Agricultural Sector Programming Models: A Review of Alternative Approaches

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International Institute for Applied Systems Analysis
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PREFACE

As part of IIASA's Food and Agricultural Program, the research project "Limits and Consequences of Food Production Technologies" addresses the question of "what long-term technical development paths are feasible and likely for increasing food production, based on the present availability of resources (including energy), the long-run feedback on the environment, and the short-run pressures reflected in current agricultural policies" (see IIASA, Research Plan 1980-1984). It is aimed at developing models that (a) describe the interactions between resources, technologies and environment in agricultural production systems and (b) provide means for the determination of policies that help countries to cope with the rising demand for food.

The paper discusses various alternative approaches based on programming models. It emphasizes the organizational aspect of the problem which is typical for agricultural production systems in both market economies and centrally planned economies. The models are basically two-level decision systems where the policy bodies influence agricultural production by formulating guidelines (prices, quotas, etc.) that allow the individual production units some flexibility in the determination of their production activities.

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AGRICULTURAL SECTOR PROGRAMMING MODELS: A REVIEW OF ALTERNATIVE APPROACHES

Roger D. Norton and Gerhard W. Schiefer

1. INTRODUCTION

The topic of this paper is the use of mathematical programming models for the solution of agricultural decision problems at a local and national level. Specifically, we consider the decision problems of policy bodies (agencies) who attempt to influence the economic pattern of agricultural activities in a context in which those activities are basically determined by decisions of individual farming units. The paper provides a selective review of the literature and issues in this area, with special emphasis on questions of appropriate model structures for replication of sector behavior and on the reliability of computed results.

It is assumed that the policy bodies may be able to influence farm decisions by restricting their decision possibilities or altering their relative profitability, but that the farms otherwise are free to make their own choices within the limits of economic and technological constraints. The policy bodies, therefore, must attempt to achieve the results they desire in an indirect manner, without certain knowledge of how farmers will respond to their new set of external parameters. In addition there are policy restrictions on the agencies' actions: a budget constraint sets limits to their influence, and political and social attitudes restrict the degree to which many particular kinds of policy instruments may be used.

In all cases, the policy body is faced with the necessity of solving two problems. One is to forecast how farmers would react to various hypothetical policy actions, and the other is to select the most appropriate combination of those possible actions, given the policy goals and constraints and the conditional forecasts of outcomes. The forecasting problem sometimes is also called the "positive" or "descriptive" problem, and the policy problem is referred to as the "normative" or "prescriptive" problem.

The reliability of the forecasts is a central problem for an agency striving to conduct realistic policy planning. Traditionally, econometric methods have been used to solve the forecasting problem, but more recently mathematical programming methods have been employed as well. Econometric response functions have obvious advantages: under proper estimation techniques, they can extract the maximum amount of information possible out of the statistical data, and also the estimation procedure provides indicators of the statistical reliability of the estimates. However, they also have four significant disadvantages. First, in multi-product (and/or multi-regional) situations, there may not be sufficient degrees of freedom to estimate parameters indicating the degree of inter-relatedness among crops' production levels (cross-supply elasticities). Second, as emphasized recently by Shumway and Chang (1977), because the estimates are valid only over the historically-experienced range of variation, they may not be applicable for the analysis of proposed policy changes which involve significant departures from historical trends. Third, econometric models cannot include inequality constraints such as seasonal land constraints (see Henderson (1959) for an early treatment of agricultural land use via optimization models). Fourth, econometric models typically do not provide much complementary information on the movement of other variables of interest. For example, in the case of a model of crop supply response, policy makers may wish to base their crop pricing decisions not only on the conditional forecasts of output responses, but also on projected movements in seasonal employment, land values, export earnings, farm income by regional and farm size class, etc.

The cross-sectionally based activity analysis models, which are solved by mathematical programming methods, usually can satisfy these four criticisms, while remaining less satisfactory than econometric methods as regards fidelity to historical data and availability of objective measures of reliability ofr their forecasts.

The focus of this paper is on the use of mathematical programming methods for the forecasting problem alone and also for the combined forecasting-and-policy problem. The paper attempts to delineate the considerations which lead to definition of appropriate model structures, and also approaches for the identification and improvement of the reliability of computed results are discussed. We do not attempt to evaluate the policy applications which have been made with agricultural programming models, for such an evaluation would have to take account of the institutional setting of each application. But we do comment on methodologies from the viewpoint of their suitability for potential applications.

The next section is devoted to clarification of some important distinctions among classes of models, distinctions which arise from the economic interpretations which may be assigned to their solutions. After that, the succeeding sections review key issues of model specification and discuss considerations regarding model reliability. Finally, we return to the theme of different levels of analysis and then offer some concluding remarks.

2. MODELS OF MULTI-LEVEL DECISION SITUATIONS

When the descriptive problem of simulating farmer or sector behavior is set up as an optimization model,* then the complete policy problem is in fact a multi-level optimization problem. One level (or levels) attempts to describe farmer decision processes, subject to technological and market constraints and also subject to specified government policy interventions. The other level concerns selection from among alternative policies, in light of simulated farmer reactions and a policy objective function, and subject of course to a government budget constraint. For the two-level case, a typical policy problem may be written as follows, in the notation of Candler and Norton (1977):

$$\max_{x_2} f_2 = (c_2' x_o) \tag{1}$$

subject to

$$f_1 = (c_1' x_1) \rightarrow \max_{x_1 | x_2} \tag{2}$$

$$A_{11} x_1 + A_{12} x_2 \leq b \tag{3}$$

$$- I x_o + A_{21} x_1 + A_{22} x_2 = 0 \tag{4}$$

$$x_o , x_1 , x_2 \leq 0 \tag{5}$$

where

x_o is a vector of target variables

x_1 is a vector of descriptive variables

x_2 is a vector of policy variables

f_2 is a policy objective function

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*The next section discusses issues which arise from the use of optimization models to simulate sector behavior.

f_1 is a descriptive objective function

b is a vector of resource endowments and other constraint values for the descriptive problem

c_1 is the set of coefficients in the descriptive objective function for farmers

c_2 is a set of policy weights

In this reasonable typical case, the model is structured so that

a) Only the target variables x_o are of interest in the policymakers' objective function;
b) Only the descriptive choice variables x_1 affect the descriptive objective function;
c) A_{11} is a technological matrix of unit resource requirements;
d) The matrix A_{12} expresses the effect of the policy variables x_2 on resource availability (a policy which increases resource availability, such as investment in new irrigation supplies, is represented by a negative element of A_{12});
e) The matrix A_{21} represents the effects of descriptive variables x_1 on target variables x_o (e.g., if greater employment is a policy target, then A_{21} may contain coefficients showing the labor input requirements per unit of production of each crop); and
f) A_{22} is a matrix of the direct effects of the policy variables x_2 on the impact variables x_o (in many cases, this matrix would be zero and so policies would have to achieve their impacts indirectly, viz., through the matrices A_{12} and A_{21}).

The domains of maximization of f_2 and f_1 are assumed to differ. For example, the policy choice variables may be levels and types of public investment whereas the decentralized (descriptive) choice variables may be crop planting patterns and total acreage seeded. If the domains of maximization did not differ, i.e., if policy makers directly controlled all of the descriptive variables, then the problem would collapse to a single mathematical programming problem and the analogous decision-making situation would be completely centralized planning.

For the solution of the multi-level optimization problem in a decentralized setting, the policy body faces two main problems. First, it has to collect information about the farms decision problems. Second, it has to compute a solution to its policy problem. There are, in principle, two ways how the policy body can find a solution to its policy optimization problem:

1. It could collect all information about the farms' decision problems that might be of interest in a "one step" communication process, formulate the complete optimization problem as shown above, and attempt to compute a solution to this multi-objective mathematical programming problem. This is a solution which optimizes the policy objective function on the basis of simultaneous considerations of the farmers' reactions to each possible policy.
2. It could collect information about the farms' decision problems via a "multi-step" communication process which eventually leads to a solution of the complete optimization problem. In such a process, in each step the policy body communicates a possible policy strategy to the farmers and then records what their response would be.

In principle, the second approach reduces the amount of information that has to be collected from farmers by the policy body, as only information that is of relevance for the solution of the policy problem has to be collected. In other words, only farmers' responses to specific policies are collected, rather than attempting to describe farmers' decision functions. It requires, however, a repeated information exchange with the farms before the policy is established, and this may be difficult in practice.

In cases where a "one-step" communication process has been realized, the policy body has to solve a mathematical problem of the form discussed above. In mathematical programming, procedures for the solution of programming problems that result from multi-level decision problems have been developed, beginning with the "decomposition principle" of Dantzig and Wolfe (1961). However, these decomposition methods were primarily designed for the solution of multi-level decision problems that could be formulated as a single large mathematical programming problem with a single objective function only. In terms of the mathematical problem discussed above, they were designed for the solution of problems there the domains of maximization of f_2 and f_1 do not differ.

There are, however, some truly multi-level algorithms, but their variables are aimed at changing the constraint set of the farms' decision problems rather than their objective function. Examples are found in the procedures of Weitzman (1970) and Candler and Townsley (1978). These procedures involve an iterative process of information exchange between different programming models, and convergence may take a very long time. For an overview of some of these procedures see, among others, Lasdon (1970) and Ruefli (1974).

Thus far, no completely satisfactory practical solution procedure to the multi-level optimization problem exists. In recognition of this fact, Kornai (1969a) suggested heuristic procedures which cannot be guaranteed to converge to a solution of the complete problem but which nonetheless are likely to lead to improvement over some initial "plan" values.

On the institutional side, decomposition procedures have a very real counterpart in the exchanges of information between agencies at different levels of decision making, i.e., in the organization of a "multi-step" communication process as discussed above. Appropriate procedures were discussed by, among others, Malinvaud (1967) and Heal (1973). Extensive empirical multi-level studies have been carried out, as in Goreux and Manne (1973) and Goreux (1977), but except for the case of Hungary (Kornai 1969b), they have not yet been successfully applied, primarily because of the necessity for repeated information exchange. Nevertheless, the results of some experimental studies (Kutcher 1973; Burton and Obel 1978; Christensen and Obel 1978; Schiefer 1978b; Ljung and Selmer 1979) or the comprehensive discussion in Dirickx and Jennergren (1979) indicate that they might soon be efficient enough to be used in real decision-making processes, at least in two-level decision situations where the number of participants is not too large. The sector-farm multi-level problem, however, is inherently more difficult to solve by actual iterations among the participants.

Lacking a sure method of solving the multi-level problem either mathematically or institutionally, agricultural economists have emphasized specification of the descriptive model alone, in order to permit policy planning bodies' simulation of farmer response to possible policy initiatives. Policy experiments have been conducted by formulating <u>ex ante</u> several possible policy packages and then testing for their probable consequences by inserting them into the parameter structure of the descriptive model (Duloy and Norton 1973b; Bassoco and Norton 1975).

While a "policy objective function" is sometimes attached to the constraint set of the descriptive problem, thereby creating a normative model, it should be clear that such a model does not represent completely either the policy problem or the descriptive problem. Such models do not contain representations of specific policy instruments whose use might lead to the outcome suggested by the model, and therefore there is no indication of whether in fact the outcome is feasible, given political limits on policy actions and also given farmers' own preferences.

These models are useful, nevertheless, for defining quantitatively the potentials of the sector--the physical frontiers of the production possibilities set. While unfortunately these frontiers may lie significantly beyond the politically attainable points, an advantage of the models is that no significant reliability (validation) question arises. If the agronomic production coefficients and the technological constraint set are regarded as correct, then the computed frontiers are correct. Of course, there are measurement errors and each coefficient may be regarded as a random variable subject to a probability distribution, but the possible errors of measurement in this regard usually are much smaller in magnitude than the possible errors introduced in other models by assumptions about farmers' and consumers' decision rules. An example is reported in Folkesson (1973) whose experiments were aimed at determining the potential of Swedish agriculture in (what he called) an "emergency situation".

In the following sections, we expand the discussion of the principal programming formulations which have been used for the descriptive problem, along with issues of reliability evaluation.

3. PROGRAMMING FORMULATIONS FOR THE DESCRIPTIVE PROBLEM

3.1. Overview

In the first generation of aggregate mathematical programming models for agriculture, the specification was almost entirely oriented to the production side--even in sector-wide models, demand was simplified to fixed quantities or fixed prices--and representative farm-level data were used for the technological parameters. Usually, a profit maximizing or cost-minimizing objective function was used. An excellent early example was the multi-regional cost minimizing model of Heady and Egbert (1964). Similar models have been developed for Sweden (Folkesson 1968), Germany (Heidhues 1966), Thailand (Faber et al. 1978), and other countries. While cost minimization or profit maximization under a fixed-price regime may be appropriate at the farm or local level, a complete representation of sector-wide behavior must take into account price endogeneity, i.e., downward-sloping demand curves.*

Beginning with the spatial equilibrium work of Takayama and Judge (1964, 1971), followed by sector and regional models for France (Farhi and Vercueil 1969; Tirel 1971), Russia (Mash and Kiselev 1971), Mexico (Duloy and Norton 1973a), Central America (Cappi et al. 1978), the EEC (Weindlmaier and Tarditi 1976), and other countries, agricultural economists exploited the possibility of using price-endogenous optimization models of simulate farmer and consumer behavior in the aggregate. This is achieved by specifying an objective function whose maximization yields the outcome of a determined market form, e.g., competitive or monopolistic. The existence of such a function had been pointed out many years earlier by Enke (1951) and Samuelson (1952), but the idea was first applied in the context of agricultural sector

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*Although international trade possibilities may limit the range within which the domestic price may move for some products, nevertheless for most agricultural products the existence of transportation costs and tariffs usually allows substantial latitude for domestic price movements.

models. The maximand may be interpreted as the sum of producer and consumer surplus. There has been a long debate over the use of Marshallian surpluses as welfare measures (Burns 1973), but in the context of sector models we are interested primarily in their use to simulate a market equilibrium and not in their welfare interpretation.

The basic optimizing market equilibrium formulation embodies the assumptions that producers are profit maximizers and that consumers' behavior is adequately described by a set of demand functions in the space of prices and quantities. Producers' supply functions are represented implicitly via specifications of their technological alternatives, the constraint set, and the objective function.

The Enke-Samuelson market simulating maximand may be interpreted as the sum of area between the demand and (implicit) supply function for each product market which is treated in the model, less costs of inputs purchased from outside the sector and less the economic rent accruing to the sector's fixed resources. When a monopolistic market is being represented, the marginal revenue function is substituted for the demand function. Factor supplies may be represented as perfectly elastic, perfectly inelastic, or in between these extremes.

Either quadratic programming (Takayama and Judge 1971) or linear programming (Duloy and Norton 1975) may be used to solve these models. By making use of a demand function, or inverse demand function of, for example, the form

$$P = D + EQ$$

where P is a vector of prices, Q of quantities produced and D and E a vector and matrix of parameters, respectively, the model may be expressed in terms of prices alone or quantities alone. In the latter case we get:

$$\text{Max } f_1 = \sum_i d_i x_i + \sum_i \sum_j 0.5 e_{ij} x_j^2 x_j^2 - \sum_i c_i q_i \tag{6}$$

subject to

$$x_i - q_i \leq 0 \tag{7}$$

$$AQ \leq b \tag{8}$$

$$X, Q \leq 0 \tag{9}$$

where

$x_i \in X$ are quantities sold,

$q_i \in Q$ are quantities produced,

$d_i \in D$

$e_{ij} \in E$

b is the vector of resource availabilities and behavioral constraints,

A is a technology and behavioral constraint matrix, and

c_i are the unit production costs for unconstrained input purchases.

Special assumptions are required on the demand matrix in order for this problem to be solvable; see section 3.21 below.

Variations have been made on this basic specification. For example, Freund (1956), Hazell (1971), Boussard and Petit (1967), Dillon and Anderson (1971), Maruyama (1972) and many others have introduced the assumption that producers are risk-averse in farm-level models, and Hazell and Scandizzo (1974) have extended that theory to sector optimizing models. (The risk question is examined in more detail in section 3.23.) As another example of departures from pure profit maximization, Kutcher and Scandizzo (1976) explored the difference between landlord and tenant production decision rules in the same kind of model.

Prior to , and in parallel with these developments with "instantaneous equilibrium" models, other authors had attempted to increase the behavioral realism of optimization models in different ways. Day (1963a) and others (Heidhues 1966; de Haen 1971; Andersen et al. 1974; Doppler 1974; Anderson and Stryg 1976; Hörner 1972) posited a degree of cautiousness in farmers' decisions in the form of limits to the amount by which production patterns could change in a given time period. This of course is a form of risk aversion also. Some authors specified an iterative adjustment of farm decisions in order to conform with regional constraints (Swanson 1971). In other cases a profit-maximizing model for a representative farm has been embedded as a submodel in a broader system which includes consumer demand functions and policy variables. Iterations are conducted to achieve consistency among the different parts of the system. In the static context, this approach is comparable to a policy-constrained Takayama-Judge model if the iterations are carried to convergence within a single time period. But in fact the iterative "simulation" models sometimes are used in order to incorporate lags explicitly in the price adjustment process, and hence full equilibrium may not be attained in any one time period. In this respect, this class of models may be distinguished from the market equilibrium models. Examples are found in de Haen and Lee (1972), Manetsch (1971), and Rossmiller (1978).

Another class of sector wide programming models is based upon identification of representative farms. Called "aggregative programming," it consists of independent specification and solution of many models for particular farms, with subsequent aggregation of solution results. This approach permits flexibility in the handling of the behavioral specification of individual farms, but it does not permit endogenous price calculations, except in a recursive manner with a lag. Large-scale examples of aggregative programming models have been constructed for England (Thomson and Buckwell 1979) and Australia (Walker and Dillion 1976; Wicks et al. 1978). The English model used fixed prices, from the Common Market, for analysis of the effect of Common Market entry on English agriculture.

The market equilibrium models successfully introduced into sector analysis a new element of economic theory and reality, the aggregate behavior of markets with respect to prices and quantities. They have been used primarily for comparative statics analysis thus far; a notable exception is the quarterly multiperiod pork sector equilibrium model of Pieri, Meilke, and MacAulay (1977). In cases of projections over time, it should be borne in mind that so far the instantaneous equilibrium models have not included the lags and obstructions to the market's process of adjusting to a new equilibrium. Hence the equilibrium position projected for a given year may not be fully realized in that year. The recursive classes of models mentioned two paragraphs above are stronger with regard to specification of lag structures, but often the empirical basis for lag assumptions is weak, e.g., the simple assumption that this year's planting decisions are governed by last year's price.

Dynamic issues aside, the instantaneous equilibrium models have an attribute which can be quite useful for policy analysis: they define a conditional equilibrium toward which the market system tends, conditional upon specified policy instrument values. Several different equilibria of this type may be delineated with the aid of the model, in order to compare the relative impetus given to the economy by alternative possible policy actions. Each "policy package" usually has multiple impacts, and it is helpful to compare the patterns of such impacts, sometimes even in qualitative form (Duloy and Norton 1973b). When these models are "applied," it is usually in this comparative sense.

The introduction of assumptions about producers' decision rules and consumer behavior also increased the scope for error in specification and estimation. This issue is discussed more fully in section 4 below.

3.2. Specific Problems*

In broad terms, a descriptive sector model comprises five structural elements which describe a) the technology set representing the production alternatives, b) the resource limitations, c) the economic environment including the consumer demand specification and the specific market conditions (e.g., competitive domestic market, existence of opportunities for international trade), d) the producers' preferences, and e) the policy environment (subsidies, taxes, and controls). In these models, the resource endowments and the policy regime (i.e., the elements of b) and e)) are represented via parameters and exogenous variables.

The major issues to be faced in the construction of these models concern the specification of consumers' behavior on the demand side and, on the supply side, the specification of the producers' decision alternatives and preferences.

3.2.1. Specification of Consumers's Behavior

On the demand side, there are two questions to be resolved in the construction of these models. One of course is the question of specification in a programming tableau (i.e., the structure of E). For the specification issue, the major concern is with cross-price effects. At present, the two most common alternatives are a) assuming zero cross-price elasticities, and b) assuming symmetric cross-price terms in a quadratic programming model. Also, for linear programming models, alternative fixed-coefficients "demand mixes" have been used (Duloy and Norton 1975). In the quadratic case, the integrability problem is encountered and it forces the symmetry assumption (Zusman 1969). If integrability is not assured, the sum of the first two terms in objective function (6) may not represent the integral under the demand function. Zusman pointed out that the symmetry requirement is indeed strong, and with empirical demand systems will be met only if the goods are "closely related in demand, have low income elasticities, and constitute a minor share of the consumer's expenditures" (Zusman 1969:55).

An alternative approach, proposed by Plessner and Heady (1965)†, involves explicit specification of both prices and quantities in the primal program. The objective function no longer represents the sum of producer and consumer surpluses, but rather the excess of consumer expenditure over the sum of factor incomes plus outlays on purchased inputs. The objective function includes a quadratic term, but since it is not derived from an integration process, the symmetry assumption (integrability requirement) may be dropped. This would appear to be a noteworthy advantage over the Samuelson-Takayama-Judge approach, and it merits more empirical exploration that it has received. A

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*For a different discussion of some of the points in this section, see the recent review paper by McCarl and Spreen (1980).

†We are grateful to Professor B. McCarl for calling our attention to this article.

practical disadvantage is that it requires much larger constraint sets, in terms of the number of rows. The main additional rows are those representing marginal-cost pricing (outputs priced at the sum of input and factor costs) for each product and production technology. However, for relatively small models, this disadvantage may not be so serious.

A related approach which also would not require the symmetry assumption is linear complementarity programming (Cottle and Dantzig 1968). When applied to the market equilibrium problem, it too requires explicit marginal-cost pricing equations. It has been applied recently to a complete sector model, not for agriculture but for the world steel economy (Hashimoto 1979).

3.2.2. Specification of Producers' Decision Alternatives

On the product supply side, the major issues concern specification in a sufficiently realistic and flexible manner the activity-analysis production vectors. Realism here refers to accuracy of production coefficients but also to sufficient disaggregation of inputs so that the truly binding constraints are represented. For example, if an annual land constraint, instead of monthly or seasonal constraints, is used, then the model probably will not reflect the peak-season competition for resources which largely determines the cropping patterns. Sufficient flexibility means an adequate number of technological alternatives so that the model's supply structure may respond to movements in relative prices. In terms of the programming tableaux, this means a production tableau which is markedly rectangular--which has many more columns than rows. Some of the existing models may be criticized for having a square, or virtually square, production submatrix. The most prominent sources of variation in production technique are degree of mechanization, fertilization levels, irrigation levels, and the possibilities of varying cropping patterns over soil classes, farm size classes, and regions.

In general, specification of an adequate set of production activities for a particular area means going beyond those activities actually observed, and inferring new techniques, partly by referring to experiment station data and also to techniques observed in other districts. A good example of construction of such a production set is found in the work of Bassoco and Rendon (1973). Use of discrete approximations to econometric productions unfortunately would appear to be ruled out, since the (dated) inputs to each production process often are quite numerous.

A major area which usually is not well treated in sector models is the labor market. Typically, the other agricultural factors, land and irrigation water, are specified with much more care. At a minimum, distinctions can be made between farm family labor and hired labor, with corresponding wage differentials. In the context of the cropping year, farmers are not as mobile as day laborers, and this difference in mobility should be reflected in differing short-run opportunity wages (Duloy and

Norton (1973b). In cases where the elasticity of day labor supply is not infinite, appropriate objective function terms can be defined so that the model gives a factor market equilibrium which corresponds to the product market equilibrium. Hazell (1979b) has presented the model structures required to achieve this and he has applied them to the rice production sector of Malaysia (Hazell 1979a). For the land market, in at least one case (Thomson and Buckwell 1979), the size distribution of farms has been changed over time in a model via application of a Markov chain rule.

Via parametric variations, sector programming models can be used to generate capital-labor substitution possibilities. This has been done for Mexico by Bassoco and Norton (1975) and Howell (1979) and for the Ivory Coast by Goreux and Vaurs (1977). Machinery labor substitution also figured importantly in the district-level analyses of Husain and Inman (1977). The market-simulating programming model will not generate isoquants in the usual sense, and hence elasticities of factor substitution in the usual sense cannot be measured. However, it does generate response functions to movements in factor prices, and these functions may be more relevant for policy decisions than isoquants are.

3.2.3. Specification of Producers' Preferences

The representation of producers' preferences is an area undergoing an interesting evolution. From an initial profit-maximizing or cost-minimizing objective, the specifications have been broadened to include risk aversion and home retentions for consumption (Bassoco and Norton 1975; Kutcher and Scandizzo 1979), and in some cases off-farm labor allocation choices (Benito 1968; Benito 1979) are explicitly allowed for. Also, in dynamic models, issues of preferences over time must be confronted.

In the treatment of risk, for stochastic yields and/or prices the principal practical alternatives facing the model builder appear to be safety-first and mean-variance decision rules. The safety-first specifications arose from the view that farmers attempt to maximize profits subject to keeping the possibility of financial ruin at a negligible level. Roy (1952) developed an early safety-first specification for the general asset holding problem. Following Roumasset (1976:37) (who attributes his discussion to Pyle and Turnovsky (1970) and Day, Aigner, and Smith (1971), three kinds of safety-first rules may be distinguished:

1. The "safety principle," under which the probability of the objective function value falling below a pre-specified critical level is minimized. This corresponds to Roy's formulation.
2. "Chance constrained programming," under which the objective function is maximized subject to a constraint which says that the probability of the objective function falling below a certain critical level must be less than or

equal to some given value α. The 1955 study of Tintner (1955) and the 1959 paper of Charnes and Cooper (1959) are the seminal works in this approach.

3. The "safety-fixed principle," or maximization of the minimum return which can be attained with the least probability α. This specification was introduced by Kataoka (1963) in 1963.

If the mean, standard deviation, and functional form* of the probability distribution of the objective function values are known, then the probabilistic statements in the various approaches can be transformed into deterministic statements in a nonlinear programming format. A linearized approach has been developed in (Hazell 1979a) and (Schiefer 1978a). Roumasset (1976) points out that the safety principle and the safety-fixed principle are both totally insensitive to variations in expected profits of the expected value of other typical objective functions. In other words, a small increase in the riskiness of the outcome cannot be offset by a large increase in the expected value of the objective function.

In all three of these specifications, either the critical objective function level (say, net income level)--the "disaster" or "ruin" level--or the maximum acceptable probability α with which ruin may be contemplated, or both parameters, are specified exogenously. Boussard (1969) considers this an advantage, but some writers would prefer that the degree of risk aversion be considered a function of the expected income level, as Arrow (1971) demeonstrates to be the case for a large class of utility functions. The mean-variance (E,V) specification of Markowitz (1959) and Freund (1956) does have this particular attribute, and it has been used in a large number of farm-level studies, e.g., Lin, Dean, And Moore (1974), Anderson, Dillon, and Hardaker (1977), and Chen and Baker (1974).

Hazell (1971) showed how to linearize the (E,V) model for farm-level studies, and Hazell and Scandizzo (1974) applied the linearization to the case of computing a market equilibrium when suppliers are confronted with risk. Tsiang (1972) has shown that the linearized (E, σ) model is justifiable under circumstances where the risk is small relative to total wealth. However, it should be noted that Tsiang's condition is likely to be violated by the situations of most smallholding farmers. More recently, Levy and Markowitz (1979) have shown that (E,V) functions can be made very close approximations to expected utility functions by judicious choice of approximating procedures. While many farm level (E,V) applications have been made, it appears that thus far only three tests of it at the regional or sector level have been carried out (Hazell et al. 1979; Hazell and Scandizzo 1979; Pomareda and Simmons 1979).

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*In cases where the functional form is not known, Tchebychev's inequality can be used for the transformation. It is, however, a very conservative measure as it is based on the "worst possible case" (see Hillier (1967)).

For the (E,V) model, a major issue is the estimation of the risk aversion coefficient, the marginal rate of substitution between expected net income and its standard deviation. Most researchers have simply simulated the consequences for the cropping pattern of different values of this parameter and then selected the value which gave the best crop fit (Hazell et al. 1979; Wiens 1976; Pomareda and Simmons 1979). However, in several cases direct farmer interviews have been used to define risk-averse utility functions (Officer and Halter 1968; Lin, Dean and Moore 1973; O'Mara 1979), and at a more aggregate level at least two studies (Moscardi and de Janvry 1977) report estimates of the risk-aversion parameter. These last studies tend to confirm the programming-simulation results which suggest that the value of the marginal rate of substitution between expected net income and its standard deviation lies between 0.5 and 2.0. At the sector level another issue is that the nature of the market equilibrium under risky production depends on how farmers form their anticipations. Hazell and Scandizzo (1977) have explored the consequences of two anticipations processes, for prices only and for revenues, in farmer decision-making.

Baumol (1974), Roumasset (1976) and others have proposed to combine the (E,V) rule with some of the safety-first rules in the spirit of lexicographic preference orderings.

Boussard and Petit (1967) developed the "focus-loss" risk specification and applied it to farmers in southern France. Essentially, application of the focus-loss procedure requires determination of what the monetary loss per hectare could be for each cropping activity in a subjectively determined "bad year," which is not necessarily the worst conceivable year. Then the optimal cropping pattern is constrained to be such that the weighted sum of the potential losses, weighted by the hectares in the optimal plan, does not exceed a pre-specified level. Kennedy and Francisco (1974) have made further applications of the focus-loss method. While ingenious, focus-loss has the serious drawback of ignoring the covariance among cropping activities' net incomes.

Empirical comparisons of the linearized (E,V) and focus-loss approaches have been made by at least two authors, Boussard (1969) and Wicks (1978). Neither formulation performed decisively better although Wicks found the linearized (E, V) model to be slightly superior. The application of safety-first and related rules is not restricted to situations with stochastic elements in the objective function of a model only. In farm models most applications of, for example, chance-constrained programming concern stochastic resource availabilities (Donaldson 1968; Rae 1971a; Boisvert 1976). In these studies, the decision-maker wishes to assure with a given probability that sufficient resources will be available for his planned program of actions.

Cocks (1968), Rae (1971a, 1971b) and Maruyama (1972) considered the case of discrete probability distributions for the stochastic parameters, and Maruyama included undertainty in three elements of the farm model: the objective function coefficients, the resource endowments, and the input requirements parameters. To date, stochastic constraints do not appear to have been incorporated into sector-wide models.

The consideration of chance constraints in a multi-period optimization problem has been discussed in (1978a), but the authors are not aware of any multi-period applications. For further reading on risk, an excellent review of decision theory under risk has been provided by Dillon (1971), and a review of risk in agricultural programming models is given by Boussard (1979).

3.3. Dynamic Aspects

Thus far, we have neglected problems that result from the dynamic aspects of the decision problems. Many farm decisions, however, are based upon present-value considerations, expected intertemporal yield changes, and other dynamic elements. To date, few sector or large regional models have included investment decisions in a multi-period context, (exceptions are the models of Day and Singh (1977), Müller et al. (1974), and Singh and Ahn (1978)).

Many intertemporal farm-level models have been built, however, beginning with the pioneering effort of Loftsgard and Heady (1959). To mention two later examples, Abalu (1974) added the salvage value of tree stumps to the decision calculation, and Willis and Hanlon (1976) specified cash flow constraints over time for the case of an apple orchard. Irwin (1968) and Kennes and Hazell (1977) provide reviews of multi-period models for farm decisions. Kennes and Hazell, who dealt exclusively with tree crops, conclude that the state of the art still is unsatisfactory, and they particularly urge a reexamination of the objective function. They feel that the sum of discounted net returns over time does not adequately represent producers' concerns, and they urge incorporation of factors such as yield uncertainty over time, inheritance obligations, and life-cycle savings behavior considerations.

In some cases, intertemporal constraints may be as important as the maximand. For example, meeting cash flow requirements during the gestation period of tree crops often inhibits their adoption by low-income farmers. Similarly, in livestock sectors, year-by-year requirements of livestock feed, management, and working capital are affected by the decisions to move from one herd size or composition to another. Biological constraints may limit the speed at which the transition can be made etc. In light of these considerations, Müller et al. (1974) incorporated constraints on annual growth rates into a large multi-period model of the agriculture and agribusiness sector of a Yugoslavian region. However, most sector models have passed over many of these considerations by specifying only alternative steady-state possibilities with respect to livestock, tree crops, and other investment opportunities.

The steady-state specifications are defined in terms of annualized input and output flows, and effectively long-cycle cropes and livestock are treated in the same manner as annual crops. This approach to the problem ignores the important fact that in the case of the long-cycle crops the investment decision largely determines the annual flow of production, and in the case of livestock the investment and production decisions are closely interrelated.

At the sector level, for descriptive models the choices would appear to be the following: define a price or revenue expectations function, and then specify the investment rule in terms of those expectations, or assume perfect foresight and hope that "on the average" it is not too erroneous an assumption. The first procedure leads to a recursive sequence of static optimization models, and the second leads to an intertemporal optimization model. Examples of recursive sector-wide or regional programming models are those of Andersen et al. (1974), Day and Singh (1977), Doppler (1974), de Haen (1971), Hörner (1972), Martin and Zwart (1975), and Thomson and Buckwell (1979).

In intertemporal optimization models, the seeming unrealism of the perfect foresight assumption can be moderated by conducting parametric solutions under different institutional bounds on investment levels, as was done by Müller et al. (1974) and with the above-mentioned linear complementary programming model for the world steel sector (Hashimoto 1979). That steel model's investment function operates so that investment occurs in a period when, allowing for gestation lags, the future (endogenous) shadow price of installed capacity rises to the level of the (exogenous) discounted price of capital. When investment occurs, then the cost of capital affects the product price via the marginal-cost pricing equations for the new vintages.

However, sector-wide multi-period models typically sidestep the investment issue either by using econometrically-estimated long-run supply functions (Pieri, Meilke, and MacAulay 1977) or by placing exogenous bounds on investment levels (Singh and Ahn 1978). Nevertheless, these procedures have been useful for analyzing issues of structural change. As noted earlier, a different approach for the consideration of dynamic aspects in models has been realized (Thomson and Buckwell 1979) where Markov chain rules have been used for modelling changes in the size distribution of farms over time.

4. MODEL VALIDATION

To be of use for an agency in its attempt to solve its decision problem, the reliability of the model's computed results has to be determined, especially with regard to the agency's objectives. The degree of the model's reliability for actual use depends on two considerations:

1. The descriptive error in the model's solutions, and
2. the stability of the results with respect to the agency's objectives.

The latter refers to the relationship between descriptive errors and the computed value of the agency's objects--the policy targets. For example, if the policy planners are particularly interested in agricultural employment, then it may not matter if the model results show compensating errors in the acreage of two crops which are comparable in labor-intensity of production.

Nugent (1970) divided the descriptive error into two components: that due to "errors and omissions" in the model, and that due to "market imperfections" which prevents the real economy from attaining the equilibrium or optimum described by the model's equations. In aggregate agricultural models, in most instances the assumption of a competitive market--subject to any existing government controls--is regarded as a reasonable accurate characterization of reality. With a few exceptions, there are too many producers for monopolistic or oligopolistic behavior to be sustained. Hence in discussions of validating agricultural models, most of the emphasis has been placed on the "errors and omissions" component.

The validation question does not really arise in the context of a single farm model. All coefficients and restraints are subject to direct verification, and the maximand is whatever the farmer would like it to be. In fact, usually the farmer would like to investigate the consequences of adopting different maximands. (An outstanding example of linear programming packages being placed at the disposal of farmers for improvement of their operations is found in the extension work of Purdue University (McCarl et al. 1977).

In regional and sector models, the principal sources of error typically include the following:

a) aggregation over a large number of non-homogenous producers;
b) errors in specification and estimation of commodity demand functions;
c) errors in the treatment of factor markets;
d) errors in the specification of the objective functions;
e) insufficiently detailed seasonality of input specifications;
f) other omissions of behavioral and technical constraints;
g) errors in production coefficients and estimated resource availabilities and other technical parameters.

As regards aggregation, there is a substantial literature on ways of reducing aggregation bias, beginning with the early paper of Day (1963b). He investigated the conditions for exact aggregation and classified them into three groups which require that aggregated farms be homogeneous with respect to their decision rules, their factor proportions, and their technical production possibilities. Miller (1966) and Lee (1966) also studied the requirements for exact aggregation, and Buckwell and Hazell (1962) and Kennedy (1974) pursued the question of how to minimize aggregation bias, by grouping farms via clustering techniques and econometric methods.

In practice, grouping farms by the following four kinds of distinctions has been found to be important for reducing aggregation bias:

a) irrigated-nonirrigated categories;
b) farm size categories (labor-land ratio groupings);
c) basic cropping regime categories (e.g., livestock vs. annual crops); and
d) spatial categories.

The spatial categories can be useful for capturing other sources of yield variation, such as differences in rainfall and altitude.

Although it would not be practical for models which represent large numbers of producers, it is worth noting the suggestion of Lin, Dean, and Moore (1974) that farms should be aggregated according to the characteristics of the utility functions of the farmers.

In general, aggregating non-homogeneous production units tends to overstate the production potential of the sector. While this bias must be recognized, it should be mentioned that it is not necessarily large in aggregate magnitude. In the Mexican case, for example, it was found that the model overstated a quantum index of production for the sector by about 12% (Bassoco and Norton 1975). In the cited study, it was pointed out that this error arises from three kinds of sources: a) aggregation bias, b) omission of important restrictions, and c) omission of some real market imperfections. The comparable overstatement for Kutcher and Scandizzo's multi-crop model of Northeast Brazil (1979) was 8%. Given that aggregation was one of three sources of this error, its induced bias in this case was perhaps not as large as it sometimes feared. Nevertheless, at the level of crops and other specific variables, the aggregation error usually is more pronounced. Tests of aggregation errors in sector linear programming models have been performed for Mexico and Portugal. For Mexico, Duloy and Norton (1973a) found that aggregation of land, labor, and irrigation constraints from a monthly to a quarterly basis produced substantial errors in many variables. For Portugal, Egbert and Kim (1975) found important errors associated with spatial aggregation over producing regions.

To validate a model once it has been constructed, the most common procedure is to compare the solution with reality in terms of acreage, production, and prices. One measure of goodness-of-fit is the mean absolute deviation (MAD) or, in terms which better facilitate comparisons, the percentage absolute deviation (PAD). The latter measure is defined as the average (over crops) absolute deviation between the model results and the data, divided by the average actual value. For four price-endogeneous, multi-crop medels, the PADs are as follows:

Table 1. Validation Measures for Agricultural Sector Models

Model name	Country	Reference	Concept	PAD
CHAC	Mexico	Bassoco and Norton 1975	Production	13.4%
MAAGAP	Philippines	Kunkel et al. 1978	Acreage	9.1%
--	N.E. Brazil	Kutcher and Scandizzo 1976	Production	8.2%
MOCA	Costa Rica	Cappi et al. 1978	Production	7.0%
	El Salvador			12.0%
	Guatemala			7.1%
	Honduras			9.3%
	Nicaragua			8.7%

MOCA is a multi-country model for Central America with international trading possibilities. The PADs are reported directly in the cited reference by Cappi et al. For the other cases, the PADs were computed from information available in the studies. More extensive validation tests with Mexican regional models are reported in Hazell et al. (1979), Howell (1979), Kutcher (1979), and Pomareda and Simmons (1979). The validation results in Table 1 may appear encouraging, but it should be borne in mind that, owing to the fact that agricultural demand elasticities typically are less than unity in absolute value, the price deviations generally are greater than the quantity deviations.

Dynamic validation, or historical "backcasting" has rarely been attempted with agricultural sector models; a notable exception are the studies of Pieri, Meilke, and MacAulay (1977) and Martin and Zwart (1975). It clearly ought to be attempted more often, for it would assist in the evaluation of the supply responsiveness of the models.

The sensitivity of policy target variables to model errors is a topic which is not often explored formally. In descriptive optimization models, tests could be performed to measure the effect of errors (say, variations in parameter values) on variables which are of interest to policy makers. If a probability distribution of data values could be obtained, then via repetitive solutions of the model under random drawings from the data distribution, a probability distribution of target variable values could be traced out. The effects of possible errors in the model may be highly significant, even if the possible errors may seem rather small. This has been demonstrated with a programming model for the agricultural sector of northern Germany. In that example, a possible error in the objective function was simulated by reducing the objective function value to a value which was 3% below the optimal value. With regard to a group of livestock-oriented regions this was compatible with a 16% to 55% variation in the percentage of the total herd located in these regions (Schiefer 1977).

These results apply to normative models as well, where it can be useful to examine the consequences of nearly optimal solutions in the spirit of Kornai (1969a).

If we adopt the viewpoint of the decision maker, some decisions on the model structure can be determined via reliability testing. The effects of seasonal disaggregation in one instance were mentioned earlier; in fact, the optimal degree of disaggregation (up to the limits imposed by data and computing systems) could be determined by validation tests and by tests of the sensitivity of target variables to further disaggregation. (The models described in Bauersachs (1972) and Henrichsmeyer and de Haen (1972) provide examples which differ mainly in the degree of spatial disaggregation.) Such procedures would be expensive, but if a model were to be utilized frequently over a relatively long period of time, then it might be worth incurring the costs.

5. OTHER LEVELS OF ANALYSIS

The farm and the sector are natural levels of analysis, and accordingly they have received the bulk of the attention from agricultural model builders. Two other levels also have been mentioned in the foregoing discussion: the multi-country and regional (district) levels. Bawden (1966) extended the Takayama-Judge spatial equilibrium model to include international trade possibilities, and he showed how it may be usefully employed to analyze the multi-country impacts of national policy changes in agriculture. The objective function becomes the sum of individual country objective functions, less international transport costs. Accounting balance equations are added for the trade flows. The spatial equilibrium approach eliminates the necessity of attempting to estimate import demand functions and export supply functions.

Cappi et al. (1978) constructed an international spatial equilibrium model for the agricultural sectors of five Central American countries, and they used it to analyze the incidence of benefits and costs of further steps toward economic integration in the Central American Common Market. Their procedure was essentially that of Bawden, except that linearizations were used to convert the model to linear programming format. For the pork sector alone, Pieri, Meilke, and MacAulay (1977) developed a spatial equilibrium model to analyze Japanese-North American production, trade and consumption.

For sector models, a complete sector normally is defined by complete coverage of the commodity balance equation for related products--by representation of all supply sources and all uses of the products. Factors markets usually are not represented as completely, although some assumptions may be made about the opportunity cost of factors in employment outside the sector. At the subsector level (region or district), product markets also are represented incompletely. When the region's share of sector-wide supply is not insignificant, then a question arises about the nature of the demand schedules facing the region's producers.

As Kutcher (1979) has shown, the relation between the national demand elasticities (evaluated at the observed prices and quantities) and the appropriate demand elasticities for the retional market depends on the assumptions made about producers' supply responsiveness in other regions. At one extreme, if producers elsewhere do not respond to price changes in the local market under consideration, then the local market demand elasticity values are the national values divided by the region's share of national production; i.e., the national and regional demand slopes are identical at the observed points. On the other hand, if supply elasticities elsewhere are equal to those of the region, then the local demand elasticities must be identical in value, when evaluated at the locally observed quantities, with the national elasticities, when the latter are evaluated at the observed national quantities.

When projections are made over time, then questions of the future local share in national markets arise. For example, if arable land and/or irrigation supplies in the sector are being expanded, then the market share of a region with fixed land and water endowments will decline. Regional market demand curves for future years which take into account changing market shares may in fact be derived from projections of the agricultural-nonagricultural terms of trade (Bassoco et al. 1979).

It is difficult to disaggregate sectoral models sufficiently so that specific investment projects may be included, but the regional/district level is appropriate for project identification analyses. For public investments in irrigation, land improvement, agricultural machinery, etc., use of annualized benefits and costs may be admissible in cases where the principal concern is to rank project components in terms of rate of return and to define the project package (Bassoco, Norton and Silos 1974; Husain and Inman 1977; Bassoco et al. 1979). Hazell (1979a) has carried out an extensive programming model study of the impacts over time of an irrigation project on a rice-growing zone of Malaysia. Little use, however, has been made of programming models for integrated rural development projects, owing to the difficulty of quantifying some of the benefits. A step in this direction has been taken by Benito (1968, 1979).

Apart from the extensive Hungarian multi-level planning experience, there is not much literature on the linkage between agricultural optimizing models and economy-wide analyses or models. In the simulation context, iterative linkages have been made (Byerlee and Halter 1974), even when the agricultural model is a programming model and the others are not (Rossmiller 1978). In the purely optimizing context, a linkage between sector and national models was made for Mexico (Duloy and Norton 1973c). In that case, the sector model was solved under varying assumptions about factor prices (of capital, labor, foreign exchange) in order to generate a set of alternative agricultural technology vectors for inclusion in an economy-wide programming model. The assumptions on factor prices had been derived from a prior set of solutions of the economy-wide model. Perhaps the principal methodological conclusion of that exercise was that it was much more relevant for economy-wide analysis than for sector analysis. The reason is that the economy-wide model was altered significantly by the inclusion of new information from the sector model on factor substitution possibilities, but for sector studies it is a simple matter to dispense with the economy-wide model and to perform sensitivity analysis on a few plausible alternative values of key parameters such as the foreign exchange rate. In other words, the flow of information regarding technical production possibilities was more helpful than particular solution results regarding factor prices.

6. CONCLUDING REMARKS

Agricultural programming models are relatively new constructs, and from the foregoing it can be seen that they have stirred considerable ferment in the profession in the last two decades. Some applications have been made, perhaps prematurely, but the attempt to confront actual decision problems certainly has contributed to the improvement of methodology and, we suspect, has assisted in the clarification of a number of choice situations. A by-product of attempted applications sometimes is the re-definition of data-gathering priorities: statistics are not costless to collect, and a model can help to determine which parameters are most critical for decision-making, i.e., those parameters for which errors in estimation have the greatest consequence for decisions.

In reviewing what is essentially an applied field, mention must be made of the interaction with the computer and the data, a process which sometimes is referred to generally as "model management." It is beyond the scope of this paper to pursue this issue, but it should be mentioned that adequate model management procedures are usually essential to the success of model applications (Kutcher and Meeraus 1979; Thomson and Buckwell 1979). To be useful in a policy setting, a model must be solved many times, usually hundreds of times, and its structure and parameter values frequently are altered. The minimum necessary pieces of software, apart from the solution routine, are a matrix generator and a report generator. Experience has shown that the presence of these tools can reduce a process which requires months to one which requires days or weeks at most (Norton and Solis 1979a).

Imperfect and abstract though they may be, models sometimes provide the policy maker's best--and most consistent--picture of reality. The consistency dimension is important and hard to ensure without a model when many variables are being considered simultaneously. From a policy viewpoint, one of the practical virtues of sector and regional models is their specification at a realistic level of detail--for example, with the prices of maize and wheat instead of an overall agricultural price index. This enables persons outside the circle of the model builders to comment on and contribute to the process of building and using the model. Model builders can enhance the usefulness of their tools by encouraging use of judgments, on specifications and parameter values, where appropriate. The judgments always should be made explicit and whenever possible sensitivity tests should be performed, but there is nothing inherently unscientific about the use of judgmental information in decision models so long as it is recorded as such. In this way, a policy maker can come to better understand the consequences of his own "mental model."

We would like to close this brief review by mentioning six areas in which the present methodology seems to be far from adequate in terms of the problems to which these models are addressed. The first three combine algorithmic or computational difficulties with specification issues, and the second three require greater ingenuity in applying existing theory or modifications of theory. The first group of topics includes multi-level models, inter-sectoral linkages, and multi-period price-endogenous models. Efficient multi-level algorithms would permit combining the normative and descriptive problems and scanning the policy feasibility space more thoroughly. Incorporating intersectoral linkages means, for example, allowing for agricultural-nonagricultural income multipliers and shifting the price-elastic demand curves to reflect endogenous income changes. It also means specifying factor supply functions facing agriculture. Improving multi-period models requires advances in understanding of dynamic decisions and also adequate recognition of the role of expectations and lagged responses, as noted earlier.

The other three topics which are particularly slighted in the present generation of models are land tenure systems, factor markets, and integrated farm household decisions. Apart from the Kutcher-Scandizzo (1976) study for northeast Brazil, there has been no attempt to capture the influence of land tenure considerations in the specification of a mathematical programming objective function. Factor prices are generally not endogenous to sector models, and land market is simply ignored. Yet renting-in and renting-out behavior, and other forms of land transactions, can affect the rural income distribution and crop supply responsiveness as well. Hazell's studies on factor markets in programming models (1979a, 1979b) provide some guidance in this area. Finally, apart from a paper by Boussard (1971), little has been done to incorporate farm household decisions on consumption, production, and labor allocation in an integrated model. These decisions are especially inter-related for subsistence farmers.

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