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**Foster, G.R., Lane, L.J., Nowlin, J.D., Laflen,
J.M. and Young, R.A.**

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A MODEL TO ESTIMATE SEDIMENT
YIELD FROM FIELD-SIZED AREAS:
DEVELOPMENT OF MODEL

G.R. Foster
L.J. Lane
J.D. Nowlin
J.M. Laflen
R.A. Young

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

G.R. FOSTER is a Hydraulic Engineer with the United States Department of Agriculture, Science and Education Administration, Agricultural Research (USDA-SEA-AR) and is an Associate Professor with the Agricultural Engineering Department at Purdue University, West Lafayette, Indiana 47907, USA.

L.J. LANE is a Hydrologist with the USDA-SEA-AR, Southwest Rangeland Watershed Research Center, 442 East Seventh, Tucson, Arizona 85705, USA.

J.D. NOWLIN is a Computer Programmer with the Agricultural Engineering Department at Purdue University, West Lafayette, Indiana 47907, USA.

J.M. LAFLEN is an Agricultural Engineer with the USDA-SEA-AR, Agricultural Engineering Department at Iowa State University, Ames, Iowa 50011, USA.

R.A. YOUNG is an Agricultural Engineer with the USDA-SEA-AR, North Central Soil Conservation Research Center, Morris, Minnesota 56267, USA.

PREFACE

The erosion of soil by water is one of the major undesirable consequences of agriculture, as soil loss leads to a decrease in the natural productivity of the agroecosystem. Thus, there is an urgent need to develop methods to plan for the control of sediment yield. The model presented in this paper seems useful for checking the current situation with respect to soil erosion and sediment yield for a field-sized area, and for trying various management alternatives to control the problem. While the model is based on previous experience from experimental studies and modelling of water erosion and sedimentation, it goes one step further.

This paper was prepared as a contribution to our collaborative efforts with the U.S. Department of Agriculture, Science and Education Administration, Agricultural Research. It fulfills the research objectives of the IIASA task "Environmental Problems of Agriculture."

Gennady N. Golubev
Task Leader
Environmental Problems
of Agriculture

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The erosion/sediment yield model described here is a component of a comprehensive field-scale model including hydrology, erosion, pesticide, and nutrient components developed by USDA-SEA-AR scientists under the leadership of W.G. Knisel, Jr. Additional information on the comprehensive model is given by Knisel (1978). Additional support for this project, including computer programming by V.A. Ferreira and typing by E.S. Schield, J.J. Rocha and K.L. Mellor, was provided by K.G. Renard of the Southwest Watershed Research Center, USDA-SEA-AR, Tucson, Arizona and by W.C. Moldenhauer of the Erosion Research Unit, USDA-SEA-AR, Lafayette, Indiana.

ABSTRACT

A tool for evaluating sediment yield from field-sized areas is needed for planning management practices to control sediment yield. We developed a reasonably simple simulation model which incorporates fundamental principles of erosion, deposition, and sediment transport mechanics. The model summarizes the state-of-the-art in erosion and sediment yield modeling with appropriate simplifications required to couple the governing equations.

Limited testing showed that the procedures developed here give improved estimates over the Universal Soil Loss Equation. Specific components of the model were tested using experimental data from overland flow, erodible channel, and impoundment studies. These results suggest that the model produces reasonable estimates of erosion, sediment transport, and deposition under a variety of circumstances common to field-scale areas.

Alternative management practices such as conservation tillage, terracing, and contouring can be evaluated separately or in combination to determine their influence on sediment yield. Given a particular location with specified characteristics for climate, soils, topography, and crops, the model provides a means of evaluating alternative management practices to suit a particular farming operation.

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A MODEL TO ESTIMATE SEDIMENT YIELD FROM FIELD-SIZED AREAS: DEVELOPMENT OF MODEL

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INTRODUCTION

Estimates of erosion and sediment yield on field-sized areas are needed to wisely select best management practices to control erosion for maintenance of soil productivity and control of sediment yield to prevent excessive degradation of water quality. A field is a typical management unit for farmers and each field has specific conditions upon which the selection of a management practice should be based. Soil conservationists have used the Universal Soil Loss Equation (USLE) (Wischmeier and Smith 1978) for several years to select practices specifically tailored to a given farmer's situation. Consequently, if sediment yield tolerances for maintenance of water quality are established for given local areas, best management practices can then be selected based on a given farmer's needs and the tolerable water loading for fields in his area using a model such as the one described herein (Foster 1979).

Sediment yield is a function of detachment of soil particles and the subsequent transport of these particles (sediment). On a given field, either detachment or sediment transport capacity may limit sediment yield depending on topography, soil characteristics, cover, and rainfall/runoff rates and amounts. Control of sediment yield by detachment or transport can change from season to season, from storm to storm, and even within a storm. The mathematical relationship for detachment is different from the one for transport, so they cannot be lumped into a single equation. Since erosion and transport for each storm are best considered separately, lumped equations such as the USLE (an erosion equation) or Williams' (1975) modified USLE (a flow transport sediment yield equation) cannot give the best results over a broad range of conditions on field-sized areas. Furthermore, the interrelation between detachment and transport is nonlinear and interactive for each storm which prevents using separate equations to linearly accumulate detachment or sediment transport capacity over several

storms. Therefore, to simulate erosion and sediment yield on an individual storm basis and to satisfy the need for a continuous simulation model, a rather fundamental approach was selected where separate equations are used for detachment and sediment transport.

A number of fundamentally based models (e.g., Beasley et al. 1977, Li 1977) compute erosion and transport at various times during the runoff event. Although these models are powerful, their excessive use of computer time practically prohibits simulating 20 to 30 years of record. Our model uses characteristic rainfall and runoff factors for a storm to compute erosion and sediment transport for that storm. In terms of computational time, this amounts to a single time step for models that simulate over the entire runoff event.

The model is intended to be useful without calibration or collection of research data to determine parameter values. Therefore, established relationships such as the USLE were modified and used in the model.

OVERVIEW OF THE MODEL

Every model is a representation and a simplification of the prototype. Various techniques, including planes and channels (Li 1977), square grids (Beasley et al. 1977), converging sections (Smith 1977), and stream tubes (Onstad and Foster 1975) have been used. Most erosion-sediment yield models have adequate degrees of freedom to fit observed data. Some models, depending on their representation scheme, distort parameter values more than do others. Distortion of parameter values greatly reduces their transferability from one area to another (Lane et al. 1975). An objective in this model development was to represent the field in a way that minimizes parameter distortion.

Hydrologic input to the erosion component consists of rainfall volume, rainfall erosivity, runoff volume, and peak rate of runoff. These terms drive soil detachment and subsequent transport by overland and open channel flow.

Overland flow, channel flow, and impoundment (pond) elements are used to represent major features of a field. The user selects the best combination of elements and enters the appropriate sequence number according to Table 1. The model (computer program) calls the elements in the proper sequence. Typical systems that the model can represent are illustrated in figure 1.

Table 1. Possible elements and their calling sequence used to represent field-sized areas.

Sequence number	Elements and their sequence
1	Overland
2	Overland-Pond
3	Overland-Channel
4	Overland-Channel-Channel
5	Overland-Channel-Pond
6	Overland-Channel-Channel-Pond

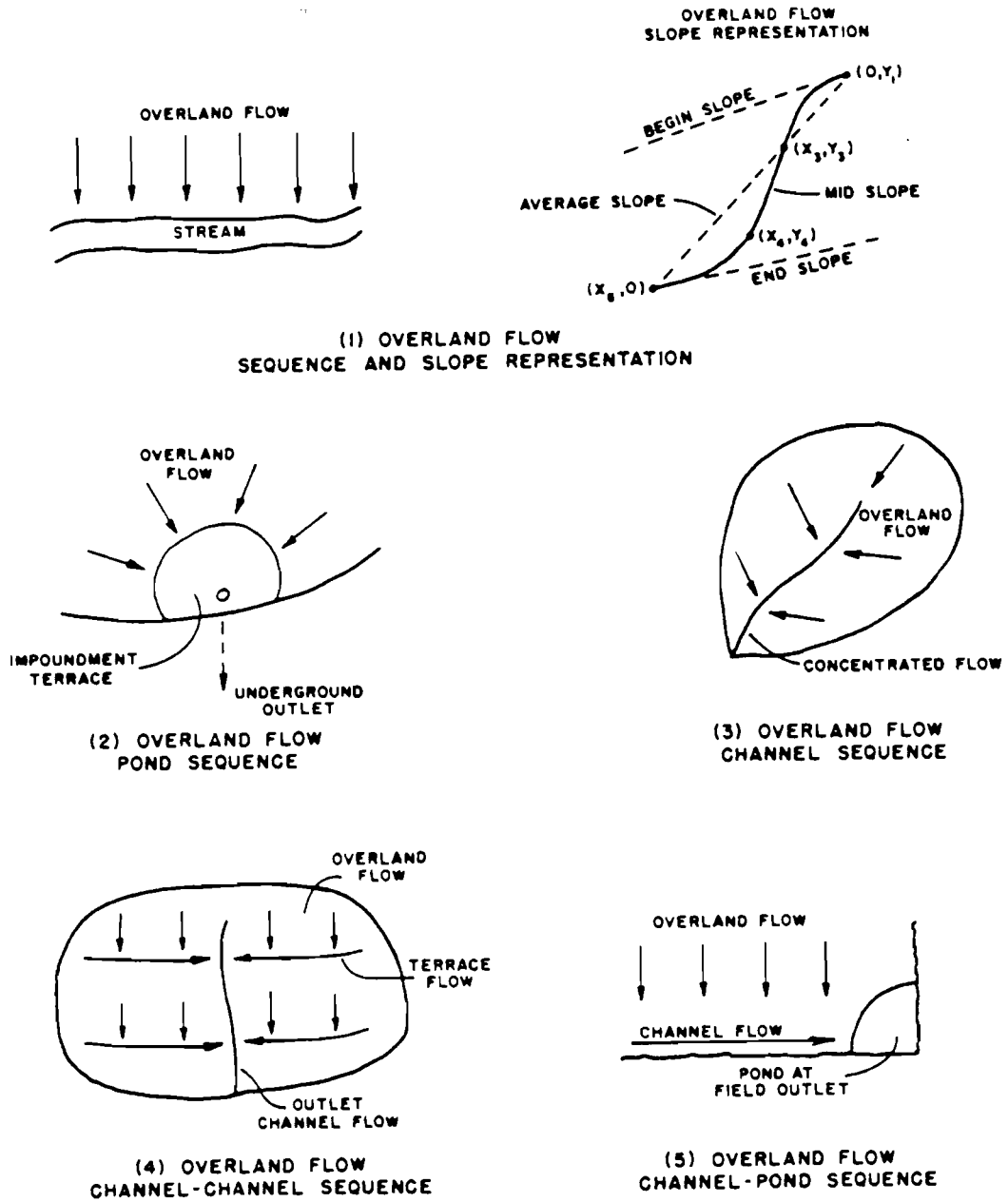


Figure 1. Schematic representation of typical field systems in the field-scale erosion/sediment yield model.

Computations begin in the uppermost element, which is always the overland flow element, and proceed downstream. Sediment concentration for each particle type is the output from each element, which becomes the input to the next element in the sequence.

BASIC CONCEPTS

Sediment load is assumed to be limited by either the amount of sediment made available by detachment or by transport capacity (Foster and Meyer 1975). Also quasi-steady state is assumed so that a rainfall and a runoff rate characteristic of each storm can be used in the computations.

BASIC EQUATIONS

The equation for continuity of mass for sediment movement downslope is expressed by:

$$dq_s/dx = D_L + D_F \quad (1)$$

where q_s = sediment load per unit width per unit time, x = distance, D_L = lateral inflow of sediment, and D_F = detachment or deposition by flow. Deletion of time terms from equation 1 is possible by the quasi-steady state assumption. The major sequence of computations is illustrated in figure 2.

Lateral sediment inflow is from interrill erosion on overland flow elements, or it is from overland flow (or a channel if two channel elements are in the sequence) for the channel elements. Flow in rills on overland flow areas or in channels transports the sediment load downstream. Lateral sediment inflow is assumed regardless of whether the flow is detaching or depositing sediment.

For a segment, either on the profile for the overland flow element or in a channel, the model computes an initial potential sediment load, which is the sum of the sediment load from the immediate upslope segment plus that added by lateral inflow within the segment. If this potential load is less than the flow's transport capacity, detachment occurs at the lesser of either the detachment capacity rate or the rate which will just fill transport capacity. When detachment by flow occurs, it adds particles having the particle size distribution for detached sediment given as input. No sorting is allowed during detachment.

If the initial potential sediment load is greater than the transport capacity, deposition is assumed to occur at the rate of:

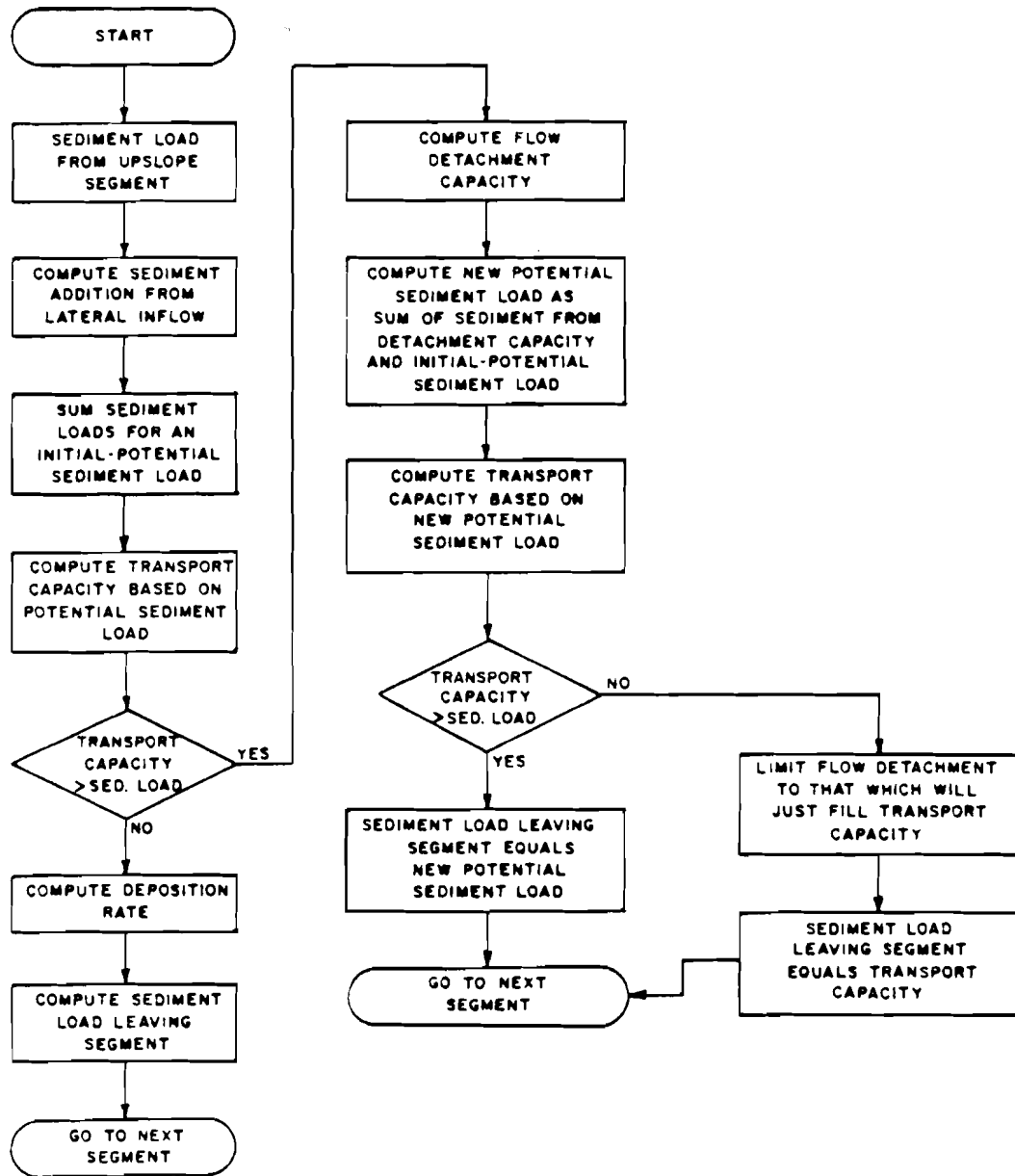


Figure 2. Flow chart for detachment-transport-deposition computations within a segment of overland flow or concentrated flow elements.

$$D = \alpha(T_C - q_S) \quad (2)$$

where D = deposition rate (mass/unit area/unit time), α = a first order reaction coefficient (length^{-1}), and T_C = transport capacity (mass/unit width/unit time). The coefficient α is estimated from:

$$\alpha = \epsilon V_S / q_L x \quad (3)$$

where $\epsilon = 0.5$ for overland flow (Davis 1978) and 1.0 for channel flow (Einstein 1968), V_S = particle fall velocity, and $q_L x = q_w$ = discharge rate of runoff per unit width (volume/width/time). Fall velocity is estimated assuming standard drag relationships for a sphere of a given diameter and density falling in still water.

Detachment-Deposition Limiting Cases

Four possible cases may exist for a segment: (1) deposition may occur over the entire segment; (2) detachment by flow in the upper end and deposition in the lower end may occur (but not necessarily) when transport capacity decreases within a segment; (3) deposition on the upper end and detachment by flow in the lower end may occur (but not necessarily) when transport capacity increases within the segment; (4) detachment by flow may occur all along the segment.

Case 1 occurs when $T_C < q_S$ all along the segment. Where deposition occurs over the entire segment length, deposition rate is:

$$D = [\phi / (1 + \phi)] (dT_C / dx - D_L) [1 - (x_U / x)^{1 + \phi}] + D_U (x_U / x)^{1 + \phi} \quad (4)$$

where:

$$\phi = \epsilon V_S / q_L \quad (5)$$

where dT_C / dx is assumed constant over the segment and D_U = deposition rate at x_U .

The deposition rate D_U may be estimated from:

$$D_U = \alpha(T_{CU} - q_{SU}) \quad (6)$$

where T_{cu} and q_{su} = respectively the transport capacity and sediment load at x_u . Sediment load at x is:

$$q_s = T_c - D/\alpha \quad (7)$$

Case 2 occurs when $T_{cu} > q_{su}$, $dT_c/dx < 0$, and T_c becomes less than q_s within the segment. If $dT_c/dx < 0$ for a segment where $T_{cu} > q_{su}$, T_c may decrease below q_s within the segment. The point where $q_s = T_c$ is determined as x_{db} which is used for x_u in equation 5, with $D_u = 0$. Deposition and sediment load are computed from equations 4, 5, and 7.

Case 3 occurs when $T_{cu} < q_{su}$, $dT_c/dx > 0$, and T_c becomes greater than q_s within the segment. In situations like a grass buffer strip, the transport capacity at the upper edge may drop abruptly below the sediment load. Within the upper end of the strip, the sediment load decreases due to deposition while the transport capacity increases from the point of the abrupt decrease. Somewhere upslope from the lower edge of the strip, the sediment load equals the transport capacity. At this point, x_{de} , deposition ends, i.e., $D_u = 0$ and, $T_c = q_s$. Downslope, detachment by flow occurs. The point where deposition ends is given by:

$$x_{de} = x_u \left\{ 1 - [(1+\phi)/\phi][D_u/(dT_c/dx - D_L)] \right\}^{1/(1+\phi)} \quad (8)$$

where:

$$D_u = \alpha (T_{cu} - q_{su}) \quad (9)$$

and T_{cu} = transport capacity after the abrupt decrease at x_u and q_{su} = sediment load at x_u . Continuity of sediment load is maintained, but D may be discontinuous at segment ends.

Downslope from x_{de} , where flow detachment occurs, the sediment load is given by:

$$q_s = (D_{Fu} + D_{Lu} + D_{FL} + D_{LL})\Delta x/2 + q_{su} \quad (10)$$

where the second subscript u or L indicates upper or lower and Δx = length of the segment where detachment by flow is occurring. In this case, Δx is from x_{de} to the lower end of the segment; q_{su} is at x_{de} , which is T_c at x_{de} ; $D_{Fu} = 0$ at x_{de} ; and D_{FL} is either detachment capacity at x or that which will just fill the transport capacity.

Case 4 occurs when $T_c > q_s$ over the entire segment. Sediment load is computed with equation 10.

The equation for sediment transport capacity (discussed later) shifts total transport capacity among the various particle types. If transport capacity exceeds availability for one particle while it is less for another, transport capacity is shifted from the particle type having the excess to the one having the deficit. Furthermore, logic checks within the model prevent simultaneous deposition and detachment of particles by flow.

Eroded sediment is a mixture of particles having various sizes and densities. The distribution is broken into classes, with each class represented by a particle diameter and density. Equations 4-10 are solved for each particle type within the given constraints.

Sediment Characteristics

Sediment eroded on field-sized areas is a mixture of primary particles and aggregates (conglomerates of primary particles). The distribution of these particles as they are detached is a function of soil properties, management, and rainfall and runoff characteristics. If deposition occurs, usually the coarse and dense particles are deposited first, leaving a finer sediment mixture. The input to the model is the distribution of the sediment as it is detached; the model calculates a new distribution if it calculates that deposition occurs.

Based on our survey of existing data, values given in Table 2 are typical of many Midwestern soils.

If the particle distribution is not known, the model assumes five particle types, and estimates the distribution from the primary particle size distribution.

$$PSA = (1.0 - ORCL)^{2.49} ORSA \quad (11)$$

$$PSI = 0.13 ORSI \quad (12)$$

$$PCL = 0.2 ORCL \quad (13)$$

$$SAG = \begin{cases} 2 ORCL & \text{for } ORCL < 0.25 \end{cases} \quad (14)$$

$$SAG = \begin{cases} 0.28(ORCL - 0.25) + 0.5 & \text{for } 0.25 \leq ORCL \leq 0.50 \end{cases} \quad (15)$$

$$SAG = \begin{cases} 0.57 & \text{for } 0.5 < ORCL \end{cases} \quad (16)$$

$$LAG = 1.0 - PSA - PSI - PCL - SAG \quad (17)$$

If $LAG < 0.0$, multiply all others by the same ratio to make $LAG = 0.0$ (18)

Table 2. Sediment characteristics assumed for detached sediment before deposition. Assumed typical of many Midwestern silt loam soils.

Particle type	Diameter (mm)	Specific gravity	Fraction of total amount (mass basis)
Primary clay	0.002	2.60	0.05
Primary silt	0.010	2.65	0.08
Small aggregates	0.030	1.80	0.50
Large aggregates	0.500	1.60	0.31
Primary sand	0.200	2.65	0.06

where ORCL, ORSI, and ORSA are, respectively, fractions for primary clay, silt, and sand in the original soil mass, and PCL, PSI, PSA, SAG, and LAG are, respectively, fractions for primary clay, silt, sand, and small and large aggregates in the detached sediment.

The diameters for the particles are given by:

$$DPCL = 0.002 \text{ mm} \quad (19)$$

$$DPSI = 0.010 \text{ mm} \quad (20)$$

$$DPSA = 0.20 \text{ mm} \quad (21)$$

$$DSAG = \begin{cases} 0.03 \text{ mm} & \text{for } ORCL < 0.25 \quad (22) \\ 0.20(ORCL - 0.25) + 0.03 \text{ mm} & \text{for } 0.25 \leq ORCL \leq 0.60 \quad (23) \\ 0.1 \text{ mm} & \text{for } 0.60 < ORCL \quad (24) \end{cases}$$

$$DLAG = 2(ORCL) \text{ mm} \quad (25)$$

where DPCL, DPSI, DPSA, DSAG, and DLAG are, respectively, the diameters of the primary clay, silt, and sand, and the small and large aggregates in sediment. The assumed specific gravities are shown in Table 2. The primary particle composition of the sediment load is estimated from:

Small aggregates:

$$CLSAG = SAG \cdot ORCL / (ORCL + ORSI) \quad (26)$$

$$\text{SISAG} = \text{SAG} \cdot \text{ORSI} / (\text{ORCL} + \text{ORSI}) \quad (27)$$

$$\text{SASAG} = 0.0 \quad (28)$$

Large aggregates:

$$\text{CLLAG} = \text{ORCL} - \text{PCL} - \text{CLSAG} \quad (29)$$

$$\text{SILAG} = \text{ORSI} - \text{PSI} - \text{SISAG} \quad (30)$$

$$\text{SALAG} = \text{ORSA} - \text{PSA} \quad (31)$$

where CLSAG, SISAG, and SASAG are fractions of the total for the sediment of, respectively, primary clay, silt, and sand in the small aggregates in the sediment load, and CLLAG, SILAG, and SALAG are corresponding fractions for the large aggregates.

If the clay in the large aggregate expressed as a fraction for that particle alone is less than 0.5 times ORCL, the distribution of the particle types is recomputed so that this constraint can be met. A sum SUMPRI is computed whereby:

$$\text{SUMPRI} = \text{PCL} + \text{PSI} + \text{PSA} \quad (32)$$

The fractions PSA, PSI, PCL are not changed. The new SAG is:

$$\text{SAG} = (0.3 + 0.5 \text{ SUMPRI})(\text{ORCL} + \text{ORSI}) / [1 - 0.5(\text{ORCL} + \text{ORSI})] \quad (33)$$

Equation 33 is derived given previously determined values for PCL, PSI, and PSA; the sum of primary clay fractions for the total sediment equals the clay fraction in the original soil; and the assumption that the fraction of primary clay in LAG equals one half of the primary clay in the original soil.

The model also computes an enrichment ratio using specific surface areas for organic matter, clay, silt, and sand. Organic matter is distributed among the particle types based on the proportion of primary clay in each type. Enrichment ratio is the ratio of the total specific surface area for the sediment to that for the original soil.

Although these relationships are approximations to the data found in the literature (R. A. Young 1978 personal communication, USDA-SEA-AR, Morris, Minnesota), they represent the general trends.

OVERLAND FLOW ELEMENT

Detachment Equation

Detachment on interrill and rill areas and transport and deposition by rill flow are the erosion-transport processes on the overland flow element. Detachment is described by a modified USLE written as:

$$D_{Li} = 4.57(EI)(s + 0.014) KCP (\sigma_p/V_u) \quad (34)$$

and

$$D_{Fr} = (6.86 \times 10^6) m V_u \sigma_p^{1/3} (x/22.1)^{m-1} s^2 KCP(\sigma_p/V_u) \quad (35)$$

where D_{Li} = interrill detachment rate ($g/m^2/s$), D_{Fr} = rill detachment capacity rate ($g/m^2/s$), EI = Wischmeier's rainfall erosivity expressed as total rain storm energy times maximum 30-minute intensity (N/h), x = distance downslope (m), s = sine of slope angle, m = slope length exponent, K = USLE soil erodibility factor [$g\ h/(N\ m^2)$], C = soil loss ratio of the USLE cover-management factor, P = USLE contouring factor, V_u = runoff volume/area (m), and σ_p = peak runoff rate expressed as volume/area/time (m/s). The units on the USLE K (Wischmeier et al. 1971) must be carefully noted. Multiplication of K in standard English units by 131.7 gives a metric K having units of $g\ h/(N\ m^2)$.

Only the contouring part of the USLE P factor is used. Other P factor effects such as strip cropping and deposition in terrace channels are accounted for directly in the model. The model more accurately represents these factors than do the broad averages given for the USLE (Wischmeier and Smith 1978).

Storm Erosivity

The hydrologic processes of rainfall and runoff drive the erosion-transport processes. Storm EI , volume of runoff, and peak discharge are the variables used to characterize hydrologic inputs. Values for these factors are generated by a hydrologic model, or observed data may be used. Techniques are commonly available for estimating the runoff factors (e.g., Schwab et al. 1966). An approximate estimate of storm EI is (Lombardi 1979):

$$EI = 0.103 V_R^{1.51} \quad (36)$$

where EI = storm EI (N/h) and V_R = volume of rainfall (mm). Multiplication of EI in standard English units by 1.702 gives a metric EI having

units of N/h. Equation 36 was developed by regression analysis from about 2,700 data points used in the development of the USLE and has a coefficient of determination (R^2) of 0.56. This relationship should be used only as a last resort.

Since rainfall energy for higher intensities does not vary greatly with intensity (Wischmeier and Smith 1978), the approximate rainfall energy per unit rainfall is $27.6 \text{ J/m}^2/\text{mm}$ of rain. An estimate of storm EI (N/h) is:

$$EI = 0.0276 V_R I \quad (37)$$

where I = maximum 30 minute intensity (mm/h). If the rainfall hyetograph is available, storm EI can be computed from:

$$e = 11.9 + 8.73 \log_{10} i \quad (38)$$

where e = rainfall energy per unit of rainfall ($\text{J/m}^2/\text{mm}$ of rain) and i = rainfall intensity (mm/h). The difference between EI computed from equation 37 or computed from equation 38 where increment energies are computed and summed is negligible.

Slope Length Exponent

For slopes less than 50 m, the slope length exponent m is set to 2, but for slopes longer than 50 m, m is limited by:

$$m = 1.0 + 3.912/\ln(x) \quad (39)$$

This limit avoids excessive erosion for very long slopes (Foster et al. 1977).

Sediment Transport Capacity

The Yalin sediment transport equation (Yalin 1963) is used to describe sediment transport capacity. It gave reasonable results when compared with experimental data for deposition of sand and coal by overland flow in a laboratory study (Foster and Huggins 1977, Davis 1978) and unpublished field plot data (Table 3). The Yalin equation was modified to distribute transport capacity among the various particle types. The discussion of the method given below is abstracted from Foster and Meyer (1972), Davis (1978), and Khaleel et al. (1980).

Table 3. Sediment yield in overland flow from concave slopes.

Sediment		Shear stress	Transport rates	
Diameter	Specific gravity		Observed	Calculated using Yalin equation
(mm)		(N/m ²)	(g/m/s)	(g/m/s)
0.342	2.65	0.52	5.6	4.2
0.342	2.65	0.76	19.7	13.4
0.150	2.65	0.55	5.2	9.0
0.150	2.65	0.70	18.8	17.6
0.342	2.65	0.40	2.2	1.4
0.342	2.65	0.60	12.8	6.6
0.342	1.60	0.30	3.5	6.0
0.342	1.60	0.42	13.7	14.4
0.156	1.67	0.30	3.8	6.1
0.156	1.67	0.40	13.3	12.7
Eroded from Barnes loam, field plots		0.33	3.3	2.8
Eroded from Miami silt loam, field plots		0.51	4.8	7.0
Eroded from Miami silt loam, field plots		0.35	2.5	3.1
Eroded from Miami silt loam, field plots		--	1.4	--

Source: Neibling and Foster (1980).

The Yalin equation is given by:

$$W_s / S_g \rho_w d V_* = 0.635 \delta [1 - (1/\sigma) \ln (1 + \sigma)] = P_s \quad (40)$$

where

$$\sigma = A \delta \quad (41)$$

$$\delta = (Y/Y_{cr}) - 1 \quad (\text{when } Y < Y_{cr}, \delta = 0) \quad (42)$$

$$A = 2.45 (S_g)^{-0.4} (Y_{cr})^{1/2} \quad (43)$$

$$Y = v_*^2 / (S_g - 1) g d \quad (44)$$

$$V_* = (gRS_f)^{1/2} \quad (45)$$

where V_* = shear velocity $=(\tau/\rho_w)^{1/2}$, τ = shear stress, g = acceleration due to gravity, ρ_w = mass density of the fluid, R = hydraulic radius, S_f = slope of the energy gradeline, S_g = particle specific gravity, d = particle diameter, Y_{cr} = critical lift force given by the Shields' diagram extended to small particle Reynolds numbers (Mantz 1977), and W_s = transport capacity (mass/unit time/unit flow width). The constant 0.635 and Shields' diagram were empirically derived.

The sediment load may have fewer particles of a given type than the flow's transport capacity for that type. At the same time, the sediment load of other particle types may exceed the flow's transport capacity for those types. The excess transport capacity for the deficit types is assumed to be available to increase the transport capacity for the types where available sediment exceeds transport capacity.

The Yalin equation was modified to shift excess transport capacity. For large sediment loads (sediment loads for each particle type clearly in excess of the respective transport capacity for each particle type), or for small loads (sediment loads for each particle type clearly less than the respective transport capacity for each particle type), the flow's transport capacity is distributed among the available particle types based on particle size and density and flow characteristics (Foster and Meyer 1972).

Yalin assumed that the number of particles in transport is proportional to δ . For a mixture, the number of particles of a given type i is assumed to be proportional to δ_i . Values of δ_i for each particle type in a mixture are calculated and summed to give a total:

$$T = \sum_{i=1}^{n_s} \delta_i \quad (46)$$

where n_s = number of particle types in the mixture. The number of transported particles of type i in a mixture is given as:

$$(N_e)_i = N_i (\delta_i/T) \quad (47)$$

where N_i = number of particles transported in sediment of uniform type i for a δ_i .

As derived by Yalin, the nondimensional transport, P_s , of equation 40 is proportional to the number of particles in transport.

Then,

$$(P_e)_i = (P_s)_i \delta_i/T \quad (48)$$

where $(P_e)_i$ = the effective P for particle type i in a mixture, and $(P_s)_i$ is the P_s calculated for uniform material of type i. The transport capacity W_{sj} of each particle type in a mixture is then expressed by:

$$W_{sj} = (P_e)_i (S_g)_i \rho_w d_i V_* \quad (49)$$

This is the transport capacity assuming that the supply of all particle types is either greater than or less than their respective W_{sj} . When availability of some types is greater than their W_{sj} and others are less than their W_{sj} , transport capacity shifts from those types where supply is less than capacity so that all of the total transport capacity is used.

The steps given below are followed to redistribute the transport capacity when excesses and deficits occur.

1. For those particles where $W_{sj} \geq q_{sj}$ (q_{sj} = sediment load for particle type i), compute the actual required P_{ireq} from equation 40, i.e.:

$$P_{ireq} = q_{sj} / (S_g)_i \rho_w d_i V_* \quad (50)$$

and assign $T_{ci} = W_{sj}$.

2. For those particle types where $W_{sj} \geq q_{sj}$, the sum:

$$SPT = \sum_{i=1}^{n_s} (P_{ireq} / P_i) k_i \quad (51)$$

is computed where $k_i = 1$ if $W_{sj} \geq q_{sj}$ and $k_i = 0$ if $W_{sj} < q_{sj}$. The sum SPT represents the fraction of the total transport capacity used by those particle types where sediment availability is less than transport capacity.

3. The excess, expressed as a fraction of the total, to be distributed is:

$$E_{xc} = 1 - SPT \quad (52)$$

4. For those particle types where $W_{sj} < q_{sj}$, sum δ_i as:

$$SDLT = \sum_{i=1}^{n_s} \delta_i l_i \quad (53)$$

where $l_i = 0$ if $W_{sj} \geq q_{sj}$ and $l_i = 1$ if $W_{sj} < q_{sj}$.

5. The excess is distributed according to the distribution of δ_i among these particle types, i.e.:

$$T_{ci} = (\delta_i/SDLT)(E_{xc})(P_i)(S_g)_i \rho_w d_i V_* l_i \quad (54)$$

For the other particle types:

$$T_{ci} = q_{si} k_i \quad (55)$$

6. Repeat steps 1-5 until either all $T_{ci} \leq q_{si}$ or all $T_{ci} \geq q_{si}$. When the former occurs, the proper T_{ci} 's have been found. If the latter occurs, one particle type will have all of the excess transport capacity. The excess for this one type should be equally distributed among all of the types. This is done by:

$$SMUS = \sum_{i=1}^n (P_{ireq}/P_i) \quad (56)$$

$$T_{ci} = (1.0/SMUS)q_{si} \quad (57)$$

Conversion from Storm to Rate Basis

Without the (σ_p/V_u) term, equations 34 and 35, as originally developed (Foster et al. 1977) were on a storm basis, whereas the transport equation is on an instantaneous rate basis. The equations are combined by assuming that computed sediment concentration represents an average for the runoff event, and that peak discharge represents a characteristic discharge that can be used to compute the average concentration.

Since most field-sized areas are relatively small, time of concentration for the runoff is usually small and is assumed to be less than rainfall duration. Thus, for a given storm, discharge at a location is assumed to be directly proportional to upstream drainage area.

Shear Stress

The transport equation requires an estimate of the runoff's shear stress. The sediment transport concept (Graf 1971), where shear is divided between form roughness and grain roughness, is used to estimate the shear stress acting on the soil, the portion assumed responsible for sediment transport. Mulch or vegetation reduces this stress. The shear stress acting on the soil, τ_{soil} , is estimated by:

$$\tau_{soil} = \gamma y s (n_{bov}/n_{cov})^{0.9} \quad (58)$$

where γ = weight density of water; y = flow depth for bare, smooth soil; s = sine of slope angle; n_{bov} = Manning's n for bare soil (0.01 for overland flow and 0.03 for channel flow assumed); and n_{cov} = total Manning's n for rough surfaces or soil covered by mulch or vegetation. Flow depth is estimated by the Manning equation as:

$$y = [q_w n_{bov} / s^{1/2}]^{0.6} \quad (59)$$

where q_w = discharge rate per unit width. Although the Darcy-Weisbach equation with a varying friction factor for laminar flow might be more accurate for y in some cases, most users are better acquainted with estimating Manning's n . The error in estimating a value for the roughness factor is probably greater than the error in using the Manning equation for laminar flow.

Slope Segments

Computations begin at the upper end of the slope. Sediment is routed downslope much the same as it is in most erosion models. Computed output is the sediment concentration for each particle type. Concentration multiplied by the runoff volume and overland flow area represented by the overland flow profile gives the sediment yield for the storm on the overland area of the field.

The overland flow area is represented by a typical land profile selected from several possible overland flow paths. Its shape may be uniform, convex, concave, or a combination of these shapes. Inputs are total slope length, average steepness, the slope at the upper end of the profile, the slope at the lower end of the profile and location of the end points of a miduniform section.

Given this minimum of information, the model establishes segments along the profile. The procedure is illustrated by the convex shape shown in figure 3. Coordinates of points A, C, and D are given, as are slopes s_b and s_m . A quadratic curve will pass through point C tangent to line CD and through point E tangent to line AB. The location of point E is the intersection of a line having a slope equal to the average of s_b and s_m with line AB. If X_2 is less than X_1 , X_3 is shifted downslope so that $X_1 = X_2$.

Each uniform section is one segment. In figure 3, AE and CD are segments. Convex sections like EC are divided into only three segments, because detachment and transport computations are not especially sensitive to the number of segments on convex slopes. Concave segments are divided into 10 segments because deposition computations on concave slopes are especially sensitive to the number of segments. Furthermore, several segments are required to accurately determine where deposition begins.

Additional segment ends are designated where K , C , P , or n change. Given locations where these changes occur as input, the model computes the coordinates for all the segments for the overland flow slope.

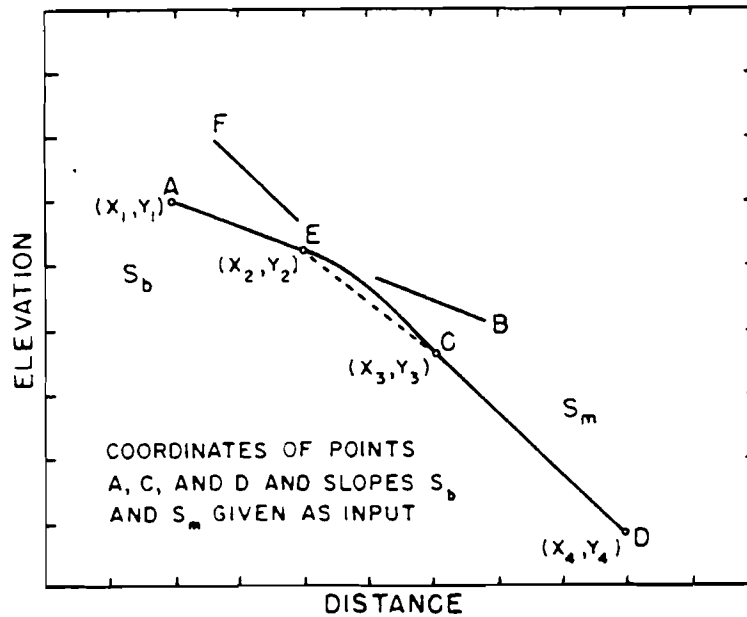


Figure 3. Representation of convex slope profile for overland flow.

Selection of Parameter Values

Slope length is, perhaps, the most difficult of the overland flow parameters to estimate. Williams and Berndt's (1977) contour method is a possible technique to use. Another is to sketch flow lines from the watershed boundary to concentrated flow. Topography in most fields converges overland flow into concentrated flow within about 100 m. Certainly a grass waterway (or a similar flow concentration without a constructed waterway where erosion may or may not be a problem), a terrace channel, or a diversion is the end of overland slope length.

Values for the parameters K , C (soil loss ratio), and P (contouring) are selected from Wischmeier and Smith (1978) according to crop stage. Values for Manning's n_{cov} may be selected from Lane et al. (1975) or from Foster et al. (1980).

CHANNEL ELEMENT

The channel element is used to represent flow in terrace channels, diversions, major flow concentrations where topography has caused overland flow to converge, grass waterways, row middles or graded rows, tail ditches, and other similar channels. The channel element does not describe gully or large channel erosion.

Except that shear stress and detachment by flow are estimated differently, the same concepts and equations are used in both the channel and overland flow elements. Discharge along the channel is assumed to vary directly with upstream drainage area. A discharge greater than zero is permitted at the upper end to account for upland contributing areas. As with

the overland flow element, changes in the controlling variables along the channel are allowed. Thus, a channel with a decreasing slope or a change in cover can be analyzed.

Spatially Varied Flow Equations

Flow in most channels in fields is spatially varied, especially for outlets restricted by ridges and heavy vegetation, and for very flat terrace channels. Also, discharge generally increases along the channel. The model approximates the slope of the energy gradeline (friction slope) along the channel using a set of normalized curves and assuming steady flow at peak discharge. As an alternative, the model will set the friction slope equal to the channel slope.

The equation for spatially varied flow (Chow 1959) with increasing discharge in a triangular channel may be normalized as:

$$dy_*/dx_* = [S_* - C_2 x_*^2/y_*^{16/3} - C_3 x_*/y_*^4]/[1 - C_3 x_*^2/y_*^5] \quad (60)$$

where $y_* = y/y_e$, y = flow depth, y_e = flow depth at the end of the channel, $S_* = sL_{eff}/y_e$, s = channel slope, x = distance along channel, $x_* = x/L_{eff}$, and L_{eff} = effective channel length (i.e., the length of the channel if it is extended upslope to where discharge would be zero with the given lateral inflow rate). Constants C_1 , C_2 , and C_3 are given by:

$$C_1 = [z^{5/2}/2(z^2 + 1)^{1/2}]^{2/3} \quad (61)$$

$$C_2 = [Q_e n L_{eff}^{1/2}/C_1 y_e^{19/6}]^2 \quad (62)$$

$$C_3 = 2 \beta Q_e^2/g z^2 y_e^5 \quad (63)$$

where n = Manning's n , z = side slope of channel, Q_e = discharge at end of channel, β = energy coefficient (1.56 used from McCool et al. 1966), and g = acceleration due to gravity. Equation 60 was solved for a range of typical values of C_1 , C_2 , and C_3 for subcritical flow. The curves given by equations 64-73 were fitted by regression to the solutions.

Range of C_3 : $C_3 > 0.3$

Where $0.0 \leq S_* \leq 1.2$ and $x_* \leq 0.9$

$$SSF = 0.2777 - 3.3110 x + 9.1683 x^2 - 8.9551x^3 \quad (64)$$

Where $1.2 \leq S_* \leq 4.8$ and $x_* \leq 0.9$

$$\text{SSF} = 2.6002 - 8.0678x_* + 15.6502x_*^2 - 11.7998x_*^3 \quad (65)$$

Where $4.8 < S_* \leq 20.0$ and $x_* \leq 0.9$

$$\text{SSF} = 3.8532 - 12.9501x_* + 21.1788x_*^2 - 12.1143x_*^3 \quad (66)$$

Where $20.0 < S_*$,

$$\text{SSF} = 0.0 \quad (67)$$

Range of C_3 : $0.3 \geq C_3 \geq 0.03$

Where $S_* > 0$ and $x_* \leq 0.8$,

$$\text{SSF} = 2.0553 - 6.9875x_* + 11.418x_*^2 - 6.4588x_*^3 \quad (68)$$

Where $S_* = 0$ and $x_* \leq 0.9$,

$$\text{SSF} = 0.0392 - 0.4774x_* + 1.0775x_*^2 - 1.3694x_*^3 \quad (69)$$

Range of C_3 : $0.03 > C_3 \geq 0.007$

Where $S_* > 0_*$ and $x_* \leq 0.8$,

$$\text{SSF} = 1.5386 - 5.2042x_* + 8.4477x_*^2 - 4.740x_*^3 \quad (70)$$

Where $S_* = 0.0$ and $x_* \leq 0.9_*$,

$$\text{SSF} = 0.0014 - 0.0162x_* - 0.0926x_*^2 - 0.0377x_*^3 \quad (71)$$

Range of C_3 : $0.007 > C_3$

Where $S_* > 0$ and $x_* < 0.7$,

$$\text{SSF} = 1.2742 - 4.7020x_* + 8.4755x_*^2 - 5.3332x_*^3 \quad (72)$$

Where $S_* = 0$ and $x_* \leq 0.9$,

$$SSF = -0.0363x_*^2 \quad (73)$$

With these values of SSF, the friction slope is:

$$S_f = (S_* - SSF) y_e / L_{eff} \quad (74)$$

Flow depth, y_e , at the end of the channel is estimated by assuming at the user's option, either critical depth, depth of uniform flow in an outlet control channel, or depth from a rating curve.

A triangular channel section was used to develop the friction slope curves because the equations are simple. In the model, a triangular channel must be used to estimate the slope of the energy gradeline, but the user may select a triangular, rectangular, or "naturally eroded" section in other computational components of the channel element.

Concentrated Flow Detachment

In the spring after planting, concentrated flow from intense rains on a freshly prepared seedbed often erodes through the finely tilled layer to the depth of secondary tillage, or perhaps, primary tillage. Once the channel erodes to the nonerodible layer, it widens at a decreasing rate.

Data from observed rill erosion (Lane and Foster 1980) suggests that detachment capacity ($\text{kg}/\text{m}^2/\text{s}$) by flow over a loosely tilled seedbed may be described by:

$$D_{Fc} = K_{ch} (1.35 \bar{\tau} - \tau_{cr})^{1.05} \quad (75)$$

where K_{ch} = an erodibility factor [$(\text{m}^2/\text{N})^{1.05} (\text{kg}/\text{m}^2/\text{s})$], $\bar{\tau}$ = average shear stress (N/m^2) of the flow in the channel, and τ_{cr} = a critical shear stress below which erosion is negligible. Critical shear stress seems to increase greatly over the year as the soil consolidates (Graf 1971).

Shear stress is assumed to be triangularly distributed in time during the runoff event in order to estimate the time that shear stress is greater than the critical shear stress. For the time that shear stress is greater than critical shear stress, shear stress is assumed constant and equal to peak shear stress for the storm.

Until the channel reaches the nonerodible layer, an active channel is assumed that is rectangular with the width obtained from figures 4 and 5 and equations 76 and 77. The solution requires finding a value of x_c .

Given the discharge Q , Manning's n , friction slope S_f , a value $g(x_c)$ is calculated from:

$$g(x_c) = (Qn/S_f^{1/2})^{3/8} (\gamma S_f / \tau_{cr}) \quad (76)$$

Given a particular value $g(x_c)$, a value of x_c is obtained from figure 4. Having determined x_c , a value for $R_* = \text{hydraulic radius/wetted perimeter}$ and $W_* = \text{width/wetted perimeter}$ is read from figure 5. The width of the channel is then calculated from:

$$W_{ac} = (Qn/S_f^{1/2})^{3/8} (W_*/R_*^{5/8}) \quad (77)$$

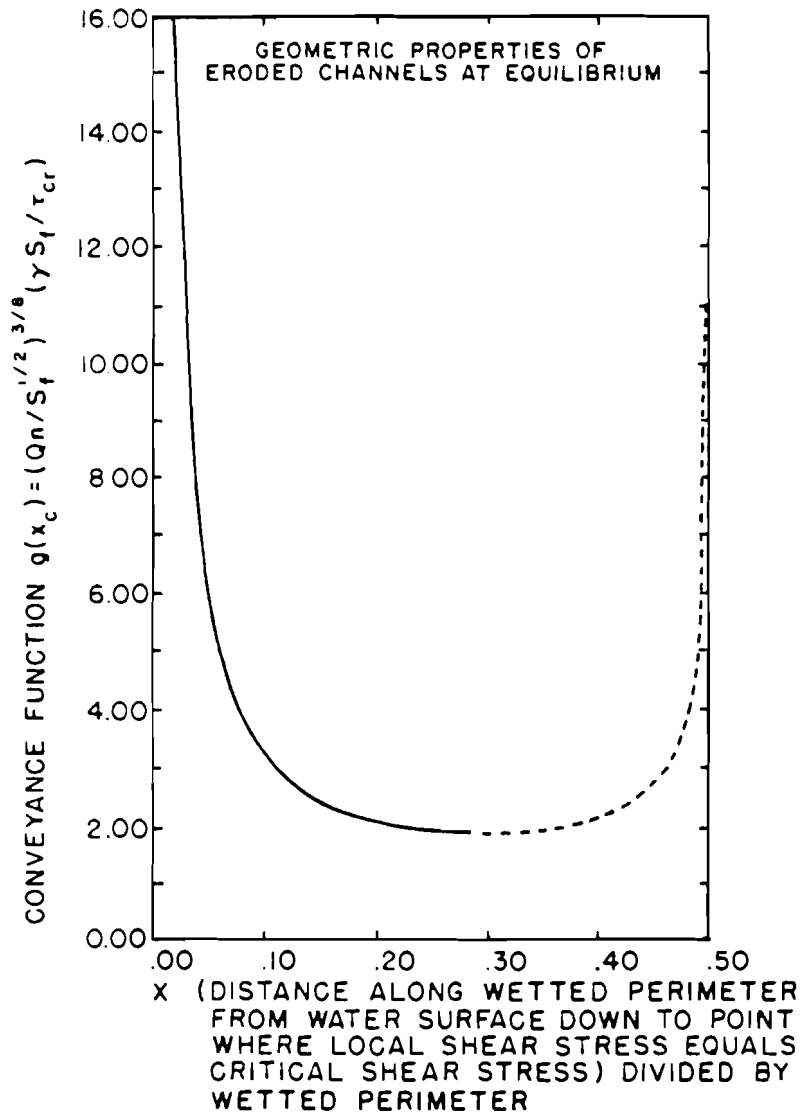


Figure 4. Function $g(x_c)$ for equilibrium eroded channel.

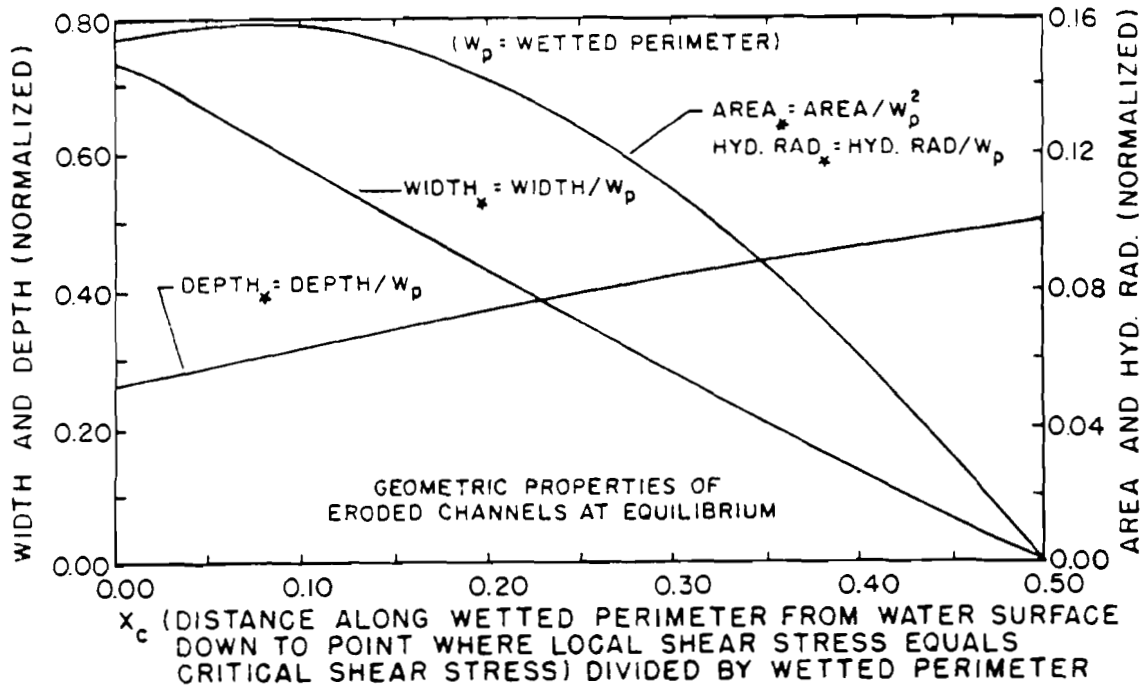


Figure 5. Equilibrium eroded channel geometric properties.

The functions shown in figures 4 and 5 are stored piecewise in the model.

The channel moves downward at the rate d_{ch} :

$$d_{ch} = D_{Fc} / \rho_{soil} = K_{ch} (1.35 \bar{\tau} - \tau_{cr})^{1.05} / \rho_{soil} \quad (78)$$

where D_{Fc} = erosion rate (mass/unit area/unit time), calculated using the maximum shear stress and ρ_{soil} = mass density of the soil in place. The erosion rate in the channel is:

$$E_{ch} = W_{ac} K_{ch} (1.35 \bar{\tau} - \tau_{cr})^{1.05} \quad (79)$$

where E_{ch} is the rate of soil loss per unit channel length (mass/unit/length/unit time).

Once the channel hits the nonerodible boundary, the erosion rate begins to decrease with time. The width W of the channel at any time after the channel has eroded to the nonerodible layer is estimated from:

$$\omega_* = 1 - \exp(-t_*) \quad (80)$$

where:

$$\omega_* = (W - W_i)/(W_f - W_i) \quad (81)$$

$$t_* = (t - t_i)(dW/dt)_i/(W_f - W_i) \quad (82)$$

where W_i = width at $t = t_i$, W = width at t , W_f = final eroded width for $t \rightarrow \infty$, and the given Q , t = time, and $(dW/dt)_i$ = rate that channel widens at $t = t_i$. The initial widening rate is given by:

$$(dW/dt)_i = 2 K_{ch} (\tau_b - \tau_{cr})^{1.05} / \rho_{soil} \quad (83)$$

where ρ_{soil} = mass density of the soil in place and τ_b is given by:

$$f(x_b) = \tau_b / \bar{\tau} = \tau_m [(8x_b)^{1/2} - 2x_b] \quad (84)$$

$$\text{and: } \tau_m = \tau_{max} / \bar{\tau} = 1.35 \quad (85)$$

where x_b = flow depth/wetted perimeter, and τ_{max} = maximum shear stress at center of channel.

The final width W_f is determined by finding the x_{cf} that gives:

$$\left[\frac{Q n}{S_f^{1/2}} \right]^{3/8} \frac{\gamma S_f}{\tau_{cr}} = \frac{1}{[x_{cf}(1 - 2x_{cf})^{3/8} f(x_{cf})]} \quad (86)$$

where $f(x_{cf})$ is the function given by equation 84 and 85 and evaluated at x_{cf} .

The final width is:

$$W_f = \left[\frac{Q n}{S_f^{1/2}} \right]^{3/8} \left[\frac{1 - 2x_{cf}}{x_{cf}^{5/3}} \right]^{3/8} \quad (87)$$

Sediment Transport and Partitioning of Shear Stress

Sediment transport capacity for the channel is described using exactly the same form of the Yalin equation that was used in the overland flow element.

The shear stress acting on the soil is the shear stress used to compute detachment and transport. Grass and mulch reduce this stress. Total shear is divided into that acting on the vegetation or mulch and that acting on the soil using sediment transport theory (Graf 1971).

First, velocity is estimated using n_t , the total Manning's n . See Chow (1959) and Ree and Crow (1977) for estimates of n_t . The hydraulic radius due to the soil is:

$$R_{\text{soil}} = (V n_{\text{bch}} / S_f^{1/2})^{3/2} \quad (88)$$

where V = flow velocity, n_{bch} = Manning's n for a bare channel, and S_f = friction slope. Shear stress acting on the soil is:

$$\tau_{\text{soil}} = \gamma R_{\text{soil}} S_f, \quad (89)$$

and shear stress acting on the cover is:

$$\tau_{\text{cov}} = \gamma [V(n_t - n_{\text{bch}}) S_f^{1/2}]^{3/2} S_f. \quad (90)$$

If τ_{cov} exceeds the shear stress at which the cover starts to move, the cover fails, thus increasing the flow's shear stress on the soil.

Variations in parameters such as Manning's n and slope along the channel can be considered. In addition, the model breaks the channel into segments that are $L_{\text{eff}}/10$ long. Calculations begin at the upper end of the channel and proceed downstream.

IMPOUNDMENT (POND) ELEMENT

The impoundment (pond) element describes deposition behind impoundments (including parallel tile outlet terraces) that drain after each storm.

Deposition is the main sedimentation process that occurs in impoundments. Since transport capacity in the impoundments is essentially nonexistent, the amount of sediment trapped in an impoundment is basically a function of time available for sediment to settle to the bottom before flow

leaves the impoundment. The equations for the pond element were developed from regression analyses that fit relationships to output from a more complex model which was previously validated with field data (Laflen and Johnson 1976, Laflen et al. 1978).

Trapping of Sediment

The fraction of particles of a specific size and density that passes through the impoundment is:

$$f_{pi} = A_1 \exp(B_1 d_{eqi}) \quad (91)$$

where f_{pi} = fraction passing through pond for particle type i , A_1 and B_1 = coefficients given below, and d_{eqi} = the equivalent sand diameter (in microns) of particle type i . The particle types in the model represent classes rather than specific particles. Therefore, equation 91 was integrated over the class range and divided by the class width to obtain average for the class as:

$$F_{pi} = A_1 [\exp(B_1 d_u) - \exp(B_1 d_l)] / (B_1 \Delta d) \quad (92)$$

where F_{pi} = fraction passed for particle class i . The equivalent sand diameters are arranged in ascending order, and d_u is the d_{eqi} for the class and d_l is the next smallest d_{eqi} . The diameters d_u and d_l are not centered around d_{eqi} because d_{eqi} is assumed to represent the maximum diameter in the class. Values of F_{pi} are limited to a maximum of 1.0. The coefficients A_1 and B_1 are given by:

$$A_1 = 1.136 \exp(Z_s) \quad (93)$$

$$B_1 = -0.152 \exp(Y_s) \quad (94)$$

with Z_s and Y_s in turn given by:

$$\begin{aligned} Z_s = & (-6.68 \times 10^{-6}) f_a (0.3048)^{B'-2} - 0.0903B' \\ & + (1.19 \times 10^{-4}) C_{or} - (1.21 \times 10^{-4}) V_{ro} - 0.0185I_p \end{aligned} \quad (95)$$

$$\begin{aligned} Y_s = & (3.28 \times 10^{-5}) f_a (0.3048)^{B'-2} + 0.123B' - (2.4 \times 10^{-4}) C_{or} \\ & + (2.86 \times 10^{-4}) V_{ro} - 0.0108I_p \end{aligned} \quad (96)$$

where f_a and B' = coefficient and exponent in a power equation relating surface area to depth $S_a = f_a y_p^{B'}$, y_p = depth in pond (m), S_a = surface area (m^2), V = volume of runoff (m^3), and I_p = infiltration rate in the pond (mm/h). The coefficient C_{or} is related to the orifice in the pipe outlet by:

$$C_{or} = 0.15 d_{or}^2 \quad (97)$$

where d_{or} = diameter of the orifice (mm). Also, the coefficient C_{or} is related to discharge and the depth above the outlet point by:

$$C_{or} = (7.02 \times 10^4) Q_p / y_d^{1/2} \quad (98)$$

where Q_p = discharge (m^3/s) and y_p = depth (m).

Runoff Reduction

All of the water which enters the pond will not leave. The volume leaving is estimated by:

$$V_{out} = 0.95 V_{in} \exp(Z_r) \quad (99)$$

where V_{out} = volume of runoff discharged, V_{in} = volume of runoff reaching the pond, and Z_r is given by:

$$Z_r = (9.29 \times 10^{-6}) f_a (0.3048)^{B'-2} + 0.0282' + (1.25 \times 10^{-4}) C_{or} - (1.09 \times 10^{-4}) V_{ro} - 0.0304 I_p \quad (100)$$

$$\text{If } I_p = 0.0, V_{out} = V_{in} \quad (101)$$

$$\text{If } V_{out} > V_{in}, V_{out} = V_{in} \quad (102)$$

are additional constraints on V_{out} from equation 99.

VALIDATION OF THE MODEL

Comparison with Other Models

The validity of the model can be partially assessed by comparing it with other models that might be used in this application. The erosion relationships in the overland flow element gave good results for a watershed at Treynor, Iowa. Estimates were considerably better than those from the USLE using storm EI (Foster et al. 1977) and better than those obtained from a procedure using runoff volume and peak discharge alone as an erosivity factor in the USLE (Onstad et al. 1976). Both rainfall and runoff appear to be important for estimating detachment on overland flow areas. More comprehensive models like ARM (Donigian and Crawford 1976) or ANSWERS (Beasley et al. 1977) use modifications of the USLE or require data for calibration or both. This model preserves the USLE form when erosion is simulated over a range of storms and slope steepnesses and lengths. On long-term simulation for uniform slopes, the model produces results comparable with those of the USLE. Information to select overland flow erosion parameters is as readily available for this model as it is for the USLE.

Comparison of Output from Model with Observed Data

The validity of the model has been partially assessed by comparing output from the model with observed measured sediment yield from concave field plots under simulated rainfall, single terrace watersheds, small watersheds with impoundment terraces, and a small watershed with conservation tillage. The simulations were made using measured rainfall and runoff values. Parameter values were selected from Foster et al. (1980) without calibration, except as noted.

Concave Plots

Three concave plots 10.7 m long were carefully shaped in a soil where soil properties were uniform within the depth of shaping. Slope along the plots continuously decreased from 18% at the upper end to 0% at the lower end. Simulated rainfall at 64 mm/h was used to detach and transport sediment (Neibling and Foster 1980). The measured particle distribution of the sediment was used as input to the model. The soil erodibility factor and Manning's n were adjusted in the model to give observed soil loss entering the deposition area at the lower end of the plots. The estimated sediment yield for the 8.8 m plot was 3.9 g/m/s compared with 2.5 g/m/s observed. For the 10.7 m plot, the estimated and observed values were 1.7 and 1.4 g/m/s/respectively.

Single Terrace Watersheds

Soil loss was simulated for 8 years of data from small, single terrace watersheds at Guthrie, Oklahoma (Daniel et al. 1943). The simulations were made without calibration. Table 4 gives computed and measured results.

Impoundment Terraces

Soil loss was simulated for selected storms representing a range of rainfall and runoff characteristics for three locations in Iowa: Eldora,

Charles City, and Guthrie Center, from an impoundment terrace study (Lafren et al. 1972). The model was run without calibration. The results are given in Table 5.

Table 4. Comparison of simulated sediment yield from single terrace watersheds with measured values.

Terrace	Grade	Sediment yield	
		Simulated	Observed
		(kg/m ²)	(kg/m ²)
2B	Variable, 0.0033 at outlet to 0.0 at upper end	6.4	12.2
3B	Variable, 0.005 at outlet to 0.0 at upper end	11.9	13.8
3C	Constant, 0.005	10.6	12.1
5C	Constant, 0.0017	4.6	4.8

Source: Measured data from Daniel et al. (1943).

Table 5. Summary of observed and simulated sediment yield from impoundment terraces in Iowa.

Watershed	Area (ha)	Julian date	Observed	Computed
			sediment yield (kg)	sediment yield (kg)
Charles City	1.9	70147	542	24
		70152	33	6
		70244	2	72
		70323	26	2
		71151	127	133
		71157	95	72
Eldora	0.73	68198	128	68
		68220	26	25
		69187	479	251
		69232	56	103
		71163	152	63
Guthrie Center	0.57	69207	116	124
		69249	10	40
		70144	55	29
		70162	90	56
		70167	10	13
		70229	5	24

Small Watershed

Simulations were run without calibration for approximately 2 1/2 years of data from the P2 watershed at Watkinsville, Georgia in a conservation tillage system for corn (Smith et al. 1978). Deposition in the backwater from the flume at the watershed outlet was modeled. Deposition measured in the flume backwater was about equal to the measured sediment yield on a similar nearby watershed (Langdale et al. 1979). The computed total sediment yield for the period of record was 1.47 kg/m^2 , while the measured value was 1.85 kg/m^2 .

Overland Flow Sediment Transport

The overland flow sediment transport estimates may be in error by a factor of two as indicated in Table 3. However, the sediment transport equations used by other models have not been tested against field data where deposition was known with certainty to be limiting sediment load. Overland flow conditions are outside the range of most sediment transport equations developed for streamflow, and consequently, many give results greatly in error for overland flow (Neibling and Foster 1980). Given the present state-of-the-art, we believe that the transport relationship used in this model is as adequate as any available.

Channel Erosion

The channel erosion relationships are the ones most likely to be in error even though they fit data from a rill erosion study very well (Lane and Foster 1980). Data from the rills (200 mm wide) may not scale up to channel size (i.e., 2 m wide). However, computed final channel widths agreed well with observed widths for a wide range of streams. While the channel erosion rate for a single storm may be in error, the upper limit for annual channel erosion should be reasonable for soils having a nonerrodible layer beneath the soil surface.

Proven parameter values for the channel soil erodibility and critical shear stress are not available. Our model considers the decay in erosion with time due to previous erosion; most models with the exception of Bruce et al. (1975) do not. This component of our model may require calibration.

Backwater

Most erosion models as applied to fields use a kinematic runoff simulation model to generate values for hydraulic variables. That is, friction slope is set equal to the channel slope. This does not allow modeling of deposition in a backwater area at the field outlet. Such deposition occurs often however, and is important in estimating chemical yields associated with enrichment of fine sediment during deposition. The solutions to the spatially varied flow equations described earlier account for these outlet controls and thus can be used to simulate sediment deposition.

SIMULATION COSTS

Comprehensive models that simulate erosion over space and over time through a runoff event are potentially more powerful than our model.

However, ours can still analyze very detailed downslope spatial variability (soil, slope, cover, etc.). The expected slight improvement in estimates with a more comprehensive model may not offset the additional costs for computing, and moreover, many of the comprehensive models use lumped parameters that prevent them from considering slope shape and buffer strips, for example, which can be analyzed with our model.

While computer costs vary from site to site and change often, rough estimates are, nonetheless, important for qualitative comparisons. Using the CDC 6500 Computer at Purdue University (mention of a specific product name does not imply endorsement), simulation costs for our model were on the order of \$0.10 per storm event. Therefore, the erosion/sediment yield model can simulate individual storm events for a cost of about \$1 to \$3 per year. Although the model is quite comprehensive, the programming is efficient, and simulation costs are not prohibitive.

SUMMARY AND CONCLUSIONS

An erosion/sediment yield model for field-sized areas was developed for use on a storm-by-storm basis. The overall objective was to develop a model, incorporating fundamental erosion/sediment transport relationships, to evaluate best management practices. Although the procedure does not consider changes in parameter values within individual storms, it does allow these parameters to change from storm-to-storm throughout the season. Moreover, parameters of the model allow for distribution of field characteristics along an overland flow slope and along waterways. Many of the model parameters are selected using tested methods developed for the well known Universal Soil Loss Equation. For this reason, we feel that the model has immediate applications without the need for extensive calibration.

Limited testing has shown that the procedures developed here give better estimates than the USLE and modified USLE procedures. Specific components of the model were tested using experimental data from overland flow, erodible channel, and impoundment studies. Initial results suggested that the model produces reasonable estimates of erosion and sediment yield from field sized areas and is a powerful tool for analyzing the influence of alternate management practices.

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APPENDIX A: LIST OF SYMBOLS

- A_1 - Coefficient in equation for deposition in an impoundment,
- B' - Exponent in surface area-depth relationship for an impoundment,
- B_1 - Exponent in equation for deposition in an impoundment,
- C - Soil loss ratio of USLE cover-management factor,
- C' - Constant of integration in equation for deposition by flow,
- CLLAG - Clay content of large aggregates, fraction of total sediment,
- CLSAG - Clay content of small aggregates, fraction of total sediment,
- C_{or} - Orifice coefficient for drainage from impoundment,
- C_1 - Coefficient in gradually varied flow equations,
- C_2 - Coefficient in gradually varied flow equations,
- C_3 - Coefficient in gradually varied flow equations,
- d - Diameter of a sediment particle
- d_{ch} - Rate that channel erodes downward, (depth/time),
- d_{eqi} - Equivalent sand diameter of a sediment particle,
- d_1 - Equivalent sand diameter of lower end of a sediment particle class,
- d_{or} - Diameter of orifice in an impoundment drain,
- d_u - Equivalent sand diameter of upper end of a sediment particle class,
- D - Rate of deposition by flow (mass/area/time),
- D_F - Rate of detachment or deposition by flow (mass/area/time),
- D_{Fc} - Rate of sediment detachment by flow in channels, (mass/area/time),

- D_{FL} - Rate of detachment or deposition by flow at lower end of a segment, (mass/area/time),
- D_{Fr} - Rate of sediment detachment by rill erosion, (mass/area/time),
- D_{Fu} - Rate of detachment or deposition by flow at upper end of a segment, (mass/area/time),
- D_L - Rate of lateral inflow of sediment, (mass/area/time),
- D_{Li} - Rate of sediment from interrill areas, (mass/area/time),
- D_{LL} - Rate of lateral inflow of sediment at lower end of a segment, (mass/area/time),
- D_{Lu} - Rate of lateral inflow of sediment at upper end of a segment, (mass/area/time),
- D_u - Rate of deposition by flow at upper end of a segment, (mass/area/time),
- DLAG - Diameter of large aggregate sediment particles,
- DPCL - Diameter of primary clay sediment particles,
- DPSA - Diameter of primary sand sediment particles,
- DPSI - Diameter of primary silt sediment particles,
- DSAG - Diameter of small aggregate sediment particles,
- e - Energy per unit of rainfall, (force distance/area depth),
- E_{ch} - Erosion rate per unit length of channel, (mass/time/length of channel)
- E_{xc} - Excess nondimensional transport capacity,
- EI - Rainfall erosivity, total storm energy times maximum 30 minute intensity,
- f_a - Coefficient in surface area-depth relationship for impoundment,
- f_{pi} - Fraction of a particular particle size deposited in an impoundment,
- $f(x_b)$ - Shear stress distribution around a channel,
- F_{pi} - Fraction of a particular particle class deposited in an impoundment,
- g - Acceleration due to gravity,
- $g(x_c)$ - Conveyance function for flow in a channel eroded to an equilibrium,
- i - Rainfall intensity,
- I - Maximum 30 minute intensity,
- I_p - Infiltration rate through boundary of an impoundment,
- k - Coefficient in transport capacity fraction summation, 0 or 1,
- K - Soil erodibility factor for the USLE,
- K_{ch} - Soil erodibility factor for the USLE,
- l - Coefficient in shear stress sum, 0 or 1,
- L_{eff} - Effective length of channel,

- LAG - Fraction of sediment made up of large aggregates,
- m - Slope length exponent for rill erosion,
- n - Manning's n,
- n_{bch} - Manning's n for a bare channel,
- n_{bov} - Manning's n for a bare overland flow surface,
- n_{cov} - Manning's n for a covered overland flow surface,
- n_s - Number of particle classes in sediment mixture,
- n_t - Total Manning's n,
- N - Number of sediment particles in a uniform sediment,
- N_e - Number of sediment particles in a given class,
- ORCL - Fraction of original soil made up of primary clay,
- ORSA - Fraction of original soil made up of primary sand,
- ORSI - Fraction of original soil made up of primary silt,
- P - Contouring component of USLE supporting practices factor,
- P_e - Effective nondimensional transport capacity for a particle class in a mixture,
- P_{ireq} - Required nondimensional transport capacity for a particle class in a mixture,
- P_s - Nondimensional transport capacity,
- PCL - Fraction of sediment made up of primary clay,
- PSA - Fraction of sediment made up of primary sand,
- PSI - Fraction of sediment made up of primary silt,
- q_s - Rate of sediment discharge per unit width, (mass/time/width),
- q_{su} - Rate of sediment discharge at upper end of a segment, (mass/time/width),
- q_w - Rate of runoff discharge per unit width (volume/time/width),
- Q - Discharge rate, (volume/time),
- Q_e - Discharge rate at end of channel, (volume/time),
- Q_p - Peak discharge rate, (volume/time),
- R - Hydraulic radius,
- R_* - Ratio of hydraulic radius to wetted perimeter,
- R_{soil} - Hydraulic radius due to soil,
- s - Sine of angle of slope,
- s_b - Sine of slope angle of land profile at upper end,
- s_e - Sine of slope angle of land profile at lower end,
- s_m - Sine of slope angle of land profile at miduniform section,
- s_* - Normalized slope along a channel,
- S_a - Surface area in an impoundment,

- S_f - Friction slope for flow hydraulics in a channel,
- S_g - Specific gravity of a sediment particle,
- SAG - Fraction of sediment made up of small aggregates,
- SALAG - Fraction of sand in large aggregates, fraction of total sediment,
- SASAG - Fraction of sand in small aggregates, fraction of total sediment,
- SDLT - Sum of excess shear stress,
- SILAG - Fraction of silt in large aggregates, fraction of total sediment,
- SISAG - Fraction of silt in small aggregates, fraction of total sediment,
- SMUS - Fraction of total transport capacity used by sediment load,
- SPT - Fraction of total transport capacity used by particles having excess transport capacity,
- SSF - A component of the normalized friction slope for channels,
- SUMPRI- Sum of fractions for primary clay, silt, and sand in sediment,
- t - Time,
- t_* - Normalized time for channel erosion,
- t_i - Initial time,
- T - Summation of normalized excess shear stress for sediment transport,
- T_c - Transport capacity,
- T_{cu} - Transport capacity at upper end of a segment,
- V - Flow velocity,
- V_* - Shear velocity,
- V_{in} - Runoff volume into impoundment,
- V_{out} - Runoff volume out of an impoundment,
- V_{ro} - Runoff volume,
- V_R - Rainfall volume per unit area, (depth),
- V_s - Fall velocity,
- V_u - Runoff volume per unit area, (depth),
- W - Channel width,
- W_* - Normalized channel width,
- W_{ac} - Width of an eroding channel at equilibrium,
- W_f - Final eroded channel width,
- W_i - Initial channel width,
- W_p - Wetted perimeter,
- W_s - Transport capacity before adjustment for a sediment mixture,
- x - Distance,
- x_b - Normalized distance around wetted perimeter to nonerodible boundary,
- x_c - Normalized distance around wetted perimeter to where $\tau = \tau_{cr}$,

- x_{cf} - Normalized distance around wetted perimeter where $\tau = \tau_{cr}$ at nonerrodible boundary,
- x_{db} - Location where deposition begins,
- x_{de} - Location where deposition ends,
- x_u - Location of upper end of segment,
- x_* - Normalized distance along channel,
- X_1, X_2, X_3, X_4 - Points along a land profile,
- y - Flow depth,
- y_* - Normalized flow depth,
- y_e - Flow depth at end of channel,
- y_p - Depth in impoundment,
- Y - Normalized shear stress,
- Y_{cr} - Ordinate from Shields diagram,
- Y_s - Exponent in deposition equation for an impoundment,
- z - Channel sideslope,
- Z_r - Exponent in equation for runoff reduction by an impoundment,
- Z_s - Exponent in equation for deposition in an impoundment,
- α - Reaction coefficient for deposition by flow,
- β - Energy coefficient in spatially varied flow equations,
- δ - Dimensionless excess shear stress,
- Δd - Width of a particle class,
- Δx - Segment length,
- γ - Weight density of water,
- ϵ - Coefficient in equation for α ,
- ρ_{soil} - Mass density of soil in place,
- ρ_w - Mass density of water,
- σ - Factor in Yalin sediment transport equation,
- σ_p - Peak runoff rate, (depth/time),
- τ - Shear stress,
- $\bar{\tau}$ - Average shear stress around wetted parimeter,
- τ_b - Shear stress in a channel at a nonerrodible boundary,
- τ_{cov} - Shear stress due to soil cover,
- τ_{cr} - Critical shear stress,
- τ_m - Normalized maximum shear stress in a channel,
- τ_{max} - Maximum shear stress in a channel,
- τ_{soil} - Shear stress acting on soil,
- ω_* - Normalized width for channel after it erodes to a nonerrodible layer.