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PREFACE

For several years the activities of the Regional Development Task at the International Institute for Applied Systems Analysis (IIASA) have been directed towards the development of a system of regional models, the elements of which were elaborated over the period 1977-1979. The final stage of the work, which involves the coordination of these individually developed models, is now nearing completion. However, before this system can become fully operational, three major problems have to be overcome. They concern the modeling approach, level of aggregation, and method of coordination to be used. The linkage problem is examined in this paper.

LINKAGE OF REGIONAL MODELS

Murat Albegov*
Alexander Umnov

INTRODUCTION

Regions** have complex economic structures and, in most cases, a specific set of future development problems. The number of aggregated sectors of a regional economy can include 10 upwards. If sectoral development is considered in multidimensional terms, it requires the solution of numerous problems. It is, therefore, impossible to describe a system of models that embraces all problems and is appropriate for all regions. However, in general only a small number of key sectors of the regional economy influence its future development.

An approach that seems to be suitable for dealing with all types of regions is one that includes module-type descriptions of all the more important sectors of a regional economy. A limited number of these modules can then be selected, adapted, and linked to form a system of models that reflects the urgent development problems of the region under analysis. This approach implies that each module should be sufficiently general to be widely applicable and yet at the same time flexible enough for adaptation to the specific problems of different regions.

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**In this paper the region is treated as a unified territory, which is homogeneous with respect to economic, social, environmental, and institutional problems.

The advantage of adopting the approach described above is clear: the general sectoral modules serve as a basis for developing widely applicable models. The model system should then be formed only from those modules that are essential for solving the problems of the given region.

GENERAL APPROACH

There are many approaches to regional development modeling. One possible method of classifying these approaches is to examine the sequence of analysis adopted. External or internal problems are generally the starting points for an iterative procedure. Thereafter, the 'bottom-up' approach is used (for details, see Andersson and Philipov 1979, pp. 33-69).

This approach is based on the assumption that the marginal costs for commodities produced and resources used as well as the data for determining regional in- and out-migration flows (average national salary, dwelling space per capita, etc.) is known (Figure 1). The starting point is the analysis of the regional specialization problem. At level 2, intraregional location problems are solved, followed by an analysis of labor and financial balance problems at level 3. Finally, at level 4, problems connected with environmental quality control as well as settlements and service provision are considered. In this scheme, coordination between levels I and II and levels III and IV is essentially that of estimating future regional economic growth and the size of the labor force.

As can be seen from Figure 1, the scheme includes many blocks (models) and is rather complicated to compute. However, the number of urgent problems to be solved in a given region is usually fairly small. For example, a discussion between IIASA members and local decision makers for the Silistra region (Bulgaria) revealed that there are only six objectives for this region:

1. To maximize regional agricultural production. This should involve not only the maximization of meat and grain production, for which the area is particularly well suited, but also the increase of local crop production (apricots, grapes, and vegetables).

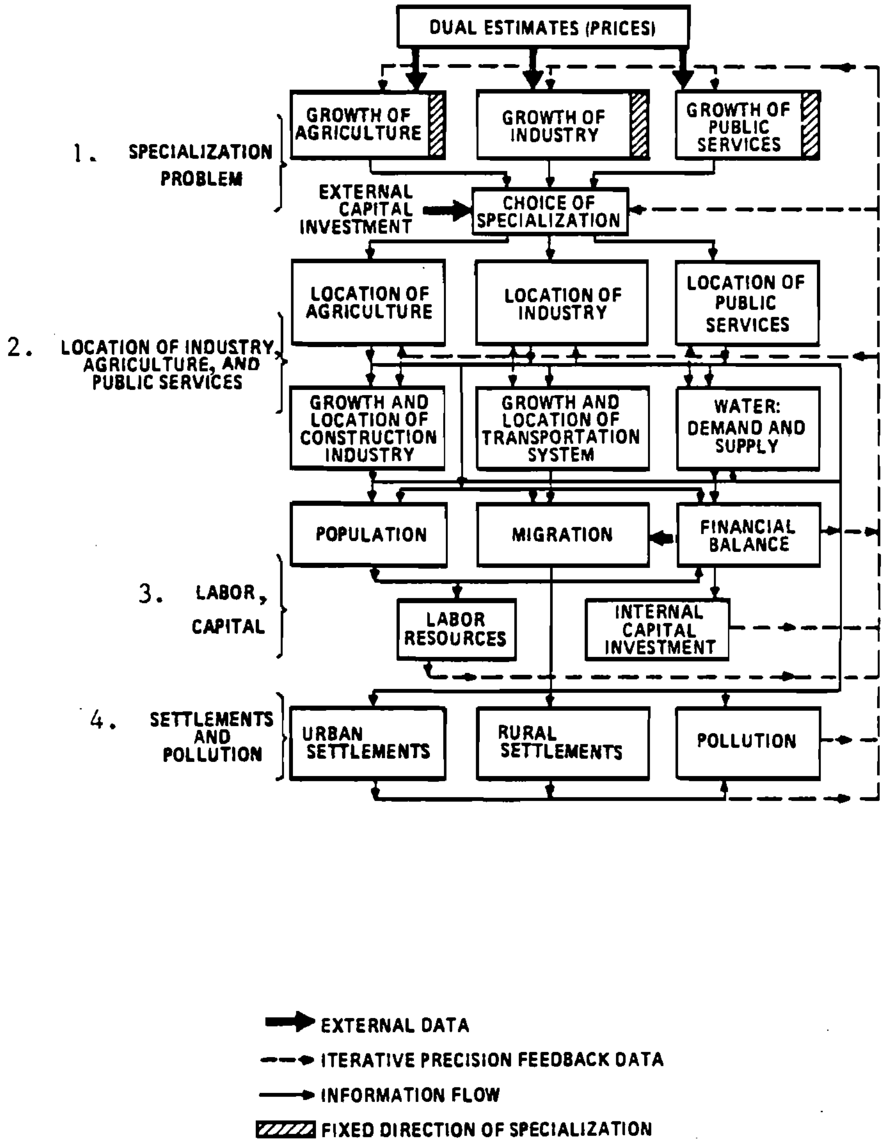


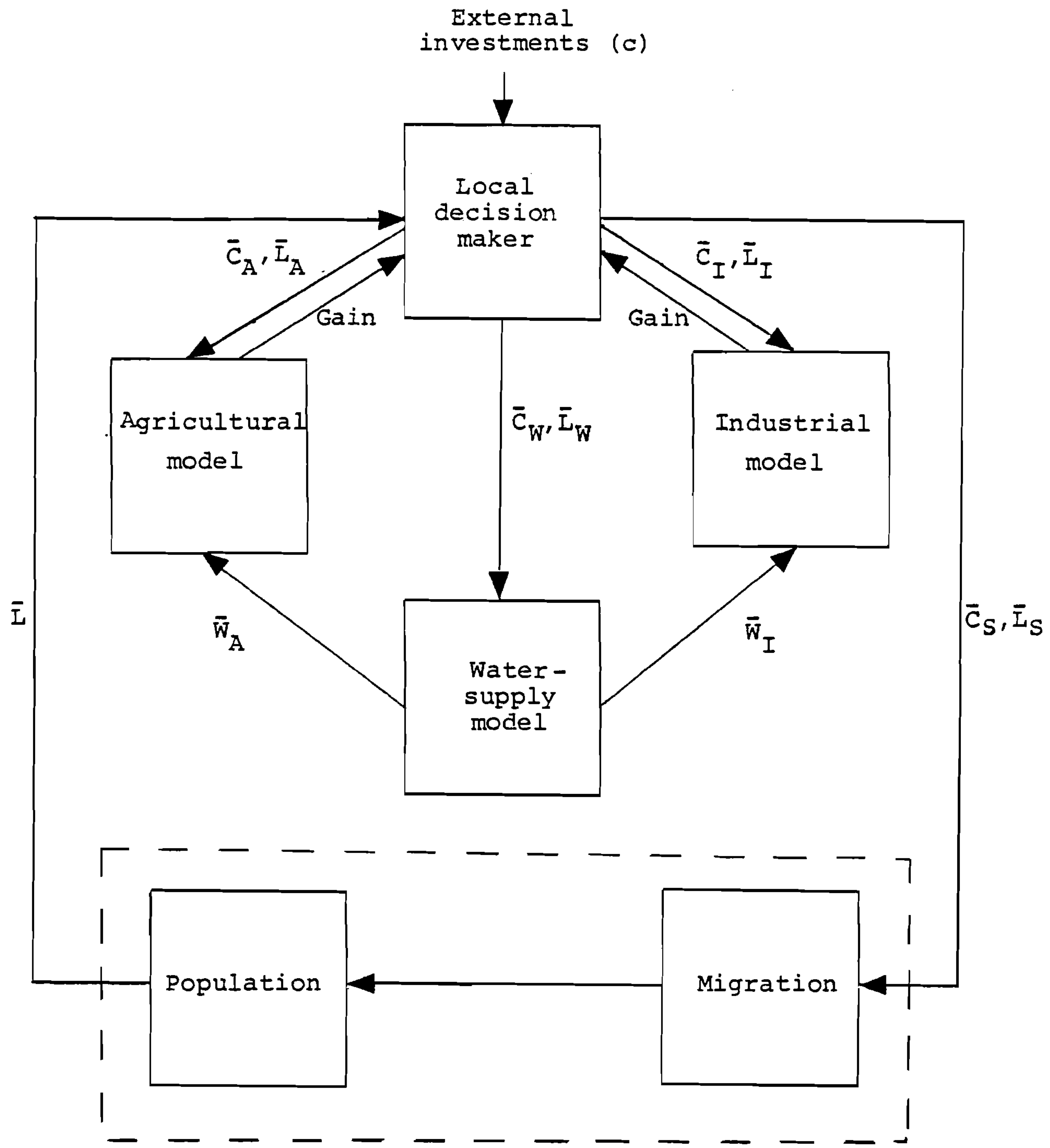
Figure 1. The bottom-up approach.

2. To develop an irrigation system that will enable local agriculture to achieve optimal production efficiency.
3. To develop local industry to complement local agriculture. This should include the development of some branches of industry that have the potential for growth in the region and that would help to balance labor demand and supply.
4. To maximize the productive use of labor resources in local agriculture, thereby restricting rural-urban migration.
5. To develop a system of settlements and public services. Above all, the existing stock of rural dwellings should be fully utilized and the road network, the health care system, etc. should be improved.
6. To develop local agriculture and industry such that no serious environmental problems result and to create a recreational area in the region.

Thus, it is clear that for the Silistra region it is essential to coordinate analyses of regional agriculture, industry, water-supply, services, and migration. It should be remembered that the decision maker may wish to change the distribution of some resources (for example, capital investments) in the model or to assess (using the computer) the consequences of different policies, etc.

The scheme presented in Figure 2 shows the individual regional models that were linked to form a system. This scheme allows the gain from industrial and agricultural activities to be maximized after different types of resources (including external investments) and productive activities have been balanced. Three types of resources are included:

-- capital investments (which are shared between production and services;
-- labor resources (for which equilibrium can be achieved by regulating the share of services);
-- water resources, which should satisfy agricultural and industrial demands (water consumption in the settlement system is fixed).



$\bar{C}_A, \bar{C}_I, \bar{C}_W, \bar{C}_S$ = vectors of interregional distribution of capital investments to the agricultural, industrial, water-supply, and service sectors;

$\bar{L}_A, \bar{L}_I, \bar{L}_W, \bar{L}_S$ = vectors of interregional distribution of labor to the agricultural, industrial, water-supply, and service sectors;

$\bar{W}_A, \bar{W}_I$ = subregional water flows to agriculture and industry;

$\bar{L}$ = vector of subregional labor;

→ = information flows.

Figure 2. A simplified system of regional models.

In this scheme, although the individual blocks may be sufficiently detailed to describe real sectoral problems, it is generally necessary to choose an appropriate level of aggregation for each one in order to make the whole system operational. Both detailed and aggregated sectoral models may be used to complement this scheme.

As can be seen from Figure 2, it is necessary for the local decision maker to supply the models with intraregional data on the distribution of capital investments, labor, water resources, production patterns, etc. The distribution for a given case obviously depends on the natural and economic conditions in the region, but in general the region should be divided into 10-20 subregions.

This scheme is relatively flexible because it permits changes to be made to the resource allocation, the addition of constraints, or variation of the objective function:

1. The shares of the productive and service sectors may be changed by the decision maker.
2. The objective function coefficients may be weighted in accordance with the decision maker's preferences.
3. Constraints on resource consumption by certain sectors could be included.
4. The specification of certain goods produced in some sectors can easily be introduced (for example, to attain the predetermined production targets).
5. The scheme and/or the coordination procedures could be changed to correspond to the specific set of problems of a given region.

Requirement 5 implies that each module in the set represents a general description of a particular sector of the regional economy.

COMPLETED MODULES

Work on generally applicable descriptions of the most important sectors of the regional economy began in 1977. Since this work could not be fulfilled by IIASA's Regional Development Task (RD) alone, Task members, while continuing their own activities, made an attempt to find suitable models completed by other groups at IIASA or outside the Institute. The main criteria for the selection was that the models should be generally applicable and supplied with the necessary computer software.

As a result the following combination of models was used:

-- Generalized Regional Agricultural Model (GRAM), elaborated in RD;
-- Regional Water-Supply Model, elaborated jointly in RD and the Resources and Environment Area (IIASA);
-- Migration Model, elaborated jointly in RD and the Human Settlements and Services Area (IIASA);
-- Model of Population Growth, elaborated in HSS;
-- Generalized Industrial Model, elaborated in Moscow at the Central Economics and Mathematics Institute (CEMI).

Although only these five models were included in the system of regional models, it is possible to add others as required.

Generalized Regional Agricultural Model

The Generalized Regional Agricultural Model (GRAM) has already been presented in detail in Albegov (1979), therefore only its main features are discussed below.

GRAM was originally developed for intraregional agricultural problem analysis in RD's Silistra and Notec case studies. It is a general model and is not intended to replace specialized agricultural models designed to solve specific problems. Rather it should be treated as a tool for examining general agricultural problems in the framework of comprehensive regional analysis. The character of this model is revealed in the variables it contains, as given below:

P_{iprl} = volume of crop i purchased for animal feed on market l by property p in subregion r;

Q_{iprl} = volume of crop i purchased for human consumption on market l by property p in subregion r;

Q_{mprl} = volume of livestock product m purchased for human consumption on market l by property p in subregion r;

R_{iprl} = volume of crop i sold on market l by property p in subregion r;

R_{mprl} = volume of livestock product m sold on market l by property p in subregion r;

W_{ipr} = human consumption of crop i on property p in subregion r;

W_{mpr} = human consumption of livestock product m on property p in subregion r;

$X_{iprs\alpha}$ = volume of first harvest of crop i on property p on land α in subregion r, when technology s is used;

$X_{jkprs'}$ = number of livestock j of specialization k on property p in subregion r, when technology s' is used;

$Y_{iprs\alpha}$ = volume of second harvest of crop i on property p on land α in subregion r, when technology s is used;

Z_{ipr} = consumption by livestock of crop i on property p in subregion r;

Z_{mpr} = consumption by livestock of livestock product m on property p in subregion r.

The set of constraints contained in GRAM relates to land-use conditions, the forage balance, human consumption, production limits, etc. (Table 1). Each group of constraints consists in several inequalities; take, for example, land use (a, b, c, d, e):

-- constraint on arable land for the region as a whole;
-- constraint on arable land according to types of property;
-- constraint on area of land occupied by plants;
-- constraint on area of land that can be improved by irrigation, terracing, etc.;
-- constraint on area of pastures and meadows.

The model can be used to analyze the following problems:

-- regional agricultural specialization;
-- different types of production (crop, livestock, market gardening, etc.) in disaggregated form;

Table 1. List of constraints included in GRAM.

Type of Equation	VARIABLES: Plants $X_{iprs\alpha}$	Plants $Y_{ipr s\alpha}$	Livestock $X_{jkprs'}$	Human Consumption W_{ipr}	Human Consumption W_{mpr}	Livestock Consumption Z_{ipr}	Livestock Consumption Z_{mpr}	Purchases Q_{iprl}	Purchases Q_{mprl}	Purchases P_{iprl}	Sale R_{iprl}	Sale R_{mprl}	CONSTRAINTS: Lowest	Highest	Equity
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Land use															
a)	1													L_{pr}	
b)	1												$L_{pr\alpha}$	$L_{pr\alpha}$	
c)	1												L_{ipr}	$L_{pr\alpha}$	
d)	1												$L_{prs\alpha}$	$L_{prs\alpha}$	
e)	1													L_{mpr}	
Forage balance															
a)	1	1		-1		-1		1	1		-1				0
b)	1	-1											0		
c)			1		-1		-1		1						0
d)			-1			1	1				1		0		
Human consumption															
a)				1									F_i		
b)					1								F_m		
Production limits															
a)	1	1											F_{ipr}	F_{ipr}	
b)			1										M_{jpr}	M_{jpr}	
Resource constraints															
a)	1	1	1											B_{pr}	
b)	1	1	1											B	
c)	1	1	1											D_{pr}	
d)	1	1	1											$\hat{D}_{pr}$	
e)	1	1	1											D	
f)	1	1	1											$\hat{D}$	
g)	1	1												E	
h)	1	1	1											G_f	
i)	1	1	1											E	
j)	1	1	1											G_{fpr}	
Purchases															
a)									1					H_{il}	
b)							1							I_{il}	
c)								1						I_{ml}	
Sales															
a)											1			$\bar{I}_{il}$	
b)												1		$\bar{I}_{ml}$	
Financial constraints															
a)	1	1	1											C_{pr}	
b)	1	1	1											C	
c)	-1	-1	-1					-1	-1	-1	1	1	$\bar{W}_p B_{pr}$		

Notation to Table 1.

B = maximum amount of labor available in the whole region;

B_{pr} = maximum amount of labor available on property p in subregion r;

C = total (external and internal) capital investment available for regional agriculture;

C_{pr} = total (external and internal) capital investment available for agriculture for property p in subregion r;

D = maximum annual water supply available in the whole region;

$\hat{D}$ = maximum water supply available at peak periods in the whole region;

D_{pr} = maximum annual water supply available for property p in subregion r;

$\hat{D}_{pr}$ = maximum water supply available at peak periods for property p in subregion r;

E = maximum amount of agricultural machinery available for the whole region;

F_i, F_m = consumption of crop i and livestock product m, respectively, in the whole region;

F_{ipr} = production of crop i on property p in subregion r;

G_f = maximum volume of fertilizer f available in the whole region;

G_{fpr} = maximum volume of fertilizer f available for property p in subregion r;

H_{il} = maximum volume of external purchases of crop i on market l for livestock in the whole region;

I_{il}, I_{ml} = maximum volume of external purchases of crop i and livestock product m, respectively, on market l for human consumption in the whole region;

$\bar{I}_{il}$, $\bar{I}_{ml}$ = sale limitation of crop i and livestock product m, respectively, on market l;

L_{ipr}, L_{mpr} = area of land (state, collective, or private) that, in accordance with crop rotation, could be used for crop i and livestock product m, respectively, on property p in subregion r;

L_{pr} = maximum area of arable land on property p in subregion r;

$L_{pr\alpha}$ = area of land α available on property p in subregion r;

$L_{prs\alpha}$ = minimum and maximum area of land α on property p in subregion r that can be improved using technology s;

M_{jpr} = production of livestock j on property p in subregion r;

$\bar{W}_p$ = minimum wage level per capita on property p.

-- land-use problems, with reference to irrigation, drainage, etc.;
-- choice of animal-feed compositions (protein, rough and green forage, etc.);
-- choice of crop-rotation conditions;
-- availability of regional supplies of labor, capital investment, fertilizers, water, etc.

Specially elaborated growth operators help to generate GRAM's matrix, which includes hundreds of inequalities.

Regional Industrial Model

The model developed by teams at the Central Institute of Economics and Mathematics in Moscow was used as a prototype for the Generalized Industrial Model (Mednitsky 1978). Descriptions of many resources and final products, nonlinear dependencies of costs on production scale, transportation of different products, etc. may be included in this model.

To describe the main ideas of their model, which is modified with respect to intraregional problems, the following notations were introduced:

i = index of product;

l = possible location of production units within the region under analysis;

s = points where demand is concentrated (within the region and on the boundaries);

r = variants in production unit capacity;

E = rate of return on capital investment;

I_1 = set of transportable commodities;

I_2 = set of nontransportable commodities;

z_i^0 = final demand (within and outside the region) for product i;

a_i^l = fixed demand for transportable commodities i at point l;

c^i_{lr} = unit cost at point l for production of commodity i under variant r;

K^i_{lr} = capital investment per unit of particular commodity i at point l under variant r;

f^i_{lr} = local resources available for producing commodity i under variant r at point l;

T^i_{ls} = cost of transportation (of particular commodity i) from point l to point s;

Z^i_l = consumption of local resources i at point l;

A^i_{lr} = matrix of inputs of transportable commodities;

F^i_{lr} = matrix of inputs of nontransportable commodities;

B^i_{lr} = matrix of outputs of commodities;

L^i_{lr} = vector of intensity of production of commodity i under variant r at point l;

U^i_{ls} = vector of volume of transport of commodity i from point l to point s;

σ^i_{lr} = integer variables that indicate whether variant r should be used at point l for producing commodity i.

The following constraints are included in the model. Demand for transportable resources within or outside the region under analysis should be satisfied:

$$\sum_{l,r} B^i_{lr} L^i_{lr} \geq Z^0_i, \; i \in I_1 \quad . \tag{1}$$

Local demand for nontransportable resources should also be satisfied:

$$\sum_r B^i_{lr} L^i_{lr} \geq Z^i_l, \; i \in I_2 \quad . \tag{2}$$

The transport volume must correspond to the amount of transportable commodities at each point of production:

$$\sum_r B^i_{lr} L^i_{lr} = \sum_s U^i_{ls}, \; i \in I_1 \quad . \tag{3}$$

Fixed demand for transportable commodities and additional demand from new enterprises at each point should be satisfied:

$$a^l_i + \sum_r A^i_{lr} L^i_{lr} = \sum_s U^i_{ls}, \; i \in I_1 \quad . \tag{4}$$

Local consumption of nontransportable resources should be confined to the available supply:

$$F^i_{lr} L^i_{lr} \le f^i_{lr} \sigma^i_{lr}, \; i \in I_2 \quad . \tag{5}$$

Variable are nonnegative and some are integers:

$$L^i_{lr} \ge 0 \quad , \quad U^i_{ls} \ge 0 \quad , \quad \sigma^i_{lr} = \{0 \text{ or } 1\} \quad . \tag{6}$$

It is possible to modify the objective function; the modification most frequently used will be minimization of production and transportation costs:

$$\min \{ \sum_{l,r} [(c^i_{lr}, L^i_{lr}) + EK^i_{lr} \sigma^i_{lr}] + \sum_{l,s} (T^i_{ls}, U^i_{ls}) \} \quad . \tag{7}$$

The model (1)-(7) may be useful in several cases. But if it is inconvenient for a particular case, a special model may be developed for inclusion in the model system.

Water-Supply Model

The Water-Supply Model was described in detail in Albegov and Chernyatin (1978), therefore only its main characteristics are presented below. The principal assumptions are:

1. The water requirements, which are distributed over time (by seasons) and space, are predetermined by the location of industrial and agricultural activities.
2. Water resources for the water-distribution systems are unlimited (mainstream water regulation is not analyzed here).

3. All water users consume water resources irreversibly.
4. Only within-year water-resource regulation is considered.
5. Time delays for water transit are not taken into account.

The main goal of the model is to meet water requirements for a given period with minimum costs. Water-quality problems are not considered. The equations of this model are derived by applying a mass balance for every node and every reservoir, upper and lower bounds for nodes, reservoirs, pumping stations, and canals are specified. The objective function is to minimize the sum of reduced costs for construction and operation.

This model has the following advantages:

1. Any configurations of the system can be considered.
2. Regional space may be represented by a number of subregions.
3. The model takes into account seasonal irregularities in water consumption.
4. The matrix growth operator facilitates implementation of the model.

Population-Growth and Migration Models

Because of the intraregional character of the analysis, sequential labor-force analysis is required at a regional as well as at a subregional level. For this reason, the following set of calculations should be performed, using the population and migration models:

- -- calculation of in- and out-migration for the region as a whole;
- -- calculation of the future population for the region as a whole;
- -- calculation of the future population for the multisubregional system;
- -- calculation of future regional and subregional labor.

The population-growth and migration models, elaborated in HSS (Willekens and Rogers 1978), are rather general. Nevertheless, the migration model may require certain modifications depending on the conditions of the region under analysis.

For example, after some investigations (Andersson and Philipov 1979) it was decided that the migration model used for the Silistra region in Bulgaria should take the form

$$P_{ij} = \frac{\exp(v_j)}{\exp(v_i)+\exp(v_j)} = \frac{1}{\exp(v_i-v_j)+1} \quad , \tag{8}$$

where

P_{ij} = probability of moves from region i to region j;

v_i, v_j = utilities for region i and j, respectively.

The form of the function v suggested is

$$v_i = \sum_{k=1}^{n} \alpha_{ik} X_{ik} + \beta_i \quad , \tag{9}$$

where

X_{ik} = characteristics of region i;

α_{ik}, β_i = coefficients to be estimated by an econometric approach.

The results of the regional migration model can be plugged into the regional population-growth model to obtain a forecast of the total regional population. The regional migration rate can be changed annually, depending on the results of the migration model runs. The age and sex structure of migrants can be assumed to be the same as for the previously observed period.

Taking the data on regional population growth as given, intraregional population growth can be analyzed. The Willekens/Rogers model (1978) can be used for this purpose:

$$\{\underline{K}^{t+1}\} = G\{K^t\} \quad , \tag{10}$$

where

$\{\underline{K}^t\}$ = age and subregional distribution of the population at time t;

$\underline{G}$ = multiregional (in this case, multisubregional matrix growth operator or generalized Leslie matrix);

t+1 = time period following t (usually 5-year periods are analyzed).

From the results for each time period and each subregion, the population number and its age structure (and if necessary, its sex structure) can be obtained.

Regional and subregional population and its age/sex structure forms the basis for assessing the labor force. Subregional labor can easily be calculated by accounting for the possible changes in the proportion of the total population constituted by the labor force and should be considered as a constraint on regional growth.

MODEL LINKAGE

The idea of a model-linkage procedure was described in Umnov (1979) and, therefore, it will only briefly be discussed here to aid the reader's general understanding of the calculation procedure and the possibilities offered by the use of this method.

The linkage models are formally described by two sets of numbers. The first set, 'variables,' presents the state of the subject to be modeled. The second set, 'parameters,' gives the external conditions of the subject, i.e. the state of its 'environment.' Only finite-dimensional optimization models are considered. It is assumed that there is a common criterion, which is expressed by the variables and parameters, for all models. The aim is to find values of the variables and parameters that are optimal for the common objective, subject to the models that are to be used as independent software units.

The main idea of the linkage procedure may be formulated as follows. Since the optimal state of the model (in the sense of its objective) generally depends on the values of its parameters, it is possible to assume (with some additional conditions) that there exist parameter values that will provide all the models with the optimal state for the common objective; for example, when the common criterion is a convex function of the objectives of the models.

Let us assume that the linked models can be written in the form:

Minimize with respect to $x^k \in E^{n^k}$

$$f^k(x^k,v) \quad , \tag{11}$$

subject to

$$Y_s^k(x^k,v) \geq 0 \quad , \qquad s = \overline{1,m^k}^{*} \quad ,$$

$$v \in E^L \quad , \tag{12}$$

where

x^k = variable vector of the model k;

v = linkage parameter vector (common for all models);

m^k = number of constraints of model k;

L = number of linkage parameters.**

We shall also assume that all functions $f^k(x^k,v)$ and $Y_s^k(x^k,v)$ are defined and are sufficiently time differentiable with respect to all their arguments.

It is possible that a set of constraints for components of the linkage vector v exists. Let it be

$$R_q(v) \geq 0 \quad , \qquad q = \overline{1,M} \quad , \tag{13}$$

where M is the number of constraints that we refer to as common constraints.

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*The system of constraints may also contain equalities, but this does not present any problems.

**Vector v contains only parameters that are used for linkage.

As mentioned above, the common criterion must be a convex function of the models' objectives. However, without losing the general character of the scheme, we can consider the common objective as a linear combination of all these objectives, which has positive weight coefficients in the form

$$\sum_{k=1}^{N} \lambda_k f^k(x^k,v) \quad , \tag{14}$$

where N is the number of linked models.

We can now formulate the mathematical programming problem.

Minimize with respect to all x^k and v

$$\sum_{k=1}^{N} \lambda_k f^k(x^k,v) \quad , \tag{15}$$

subject to

$$Y_s^k(x^k,v) \geq 0 \quad , \qquad s = \overline{1,m^k} \quad , \tag{16}$$

$$k = \overline{1,N} \quad ,$$

$$R_q(v) \geq 0 \quad , \qquad q = \overline{1,M} \quad , \tag{17}$$

the solution of which $\{x^{*k},v^*\}$ gives us the desirable values of the variables and parameters.

Problem (15) has $L + \sum_{k=1}^{N} n^k$ variables and $M + \sum_{k=1}^{N} m^k$ constraints. Thus, its dimensions are sufficiently large even for the simplest practical case. Our aim is to try to simplify problem (15) as far as possible, using the solutions to problems (11) for the fixed values of the linkage parameters.

Let the dependencies of the optimal x^k of v be expressed by $x^{*k}(v)$. Substituting them into (15), we have a new problem.

Minimize with respect $v \in E^L$ (only)

$$\sum_{k=1}^{N} \lambda_k f^k(x^{*k}(v),v) \quad , \tag{18}$$

subject to

$$R_q(v) \geq 0 \quad , \quad q = \overline{1,M} \quad . \tag{19}$$

Constraints $Y_s^k(x^{*k}(v),v) \geq 0$ are omitted here because $x^{*k}(v)$ are the solutions of (11) and all constraints of the problems are to be satisfied by their solutions.

As Geoffrion (1970) has shown, v^* is the solution of problem (18). In other words, the following relations are valid:

$$x^{*k} = x^{*k}(v^*) \quad . \tag{20}$$

Thus, we can independently obtain all optimal (in the usual sense) points for models (11), as soon as we find the solution to problem (18).

Problem (18) is the central consideration. The procedure for solving this problem was previously referred to as the linkage process. Therefore, the method of solution will define the content and volume of informational exchange between the linked models.

Although problem (18) has formally fewer dimensions than (15), there are difficulties (in addition to the usual problems encountered) that prevent us from using standard schemes for solution:

1. It is impossible to find explicit expressions for $x^{*k}(v)$, except perhaps in some cases of no practical interest.
2. Functions $x^{*k}(v)$ are not defined for any v satisfying (12), since problems (11) can have no feasible solution for some v.
3. Functions $x^{*k}(v)$ are not differentiable at some points of E^L.

It is necessary to emphasize that all the difficulties result from the distributed scheme. The simplest way of avoiding them is to merge all models (11). However, we consider a situation in which this is unreasonable or, simply, impossible. Therefore, we need to find another approach to solving problem (18).

There have been attempts to solve such types of problems, where each of the difficulties mentioned above was overcome by different methods (Geoffrion 1970, Ermoliev 1980). Here another approach, which permits us to resolve all the obstacles by means of a single method, will be considered.

The approach consists in substituting into (15), instead of $x^{*k}(v)$, new functions $x^{-k}(v)$, which:

-- are defined at any $v \in E^L$;

-- are differentiable for all $v \in E^L$;

-- have values, which are close to values of $x^{*k}(v)$ for all v, where $x^{*k}(v)$ exists.

Instead of using functions $\bar{x}^k(v)$, we can use the solutions of problems (11), which are found by employing a 'smooth' version of the Penalty Function Method, or the SUMT (see, for example, Fiacco and McCormick 1968). The method replaces problems (11) by an unconstrainedminimization of the following auxiliary function:

$$E^k(x^k,v) = f^k(x^k,v) + \sum_{s=1}^{m^k} P(T,Y_s^k(x^k,v)) \quad , \tag{21}$$

where $P(T,\alpha)$ is the penalty function, which satisfies the relation

$$\lim_{T\to+0} P(T,\alpha) = \begin{cases} 0, & \text{for } \alpha > 0 \\ +\infty, & \text{for } \alpha < 0 \end{cases} \quad . \tag{22}$$

In other words, $\bar{\infty}(v)$ is a point at which function (21) has a minimum.

From the properties of the SUMT, $\bar{x}^k(v)$ are defined for all $v \in E^L$, because the auxiliary functions (21) have the minimum both for feasible and infeasible problems (11).

For all points, where $x^{*k}(v)$ exist, the following equality is valid:

$$\lim_{T\to+0} \bar{x}^k(T,v) = x^{*k}(v) \quad . \tag{23}$$

Therefore, values of $\bar{x}^k(v)$ and $x^{*k}(v)$ are close.

The stationary condition for function (21) is

$$\operatorname{grad}_{x^k} E^k(\bar{x}^k, v) = 0 \quad . \tag{24}$$

If all functions $f^k(x^k,v)$, $Y_s^k(x^k,v)$, and $P(T,\alpha)$ are sufficiently smooth, it would be possible to apply the well-known implicit function's theorem to equation (24) and to discover that $\bar{x}^k(v)$ is differentiable for any $v \in E^L$.

It is as difficult to find the explicit form of $\bar{x}^k(v)$ as it is to find the explicit form of $x^{*k}(v)$. Hence, we use a numerical algorithm that does not require $x^{*k}(v)$ to be stated explicitly, but needs only some numerical evaluations (such as values of functions and their derivatives), to solve problem (18). Finally, we obtain a new problem.

Minimize with respect to $v \in E^L$

$$\sum_{k=1}^{N} \lambda_k f^k(\bar{x}^k(v), v) \quad , \tag{25}$$

subject to

$$R_q(v) \geq 0 \quad , \qquad q = \overline{1,M} \quad , \tag{26}$$

where $\bar{x}^k(v)$ are the solutions of (24).

The direct solution of the problem may require 'know-how' to calculate values of the first (and perhaps the second) derivatives of $\bar{x}^k(v)$. It is a difficult computational problem, but there is a way of simplifying the procedure slightly.

Let us return to problem (15), which we also solve by the same 'smooth' version of the SUMT. The auxiliary function in this case will be written as

$$E = \sum_{k=1}^{N} \lambda_k f^k(x^k, v) + \sum_{k=1}^{N} \lambda_k \sum_{s=1}^{m^k} P(T, Y_s^k(x^k, v)) + \sum_{q=1}^{M} P(T, R_q(v)) \quad . \tag{27}$$

Multiplying the penalty terms of E^k by positive number λ_κ does not change the situation.

Let us substitute $\bar{x}^k(v)$ into E to reduce its dimensions:

$$\bar{E}(T,v) = w(T,v) + \sum_{k=1}^{N} \lambda_k E^k(T,\bar{x}^k(v), v) \quad , \tag{28}$$

and let $\bar{v}(T)$ be a minimum point of (28). Then

$$\lim_{T \to +0} \bar{v}(T) = v^* \quad , \tag{29}$$

i.e., we can find the optimum (with some small errors) by minimizing function (28). The problem of accuracy is theoretically interesting and difficult (see Umnov 1974, 1975, and 1979), but has little practical value in the scheme.

Now we give the formula for calculating the first and second partial derivatives of $\bar{E}$ with respect to components of v.

For the first derivatives:

$$\frac{\partial \bar{E}}{\partial v_r} = \frac{\partial E}{\partial v_r} + \sum_{k=1}^{N} \sum_{i=1}^{n^k} \frac{\partial E}{\partial x_i^k} \frac{\partial \bar{x}_i^k}{\partial v_r} \quad . \tag{30}$$

But taking (24) into consideration, we obtain simply

$$\frac{\partial \bar{E}}{\partial v_r} = \frac{\partial E}{\partial v_r} \quad , \qquad \text{for all } r = \overline{1,L} \quad . \tag{31}$$

In an analogous way, for the second partial derivatives:

$$\frac{\partial^2 \bar{E}}{\partial v_r \partial v_p} = \frac{\partial^2 E}{\partial v_r \partial v_p} + \sum_{k=1}^{N} \sum_{i=1}^{n^k} \frac{\partial^2 E}{\partial v_r \partial x_i^k} \frac{\partial \bar{x}_i^k}{\partial v_p} + \sum_{k=1}^{N} \sum_{i=1}^{n^k} \frac{\partial E}{\partial x_i^k} \frac{\partial^2 \bar{x}_i^k}{\partial v_r \partial v_p}$$

$$+ \sum_{k=1}^{N} \sum_{i=1}^{n^k} \left(\frac{\partial \bar{x}_i^k}{\partial v_r} \frac{\partial^2 E}{\partial x_i^k \partial v_p} + \sum_{j=1}^{n^k} \frac{\partial^2 E}{\partial x_i^k \partial x_j^k} \frac{\partial \bar{x}_j^k}{\partial v_p} \right) \quad . \tag{32}$$

Once again using (24) and taking into account that, after differentiation of (24) with respect to v_p, we will have the relation

$$\sum_{j=1}^{n^k} \frac{\partial^2 E^k}{\partial x_i^k \partial x_j^k} \frac{\partial \bar{x}_j^k}{\partial v_p} = - \frac{\partial^2 E^k}{\partial v_p \partial x_i^k} \quad , \quad \text{for all } p = \overline{1,L} \text{ and } K = \overline{1,N} \quad , \tag{33}$$

we then find that

$$\frac{\partial^2 \bar{E}}{\partial v_r \partial v_p} = \frac{\partial^2 E}{\partial v_r \partial v_p} + \sum_{k=1}^{N} \sum_{i=1}^{n^k} \frac{\partial^2 E}{\partial v_r \partial x_i^k} \frac{\partial \bar{x}_i^k}{\partial v_p} \quad , \tag{34}$$

for all $r = \overline{1,L}$ and $p = \overline{1,L}$,

because

$$\frac{\partial E}{\partial x_i^k} = \frac{\partial E^k}{\partial x_i^k} \quad . \tag{35}$$

The formulae (31) and (34) allow us to minimize (28) by any of the standard procedures, using partial derivatives of the first and second order of $\bar{E}$. However, these procedure require use of the first order only to obtain values of components for $\bar{x}^k(v)$ and the matrix $\frac{\partial \bar{x}^k}{\partial v}$, which is called the sensitive matrix.

The linkage scheme consists in the following procedure. For a starting point $v \in E^L$, all models independently generate $\bar{x}^k(v)$, $E^k(\bar{x}^k(v), v)$, and the sensitive matrix $\frac{\partial \bar{x}^k}{\partial v}$. On the basis of this information, the central processor finds a new point in E^L according to the optimizational procedure selected. Usually, the iteration can be written as

$$v = v_o + s\omega \quad , \tag{36}$$

where

v_o = the starting point;

v = the new 'better' point;

ω = the direction of minimization;

s = the length of the step along ω.

s and ω are calculated on the basis of values of the function minimized, its gradient, and perhaps even, its gessian. If the terminating condition is not satisfied at point v, the iteration is repeated.

Procedures such as (36) are usual for optimizational schemes. However, the distributed nature of the problem makes us re-evaluate some of the standard views on these procedures. We also have to take into account some of the specific features of the linked models.

Let us consider the scheme given for the case of linear optimizational models. We assume that:

1. The common constraints are linear with respect to v.
2. The linkage parameters are included only in the free terms of the internal constraints of the models.
3. There is no software that enables a version of the SUMT to be used; only standard simplex procedures are available.

Then, each of the linked models may be formulated as:

Minimize with respect to $x^k \in E^{n^k}$

$$\sum_{i=1}^{n^k} p_i^k x_i^k \quad , \tag{37}$$

subject to

$$- b_s^k - c_s^k v_{\rho(k,s)} + \sum_{j=1}^{n^k} a_{sj}^k x_j^k \geq 0 \quad , \qquad s = \overline{1,m^k} \quad , \tag{38}$$

where

$p_i^k,\ b_s^k,\ c_s^k,\ a_{sj}^k$ = constants for all values of their indices;

$\rho(k,s)$ = an index function, which equals the index of the component of v contained in the s-th constraint of model k; if $c_s^k = 0$, $\rho(k,s) = 0$.

The set of common constraints is

$$-F_q + \sum_{r=1}^{L} D_{qr} v_r \geq 0 \quad , \qquad q = \overline{1,M} \quad , \tag{39}$$

where F_q and D_{qr} are given constants.

It is then necessary to choose an algorithm for scheme (36). Let it be a modification of the well-known Newton method, which requires partial derivatives of $\bar{E}$ to be calculated for the first and second order.

Formally, using formula (31), we have

$$\frac{\partial \bar{E}}{\partial v_r} = \sum_{q=1}^{M} D_{qr} \frac{\partial P}{\partial R_q} - \sum_{k=1}^{N} \lambda_k \sum_{s=1}^{m^k} c_s^k \frac{\partial P}{\partial y_s^k} \delta_{r\rho(k,s)} \quad , \tag{40}$$

where δ_{ij} is equal to 1, if i is equal to j, and is zero otherwise. In this expression it is necessary to determine the terms $\frac{\partial P}{\partial y_s^k}$, since only they are related to the 'smooth' version of the SUMT for problems (37)-(38). This may be done with the help of the Fiacco and McCormick theorem (1968).

If gradients of active constraining functions (i.e. $y_s^k(x^k,v)$) are linearly independent, then the relation

$$\lim_{T \to +0} \frac{\partial P}{\partial y_s^k} = -u_s^k \tag{41}$$

takes place, where u_s^k is the Lagrangian multiplier for constraints s of model k.

In the opposite case, we should use several iterations of the Newton method to solve problems (24), with the starting points given by the Simplex method, and to calculate $\frac{\partial P}{\partial y_s^k}$ directly. We will now describe how elements of the sensitive matrix can be found.

In the linear case

$$\frac{\partial^2 E^k}{\partial x_i^k \partial x_j^k} = \sum_{s=1}^{m^k} a_{si}^k a_{sj}^k \frac{\partial^2 P}{\partial (y_s^k)^2} \quad , \tag{42}$$

$$\frac{\partial^2 E^k}{\partial x_i^k \partial x_j^k} = - \sum_{s=1}^{m^k} c_s^k a_{si}^k \frac{\partial^2 P}{\partial (y_s^k)^2} \delta_{r\rho(k,s)} \quad . \tag{43}$$

Taking into consideration proved relations (Umnov 1975):

$$\lim_{T \to +0} \frac{\partial^2 P}{\partial (y_s^k)^2} = + \infty \quad , \qquad \text{for active } s \quad , \tag{44}$$

and

$$\lim_{T \to +0} \frac{\partial^2 P}{\partial (y_s^k)^2} = 0 \quad , \quad \text{for nonactive } s \quad , \tag{45}$$

we obtain the systems of linear equations for desirable components of $\frac{\partial x^k}{\partial v}$:

$$\sum_{j=1}^{n^k} \left(\sum_{s \in \Omega^k} a_{si}^k a_{sj}^k \right) \frac{\partial \bar{x}_j^k}{\partial v_r} = \sum_{s \in \Omega^k} c_s^k a_{si}^k \delta_{r\rho(k,s)} \quad , \tag{46}$$

$$\text{for all } i = \overline{1,n^k} \text{ and } r = \overline{1,L} \quad ,$$

where Ω^k is the set of indices for the active constraints of model k.

For the partial derivatives of the second order we can use the same ideas without any theoretical innovations.

To complete our consideration of the distributed system of linear models, we have to solve the problem of how to choose the length of the step along the minimizing direction ω. As mentioned above, it is not reasonable to use methods that are based on testing a large number of sample points. It is better to use no sample points.

In the proposed approach we only have the possibility of evaluating the length on the basis of information already obtained. The desirable evaluation is equal to the minimum of the following three numbers:

-- the norm of ω, i.e. $\| \omega \|$;

-- the value of the step $\check{s}$, by which at least one of the nonactive common constraints becomes active;

-- the value of the step $\hat{s}$, by which at least one of the nonactive constraints belonging to the linked models becomes active.

This evaluation ensures a decrease in the value of $\bar{E}$ and, hence, the convergence of the whole procedure (Pshenitshnij and Danilin 1975).

The values of $\check{s}$ and $\hat{s}$ may be found in the following way. Let $v = v_o + \check{s}\omega$, then

$$R_q(v) = R_q(v_o + \check{s}\omega) = R_q(v_o) + \check{s} \sum_{r=1}^{L} D_{qr}\omega_r \quad . \tag{47}$$

We consider only those q for which

$$R_q(v_o) > 0 \text{ and } \sum_{r=1}^{L} D_{qr}\omega_r < 0 \quad . \tag{48}$$

Hence

$$\check{s} = \min_q \left\{ \frac{R_q(v_o)}{\sum_{r=1}^{L} D_{qr}\omega_r} \right\} . \tag{49}$$

In an analogous way, we can obtain

$$y_\tau^k(v_o + \hat{s}\omega) = y_\tau^k + \hat{s}\Phi_\tau^k \quad , \tag{50}$$

where, for example, in the case of independence, the gradients of active constraints

$$\Phi_\tau^k = -c_\tau^k \omega_{\rho(k,\tau)} + \sum_{i\in\theta^k} \omega_i \sum_{j=1}^{n^k} a_{\tau j}^k \frac{\partial \bar{x}_j^k}{\partial v_i} \quad . \tag{51}$$

In other cases Φ_τ^k can be evaluated by using a test point along the direction ω.

This implies that

$$\hat{s} = \min_{k,\tau} \left\{ - \frac{y_\tau^k(v_o)}{\Phi_\tau^k} \right\} \quad \text{for all } k \text{ and } \tau \quad , \tag{52}$$

for which

$$y_\tau^k(v_o) > 0 \text{ and } \Phi_\tau^k < 0 \quad . \tag{53}$$

TEST CASE

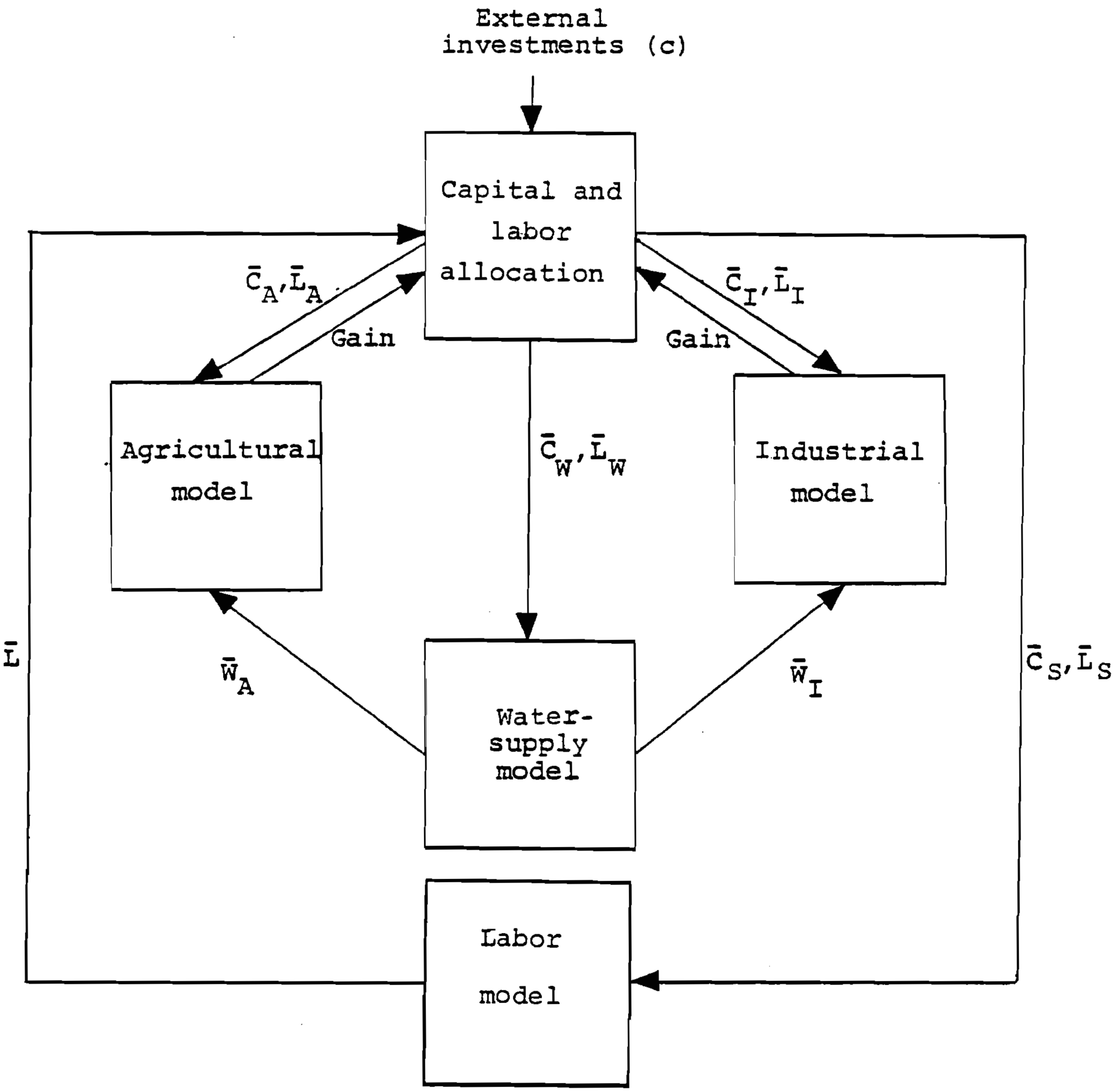
A special example supplied with good synthetic data was prepared in order to test the model system. The region under analysis was divided into three subregions, which contained the following sectors: agriculture, industry, water supply, and labor.

Coordination of the sectors is shown in Figure 3, which differs from Figure 2 in the following way. A labor model replaces the population and migration models (which in Figure 2 depend on capital and labor allocation). In the labor model, the number of employees is dependent only on capital investments directed to the service sector. The characteristics of each sector are discussed below.

Agriculture

For each of the three subregions four types of crop and two types of technology (with water-consumption variants per crop unit) are considered. The following constraints are also assumed:

-- constraint on the land available for agriculture;
-- constraint on the choice of technology;
-- constraint on water available for basic consumption;
-- constraint on water available for peak-period consumption;



$\bar{C}_A, \bar{C}_I, \bar{C}_W, \bar{C}_S$ = vectors of interregional distribution of capital investments to the agricultural, industrial, water-supply, and service sectors;

$\bar{L}_A, \bar{L}_I, \bar{L}_W, \bar{L}_S$ = vectors of interregional distribution of labor to the agricultural, industrial, water-supply, and service sectors;

$\bar{W}_A, \bar{W}_I$ = subregional water flows to agriculture and industry;

$\bar{L}$ = vector of subregional labor;

→ = information flows.

Figure 3. The tested system of regional models.

-- constraint on capital investments;

-- constraint on labor.

Total production volume is not fixed.

GRAM is a standard linear model, which may be used in combination with other models. Using a system of models, it is possible to determine how changes in the agricultural model (productivity, efficiency, technology, etc.) could change the optimal solution for the region as a whole.

Industry

Four types of industrial enterprise and two types of technology (with water-consumption variants per production unit) are specified for each of the three subregions. Total consumption of production by both possible technologies in all three subregions is fixed on the upper level. The model's objective function with respect to each product is:

Minimize

$$\left[\sum_{l,r} (C_{l,r} L_{l,r}) + EK_{L,r} \delta_{l,r} \right] , \tag{54}$$

subject to

$$\sum_{l,r} B_{l,r} L_{l,r} \geq Z^0 , \tag{55}$$

$$A_{l,r} L_{l,r} = U_{l,r} , \tag{56}$$

$$L_{l,r} \geq 0 . \tag{57}$$

The notation used for the model (54)-(57) is the same as for (1)-(7). The matrix of inputs includes data on water, labor, and capital investment required per unit of production. Index r is introduced to describe the possibilities for choosing the production technology.

Water Supply

The scheme presented in Figure 4 was used to analyze regional water-supply problems. This scheme was based on the water-supply

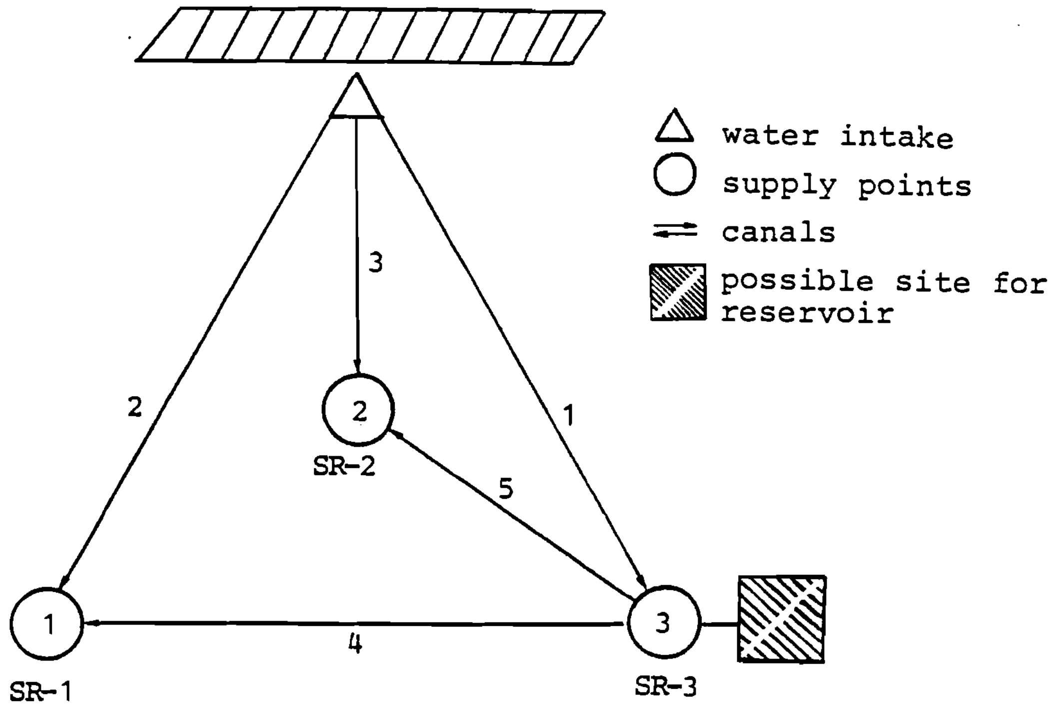


Figure 4. Water-supply scheme.

system in one of the case-study regions and it includes: one water intake, five possible canals, three supply points, and one point at which a reservoir can be constructed.

A sequential description of the pumping station and canal capacities is used, and water demand is considered as consumption. Industrial consumption is assumed to be equally distributed throughout the year. Agricultural consumption is considered to be irregular and is thus classified into three periods: April-May, June-September, October-March. During the third period no irrigation is necessary, the water-supply system can therefore be used to fill the reservoir.

The configuration and location of the water-supply system should correspond to the scale and location of industrial and agricultural activities. However, to a certain extent it may also influence their development and the technology they use.

Labor

A simplified scheme for determining the size of the labor force was adopted. In every subregion lower and upper limits were introduced for labor use in the industrial and agricultural sectors. The number of employees within these limits can be regulated by the size of capital investments in the rural and urban service sector (it was assumed that the number of employees in the service sector is mainly determined by the scale of capital investments). As a result, the total size of the labor force of sector s (industry or agriculture) in subregion 2 is

$$L_{s2}^{0} \leq L_{s2} \leq L_{s2}^{0} + K_{s2}C_{s2}^{serv} \leq L_{s2}^{h} \quad , \tag{58}$$

where

L_{s2}^{0}, L_{s2}^{h} = lower and upper limits for labor in sector s of subregion 2;

L_{s2} = number of employees in sector s of subregion 2;

K_{s2} = rate of increase in the number of employees per unit of capital investment in the rural or urban service sector of subregion 2;

C_{s2}^{serv} = volume of capita investments for the service sector.

RESULTS OF THE CALCULATIONS

Several dozen calculations were made to prove that the model system can successfully cope with changes in the following data:

-- coefficient of objective functions of all included optimization models;
-- matrix of conditions of every optimization model;
-- parameters of nonoptimization migration models.

The main series of calculations was performed to obtain a picture of the changes in regional activities resulting from changes in external capital investments and the number of employees.

The generalized results of these calculations are shown in Figure 5, where capital investments vary from 0 to 350 (millions

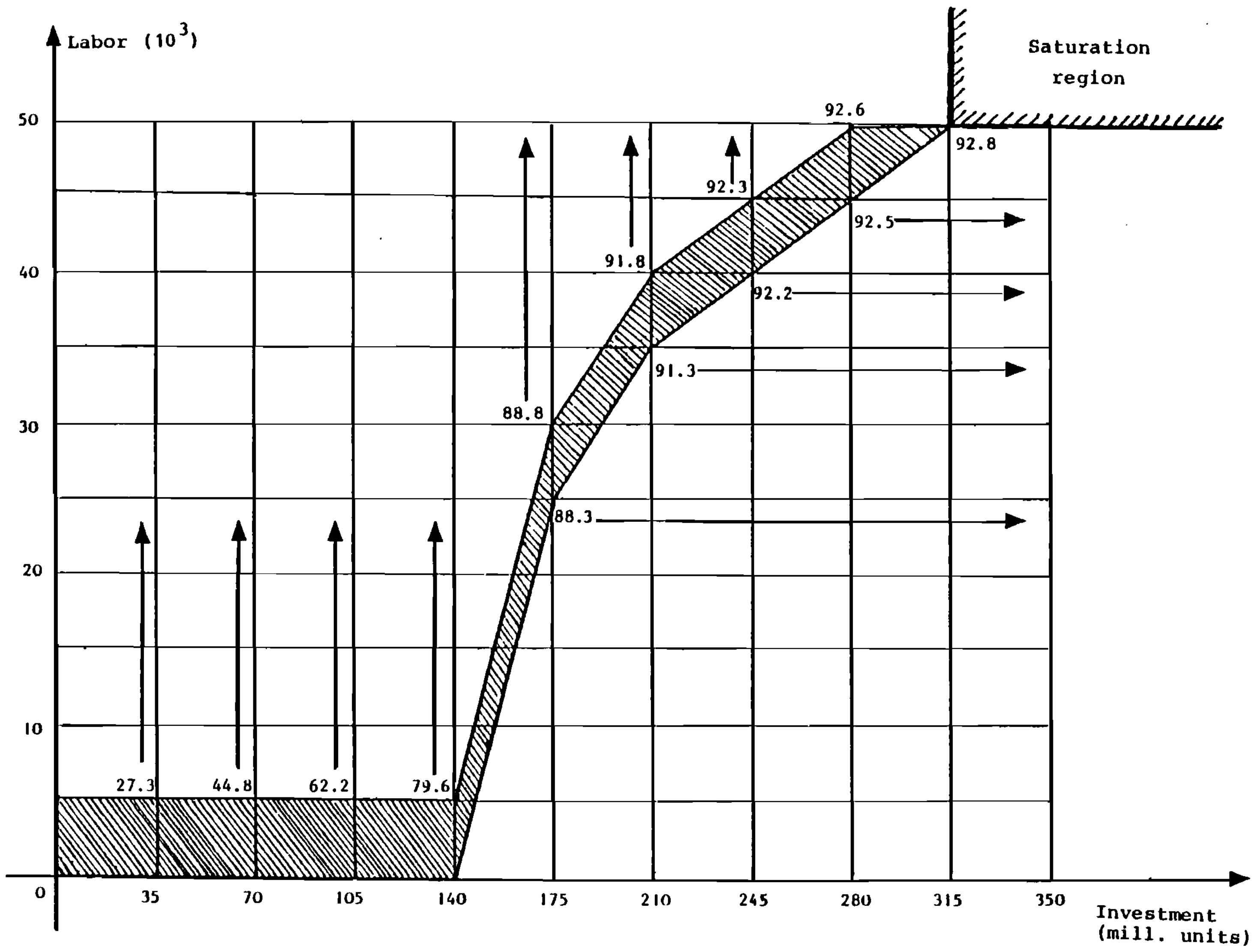


Figure 5. Changes in regional activities resulting from changes in external capital investments and the number of employees.

of conditional money units) and labor varies from 0 to 50×10^3 persons. They indicate a rather surprising situation: only in relatively small areas are the results dependent on both capital investments and labor. There exists a large area of saturation by labor or by capital investments.

An example of the results of the calculations is presented in Table 2. The results are as follows: the value of the objective function is 59,695 units (shown in the middle of Table 2). Capital investments are directed only to industrial activities in the first region (84,258 units) and to supporting water-supply sectors (1,032 units), as is shown on the upper left of Table 2.

Thus, there is industrial activity in subregion 1 only, where commodities 1 and 2 are produced (1,000 and 6.33 units, respectively), and for both commodities technology requiring heavy water consumption is used (see the upper right of Table 2). In the bottom part of Table 2 the data on water-supply systems are shown. Construction of Canal 2 only is required, and this canal is used only during the vegetation period. No reservoir is needed. The system of dual estimates is also presented. This is taken from the agricultural model (for peak and basic agricultural water demand), from the industrial model (for industrial water demand), and from the general block, which includes the constraints on common resources.

Below the results of four other calculations for different combinations of investments and labor are shown (Tables 3, 4, 5, and 6). These combinations are:

Capital investments (10^3):	140, 160, 180, 350;
Labor (10^3):	20, 15, 32.5, 50.

These results indicate that an increase in capital investment (provided there is sufficient labor) leads first to an increase in industrial activities in the first subregion, then to deployment of agricultural activities in all subregions.

For products 1 and 2 the technology with heavy water consumption was effective when irrigation appeared to be unjustified for agricultural production.

Table 2. Results of the calculations given investment and labor resources of 100,000 monetary units and 50,000 persons.

RESOURCES	Subregion 1	2	3
Investment:			
Industry	84,257.92	0	0
Agriculture	0	0	0
Water supply	7,316.29	0	0
Labor:			
Industry	1,416.29	0	0
Agriculture	0	0	0
Water Supply:			
Industry	1,031.63	0	0
Base agriculture	0	0	0
Peak agriculture	0	0	0
Objective Function Value	59,695.00		

PRODUCTION*	Sector	Technology	Subregion 1	2	3
INDUSTRY					
	1	ω	1,000.00	0	0
		o	0	0	0
	2	ω	6.33	0	0
		o	0	0	0
	3	ω	0	0	0
		o	0	0	0
	4	ω	0	0	0
		o	0	0	0
AGRICULTURE					
	1	ω	0	0	0
		o	0	0	0
	2	ω	0	0	0
		o	0	0	0
	3	ω	0	0	0
		o	0	0	0
	4	ω	0	0	0
		o	0	0	0

WATER-SUPPLY SYSTEM

Period	Canal 1	Canal 2	Canal 3	Canal 4	Canal 5	Reservoir
1	0	731.63	0	0	0	0
2	0	0	0	0	0	0
3	0	300.00	0	0	0	0
Maximum capacity	0	731.63	0	0	0	0

DUAL ESTIMATES

Subregion	Peak water price	Base water price	Industrial water price	Centrally determined price
1	5.077	5.097	5.097	5.097
2	5.077	4.140	5.097	5.097
3	5.077	4.109	5.097	5.097

*ω refers to technology requiring heavy water consumption; o refers to technology with light water consumption.

Table 3. Results of the calculations given investment and labor resources of 140,000 monetary units and 20,000 persons.

RESOURCES

	Subregion 1	Subregion 2	Subregion 3
Investment:			
Industry	120,457.01	0	0
Agriculture	0	0	0
Water supply	7,497.29	0	0
Labor:			
Industry	1,597.29	0	0
Agriculture	0	0	0
Water Supply:			
Industry	1,049.73	0	0
Base agriculture	0	0	0
Peak agriculture	0	0	0
Objective Function Value	79,602.00		

PRODUCTION*

	Sector		Subregion 1	Subregion 2	Subregion 3
INDUSTRY					
	1	ω	1,000.00	0	0
		o	0	0	0
	2	ω	9.95	0	0
		o	0	0	0
	3	ω	0	0	0
		o	0	0	0
	4	ω	0	0	0
		o	0	0	0
AGRICULTURE					
	1	ω	0	0	0
		o	0	0	0
	2	ω	0	0	0
		o	0	0	0
	3	ω	0	0	0
		o	0	0	0
	4	ω	0	0	0
		o	0	0	0

WATER-SUPPLY SYSTEM

Period	Canal 1	Canal 2	Canal 3	Canal 4	Canal 5	Reservoir
1	0	749.73	0	0	0	0
2	0	0	0	0	0	0
3	0	300.00	0	0	0	0
Maximum capacity	0	749.73	0	0	0	0

DUAL ESTIMATES

Subregion	Peak water price	Base water price	Industrial water price	Centrally determined price
1	5.077	5.097	5.097	5.097
2	5.077	4.140	5.097	5.097
3	5.077	4.109	5.097	5.097

*ω refers to technology requiring heavy water consumption; o refers to technology with light water consumption.

Table 4. Results of the calculations given investment and labor resources of 160,000 monetary units and 15,000 persons.

RESOURCES

	Subregion 1	Subregion 2	Subregion 3
Investment:			
Industry	129,692.88	0	0
Agriculture	75.00	4,080.00	2,515.75
Water supply	10,000.00	0	0
Labor:			
Industry	1,754.49	0	0
Agriculture	54.00	8,160.00	5,031.51
Water Supply:			
Industry	1,270.00	0	0
Base agriculture	180.00	0	0
Peak agriculture	150.00	0	0
Objective Function Value	84,859.00		

PRODUCTION*

	Sector		Subregion 1	Subregion 2	Subregion 3
INDUSTRY					
	1	ω	1,000.00	0	0
		o	0	0	0
	2	ω	1.57	0	0
		o	8.43	0	0
	3	ω	0	0	0
		o	0.05	0	0
	4	ω	0	0	0
		o	0	0	0
AGRICULTURE					
	1	ω	300.00	0	0
		o	0	0	0
	2	ω	0	0	0
		o	0	0	0
	3	ω	0	0	0
		o	0	13,600.00	8,385.84
	4	ω	0	0	0
		o	0	0	0

WATER-SUPPLY SYSTEM

Period	Canal 1	Canal 2	Canal 3	Canal 4	Canal 5	Reservoir
1	0	1,000.00	0	0	0	0
2	0	300.00	0	0	0	0
3	0	300.00	0	0	0	0
Maximum capacity	0	1,000.00	0	0	0	0

DUAL ESTIMATES

Subregion	Peak water price	Base water price	Industrial water price	Centrally determined price
1	2.344	2.929	2.929	2.929
2	2.344	1.921	2.929	2.929
3	2.344	1.906	2.929	2.929

*ω refers to technology requiring heavy water consumption; o refers to technology with light water consumption.

Table 5. Results of the calculations given investment and labor resources of 180,000 monetary units and 32,500 persons.

RESOURCES

	Subregion 1	Subregion 2	Subregion 3
Investment:			
Industry	131,103.99	0	0
Agriculture	7,461.05	8,645.74	7,334.66
Water supply	10,000.00	0	0
Labor:			
Industry	1,660.00	0	0
Agriculture	10,746.11	10,864.57	9,229.32
Water Supply			
Industry	1,270.00	0	0
Base agriculture	180.00	0	0
Peak agriculture	150.00	0	0
Objective Function Value	89,815.00		

PRODUCTION*

	Sector		Subregion 1	Subregion 2	Subregion 3
INDUSTRY					
	1	ω	1,000.00	0	0
		o	0	0	0
	2	ω	0	0	0
		o	10.00	0	0
	3	ω	0	0	0
		o	0	0	0
	4	ω	0.02	0	0
		o	0	0	0
AGRICULTURE					
	1	ω	300.00	0	0
		o	20,400.00	23,800.00	27,200.00
	2	ω	0	0	0
		o	0	16,669.15	0
	3	ω	0	0	0
		o	11,020.17	7,396.10	6,315.53
	4	ω	0	0	0
		o	0	0	0

WATER-SUPPLY SYSTEM

Period	Canal 1	Canal 2	Canal 3	Canal 4	Canal 5	Reservoir
1	0	1,000.00	0	0	0	0
2	0	300.00	0	0	0	0
3	0	300.00	0	0	0	0
Maximum capacity	0	1,000.00	0	0	0	0

DUAL ESTIMATES

Subregion	Peak water price	Base water price	Industrial water price	Centrally determined price
1	2.412	2.990	2.990	2.990
2	2.412	1.982	2.990	2.990
3	2.412	1.969	2.990	2.990

*ω refers to technology requiring heavy water consumption; o refers to technology with light water consumption.

Table 6. Results of the calculations given investment and labor resources of 350,000 monetary units and 50,000 persons.

RESOURCES

	Subregion 1	Subregion 2	Subregion 3
Investment:			
Industry	131,103.99	0	0
Agriculture	42,940.12	51,308.85	34,403,12
Water supply	10,000.00	0	0
Labor:			
Industry	1,660.00	0	0
Agriculture	14,294.01	15,130.88	15,160.47
Water Supply:			
Industry	1,270.00	0	0
Base agriculture	180.00	0	0
Peak agriculture	150.00	0	0
Objective Function Value	92,750.00		

PRODUCTION*

	Sector		Subregion 1	Subregion 2	Subregion 3
INDUSTRY					
	1	ω	1,000.00	0	0
		o	0	0	0
	2	ω	10.00	0	0
		o	0	0	0
	3	ω	0	0	0
		o	0	0	0
	4	ω	0.02	0	0
		o	0	0	0
AGRICULTURE					
	1	ω	300.00	0	0
		o	20,400.00	23,800.00	27,200.00
	2	ω	0	0	0
		o	20,000.12	22,108.84	15,604.68
	3	ω	0	0	0
		o	13,600.00	13,600.00	13,600.00
	4	ω	0	0	0
		o	0	0	0

WATER-SUPPLY SYSTEM

Period	Canal 1	Canal 2	Canal 3	Canal 4	Canal 5	Reservoir
1	0	1,000.00	0	0	0	0
2	0	300.00	0	0	0	0
3	0	300.00	0	0	0	0
Maximum capacity	0	1,000.00	0	0	0	0

DUAL ESTIMATES

Subregion	Peak water price	Base water price	Industrial water price	Centrally determined price
1	3.813	4.200	4.200	4.200
2	3.813	3.189	4.200	4.200
3	3.813	3.208	4.200	4.200

*ω refers to technology requiring heavy water consumption; o refers to technology with light water consumption.

If changes in the system of dual estimates (DE) of water are analyzed, then it should be emphasized that, whereas for the first capital-labor combination (140-20) the DE is 5.097 (per unit), for the more intensive variant (160-15), the water DE falls considerably from this level. The reason for this becomes clear if the testing of the water-system capacities is taken into account: in the 140-20 variant the productivity of Canal 2 is not fully utilized. After that, water DE are increased with the increase in regional activities.

Because of the experimental character of the data used, discussion of the results was intentionally restricted. Nevertheless, even a short description of the results obtained clearly shows the workability of the proposed model system.

REFERENCES

Albegov, M. 1979. Generalized Regional Agriculture Model (GRAM): Basic Version. WP-79-93. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Albegov, M. and V. Chernyatin. 1978. An Approach to the Construction of the Regional Water Resource Model. RM-78-59. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Andersson, Å.E. and D. Philipov (eds). 1979. Proceedings of Task Force Meeting I on Regional Development Planning for the Silistra Region (Bulgaria). CP-79-7. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Ermoliev, Iu.M. 1980. Some Problems of Linkage Systems. WP-80-102. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Fiacco, A.V. and G.P. McCormick. 1968. Nonlinear Programming: Sequential Unconstrained Minimization Techniques. New York: Wiley.

Geoffrion, A.M. 1970. Primal resource--Directive approaches for optimizing nonlinear decomposable systems. Operations Research 18 (3): 375-403.

Mednitsky, V. 1978. Special Methods for Optimizing the Problem Solution. Moscow: Nauka. (In Russian)

Pshenitshnij, B.N. and Iu.M. Danilin. 1975. Numerical Methods for Extremal Problems. Moscow: Nauka.

Umnov, A.E. 1974. Iterative linear extrapolation in the Penalty Functions Method. Journal of Computer Mathematics and Mathematical Physics 6.