00669005	0.00511934	0.00316203	0.00533238	0.00483678	0.01053381	0.89262366	0.00301712
6	0.00760536	0.00658318	0.00481295	0.00798150	0.00743731	0.00917391	0.00597207	0.90365133
7	0.93925267	0.03426847	0.01804665	0.02075065	0.01536015	0.02012953	0.02062355	0.02051643
7	0.00734370	0.89038229	0.01219472	0.01374623	0.00745568	0.00689337	0.00362900	0.00464855
7	0.00660252	0.01685432	0.91310530	0.00817605	0.00644394	0.01293288	0.01027351	0.00396701
7	0.00323046	0.00976386	0.00303001	0.87389851	0.00968990	0.00390936	0.00232980	0.00256952
7	0.02270470	0.02632396	0.02393674	0.05798476	0.93121660	0.06073812	0.02097972	0.02354264
7	0.00610784	0.00977245	0.01166137	0.01261501	0.01752890	0.87841773	0.00925777	0.00558749
7	0.00435663	0.00421628	0.00507073	0.00434937	0.00316903	0.00633181	0.92526704	0.00160704
7	0.00493295	0.00397473	0.00346881	0.00431485	0.00455271	0.00648138	0.00278682	0.93118399
8	0.95174670	0.02212429	0.01423571	0.01237253	0.01065214	0.01236941	0.01413963	0.01510678
8	0.00572818	0.92134001	0.00794365	0.00824737	0.00478985	0.00505193	0.00301480	0.00376631
8	0.00414737	0.01123156	0.93209559	0.00438367	0.00429567	0.00999656	0.00870406	0.00254694
8	0.00221433	0.00693204	0.00265052	0.91050774	0.00733510	0.00317661	0.00180931	0.00174565
8	0.01643516	0.01812363	0.01833914	0.03969535	0.94646364	0.04293479	0.01634135	0.01372352
8	0.00461491	0.00731355	0.00947323	0.01151342	0.01329308	0.91109586	0.00771892	0.00380126
8	0.00317135	0.00174172	0.00390648	0.00204347	0.00221215	0.00403347	0.93743318	0.00113246
8	0.00286926	0.00261158	0.00333385	0.00389028	0.00321873	0.00434860	0.00209578	0.94795328
9	0.95416856	0.01451393	0.00998204	0.00906133	0.00788331	0.00786829	0.00944304	0.00935228
9	0.00442283	0.93824136	0.00550183	0.00623531	0.00359005	0.00300222	0.00237208	0.00274748
9	0.00321011	0.00669322	0.94054355	0.00254359	0.00357054	0.00707998	0.00572608	0.00194800
9	0.00173072	0.00450566	0.00211853	0.92896384	0.00629583	0.00228111	0.00094009	0.00123022
9	0.01127716	0.01366838	0.01361586	0.02995207	0.94808394	0.03232533	0.01153521	0.00935566
9	0.00399801	0.00529243	0.00320646	0.00702116	0.01287447	0.92864645	0.00636524	0.00274200
9	0.00325351	0.00151529	0.00410202	0.00207936	0.00209652	0.00354408	0.94675285	0.00098737
9	0.00242514	0.00155235	0.00220253	0.00219174	0.00279372	0.00260902	0.00127809	0.95386082

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
10	0.95109856	0.01256779	0.00742708	0.00915993	0.00619257	0.00619670	0.00663281	0.00513298
10	0.00363563	0.93480796	0.00402433	0.00652457	0.00268301	0.00211743	0.00202527	0.00197946
10	0.00260064	0.00501053	0.94057679	0.00139563	0.00290095	0.00633894	0.00384309	0.00146836
10	0.00146365	0.00380642	0.00188653	0.92350674	0.00536942	0.00213738	0.00041840	0.00085286
10	0.00787097	0.01379434	0.01103210	0.03112315	0.94433951	0.03238789	0.00867099	0.00607578
10	0.00363748	0.00506030	0.00782599	0.00558040	0.01338715	0.92603278	0.00564575	0.00190612
10	0.00331869	0.00172078	0.00457991	0.00272068	0.00217463	0.00338953	0.94725412	0.00086012
10	0.00219605	0.00120056	0.00143867	0.00150763	0.00265872	0.00183669	0.00073448	0.95354623
11	0.94275081	0.01127167	0.00683515	0.00720043	0.00622060	0.00531774	0.00602654	0.00423937
11	0.00313409	0.92874002	0.00366316	0.00513045	0.00271169	0.00182071	0.00179536	0.00159602
11	0.00218557	0.00435755	0.93313265	0.00153127	0.00207693	0.00533860	0.00354307	0.00113990
11	0.00125355	0.00351525	0.00170805	0.92597836	0.00614439	0.00194032	0.00042336	0.00071004
11	0.00653275	0.01200236	0.00988275	0.02457889	0.93457359	0.02810543	0.00798537	0.00499503
11	0.00309476	0.00468542	0.00752630	0.00437185	0.01379835	0.92270351	0.00539241	0.00155401
11	0.00281767	0.00152369	0.00419055	0.00222541	0.00217601	0.00337335	0.93851340	0.00068893
11	0.00183346	0.00115452	0.00141354	0.00116317	0.00265009	0.00160477	0.00065923	0.94403094
12	0.92553035	0.01091851	0.00725579	0.00466633	0.00603448	0.00333263	0.00556387	0.00399640
12	0.00303767	0.91457462	0.00317934	0.00306674	0.00269498	0.00111532	0.00115142	0.00103916
12	0.00203378	0.00304703	0.91321318	0.00223012	0.00170261	0.00273976	0.00376775	0.00098985
12	0.00103274	0.00330273	0.00141138	0.92414320	0.00673035	0.00139152	0.00098137	0.00062295
12	0.00553593	0.00854662	0.00851193	0.01693964	0.91765958	0.02315459	0.00867906	0.00463271
12	0.00291966	0.00568701	0.01046239	0.00202447	0.01482326	0.91547382	0.00688056	0.00113079
12	0.00297615	0.00132332	0.00433157	0.00199125	0.00204385	0.00290962	0.91893309	0.00047293
12	0.00154957	0.00159209	0.00202704	0.00033525	0.00234591	0.00135464	0.00037016	0.92549283
13	0.89333923	0.01094725	0.00627562	0.00476851	0.00550186	0.00431319	0.00502589	0.00322133
13	0.00315336	0.88409799	0.00232903	0.00400854	0.00278492	0.00124781	0.00060833	0.00053913
13	0.00208547	0.00329732	0.88630319	0.00269634	0.00200393	0.00297640	0.00374936	0.00067466
13	0.00085620	0.00401097	0.00095221	0.89256329	0.00611093	0.00195623	0.00124137	0.00037422
13	0.00574328	0.00716343	0.00613058	0.01326879	0.89128882	0.02366152	0.00705931	0.00390120
13	0.00266630	0.00621954	0.01061933	0.00233257	0.01494056	0.88509256	0.00536036	0.00080392
13	0.00249705	0.00164134	0.00371330	0.00161365	0.00168903	0.00256228	0.88967717	0.00035539
13	0.00127428	0.00148900	0.00209236	0.00076981	0.00185130	0.00135054	0.00022693	0.89596742
14	0.84023291	0.00837939	0.00320962	0.00515102	0.00427534	0.00451687	0.00531629	0.00235578
14	0.00280930	0.83359617	0.00120217	0.00546333	0.00237794	0.00143012	0.00082058	0.00046376
14	0.00193675	0.00339050	0.84321922	0.00198532	0.00132743	0.00414926	0.00409578	0.00046722
14	0.00072769	0.00313378	0.00049388	0.84159189	0.00411422	0.00143799	0.00114054	0.00017742
14	0.00595303	0.00674472	0.00322553	0.01998913	0.85316360	0.02077747	0.00549098	0.00307461
14	0.00220845	0.00476326	0.00551743	0.00363854	0.01173699	0.83942729	0.00395444	0.00066338
14	0.00158511	0.00167239	0.00191550	0.00063147	0.00125633	0.00170151	0.83771044	0.00037368
14	0.00099282	0.00068445	0.00108057	0.00062916	0.00136865	0.00111732	0.00048419	0.84429950

Age Group	North	E.Midland	W.Midland	E.A.	S.E.	S.W.	Wales	Scotland
15	0.76212937	0.00751753	0.00235104	0.00454730	0.00310438	0.00391435	0.00675257	0.00249501
15	0.00237234	0.76603067	0.00086701	0.00482429	0.00173674	0.00124995	0.00099889	0.00050257
15	0.00165243	0.00327641	0.77351207	0.00175344	0.00132583	0.00350056	0.00467793	0.00050336
15	0.00064481	0.00276188	0.00034656	0.77695853	0.00303675	0.00127384	0.00133319	0.00021686
15	0.00511027	0.00570436	0.00229995	0.01781184	0.78726536	0.01763343	0.00632970	0.00310456
15	0.00191417	0.00403552	0.00392907	0.00319056	0.00855277	0.76912445	0.00452781	0.00066025
15	0.00134744	0.00140327	0.00135120	0.00054840	0.00095221	0.00143897	0.75409222	0.00037239
15	0.00087042	0.00054566	0.00075900	0.00054360	0.00098721	0.00093795	0.00054595	0.75855839
16	0.64763387	0.00610838	0.00225814	0.00319335	0.00271452	0.00338325	0.00610351	0.00252142
16	0.00206459	0.65756541	0.00086636	0.00334615	0.00153165	0.00112822	0.00087336	0.00049138
16	0.00142660	0.00270868	0.65787524	0.00117763	0.00116413	0.00306183	0.00424770	0.00049091
16	0.00058561	0.00229210	0.00032281	0.68526125	0.00270048	0.00115979	0.00123443	0.00022852
16	0.00453359	0.00474139	0.00226448	0.01257073	0.68105000	0.01547889	0.00578546	0.00320911
16	0.00167579	0.00333319	0.00385583	0.00225021	0.00755663	0.66115373	0.00410368	0.00069978
16	0.00112224	0.00117409	0.00130311	0.00037039	0.00083120	0.00122140	0.64003307	0.00037684
16	0.00061154	0.00042367	0.00075865	0.00036678	0.00086066	0.00080569	0.00047938	0.63779616
17	0.65063433	0.01038400	0.00366298	0.00553506	0.00470357	0.00536875	0.00950089	0.00400057
17	0.00356675	0.70122641	0.00135101	0.00620072	0.00273023	0.00188857	0.00147589	0.00074701
17	0.00238243	0.00472832	0.63534517	0.00209590	0.00210210	0.00519869	0.00689068	0.00060591
17	0.00112334	0.00433046	0.00075257	0.81379389	0.00532952	0.00215145	0.00230176	0.00036524
17	0.00334235	0.00357538	0.00413353	0.02493373	0.78750733	0.02858902	0.01004298	0.00622775
17	0.00298403	0.00584523	0.00676152	0.00452010	0.01339746	0.72665900	0.00693448	0.00131672
17	0.00179041	0.00208103	0.00216547	0.00089443	0.00145667	0.00213479	0.64415801	0.00064099
17	0.00141276	0.00101682	0.00119282	0.00087819	0.00148216	0.00129367	0.00097518	0.63666618

REFERENCES

Andersson, A.E., and I. Holmberg (1980) *Migration and Settlement: 3. Sweden.* RR-80-5. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Coale, A.J. (1972) *The Growth and Structure of Human Populations: A Mathematical Investigation.* Princeton, N.J.: Princeton University Press.

Feeney, G.M. (1970) Stable Age by Region Distribution. *Demography* 7:341-348.

Gantmacher, F.R. (1960) *The Theory of Matrices.* New York: Chelsea Publishing Company.

Keyfitz, N. (1968) *Introduction to the Mathematics of Population.* Reading, Massachusetts: Addison-Wesley Publishing Company.

Liaw, K.L. (1978a) Derivation and characterization of the analytic solution of the generalized Rogers model of interregional demographic growth. Unpublished paper, Department of Geography, McMaster University, Hamilton, Ontario, Canada.

Liaw, K.L. (1978b) Dynamic Properties of the 1966-1971 Canadian Population System. *Environment and Planning A* 10:389-398.

Liaw, K.L. (1979) Implications of Eliminating Foreign Migration upon the Dynamic Properties of the Canadian Interregional Population System. *Modeling and Simulation.* Proceedings of the Tenth Annual Pittsburgh Conference, University of Pittsburgh, Vol. 10, Part 4, pp. 1369-1375.

Liaw, K.L. (1980a) Multistate Dynamics: the Convergence of an Age-by-Region Population System. *Environment and Planning A* 12:589-613.

Liaw, K.L. (1980b) The Robustness of the Analytic Solution of a Time-Homogeneous Spatial Population System: A Canadian Case Study. Part 4: Social Economic Systems. *Modeling and Simulation*, Vol. 11. Proceedings of the Eleventh Annual Pittsburgh Conference, University of Pittsburgh, pp. 1253-1258.

Liaw, K.L., V. Aresta, and K. George (1979) Major Cyclical Components of the 1966-1971 Canadian Interregional Population System. *Geographical Analysis* 11:109-119.

Rees, P.H. (1979) *Migration and Settlement: 1. United Kingdom.* RR-79-3. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Rogers, A. (1975) *Introduction to Multiregional Mathematical Demography.* New York: John Wiley & Sons.

Rogers, A. (1979) Migration Patterns and Population Redistribution. *Regional Science and Urban Economics* 9:275-310.

Soboleva, S. (1980) *Migration and Settlement: 8. Soviet Union.* RR-80-36. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Theil, H. (1971) *Principles of Econometrics.* New York: John Wiley and Sons.

RELATED IIASA PUBLICATIONS

Andrei Rogers (editor), *Migration and Settlement: Selected Essays*. RR-78-6. Reprinted from a special issue of *Environment and Planning A*.

Andrei Rogers and Frans Willekens, *Migration and Settlement: Measurement and Analysis*. RR-78-13.

Frans Willekens and Andrei Rogers, *Spatial Population Analysis: Methods and Computer Programs*. RR-78-18.

Andrei Rogers, *Migration Patterns and Population Redistribution*. RR-80-7. Reprinted from *Regional Science and Urban Economics*.

Andrei Rogers (editor), *Essays in Multistate Mathematical Demography*. RR-80-10. Reprinted from a special issue of *Environment and Planning A*.

Nathan Keyfitz, *Multidimensionality in Population Analysis*. RR-80-33. Reprinted from *Sociological Methodology 1980*.

Andrei Rogers (editor), *Advances in Multiregional Demography*. RR-81-6.