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A Simple Method of Measuring the Increase of Life Expectancy When a Fixed Percent of Deaths from Certain Causes are Eliminated

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IIASA Collaborative Paper
December 1980

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FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

In this paper, Professor Nanjo, from the Fukushima Medical College, Japan, generalizes Keyfitz's method for measuring the increase of life expectancy due to a marginal reduction in any one cause of death. He relaxes Keyfitz's assumption that the number of deaths in each age group is decreased at a fixed rate and goes on to derive a mathematical formulation that leads to an improved approximation.

Recent publications in the Health Care Systems Task are listed at the end of this report.

Andrei Rogers
Chairman
Human Settlements
and Services Area

ACKNOWLEDGMENTS

I am very grateful to Prof. N. Keyfitz for having read the draft of this paper and having made some kind comments on it. Further, I am much obliged to Dr. A. Rogers who has been so kind as to propose some changes in the draft and make some useful suggestions on it. This paper could never have been completed without their invaluable help.

ABSTRACT

The effect that one type of medical improvement will have on life expectancies is often computed using a life table. In classical methods, such as Greville's, the increase in life expectancy has been dealt with by assuming that deaths from a particular cause have been eradicated. Keyfitz derived a parameter that measures the increase in life expectancy by a marginal reduction in any cause of death. The parameter is additive in several causes and useful for various studies of causes of death.

This paper is a generalization of Keyfitz's idea and deals with a case where some percent of the deaths from a particular cause are eliminated, not necessarily uniformly in all age intervals.

CONTENTS

A SIMPLE METHOD OF MEASURING THE INCREASE OF LIFE EXPECTANCY WHEN A FIXED PERCENT OF DEATHS FROM CERTAIN CAUSES ARE ELIMINATED

1. INTRODUCTION

Life tables are often used in the analysis of the increase in life expectancy when certain death causes are eradicated. With this tool we are able to obtain the difference between the life expectancy for all death causes and the one calculated on the assumption that deaths from a certain death cause have been eliminated. There are several methods for calculating these life tables, among them Greville's methods (Greville,1948 and 1954), and those of Preston et al.(1972) are well-known. However, when we use these traditional methods, the following points are questioned.

(a) Usually

$$d_A + d_B < d_{A+B}$$

when d_A, d_B, and d_{A+B} denote, respectively, the increase of life expectancy assuming that death causes A, B, and A+B have been eradicated. In this case equality does not hold. That is to say, the increase of life expectancy is not additive with the two causes of death.

(b) Greville's and other traditional methods discuss the case in which deaths from a certain cause have been eradicated. We are more concerned here with the case in which some percent of the deaths from several causes are decreased. In the former case, for example, one hundredth of an increase in life expectancy, assuming that deaths from cause A have been eradicated, cannot be used as the increase of life expectancy when one percent of deaths from death cause A has been eliminated.

(c) The traditional methods assume that several death causes are independent.

In our case the assumption in (c) cannot be avoided. Points (a) and (b), however, have been discussed by Keyfitz (1977a). He has derived a parameter that measures how much life expectancy is increased due to a marginal reduction in any cause of death. This parameter is additive when several causes of death are considered.

Our method is a generalization of Keyfitz's idea. When some percent of the deaths from certain causes are eliminated, not necessarily uniformly in all the age intervals, we can easily get the life expectancy, based on a given life table, by using the sets of parameters that have been obtained beforehand. Our method is also additive for several causes and has interesting applicability to the study of causes of death.

2. GENERALIZATION OF KEYFITZ'S RESULTS

To begin with, I will explain briefly Keyfitz's idea (Keyfitz,1977a,b). If the chance of dying in the time interval dx of a year for a person who has reached age x has been $\mu(x)dx$, suppose that this is changed to $\mu'(x)dx = \mu(x)(1+\delta)dx$. In this case, δ will be a small negative quantity, typically -0.01, representing 1 percent improvement in all causes at all ages. The probability of living to age x then changes from

$$l(x) = \exp[-\int_0^x \mu(x)dx]$$

to

$$l'(x) = \exp[-\int_0^x \mu(x)(1+\delta)dx] = l(x)^{1+\delta}$$

this being approximately

$$l(x)[1+\delta \ln l(x)]$$

in the neighborhood of $\delta = 0$ by Taylor's expansion. The life expectancy at birth changes from

$$\mathring{e}_0 = \int_0^\omega l(x)dx$$

to

$$\mathring{e}'_0 = \int_0^\omega l'(x)dx$$

Therefore we have

$$(\mathring{e}'_0 - \mathring{e}_0)/\mathring{e}_0 = \int_0^\omega [l'(x)-l(x)]dx/\int_0^\omega l(x)dx$$

$$= -\delta[\int_0^\omega -l(x)\ln l(x)dx/\int_0^\omega l(x)dx] = -\delta H,$$

where

$$H = -\int_0^\omega l(x)\ln l(x)dx/\int_0^\omega l(x)dx$$

This expression shows that the effect of the elimination of 1 percent of deaths on the life expectancy will increase $\mathring{e}_0$ by H percent.

Similarly, suppose the age specific death rate from the cause i changes from

$$\mu^{(i)}(x)$$

to

$$\mu^{(i)}(x)(1+\delta)$$

The probability of living to age x then changes from l(x) to

$$l(x)[l^{(i)}(x)]^{\delta} \doteq l(x)[1 + \delta \ln \quad l^{(i)}(x)]$$

where $l^{(i)}(x)$ is the probability of living to age x in the face of risks from the cause i alone. Therefore, in the same way as we mentioned above, we have

$$(\mathring{e}_0' - \mathring{e}_0)/\mathring{e}_0 = -\delta H^{(i)}$$

where

$$H^{(i)} = -\int_0^{\omega} l(x) \ln \quad l^{(i)}(x)dx / \int_0^{\omega} l(x)dx$$

This shows that the effect of the elimination of 1 percent of deaths from the i-th cause on the life expectancy will increase $\mathring{e}_0$ by $H^{(i)}$ percent.

Keyfitz's idea is based on an assumption that the number of deaths at each age group is decreased at a fixed rate. This assumption is, however, often unsuitable for real situations. Here, therefore, let us make a general assumption that the rate of decrease in deaths at each age group is not necessarily fixed.

According to Greville, it is known that the probability of living n year after age x in a life table from which the cause i is completely eliminated is closely approximated from

$${}_nP_x^{(-i)} = {}_nP_x^{\,1-{}_n\gamma_x^{(i)}}$$

where ${}_nP_x$ relate to the life table with all causes present, and

$${}_n\gamma_x^{(i)} = {}_nD_x^{(i)} / {}_nD_x$$

in which ${}_nD_x$ and ${}_nD_x^{(i)}$ represent respectively the number of deaths from all the causes and from the cause i alone in the age group (x, x+n-1). In this case, if the number of deaths from the cause i is not eradicated but decreased by $-100\delta_x$%, it can be shown that the probability of living in the age interval is

$${}_nP_x^{\,1+{}_n\gamma_x^{(i)}\delta_x} \qquad (\delta_x < 0)$$

as in Greville's expression (Greville, 1948).
Therefore, if the deaths from the cause i are decreased at the rate of $-\delta_0$, $-\delta_5$, ..., $-\delta_{80}$, $-\delta_{85}$ for the age interval 0~4, 5~9, ..., 80~84, 85+ respectively, the probability of living to age x is

$$l'(x) = \left[{}_5P_0^{\,1+{}_5\gamma_0^{(i)}\delta_0}\right]\left[{}_5P_5^{\,1+{}_5\gamma_5^{(i)}\delta_5}\right]\cdots\left[{}_5P_{x-5}^{\,1+{}_5\gamma_{x-5}^{(i)}\delta_{x-5}}\right]$$

$$= \left[{}_5P_0 \cdot {}_5P_5 \cdots {}_5P_{x-5}\right] \cdot \left[{}_5P_0^{\,{}_5\gamma_0^{(i)}}\right]^{\delta_0} \cdots \left[{}_5P_{x-5}^{\,{}_5\gamma_{x-5}^{(i)}}\right]^{\delta_{x-5}} \qquad (1)$$

This is equivalent to Keyfitz's expression, $l(x)\,[l^{(i)}(x)]^{\delta}$, if $\delta_0 = \delta_5 = \ldots = \delta$. Here, using Taylor's theorem as above to hold linearity, we have

$$l'(x) \doteqdot {}_5P_0\,{}_5P_5\cdots{}_5P_{x-5}(1+\delta_0 \ln {}_5P_0^{\,{}_5\gamma_0^{(i)}})(1+\delta_5 \ln {}_5P_5^{\,{}_5\gamma_5^{(i)}})\ldots$$

$$(1+\delta_{x-5} \ln {}_5P_{x-5}^{\,{}_5\gamma_{x-5}^{(i)}})$$

$$\doteqdot ({}_5P_0\cdots{}_5P_{x-5})(1+\delta_0 \ln {}_5P_0^{\,{}_5\gamma_0^{(i)}} + \ldots + \delta_{x-5} \ln {}_5P_{x-5}^{\,{}_5\gamma_{x-5}^{(i)}}) \qquad (2)$$

This is a closely approximated expression when each δ_k is small. Thus

$$\overset{\circ}{e}{}'_0 - \overset{\circ}{e}_0 = \int_0^{\omega} l'(x)dx - \int_0^{\omega} l(x)dx \tag{3}$$

can be expressed in the linear form

$$(-\delta_0)C_0^{(i)} + (-\delta_5)C_5^{(i)} + \ldots + (-\delta_{85})C_{85}^{(i)} \tag{3a}$$

where $C_x^{(i)}$ can be calculated from equation (2). Therefore, from this expression we can obtain the quantity of increase in the life expectancy at birth $\overset{\circ}{e}{}'_0 - \overset{\circ}{e}_0$ at once, if we have coefficients $C_0^{(i)}, \ldots, C_{85}^{(i)}$ for the cause of death i beforehand.

3. OUR METHOD OF COMPUTATION

For x=0,5,10,... we denote l(x), l'(x) by l_x, l'_x respectively, and then write

$$_5d_x = l_x - l_{x+5} \tag{4}$$

$$\beta_x = \gamma_x \ln {}_5P_x = {}_5\gamma_x^{(i)} \delta_x \ln {}_5P_x \tag{5}$$

$$l'_x = l_x \; (1+\beta_0 + \beta_5 + \ldots + \beta_{x-5}) \; [\text{cf.}(2)] \tag{6}$$

$$_5d'_x = l'_x - l'_{x+5} \tag{7}$$

Let $_nL_x$ be the total person-years lived by the stationary population in the age-interval x to x+n. Then

$$_nA_x = \frac{_nL_x - n l_{x+n}}{_nd_x} \tag{8}$$

is the average number of years lived in the age-interval x to x+n by those who die in it. Using this ${}_nA_x$, we estimate ${}_nL'_x$, as is often done, by

$$ {}_nL'_x = nl'_{x+n} + {}_nA_x \cdot {}_nd'_x \tag{9} $$

Here for the sake of brevity, we use $A_x, d_x, d'_x, \gamma_x^{(i)}$, L_x and L'_x in place of ${}_5A_x$, ${}_5d'_x$, ${}_5\gamma_x^{(i)}$, ${}_5L_x$, and ${}_5L'_x$. In the last age-interval, we use the age-interval of ages over x. For example, $L_{100} = {}_\infty L_{100}$. Then we have

$$ L'_0 = 5l'_5 + A_0 d'_0 = 5l_5(1+\beta_0) + A_0\{l_0 - l_5(1+\beta_0)\} $$

$$ = 5l_5(1+\beta_0) + A_0(d_0 - l_5\beta_0) \qquad (\because l_0 = l'_0) $$

Similarly

$$ L'_5 = 5l'_{10} + A_5 d'_5 = 5l_{10}(1+\beta_0+\beta_5) + A_5\{d_5(1+\beta_0) - l_{10}\beta_5\} $$

$$ L'_{10} = 5l'_{10} + A_{10}d'_{10} = 5l_{15}(1+\beta_0+\beta_5+\beta_{10}) + A_{10}\{d_{10}(1+\beta_0+\beta_5) - l_{15}\beta_{10}\} \tag{10} $$

....

$$ L'_{95} = 5l'_{100} + A_{95}d'_{95} = 5l_{100}(1+\beta_0+\ldots+\beta_{95}) + A_{95}\{d_{95}(1+\beta_0+\ldots+\beta_{90}) - l_{100}\beta_{95}\} $$

In the last age-interval

$$ L'_{100} \doteq L_{100}(1+\beta_0+\ldots+\beta_{95})/(1+\gamma_{100}) $$

This is equivalent to Greville's expression (Greville, 1954) used to get the life expectancy $\mathring{e}_x$ at age x in the last age-interval (x,∞) when a death cause has been eradicated. This is approximately equal to

$$L_{100}(1 + \beta_0 + \dots + \beta_{95})(1 - \gamma_{100}) \tag{11}$$

$$= L_{100}\left\{(1 + \beta_0 + \dots + \beta_{95}) - \gamma_{100} - \gamma_{100}(\beta_0 + \dots + \beta_{95})\right\}$$

$$\doteq L_{100}\ (1+\beta_0 + \dots + \beta_{95}) - \gamma_{100}L_{100}$$

if γ_k $(k = 0, 5, 10, \dots)$ is small.

Therefore we have

$$L_0' - L_0 = 5l_5(\beta_0) + A_0(-l_5\beta_0) \tag{12}$$

$$= (5-A_0)l_5\beta_0$$

$$L_5' - L_5 = 5l_{10}(\beta_0+\beta_5) + A_5\{d_5\beta_0 - l_{10}\beta_5\} \tag{13}$$

$$= L_5\beta_0 + 5l_{10}\beta_5 - A_5 l_{10}\beta_5$$

$$= L_5\beta_0 + (5-A_5)l_{10}\beta_5$$

$$L_{10}' - L_{10} = 5l_{15}(\beta_0+\beta_5+\beta_{10}) + A_{10}\{d_{10}(\beta_0+\beta_5) - l_{15}\beta_{10}\}$$

$$= L_{10}(\beta_0+\beta_5) + 5l_{15}\beta_{10} - A_{10}l_{15}\beta_{10} \tag{14}$$

$$= L_{10}(\beta_0+\beta_5) + (5-A_{10})l_{15}\beta_{10}$$

....

$$L_{95} - L_{95} = 5l_{100}(\beta_0 + \dots + \beta_{95}) + A_{95}\{d_{95}(\beta_0 + \dots + \beta_{90}) - l_{100}\beta_{95}\}$$

$$= L_{95}(\beta_0 + \dots + \beta_{90}) = (5-A_{95})l_{100}\beta_{95} \tag{15}$$

and

$$L'_{100} - L_{100} = L_{100}(\beta_0 + \dots + \beta_{95}) - \gamma_{100}L_{100} \tag{16}$$

Sum up both sides of the expressions (12) to (16) and divide the total by l_0. Then if we use

$$T_x = L_x + L_{x+5} + \ldots, \quad T'_x = L'_x + L'_{x+5} + \ldots,$$

we obtain

$$\overset{\circ}{e}'_0 - \overset{\circ}{e}_0 = \frac{1}{l_0}(T'_0 - T_0) \tag{17}$$

$$= \frac{1}{l_0}\Big[\beta_0\{T_5+(5-A_0)l_5\}$$

$$+ \beta_5\{T_{10}+(5-A_5)l_{10}\}$$

$$+ \beta_{10}\{T_{15}+(5-A_{10})l_{15}\} + \ldots$$

$$+ \beta_{95}\{T_{100}+(5-A_{95})l_{100}\} - \gamma_{100}T_{100}\Big]$$

If we write

$$C_x^{(i)} = -\frac{\gamma_x^{(i)} \ln {}_5P_x}{l_0}\{T_{x+5}+(5-A_x)l_{x+5}\} \quad (x=0,5,10,\ldots,95)$$

$$C_{100}^{(i)} = \gamma_{100}^{(i)} T_{100}/l_0 \tag{18}$$

and if we refer to the expression (5), we obtain

$$\overset{\circ}{e}'_0 - \overset{\circ}{e}_0 = (-\delta_0)C_0^{(i)} + (-\delta_5)C_5^{(i)} + \ldots + (-\delta_{100})C_{100}^{(i)} \tag{19}$$

In this expression, if $C_{85}^{(i)} + C_{90}^{(i)} + \ldots + C_{100}^{(i)}$ is denoted by $C_{85}^{(i)}$ we obtain the expression (3a) in section 2. In the case of $\overset{\circ}{e}'_{60} - \overset{\circ}{e}_{60}$, as in the case of age 0, we start from l_{60} for age 60 and obtain

$$\overset{\circ}{e}{}'_{60} - \overset{\circ}{e}_{60} = \frac{1}{l_{60}}[\beta_{60}\{T_{65}+(5-A_{60})l_{65}\}$$

$$+ \beta_{65}\{T_{70}+(5-A_{65})l_{70}\} + \ldots$$

$$+ \beta_{95}\{T_{100}+(5-A_{95})l_{100}\}-\gamma_{100}T_{100}] \tag{20}$$

Therefore if we write

$$C_x^{(i)} = -\frac{\gamma_x^{(i)}\ln {}_5P_x}{l_{60}}\{T_{x+5}+(5-A_x)l_{x+5}\} \quad (x=60,65,\ldots,95)$$

$$C_{100}^{(i)} = \gamma_{100}^{(i)}T_{100}/l_{60} \tag{21}$$

we obtain

$$\overset{\circ}{e}{}'_{60} - \overset{\circ}{e}_{60} = (-\delta_{60})C_{60}^{(i)}+\ldots+(-\delta_{100})C_{100}^{(i)} \tag{22}$$

Now if the coefficients $C_x^{(i)}$ are computed for $x = 0,5,\ldots,85$, or $x = 60,65,\ldots,85$, it seems to serve our purpose sufficiently. However, there is some doubt about the expression in the treatment of the last coefficient:

$$C_{85}^{(i)} = \gamma_{85}^{(i)}T_{85}/l_0, \text{ or } \quad C_{85}^{(i)} = \gamma_{85}^{(i)}T_{85}/l_{60}.$$

Therefore we computed $C_x^{(i)}$ to as advanced an age as possible, that is, to 100 years of age as shown above. In this case, it should be noted that $C_{100}^{(i)}$ is extremely small. In Japan, the data necessary for this computation are available.

In its practical use, we can use the table that summed up the figures of ages 85 and over form the table calculated in the above-mentioned way (cf. Table in Section 5 and Appendix tables).

4. RELATION BETWEEN OUR RESULTS AND THE RESULTS OF KEYFITZ

According to Keyfitz (1968, p.342), it is known that the effect on $\overset{\circ}{e}_0$ of $\Delta\,{}_5q_x = {}_5q'_x - {}_5q_x$ when the probability of dying in the age-interval (x,x+4) changed from ${}_5q_x$ to ${}_5q'_x$ is approximately

$$\overset{\circ}{e}{}'_0 - \overset{\circ}{e}_0 \doteqdot -\frac{l_x}{l_0}(\overset{\circ}{e}_{x+5}+5-A_x)\Delta\,{}_5q_x \tag{23}$$

And if we take out only an age-interval (x, x+4) from the expression (17), we obtain

$$\begin{aligned}\overset{\circ}{e}{}'_0 - \overset{\circ}{e}_0 &= -\frac{1}{l_0}\beta_x\{T_{x+5}+(5-A_x)l_{x+5}\}\\ &= \frac{1}{l_0}\beta_x l_{x+5}(\overset{\circ}{e}_{x+5}+5-A_x)\end{aligned} \tag{24}$$

The expression (24) is one which was obtained by quite a different idea from (23), but both of them give much the same result. This is shown in the following way. If $\delta_x\gamma_x^{(i)}$ is small

$${}_5P_x^{\,1+\delta_x\gamma_x^{(i)}} \doteqdot {}_5P_x(1+\delta_x\gamma_x^{(i)}\ln{}_5P_x) \tag{25}$$

Therefore, if we denote ${}_5P_x^{\,1+\delta_x\gamma_x^{(i)}} - {}_5P_x$ by Δ_5P_x, we obtain from the expression (25)

$$\Delta_5P_x = \delta_x\gamma_x^{(i)}\,{}_5P_x\ln{}_5P_x$$

i.e.

$$-\Delta_5q_x = \delta_x\gamma_x^{(i)}\frac{l_{x+5}}{l_x}\ln{}_5P_x \qquad (\because {}_5P_x=1-{}_5q_x)$$

therefore

$$-l_x \Delta_5 q_x = \delta_x \gamma_x^{(i)} \log {}_5P_x \; l_{x+5} = \beta_x l_{x+5}$$

so we get our result.

5. AN APPLICATION AND COMMENTS

As an example, let me show part of the table (Table 1) of $C_k^{(i)}$ for males, which is taken out of the Table A2 (Appendix) computed by our method, using the life table and the statistics of mortality of Japan for 1970.

Table 1. The coefficients $C_k^{(i)}$ for malignant neoplasms and cerebrovascular disease on Japanese males, 1970

Start of age interval	coefficients	Causes of death B19[a]	B30[b]
Total	C	2.02632	2.46930
0	C_0	0.02698	0.00530
5	C_5	0.01705	0.00148
10	C_{10}	0.01338	0.00158
15	C_{15}	0.01855	0.00352
20	C_{20}	0.02115	0.00448
25	C_{25}	0.02680	0.00958
30	C_{30}	0.03874	0.01798
35	C_{35}	0.05910	0.04426
40	C_{40}	0.08999	0.07757
45	C_{45}	0.13889	0.10699
50	C_{50}	0.19812	0.16027
55	C_{55}	0.26542	0.23360
60	C_{60}	0.31843	0.33045
65	C_{65}	0.32470	0.41371
70	C_{70}	0.25360	0.44192
75	C_{75}	0.14763	0.35570
80	C_{80}	0.05458	0.18977
85	C_{85}	0.01322	0.07116

[a] B19: Malignant Neoplasms

[b] B30: Cerebrovascular Disease

According to the table above, if the number of deaths or death rate from B19 at all ages is decreased by 3 percent, we have only to multiply C by 0.03 to get the increment of $\overset{\circ}{e}_0$: $2.02632 \times 0.03 = 0.061$. As another example, if the death rate from B19 in 1970 is decreased by 4 percent at ages over 50 and by 2 percent at the other ages, $0.02 \times (C_0+\ldots+C_{45}) + 0.04 \times (C_{50}+\ldots+C_{85}) = 0.072$, that is, $\overset{\circ}{e}_0$ will increase by 0.072 of a year.

And if the death rate from B30 decreases by 3 percent at ages over 60,

$$0.03 \times (C_{60} + C_{65} + \ldots + C_{85}) = 0.054$$

that is, $\overset{\circ}{e}_0$ will increase by 0.054 of a year. If we want to take these two cases together, we have only to sum up the two results above, so we have

$$0.072 + 0.054 = 0.126 \text{ year}$$

In our computation we also assume the independence of death causes as do the traditional methods. [Assumption (c) in section 1]. And the effect of reduction of deaths from a death cause upon the life expectancy is extremely small. If the assumption of independence is not built up, the effect will become still smaller. In fact, however, the independence does not exist.

Keyfitz's and our methods should be used for the case of marginal reduction but not for that of the eradication of deaths from a particular cause. However, if these methods are to be used for the latter case, the effect of the eradication of deaths from a cause upon the life expectancy obtained by Keyfitz's method will be a little smaller than that of Greville's method, and the effect obtained by our method will be very much smaller than that of Keyfitz's method. If we take the assumption of independence into account, we can consider our result to be the upper limit of the effect of the elimination of deaths from a cause upon the expectation of life.

6. CONCLUSION

It has been said that the traditional methods are inadequate for measuring the effect of the marginal reduction of deaths from a particular cause upon the expectation of life. We generalized Keyfitz's method, which was devised to improve these methods. By our method we can easily calculate the effect of some percent elimination of deaths from a death cause in any age-interval upon life expectancy. And the appended tables computed for this will be of much use for the study of main death causes. By means of our method, also, we can easily get the parameters which are equivalent to Keyfitz's parameter $H^{(i)}$ for ages over 0 and $H_{60}^{(i)}$ for ages over 60.

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APPENDIX

Table A1. A list of 12 causes of death according to the international classification of diseases.

B List Number	Detailed list number	
B 4	008,009	Enteritis and other diarrhoeal diseases
B 46 e	535,561,563	Gastritis, duodenitis and chronic gastro-enteritis
B 5	010-012	Tuberculosis of respiratory system
B 6	013-019	Other tuberculosis, including late effects
B 19	140-209	Malignant neoplasms, including neoplasms of lymphatic and haematopoietic tissue
B 26	393-398	Chronic rheumatic heart disease
B 28	410-414	Ischaemic heart disease
B 29	420-429	Other forms of heart disease
B 27	400-404	Hypertensive disease
B 30	430-438	Cerebrovascular disease
B 32	480,486	Pneumonia
B 33 a	490,491	Bronchitis
B 46 d	466	Acute bronchitis and bronchiolitis
B 37	571	Cirrhosis of liver
B 38	580,584	Nephritis and nephrosis
B 45 a	794	Senility without mention of psychosis
B E 47	E810,E823	Motor vehicle accidents
B E 48	E800-E807, E825,E949	All other accidents
B E 49	E950,E959	Suicide

Notes on Tables A2, A3, A4, and A5

1. Data are Complete Life Tables and Death Statistics published by Department of Statistics and Information, Ministry of Health and Welfare, Japan.
2. Causes of death are based on International B List Number. (cf. Table A1)
3. Each figure in the row of Total is the sum of figures for age 0 to 100+ in the corresponding column.
4. Each figure in the row of 85+ is the sum of figures for age 85 to 100+ in the corresponding column.
5. Each figure in the row of G-Method shows the increment of $\overset{\circ}{e}_0$ by Greville's method in case of a cause being eradicated, and each figure should be multiplied by 10^2.
6. Each figure in the last column is the sum of figures in the corresponding row and should be used for a small quantity of change in the death rate from all causes of death.

Table A2. The effect of 1% change in number of deaths by age-interval and by cause of death on $^{o}e_{0}$, Japanese Males, 1970. (This should be used for a small quantity of change)

(x 10^{-2})

Causes of death	B4, B46 e	B5, B6	B19	B26 B28, B29	B27	B30	B32, B33 a B46 d	B37	B38	B45 a	BE47 BE48	BE49	Others	All causes
Total	0.11617	0.33527	2.02632	1.21226	0.19930	2.46930	0.56414	0.29076	0.16577	0.22772	1.45295	0.33354	2.53024	11.92375
(Age)														
0	0.04944	0.00177	0.02698	0.01589	0.0	0.00530	0.15694	0.00085	0.00212	0.0	0.18527	0.0	0.90345	1.34800
5	0.00133	0.00044	0.01705	0.00245	0.0	0.00148	0.00927	0.00037	0.00422	0.0	0.09369	0.0	0.04640	0.17671
10	0.00043	0.00079	0.01338	0.00446	0.00007	0.00156	0.00511	0.00058	0.00460	0.0	0.04367	0.00288	0.04130	0.11836
15	0.00056	0.00140	0.01855	0.00972	0.00045	0.00352	0.00637	0.00061	0.00922	0.0	0.15719	0.02201	0.04765	0.27723
20	0.00105	0.00315	0.02115	0.01434	0.00046	0.00448	0.00548	0.00091	0.01174	0.0	0.15020	0.04541	0.05285	0.31122
25	0.00103	0.00455	0.02680	0.02108	0.00146	0.00958	0.00526	0.00225	0.01474	0.0	0.11548	0.04633	0.05671	0.30528
30	0.00103	0.01020	0.03674	0.02651	0.00144	0.01798	0.00485	0.00890	0.01164	0.0	0.10066	0.03429	0.06597	0.32221
35	0.00124	0.01629	0.05910	0.03637	0.00364	0.04426	0.00756	0.02329	0.01100	0.0	0.09823	0.03161	0.07951	0.41410
40	0.00172	0.02460	0.08999	0.04756	0.00456	0.07757	0.00948	0.03165	0.01019	0.0	0.09414	0.02347	0.08764	0.50268
45	0.00179	0.03173	0.13689	0.05951	0.00586	0.10599	0.01296	0.03195	0.01108	0.0	0.08533	0.02207	0.09893	0.60708
50	0.00230	0.03555	0.19812	0.08011	0.01042	0.16027	0.01674	0.03794	0.01231	0.00009	0.07960	0.02210	0.11169	0.76743
55	0.00344	0.03910	0.25542	0.11039	0.01538	0.23360	0.02556	0.04064	0.01263	0.00037	0.07111	0.02204	0.13637	0.97604
60	0.00526	0.04327	0.31843	0.14592	0.02077	0.33045	0.03975	0.03631	0.01234	0.00214	0.05894	0.01970	0.16505	1.19653
65	0.00723	0.04720	0.32470	0.16730	0.03044	0.41371	0.05687	0.02935	0.01207	0.00773	0.04643	0.01646	0.16755	1.34703
70	0.00953	0.03755	0.25350	0.17319	0.03573	0.44192	0.06846	0.02221	0.01066	0.02507	0.03514	0.01241	0.15462	1.31031
75	0.01220	0.02343	0.14763	0.15048	0.03453	0.35570	0.06531	0.01453	0.00792	0.05415	0.02212	0.00765	0.14380	1.03967
80	0.01002	0.00977	0.05458	0.09639	0.02276	0.16977	0.04396	0.00642	0.00565	0.07149	0.01090	0.00342	0.03242	0.60593
85	0.00514	0.00220	0.01165	0.04046	0.00946	0.05990	0.01931	0.00153	0.00186	0.04894	0.00398	0.00129	0.02979	0.23570
90	0.00131	0.00026	0.00126	0.00910	0.00169	0.01039	0.00444	0.00025	0.00035	0.01566	0.00060	0.00018	0.00568	0.05136
95	0.00011	0.00002	0.00009	0.00086	0.00010	0.00085	0.00038	0.00001	0.00003	0.00174	0.00006	0.00002	0.00057	0.00466
100	0.0	0.0	0.00003	0.00013	0.0	0.00001	0.00010	0.0	0.0	0.00015	0.00001	0.0	0.00010	0.00053
85+	0.00656	0.00248	0.01322	0.05057	0.01124	0.07116	0.02422	0.00179	0.00224	0.06669	0.00465	0.00149	0.03614	0.29246
C-Method	0.11687	0.33924	2.19353	1.31925	0.20326	2.98977	0.58113	0.29322	0.16640	0.27232	1.47884	0.33531	2.67108	12.96022

Table A3 . The effect of 1% change in number of deaths by age-interval and by cause of death on e_{60}, Japanese Males, 1970. (This should be used for a small quantity of change)

($x\ 10^{-2}$)

Total	0.06299	0.20303	1.37940	0.97222	0.[illegible]9283	2.23590	0.37031	0.13719	0.06237	0.28188	0.22100	0.07606	0.99197	7.18716
(Age) 60	0.00652	0.05367	0.39495	0.18098	0.02576	0.40986	0.04930	0.04504	0.01530	0.00265	0.07310	0.02443	0.20472	1.48628
65	0.00697	0.05855	0.40272	0.20750	0.03775	0.51312	0.07053	0.03640	0.01497	0.00958	0.05759	0.02041	0.23262	1.67072
70	0.01182	0.04657	0.31454	0.21481	0.04431	0.54812	0.08492	0.02755	0.01322	0.03109	0.04359	0.01539	0.22923	1.62516
75	0.01513	0.02906	0.18310	0.18664	0.04283	0.44117	0.08100	0.01802	0.00983	0.06717	0.02743	0.00975	0.17535	1.28949
80	0.01242	0.01211	0.06769	0.11955	0.02823	0.23537	0.05452	0.00796	0.00626	0.08867	0.01352	0.00424	0.10223	0,75277
85	0.00637	0.00273	0.01470	0.05019	0.01173	0.07429	0.02395	0.00190	0.00231	0.06070	0,00493	0.00160	0.03694	0.29234
90	0.00163	0.00032	0.00156	0.01129	0.00209	0.01289	0.00550	0.00031	0.00044	0.01967	0.00074	0.00022	0.00705	0,06371
95	0.00013	0.00002	0.00011	0.00110	0.00013	0.00106	0.00047	0.00001	0.00004	0.00216	0,00008	0.00002	0.00071	0,00603
100	0.0	0.0	0.00003	0.00016	0.0	0.00002	0.00012	0.0	0.0	0.00019	0.00002	0.0	0.00012	0,00066
85+	0.00814	0.00307	0.01640	0.06273	0.01394	0.08826	0.03004	0.00222	0.00278	0.08271	0,00577	0.00185	0.04482	0,36273

Table A4. The effect of 1% change in number of deaths by age-interval and by cause of death on $^{o}e_{0}$, Japanese females, 1970. (This should be used for a small quantity of change)

(x 10^{-2})

Causes of death	B4, B46 e	B5, B6	B19	B26 B28 B29	B27	B30	B32 B33 a B46 d	B37	B38	B45 a	BE47 BE48	BE49	Others	All causes
Total (Age)	0.15153	0.16936	1.93511	1.26074	0.26262	2.42250	0.55871	0.14076	0.16928	0.50540	0.50014	0.29228	2.34450	10.73294
0	0.03985	0.00308	0.02313	0.01518	0.0	0.00325	0.15152	0.00130	0.00187	0.0	0.13505	0.0	0.74528	1.11950
5	0.00176	0.00067	0.01399	0.00218	0.0	0.00092	0.00989	0.00025	0.00235	0.0	0.03972	0.0	0.04768	0.11941
10	0.00057	0.00024	0.01236	0.00428	0.0	0.00137	0.00670	0.00040	0.00339	0.0	0.01373	0.00121	0.03432	0.07858
15	0.00068	0.00101	0.01555	0.00676	0.0	0.00303	0.00613	0.00051	0.00872	0.0	0.02263	0.01947	0.03938	0.12407
20	0.00076	0.00267	0.01958	0.01118	0.00086	0.00483	0.00529	0.00065	0.00871	0.0	0.02686	0.04324	0.05492	0.17957
25	0.00134	0.00581	0.03253	0.01470	0.00077	0.00586	0.00663	0.00139	0.00930	0.0	0.01783	0.03602	0.06000	0.19877
30	0.00175	0.01059	0.05246	0.01908	0.00108	0.01059	0.00653	0.00216	0.01018	0.0	0.01558	0.02633	0.06464	0.22078
35	0.00150	0.01517	0.07984	0.02552	0.00155	0.01784	0.00726	0.00492	0.01114	0.0	0.01943	0.02173	0.06434	0.27028
40	0.00155	0.01960	0.11813	0.03562	0.00323	0.03562	0.00942	0.00691	0.01083	0.0	0.02065	0.01719	0.06796	0.34659
45	0.00207	0.01913	0.16034	0.04835	0.00459	0.06473	0.00945	0.01121	0.01206	0.0	0.01900	0.01742	0.07671	0.44507
50	0.00211	0.01932	0.19703	0.06232	0.00815	0.11250	0.01406	0.01342	0.01108	0.0	0.02212	0.01850	0.09423	0.57543
55	0.00448	0.01664	0.23303	0.08515	0.01042	0.16400	0.01698	0.01935	0.01321	0.00097	0.02282	0.01816	0.10983	0.71707
60	0.00476	0.01730	0.26198	0.11455	0.02160	0.25009	0.02912	0.02006	0.01370	0.00207	0.02436	0.01719	0.13930	0.91608
65	0.00824	0.01654	0.24827	0.15208	0.03111	0.35120	0.04235	0.01774	0.01364	0.01010	0.02561	0.01657	0.16035	1.10379
70	0.01415	0.01693	0.22331	0.18653	0.04740	0.45453	0.06084	0.01761	0.01507	0.03701	0.02594	0.01669	0.18925	1.30906
75	0.02212	0.01223	0.15410	0.19779	0.05648	0.46604	0.06999	0.01327	0.01189	0.09791	0.02369	0.01202	0.17091	1.31445
80	0.02292	0.00595	0.06651	0.16087	0.04777	0.31467	0.05817	0.00714	0.00807	0.15460	0.01572	0.00570	0.12677	0.99588
85	0.01513	0.00215	0.01795	0.08620	0.02205	0.12846	0.03376	0.00213	0.00332	0.13431	0.00714	0.00232	0.05962	0.51454
90	0.00482	0.00033	0.00258	0.02619	0.00496	0.02922	0.01041	0.00033	0.00057	0.05549	0.00193	0.00045	0.01684	0.15412
95	0.00063	0.0	0.00034	0.00394	0.00050	0.00345	0.00165	0.00001	0.00011	0.01117	0.00019	0.00005	0.00254	0.02458
100	0.00015	0.0	0.00010	0.00049	0.00010	0.00049	0.00054	0.0	0.00008	0.00175	0.00008	0.0	0.00044	0.00422
85+	0.02073	0.00246	0.02097	0.11682	0.02761	0.16161	0.04639	0.00248	0.00408	0.20273	0.00934	0.00282	0.07944	0.69747
G-Method	0.15323	0.19016	2.03905	1.37906	0.26904	2.88845	0.57280	0.14138	0.16986	0.67131	0.50344	0.29356	2.46367	11.73502

Table A5. The effect of 1% change in number of deaths by age-interval and by cause of death on $^{o}e_{60}$, Japanese females, 1970. (This should be used for a small quantity of change)

($\times 10^{-2}$)

Total	0.10569	0.08579	1.11139	1.05828	0.26384	2.27268	0.34898	0.08906	0.07558	0.57373	0.14178	0.08301	0.99863	7.20842
(Age)														
60	0.00541	0.01968	0.29798	0.13029	0.02457	0.26445	0.03312	0.02282	0.01556	0.00236	0.02771	0.01955	0.15843	1.04193
65	0.00937	0.02109	0.28238	0.17297	0.03538	0.39945	0.04616	0.02018	0.01551	0.01149	0.02913	0.02112	0.16920	1.25543
70	0.01609	0.02153	0.25399	0.21420	0.05391	0.51098	0.06919	0.02003	0.01715	0.04210	0.02950	0.01898	0.21525	1.48891
75	0.02516	0.01391	0.17527	0.22496	0.06424	0.53007	0.07960	0.01509	0.01352	0.11136	0.02695	0.01367	0.20122	1.49503
80	0.02807	0.00677	0.07792	0.13298	0.05433	0.35791	0.06016	0.00812	0.00918	0.17584	0.01788	0.00648	0.14418	1.13383
85	0.01721	0.00243	0.02042	0.09805	0.02508	0.14611	0.03842	0.00243	0.00377	0.15277	0.00812	0.00264	0.06781	0.58523
90	0.00549	0.00038	0.00293	0.02979	0.00564	0.03323	0.01184	0.00038	0.00065	0.06312	0.00219	0.00051	0.01915	0.17530
95	0.00071	0.0	0.00038	0.00448	0.00057	0.00392	0.00188	0.00001	0.00013	0.01270	0.00021	0.00006	0.00259	0.02796
100	0.00016	0.0	0.00012	0.00056	0.00012	0.00056	0.00061	0.0	0.00009	0.00199	0.00009	0.0	0.00050	0.00480
85+	0.02558	0.00280	0.02335	0.13287	0.03140	0.18382	0.05276	0.00282	0.00463	0.23058	0.01062	0.00320	0.09035	0.79329

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