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Modeling the Regional Demoeconomic Development

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FOREWORD

Although many separate models of economic and demographic development exist, it is apparent that up to the present there has been little coordination between them (and, in general, between these and other sectoral models). Thus, the exploration of methods for combining such models is of prime importance. In this paper, R. Dobrinski presents such a demoeconomic model, which is based on a general description of regional economic growth. The development of his model is useful, though initial, work dealing with a particular region (Silistra, Bulgaria) which provides an excellent framework for more detailed sectoral and subregional analysis.

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Task Leader
REGIONAL DEVELOPMENT

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ABSTRACT

The demographic and economic development of a country as well as of a single region are closely linked together. The available labor resources and migration processes are essential factors for the economic growth. On the other hand, demographic growth and migration are strongly influenced by the economic development. The joint modeling of the demographic and economic development is an important problem taking into consideration the intensive urbanization in many countries of the world.

In this paper a regional demoeconomic model is described and some results of its practical application are presented. Although the model is quite simplified it gives some insight into the mutual influence of the demographic and economic factors for the development of the region.

MODELING THE REGIONAL DEMOECONOMIC DEVELOPMENT

Rumen Dobrinsky

I. INTRODUCTION

The demographic and economic development of a country as well as of a single region are closely linked together. The available labor resources and migration processes are essential factors for the economic growth. On the other hand, demographic growth and migration are strongly influenced by the economic development. This mutual influence becomes extremely important nowadays when many countries in the world undergo a process of intensive urbanization. The joint modeling of the demographic and economic development therefore becomes an urgent problem.

There exist various approaches to the modeling of demoeconomic development. A comprehensive survey of the theories and models of regional demoeconomic growth for countries with market economies is presented in J. Ledent (1978). Recently successful attempts to create models of demoeconomic growth in developing countries were preformed by D. Colosio (1979) and A. Kelley and J. Williamson (1979). An example of a demoeconomic model of a country with a socialist type of economy is the demoeconometric model of Poland, developed by Z. Pawlowski (1979).

In this paper a regional demoeconomic model for a region in a socialist country is described. The demographic part of the model consists of three subsectors: population, migrations and labor force. In its turn the economic part of the model treats three aggregated sectors of the regional economy, namely industries, agriculture and the non-productive sector, each of which includes three subsectors: capital funds, gross output* and incomes. It is accepted that migrations within the region as well as between the region and the rest of the country depend on the differences in the socioeconomic conditions of life. In the model the factors influencing migrations are defined according to the results of the economic activity in the region and those for the rest of the country.

A dynamic simulation model was constructed based on the assumptions above which yields yearly results for the demographic and economic development in the region. The model was evaluated and tested with data for the Silistra region in Bulgaria.

Four scenarios for different economic policies in the region have been assessed with regard to their demographic and socioeconomic consequences. Some of the results obtained through the model are also presented in the paper.

II. SUBMODEL OF THE REGIONAL DEMOGRAPHIC GROWTH

Multiregional methods are used to model the regional demographic development (see A. Rogers and D. Philipov 1979) and three types of population are taken into consideration: urban population in the region P_u, rural population in the region P_r, and population in the rest of the country P_c.

An essential point in the multiregional method is that all migrations are treated as out-migration and only out-migration coefficients are being defined. Accordingly, in our case, the three types of population at year t will be described as follows:

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*Besides the non-productive sector.

$$P_u(t) = [1 + g_u(t) - O_{ur}(t) - O_{uc}(t)]P_u(t-1) + O_{ru}(t)P_r(t-1) + O_{cu}(t)P_c(t-1) \quad (1)$$

$$P_r(t) = [1 + g_r(t) - O_{ru}(t) - O_{rc}(t)]P_r(t-1) + O_{ur}(t)P_u(t-1) + O_{cr}(t)P_c(t-1) \quad (2)$$

$$P_c(t) = [1 + g_c(t) - O_{cu}(t) - O_{cr}(t)]P_c(t-1) + O_{uc}(t)P_u(t-1) + O_{rc}(t)P_r(t-1) \quad (3)$$

where:

$g_i(t)$ are the rates of natural growth (fertility minus mortality), $i \varepsilon \{u,r,c\}$

$O_{ij}(t)$ are the corresponding outmigration coefficients, $i,j \varepsilon \{u,r,c\}$*.

Using the same approach we can define the net migrations between the region and the rest of the country N_{RC} and between the rural and urban areas within the region N_{ru}

$$N_{RC}(t) = O_{uc}(t)P_u(t-1) + O_{rc}(t)P_r(t-1) - O_{cu}(t)P_c(t-1) - O_{cr}(t)P_c(t-1) \quad (4)$$

$$N_{ru}(t) = O_{ru}(t)P_r(t-1) - O_{ur}(t)P_u(t-1) \quad . \quad (5)$$

A major problem in the application of this approach is the determination of the migration coefficients. In the long run it comes to the determination of the factors that influence migration processes. B. Mihailov (1979) points out several such factors:

*All equations and parameters are described in detail in Appendix A.

- demographic processes (fertility and mortality)
- economics
- social conditions
- political constraints
- psychological motivations
- spatiality.

In most demoeconomic studies [see also P. Frick and A. LaBella (1977), A. Andersson and A. LaBella (1979), D. Colosio (1979), A. Kelley (1979)] it is assumed that the main factors determining migration from one area to another are the differentials between the social and economic conditions of life in the areas.

A similar approach was used in our model. The important point is to select such a set of socioeconomic factors that can be defined in the economic sector of the model. It was assumed that the migration coefficients are a function of:

$$o_{ij}(t) = f[\Delta W^{*}_{RC}(t),\ \Delta g^{W}_{RC}(t),\ \Delta C^{*}_{nRC}(t),\ \Delta W^{*}_{ai}(t),\ \Delta i_{RC}(t)]$$

$$i,j \in \{u,r,c\} \qquad (6)$$

where:

$\Delta W^{*}_{RC}(t)$ is the differential in the average per capita income between the region and the rest of the country for 3 years preceding t.

$\Delta g^{W}_{RC}(t)$ is the differential in the rates of growth of the average per capita income between the region and the rest of the country for the year preceding t

$\Delta C^{*}_{nRC}(t)$ is the differential in the per capita capital investments in the non-productive sector* between the region and the rest of the country for 3 years preceding t

$\Delta W^{*}_{ia}(t)$ is the differential in the average per capita income between industries and agriculture within the region for 3 years preceding t

*The non-productive sector includes such subsectors as education, medical service, culture, kindergartens, etc.

$\Delta i_{RC}(t)$ is the differential in the degrees of industrialization between the region and the rest of the country for the year preceding t. (The degree of industrialization is measured as the ratio of the gross output in industries and the total gross product.)

The relationships between the migration coefficients and the migration factors can be assessed through regression analysis using historical data. The type of equations used is shown in Appendix A.

Knowing the size of the urban and rural population in the region for each year, one can define the number of employees in industries and agriculture as a portion of the urban and rural population correspondingly, the coefficients also being assessed using historical data.

III. SUBMODEL OF THE REGIONAL ECONOMIC DEVELOPMENT

As mentioned earlier, three aggregated sectors of the regional economy are described in the model: industries, agriculture and the non-productive sector. Subsectors such as industry proper, construction, transportation, communications, trade and some other subsectors of the material production are included in the industrial sector. Agriculture comprises agriculture proper and forestry, while the non-productive sector includes subsectors such as education, medical service, culture, management and some other services. It is assumed that the whole gross regional product is created in the productive sectors: industries and agriculture.

Production functions of the Cobb-Douglas type are used to describe the gross output in the productive sectors:

$$G_j(t) = C_j(t)^{\alpha_j(t)} \; L_j(t)^{1-\alpha_j(t)} \quad , \quad j \in \{i,a\} \; . \qquad (7)$$

The number of employees in each sector $L_j(t)$ is defined in the demographic submodel while the value of the capital funds is derived from the following equation:

$$C_j(t) = \beta_j(t)C_j(t-1) + \Delta C_j^*(t) \quad , \quad j \in \{i,a\} \tag{8}$$

where:

$\beta_j(t)$ is the annual discount rate of capital funds

$\Delta C_j^*(t)$ is the amount of the newly commissioned capital funds in year t.

It is assumed that $\Delta C_j^*(t)$ is a function of the capital investments in previous years $\Delta C_j(t-p)$; besides, all capital investments are put into commission as capital funds within four years--$p = 1 \div 4$.

$$\Delta C_j^*(t) = f[\Delta C_j(t-1), \Delta C_j(t-2), \Delta C_j(t-3), \Delta C_j(t-4)] \tag{9}$$

In the model the value of $\Delta C_j^*(t)$ is derived from the values of $\Delta C_j(t-p)$, $p = 1 \div 4$ through a random process, assuming that a portion $r_p \Delta C_j(t-p)$, $p = 1 \div 4$, of the previous capital investments is put into commission at year t, where r_p is a random number, $r_p \in (0,1)$.

The amounts of the capital investments in the three sectors for each year are assumed to be portions of the corresponding gross outputs

$$\Delta C_j(t) = \psi_j(t)G_j(t) \quad , \quad j \in \{i,a\} \tag{10}$$

$$\Delta C_n(t) = \psi_n(t)G(t) \tag{11}$$

where:

$G_i(t)$, $G_a(t)$ are the gross outputs in industries and agriculture

$G(t)$ is the gross regional product: $G(t) = G_i(t) + G_a(t)$.

The coefficients $\psi_i(t)$, $\psi_a(t)$, $\psi_n(t)$ are used as control variables in the model to form different scenarios of the economic development in the region.

The average income of the employees in the different sectors $W_j(t)$ is assumed to be a function of the average income in the previous year and of the growth rate of per capita gross output in the corresponding sector $r_{G_j}(t)$:

$$W_j(t) = W_j(t-1)[1 + \Omega_j(t) r_{G_j}(t)] \quad , \quad j \in \{i,a\} \tag{12}$$

$$W_n(t) = W_n(t-1)[1 + \Omega_n(t) r_G(t)] \tag{13}$$

where:

$$r_{G_j}(t) = \frac{h_j(t) - h_j(t-1)}{h_j(t)} \quad , \quad j \in \{i,a\} \tag{14}$$

$$r_G(t) = \frac{h(t) - h(t-1)}{h(t)} \tag{15}$$

$$h_j(t) = \frac{G_j(t)}{L_j(t)} = C_j(t)^{\alpha_j(t)} L_j(t)^{-\alpha_j(t)} \tag{16}$$

$$j \in \{i,a\}$$

$$h(t) = \frac{G_i(t) + G_a(t)}{L_i(t) + L_a(t)} \quad . \tag{17}$$

The coefficients $\Omega_i(t)$, $\Omega_a(t)$, $\Omega_n(t)$ are also used as control variables in the model to form different scenarios of changing the income patterns in the region.

Having the values of the gross outputs and the average incomes for a given year, we can define the values of the migration factors for the following year

$$\Delta W^*_{RC}(t+1), \ \Delta g^W_{RC}(t+1), \ \Delta C^*_{nRC}(t+1), \ \Delta W^*_{ai}(t+1), \ \Delta i_{RC}(t+1) \ .$$

The corresponding mathematical expressions are given in Appendix A. It should be noted that planned values are used for all parameters concerning the rest of the country.

IV. MODELING THE DEMOECONOMIC DEVELOPMENT OF THE SILISTRA REGION IN BULGARIA

The outlined approach was applied to model the future demoeconomic development of the Silistra region in Bulgaria.

The Silistra region is in the outlying northeastern part of Bulgaria and occupies an area of 2860 km^2 or 2.6% of Bulgaria. In 1977 its population was about 176,400 persons (2% of Bulgaria's total population), 81,250 of which are living in urban areas and 95,150 in rural areas. In the same year the region produced about 1.75% of the country's GNP (1.36% in industries and 4.3% in agriculture). Also, 47.3% of the working population was occupied in industries, 39.5% in agriculture and 13.2% in the non-productive sector. The Silistra region is a major agricultural producer but industrially it is still less developed than the rest of the country, although in the last few years it underwent substantial industrial development. On the whole, the rate of economical development in the region is slightly lower than the rate for the country as a whole.

The natural population growth rate in the region (average of 7.8 for the period 1972-1977) is higher than that for the country as a whole (6.3 for the same period), but the region develops with a negative net migration balance. With the rapid industrialization, intensive urbanization takes place within the region with migration flows to the towns in the region and mainly towards Silistra.

Four different scenarios were created to study the demoeconomic development of the Silistra region under four different economic policies until 1990.*

According to Scenario I the existing trends of economical development continue until 1990, the Silistra region developing with a lower rate than the country as a whole.

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*The sources of data used for evaluation of the model parameters are the Statistical Yearbooks of the People's Republic of Bulgaria and the Population Yearbook of the People's Republic of Bulgaria.

Scenario II assumes that the Silistra region develops with the same rates as the country as a whole.

According to Scenarios III and IV the Silistra region develops with a higher rate than the country as a whole, while in Scenario III preference is given to the industrial development (higher rate of industrial development; agriculture develops with the rates of Scenario II), Scenario IV favors agriculture (higher rate of agricultural development; industries develop with the rates of Scenario II).

Some of the results for these four scenarios are shown in Tables 1, 2, 3 and Figures 1, 2, 3.

Table 1. Population in the region.

Year		1980	1985	1990
Scenario I	$P_u(t)$	87,650	97,550	107,510
	$P_r(t)$	90,530	83,470	75,870
	P(t)	178,180	181,020	183,380
Scenario II	$P_u(t)$	87,640	97,280	107,140
	$P_r(t)$	90,550	83,860	76,740
	P(t)	178,190	181,140	183,880
Scenario III	$P_u(t)$	87,640	97,360	107,910
	$P_r(t)$	90,550	83,800	76,170
	P(t)	178,190	181,160	184,080
Scenario IV	$P_u(t)$	87,600	96,320	104,040
	$P_r(t)$	90,610	84,990	80,110
	P(t)	178,210	181,310	184,150

Table 1 gives the size of the urban, rural and total population in the region and Table 2 - the relative part (in %) of the working population, occupied in the three economic sectors according to the four scenarios.

Table 2. Percentage of the working population in the three sectors.

Year		1980	1985	1990
Scenario I	Ind.	51.4	58.1	63.0
	Agr.	34.6	26.8	21.2
	Non-prod.	14.0	15.1	15.8
Scenario II	Ind.	51.4	58.0	62.7
	Agr.	34.6	26.9	21.4
	Non-prod.	14.0	15.1	15.9
Scenario III	Ind.	51.4	58.2	63.3
	Agr.	34.6	26.7	21.0
	Non-prod.	14.0	15.1	15.7
Scenario IV	Ind.	51.4	57.6	61.5
	Agr.	34.6	27.3	22.5
	Non-prod.	14.0	15.1	16.0

The annual net migrations from the region to the rest of the country and within the region for the period until 1990 are shown in Figure 1a and b. Smooth curves are drawn rather than broken lines.

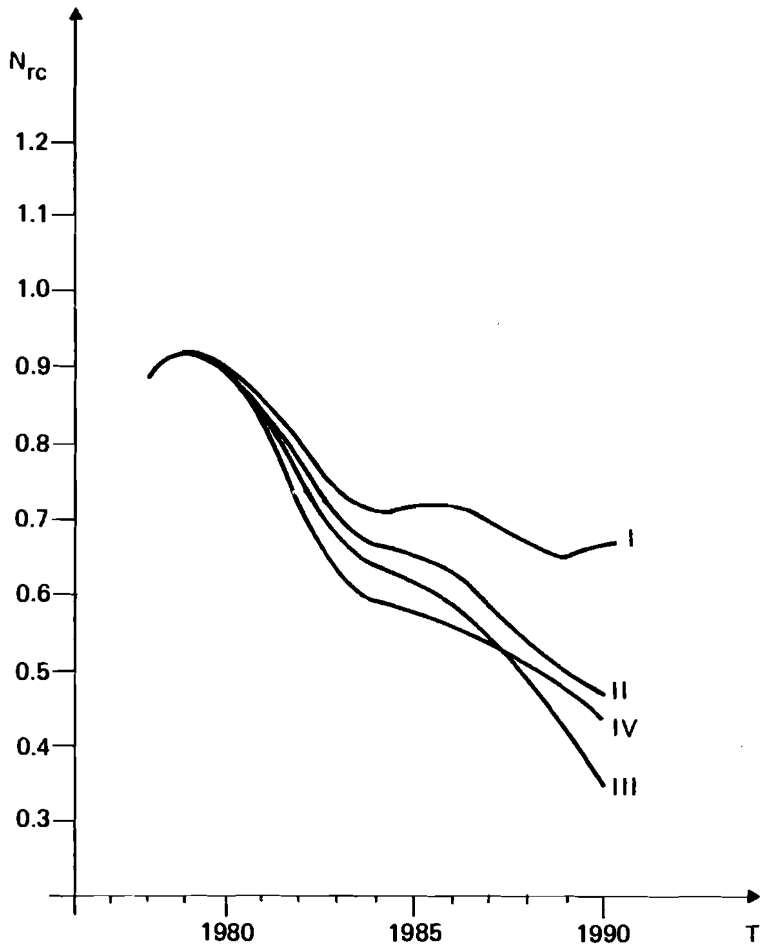
As can be seen from the tables and from the figure, Scenarios I, II, and III do not differ significantly with respect to the population sizes, the structure of the working population and the internal migration, as they all follow the general trend of development. Whereas in Scenario IV, the urbanization trend slows down and a number of people willing to leave rural areas for urban areas within the region decreases as the living conditions in the rural parts of the region improve more rapidly.

As for the net migrations from the region to the rest of the country in Scenario I, they remain rather high during the whole period, while for the rest of the scenarios they decrease rapidly.

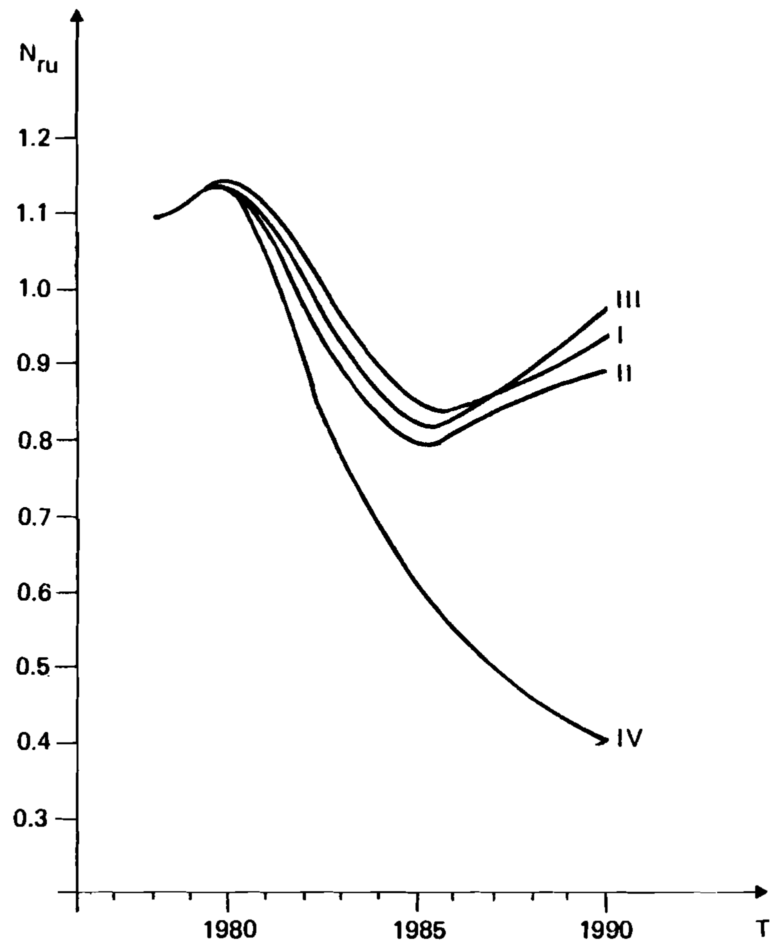
Table 3 depicts the change in the degree of industrialization in the region (the ratio of the gross output in industries to the gross regional product) for the four scenarios.

Table 3. Change in the degree of industrialization in the region (%).

Year	1980	1985	1990
Scenario I	69.4	74.2	80.0
Scenario II	69.4	74.2	80.2
Scenario III	69.4	74.6	81.3
Scenario IV	69.4	74.1	79.7



a)



b)

Figure 1. Net migrations a) from the region to the rest of the country; b) from rural areas to urban areas within the region (the average annual values for the period 1972 - 1977 = 2.0).

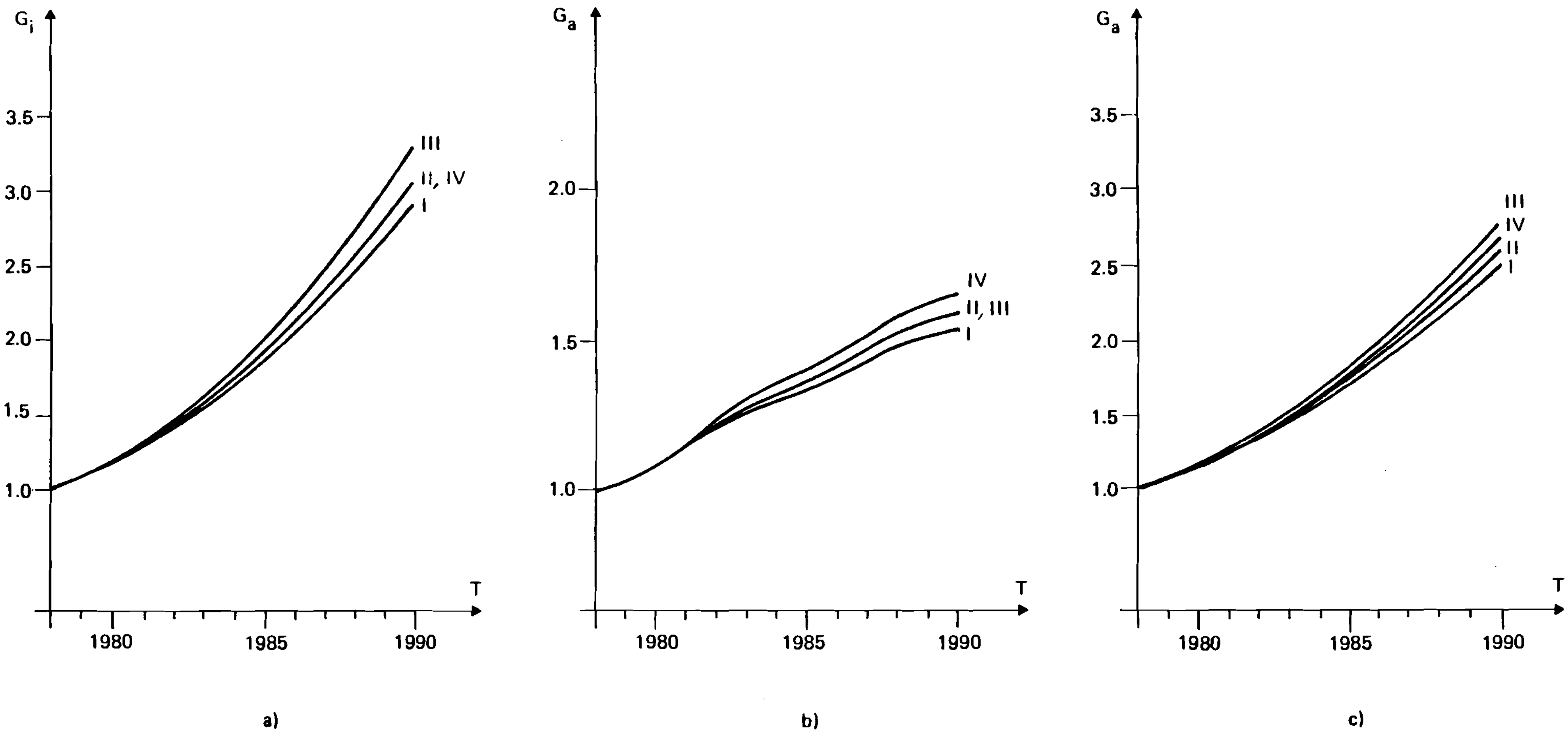
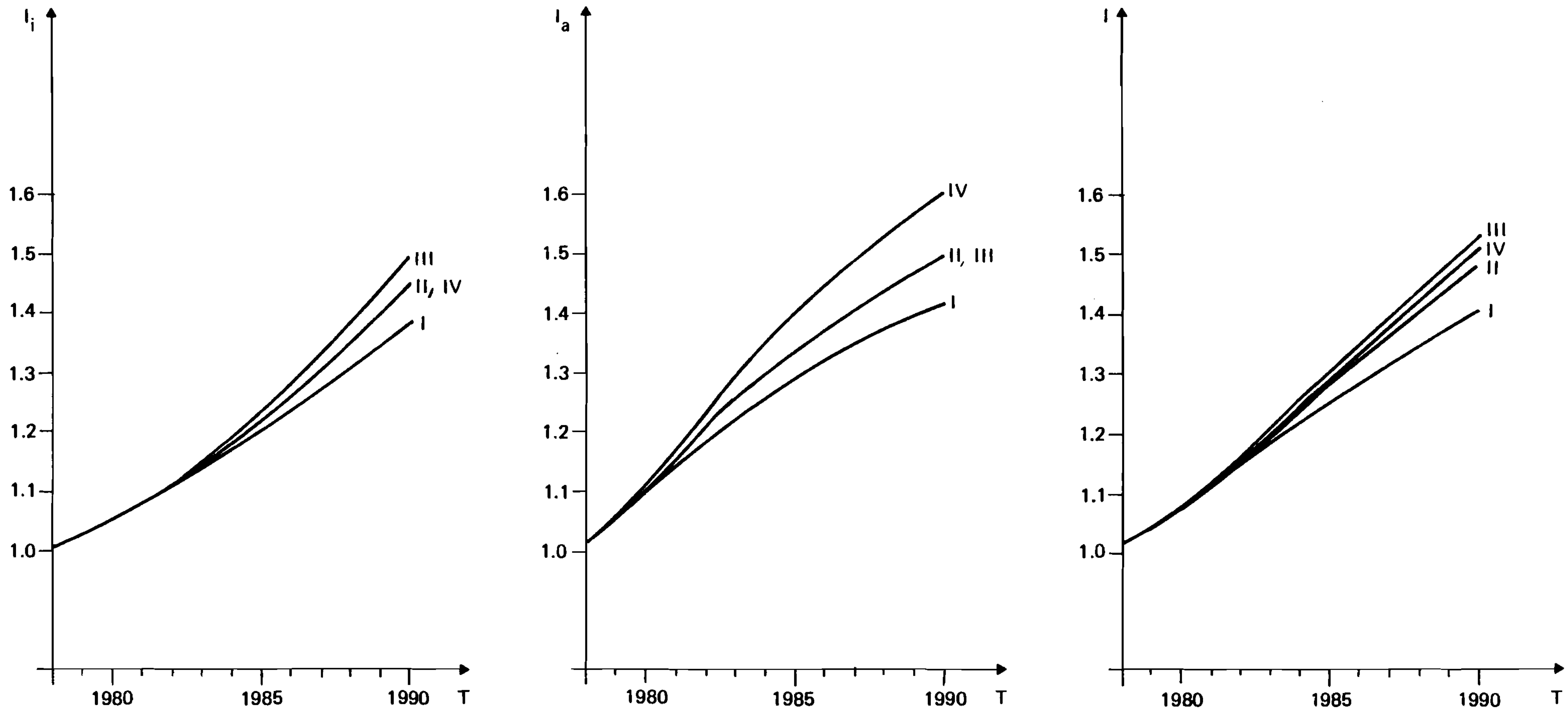


Figure 2. Growth of gross outputs; a) in industries; b) in agriculture; c) total GRP (1977 = 1.0).



a) b)

Figure 3. Growth of per capita incomes in the region a) in industries; b) in agriculture; c) total (1977 = 1.0).

Figure 2 shows the growth of the gross outputs in industries, agriculture and the total gross regional product and Figure 3 reflects the growth of per capita increase incomes in the region. As can be seen, the GRP and the average per capita income in the region reach highest values in Scenario III, whereas the overall results for Scenario IV are worse.

V. CONCLUSIONS

In this paper the principles of creating a regional demo-economic model have been discussed. The practical application of the suggested approach is shown in the example of modeling the demoeconomic development of the Silistra region in Bulgaria.

Although the model is quite simplified it gives some insight into the mutual influence of the demographic and economic factors for the development of the region. This might assist planning agencies towards a better understanding and searching for efficient regional demoeconomic development.

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APPENDIX A

I. MODEL EQUATIONS

$$O_{rc}(t) = f[\Delta W^{*}_{RC}(t),\ \Delta g^{W}_{RC}(t),\ \Delta C^{*}_{nRC}(t),\ \Delta i_{RC}(t)] \tag{1}$$

$$O_{uc}(t) = f[\Delta W^{*}_{RC}(t),\ \Delta g^{W}_{RC}(t),\ \Delta C^{*}_{nRC}(t),\ \Delta i_{RC}(t)] \tag{2}$$

$$O_{cr}(t) = f[\Delta W^{*}_{RC}(t),\ \Delta C^{*}_{nRC}(t),\ \Delta i_{RC}(t)] \tag{3}$$

$$O_{cu}(t) = f[\Delta W^{*}_{RC}(t),\ \Delta C^{*}_{nRC}(t),\ \Delta i_{RC}(t)] \tag{4}$$

$$O_{ru}(t) = f[\Delta W^{*}_{ai}(t),\ \Delta i_{RC}(t)] \tag{5}$$

$$O_{ur}(t) = f[\Delta W^{*}_{ai}(t),\ \Delta i_{RC}(t)] \tag{6}$$

$$P_u(t) = [1 + g_u(t) - O_{ur}(t) - O_{uc}(t)]P_u(t-1) + O_{ru}(t)P_r(t-1) + O_{cu}(t)P_c(t-1) \tag{7}$$

$$P_r(t) = [1 + g_r(t) - O_{ru}(t) - O_{rc}(t)]P_r(t-1) + O_{ur}(t)P_u(t-1) + O_{cr}(t)P_c(t-1) \tag{8}$$

$$P_c(t) = [1 + g_c(t) - O_{cu}(t) - O_{cr}(t)]P_c(t-1) + O_{uc}(t)P_u(t-1) + O_{rc}(t)P_r(t-1) \quad (9)$$

$$P(t) = P_u(t) + P_r(t) \quad (10)$$

$$N_{RC}(t) = O_{uc}(t)P_u(t-1) + O_{rc}(t)P_r(t-1) - O_{cu}(t)P_c(t-1) - O_{cr}(t)P_c(t-1) \quad (11)$$

$$N_{ru}(t) = O_{ru}(t)P_r(t-1) - O_{ur}(t)P_u(t-1) \quad (12)$$

$$P_{um}(t) = 0.5[P_u(t-1) + P_u(t)] \quad (13)$$

$$P_{rm}(t) = 0.5[P_r(t-1) + P_r(t)] \quad (14)$$

$$P_m(t) = 0.5[P(t-1) + P(t)] \quad (15)$$

$$L_i(t) = l_i(t)P_{um}(t) \quad (16)$$

$$L_a(t) = l_a(t)P_{rm}(t) \quad (17)$$

$$L_n(t) = l_n(t)P_m(t) \quad (18)$$

$$\Delta C_i^*(t) = \text{ran}[\Delta C_i(t-1), \Delta C_i(t-2), \Delta C_i(t-3), \Delta C_i(t-4)] \quad (19)$$

$$\Delta C_a^*(t) = \text{ran}[\Delta C_a(t-1), \Delta C_a(t-2), \Delta C_a(t-3), \Delta C_a(t-4)] \quad (20)$$

$$\Delta C_n^*(t) = \text{ran}[\Delta C_n(t-1), \Delta C_n(t-2), \Delta C_n(t-3), \Delta C_n(t-4)] \quad (21)$$

$$C_i(t) = \beta_i(t)C_i(t-1) + \Delta C_i^*(t) \quad (22)$$

$$C_a(t) = \beta_a(t)C_a(t-1) + \Delta C_a^*(t) \quad (23)$$

$$C_n(t) = \beta_n(t)C_n(t-1) + \Delta C_n^*(t) \quad (24)$$

$$C_{im}(t) = 0.5[C_i(t-1) + C_i(t)] \tag{25}$$

$$C_{am}(t) = 0.5[C_a(t-1) + C_a(t)] \tag{26}$$

$$G_i(t) = C_{im}(t)^{\alpha_i(t)} L_i^{1-\alpha_i(t)} \tag{27}$$

$$G_a(t) = C_{am}(t)^{\alpha_a(t)} L_a^{1-\alpha_a(t)} \tag{28}$$

$$G(t) = G_i(t) + G_a(t) \tag{29}$$

$$h_i(t) = \frac{G_i(t)}{L_i(t)} = C_{im}(t)^{\alpha_i(t)} L_i(t)^{-\alpha_i(t)} \tag{30}$$

$$h_a(t) = \frac{G_a(t)}{L_a(t)} = C_{am}(t)^{\alpha_a(t)} L_a(t)^{-\alpha_a(t)} \tag{31}$$

$$h(t) = \frac{G(t)}{L_i(t) + L_a(t)} \tag{32}$$

$$W_i(t) = W_i(t-1)\left[1 + \Omega_i(t)\ \frac{h_i(t) - h_i(t-1)}{h_i(t-1)}\right] \tag{33}$$

$$W_a(t) = W_a(t-1)\left[1 + \Omega_a(t)\ \frac{h_a(t) - h_a(t-1)}{h_a(t-1)}\right] \tag{34}$$

$$W_n(t) = W_n(t-1)\left[1 + \Omega_n(t)\ \frac{h_n(t) - h_n(t-1)}{h_n(t-1)}\right] \tag{35}$$

$$W(t) = \frac{L_i(t)W_i(t) + L_a(t)W_a(t) + L_u(t)W_n(t)}{L_i(t) + L_a(t) + L_n(t)} \tag{36}$$

$$\Delta C_i(t) = \psi_i(t)G_i(t) \tag{37}$$

$$\Delta C_a(t) = \psi_a(t)G_a(t) \tag{38}$$

$$\Delta C_n(t) = \psi_n(t)G(t) \quad , \quad C_n'(t) = \frac{C_n(t)}{P(t)} \tag{39}$$

$$W_R^*(t) = \frac{1}{3}[W(t) + W(t-1) + W(t-2)] \tag{40}$$

$$C^{*}_{nR}(t) = \frac{1}{3}[\Delta C'_{n}(t) + \Delta C'_{n}(t-1) + \Delta C'_{n}(t-2)] \quad (41)$$

$$W^{*}_{i}(t) = \frac{1}{3}[W_{i}(t) + W_{i}(t-1) + W_{i}(t-2)] \quad (42)$$

$$W^{*}_{a}(t) = \frac{1}{3}[W_{a}(t) + W_{a}(t-1) + W_{a}(t-2)] \quad (43)$$

$$i_{R}(t) = \frac{G_{i}(t)}{G(t)} \quad (44)$$

$$\Delta W^{*}_{RC}(t+1) = \frac{W^{*}_{c}(t) - W^{*}_{R}(t)}{W^{*}_{c}(t)} \quad (45)$$

$$\Delta g^{*}_{RC}(t+1) = \frac{[W(t) - W(t-1)]\ W_{c}(t)}{W(t-1)\,[W_{c}(t) - W_{c}(t-1)]} \quad (46)$$

$$\Delta C^{*}_{nRC}(t+1) = \frac{\Delta C^{*}_{nc}(t) - \Delta C_{nR}(t)}{\Delta C^{*}_{nc}(t)} \quad (47)$$

$$\Delta W^{*}_{ai}(t+1) = \frac{W^{*}_{i}(t) - W^{*}_{a}(t)}{W^{*}_{i}(t)} \quad (48)$$

$$\Delta i_{RC}(t+1) = \frac{i_{c}(t) - i_{R}(t)}{i_{c}(t)} \quad (49)$$

II. VARIABLES

$O_{ru}(t)$ - outmigration coefficient from rural to urban areas within the region (the ratio of the outmigrants to the total rural population) in year t

$O_{ur}(t)$ - urban to rural outmigration coefficient

$O_{rc}(t)$ - rural to the rest of the country outmigration coefficient

$O_{cr}(t)$ - the rest of the country to rural areas in the region outmigration coefficient

$O_{uc}(t)$ - urban to the rest of the country outmigration coefficient

$O_{cu}(t)$ - the rest of the country to urban areas in the region outmigration coefficient

$P(t)$ - total population in the region by the end of year t

$P_u(t)$ - urban population in the region by the end of year t

$P_r(t)$ - rural population in the region by the end of year t

$P_c(t)$ - population in the rest of the country by the end of year t

$g_u(t)$, $g_r(t)$, $g_c(t)$ - natural rates of growth of the urban and rural population in the region, and in the rest of the country, correspondingly

$N_{RC}(t)$ - net migration from the region to the rest of the country in year t

$N_{ru}(t)$ - net migration from rural to urban areas within the region in year t

$P_{um}(t)$, $P_{rm}(t)$, $P_m(t)$ - mid-year value of the urban, rural and total population in the region for year t

$L_i(t)$, $L_a(t)$, $L_n(t)$ - number of employees in industries, in agriculture, and in the non-productive sector in the region in year t

$\Delta C_i^*(t)$, $\Delta C_a^*(t)$, $\Delta C_n^*(t)$ - value of the newly commissioned capital funds in industries, in agriculture and in the non-productive sector in the region in year t

ran - random function

$\Delta C_i(t)$, $\Delta C_a(t)$, $\Delta C_n(t)$ - values of capital investments in industries, in agriculture and in the non-productive sector in the region in year t

$C_i(t)$, $C_a(t)$, $\Delta C_n(t)$ - values of the capital funds in industries, in agriculture and in the non-productive sector in the region by the end of year t

$\beta_i(t)$, $\beta_a(t)$, $\beta_n(t)$ - annual discount rate of capital funds in industries, in agriculture and in the non-productive sector in the region in year t

$C_{im}(t)$, $C_{am}(t)$ - mid-year values of $C_i(t)$ and $C_a(t)$

$G_i(t)$, $G_a(t)$ - gross output in industries and agriculture in the region in year t

$\alpha_i(t)$, $\alpha_a(t)$ - coefficients

$G(t)$ - gross regional product in year t

$h_i(t)$, $h_a(t)$, $h(t)$ - per capita gross output in industries, in agriculture, and average for the material sectors in the region in year t

$W_i(t)$, $W_a(t)$, $W_n(t)$, $W(t)$ - per capita income of employees in industries, in agriculture, in the non-productive sector and average for the region in year t

$\Omega_i(t)$, $\Omega_a(t)$, $\Omega_n(t)$ - coefficients

$\psi_i(t)$, $\psi_a(t)$, $\psi_n(t)$ - coefficients

$\Delta C_n'(t)$ - per capita value of $\Delta C_n(t)$

$W_R^*(t)$, $W_c^*(t)$ - average income of one employee in the region and in the rest of the country for the last three years

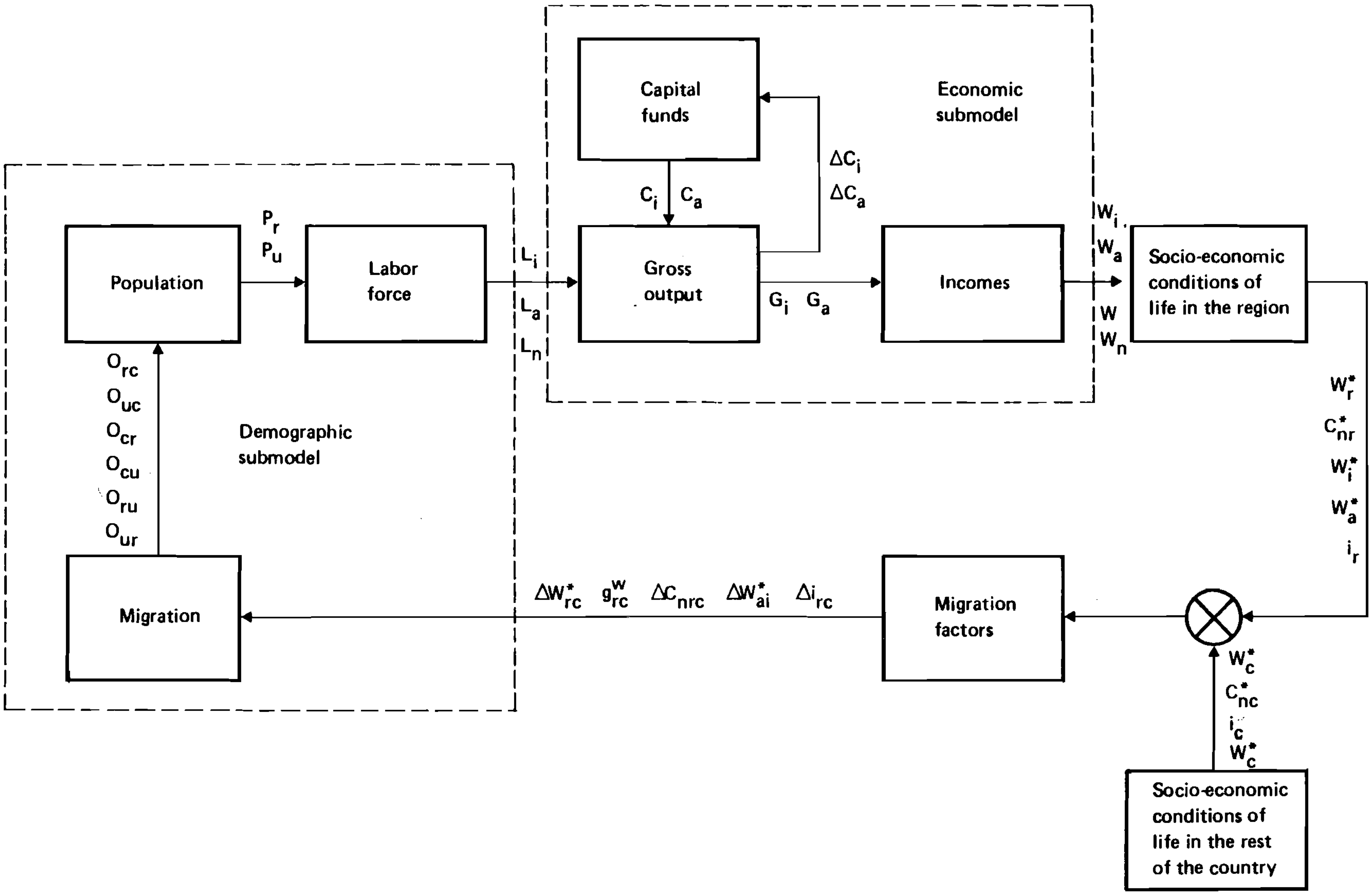
$C_{nR}'(t)$, $C_{nc}'(t)$ - average per capita capital investments in the non-productive sector in the region and in the rest of the country for the last three years

$W_i^*(t)$, $W_a^*(t)$ - average income of one employee in industries and in agriculture in the region for the last three years

$\Delta i_R(t)$, $\Delta i_c(t)$ - degree of industrialization in the region and in the rest of the country

$\Delta W_{RC}^*(t)$, $\Delta g_{RC}^W(t)$, $\Delta C_{nRC}^*(t)$, $\Delta W_{ai}^*(t)$, $\Delta i_{RC}(t)$ are described in the text

APPENDIX B: LOGICAL STRUCTURE OF THE SIMULATION MODEL



Capital funds
Economic submodel
ΔC_i
ΔC_a
C_i C_a
Population
P_r
P_u
Labor force
L_i
L_a
L_n
Gross output
G_i G_a
Incomes
W_i
W_a
W
W_n
Socio-economic conditions of life in the region
W_r^*
C_{nr}^*
W_i^*
W_a^*
i_r
O_{rc}
O_{uc}
O_{cr}
O_{cu}
O_{ru}
O_{ur}
Demographic submodel
Migration
ΔW_{rc}^* g_{rc}^w ΔC_{nrc} ΔW_{ai}^* Δi_{rc}
Migration factors
W_c^*
C_{nc}^*
i_c
W_c^*
Socio-economic conditions of life in the rest of the country