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ABSTRACT

This paper summarizes the contents of a lecture presented by Dr. Borodkin of the Institute of Economics and Industrial Engineering of the Siberian Branch of the Academy of Sciences in the Soviet Union. In it he argues for a demoeconomic perspective of rural development and outlines a model designed in his Institute that reflects such a perspective.

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CONTENTS

1. INTRODUCTION

Usually problems of population and economic growth are solved separately. In some cases there are constraining linkages between demography and economics. For instance, the population of a country is considered to be the source of its labor force and therefore acts as a limitation on economic growth. In other cases economic growth is considered a constraint on various aspects of the population's living conditions or its size.

It is obvious that economics and demography are two parts of the whole. Therefore, many scholars are interested in formulating problems under which population and economic growth are interconnected. This paper focuses on the construction of such a model. Although this paper is not the first, work in this area has only been done in the last 10 or 15 years. The papers of Kelley and Williamson (1979), Andersson and Philipov (1980), and Gordon and Ledent (1980) are the most recent examples. These papers contain an extensive list of works in which population and economic growth are connected. There are also many publications on this topic in Russian.

The interaction between population and economics can be described for various levels or parts of a society or a country.

For instance, we can consider a national economy and population as a whole and, in the case of a closed country, formulate the problem of balanced growth. We can consider some part of the country such as an administrative or economic subregion and evaluate the causes of out- or inmigration of people and/or commodities. We can also construct a model that describes some balanced interaction between people and the economy and use it as an analytical tool. In some cases such a model is not only an analytical tool, but also is capable of simulating parts of or the entire country.

In the following, I shall use the term "region" to mean some part of the economy and the population organically connected and interacting. We will deal with the rural region consisting of the people and the agricultural, construction, and services sectors of the economy. Agricultural and construction sectors will be named in some cases by industries. The agricultural sector provides food for the total regional population as well as the exportation of food to other regions. The construction sector provides the rural region (i.e., the three sectors of the economy) with buildings. Services deal only with people. Each sector needs manpower of definite qualifications. The development of each sector depends on capital investment and manpower.

Manpower demand depends on capital investments and past trends, but manpower supply is connected with population--its size and structure, determined by past natural growth and migration.

Some conditions that will be discussed below allow us to state that the balance between industries and services--or between capital investments into industries and services--strongly influences migration for a region, particularly for a rural region. Hence, a particular distribution of investments between sectors of the economy can affect migration and constrain the development of these sectors. Thus we can formulate the following problems: to distribute capital investments in such a way as to guarantee a balance between industries, services, and population; or to evaluate the influence of an imbalance on migration and therefore on the development of industries and services.

In order to analyze these problems, we have developed a model that deals with the dynamics of the links between the three sectors of the economy, capital investments, and population. The model consists of two parts. The first part describes the economy using the three sectors. The second part connects this situation with population size through the correlation between capital investments in industries and services. The first part will "produce" jobs in the three sectors; the second part links the population and jobs and "washes out" all the surplus of population that is not provided with jobs because of an imbalance between the capital investments to industries and services.

The main problem is as follows: How can one determine the reaction of the migrant to this imbalance? Also, how does population size affect the migrant's reaction?

We will briefly describe the analytical part of the model, which we have used to forecast West Siberian rural development for the next 20 years.

2. THE FIRST PART OF THE MODEL

The first part of the model "produces" jobs on the basis of the following assumptions:

1. Investments are fully related to physical stocks.
2. The immovable part* of the stocks is produced by the rural construction industry alone.
3. Capital investments in each sphere are divided into two flows: one for the acquisition of equipment and machinery, and the other for the financing of capital construction. The former creates a movable part of stocks, and each invested rouble produces an incremental rouble of stock. The other flow creates an immovable part, but

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*According to Soviet terminology, total stocks are subdivided into two parts. The term for the first part is "movable stocks". These consist of machines, tractors, and so on. The second part is "immovable stocks". These include all capital constructions, such as buildings. This subdivision is helpful in many cases because immovable stocks are created by a special sector of the economy: capital construction.

each invested rouble produces less than an incremental rouble of stock because of construction lag, changed grades of building materials, and a rise in the cost of machinery.

4. There exists a positive growth of immovable stocks in all three spheres at each given time point.

5. Depleting stocks are immediately recovered to their former amount, and the means required for recovery are not included in capital investments.

Our model uses the following notations:

K_1 - investment flow into proper agricultural and capital construction

K_2 - investment flow into services

$K_1 + K_2 = K$ - total flow of capital investments

R_1, R_2, R_3 - number of jobs in the proper agricultural sphere, services, and capital construction, respectively

$\beta_1, \beta_2, \beta_3$ - expenditures per one incremental rouble of immovable stock in each of the above-mentioned spheres, respectively

F_1, F_2, F_3 - stocks in each sphere, respectively

W_1, W_2, W_3 - total stock per one job in each sphere

ϕ_1, ϕ_2, ϕ_3 - share of immovable stock in fixed assets in each of the three above-mentioned spheres, respectively

It is assumed that all the above-listed variables are continuous and at least twice-differentiable over time. Time derivatives will be denoted as points above appropriate letters when these cannot lead to misunderstanding.

The first three assumptions allow us to write the following equation:

$$\sum_{i=1}^{3} \frac{d[F_i(1 + \phi_i(\beta_i - 1))]}{dt} = K$$

or, assuming $\alpha_i = 1 + \phi_i(\beta_i - 1)$,

$$\frac{d(\alpha_i F_i)}{dt} + \frac{d(\alpha_2 F_2)}{dt} + \frac{d(\alpha_3 F_3)}{dt} = K \qquad (1)$$

In the service sector (unlike agriculture and construction), a considerable part of the stocks are an object of activity rather than an instrument. This creates only a limited number of jobs for their maintenance. The rest of the jobs are related to population size and structure. However, creating any job vacancy in services calls for certain investments and therefore is related quantitatively through network indicators of the size of the stocks.

The third assumption concerning capital investments can be used in many ways. We used it in the following two ways.

First, the increment of immovable stocks is a product of one construction job multiplied by the number of jobs used for the creation of the immovable part of the stocks in this sphere. It seems natural to assume that:

1. Productivity of one construction job is the same for all three spheres.
2. The share of construction jobs creating the immovable part of the stocks of one sphere equals the share of incremental immovables in this sphere in the total growth of immovable stocks.

If we denote the productivity of one construction job by a, then

$$\frac{d(\phi_1 F_1)}{dt} + \frac{d(\phi_2 F_2)}{dt} + \frac{d(\phi_3 F_3)}{dt} = aR_3 \qquad (2)$$

Second, the increment of immovables in each sphere can be described by a Cobb-Douglas function where the construction stocks and number of jobs are considered. Then, the fact that one part of the stocks and the construction jobs are common to all spheres and the other part is meant for a particular sphere will be accounted for in the standardizing factor. Then

$$\frac{d(\phi_i F_i)}{dt} = b_i F_3^{\gamma_1} R_3^{\gamma_2}, \qquad \gamma_1 + \gamma_2 = 1;\ \gamma_1, \gamma_2 \geq 0$$

But $R_3 = \frac{F_3}{W_3}$, whence

$$\frac{d(\phi_i F_i)}{dt} = b_i F_3 W_3^{-\gamma_2}$$

Consequently,

$$\sum_{i=1}^{3} \frac{W_3^{\gamma_2}}{b_i} \cdot \frac{d(\phi_i F_i)}{dt} = F_3 \qquad (3)$$

Then, obviously,

$$K_1 = \frac{d(\alpha_1 F_1)}{dt} + \frac{d(\alpha_3 F_3)}{dt} \qquad (4)$$

and

$$K_2 = \frac{d(\alpha_2 F_2)}{dt} \qquad (5)$$

and

$$K = K_1 + K_2 \qquad (6)$$

Thus, equations (1) and (3)-(6) form the first part of the model.

3. THE SECOND PART OF THE MODEL

This part, as previously mentioned, links jobs, the imbalance of capital investment into industries and services, and migration. We use the following assumption. Urban-rural migration is negligible compared to rural-urban migration, and net migration is essentially a flow of city-bound migrants. This flow quantitatively consists of two parts. The first is equal to the natural population growth. The second is formed by the imbalance of changes in working and living conditions in the services. In the simplest case this second part is a linear function of capital investments into rural industries and services.

Let

P be population size

I be natural population growth

h be the share of population that is employed

μ_1, μ_2, μ_3 be the manpower demand of each job in the three sectors, respectively

The relationship between population employment and available jobs is written simply as

$$\mu_1 R_1 + \mu_2 R_2 + \mu_3 R_3 = hP \tag{7}$$

According to the assumption for the simplest case,

$$\dot{P} = \eta_1 K_1 + \eta_2 K_2 + I \leq 0 \tag{8}$$

Quantities η_1 and η_2 are unknown, but we formulate some considerations allowing us to determine the signs of η_1 and η_2. Demographers often consider that migration takes place because of a desire for an improvement of working conditions adequate to living conditions in the service sector. We think this viewpoint is justified. In fact, as industries develop higher technical sophistication, a higher-skilled labor force is needed. Skill upgrading is connected with higher educational attainments and with a higher culture in general. A higher education level leads to a higher pay and a higher culture and therefore requires growth in all spheres of living activity, including the service sector of the economy.

The satisfaction of additional needs is partly achieved through general growth of services on regional and national scales (better communications, mass media, urban services, and the like), and partly through the growth of the particular sector's services. Rural services are rigidly attached to the place of residence. Their growth, therefore, plays more of a role in the satisfaction of incremental needs, the less is the accessibility of nonspecialized, nonsectoral services.

In Siberia, the Far East, and in the north of the European part of the USSR this situation exists. For similar regions, therefore, $\dot{P} \leq 0$. And one can suppose that the more urbanized a region is, the more "wrong" is assumption (4) and the more efficient some other factors become, for example, urban services, availability of vacancies in cities, etc. In the roughest approximation we shall consider $\eta_1 = -\eta_2$, and in the final form

$$\dot{P} = -\eta(K_1 - K_2) - I \qquad (9)$$

4. THE MODEL AS A WHOLE

The differential equations (1), (3)-(6), (7) and (9) form a system. In this system K, μ_i, ϕ_i, a, W_3, η, and I, as well as the initial values for t = 0, K^o, F_i^o, and P^o are assumed to be known. The variables P, K_1, K_2, F_i, R_i, and h are sought. Certainly, other possible combinations are predetermined and sought for variables and functions. For example, if the value h for each time point is fixed, it is possible, as will be seen later, to find the value of K. An analytical solution of the system, however, is very difficult because of the coefficients being functions of time, although a numerical solution is in fact possible.

With this in mind we shall pass to the solution of the system assuming time-independence of the coefficients ϕ_i, a, W_3, h, and μ_i. This assumption is not very artificial since the solution results will be extended only to some short time intervals. In equation (9) we can omit item I if natural growth is known, and we deduct it beforehand since in this case, change in population size is only a result of migration. In equation (2) we assume

$$S_i = \frac{\phi_i}{a} W_i$$

and in equation (1) $\gamma_i = \alpha_i W_i$. In summary, we have the following system of equations:

$$\gamma_1 \dot{R}_1 + \gamma_2 \dot{R}_2 + \gamma_3 \dot{R}_3 = K \qquad (10)$$

$$S_1 \dot{R}_1 + S_2 \dot{R}_2 + S_3 \dot{R}_3 = R_3 \qquad (11)$$

$$\mu_1 R_1 + \mu_2 R_2 + \mu_3 R_3 = hP \qquad (12)$$

$$\dot{P} = -\eta(\gamma_1 \dot{R}_1 + \gamma_3 \dot{R}_3 - \gamma_2 \dot{R}_2) \qquad (13)$$

The solution is simple. First, from equations (10) and (11) we find values $\dot{R}_1$ and $\dot{R}_2$ as functions of R_3, $\dot{R}_3$, and K. From equations (12) and (13) we exclude P, having first integrated (13) over time. After finding the integral $\dot{R}_1$ and $\dot{R}_2$, we substitute it into a system obtained from (12) and (13). Differentiating again the substitution result, we obtain the following differential equation:

$$\dot{R}_3 + mR_3 = nK \qquad (14)$$

where

$$m = \frac{\gamma_1\mu_2 - \gamma_2\mu_1}{(\gamma_2 S_3 - \gamma_3 S_2)\mu_1 + (\gamma_3 S_1 - \gamma_1 S_3)\mu_2 + (\gamma_1 S_2 - \gamma_2 S_1)\mu_3}$$

$$n = - \frac{S_1\mu_2 + S_2\mu_1}{(\gamma_2 S_3 - \gamma_3 S_2)\mu_1 + (\gamma_3 S_1 - \gamma_1 S_3)\mu_2 + (\gamma_1 S_2 - \gamma_2 S_1)\mu_3}$$

The solution of equation (14) is obvious:

$$R_3 = e^{-mt}\left(R_3^o + n \int_o^t Ke^{mu}\, du\right) \tag{15}$$

Assuming that capital investments are changing during the time interval $\{o,t\}$ at a constant rate C, we have

$$R_3 = R_3^o e^{-mt} + \frac{nKo}{c + m}(e^{(m+c)t} - 1)e^{-mt} \tag{15'}$$

The values of R_1 and R_2 can be obtained by using the following formulas:

$$R_1 = \frac{S_2\tilde{K} - \gamma_2\tilde{R}_3 + (\gamma_2 S_3 - \gamma_3 S_2)(R_3 - R_3^o)}{\gamma_1 S_2 - \gamma_2 S_1}$$

$$R_2 = \frac{\gamma_1\tilde{R}_3 - S_1\tilde{K} + (\gamma_3 S_1 - \gamma_1 S_3)(R_3 - R_3^o)}{\gamma_1 S_2 - \gamma_2 S_1}$$

where

$$\tilde{K} = \int_o^t K(u)\,du, \qquad \tilde{R}_3 = \int_o^t R_3(u)\,du$$

Then, from relationship (13) population size is found:

$$P = \frac{\tilde{K}(s_2 - s_1) + \tilde{R}_3(\gamma_1 - \gamma_2) + (R_3 - R_3^o)(\gamma_2 s_3 + \gamma_3 s_1 - \gamma_3 s_2 - \gamma_1 s_3) + R_3(\gamma_1 s_2 - \gamma_2 s_1)}{\gamma_1 s_2 - \gamma_2 s_1}$$

The behavior of population size as a response to investments is fully determined by the relationship between the coefficients of the equations. Indeed,

$$\frac{dP}{dK} = \frac{K(c + m)(s_2 - s_1) + \eta(\phi_1 - \phi_2) + nc(\phi_2 s_3 + \phi_3 s_1 - \phi_3 s_2 - \phi_1 s_3 - \phi_2 s_1)}{c(\phi_1 s_2 - \phi_2 s_1)(c + m)}$$

How can we find out how the distribution of investments between the industries (K_1) and services (K_2) depends upon investment levels? Since

$$K_1 - K_2 = -\frac{\dot{P}}{\eta}$$

then

$$\frac{d(K_1 - K_2)}{dk} = -\frac{d\dot{P}}{\eta dk} = \frac{K(s_1 - s_2)}{\eta(\gamma_1 s_2 - \gamma_2 s_1)}$$

Returning to the initial notation,

$$\frac{d(K_1 - K_2)}{dk} = \frac{K(\phi_1 W_1 - \phi_2 W_2)}{\eta W_1 W_2 [\phi_2 - \phi_1 + \phi_1 \phi_2 (\beta_1 - \beta_2)]}$$

Obviously, the percentage of machines and equipment, as well as expenditures per incremental unit of immovable stocks in the industries sector, is higher than in the services sector. It follows that the sign of this derivative is determined by the sign of the relationship $(\phi_1 W_1 - \phi_2 W_2)$. At present, the situation in the rural services is such that $\phi_2 \simeq 1$. Consequently, if $W_2 \geq \phi_1 W_1$, then

$$\frac{dK_2}{dk} > \frac{dK_1}{dk}$$

since with investment growth, a predominant share of the investments will be put into services.

If the requirement is made that there will always be a relationship between $\eta \geq 0$ and $K_1 - K_2 \geq 0$, it makes sense to pose a question about the maximum size of investments to be effectively used (i.e., turned into stocks provided with manpower) in the region. This quantity will be referred to as maximum investments and denoted by K_{max}. For fixed coefficients of the above system, it is obvious that investments achieve their maximum with $K_1 = K_2$. We will seek maximum investments with the help of the parameter Z, whose function will be investments and assets. Finding investments in the system as a function of this parameter and system coefficients, we will integrate them over the interval $[o, \infty]$ and obtain optimal investments. Consider the following equations:

$$\gamma_1 \frac{dR_1(Z)}{dZ} + \gamma_2 \frac{dR_2(Z)}{dZ} + \gamma_3 \frac{dR_3(Z)}{dZ} = K(Z) \qquad (16)$$

$$\gamma_1 \frac{dR_1(Z)}{dZ} + \gamma_3 \frac{dR_3(Z)}{dZ} = \gamma_2 \frac{dR_2(Z)}{dZ} \qquad (17)$$

$$S_1 \frac{dR_1(Z)}{dZ} + S_2 \frac{dR_2(Z)}{dZ} + S_3 \frac{dR_3(Z)}{dZ} = R_3(Z) \qquad (18)$$

$$\mu_1 \frac{dR_1(Z)}{dZ} + \mu_2 \frac{dR_2(Z)}{dZ} + \mu_3 \frac{dR_3(Z)}{dZ} = 0 \qquad (19)$$

From equations (17)-(19) we obtain a homogeneous differential equation for $R_3(Z)$ and when solving it, we obtain

$$R_3(Z) = ce^{-AZ}$$

where

$$A = \frac{\gamma_1\mu_2 + \gamma_2\mu_1}{S_1\gamma_3\mu_2 + S_1\gamma_2\mu_3 + S_2\gamma_1\mu_3 + S_2\gamma_3\mu_1 - S_3\gamma_1\mu_2 - S_3\gamma_2\mu_1}$$

Since in the services the share of immovable stocks is close to unity, and in construction it is close to zero, $K_1 - K_2 = 0$, $A > 0$. Finding values $\frac{dR_i(Z)}{dz}$ and substituting them into (16), we obtain

$$K(Z) = \left(\frac{\gamma_1\gamma_3\mu_2 + \gamma_1\gamma_2\mu_3 + \gamma_1\gamma_2\mu_3 + \gamma_2\gamma_3\mu_1}{\gamma_1\mu_2 + \gamma_2\mu_1} - \gamma_3\right) ACE^{-AZ}$$

Hence

$$K_{max} = c\left(\frac{\gamma_1\gamma_3\mu_2 + 2\gamma_1\gamma_2\mu_3 + \gamma_2\gamma_3\mu_1}{\gamma_1\mu_2 + \gamma_2\mu_1} - \gamma_3\right)$$

Now we can answer the question of how the distribution of investments into industries and services is associated with the difference of actual investments from the maximum. This difference can be estimated either as the relationship $\frac{K}{K_{max}}$ or as the difference $(K_{max} - K)$. For the former estimation

$$\frac{d(K_1 - K_2)}{d\, K/K_{max}} = \frac{K}{K_{max}} \text{ const}$$

For the latter,

$$\frac{d(K_2 - K_2)}{d(K_{max} - K)} = - \frac{d(K_1 - K_2)}{dK}$$

So far, we have supposed that stock depletion is immediately replaced. This assumption is not very realistic. Now we shall consider a model with the real depletion of stocks. We assume that the amount of depleted stocks at a time point t is a part of stocks at this very point in time. Denote the receipt of stocks by F^+, and depletion by F^-. Obviously,

$$\dot{F}_i = F_i^+ - F_i^-$$

and if $F_i^- = g_i F_i$,

$$F_i^+ = \dot{F}_i + g_i F_i$$

where $g_i = g_i^m(1 - \phi_i) + g_i^{im}\phi_i$ and g_i^m, g_i^{im} are the shares of movable and immovable stocks, respectively.

It is obvious now that equations (10), (11), and (13) are dealing with increasing (new) jobs. After substituting R_i^+ for $\dot{R}_i$ in (10), (11), and (13), after substituting the expression in (20) for R_i^+, and after excluding $\dot{P}$ from equations (12) and (13), we obtain the following differential equations:

$$\sum_{i=1}^{3} a_{ij} \dot{R}_i + \sum_{i=1}^{3} b_{ij} R_i = c_j, \qquad j = 1,2,3$$

$$c_1 = K_i, \qquad c_2 = c_3 = 0$$

In the general case, depending on the relationship of coefficients c_{ij} and b_{ij}, the solution of a homogeneous system describes either the exponential or the fluctuational change of R_i. Investments then act as a disturbing factor able to both intensify and slacken the fluctuations depending on the nature of function K(t). Thus in reality fluctuations in the number of jobs can take place and will be associated with these equations. We can also show that the variable $\dot{P}$, i.e., net migration change, is liable to fluctuations. It is also interesting to note that if a solution with fluctuations for one sphere is obtained, and we artificially (i.e., not according to the solution) transfer to it part of the investments from other spheres to smooth these fluctuations, fluctuations will arise in other spheres. All these effects can be observed in actual statistics.

5. CONCLUSION

On the basis of this model in the Institute of Economics and Industrial Engineering (Novosibirsk, USSR), the problem of balanced capital investment distribution between agriculture, construction, and services for West Siberia was analyzed. The agricultural sphere was presented using nine food sectors. The model gave us the opportunity to determine the optimal number and structure of jobs in each sector for the next 20 years. Fortunately, we were able to determine an adequate migrant reaction to changes in the economy and, therefore, the results of our forecasts and demographic forecast were nearly the same. On the other hand, the the second model--the demo-educational model--was used for solving the supply-demand problem for manpower. A hypothetical demand structure was formed by the above-mentioned model and the supply needed was derived by the demo-educational model. Supply-demand imbalance was eliminated through special education programs.

The next step in modeling will be the construction of a similar model for the urban region in such a way that rural and urban regions will be linked through flows of migrants and commodities.

The final step will be the construction of a multiregional two-sector model. Usually in all of these models, the demo-educational part is a demographic model. It will be necessary to use multiregional population projections (Rogers 1975).

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