



Applications of Catastrophe Theory to Water Quality Modelling -- A Review

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APPLICATIONS OF CATASTROPHE THEORY
TO WATER QUALITY MODELLING
A Review

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PREFACE

Attention has recently been drawn to the potential for applications of catastrophe theory in the field of water quality modelling. The first author, being one of the first to work in this field, and the second author, representing the active interest of IIASA's Resources and Environment Area in new methods of systems analysis, have joined together to prepare a critical appraisal of the applications published so far. The review was completed during a short visit of the first author to IIASA within the framework of REN Area's Task 2 on Models for Environmental Control and Management. This Collaborative Paper reports on that brief review.

The reader who is not familiar with catastrophe theory will find it helpful to read the excellent tutorial paper on 'Catastrophe Theory Applied to Ecological Systems' by Dixon D. Jones, published in *Simulation*, July 1977, pp. 1-15, prior to reading this CP. Since the authors are still strongly interested in the topic discussed, they would be grateful for any kind of feedback from the reader.

APPLICATIONS OF CATASTROPHE THEORY TO WATER QUALITY MODELLING

I. INTRODUCTION

The purpose of this paper is to review published applications of catastrophe theory to water quality modelling (Duckstein, et al, 1979, Van Nguyen and Wood, 1979, and Dubois, 1979) and to locate these applications within the framework of water quality modelling and applied catastrophe theory in general.

Most water quality models have been formulated for prediction and control of particular water bodies, whether lake, stream, or estuary, and can therefore be categorized as engineering models. Application has been largely limited to lakes or estuaries where stratification creates a distinct epilimnion and hypolimnion and in which the epilimnion is well mixed (see, e.g. Di Toro, et al, 1971, 1977, Chen and Orlob, 1972). Michaelis-Menten kinetics is assumed for phytoplankton nutrient uptake and the changes in phytoplankton biomass tend to be smooth over a period of several weeks. Nutrient concentrations are low and zooplankton populations high, so that predator-prey dynamics and limited nutrient

control the phytoplankton population size. Species distinctions are either ignored or minimized.

The phenomenon of sudden algal blooms and dieoffs in response to increased nutrient loading has been known for some time (Thomas, 1960); however, only recently has attention been drawn back to the matter. Water bodies experiencing these sudden fluctuations tend to be small, with restricted inflow and outflow. Species composition is of paramount importance, since lakes with Cyanophyceae (blue-green algae) as the dominant species tend toward bloom-dieoff dynamics more strongly than those in which the Chlorophyceae (green algae) dominate. In addition, zooplankton population size is usually very low. Since many of these ponds are used for raising fish (Barica, 1973, Boyd, et al, 1975) nutrient loading is usually high in order to assure high phytoplankton populations upon which the fish feed. The importance of predicting water quality in such lakes has been the goal of applying catastrophe theory to the problem (Duckstein, et al, 1979, and Van Nguyen and Wood, 1979).

The third paper (Dubois, 1979) does not relate to pond algal dynamics. In that publication an attempt was made to explain the observed hysteresis in the relation of (river) oxygen content to temperature over the annual cycle in terms of catastrophe theory.

Since two of the three papers deal with bloom-dieoff, the characteristics of bloom-dieoff dynamics will be reviewed in somewhat more detail in the next section. Section III will discuss applied catastrophe theory and briefly describe how the water quality models described in here utilize catastrophe theory. In Section IV, the models themselves will be critically examined,

and discussed, while Section V will present a discussion of some questions not touched by the existing models but which are crucial to modelling water body ecology. Finally, Section VI will summarize the report and present conclusions.

II. BLOOM-DIEOFF DYNAMICS

Small lakes experiencing large algal blooms and dieoffs have been studied by a number of different groups.

Barica (1973, 1975, 1977) reported on a series of pothole lakes located in the southwestern Manitoba prairie. The lakes could be separated into two groups, based on their phytoplankton species types and dynamics--those which exhibited the typical bloom-dieoff sequence and were dominated by blue-green algae (Aphanizomenon flos aquae, a nitrogen fixer) and those in which the fluctuations in phytoplankton biomass were more regular and in which green algae dominated. The bloom-dieoff sequence consisted of a bloom period, in which algal biomass was high and nutrient concentration low, followed by a dieback period, in which the algal population decreased suddenly, causing the water to become anoxic and raising nutrient concentrations (particularly nitrate and ammonia) to almost toxic levels. A correlation was noted by Barica (1977) between drops in air and water temperature and the dieoffs.

Boyd et al, (1975) described the development of a phytoplankton dieoff in a catfish pond in Alabama. Again, the phytoplankton population was dominated by a blue-green nitrogen fixer (Anabaena variabilis) before the dieoff. No data is given on soluble orthophosphate concentration, although the authors mention that no orthophosphate was detectable during the anoxic stage, in

contrast to the Manitoba lakes. The authors attributed the lack of detectable orthophosphate to binding of orthophosphate to ferric ions, created by the addition of potassium permanganate to the pond. Potassium permanganate was added to help oxidize decaying algae. After the dieoff, the algal species type changed to green algae and the population remained stable. Parks, et al (1975) extended the above work to describe phytoplankton in a catfish pond over an entire season. Addition of orthophosphate fertilizer to the water at periodic intervals caused sudden blooms of algae. After a period of a week or less, the algal biomass died off quite suddenly, leaving the water in an anoxic condition. As in the previous case, Anabaena was the dominant species and no changes in orthophosphate concentration were observed, except after the first two fertilizations of the season and after the last dieoff. The authors attributed the dieoffs to floating of Anabaena to the surface, which exposes the algal cells to higher light intensities, causing death.

Finally, Schindler and co-workers (1971, 1974, 1977, 1978) describe a series of comprehensive experiments involving fertilization of lakes in the Canadian Arctic. Fertilization proceeded over a period of several years and was designed to measure the effect of nutrient addition (nitrates and ammonia, phosphates, and carbon-based nutrients) on lake trophic state. Schindler (1977) noted that the type of dominant algal species changed as the ratio of nitrogen to phosphate (N/P ratio) in the fertilizer was changed. Lakes which received additions of fertilizer having an N/P ratio of 15 had green algae as the dominant species, while those receiving with N/P ratio of 5 had blue-green algae dominant.

III. APPLIED CATASTROPHE THEORY

In order to put applications of catastrophe theory to water quality modelling into perspective, applied catastrophe theory will be briefly reviewed. For a more detailed presentation, see Zeeman (1977) and Poston and Stewart (1978). Applied catastrophe theory can be roughly broken into two parts: catastrophe modelling and catastrophe analysis. Fararo (1978) discusses both in some detail.

Catastrophe modelling is best exemplified by the work of Christopher Zeeman (1977), in which a particular dynamical system is used as a model for a real world system. The dynamical system consists of a differential equation model, with equations specified for one state variable (usually called x) and between one and 5 control variables (called, a , b , etc.). Most applications use 2 control variables. Note that, in the usual dynamical system terminology (Hirsch and Smale, 1974), all variables are state variables; but in catastrophe modelling, it is assumed that the system is in equilibrium (i.e., that transient changes have died out). Therefore, changes in the control variables are considerably slower than changes in the state variable, and the state variable becomes a solution to the equation:

$$f(x, \bar{c}) = \frac{dx}{dt} \cong 0 \quad ,$$

where x is the state variable and c is the control variable vector, and f is one of the elementary catastrophes. The result is that trajectories of the system follow flow streamlines on a manifold in the state space, the most familiar example of which is the three dimensional cusp (Zeeman, 1977).

Strictly speaking, catastrophe modelling is only applicable to gradient or gradient-like systems, i.e., those for which a potential function or Liapunov function can be constructed, in which case, the Liapunov or potential function will be:

$$\int f(x, \bar{c}) dx = V(x, \bar{c}) \quad . \quad (1)$$

However, many modelling attempts have assumed that a state function of catastrophe form applies and have left the exact nature of the differential equations unspecified. Such implicit catastrophe models have encountered heavy criticism (Sussman and Zahler, 1978), even those which have tried to fit data to a cusp surface (Zeeman, et al, 1976). An explicit equation system, in which all the differential equations are specified and which uses a catastrophe for the time derivative of the state variable, was used for modelling the nerve impulse, (Zeeman, 1977). Sussman and Zahler (1978) criticized the nerve impulse model, claiming that the predictions did not match the data very well; however, a more careful study (Stewart and Woodcock, 1978) has revealed that, although the model cannot predict all the features of the nerve impulse at the same time, selected parts could be used. At present, no application of catastrophe modelling has been published in the water quality field, although the model of Duckstein, et al, (1979) is similar, in some respects, to a catastrophe model.

Some of the most successful applications of catastrophe theory are in the physical sciences (e.g. analysis of light caustics and buckling of engineering structures) and result from application of catastrophe analysis (see Poston and Stewart, 1978 for a review). In its restricted form catastrophe analysis

consists of using a set of mathematical rules to reduce a function, parameterized by between one and five exogenous variables, through diffeomorphic variable transforms* to a canonical form. The function can then be split into two parts--a "Morse" part, in which the exogenous variables do not appear, and a non-Morse part, which includes the exogenous variables and one or two endogenous variables as well. Through further reduction, the non-Morse part can be put into the form of one of the elementary catastrophes, and information about the behavior of the system both qualitative and quantitative, modelled by the function can be obtained. Variations on this procedure are possible and Fararo (1978) has used the term catastrophe analysis to include examination of the behavior of the equilibrium set of any dynamical system. Thom (1975) calls this non-elementary or generalized catastrophe theory, and includes under this heading such phenomena as the Hopf bifurcation and limit cycles (Marsden and McCracken, 1977). The water quality models reviewed in here use a modified form of catastrophe analysis.

IV. THE MODELS

The two models of bloom-dieoff dynamics, namely Duckstein, et al (1979), and Van Nguyen and Wood (1979) were developed for two different dynamical situations and represent two different approaches to the problem. Both models are preliminary developments, and could be improved in a number of ways.

Van Nguyen and Wood (1979) used physiological equations drawn from a number of sources to develop an integrated equation of state for the phytoplankton population in a pond.

*A diffeomorphic transform is one in which the Jacobian matrix (matrix of partial derivatives) exists. For a transform to be a diffeomorphism, there must be as many variables in the transform as in the original.

The basis of the model was the Monod relationship

$$\dot{P} = (G - D)P \quad (2)$$

where

P = phytoplankton biomass,

G = growth rate coefficient,

D = death rate coefficient,

$\dot{P} = dP/dt$.

Essential is the assumption that nutrient uptake is a two-step process, i.e. first transport from the bulk to the cell, governed by a diffusion resistance coefficient, followed by uptake, modelled with a nutrient intake resistance coefficient. By incorporating the effect of nutrient concentration, light, temperature, and algal respiration, and integrating out transient effects, the equation of state for phytoplankton was derived as

$$P^3 - \beta \left[\alpha I (1 + r_d) + N_e - Rr_d \right] P^2 \quad (3)$$
$$+ \gamma \left[\alpha I (N_e - Rr_x) - RN_e \right] P + \rho \delta = 0$$

where

$$\beta = 3/2 r_d (G - D) ,$$

$$\gamma = 3/r_d (G - D)^2 ,$$

$$\rho = 3/r_d (G - D)^3 ,$$

δ = a constant of integration .

P may be viewed here as the biomass at equilibrium (i.e. at $\dot{P} = 0$), although the authors claim validity at any predefined rate. According to the authors, δ will be a polynomial function of water temperature.

Further:

- I = light influx density,
- r_d = diffusion resistance,
- α = photochemical efficiency,
- r_x = nutrient intake resistance,
- R = respiration rate,
- N_e = nutrient concentration.

The authors then suggest that the potential for bloom-dieoff dynamics follows when the polynomial coefficient of P^2 is zero, although no other reason is given for this procedure. This results in the following simplified equation of state:

$$V(P, g, h) = P^3 + g(N_e, R)P + h(T_a, I) = 0 \quad , \quad (4)$$

where g and h are functions of N_e , R , T_a (air temperature) and I . The authors noted that this equation has the form of the cusp catastrophe manifold.

The data of Barica (1977) for a shallow pond are then plotted on a cusp manifold and graphs are presented with phytoplankton biomass as a function of air temperature and ammonia nitrogen. The trajectory of this lake in the control space of ammonia nitrogen - maximum air temperature is also graphed from the

bifurcation set of the cusp equation. The authors then develop criteria for forecasting a dieoff and predicting the size of the dieoff. The size of the dieoff will be determined by:

$$\Delta P = (-3g)^{0.5} , \tag{5}$$

and the dieoff can be forecast by:

$$h_{\text{crit}}(T_a, I) = -2/27 \Delta P_{\text{max}}^3 , \tag{6}$$

according to the authors. Here h_{crit} is the critical value of the h function, which, if exceeded, will lead to a dieoff. Thus, the authors conclude that dieoffs are triggered by cool, windy weather conditions occurring for particular physiological states of the pond. The authors did not attempt to apply the predicting and forecasting equations to the data, however.

There are a number of areas in which the model could see some improvement.

- 1) In a technical sense, the functions g and h are not diffeomorphic transforms. While the domain of the transform is R^4 ($N_e - R - T_a - I$ space), the range is R^2 (g, h space) and hence the Jacobian determinant does not exist. Although some of the variables may be "dummies" in the sense that they do not affect the catastrophe behavior of the system, the important variables can not be examined without a closer examination of the variable transform. Furthermore, by setting the P^2 coefficient to zero, the authors are essentially making the variable transform degenerate (Posten and Stewart, 1978); that is, not allowing the variables to take on all possible values. A complete

coordinate transform without destroying the topological structure of the map would, however, be possible.

- 2) Calibration and validation might be difficult, since many variables are included which could prove difficult or impossible to measure. In particular, the diffusion and nutrient intake resistance might not be constant over the entire range of nutrient concentrations and temperature conditions in the environment.
- 3) Although variations in such biological parameters as nutrient intake resistance, r_x , growth and death rate, G and D , might be enough to account for the observed differences between dynamics in ponds dominated by blue-green algae and those dominated by green algae, the lack of consideration for species differences in the model could lead to problems. Since blue-green algae are nitrogen fixers, introduction of nitrogen from the atmosphere by the algae may invalidate the use of nitrogen as a control variable and mass balance. In addition, the tendency of blue-green algae to float to the surface during calm periods in the absence of vertical mixing and return to depth during windy periods, calls the model's conclusions into question. The chlorophyll a in Barica data was not measured for various depths, and therefore it is not possible to differentiate between a dieback and removal of chlorophyll a by vertical mixing, during a cool windy period.
- 4) Finally, the validity of a two step nutrient uptake, developed from physiological experiments on tomato plants, may be questioned if applied to phytoplankton growth. In

contrast to the situation in the root zone in the soil, in surface water turbulence is very instrumental in transport of nutrients from the bulk of the water to the cells.

Hence, transport diffusion limitation is unlikely in phytoplankton systems.

Duckstein, et al. (1979) developed a differential equation model from ecological considerations based on the dieoffs reported by Parks, et al. (1975). Through modification of the logistic equation (Poole, 1974) to include the effect of algal anabiosis, floating of Anabaena cells (b , a control variable) to the water surface, and varying orthophosphate concentration (a , a control variable), a differential equation for the total algal biomass (x , the state variable) was derived as

$$dx/dt = -(c_1x^3 - c_2ax + c_3b) \quad , \quad (7)$$

which has the form of the cusp catastrophe equation.

In order to model growth of the Anabaena population, the authors stated that the logistic equation could again be modified to include the effect of varying nutrient concentration in the growth term and competition from the total algal population in the dieoff term. The resulting equation for Anabaena was given as

$$db/dt = c_5ab - c_6bx \quad . \quad (8)$$

An empirical relationship was used by the authors for the orthophosphate concentration equation. The authors assumed that orthophosphate concentration would decrease in proportion to the concentration of algae, until a certain limiting (equilibrium)

concentration was reached. The equation was:

$$da/dt = -c_4 x(a - a_0) \quad , \quad (9)$$

where a_0 , the equilibrium concentration, was 0.01 mg P/l in the pond.

The authors then calibrated the system against data from Parks, et al. (1975), applying it to the top three feet of the pond, since a complete data set was only available for this region.

Graphs of the simulation were plotted against the pond data, and the authors suggested that the fit was reasonable for Anabaena and total algae, except for a dip in the data during the middle of the dieoff, which the authors attribute to a sudden increase in vertical mixing. On the other hand, the plot of the simulated orthophosphate concentration against the pond data was not very good and, instead, the authors used some laboratory data from Di Toro, et al. (1977) to calibrate the orthophosphate fit.

In order to examine critical points in the system, the authors equated Equation 7 with the negative partial derivation of a potential function with respect to x , and by integrating Equation 7, come to the conclusion that the potential function is

$$V(x, a, b) = \frac{c_1 x^4}{4} - \frac{c_2 a x^2}{2} + c_3 b x + K \quad , \quad (10)$$

which can be reduced by a co-ordinate change to the canonical equation for the cusp catastrophe

$$V(z, P_1, P_2) = \frac{z^4}{4} - \frac{P_1 z^2}{2} + \frac{P_2 z}{1} \quad , \quad (11)$$

where Z is a linear function of x , P_1 a linear function of a , and P_2 a linear function of b .

The implications of the catastrophe format for the system are discussed, namely, that once the pond is displaced from the equilibrium point by fertilization, it goes through a maximum point on the cusp manifold and returns to the equilibrium; hence, the bloom-dieoff sequence.

Improvements could be made upon the model in several areas.

- 1) As pointed out by Poston (1979), the equations for x , a , and b are all about the same "speed, that is, the controls do not vary slowly with respect to the state. The equation for x is slightly faster than the other two. For typical values of the constants however, the difference is not enough to make the Zeeman model applicable. Hence, if a potential function exists, it will be vector valued and elementary catastrophe theory is not appropriate, unless it can be proved that Equation 11 is a Liapunov function (Hirsch and Smale, 1974). The zeros of Equation 7 can still be used to determine where maxima and minima for the total algal population (x) occur, but the rigor of the model is reduced. In this respect, the model suffers from the same drawback as Zeeman's nerve impulse model (Stewart and Woodcock, 1978).
- 2) Some of the model predictions do not match very well with the data. Although the ambiguity surrounding the orthophosphate prediction could be the result of the way in which the pond data was taken, as asserted by

the authors, the equilibrium value of total algae and Anabaena in the pond is different from what is predicted.

If the system is solved for equilibrium points, the equilibrium at which all variables are non-zero is given by

$$a_e = a_o = 0.01$$

$$x_e = \frac{c_5 a_o}{c_6}$$

$$b_e = \frac{c_2 a_o^2 c_5}{c_3} - \frac{c_1 \left(\frac{c_5 a_o}{c_6} \right)^3}{c_3} .$$

Solving these equations by inserting the numerical values of the constants results in:

$$a_e = 1.0 \times 10^{-2} \quad \mu\text{gP/ml}$$

$$x_e = 7.0 \times 10^{-3} \quad \mu\text{gchl/ml}$$

$$b_e = 8.0 \times 10^{-4} \quad \mu\text{gchl/ml} ,$$

whereas the actual pond values were:

$$a_e = 1.0 \times 10^{-2} \quad \mu\text{gP/ml}$$

$$x_e = 8.75 \times 10^{-4} \quad \mu\text{gchl/ml}$$

$$b_e = 3.5 \times 10^{-4} \quad \mu\text{gchl/ml} ,$$

as calculated from data in Parks, et al. (1975).

- 3) The authors do not include the effect of changing seasonal conditions in the model. Since the pond is dominated by Anabaena only in the spring, some attempt, possibly by including time variable constants, should be made to model the effect of light and temperature conditions. In addition, from the pond data, it is

obvious that the size of the dieoffs increases as the season progresses. Inclusion of time variable constants could also significantly alter the modelling conclusions.

- 4) Finally, as in the van Nguyen and Wood model, no account was taken of the effect of changing the fertilizer N/P ratio on the algal dynamics, although the model did include the effects of blue-green algae.

The authors conclude that the dieoffs are a result of the pond dynamics, which, after the pond is displaced from equilibrium by addition of phosphate fertilizer, causes the bloom-dieoff sequence.

The model describing the oxygen dynamics to examine the effects of temperature and nutrients presented by Dubois (1979) is also based on the logistic growth equation. An analogy is used with phytoplankton growth to represent the photosynthetic oxygen production rate in terms of oxygen concentration X

$$O_2 \text{ production rate} = k_1 X(1-\alpha X) \quad (12)$$

where k_1 is the specific production rate and α a saturation constant. Furthermore a Michaelis-Menten type expression for respiration is assumed. The resulting oxygen equation is

$$\frac{dx}{dt} = a(T) + bx(1-cx) - d \frac{x}{1+x} \quad (13)$$

where the dimensionless oxygen concentration x is measured relative to the Michaelis-Menten coefficient for respiration, a(T) contains the temperature dependant oxygen saturation concentration, and b, c and d depend upon the other parameters of

the model. In the subsequent equilibrium analysis it is assumed that the temperature affects only $a(T)$, which is thus considered as the control variable. Equating dx/dt in equation 13 to zero results in a cubic equation representing a cusp surface, reducing with $a(T)$ as the only control variable to an S-curve in the x - a -plane. The jumps in oxygen concentration with temperature predicted from this analysis were then shown to occur in a dynamic simulation of equation 13, using a slowly varying annual temperature cycle.

Several improvements would be possible in this model:

- 1) The replacement of the phytoplankton biomass by oxygen concentration to express the photosynthetic oxygen production rate (equation 12) is a somewhat arbitrary procedure. Rather, a simultaneous equation for phytoplankton should be used, and the analysis redone for the coupled oxygen-phytoplankton system. Similar arguments apply to the respiration term in the oxygen balance.
- 2) In practice temperature does not only affect the oxygen saturation concentration, but also, and perhaps even stronger, the other parameters in the model. Hence, an analysis of temperature effects should include these dependencies.

V. DISCUSSION

As noted in the previous section, the existing models could be modified in a number of ways to improve their mathematical rigor and predictive accuracy. However, the fundamental conclusions of the two dieoff models were quite different.

Whereas the van Nguyen and Wood model concluded that the dieoffs were a result of changing weather conditions on a pond which was in a susceptible physiological state, the Duckstein, et al. model concluded that the dieoffs would occur for any pond in which blue-green algae occurred, if the pond was overfertilized.

The cusp equation appears in the models in different ways. In the Duckstein, et al. model, the cusp is a result of increasing the degree of the dieoff term in the logistic equation from quadratic to cubic because of algal anabiosis. Whether this step is justified is impossible to tell from present laboratory results on algal growth. In the van Nguyen and Wood model, the cubic term develops after integration to steady state, from a quadratic term in the non-steady state form. The biological basis of the cubic in the van Nguyen and Wood model is a two-step nutrient uptake process, in which the nutrient diffused first into the environmental vicinity of the algae from the larger lake and then into the algal cells from the environment. As pointed out before, the applicability of such a two-stage diffusion process to lake modelling is questionable. In the Dubois model the cubic expression results from the combination of a logistic equation for photosynthetic oxygen production with a Michaelis-Menten type equation for the ecosystem respiration. By relating these terms with oxygen directly, rather than with phytoplankton or BOD concentration, the modeling procedure is open to debate.

Whether or not catastrophe modelling and catastrophe analysis are useful for water quality modelling, depends on how

important transient effects are in the lake ecosystem. The van Nguyen and Wood model assumes that the transient dynamics in the lake are not of particular importance in bloom-dieoff lakes, since the cusp manifold gives only the equilibrium points of the system. Those systems for which elementary catastrophe theory has been most successful have included a variable which is very close to equilibrium, for example, in engineering structures, the position of a beam will move rapidly to equilibrium as the force on the beam is changed slowly (Poston and Stewart, 1978). It would be interesting to see if a standard lake model (for example, the Di Toro model (Di Toro, 1971)) displays catastrophe behavior, given a parameter set equivalent to that in the bloom-dieoff lakes. Catastrophe behavior would be indicated if the phytoplankton biomass moved very rapidly (i.e. had a very large derivative) for small changes in the environmental state variables (light, temperature, etc.). Dixon Jones (Jones, 1975) used catastrophe modelling on a simulation model of the spruce budworm with some degree of success. Alternatively, application of catastrophe analysis to a standard ecosystem model, that is, analytical reduction of a Liapunov function or a model at equilibrium to catastrophe form, with the parameter set reduced to 5 or less, might determine whether elementary catastrophe theory applies.

Inclusion of time variable effects in the Duckstein, et al. model to account for the observed increase in bloom size and disappearance of orthophosphate spikes after fertilization as the season progressed might be useful. An analytical reduction of the "potential" function, to determine whether it

actually is a Liapunov function around the system equilibria would also considerably solidify the catastrophe analysis.

Another approach might be to use implicit catastrophe modelling to fit a state equation of catastrophe form to a data set. Poston and Stewart (1978) reviewed an attempt to validate an ecological form of Zeeman's primary-secondary wave model (Zeeman, 1977) using data from New Zealand. The model is based on a fold catastrophe and describes the invasion of trees onto cleared land and the change of dominant tree species as time progresses. Similar methods might be useful for validating water quality models based on catastrophe theory. Again, the crucial question is how important transient dynamics is for the system. If the system is always in the transient state, then elementary catastrophe theory might not be appropriate.

CONCLUSIONS

The published applications of catastrophe theory to water quality modelling (Duckstein, et al. 1979, Van Nguyen and Wood, 1979, and Dubois, 1979) were reviewed and placed within the framework of applied catastrophe theory and water quality modelling. The behavior of the Duckstein, et al. model was reasonable for two of the three variables, with the trajectory of the orthophosphate variable not reflecting the pond data. The performance of van Nguyen and Wood model could not be assessed because it has not been validated. The Dubois model was able to explain in qualitative terms the observed hysteresis of river oxygen concentration to the annual change in temperature. All three models were found to contain defects, with the biological basis of being debatable. The origin of the cubic term in

the models was discussed and the importance of distinguishing transient dynamics from steady state and the implication for applying catastrophe theory to water quality systems was emphasized. Further analysis of the Duckstein, et al. model, use of catastrophe modelling and catastrophe analysis on a standard model, and attempting to fit a data set to a catastrophe state equation were suggested as directions for further research.

REFERENCES

- Barica, J. "Changes in Water Chemistry Accompanying Summer Fish Kills in Shallow, Eutrophic Lakes of Southwest Manitoba," Proceedings of the Symposium on Lakes of Western Canada, University of Alberta Water Resources Centre, Publication No. 2, pp. 227-240, 1973.
- Barica, J. Geochemistry and Nutrient Regime of Saline Eutrophic Lakes in the Erickson-Elphenstone District of Southwestern Manitoba, Technical Report No.11, Fisheries and Marine Service, Environment Canada, 1975.
- Barica, J. 1977 "Collapses of Aphanizomenon flos aquae blooms resulting in massive fish kills in eutrophic lakes: effects of weather," In: Proceedings 20th SIL Congress, Copenhagen Verlehnungen der Internationale Vereinigung der Limnologie, 20: in press.
- Boyd, C.E. "Biotic Interactions Between Different Species of Algae," Weed Science, 21, pp. 32-37, 1973.
- Chen, C.W., and G.T. Orlob, Ecologic Simulation for Aquatic Environments, Water Resources Engineers Inc., Walnut Creek, California, 1972.
- Di Toro, D.M., P.J. O'Connor and R.V. Thomann, "A Dynamic Model of the Phytoplankton Population in the Sacramento-San Joaquin Delta," Non-Equilibrium Systems in Natural Water Chemistry, Advances in Chemistry Series, 106, pp.171-180, 1971.
- Di Toro, D., R.V. Thomann, P.J. O'Connor and J.L. Mancini, "Estuarine Phytoplankton Biomass Models - Verification and Preliminary Applications," The Sea 6, John Wiley and Sons, New York, pp. 969-1020, 1977.

- Dubois, D.M. "Catastrophe Theory Applied to Water Quality Regulation of Rivers," In: State-of-the-Art in Ecological Modelling, 7, Copenhagen, pp. 751-758, 1979.
- Duckstein, L., J. Casti and J. Kempf "Modelling Phytoplankton Dynamics Using Catastrophe Theory" Water Resources Research, 15, pp. 1189-1194, 1979.
- Farano, T. "An Introduction to Catastrophes" Behavioral Science, 23, pp. 291-317, 1978.
- Hirsch, M.W., and S. Smale. Differential Equations, Dynamic Systems, and Linear Algebra, Academic Press, Inc., New York, 1974.
- Marsden, J.E., and M. McCracken. The Hopf Bifurcation and Its Applications, Springer Verlag, Applied Mathematical Sciences Series, 19, 1976.
- Parks, R.W., E. Scarsbrook and C.E. Boyd. Phytoplankton and Water Quality in a Fertilized Fish Pond, Circular 224, Agricultural Experiment Station, Auburn University, Auburn, Alabama, 16 pp., 1975.
- Poole, R.W. An Introduction to Quantitative Ecology, McGraw-Hill Series in Population Biology, New York, 473 pp., 1974.
- Poston, T. and I. Stewart. Catastrophe Theory and Its Applications, Pitman, London, 408 pp., 1978.
- Poston, T. (1979) Personal Communication.
- Schindler, D.W., H. Kling, R.V. Schmidt, J. Prokopowich, V.E. Frost, R.A. Reid and M. Capel. "Eutrophication of Lake 227 by Addition of Phosphate and Nitrate: the Second, Third, and Forth Years of Enrichment, 1970, 1971, 1972," J. Fisheries Research Board, Canada, 30, pp. 1415-1440, 1973.
- Schindler, D.W., and E.J. Fee, "Experimental Lakes Area: Whole-Lake Experiments in Eutrophication," J. Fisheries Research Board Canada, 31, pp. 932-953, 1974.
- Schindler, D.W., E.J. Fee, and T. Ruscynski, "Phosphorous Input and its Consequences for Phytoplankton Standing Crop and Production in the Experimental Lakes Area and in Similar Lakes". J. Fisheries Research Board Canada, 35, pp. 190-196, 1978.
- Schindler, D.W. "Evolution of Phosphorous Limitation in Lakes" Science, 195, pp. 260-262, 1977.
- Stewart, I. and A. Woodcock. On Zeeman's Equations for the Nerve Impulse, preprint, 1978.

- Sussman, H.J., and R.S. Zahler, "Catastrophe Theory as Applied to the Social and Biological Sciences: A Critique". Synthese, 6, pp. 117-216, 1977.
- Thom, R. Structural Stability and Morphogenesis, W.A. Benjamin, Inc., Reading, Massachusetts, 296 pp., 1975.
- Thomas, E.A. "Sauerstoff Minima und Stoffkreislaufe in Ufernaehen Oberflaechen Waesser des Zuerchersees (Cladophora und Phragmites-Gurtel)" Monats Bulletin Schweiz Verleih Gas-Wasserfachmaenner, 6, pp. 1-8, 1960.
- Van Nguyen, V., and E. Wood. "On the Morphology of Summer Algae Dynamics in Non-Stratified Lakes," Ecological Modelling, 6, pp. 117-131, 1979.
- Zeeman, E.C. Catastrophe Theory: Selected Papers 1972-1977, Addison-Wesley Publishing, Reading, Massachusetts, U.S.A., 700 pp., 1978.