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Modeling Regional Water Supply: Silistra Case Study

Chernyatin, V.

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MODELING REGIONAL WATER SUPPLY: SILISTRA CASE STUDY

V. Chernyatin

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

In 1978 it was decided that parallel to the continuation of demand studies, an attempt would be made to integrate the results of our studies on water demands with water supply considerations. This new task was named "Regional Water Management" (Task 1, Resources and Environment Area).

One of the case studies in this Task, carried out in collaboration with several Bulgarian institutions and the Regional Development Task of IIASA, is concerned with water resources management in the Silistra region of Bulgaria. This paper on modeling the water supply system in the Silistra region accompanies the earlier study on the water demands of agriculture in the same region.

Murat Albegov	Janusz Kindler
Leader	Chairman
Regional Development Task	Resources & Environment Area

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MODELING REGIONAL WATER SUPPLY: SILISTRA CASE STUDY

V. Chernyatin

1. INTRODUCTION

The IIASA's water resources research related to the Silistra Case Study started in 1977 with the modeling of agricultural water demands (Gouevsky and Maidment, 1977). In many respects, the water demand model had the character of a general agricultural model for the Silistra region. Later on, the model was extended to take into account the subdivision of the region into a number of districts with the various conditions of soil, crop structure, water supply, etc. (Gouevsky, Maidment, and Sikorski, RR-80-38, 1980). The crucial point at this stage of the study was knowing how much water supply costs in total, and what the shadow prices of water for the various districts would be. Unfortunately, it is impossible to answer these questions, even roughly, without analysis of a regional water supply system.

The second stage of the Silistra water resources related study, being the main subject of this paper, is a water supply model. The major problem to be solved here is to determine the least-cost variant of the water supply system and the shadow prices of water distributed geographically. The latter very much influences the intraregional structure and intensity of production. In this respect the water supply model reported here can be considered as an essential part of the system of regional models

(Albegov and Chernyatin, 1978). What is especially important is that the water supply model should be interrelated with other regional models. In the Silistra Case, at the end of 1979, such a coordination was done for the agricultural water demand and supply models (Chernyatin and Gouevsky, forthcoming).

This paper sums up IIASA's work on modeling of the Silistra water supply system. The water supply model presented here was developed in close cooperation with the Sofia Institute for Water Projects which is responsible for designing water resources systems in Bulgaria. It must be stressed that the Silistra region is characterized by fairly simple hydrological conditions in the region. Namely, abundance of water in the Danube river--the only source of water--allows one correctly to confine oneself to within-year regulation of water resources. This property essentially simplifies analysis of a water supply system. The developed optimization model determines basic parameters of the Silistra water supply system--capacities of reservoirs and pumping stations, and discharge capacities of canals.

Although intended primarily for the Silistra water supply system, the model actually had many properties of a general water supply model under conditions of within-year regulation of water resources. Afterwards the model was generalized to cover the whole set of irrigation systems of "Silistra type". In this general form, it is expected to be applied for the planning of many irrigation systems in the Danube lowland in Bulgaria. The first experiment in this field was crowned with success. Namely, the practical application of the modeling results lead to a considerable budget saving for the Silistra irrigation system.

Below, the purposes of modeling, the mathematical model of a regional water supply, and the results of its application for the Silistra region are described in detail.

2. SILISTRA WATER SUPPLY PROBLEMS AND PURPOSES OF MATHEMATICAL MODELING

Silistra is a region covering a 2700 km^2 area, with a population of 200,000 located in the North-Eastern part of Bulgaria. The soil quality and the number of days of sun per year make this region favorable for intensive agricultural development under irrigation. Unfortunately, it has a pronounced shortage of its internal water resources. Since no other rivers exist in the region, the bordering Danube river is the only source of water for agricultural, domestic and industrial consumption. Groundwater is available, only in small quantities, at a depth exceeding 400 metres which makes it unprofitable for production use. Furthermore, the annual rainfall is rather moderate--500 mm in average--and distributed (somewhat unfavorably) within the year with respect to the growing season.

According to the long-term hydrological forecasts, there will be no deficit of water in the Danube river at least until the year of 2000. Because of the abundance of water in the Danube river, the question of how much water to withdraw for agricultural and industrial production and for municipal use is decided solely by the economics of water use. Suffice it to say, for example, that all the irrigated areas are located at a level varying from 100 m to more than 200 m higher than the Danube river level. This means that the conveyance of irrigation water is rather expensive.

The water supply system for the Silistra region is divided into two separate sub-systems--irrigation water supply and water supply for household and industrial consumption. The reason for making such a division is the essential difference in the level of water quality demanded by different types of water users. With regard to industry these are mainly food enterprises except for some other industrial activities in the city of Silistra. However, being situated along the Danube river they have their own water intakes which are small in comparison to the total regional water requirements. As it is known, the food industry requires that the water quality be of drinking-water standards, which, of course, are higher than that required for irrigation water.

In quantitative respect, about 10 to 15% of the total regional water requirements fall to the share of household and industrial uses. The only source of water for these uses is the <u>Danube terrace water</u>, which is limited in quantity. On the other hand, irrigation water is withdrawn from the Danube river whose water quality is much worse than that of terrace water, but it is admissible for irrigation. The irrigation water supply system, in turn, represents a system of interconnected reservoirs, canals, pumping station, culverts, syphons, etc. All the above leads to the conclusion that the Silistra irrigation system is a separate and most important part of the regional water supply system. That is why this study is concerned with the irrigation water supply system only.

From the geographical point of view, the irrigation system for the Silistra region is divided into three hydraulically disconnected water supply systems for the Tutrakan, Malak Preslavets and Silistra districts, respectively (see Figure 1). The M. Preslavets irrigation system is the most representative one with respect to both the irrigation area (more than 60%) and the number of typical system elements such as reservoirs, pumping stations, canals, etc. Regarding the Tutrakan irrigation system, this project is already underway and half-built. Therefore, the M. Preslavets irrigation systems have been chosen as a pilot water project in the Silistra case. The water supply model developed for the M. Preslavets district is expected to be transferred afterwards to other irrigation systems in the Silistra region.

The work on mathematical modeling of water supply systems has been done by IIASA, in close cooperation with the Sofia Institute for Water Projects. Of course, the mathematical modeling by no means replaces the whole work of designing a water resources system. The best way to understand the purposes of modeling or, similarly, what IIASA's role was in such a collaboration, is to briefly reproduce the sequence of designing stages in the development of a water supply system as they were made by the Institute for Water Projects.

As seen from Figure 2, the designing stages for an irrigation system range from preliminary investigations to design work. We will briefly comment on them. The <u>first stage</u> includes preliminary geological, topographical, and design investigations, with the object of roughly outlining what type of water supply

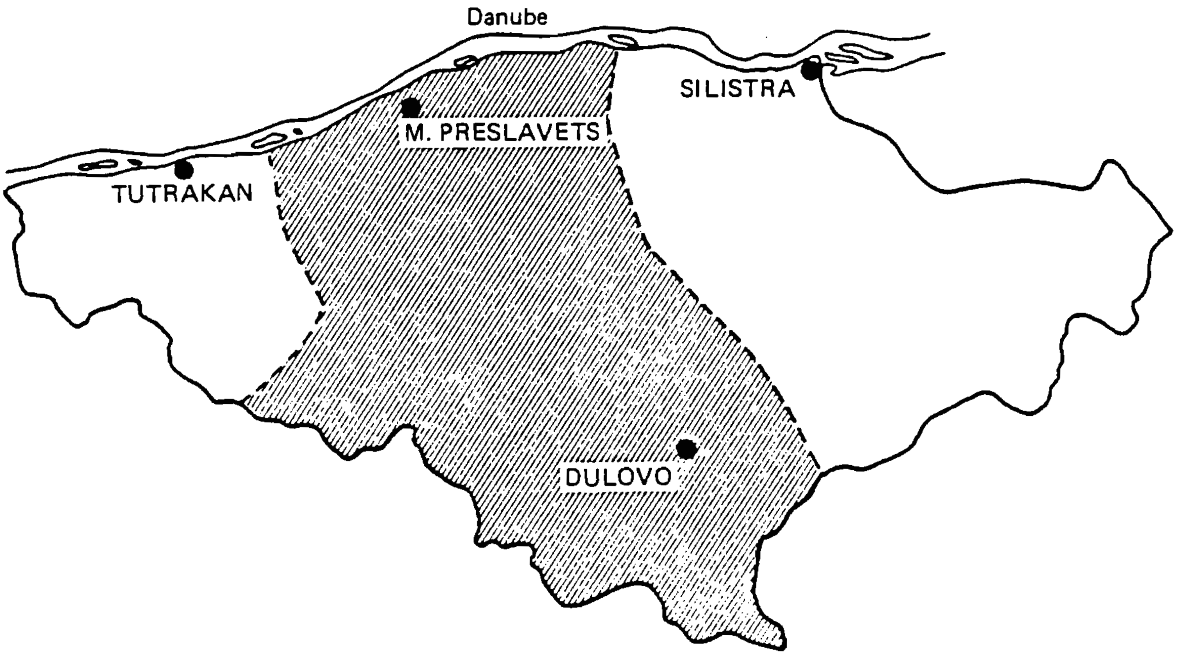


Figure 1. Silistra region.

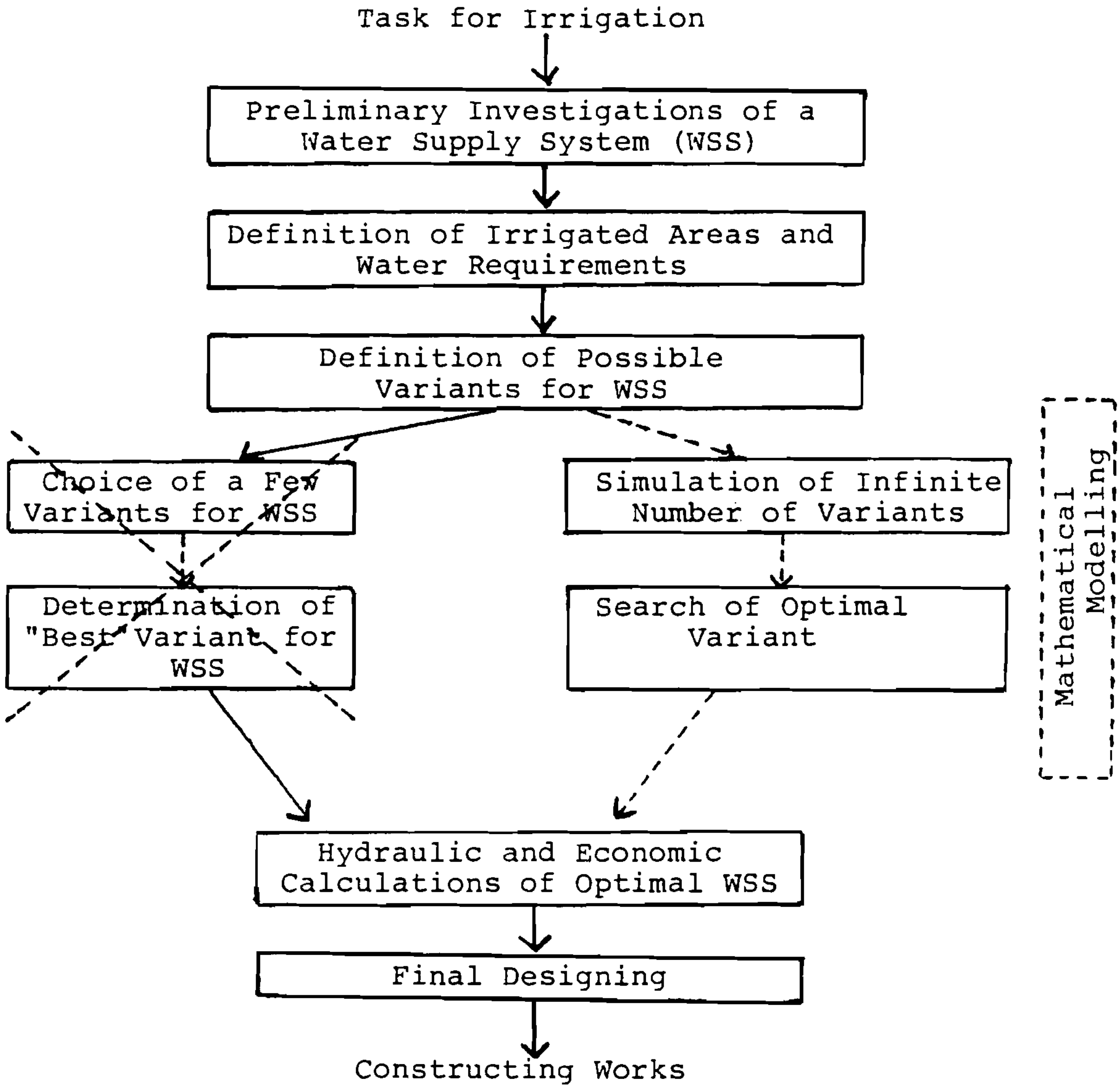


Figure 2. Sequential stages in designing of water supply system.

system will be created--with reservoir or not, with open canals or pipelines, with water conveyance by gravity or by pumping, etc. At the second stage, the land suitable for irrigation is defined and, by doing that, the water requirements are determined.

The third stage consists in the definition of a set of possible variants for a water supply system. In fact, this alternative set is infinite. Nevertheless, in view of pure practical difficulties--complicated water-balance, engineering, and economic calculations of the whole irrigation system--in the next stage the designer has to confine himself to a set of a few variants, usually no more than 4 or 5. The fifth design stage results in determination of the "best" variant for an irrigation system. The "best" here, means the optimal variant in the narrow sense of the word, as we are dealing with the optimal variant chosen from among a very limited set of possible variants of water supply systems. The measure with which to compare the different variants amongst each other is the total annual cost of a water supply system.

Looking at the design scheme presented in Figure 2, it is evident that stages 4 and 5 are rather labor-consuming and, at the same time, easily formalizable. We can give the mathematical modeling complete control over these two stages. The essential advantage of the mathematical-modeling approach here is that it allows to analyze the infinite number of variants for a water supply system. In the mathematical model, the simulation of the infinite number of variants is realized in a fairly simple manner--by the flow and mass balance constraints in all the nodes of a water network. Another stage--search for the optimal variant--is realized by the optimization procedure, which determines the least-cost variant of water supply system.

Thus, through substitution of the two conventional design stages (see Figure 2--choice of a few variants and determination of the "best" variant of a water supply system) for the two modeling stages--simulation of all potential variants and search of the optimal variant produces results which:

- o saves the designer from the multiple, labor-consuming calculations of a water supply system;
- o guarantee the selected variant to be really optimal.

The mathematical modeling effort has two additional objectives which are very important for practical application. Namely, the mathematical model should be:

(a) operational for a wide range of initial data,

(b) suitable for the rather arbitrary configurations of water supply system.

Of course, application of a mathematical model requires that the analyst has access to the computing facilities equipped with a necessary software.

3. GENERALIZED WATER SUPPLY MODEL

3.1 Basic Assumptions

Before describing the mathematical model, it is necessary to outline the range of its applicability. The best way of doing that is to present the main assumptions of the model:

1. The main goal of the water supply system under analysis is to meet water requirements, prespecified both in space and time.
2. The water supply system is determined as it is by the end of the planning period.
3. The available water resources are unlimited and can meet all water requirements.
4. Proceeding from the analysis of topographical and geological conditions, the basic scheme of the water supply system is fixed.
5. The optimal water supply system is considered to be that one which is the least-costly.
6. All water-users consume water resource irreversibly.
7. Only within-year regulation of water resources is considered.
8. The transit time delays for canals are not taken into account.

Actually these assumptions indicate the type of a water supply system which can be analyzed by the model presented below. The first step of the model building process is to construct flow network representation of the irrigation system.

3.2 Flow Network Representation of the System

The flow network consists of the following standard elements:

1. nodes,
2. arcs,
3. inputs (inflows),
4. outputs (outflows).

All of them should be interconnected in a certain sequence as it is in the real irrigation system. Though mapping a real system into a uniform network is not a matter of difficulty, nevertheless, this procedure cannot be entirely formalized. For example, in doing so, sometimes we have to introduce a number of fictitious nodes and arcs, combine a few standard elements into one unit, etc.

Next, this spatial representation has to be expanded to take into account the multi-period operation of the water supply system. This means that the flow network should have two dimensions--space and time. The time representation of the system can be realized as a layered network, where each layer corresponds to a single time period and is connected with the subsequent ones by storage arcs leaving all reservoir nodes. Since the links between the time layers are easy to be accounted for, we can confine ourselves to a detailed consideration of only one time layer of flow network, taking into account the storage arcs entering and leaving the reservoir nodes. Figure 3 shows all standard elements of a flow network and their interpretation in the terms of real elements of an irrigation system.

Any irrigation system we are dealing with in this paper is assumed to be represented by a flow network consisting only of the standard elements presented in Figure 3. By definition, any input can only be at a pumping node, which is called an input pumping node. On the other hand, any internal pumping or distributing node is assumed to have an output. The actual absence of water withdrawal in some internal pumping or distributing nodes is simulated by the output of zero capacity. For explanatory purposes, a simple example of a flow network for a single time period is shown in Figure 4.

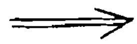
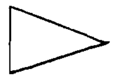
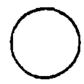
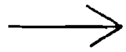
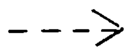
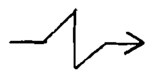
	Standard Element of the Model	Corresponding Element of the Real Water Supply System	Legend
1	Input	Water source or water inflow from other system	
2	Reservoir Node	Storage reservoir	
3	Pumping Node	Pumping station	
4	Distributing Node	Junction of two or more water flows	
5	Arc	Open canal, pipeline, culvert, syphon or any combination of those	
6	Storage Arc	Fictitious link for taking into account the transfer of water from one time period to another	
7	Output	Water withdrawal for irrigation or water outflow to other system	

Figure 3. Types and Definitions for Standard Elements of the Flow Network.

Now we should introduce the numbering system for all elements of the flow network and for all time periods. The complete numbering system is shown in Table 1 (the elements of the flow network presented in Figure 4 are numbered following these rules).

For the sake of generality, it is easy to present the uniform network in an analytical form. For these purposes, it is necessary to introduce the following notions describing the links between all nodes.

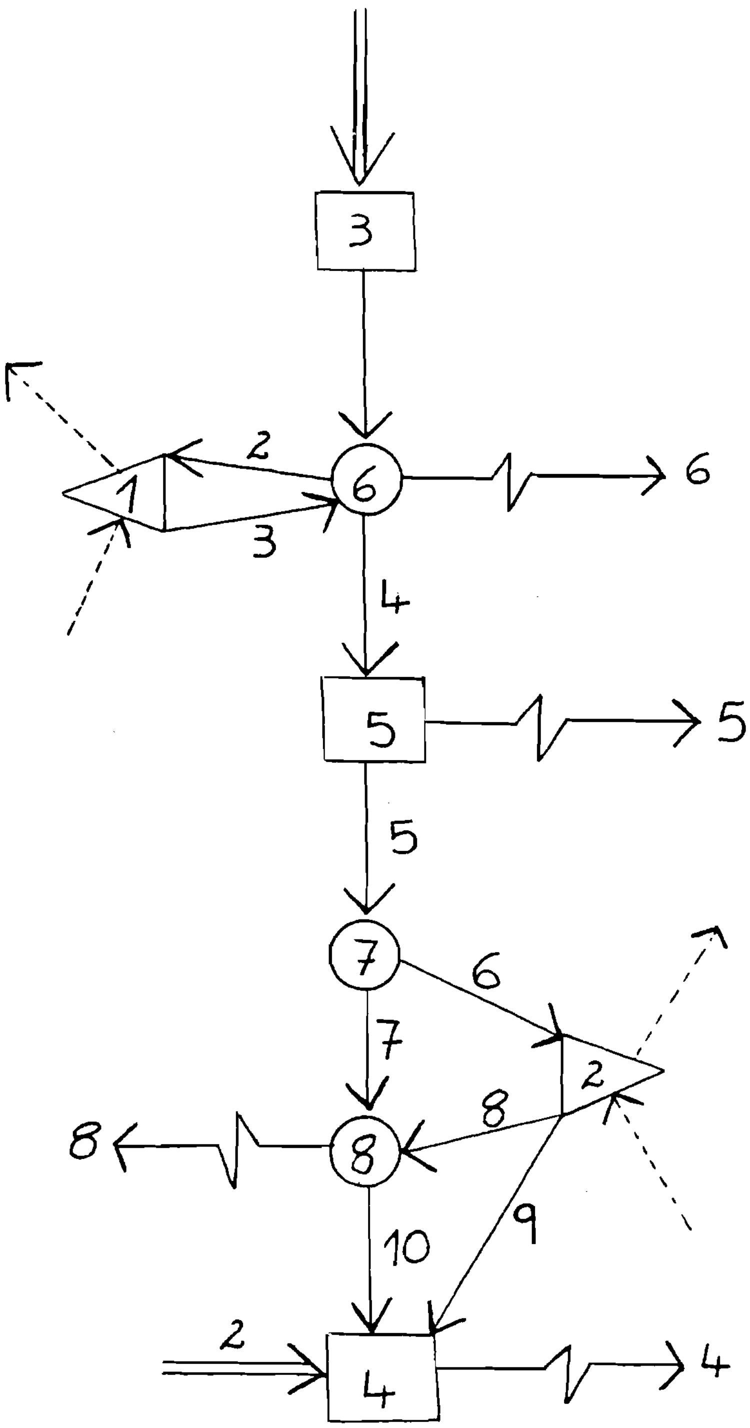


Figure 4. One-Layer Spatial Uniform Network.

Table 1. Numbering System for Standard Elements of Flow Network and for the Time Periods.

	Type of Model Elements		Number of Elements	Numbering Index	Set of Elements of a given type
1.	Inputs		s	α	1,...,s
2.	Reservoir Nodes		r	j	1,...,r
3.	Pumping Nodes	input	s	j	r+1,...,r+s
		internal	m	j	r+s+1,...,r+s
4.	Distributing Nodes		l	j	r+s+m+1,...,r+s+m+ℓ
5.	Arcs		n	i	1,...,n
6.	Outputs *		m+ℓ	j	S + r+1,...,r+s+m+ℓ
7.	Time Periods		N	k	1,...,N

* Distributing or internal pumping nodes and outputs leaving them have the same sequential number.

$I \quad = \{1,\ldots,n\}$ = set of all arc numbers,

$I_j^+ \varepsilon I$ = subset of the entering-arc numbers for node j,

$I_j^- \varepsilon I$ = subset of the leaving-arc numbers for node j.

Let us assume a real irrigation system which is mapped into a uniform flow network consist of the standard elements given in Figure 3. Then the uniform network is said to be presented analytically, if the following data are specified:

1. numbers s, r, m, l, n,
2. subsets of arc numbers I_j^+ and I_j^- for all $j = 1,\ldots,$ $r+s+m+\ell$.

For example, the analytical representation of the flow (Table 2) network shown in Figure 4 is:

$s = 2;\ r = 2;\ m = 1;\ \ell = 3;\ n = 11.$

Table 2. Subsets I_j^+ and I_j^- in the pattern network

Node Number = j	I_j^+	I_j^-
1	2	3
2	7	8, 11
3	∅*	1
4	∅	6
5	4	5
6	8, 9	10
7	1, 3	2, 4
8	5, 6	7, 9
9	10, 11	∅

* ∅ is empty set

3.3 Mathematical Description of the Generalized Model

To describe formally the water supply model, it is necessary to define the model variables; for the sake of brevity they are presented in the following table:

Table 3. Variables in the Model

Definition	Designation	Units	Type of Variable
Input flow α in period k	q_α^k	m^3/s	decision (or initial data)
Flow in arc i in period k	y_i^k	m^3/s	decision
Output flow j in period k	w_j^k	m^3/s	initial data
Capacity of reservoir j	V_j	m^3	decision
Discharge capacity of canal/ arc i	z_i	m^3/s	decision
Capacity of pumping station/ node j	x_j	m^3/s	decision
Duration of time period k	t_k	sec	initial data
Active water storage in reservoir j at the beginning of period k	s_j^k	m^3	decision

As seen from the above table, input flow q_α^k $(k=1,\ldots,N)$ from a water source α can be a decision variable or initial data. The first situation corresponds to the case where input water flow results from the solution of some optimization problem. The water sources here can be streamflow, lake or groundwater. In the second case, the input water flow is prespecified. For example, it can be an input from another irrigation system already built. On the other hand, output water flow w_j^k can be a water requirement for an irrigated area or a fixed input into another irrigation system. In both cases, this flow should be prespecified.

3.3.1 Constraints

Now we have at our disposal everything that is required to describe the model mathematically. Let us start with constraints on the decision variables. All constraints of the model are physical ones, and can be divided into the following four groups:

(1) <u>Non-negativity conditions for decision variables</u>

Being inherent in most mathematical programming problems, these constraints require that all decision variables be non-negative.

(2) <u>Flow balances at pumping and distributing nodes</u>

This set of constraints requires that flow continuity be satisfied at the network nodes. For input pumping nodes those are:

$$q_\alpha^k + \sum_{i \in I_j^+} y_i^k - \sum_{i \in I_j^-} y_i^k = 0,$$

$$\alpha = 1,\ldots,s$$
$$j = r+\alpha$$
$$k = 1,\ldots,N\ .$$

Analogously, for the distributing and internal pumping nodes we have:

$$\sum_{i \in I_j^+} y_i^k - \sum_{i \in I_j^-} y_i^k - w_j^k = 0\ ,$$

$$j = r+s+1,\ldots,r+s+m+\ell$$
$$k = 1,\ldots,N.$$

(3) <u>Mass balances for each reservoir</u>

These constraints describe the release and storage regimes for all reservoirs:

$$s_j^1 = t_N \left(\sum_{i \in I_j^+} y_i^N - \sum_{i \in I_j^-} y_i^N \right) + s_j^N\ ,$$

$$s_j^{k+1} = \left(\sum_{i \in I_j^+} y_i^k - \sum_{i \in I_j^-} y_i^k \right) + s_j^k \quad ,$$

$$j = 1,\ldots,r$$

$$k = 1,\ldots,N-1$$

The first set of these constraints is annual cycle condition for reservoirs. In other words, it reflects the fact that we consider only within-year regulation of water.

(4) Upper bounds

This set of physical constraints requires that canal and pumping station flows and active reservoir storages should not exceed their capacities. Those are:

$$t_k \sum_{i \in I_j^-} y_i^k - s_j^k \leq 0$$

$$s_j^k - V_j \leq 0 \qquad \text{(for reservoir nodes)}$$

$$j = 1,\ldots,r; \; k = 1,\ldots,N$$

$$\sum_{i \in I_j^-} y_i^k - x_j \leq 0 \qquad \text{(for pumping nodes)}$$

$$j = r+1,\ldots,r+s+m; \; k = 1,\ldots,N \quad .$$

$$y_i^k - z_i \leq 0 \qquad \text{(for canals/arcs)}$$

$$i = 1,\ldots,n; \; k = 1,\ldots,N \quad .$$

Furthermore, a number of upper bounds constraining capacities of water supply facilities should be added.

3.3.2 Objective Function

As stated above, the objective of our modeling is to find a least-cost water supply system. When the hydraulic scheme of the system is fixed, we should determine capacities of reservoirs, pumping stations, canals, and within-year regimes of their operation. In the model under analysis, the measure for the total costs associated with the establishment and operation of the water system is the generalized annual cost caused by:

- construction of reservoirs, pumping stations and canals,
- loss of the submerged arable lands,
- operation of reservoirs and canals,
- maintenance of pumping stations,
- consumption of electric energy for pumping water.

As stated above, we keep the assumption that objective function is linear with respect to capacities of reservoirs and pumping stations, and discharge capacities of canals. To express it formally, the following notions should be introduced:

a_j = increment of annual cost associated with the construction and operation of reservoir j, due to the unit increment of its capacity, lv/m^3;*

b_j = increment of annual cost associated with the construction and maintenance of pumping station j, due to the unit increment of its capacity, $lv/m^3/s$;

γ_i = increment of annual cost associated with the construction and operation of canal i, due to the unit increment of its discharge capacity, $lv/m^3/s$;

e_j = unit cost associated with electricity consumption for water pumping at node j.

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* lv is an abbreviation for leva - Bulgarian monetary unit.

In these terms, the objective function can be written as follows:

$$E = \underbrace{\sum_{j=1}^{r} a_j V_j}_{\text{cost of reservoirs}} + \underbrace{\sum_{j=r+1}^{r+s+m} b_j X_j}_{\text{cost of pumping stations}} +$$

$$+ \underbrace{\sum_{i=1}^{n} \gamma_i Z_i}_{\text{cost of canals}} + \underbrace{\sum_{j=r+\sigma+1}^{r+s+m} e_j \sum_{i \in I_j^+} \sum_{k=1}^{N} t_k y_i^k + \sum_{\alpha=1}^{\sigma} e_{r+d} \sum_{k=1}^{N} t_k q_\alpha^k}_{\text{cost of electric energy}} .$$

It must be said that objective function E describes the real annual cost of the whole water supply system with the precision of constant additives. Because constant additives do not influence the solution of an optimization problem, they are omitted.

Thus, the generalized mathematical model of water supply system under analysis is the set of constraints (1)-(4) and objective function (5) to be minimized over all decision variables. It is enough for the user of this model to know the numbers r, s, m, 1, n, N, the sets of arc numbers I_j^+ and I_j^-, and the initial data w_j^k, t_k, a_j, b_j, γ_j, e_j, for all j and k.

4. SILISTRA WATER SUPPLY MODEL*

The mathematical model of the Silistra water supply system will be derived from the general model as a special case. The detailed scheme of the (modeled) water supply system is shown in Figure 5. It consists of the following standard elements: three reservoirs, six pumping stations, twenty canals, and nine distributary nodes. In addition, the irrigation system's only water input comes from the Danube river and the twelve water outputs intended for irrigated areas.

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* By the Silistra water supply system we mean the M. Preslavets one.

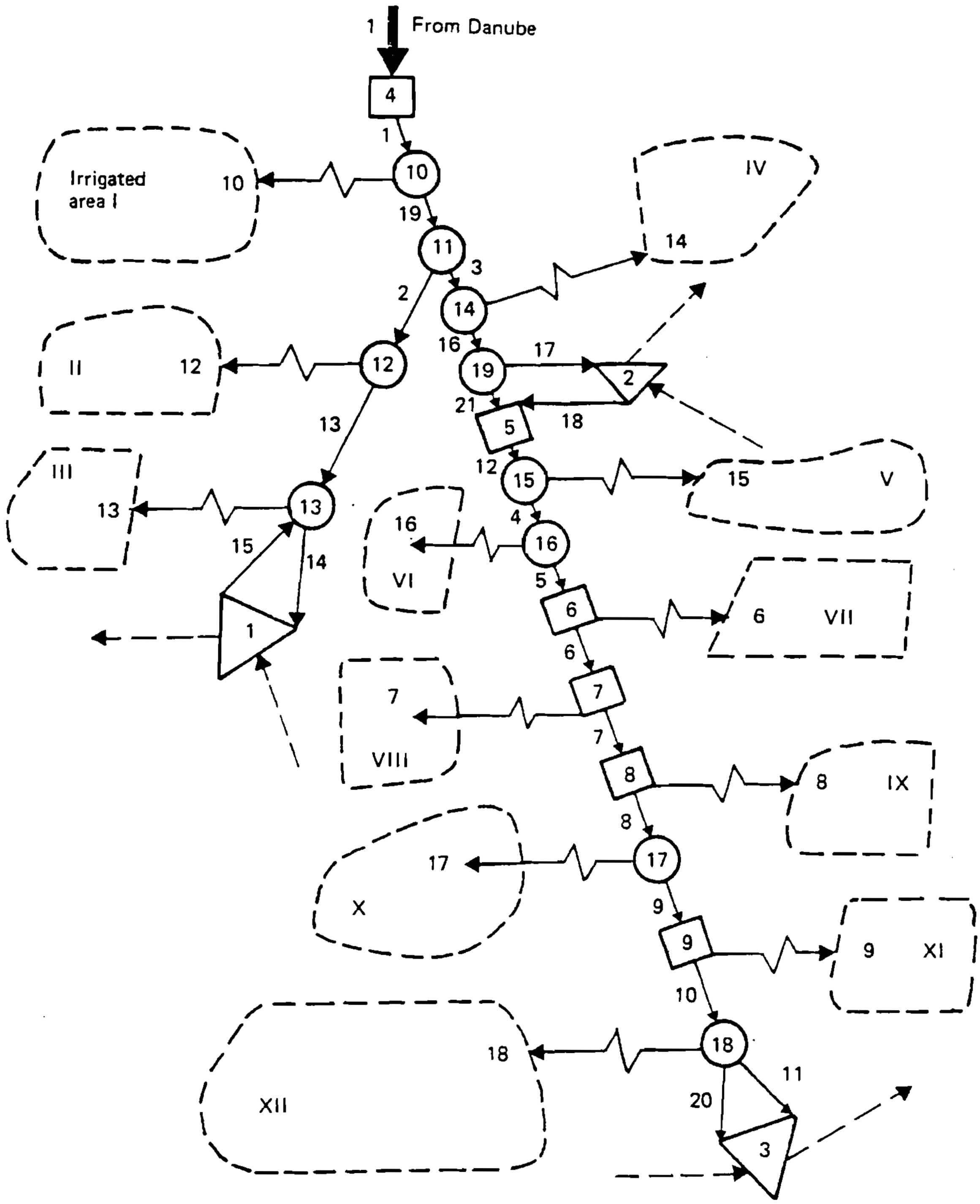

Figure 5. Scheme of the Silistra water supply system.

The Silistra water supply model is constructed under the same assumptions which we previously stated. We will briefly comment on some of them in connection with the Silistra Case. For each irrigated area, the water requirement and the generalized irrigation time-table are specified. Although the Silistra water supply system is expected to be put into operation only step-wise, the decision is made solely with respect to the fully completed system which corresponds to the end of the planning period, the year of 1990. In the Silistra site of the Danube river, the total water withdrawal does not exceed 5% of the streamflow, even in the peak period of a dry year. This allows one to regard the (available) water resources as unlimited. The irreversible use of water follows from the fact that the only user of water is irrigation. Finally, because of the small size of the Silistra region, the transit time delays are not taken into account.

As stated previously, in order to describe mathematically the model we should:

(1) specify the numbers s, r, m, l, n, N;

(2) enumerate all the standard elements into a flow network according to the rules of Figure 5;

(3) define the subsets I_j^+ and I_j^- for all nodes j=1, ..., r+s+m+ℓ. After doing this (see Figure 1), we obtain:

s = 1 = number of water inputs or input pumping nodes,

r = 3 = number of reservoir nodes,

m = 5 = number of internal pumping nodes,

1 = 10 = number of distributing nodes,

n = 21 = number of arcs.

The subsets of the entering (I_j^+) and leaving (J_j^-) arcs are in Table 4.

It is necessary to stress that in the Silistra Case the water input q_1^k is a decision variable, and the twelve water outputs w_j^k (j = 6, ..., 10, 12, ..., 18) are prespecified irrigation water requirements.

Table 4. Subsets I_j^+ and I_j^- in the Silistra Water Supply System

Node Number j	I_j^+	I_j^-
1	14	15
2	17	18
3	20	11
4	ϕ	1
5	18, 21	19
6	5	6
7	6	7
8	7	8
9	9	10
10	1	12
11	12	2, 3
12	2	13
13	13, 15	14
14	3	16
15	19	4
16	4	5
17	8	9
18	10, 11	20
19	16	17, 21

Thus, the mathematical model of the Silistra water supply system can be presented in the form of constraints (1), ..., (4) and the objective function (5). However, taking into account the peculiarities of the Silistra irrigation scheme, the model can be written much more simply. First, as seen from Figure 5, capacities of all pumping stations are equal to the discharge capacities of the respective canals. This means that some decision variables are unnecessary and can be omitted from the general model. Second, some canals are (intentionally) considered to be of zero cost. This is done for three reasons. The first is that the sizes (costs) of some canals (e.g. 13) are fully defined by those of the adjacent up-stream canals (respectively 2). The second reason is that some of the canals are artificially introduced into the scheme (e.g. 21) to present it as a uniform network. Finally, the costs of some canals (e.g. 17) are included in the costs of other facilities (reservoir 2). Everything mentioned above means that the expression (5) for objective function in the Silistra water supply model can be essentially simplified in comparison with the general case.

The mathematical model for the Silistra irrigation system is presented below in the reduced form, which corresponds to the general model (1) - (5) with the above mentioned simplifications introduced. The transformations of the general model constraints and the objective function include:

(1) elimination of the decision variables:

$g_1^k,\ y_{21}^k,\ y_{19}^k,\ X_4,\ \dots,\ X_9$,

by the relations :

$$
\begin{aligned}
q_1^k &= y_1^k \\
y_{19}^k &= y_2^k + y_3^k \\
y_{21}^k &= y_{16}^k - y_{17}^k \\
X_4 &= Z_1 \\
X_5 &= Z_{12} \\
X_6 &= Z_5 \\
X_7 &= Z_6 \\
X_8 &= Z_7 \\
X_9 &= Z_9
\end{aligned}
\qquad (1)
$$

(2) omission of the decision variables $Z_{12}, \dots, Z_{21}$ because they correspond to the canals of zero cost as explained above.

With these modifications, the model under analysis is written as follows (constraint sets and the objective function are numbered in accordance with the numbers adopted for the general model presented in section 3 of this paper):

Flow balances at pumping and distributing nodes

$$
\begin{aligned}
&y_1^k - y_2^k - y_3^k &&= w_{10}^k \\
&y_2^k - y_{13}^k &&= w_{12}^k \\
&y_{13}^k - y_{14}^k + y_{15}^k &&= w_{13}^k \\
&y_3^k - y_{16}^k &&= w_{14}^k \\
&y12_k + y_{17}^k - y_{18}^k - y_{16}^k &&= 0 \\
&y_{12}^k - y_4^k &&= w_{15}^k \qquad\qquad (2) \\
&y_4^k - y_5^k &&= w_{16}^k \qquad\qquad k = 1, \ldots, N \\
&y_5^k - y_6^k &&= w_6^k \\
&y_6^k - y_7^k &&= w_7^k \\
&y_7^k - y_8^k &&= w_8^k \\
&y_8^k - y_9^k &&= w_{17}^k \\
&y_9^k - y_{10}^k &&= w_9^k \\
&y_{10}^k + y_{11}^k - y_{20}^k &&= w_{18}^k \quad .
\end{aligned}
$$

Mass balances for reservoirs

$$
\begin{aligned}
&s_1^{k+1} - s_1^k + t_k(y_{15}^k - y_{14}^k) = 0 \qquad\qquad k = 1, \ldots, N \\
&s_1^{n+1} = s_1^1 \\
&s_2^{k+1} - s_2^k + t_k\ (y_{18}^k - y_{17}^k) = 0 \qquad\qquad (3) \\
&s_2^{n+1} = s_2^1 \\
&s_3^{k+1} - s_3^k + t_k\ (y_{11}^k - y_{20}^k) = 0 \\
&s_3^{n+1} = s_3^1 \quad .
\end{aligned}
$$

Upper bounds

$$
\begin{aligned}
&t_k y_{15}^k - s_1^k \leq 0 \\
&t_k y_{18}^k - s_2^k \leq 0 \qquad\qquad (4) \\
&t_k y_{11}^k - s_3^k \leq 0 \qquad\qquad k = 1, \ldots, N \\
&y_{16}^k - y_{17}^k \leq 0 \\
&s_j^k - v_j \leq 0 \qquad\qquad j = 1, 2, 3 \\
&y_p^k - z_p \leq 0 \qquad\qquad p = 1, \ldots, 11, 12
\end{aligned}
$$

The reduced form for the objective function is:

$$E = \sum_{j=1}^{3} a_j v_j \qquad + \qquad \sum_{p=1}^{12} \gamma_p z_p \qquad + \qquad (5)$$

cost of reservoirs *cost of canals and pumping stations*

$$+ \sum_{k=1}^{n} + t_k (e_4 y_1{}^k + e_5 y_{12}{}^k + e_6 y_5{}^k + e_7 y_6{}^k + e_8 y_7{}^k + e_9 y_9{}^k)$$

cost of energy for pumping water

The model consisting of relations (2)-(4) and the objective function (5) was implemented on a computer.

5. RESULTS OF MODELING

The mathematical model presented in Section 4 was run on the IBM 370/165 in Pisa. Before showing the results of modeling, it is necessary to present the full set of model coefficients--time periods t_k, water requirements w_j^i, and the cost coefficients a_j, e_α, γ_p.

Actually, a year was divided into the nine time periods, as shown in Table 5. While modeling, the three-month time period, December, January, and February was omitted, because during these winter months water supply system does not operate. This interruption is caused both by possible freezing of water in canals or reservoirs and by the necessity of carrying out some work on maintenance of the irrigation system.

The prespecified water requirements for all irrigated areas are shown in Table 6. There is no irrigation in the first period of four months, with interruption for the winter season, and it can only be used for storing water in reservoirs, if any. The sixth period--the first ten days of August--is a period of the most intensive irrigation for all areas.

Table 5. Division of a Year into Time-Periods.

Period Number	Months	Duration [month]	Comment
-	December January February	3	out of work
1	October November March April	4	no irrigation
2	May	1	
3	First 20 days of June	2/3	
4	Last 10 days of June	1/3	
5	July	1	
6	First 10 days of August	1/3	the most intensive irrigation
7	Last 20 days of August	2/3	
8	September	1	

Table 6. Water Requirements for All Irrigation Areas [m^3/s].

Water requirements \ Period number i	1	2	3	4	5	6	7	8
w_1^i	0	0.248	0.317	1.191	0.897	1.294	0.582	0.242
w_2^i	0	0.179	0.228	0.859	0.61	0.961	0.42	0.174
w_3^i	0	4.015	5.127	19.248	13.674	20.927	9.419	3.914
w_4^i	0	0.383	0.488	1.833	1.302	1.993	0.898	0.373
w_5^i	0	0.626	0.8	3.003	2.133	3.264	1.469	0.61
w_6^i	0	0.413	0.528	1.983	1.408	2.155	0.97	0.403
w_7^i	0	0.257	0.328	1.229	0.873	1.337	0.602	0.25
w_8^i	0	0.887	1.135	4.263	3.027	4.634	2.086	0.867
w_9^i	0	1.057	1.35	5.069	3.6	5.51	2.48	1.03
w_{10}^i	0	0.525	0.671	2.522	1.791	2.741	1.234	0.512
w_{11}^i	0	0.414	0.53	1.987	1.411	2.16	0.972	0.404
w_{12}^i	0	7.413	9.467	35.551	25.248	38.64	17.39	6.569

While running the model, the following cost coefficients were used:

$a_1 = 0.0122$	lv/m^3	$\gamma_1 = 118060$	$lv/m^3/sec.$
$a_2 = 0.101$	"	$\gamma_2 = 66896$	"
$a_3 = 0.0368$	"	$\gamma_3 = 79130$	"
$e_4 = 0.00593$	"	$\gamma_4 = 68900$	"
$e_5 = 0.000359$	"	$\gamma_5 = 128088$	"
$e_6 = 0.000282$	"	$\gamma_6 = 149418$	"
$e_7 = 0.00094$	"	$\gamma_7 = 150641$	"
$e_8 = 0.000525$	"	$\gamma_8 = 8653$	"
$e_9 = 0.00213$	"	$\gamma_9 = 105155$	"
		$\gamma_{10} = 17115$	"
		$\gamma_{11} = 67754$	"
		$\gamma_{12} = 44960$	"

All of the model coefficients presented in this section are calculated on the basis of initial data submitted by the Sofia Institute for Water Projects.

5.1 Basic Characteristics of Water Supply System

One of the main goals of modeling is to determine basic characteristics of the Silistra water supply system--capacities of reservoirs and pumping stations and discharge capacities of canals. Some of these capacities computed under the above coefficients are shown in the following table (Table 7).

As can be seen from Table 6, the time-tables of irrigation for all areas are rather irregular. Analysis of water requirements shows that ratio $w_j^i/\max w_j^i$ is constant for all irrigated areas (within 2-3%). In other words, all areas have the same irrigation time-table (see Figure 6).

If the water supply system had contained reservoirs, then all canals and pumping stations would have had the same within-year operation regimes as the time-table of irrigation. From this point of view, reservoirs are intended for equalization of the within-year operation regime for the irrigation system.

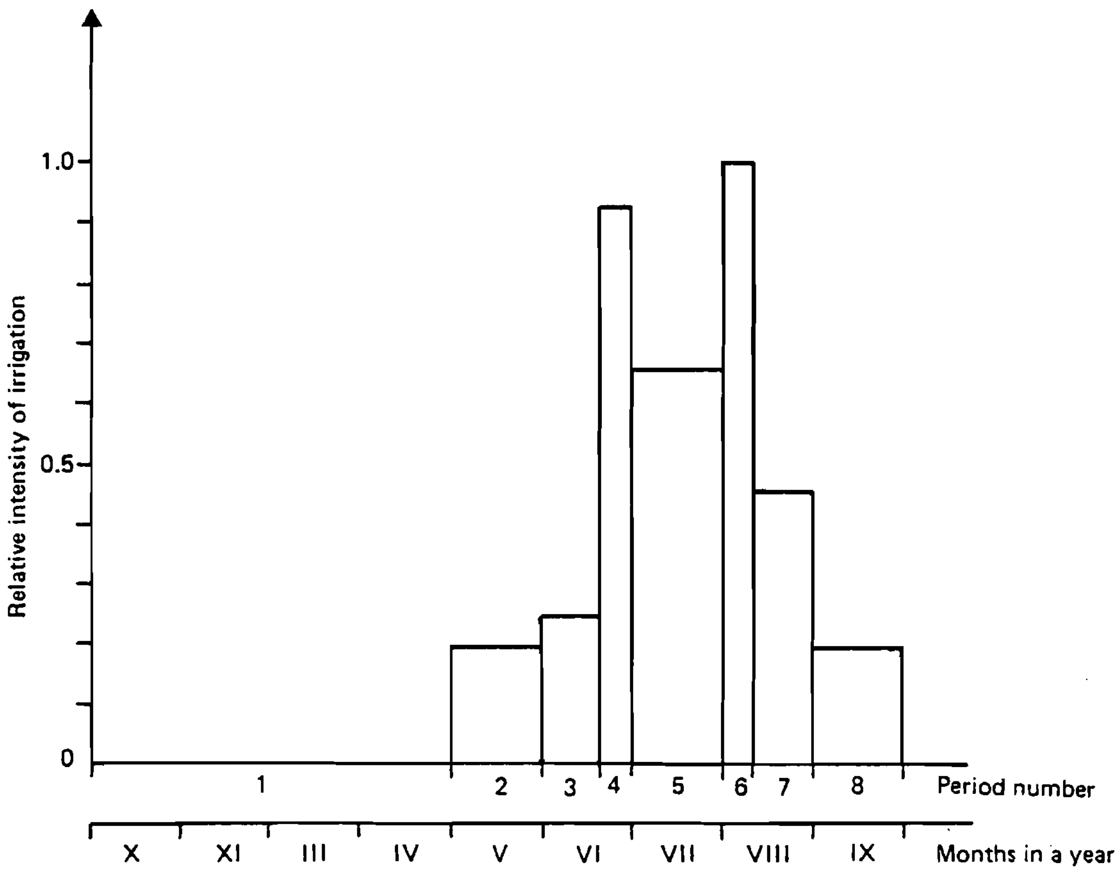

Figure 6. Time-table of irrigation.

Table 7. Basic Characteristics of the Silistra Irrigation System

Facilities	Capacities		Units
	Model Notation	Value	
Reservoir 1	V_1	5.248	$10^6 m^3$
Reservoir 2	V_2	2.459	"
Reservoir 3	V_3	131.645	"
Pumping Station 1	Z_1	38.054	m^3/s
Pumping Station 2	X	21.801	"
Pumping Station 3	Z_5	16.382	"
Pumping Station 4	Z_6	15.045	"
Pumping Station 5	Z_7	12.433	"
Pumping Station 6	Z_9	10.891	"
Canal 2	Z_2	15.813	"
Canal 3	Z_3	21.344	"
Canal 11	Z_{11}	38.64	"

Figures 7 and 8 show the operation regimes for some facilities. Comparison of Figures 6 and 8 lead to the conclusion that the reservoirs result in:

(1) equalizing the within-year operation regimes, and

(2) decreasing the maximum transient water flows,

for pumping stations and canals. The former occurs during the four-month non-irrigation period of storing water in reservoirs. For example, the operation time-table of pumping station 4 is very close to constant during all the nine working months (see Figure 8).

The decrease of the maximum transient water flows becomes possible, because in the peak irrigation periods--last 10 days of June and first 10 days of August--the water requirements are met by reservoirs as much as possible. Quantitative illustration of this point is made easy with the help of the example of the basic pumping station 1 situated on the Danube streamflow. As seen from Table 6, the total maximum water requirement

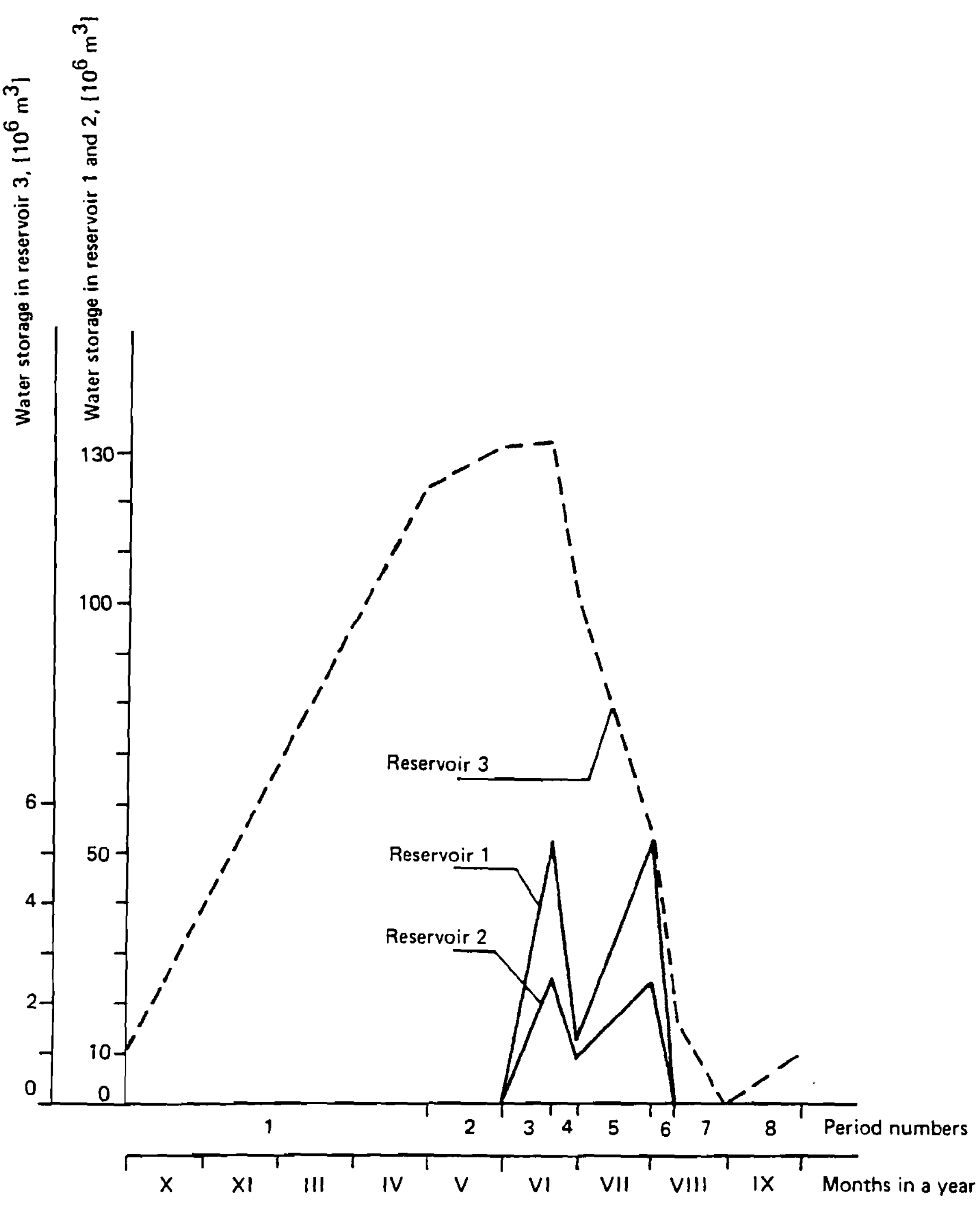

Figure 7. Within-year storing of water in reservoirs.

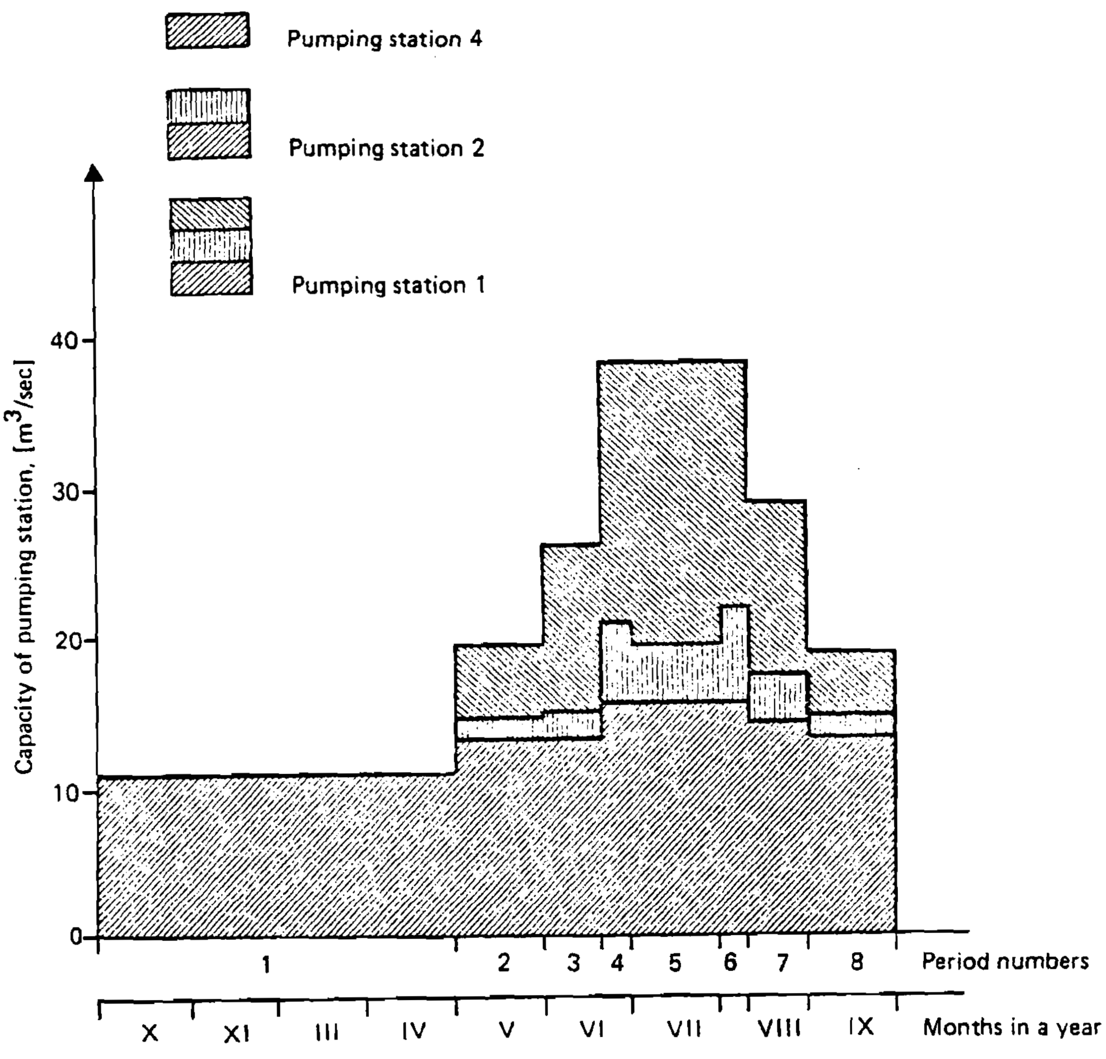

Figure 8. Within-year operation regimes for pumping stations.

corresponding to the peak irrigation period 6 is:

$$w_{max} = \sum_{j=1}^{12} w_j^6 = 85.616 \ m^3/s \quad .$$

This means if reservoirs had not existed, the capacity of the pumping station would have been equal to 85.626 m^3/s. But, as can be seen from the last table (Table 7), the optimal value for this capacity is only 38.054 m^3/s. In other words, the presence of three reservoirs decreases the capacity of the Danube pumping station by 2.25 times.

5.2 Marginal Costs of Water

Another set of modeling results is concerned with marginal cost of water. The two groups of costs are presented here--seasonal and mean annual unit costs of water. Both are obtained as the solution of the dual problem with respect to basic one.

By definition, the seasonal marginal cost (c_j^k) of water in the j-th irrigated area is the increment of the optimal value of objective function E caused by the unit increment of water consumption in this area at time period k. In principle, it is an additional cost associated with supplying the j-th area with an additional unit of water at time period k. The seasonal marginal costs of water are shown in the following table.

As can be seen from Table 8, seasonal marginal costs of water depend essentially on the geographical location of an irrigated area and the season of water consumption. For example, at the seventh irrigation period, the seasonal costs vary from 0.0059 to 0.0584 lv/m^3, or about ten times, depending on the location of the irrigated area. Analogously, in the eighth irrigated area, these costs differ about sixtysix-fold--from 0.0075 to 0.0495 lv/m^3--depending on the season.

Three tendencies are clearly observed when analyzing the seasonal marginal costs of water. Firstly, costs depend on the intensity of irrigation during a given period, rather than on

Table 8. Seasonal Marginal Costs of Water

Area Numbers \ Period Numbers	1	2	3	4	5	6	7	8	Units
1	0.59	0.59	0.59	2.89	1.01	10.7	0.59	0.59	10^{-2} lv/m^3
2	0.59	0.59	0.59	2.89	2.89	12.8	0.59	0.59	"
3	0.59	0.59	0.59	2.89	2.89	12.8	0.59	0.59	"
4	0.59	0.59	0.59	3.77	3.77	10.7	0.59	0.59	"
5	0.63	0.63	0.63	3.81	3.81	15.9	0.63	0.63	"
6	0.63	0.63	0.63	3.81	3.81	23.9	0.63	0.63	"
7	0.66	0.66	0.66	3.84	3.84	38.8	0.66	0.66	"
8	0.75	0.75	0.75	5.58	5.58	49.5	0.75	0.75	"
9	0.8	1.82	1.94	5.63	5.63	49.6	5.63	1.62	"
10	0.8	1.82	1.95	5.63	5.63	49.6	5.63	1.95	"
11	2.0	2.16	2.16	5.84	5.84	49.8	5.84	2.16	"
12	2.16	2.16	2.16	5.84	5.84	14.7	5.84	2.16	"

the period itself. In particular, at the period of the most intensive irrigation--the first 10 days of August (i=6)--the unit water costs for all irrigated areas are much higher than all other periods. Secondly, the seasonal costs of water increase, as a rule, when the distance of an irrigated area from the Danube river increases too. Thirdly, it is important that the possibility of withdrawing water for some irrigated areas directly from a reservoir influences the seasonal cost of water very much. For example, for area 12, distant from the Danube river, the seasonal cost (0.0147) is much less than in the adjacent area 11 (0.498 lv/m^3). This happens because irrigated area 12 can use water from reservoir 3, which is impossible for area 11.

For the purposes of economic analysis, the mean annual marginal costs of water are more suitable than the seasonal ones. It is natural enough to define the mean annual cost c_j, for some irrigated area j, as the weighted-mean sum of seasonal costs over all the time periods; that is:

$$c_j = \sum_{i=1}^{8} \delta_j^k \; c_j^k \quad , \tag{6}$$

where $\delta_j^i > 0$ and $\sum \delta_j^i = 1$. By definition, the weight coefficients δ_j^i are directly proportional to the amount of water taken by irrigated area j in the respective time periods. Namely,

$$\delta_j^k = \frac{t_k w_j^k}{\sum\limits_i t_k w_j^k} \quad . \tag{7}$$

As seen from the above (see Table 6), the weight coefficients for the different irrigated areas are with sufficient accuracy equal, i.e. $\delta_j^k = \delta^k$ for all j=1, ..., 12. This is a trivial consequence of that fact that all irrigated areas have the same time-table of irrigation (see Figure 6).

The weight coefficients calculated by the formula (7) are given below:

Table 9. Weight Coefficients for Marginal Costs of Water

Period Number i	1	2	3	4	5	6	7	8
Weight Coeffi-cient k	0	0.09	0.076	0.144	0.306	0.156	0.141	0.088

Using relation (6), the mean annual unit costs of water are determined as follows:

Table 10. Mean Annual Costs of Water

Area Numbers	1	2	3	4	5	6	7	8	9	10	11	12
Mean Annual Costs, 10^{-2} lv/m^3	2.67	3.57	3.53	3.6	4.54	5.69	8.03	10.5	11.5	11.7	11.8	6.16

As can be seen from above, the mean annual costs of water also depends on geographic location of an irrigated area. The costs vary about 4.5 times--from 0.0267 lv/m^3 in area 1 to 0.118 lv/m^3 in area 11. As stated before, reservoirs decrease the mean annual costs of water for irrigated areas using water directly from the reservoir. For example, in area 12, the mean annual cost equals 0.626 lv/m^3 versus 0.118 lv/m^3 in the eleventh area, which is closer to the Danube than the previous one. The same can be said about the third and second irrigated areas--0.353 lv/m^3 versus 0.0357 lv/m^3, respectively.

All the above-mentioned facts mean that the use of the average unit cost of water is not correct in economic analysis.

5.3 Sensitivity Analysis

When modeling, the response of optimal solutions to the variations in some initial data was analyzed. From one point of view, water requirements, price of land, and price of electric energy are most uncertain in the Silistra Case. We will try briefly to explain this point. As a rule, water requirements are determined proceeding from *a priori* defined unit costs of water. Since it is impossible to correctly prespecify these costs, we should make provision for the possibility of varying water requirements in a wide range. Next, the land price is a rather subjective value and therefore uncertain. It is sufficient to say that the price of land in Bulgaria is defined as a net return from the hundred-years crop-yield on this land. At the same time, in the Silistra region, one has to distinguish four categories of land depending on soil quality and topographical conditions. Lastly, the problem of today--energy--is the reason for widely varying prices of electricity. That is why we have centered on the sensitivity of the model with respect to the above-mentioned initial data.

Since meeting water requirements is the main purpose of water supply system, it should be very sensitive to those. The two types of variations in water requirements were considered--coordinated and partial. By coordinated variation in water requirements, we mean the case where all of the fractional changes $\frac{\Delta w_j^i}{w_j^i}$ in water requirements are equal for all i and j. The partial variation corresponds to an opposite case.

The response of the modeled water supply system to the coordinated variations in water requirements is shown in Figure 9. As noted from the figure, the changes in basic parameters and generalized annual cost of the irrigation system are a linear function of variation in water requirements. For example, the coordinated change in water requirements of 20% is identified with the changes in:

- o capacity of pumping station 1 of 7.6 m^3/s,
- o capacity of reservoir 1 of about 1.05 million m^3,
- o capacity of reservoir 1 of about 26.3 million m^3,
- o generalized annual system cost of 5.88 million lv/year.

Practically speaking, the marginal water costs--seasonal and mean annual--are insensitive to the coordinated variations in water requirements.

In a partial manner, water requirements were varied only for the third and twelfth irrigated areas; separately for each one. Such a choice of irrigated areas is stipulated by the fact that, among others, areas 3 and 12 have the largest water

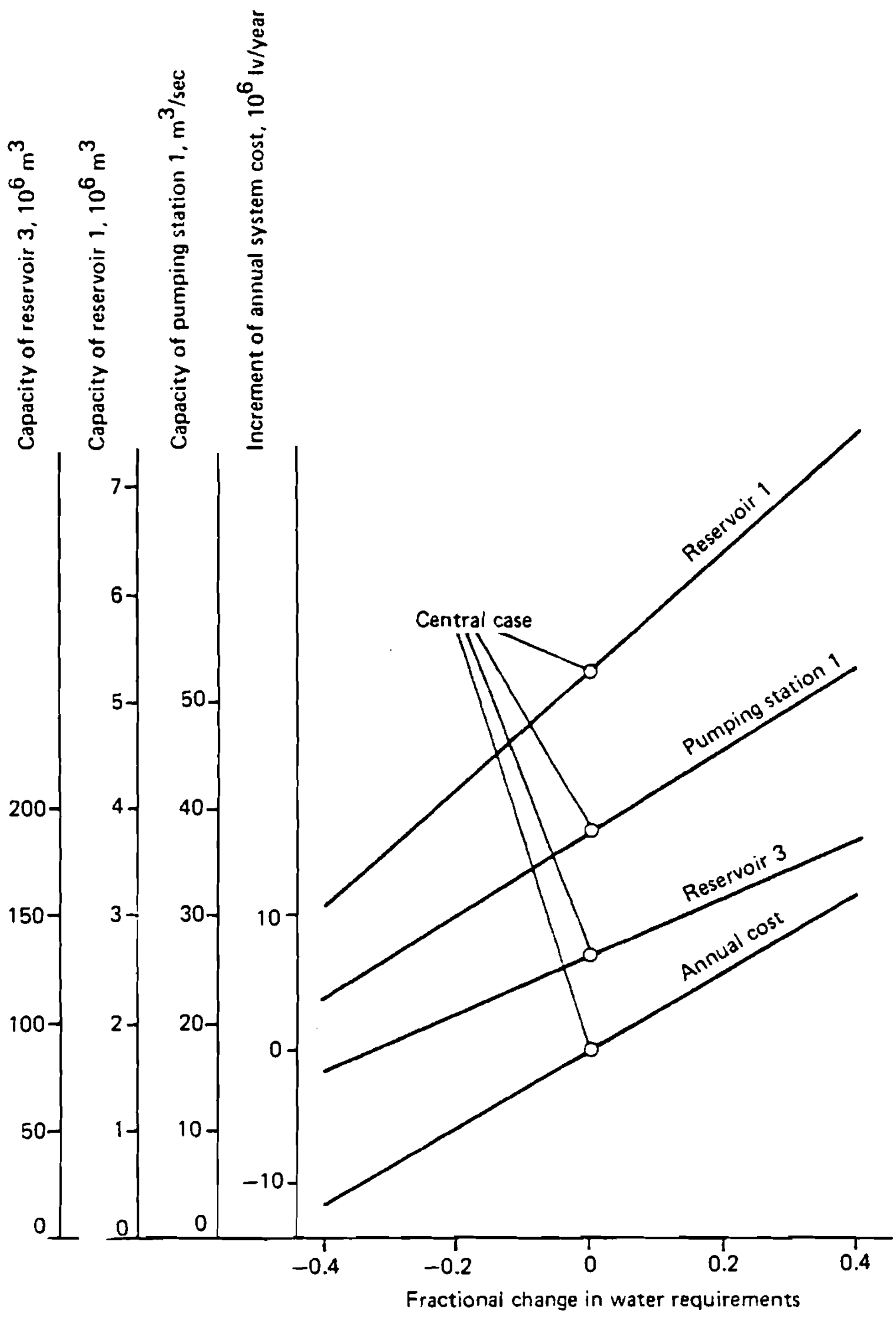

Figure 9. Sensitivity of the system to coordinated variations in water requirements.

requirements. Within a year, water requirements for an area were changed proportionally over all time-periods so that the fractional variations $\frac{\Delta w_j^i}{w_j^i}$ (j = 3 or 12) were equal for all i.

Figure 10 illustrates the results of sensitivity of some system parameters to the variations in water requirements for irrigated area 12. As can be seen from this figure, the responses of capacities of pumping station 1 and reservoir 2 to the variations in water requirements w_{12}^i are non-linear functions of those. In particular, a 20% increase of water requirement in area 12 does not influence the capacities of pumping station 1 and reservoir 2. In contrast with this, a 20% decrease in water requirement is identified with changes in capacities of pumping station 1 of 1.254 m^3/s and reservoir 2 of 1.082 million m^3.

As expected, the annual cost of the system and capacity of reservoir 3 are rather sensitive to the variations in the water requirements for area 12. Practically speaking, both are linear functions of those variations with the state coefficients of 0.1355 million lv/year/percent variation and 1.55 million m^3/percent variation, respectively. It must be said that the capacity of the reservoir that is not shown in Figure 10, does not depend on the water requirement variations ranging from -80% to 20%.

The responses of some system parameters to the variations in water requirement for irrigated area 3 are shown in Figure 11. Three curves there corresponding to the generalized annual cost of the system, and capacities of pumping station 1 and reservoir 1, are very close straight lines. Hence, a 20% increase in water requirements increases annual cost by 0.818 million lv/year, capacity of reservoir 1 by 1.012 million m^3 and in capacity of pumping station 1 by 3.013 m^3/s. Notably, capacities of all the water supply facilities situated on the branches of the water network to the right side of node 11 (see Figure 5) do not depend on the variations in water requirements for area 3.

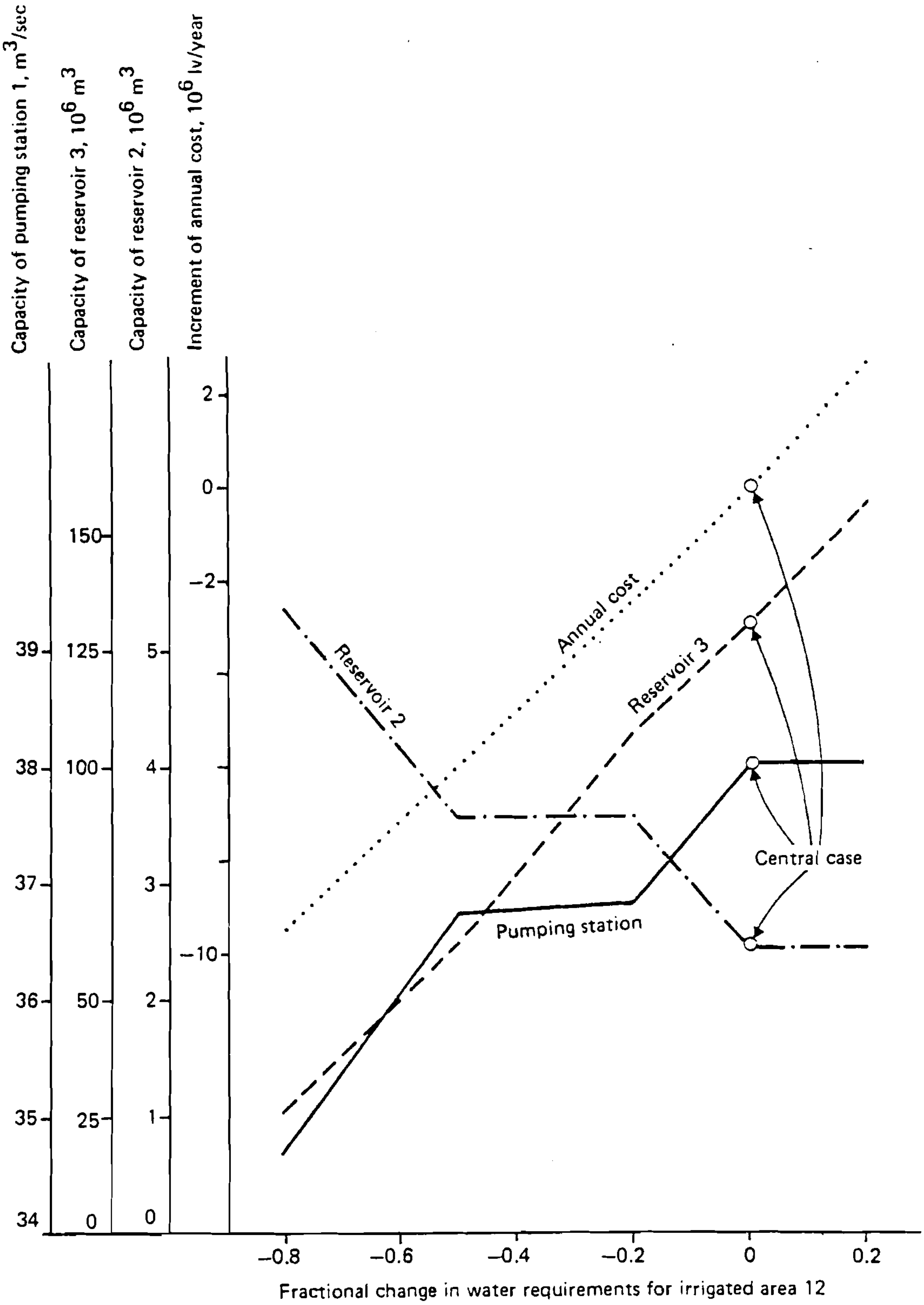

Figure 10. Sensitivity of system to partial variations in water requirements.

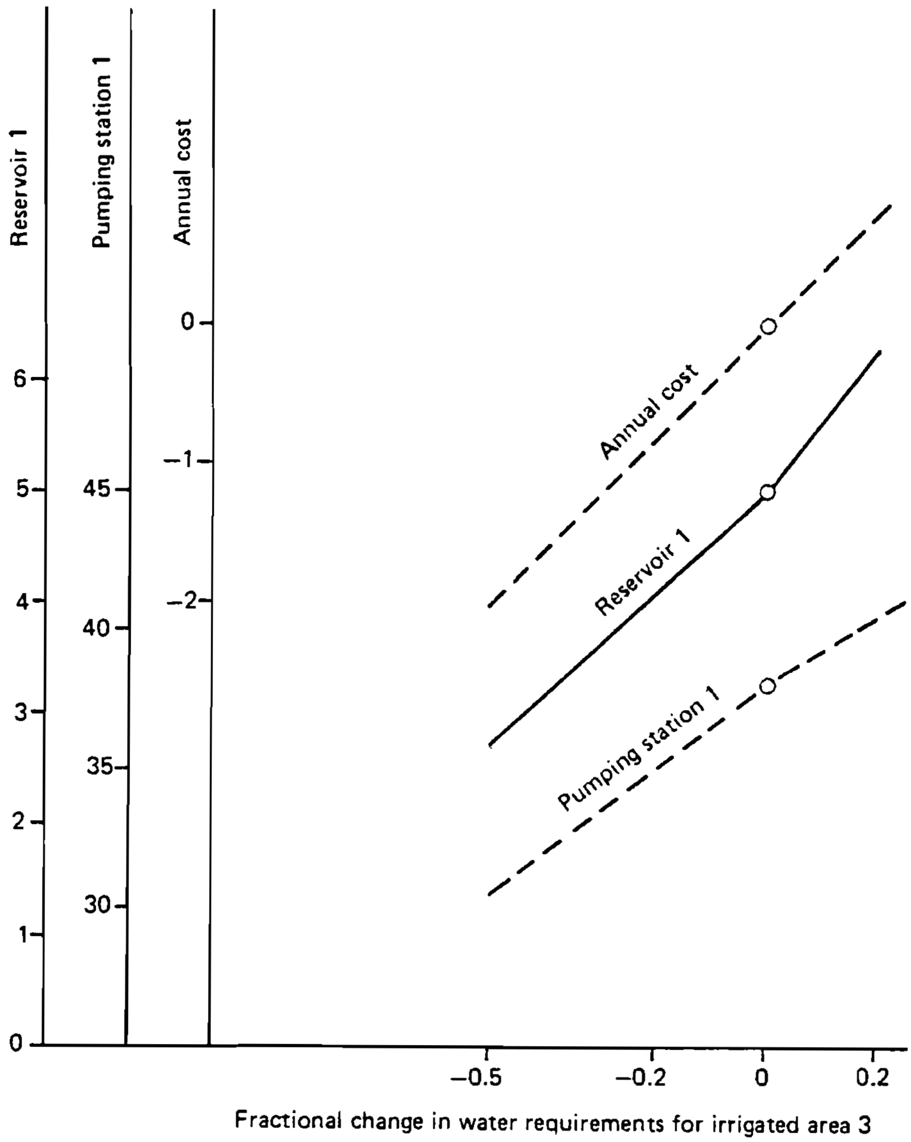

Figure 11. Sensitivity of system parameters to variations in water requirements.

We go on with sensitivity analysis of the system under uncertainty in land price. As stated above, it is a subjective value. At the same time, establishment of the water supply system entails the inevitable losses of land needed for the construction of irrigation facilities. According to the data presented by the Sofia Institute for Water Projects, up to 70% of their generalized annual costs are due to the land losses. When modeling, the following two types of lost land are taken into account:

(1) submerged by reservoirs,

(2) needed for construction of canals.

It must be said that, as distinct from reservoirs, the area of land required for construction of canals depends on canal capacities.

Figure 12 illustrates the sensitivity of water supply system with respect to the variations in land price. As can be seen from it, capacity of the Danube pumping station is actually insensitive to land price in its whole range. The capacities of reservoirs 1 and 3 are constant in the range of price ratio from 0.5 to 2.0, but then the decrease in the price ratio from 0.5 to 0.1 is identified with the changes in the capacities of 6.95% and 7.68% for reservoirs 1 and 3, respectively. The capacity of reservoir 2 is most sensitive to land price. For example, a decrease of the price ratio from 0.5 to 0.1 causes the 86.3% increase in the capacity of reservoir 2. Observing the curves in Figure 12, we can state that basic parameters of water supply system are fairly insensitive to land price in the range of price ratio from 0.5 to 1.5.

The capacities of all water supply facilities are quite insensitive to the variations in energy price. This conclusion follows from the structure of objective function (5). As can be seen from it, the total energy cost associated with pumping water is determined only by the following values:

(1) price of electric energy,

(2) amounts of water pumped by each of the six pumping stations within a year.

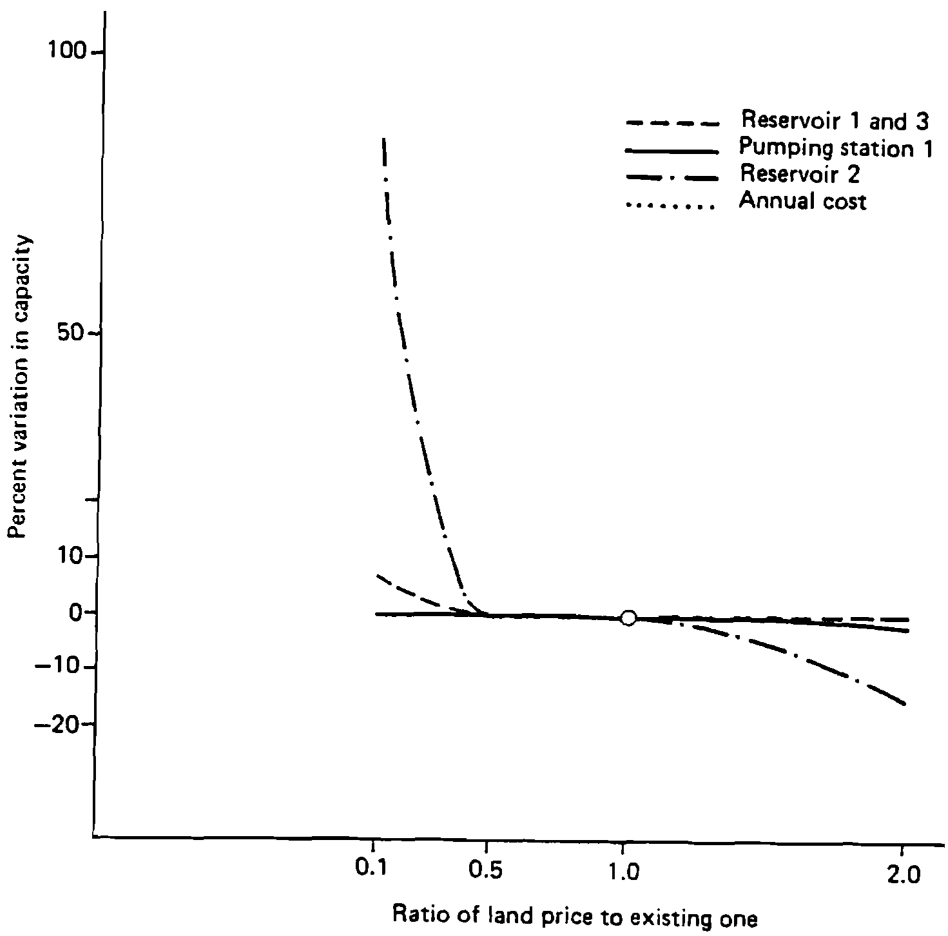

Figure 12. Sensitivity of system to variations in price of land.

Since the scheme of water supply system is fixed and water requirements are prespecified, the latter means that the energy cost does not depend on the variant of the system. In other words, the search of optimal solutions does not depend on energy price. Thus, this results in the insensitivity of the basic system parameters to every price. Of course, the generalized annual cost of the water supply system is influenced by energy price. Specifically, the annual cost of the system is a linear function of energy price, so that a 1% increase in price is identified with an increase in annual cost of 0.039 million lv/year.

Summarizing some results of sensitivity analysis, we can state that the water supply system is rather sensitive to both the coordinated and partial variations in water requirements. As to prices of land and energy, the system is fairly insensitive to them--at least in the long range.

5.4 Practical Application of the Model

The present version of the Silistra Water Supply Model was first implemented on the Pisa IBM 370/165 computer and afterwards transferred to the Sofia ICL 1904 computer. Running the model on the latter allowed the Bulgarian designers to correct basic parameters of the irrigation system due to the variations in some initial data over time.

In 1979, IIASA recommendations on the basic parameters of the Silistra water supply system were given to the Sofia Institute for Water Projects which is in charge of designing water resource systems in Bulgaria. As a result of the above, the main criteria for the practical estimation of the modeling results is the generalized annual cost of the water supply system, in spite of the fact that this annual cost of the system envisaged by the project was known--45.516 million lv/year. The point is that the generalized annual cost under modeling is not a real one due to the following. Firstly, the linear objective function describes only in approximate terms, the changes in real costs depending on the changes in decision variables. Secondly, it takes into account no additive components. As a modeling results, the optimal value of generalized annual cost of the system is equal to 29.4 million lv/year.

That is why the following way was chosen to correctly estimate the optimal solution obtained by the model run. As stated above, the model run results in capacities and operation regimes for all the water supply facilities. On the basis of this data, once more hydraulic and economic calculation of the whole irrigation system was made by the Sofia Institute for Water Projects. The generalized annual cost--39.212 million lv/year--obtained by this calculation, should be compared with that envisaged by the project. Comparison of two annual costs answers the question, which of the two variants of water supply system is better--that envisaged by the project or that determined by the optimization model. The following table illustrates some results of comparing the two variants of the water supply systems.

Table 11. Comparison of the Water Supply System Variants

Variant / Facility	Determined by Modeling	Envisaged by the Institute for Water Projects	Units
Pumping Station 1	38.054	23.22	m^3/s
Reservoir 1	5.248	26.02	million m^3
Reservoir 2	2.454	26.03	million m^3
Reservoir 3	131.645	193.80	million m^3
Canal 1	38.054	23.22	m^3/s
Canal 2	15.813	12.10	m^3/s
Canal 3	21.344	12.40	m^3/s
Generalized Annual Cost	29.400 by model run 39.212 by hydraulic calculation	45.50	million lv/year

By analyzing Table 11, we can conclude that the main distinction of the modeled variant from the projected one consists in the decrease in the capacities of all reservoirs and, on the other hand, in the respective increase in the capacities of the main canals and pumping station. The analysis of the generalized annual costs shows that the variant of the irrigation system obtained by modeling is cheaper by 6-304 million lv/year (or about 15%) in comparison with the projected one.

The more detailed cost analysis, carried out by the Sofia Institute for Water Projects points out the additional advantages of the optimal variants of the system compared to those envisaged by the project. Specifically, for the variant of the irrigation system determined by modeling, the total capital investment and cost of submerged land is 32 million lv and 8.4 million lv respectively; less than for the variant envisaged by the project.

Thus, using the optimization model in choosing the optimal variant of the Silistra water supply system has resulted in a considerable budget saving. If there had not been a projected variant, the use of the model would have saved designers from the labor-consuming hydraulic calculations for a number of preliminary variants of the water supply system. The latter advantage of the mathematical modeling approach to a water supply problem is of great importance in planning the new irrigation systems.

6. CONCLUSIONS

This paper sums up the IIASA's work on the water supply modeling in the Silistra Case Study. The major results are:

1. Development of the general water supply model suitable for irrigation systems of rather arbitrary configurations. The main objective of modeling is to determine the least-cost variant of a water supply system.

2. Development and computer implementation of the Silistra irrigation water supply model based on the general one. For a wide range of input data, the developed model allows one to determine:
   - o capacities,
   - o geographically distributed marginal costs of water, and
   - o within-year operation regimes,

   for all the water supply facilities of the Silistra irrigation system.

3. The sensivitity analysis which results in that Silistra water supply system is rather sensitive to the variations in water requirements, and fairly insensitive to the variations in prices of land and electric energy.
4. Practical application of the modeling results to the Silistra irrigation system points to the advisability of a considerable decrease in the capacities of all three reservoirs. The optimal variant obtained by modeling is about 40 million lv in capital investment, and 6.3 million lv in annual cost cheaper than that envisaged by the project.
5. The developed modeling approach is expected to be used by the water use designer as a universal tool to search for the least-cost alternatives for many irrigation systems in Bulgaria.

REFERENCES

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APPENDIX: COMPUTER OUTPUT OF THE LP SILISTRA MODEL

The linear programming model of the Silistra Irrigation was implemented on the Pisa IBM 370/165 by Andras Por from the SDS Section of IIASA. The Sesame package was used for generating the model into a MPS 360 format and was also used for the solution. Below, the identifiers for constraints (2^1) - (5^1), (horizontal section), and decision variables (vertical section), and the listing of optimal solutions are presented.

Name of Row	Definition	Reference	Unit
DEMjk	Flow balance at node j in time period k	(2^1)	m^3/s
STORik	Mass balance in reservoir i in time period k	(3^1)	m^3
BALAi	Annual cycle condition for reservoir i	(3^1)	m^3
RELik	Upper bound on water release from reservoir i in period k	(4^1)	m^3
INFLOWk	Flow balance at node 5 in period k	(2^1)	m^3/s
CONSTRK	Physical constraint on flows at node 19	(4^1)	m^3/s
UPSTOik	Upper bound on water storage in reservoir i in period k	(4^1)	M^3
UPCNnk	Upper bound on water flow in canal n in period k	(4^1)	m^3/s
FUNC	Objective function	(5^1)	lv/year

The decision variable identifiers almost coincide with their notions in the mathematical model. The only difference is that the first identification number corresponds to the lower index of a variable and the second identifier number corresponds to the upper one.

SILISTRA WATER SUPPLY   AT 18:38:22 ON 02/19/80

SOLUTION

OPTIMAL SOLUTION AT ITERATION NUMBER 332

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	29400019.	FUNC
RESTRAINTS		RHS

STUDY WATER SUPPLY

AT 18:38:22 ON 02/19/80

SOLUTION

ROWS SECTION

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	[illegible]	BS	29400019.	-29400019.	NONE	NONE	1.0000000
2	CAL01	EQ	.	.	.	.	-.00593000
3	SAL02	EQ	.	.	.	.	-.00593000
4	[illegible]	EQ	.	.	.	.	-.02163347
5	DEF1001	EQ	.	.	.	.	-61482.240
6	DEC1201	EQ	.	.	.	.	-61482.240
7	DEM1301	EQ	.	.	.	.	-61482.240
8	DEM1401	EQ	.	.	.	.	-61482.240
9	DEF1501	EQ	.	.	.	.	-65204.352
10	DEC1601	EQ	.	.	.	.	-65204.352
11	DEF0501	EQ	.	.	.	.	-68128.128
12	DEM0701	EQ	.	.	.	.	-77874.048
13	DEF0801	EQ	.	.	.	.	-83317.248
14	DEM1701	EQ	.	.	.	.	-83317.248
15	DEF0901	EQ	.	.	.	.	-207160.78
16	DEF1801	EQ	.	.	.	.	-224295.78
17	STOR101	EQ	.	.	.	.	.00593000
18	REL101	BS	.	.	NONE	.	.
19	DESL0101	BS	-5248584.0	5248584.0	NONE	.	.
20	STOR201	EQ	.	.	.	.	.00593000
21	REL201	UL	.	.	NONE	.	.
22	DESL0201	BS	-2459808.0	2459808.0	NONE	.	.
23	STOR301	EQ	.	.	.	.	.02163347
24	REL301	BS	-10157930.	10157930.	NONE	.	.
25	DESL0301	BS	-121487E03	121487E03	NONE	.	.
26	CONSTR01	BS	-10.891955	10.891955	NONE	.	.
27	[illegible]	EQ	.	.	.	.	61482.240
28	DPCE0101	BS	-27.162295	27.162295	NONE	.	.
29	DPCE0201	BS	-15.813250	15.813250	NONE	.	.
30	DPCE0301	BS	-10.452045	10.452045	NONE	.	.
31	DPCE0401	BS	-7.6450455	7.6450455	NONE	.	.
32	DPCN0501	BS	-5.4900455	5.4900455	NONE	.	.
33	DPCE0601	BS	-4.1530455	4.1530455	NONE	.	.
34	DPCM0701	BS	-1.5420000	1.5420000	NONE	.	.
35	DPCE0801	BS	-.51200000	.51200000	NONE	.	.
36	DPCE0901	UL	.	.	NONE	.	101779.69
37	DPCE1001	UL	.	.	NONE	.	17115.000
38	DPCE1101	BS	-38.640000	38.640000	NONE	.	.
39	DPCE1201	BS	-10.909045	10.909045	NONE	.	.
40	DEF1002	EQ	.24800000	.	.24800000	.24800000	-15370.560
41	DEM1202	EQ	.17900000	.	.17900000	.17900000	-15370.560
42	DEM1302	EQ	4.0150000	.	4.0150000	4.0150000	-15370.560
43	DEF1402	EQ	.38300000	.	.38300000	.38300000	-15370.560
44	DEM1502	EQ	.62600000	.	.62600000	.62600000	-16301.088
45	DEF1602	EQ	.41300000	.	.41300000	.41300000	-16301.088
46	[illegible]	EQ	.25700000	.	.25700000	.25700000	-17032.032
47	DEM0702	EQ	.88700000	.	.88700000	.88700000	-19468.512
48	DEF0802	EQ	1.0570000	.	1.0570000	1.0570000	-50552.985
49	DEM1702	EQ	.52500000	.	.52500000	.52500000	-50552.985
50	[illegible]	EQ	.41400000	.	.41400000	.41400000	-56073.945
51	DEF1802	EQ	7.4130000	.	7.4130000	7.4130000	-56073.945
52	STOR102	EQ	.	.	.	.	.00593000

SOLUTION

ROWS SECTION

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	53	REL102	BS	.	.	NONE	.	.
	54	UPST0102	BS	-5248584.0	5248584.0	NONE	.	.
	55	STOR202	EQ	.	.	.	.	.00593000
A	56	REL202	UL	.	.	NONE	.	.
	57	UPST0202	BS	-2459808.0	2459808.0	NONE	.	.
	58	STOR302	EQ	.	.	.	.	.02163347
	59	REL302	BS	-22930835.	22930835.	NONE	.	.
	60	UPST0302	BS	-8559451.6	8559451.6	NONE	.	.
	61	CONSTR02	BS	-14.616955	14.616955	NONE	.	.
	62	IREL0902	EQ	.	.	.	.	15370.560
	63	UPCM0102	BS	-18.612295	18.612295	NONE	.	.
	64	UPCM0202	BS	-11.619250	11.619250	NONE	.	.
	65	UPCM0302	BS	-6.3440455	6.3440455	NONE	.	.
	66	UPCM0402	BS	-4.5460455	4.5460455	NONE	.	.
	67	UPCM0502	BS	-2.8040455	2.8040455	NONE	.	.
	68	UPCM0602	BS	-1.7240455	1.7240455	NONE	.	.
	69	UPCM0702	UL	.	.	NONE	.	29723.673
	70	UPCM0802	BS	-.02700000	.02700000	NONE	.	.
	71	UPCM0902	BS	-.04000000	.04000000	NONE	.	.
	72	UPCM1002	BS	-.45400000	.45400000	NONE	.	.
A	73	UPCM1102	UL	.	.	NONE	.	.
	74	UPCM1202	BS	-7.1840455	7.1840455	NONE	.	.
	75	DEM1003	EQ	.31700000	.	.31700000	.31700000	-10247.040
	76	DEM1203	EQ	.22800000	.	.22800000	.22800000	-10247.040
	77	DEM1303	EQ	5.1270000	.	5.1270000	5.1270000	-10247.040
	78	DEM1403	EQ	.48800000	.	.48800000	.48800000	-10247.040
	79	DEM1503	EQ	.80000000	.	.80000000	.80000000	-10867.392
	80	DEM1603	EQ	.52800000	.	.52800000	.52800000	-10867.392
	81	DEM0603	EQ	.32800000	.	.32800000	.32800000	-11354.688
	82	DEM0703	EQ	1.1350000	.	1.1350000	1.1350000	-12979.008
	83	DEM0803	EQ	1.3500000	.	1.3500000	1.3500000	-33701.990
	84	DEM1703	EQ	.67100000	.	.67100000	.67100000	-33701.990
	85	DEM0903	EQ	.53000000	.	.53000000	.53000000	-37382.630
	86	DEM1803	EQ	9.4670000	.	9.4670000	9.4670000	-37382.630
	87	STOR103	EQ	.	.	.	.	.00593000
A	88	REL103	UL	.	.	NONE	.	.
	89	UPST0103	BS	-5248584.0	5248584.0	NONE	.	.
	90	STOR203	EQ	.	.	.	.	.00593000
A	91	REL203	UL	.	.	NONE	.	.
	92	UPST0203	BS	-2459808.0	2459808.0	NONE	.	.
	93	STOR303	EQ	.	.	.	.	.02163347
	94	REL303	BS	-64156477.	64156477.	NONE	.	.
	95	UPST0303	BS	-718769.45	718769.45	NONE	.	.
	96	CONSTR03	BS	-15.224955	15.224955	NONE	.	.
	97	IREL0803	EQ	.	.	.	.	10247.040
	98	UPCM0103	BS	-12.208420	12.208420	NONE	.	.
	99	UPCM0203	BS	-7.4208750	7.4208750	NONE	.	.
	100	UPCM0303	BS	-4.2075455	4.2075455	NONE	.	.
	101	UPCM0403	BS	-4.1120455	4.1120455	NONE	.	.
	102	UPCM0503	BS	-2.4850455	2.4850455	NONE	.	.
	103	UPCM0603	BS	-1.4760455	1.4760455	NONE	.	.
	104	UPCM0703	UL	.	.	NONE	.	19815.782

SOLUTION

ROWS SECTION

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	105	[illegible]	BS	-.32000000	.32000000	NONE	.	.
	106	[illegible]	BS	-.47900000	.47900000	NONE	.	.
	107	[illegible]	BS	-1.0090000	1.0090000	NONE	.	.
A	108	[illegible]	UL	.	.	NONE	.	.
	109	[illegible]	BS	-6.5760455	6.5760455	NONE	.	.
	110	[illegible]	EQ	1.1910000	.	1.1910000	1.1910000	-25010.520
	111	[illegible]	EQ	.85900000	.	.85900000	.85900000	-25010.520
	112	[illegible]	EQ	19.248000	.	19.248000	19.248000	-25010.520
	113	[illegible]	EQ	1.8330000	.	1.8330000	1.8330000	-32605.020
	114	[illegible]	EQ	3.0030000	.	3.0030000	3.0030000	-32915.196
	115	[illegible]	EQ	1.9830000	.	1.9830000	1.9830000	-32915.196
	116	[illegible]	EQ	1.2290000	.	1.2290000	1.2290000	-33158.844
	117	[illegible]	EQ	4.2630000	.	4.2630000	4.2630000	-48192.595
	118	[illegible]	EQ	5.0690000	.	5.0690000	5.0690000	-48646.195
	119	[illegible]	EQ	2.5220000	.	2.5220000	2.5220000	-48646.195
	120	[illegible]	EQ	1.9870000	.	1.9870000	1.9870000	-50486.515
	121	[illegible]	EQ	35.551000	.	35.551000	35.551000	-50486.515
	122	[illegible]	EQ	.	.	.	.	.02894736
A	123	[illegible]	UL	.	.	NONE	.	.
	124	DPST0104	UL	.	.	NONE	.	.02301736
	125	[illegible]	EQ	.	.	.	.	.03773729
A	126	[illegible]	UL	.	.	NONE	.	.
	127	DPST0204	UL	.	.	NONE	.	.03180729
	128	[illegible]	EQ	.	.	.	.	.05843347
	129	[illegible]	BS	-98260207.	98260207.	NONE	.	.
	130	[illegible]	UL	.	.	NONE	.	.03680000
	131	[illegible]	BS	-18.413000	18.413000	NONE	.	.
	132	[illegible]	EQ	.	.	.	.	32605.020
	133	[illegible]	UL	.	.	NONE	.	19887.000
	134	[illegible]	BS	-.29400000	.29400000	NONE	.	.
	135	[illegible]	UL	.	.	NONE	.	7594.5000
	136	[illegible]	BS	-.28000000	.28000000	NONE	.	.
	137	[illegible]	BS	-.10800000	.10800000	NONE	.	.
	138	[illegible]	UL	.	.	NONE	.	14221.591
	139	[illegible]	BS	-1.6519545	1.6519545	NONE	.	.
	140	[illegible]	BS	-5.6909545	5.6909545	NONE	.	.
	141	[illegible]	BS	-7.7009545	7.7009545	NONE	.	.
	142	[illegible]	BS	-9.6879545	9.6879545	NONE	.	.
A	143	[illegible]	UL	.	.	NONE	.	.
	144	[illegible]	BS	-.54100000	.54100000	NONE	.	.
	145	[illegible]	EQ	.89700000	.	.89700000	.89700000	-26279.560
	146	[illegible]	EQ	.61000000	.	.61000000	.61000000	-75031.560
	147	[illegible]	EQ	13.674000	.	13.674000	13.674000	-75031.560
	148	[illegible]	EQ	1.3020000	.	1.3020000	1.3020000	-97815.060
	149	[illegible]	EQ	2.1330000	.	2.1330000	2.1330000	-98745.588
	150	[illegible]	EQ	1.4080000	.	1.4080000	1.4080000	-98745.588
	151	[illegible]	EQ	.87300000	.	.87300000	.87300000	-99476.532
	152	[illegible]	EQ	3.0270000	.	3.0270000	3.0270000	-144577.78
	153	[illegible]	EQ	3.6000000	.	3.6000000	3.6000000	-145938.58
	154	[illegible]	EQ	1.7910000	.	1.7910000	1.7910000	-145938.58
	155	[illegible]	EQ	1.4110000	.	1.4110000	1.4110000	-151459.54
	156	[illegible]	EQ	25.248000	.	25.248000	25.248000	-151459.54

SOLUTION

ROWS SECTION

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	157	STOR105	EQ	.	.	.	.	.02894736
A	158	REL105	UL	.	.	NONE	.	.
	159	UPST0105	BS	-3963816.0	3963816.0	NONE	.	.
	160	STOR205	EQ	.	.	.	.	.03773729
A	161	REL205	UL	.	.	NONE	.	.
	162	UPST0205	BS	-1511136.0	1511136.0	NONE	.	.
	163	STOR305	EQ	.	.	.	.	.05843347
	164	REL305	BS	-1814478.5	1814478.5	NONE	.	.
	165	UPST0305	BS	-29675808.	29675808.	NONE	.	.
	166	CONSTR05	BS	-19.093000	19.093000	NONE	.	.
	167	IREL0W05	EQ	.	.	.	.	97815.060
	168	UPCN0105	UL	.	.	NONE	.	10909.000
	169	UPCN0205	UL	.	.	NONE	.	48752.000
	170	UPCN0305	UL	.	.	NONE	.	71535.500
	171	UPCN0405	BS	-1.2110000	1.2110000	NONE	.	.
	172	UPCN0505	BS	-.46400000	.46400000	NONE	.	.
	173	UPCN0605	UL	.	.	NONE	.	42664.773
	174	UPCN0705	BS	-.41595455	.41595455	NONE	.	.
	175	UPCN0805	BS	-2.9859545	2.9859545	NONE	.	.
	176	UPCN0905	BS	-4.2649545	4.2649545	NONE	.	.
	177	UPCN1005	BS	-5.6759545	5.6759545	NONE	.	.
A	178	UPCN1105	UL	.	.	NONE	.	.
	179	UPCN1205	BS	-2.3420000	2.3420000	NONE	.	.
	180	DEM1006	EQ	1.2940000	.	1.2940000	1.2940000	-92387.520
	181	DEM1206	EQ	.96100000	.	.96100000	.96100000	-110531.52
	182	DEM1306	EQ	20.927000	.	20.927000	20.927000	-110531.52
	183	DEM1406	EQ	1.9930000	.	1.9930000	1.9930000	-92387.520
	184	DEM1506	EQ	3.2640000	.	3.2640000	3.2640000	-137657.70
	185	DEM1606	EQ	2.1550000	.	2.1550000	2.1550000	-206557.70
	186	DEM0606	EQ	1.3370000	.	1.3370000	1.3370000	-334889.34
	187	DEM0706	EQ	4.6340000	.	4.6340000	4.6340000	-428233.14
	188	DEM0806	EQ	5.5100000	.	5.5100000	5.5100000	-428686.74
	189	DEM1706	EQ	2.7410000	.	2.7410000	2.7410000	-428686.74
	190	DEM0906	EQ	2.1600000	.	2.1600000	2.1600000	-430527.06
	191	DEM1806	EQ	38.640000	.	38.640000	38.640000	-118240.51
	192	STOR106	EQ	.	.	.	.	.00593000
	193	REL106	UL	.	.	NONE	.	.12200000
	194	UPST0106	UL	.	.	NONE	.	.09898264
	195	STOR206	EQ	.	.	.	.	.00593000
	196	REL206	UL	.	.	NONE	.	.10100000
	197	UPST0206	UL	.	.	NONE	.	.06919271
	198	STOR306	EQ	.	.	.	.	.05843347
	199	REL306	BS	-16661455.	16661455.	NONE	.	.
	200	UPST0306	BS	-81598752.	81598752.	NONE	.	.
	201	CONSTR06	BS	-18.954000	18.954000	NONE	.	.
	202	IREL0W06	EQ	.	.	.	.	92387.520
	203	UPCN0106	UL	.	.	NONE	.	87264.000
	204	UPCN0206	UL	.	.	NONE	.	18144.000
	205	UPCN0306	BS	-.39700000	.39700000	NONE	.	.
	206	UPCN0406	UL	.	.	NONE	.	68900.000
	207	UPCN0506	UL	.	.	NONE	.	128088.00
	208	UPCN0606	UL	.	.	NONE	.	92531.636

[illegible] WATER SUPPLY — AT 18:38:22 ON 02/19/80

SOLUTION

ROWS SECTION

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
209	DPCF0706	BS	-2.0229545	2.0229545	NONE	.	.
210	DPCF0806	BS	-6.5029545	6.5029545	NONE	.	.
211	DPCF0906	BS	-8.7319545	8.7319545	NONE	.	.
212	DPCF1006	BS	-10.891955	10.891955	NONE	.	.
213	DPCF1106	UL	.	.	NONE	.	67754.000
214	DPCF1206	UL	.	.	NONE	.	44960.000
215	DEF1007	EQ	.58200000	.	.58200000	.58200000	-10247.040
216	DEF1207	EQ	.42000000	.	.42000000	.42000000	-10247.040
217	DEF1307	EQ	9.4190000	.	9.4190000	9.4190000	-10247.040
218	DEF1407	EQ	.89800000	.	.89800000	.89800000	-10247.040
219	DEF1507	EQ	1.4690000	.	1.4690000	1.4690000	-10867.392
220	DEF1607	EQ	.97000000	.	.97000000	.97000000	-10867.392
221	DEF0607	EQ	.60200000	.	.60200000	.60200000	-11354.688
222	DEF0707	EQ	2.0860000	.	2.0860000	2.0860000	-12979.008
223	DEF0807	EQ	2.4800000	.	2.4800000	2.4800000	-97292.390
224	DEF1707	EQ	1.2340000	.	1.2340000	1.2340000	-97292.390
225	DEF0907	EQ	.97200000	.	.97200000	.97200000	-100973.03
226	DEF1807	EQ	17.390000	.	17.390000	17.390000	-100973.03
227	SIG0107	EQ	.	.	.	.	.00593000
228	PEL107	UL	.	.	NONE	.	.
229	DEST0107	BS	-5248584.0	5248584.0	NONE	.	.
230	SIG0207	EQ	.	.	.	.	.00593000
231	PEL207	UL	.	.	NONE	.	.
232	DEST0207	BS	-2459808.0	2459808.0	NONE	.	.
233	SIG0307	EQ	.	.	.	.	.02163347
234	PEL307	UL	.	.	NONE	.	.03680000
235	DEST0307	BS	-114984E03	114984E03	NONE	.	.
236	COEST0807	BS	-17.560955	17.560955	NONE	.	.
237	DEFL0807	EQ	.	.	.	.	10247.040
238	DPCF0107	BS	-9.1742955	9.1742955	NONE	.	.
239	DPCF0207	BS	-5.9742500	5.9742500	NONE	.	.
240	DPCF0307	BS	-2.8850455	2.8850455	NONE	.	.
241	DPCF0407	BS	-2.4450455	2.4450455	NONE	.	.
242	DPCF0507	BS	-1.2600455	1.2600455	NONE	.	.
243	DPCF0607	BS	-.52504545	.52504545	NONE	.	.
244	DPCF0707	UL	.	.	NONE	.	83406.182
245	DPCF0807	BS	-1.4500000	1.4500000	NONE	.	.
246	DPCF0907	BS	-2.1720000	2.1720000	NONE	.	.
247	DPCF1007	BS	-3.1440000	3.1440000	NONE	.	.
248	DPCF1107	BS	-28.997955	28.997955	NONE	.	.
249	DPCF1207	BS	-4.2400455	4.2400455	NONE	.	.
250	DEF1008	EQ	.24200000	.	.24200000	.24200000	-15370.560
251	DEF1208	EQ	.17400000	.	.17400000	.17400000	-15370.560
252	DEF1308	EQ	3.9140000	.	3.9140000	3.9140000	-15370.560
253	DEF1408	EQ	.37300000	.	.37300000	.37300000	-15370.560
254	DEF1508	EQ	.61000000	.	.61000000	.61000000	-16301.088
255	DEF1608	EQ	.40300000	.	.40300000	.40300000	-16301.088
256	DEF0608	EQ	.25000000	.	.25000000	.25000000	-17032.032
257	DEF0708	EQ	.86700000	.	.86700000	.86700000	-19468.512
258	DEF0808	EQ	1.0300000	.	1.0300000	1.0300000	-38524.676
259	DEF1708	EQ	.51200000	.	.51200000	.51200000	-47177.676
260	DEF0908	EQ	.40400000	.	.40400000	.40400000	-56073.945

[illegible] WATER SUPPLY

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SOLUTION

ROWS SECTION

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
261	DEF1808	EQ	6.5690000	.	6.5690000	6.5690000	-56073.945
262	STOR108	EQ	.	.	.	.	.00593000
263	REL108	BS	.	.	NONE	.	.
264	DESTO108	BS	-5248584.0	5248584.0	NONE	.	.
265	STOR208	EQ	.	.	.	.	.00593000
A 266	REL208	UL	.	.	NONE	.	.
267	DESTO208	BS	-2459808.0	2459808.0	NONE	.	.
268	STOR308	EQ	.	.	.	.	.02163347
269	REL308	BS	.	.	NONE	.	.
270	DESTO308	BS	-131645E03	131645E03	NONE	.	.
271	CONSTR08	BS	-14.563955	14.563955	NONE	.	.
272	INFLOW08	EQ	.	.	.	.	15370.560
273	DECR0108	BS	-18.787295	18.787295	NONE	.	.
274	DECR0208	BS	-11.725250	11.725250	NONE	.	.
275	DECR0308	BS	-6.4070455	6.4070455	NONE	.	.
276	DECR0408	BS	-4.5830455	4.5830455	NONE	.	.
277	DECR0508	BS	-2.8310455	2.8310455	NONE	.	.
278	DECR0608	BS	-1.7440455	1.7440455	NONE	.	.
279	DECR0708	UL	.	.	NONE	.	17695.364
280	DECR0808	UL	.	.	NONE	.	8653.0000
281	DECR0908	UL	.	.	NONE	.	3375.3091
282	DECR1008	BS	-.40400000	.40400000	NONE	.	.
283	DECR1108	BS	-38.640000	38.640000	NONE	.	.
284	DECR1208	BS	-7.2370455	7.2370455	NONE	.	.

SILISTRA WATER SUPPLY AT 18138122 ON 02/19/80

SOLUTION

COLUMNS SECTION

	NUMBER	.COLUMN	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
	285	Z01	BS	38.054250	118060.00	.	NONE	.
	286	Z02	BS	15.813250	66896.000	.	NONE	.
	287	Z03	BS	21.344000	79130.000	.	NONE	.
	288	Z04	BS	18.537000	68900.000	.	NONE	.
	289	Z05	BS	16.382000	128088.00	.	NONE	.
	290	Z06	BS	15.045000	149418.00	.	NONE	.
	291	Z07	BS	12.433955	150641.00	.	NONE	.
	292	Z08	BS	11.403955	8653.0000	.	NONE	.
	293	Z09	BS	10.891955	105155.00	.	NONE	.
	294	Z10	BS	10.891955	17115.000	.	NONE	.
	295	Z11	BS	38.640000	67754.000	.	NONE	.
	296	V1	BS	5248584.0	.12200000	.	NONE	.
	297	V2	BS	2459808.0	.10100000	.	NONE	.
	298	V3	BS	131645E03	.03680000	.	NONE	.
	299	Z12	BS	21.801000	44960.000	.	NONE	.
A	300	S109	LL	.	.	.	NONE	.
	301	S101	BS	.	.	.	NONE	.
	302	S209	BS	.	.	.	NONE	.
	303	S201	BS	.	.	.	NONE	.
	304	S309	BS	10157930.	.	.	NONE	.
	305	S301	BS	10157930.	.	.	NONE	.
	306	Y1301	BS	.	.	.	NONE	.
	307	Y1401	BS	.	.	.	NONE	.
A	308	Y1501	LL	.	.	.	NONE	.
	309	Y1601	BS	10.891955	.	.	NONE	.
	310	Y1201	BS	10.891955	3722.1120	.	NONE	.
	311	Y2001	BS	10.891955	.	.	NONE	.
A	312	Y1101	LL	.	.	.	NONE	.
	313	Y0101	BS	10.891955	51482.240	.	NONE	.
	314	Y0201	BS	.	.	.	NONE	.
	315	Y0301	BS	10.891955	.	.	NONE	.
	316	Y0401	BS	10.891955	.	.	NONE	.
	317	Y0501	BS	10.891955	2923.7760	.	NONE	.
	318	Y0601	BS	10.891955	9745.9200	.	NONE	.
	319	Y0701	BS	10.891955	5443.2000	.	NONE	.
	320	Y0801	BS	10.891955	.	.	NONE	.
	321	Y0901	BS	10.891955	22083.840	.	NONE	.
	322	Y1001	BS	10.891955	.	.	NONE	.
A	323	S102	LL	.	.	.	NONE	.
	324	S202	BS	.	.	.	NONE	.
A	325	Y1701	LL	.	.	.	NONE	.
	326	Y1801	BS	.	.	.	NONE	.
	327	S302	BS	123086E03	.	.	NONE	.
	328	Y1302	BS	4.0150000	.	.	NONE	.
A	329	Y1402	LL	.	.	.	NONE	.
	330	Y1502	BS	.	.	.	NONE	.
	331	Y1602	BS	14.616955	.	.	NONE	.
	332	Y1202	BS	14.616955	930.52800	.	NONE	.
	333	Y2002	BS	41.664955	.	.	NONE	.
	334	Y1102	BS	38.640000	.	.	NONE	.
	335	Y0102	BS	19.441955	15370.560	.	NONE	.
	336	Y0202	BS	4.1940000	.	.	NONE	.

SILISTRA WATER SUPPLY

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SOLUTION

COLUMNS SECTION

NUMBER	.COLUMN	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
337	Y0302	BS	14.999955	.	.	NONE	.
338	Y0402	BS	13.990955	.	.	NONE	.
339	Y0502	BS	13.577955	730.94400	.	NONE	.
340	Y0602	BS	13.320955	2436.4800	.	NONE	.
341	Y0702	BS	12.433955	1360.8000	.	NONE	.
342	Y0802	BS	11.376955	.	.	NONE	.
343	Y0902	BS	10.851955	5520.9600	.	NONE	.
344	Y1002	BS	10.437955	.	.	NONE	.
345	S103	LL	.	.	.	NONE	.
346	S203	LL	.	.	.	NONE	.
347	Y1702	BS	.	.	.	NONE	.
348	Y1802	BS	.	.	.	NONE	.
349	S303	BS	130926E03	.	.	NONE	.
350	Y1303	BS	8.1643750	.	.	NONE	.
351	Y1403	BS	3.0373750	.	.	NONE	.
352	Y1503	BS	.	.	.	NONE	.
353	Y1603	BS	16.648455	.	.	NONE	.
354	Y1203	BS	15.224955	620.35200	.	NONE	.
355	Y2003	BS	39.055955	.	.	NONE	.
356	Y1103	BS	38.640000	.	.	NONE	.
357	Y0103	BS	25.845830	10247.040	.	NONE	.
358	Y0203	BS	8.3923750	.	.	NONE	.
359	Y0303	BS	17.136455	.	.	NONE	.
360	Y0403	BS	14.424955	.	.	NONE	.
361	Y0503	BS	13.896955	487.29600	.	NONE	.
362	Y0603	BS	13.568955	1624.3200	.	NONE	.
363	Y0703	BS	12.433955	907.20000	.	NONE	.
364	Y0803	BS	11.083955	.	.	NONE	.
365	Y0903	BS	10.412955	3680.6400	.	NONE	.
366	Y1003	BS	9.8829545	.	.	NONE	.
367	S104	BS	5248584.0	.	.	NONE	.
368	S204	BS	2459808.0	.	.	NONE	.
369	Y1703	BS	1.4235000	.	.	NONE	.
370	Y1803	BS	.	.	.	NONE	.
371	S304	BS	131645E03	.	.	NONE	.
372	Y1304	BS	14.660250	.	.	NONE	.
373	Y1404	BS	1.4870000	.	.	NONE	.
374	Y1504	BS	6.0747500	.	.	NONE	.
375	Y1604	BS	19.511000	.	.	NONE	.
376	Y1204	BS	21.260000	310.17600	.	NONE	.
377	Y2004	BS	4.2930000	.	.	NONE	.
378	Y1104	BS	38.640000	.	.	NONE	.
379	Y0104	BS	38.054250	5123.5200	.	NONE	.
380	Y0204	BS	15.519250	.	.	NONE	.
381	Y0304	BS	21.344000	.	.	NONE	.
382	Y0404	BS	18.257000	.	.	NONE	.
383	Y0504	BS	16.274000	243.64800	.	NONE	.
384	Y0604	BS	15.045000	812.16000	.	NONE	.
385	Y0704	BS	10.782000	453.60000	.	NONE	.
386	Y0804	BS	5.7130000	.	.	NONE	.
387	Y0904	BS	3.1910000	1840.3200	.	NONE	.
388	Y1004	BS	1.2040000	.	.	NONE	.

SIETSTRA WATER SUPPLY AT 18:38:22 ON 02/19/80

SOLUTION

COLUMNS SECTION

NUMBER	.COLUMN	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
389	S105	BS	1284768.0	.	.	NONE	.
390	S205	BS	948672.00	.	.	NONE	.
391	Y1704	BS	1.0980000	.	.	NONE	.
392	Y1804	BS	2.8470000	.	.	NONE	.
393	S305	BS	101969E03	.	.	NONE	.
394	Y1305	BS	15.203250	.	.	NONE	.
395	Y1405	BS	2.0249167	.	.	NONE	.
396	Y1505	BS	.49566667	.	.	NONE	.
397	Y1605	BS	20.042000	.	.	NONE	.
398	Y1705	BS	19.459000	930.52800	.	NONE	.
399	Y2005	BS	18.608000	.	.	NONE	.
400	Y1105	BS	38.640000	.	.	NONE	.
401	Y0105	BS	38.054250	15370.560	.	NONE	.
402	Y0205	BS	15.813250	.	.	NONE	.
403	Y0305	BS	21.344000	.	.	NONE	.
404	Y0405	BS	17.326000	.	.	NONE	.
405	Y0505	BS	15.918000	730.94400	.	NONE	.
406	Y0605	BS	15.045000	2436.4800	.	NONE	.
407	Y0705	BS	12.018000	1360.8000	.	NONE	.
408	Y0805	BS	8.4180000	.	.	NONE	.
409	Y0905	BS	6.6270000	5520.9600	.	NONE	.
410	Y1005	BS	5.2160000	.	.	NONE	.
411	S106	BS	5248564.0	.	.	NONE	.
412	S206	BS	2459808.0	.	.	NONE	.
413	Y1705	BS	.94900000	.	.	NONE	.
414	Y1805	BS	.36600000	.	.	NONE	.
415	S306	BS	50046415.	.	.	NONE	.
416	Y1306	BS	14.852250	.	.	NONE	.
417	Y1406	LL	.	.	.	NONE	105408.00
418	Y1506	BS	6.0747500	.	.	NONE	.
419	Y1606	BS	18.954000	.	.	NONE	.
420	Y1706	BS	21.801000	310.17600	.	NONE	.
421	Y2006	LL	.	.	.	NONE	67754.000
422	Y1106	BS	38.640000	.	.	NONE	.
423	Y0106	BS	38.054250	5123.5200	.	NONE	.
424	Y0206	BS	15.813250	.	.	NONE	.
425	Y0306	BS	20.947000	.	.	NONE	.
426	Y0406	BS	18.537000	.	.	NONE	.
427	Y0506	BS	16.382000	243.64800	.	NONE	.
428	Y0606	BS	15.045000	812.16000	.	NONE	.
429	Y0706	BS	10.411000	453.60000	.	NONE	.
430	Y0806	BS	4.9010000	.	.	NONE	.
431	Y0906	BS	2.1600000	1840.3200	.	NONE	.
432	Y1006	LL	.	.	.	NONE	312286.55
433	S107	BS	.	.	.	NONE	.
434	S207	BS	.	.	.	NONE	.
435	Y1706	LL	.	.	.	NONE	87264.000
436	Y1806	BS	2.8470000	.	.	NONE	.
437	S307	BS	16661455.	.	.	NONE	.
438	Y1307	BS	9.4190000	.	.	NONE	.
439	Y1407	LL	.	.	.	NONE	.
440	Y1507	BS	.	.	.	NONE	.

SILISTRA WATER SUPPLY AT 18:38:22 ON 02/19/80

SOLUTION

COLUMNS SECTION

	NUMBER	.COLUMN	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
	441	Y1607	BS	17.560955	.	.	NONE	.
	442	Y1207	BS	17.560955	620.35200	.	NONE	.
	443	Y2007	LL	.	.	.	NONE	63590.400
	444	Y1107	BS	9.6420455	.	.	NONE	.
	445	Y0107	BS	28.879955	10247.040	.	NONE	.
	446	Y0207	BS	9.8390000	.	.	NONE	.
	447	Y0307	BS	18.458955	.	.	NONE	.
	448	Y0407	BS	16.091955	.	.	NONE	.
	449	Y0507	BS	15.121955	487.29600	.	NONE	.
	450	Y0607	BS	14.519955	1624.3200	.	NONE	.
	451	Y0707	BS	12.433955	907.20000	.	NONE	.
	452	Y0807	BS	9.9539545	.	.	NONE	.
	453	Y0907	BS	8.7199545	3680.6400	.	NONE	.
	454	Y1007	BS	7.7479545	.	.	NONE	.
	455	S108	BS	.	.	.	NONE	.
	456	S208	BS	.	.	.	NONE	.
A	457	Y1707	LL	.	.	.	NONE	.
	458	Y1807	BS	.	.	.	NONE	.
	459	S308	BS	.	.	.	NONE	.
	460	Y1308	BS	3.9140000	.	.	NONE	.
	461	Y1408	BS	.	.	.	NONE	.
A	462	Y1508	LL	.	.	.	NONE	.
	463	Y1608	BS	14.563955	.	.	NONE	.
	464	Y1208	BS	14.563955	930.52800	.	NONE	.
	465	Y2008	BS	3.9189545	.	.	NONE	.
A	466	Y1108	LL	.	.	.	NONE	.
	467	Y0108	BS	19.266955	15370.560	.	NONE	.
	468	Y0208	BS	4.0880000	.	.	NONE	.
	469	Y0308	BS	14.936955	.	.	NONE	.
	470	Y0408	BS	13.953955	.	.	NONE	.
	471	Y0508	BS	13.550955	730.94400	.	NONE	.
	472	Y0608	BS	13.300955	2436.4800	.	NONE	.
	473	Y0708	BS	12.433955	1360.8000	.	NONE	.
	474	Y0808	BS	11.403955	.	.	NONE	.
	475	Y0908	BS	10.891955	5520.9600	.	NONE	.
	476	Y1008	BS	10.487955	.	.	NONE	.
A	477	Y1708	LL	.	.	.	NONE	.
	478	Y1808	BS	.	.	.	NONE	.