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The IIASA Health Care Resource Allocation Submodel: DRAM Calibration for Data from the South West Health Region, UK

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THE IIASA HEALTH CARE RESOURCE ALLOCATION SUBMODEL: DRAM CALIBRATION FOR DATA FROM THE SOUTH WEST HEALTH REGION, UK

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Philip Aspden, seconded from the Operational Research Unit of the Department of Health and Social Security, UK, was a IIASA research scholar from 1979 to 1980.

FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

This paper is a second application by Philip Aspden of the DRAM (Disaggregated Resource Allocation Model). The first such paper analyzed in-patient care for Czechoslovakia using 1976 data. Here, 1975 and 1976 data for the South West Region of England have been used to successfully predict the resource allocations for 1977, thus showing how DRAM could be used to aid health care planners in their analysis of future needs.

Related publications in the Health Care Systems Task are listed at the end of this report.

Andrei Rogers
Chairman
Human Settlements
and Services Area

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ABSTRACT

In many developed countries the problem of allocating resources within the Health Care System (HCS) is perennial. Health care admininstrators are continually asking what are the consequences of changing the mix of resources. The disaggregated resource allocation model (DRAM) has been developed to assist health care administrators with this problem. The model simulates how the HCS in aggregate allocates limited supplies of resources between competing demands. The principal outputs of the model are the numbers of patients treated in different categories, and the modes and quotas of treatment they receive.

Health care planners in the South West Health Region of England are concerned about the consequences for hospital in-patient care of increasing the number of hospital doctors and decreasing the number of hospital beds. This paper indicates how DRAM could be used to assist in the solution of this problem. Parameters were estimated for a model of hospital in-patient care for the region. This model consisted of seven patient categories (general surgery, general medicine, obstetrics and gynaecology, traumatic and orthopaedic surgery, otorhinolarygology, paediatrics and ophthalmology) and two resource types (hospital beds and hospital doctors). The ability with which this model was able to reproduce actual allocations or resources had similarities with a model (of identical structure) of Czechoslovakian hospital in-patient care.

It was considered appropriate to reduce the number of patient categories to three (general surgery, general medicine, and obstetrics and gynaecology). Parameters for this three-patient-category model were re-evaluated. Within the assumed predictive accuracy of this model, it successfully predicted health care resource allocation across time and space.

The three-patient-category/two-resource type model was then used to explore the consequences of changing the mix of resources in the South West Health Region. Firstly, the consequences of changes from the existing resource levels which involved no estimated increase in running costs were considered. More general changes where this constraint no longer held, were then examined.

CONTENTS

THE IIASA HEALTH CARE RESOURCE ALLOCATION SUBMODEL: DRAM CALIBRATION FOR DATA FROM THE SOUTH WEST HEALTH REGION, UK

1. INTRODUCTION

One of the responsibilities of the South West Regional Health Authority (SWRHA), UK, is the provision of hospital in-patient care. An issue facing the health care planners in the South West Health Region (SWHR) is the determination of the appropriate mix of hospital resources for in-patient care. In the particular, the planners wish to know the consequences (in terms of changes in admission rates and resource supply levels per patient) of increasing the number of hospital doctors and decreasing the number of hospital beds.

DRAM (disaggregated resource allocation model) is designed to help answer such a question. It is one of the submodels being developed by the Health Care Systems Task at the International Institute for Applied Systems Analysis. DRAM was formulated by Gibbs (1978) and further developed by Hughes (1978a, b, c).

This working paper describes how DRAM can be used to help answer the above question. The paper begins with a brief description of DRAM (section 2). This is followed by a section describing how the DRAM variables have been defined for SWHR hospital in-patient care.

Section 4 gives details of the DRAM parameter estimation process. The next section shows how DRAM could be used to investigate the consequences of changing the mix of hospital beds and hospital doctors for hospital care in the SWHR. The methodology used in the paper is similiar to that used by Aspden and Ruznak (1980) to parameterize DRAM for hospital in-patient care in Czechoslovakia.

All the analyses described in the paper have been carried out using aggregated data from the SWHR, and with little contact with officials of the SWRHA. To a certain extent, therefore, these analyses are of an indicative nature. In appropriate places, the paper indicates where more detailed analyses would be beneficial.

2. A HEALTH CARE RESOURCE ALLOCATION MODEL - DRAM

Health services cannot be administered in a rigid, centralized way. In every country, doctors have clinical control over the treatment of their patients, and it is local medical workers who ultimately determine how to use the resources (e.g. hospital beds, nurses) available to them. The specific question underlying DRAM is: If the decision maker provides a certain mix of resources, how will the HCS allocate them?

There are two assumptions about the bahavior of the HCS in the model. First it is assumed that there is never sufficient supply of resources to meet all the potential (or ideal) demands for them. The model simulates the balance chosen by the many agents in the HCS (doctors, nurses, social workers), between different treatment categories, between alternative combinations (modes) of care within the same treatment category, and between quality of care and numbers treated. The second assumption is that the aggregate behavior of the agents in the HCS can be represented as the maximization of a utility function whose parameters can be inferred from results of previous choices. Thus when the model is parameterized, it can be used to estimate the consequences of different allocations of resources.

The variables used in DRAM are as follows:

x_{jk} = numbers of individuals in the j^{th} patient category who receive resources in the k-th mode of care (per head of population per year)

X_{jk} = the ideal number of individuals in the j^{th} patient category who should receive resources in the k-th mode of care (per head of population per year) assuming no constraint on resource availability

$y_{jk\ell}$ = supply of resource type ℓ received by each individual in the j^{th} patient category in the k^{th} mode of care

$Y_{jk\ell}$ = the ideal levels of supply of resource type ℓ for each individual in the j^{th} patient category in the k^{th} mode of care assuming no constraint on resource availability*

R_{ℓ} = the availability of resource type ℓ (per head of population per year)

C_{ℓ} = marginal cost of resource type ℓ when all demands are satisfied

The utility function (Z) used in DRAM depicts the many agents who control the allocation of health care resources as seeking to attain ideal levels of service (X) and supply (Y), but where the urge to increase the actual levels of service (x) and supply (y) decreases with increasing values of x and y. The costs of different resources are introduced so that the marginal increases in Z when ideal levels are achieved (x = X, y = Y) equal the marginal resource costs. Beyond these levels, extra resources are only useful as assets and not for treating patients.

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*In the sequel, x,y are used to denote $\{x_{jk}\}$, $\{y_{jk\ell}\}$ respectively, with a like notation for similarly subscripted variables.

These assumptions have expressed in mathematical form as follows:

$$Z(x,y) = \sum_j \sum_k g_{jk}(x_{jk}) + \sum_j \sum_k \sum_\ell x_{jk} h_{jk\ell}(y_{jk\ell}) \qquad (1)$$

subject to

$$\sum_j \sum_k x_{jk} y_{jk\ell} = R_\ell \qquad \forall \ell$$

where

(1) $$g_{jk}(x) = \frac{\sum_\ell C_\ell X_{jk} Y_{jk\ell}}{\alpha_j} \left\{ 1 - \left(\frac{x}{X_{jk}} \right)^{-\alpha_j} \right\}$$

(2) $$h_{jk\ell}(y) = \frac{C_\ell Y_{jk\ell}}{\beta_{jk\ell}} \left\{ 1 - \left(\frac{y}{Y_{jk\ell}} \right)^{-\beta_{jk\ell}} \right\}$$

(3) $\alpha_j(>0)$ is a parameter measuring the relative importance of treating the ideal number of individuals X_{jk}; higher values indicate greater importance

(4) $\beta_{jk\ell}(>0)$ is a parameter measuring the relative importance of achieving the ideal level $Y_{jk\ell}$; again, higher values indicate greater importance

Hughes (1978c) has shown that the solution of the optimization problem in equation 1 is as follows

$$y_{jk\ell} = Y_{jk\ell}(\lambda_\ell)^{\frac{-1}{\beta_{jk\ell}+1}} \qquad (2)$$

$$x_{jk} = X_{jk}(\mu_{jk})^{\frac{-1}{\alpha_j+1}} \qquad (3)$$

where μ_{jk} is a weighted sum

$$\mu_{jk} = \frac{\sum_{\ell} C_{\ell} Y_{jk\ell} \nu_{jk\ell}}{\sum_{\ell} C_{\ell} Y_{jk\ell}}$$

of the terms

$$\nu_{jk} = \left((\beta_{jk\ell} + 1)\lambda_{\ell}^{\frac{\beta_{jk\ell}}{\beta_{jk\ell}+1}} - 1 \right) / \beta_{jk\ell}.$$

and where λ_{ℓ} are the solutions of the following set of equations

$$0 = -R_{\ell} + \sum_{j}\sum_{k} X_{jk} Y_{jk\ell} (\lambda_{\ell})^{\frac{-1}{\beta_{jk\ell}+1}} (\mu_{jk})^{\frac{-1}{\alpha_j+1}} \quad \text{for all } \ell$$

The algorithm for determining the solutions (equations 2 and 3) has been developed by Hughes and Wierzbicki (1980). This algorithm has been programmed, and requires no specialized software. Experience has shown that the computer program is easily transferred from computer to computer.

3. DEFINITION OF THE DRAM VARIABLES FOR HOSPITAL IN-PATIENT CARE IN THE SOUTH WEST HEALTH REGION

3.1. Introduction

As mentioned earlier, an issue facing the SWHR health care planners, is the determination of the appropriate mix of hospital resources for in-patient care. In particular, what are the consequences (in terms of changes in admission rates and in levels of service per patient) of increasing the number of hospital doctors and reducing the number of hospital beds. The aim of this paper is to help answer this problem by parameterizing DRAM for the two resources, hospital beds and hospital doctors. Hospital in-patient data was available for the years 1975, 1976, and 1977 for each Area Health Authority (AHA) in the SWRHA. The initial approach adopted was to develop a model for data from 1975 and 1976 and test it using 1977 data. Section 3.3 considers the choice of treatment categories and section 3.4 discusses the resource measures adopted for each of the resource types. (It will be assumed that there is only one mode of care: namely, in-patient care). The following section gives a brief description of how hospital in-patient care is organized in the South West Health Region.

3.2. Hospital In-patient Care in the SWHR

In England, health care is provided by two independently administered organizations, the Regional Health Authority (RHA) and the Local Authority (LA). England is divided into 14 RHAs. They are financed by the central government and provide medical and nursing services via hospitals, clinics, and home visits. The LA, financed by local taxes and the central government, provides personal social services such as residential homes, social workers, home helps and the home meals service.

The South West Health Region covers about 7000 sq.miles in the south west of England. In shape it is long and thin, being about 250 miles long and never wider than about 70 miles. The region is rival in character with pockets of highly urbanized development at the main centers, especially Bristol and Plymouth.

Administratively the SWRHA is divided into five Area Health Authorities: Avon, Cornwall, Devon, Gloucestershire, and Somerset. All but one of these AHA are further subdivided into Health Districts. In all there are 13 Health Districts in the South West Health Region. Each District serves a population of about 250,000 and provides almost the whole range of hospital services. Types of care for which there is relatively little demand (e.g. plastic surgery) are provided in Regional or National Centers.

3.3. The Choice of Treatment Categories

Two common ways of defining hospital treatment categories are by "treatment speciality" or by the International Classification of Diseases (ICD) Code. The latter is a more detailed categorization. However, most measures of hospital resource are normally given in terms of the amount of resource available for a speciality (e.g. the available beds in the speciality of general medicine). In this study, patient categories are either single specialities or groups of specialities.

In choosing the treatment categories, it is necessary to take into account certain requirements imposed by the DRAM parameter estimation process (Appendix A). It is assumed that each AHA for each year provides an independent data point, i.e. the same utility function Z(x,y) (equation 1) holds across space and across time. One implication (others are discussed in section 4) of this is that each chosen treatment group should be self sufficient in each area, i.e. if general medicine is chosen, all (or almost all) general medicine patients should be treated in the area in which they arise. Thus treatment categories which are regarded as "regional" specialities must be excluded. In theory, with regard to all the major specialities, each AHA is meant to treat all the patients within its area. In practice patients for reasons of convenience, etc., cross boundaries to receive treatment in areas in which they do not live. Such "cross boundary patient flows" are taken into account when the Central Government allocates financial resources to the Health Regions [Department of Health and Social Security (1976)]. The SWRHA uses estimates of resident ("defined") populations adjusted for cross boundary patient flows for the planning of hospital activity in the region. These adjusted ("catchment") population estimates will be used in this paper to calculate patient category admission rates. It is assumed that the same "catchment" population estimates apply to all the treatment groups. Unfortunately, it was not possible to check this assumption. Here is an example where it would be appropriate to carry out some further analysis. A comparison of "defined" and "catchment" populations for 1977 is given in Table 1 [SWRHA (1979)].

Table 1. Comparison of "defined" and "catchment" population estimates for the South West Health Region for 1977.

	"Defined"	"Catchment"
Avon	788,400	832,400
Cornwall	413,600	324,600
Devon	947,200	1,036,600
Gloucestershire	492,700	492,700
Somerset	364,700	349,100
	3,006,600	3,035,400

Another requirement for chosen treatment categories arises from the fact that in the DRAM formulation, the resource levels are treated as continuous variables. This means that the basic unit of each resource (e.g. a hospital bed year) should be small compared to the total amount of the resource devoted to a treatment category in each year in each AHA. Hence treatment categories should not be too small.

Having taken the above into account, the following initial set of treatment categories was chosen

- General surgery (including urology)
- General medicine (including cardiology)
- Obstetrics and gynaecology
- Traumatic and orthopaedic surgery
- Otorhinolaryngology
- Paediatrics (including special care baby units)
- Ophthalmology

This is the same set of treatment categories for which Aspden and Ruznak (1980) parameterized DRAM for Czechoslovakian hospital in-patient care.

Data on the number of patients admitted to hospital in the SWHR in 1975, 1976, and 1977 for all the above patient categories were taken from the Department of Health and Social Security (DHSS) Statistical Return SH3.

3.4. The Resource Measures for the Resource Types - Hospital Beds and Hospital Doctors

In Aspden and Ruznak (1980) the question was asked: Which are the most important health care resources for hospital in-patient care? Although this paper is concerned with hospital beds and hospital doctors, this question is still appropriate. The aim of the work described in this paper is to help plan hospital in-patient care, and the implication is that the most important resource inputs are probably hospital beds and hospital doctors. However, there may be other important resource types (e.g. nurses, operating theaters, diagnostic and technical support facilities) which have an important bearing on hospital performance. It would be worthwhile investigating the importance of some of these other resource types.

Given that a model of hospital in-patient care is to be parameterized for two resource types - hospital beds and hospital doctors - it is necessary to decide how these resources are to be measured. The unit for hospital beds was taken to be available beds per 1000 population in a particular area. This means that the supply variable ($y_{jk\ell}$) is available bed-days per patient. This has the advantage over the more usual measure of occupied bed-days per patient (i.e. length of stay) of eliminating the separate estimation of occupancy rates.

With regard to hospital doctors, there are several possible measures. The aim is to find the measures which best explain the variations in admission rates and supply levels per patient. Examples of possible measures are:

(a) The number of hospital doctors (incl. anesthetists pathologists, surgeons) involved with a particular treatment category

(b) The number of hospital doctors of all grades belonging to the specialities which treat a particular treatment category (For example, if the treatment is "general medicine", then this measure would be the number of doctors within the general medicine specialty.)

(c) The number of senior hospital doctors (consultants in UK) belonging to the specialities which treat a particular treatment category

(d) The number of anesthetists involved with a particular treatment category

These measures are not exclusive, since, for instance, measures (c) and (d) could be used simultaneously. However, some of these measures may be difficult to calculate, as it would be difficult to allocate the time of a pathologist or an anesthetist to the various treatment categories. In this study measure (c) was adopted as it was the only one for which data was available at the Institute (the unit of measurement was taken to be doctor days per 1000 population - one doctor year = 225 doctor days). However, there is a difficulty associated with this measure and this concerns the number of consultants available per year per area for each of the patient categories, otorhinolaryngology, paediatrics,and ophthalmology. During 1975-1977, this figure was between 2 and 3 per year for the AHAs, Cornwall, Gloucestershire, and Somerset (the other two AHAs, averaged about seven consultants per area per treatment category over the same period). This suggests, given the requirement (arising from the fact that the measures of resource availability in DRAM are assumed to be continuous variables) in section 3.3, that these three categories should perhaps be excluded from the analysis. This will be considered later.

The above difficulty, suggests that it might be worthwhile considering the number of hospital doctors of all grades within a specialty [measure (b)] as a measure of the resource type hospital doctors. This was the measure used in Aspden and Ruznak (1980).

Data on the levels of bed supply for the seven patient categories for each AHA were taken the from DHSS Statistical Return SH3. Data on the supply of consultants for the seven patient categories were taken from SWRHA (1977, 1978, 1979).

4. PARAMETER ESTIMATION FOR DRAM - USING HOSPITAL IN-PATIENT DATA FROM THE SOUTH WEST HEALTH REGION

4.1. Introduction

The problem of calibrating DRAM hospital in-patient data from the South West Health Region is now considered. Estimates are required for three groups of parameters:

(1) The ideal levels X,Y at which patients would be admitted and receive resources, if there were no constraints on resource availability

(2) The power parameters α, β which reflect the relative importance of achieving the ideal levels X and Y (For instance, if an α is relatively high then it is relatively more important to treat the corresponding X.)

(3) The relative costs, C, of the different resources - in this case hospital beds and hospital doctors

In what follows the parameter $\{X,Y,\alpha,\beta\}$ will be estimated from actual allocations of resources. However, if estimates of the ideal levels (X,Y) derived from morbidity surveys and surveys of clinical opinion were available, then these could have been used. The cost parameters will be determined exogenously.

In estimating the parameter set $\{X,Y,\alpha,\beta\}$ the approach of Hughes (1978c) will be followed. This is described in

Appendix A. The approach assumes that each AHA for each year provides an independent data point, i.e. the same utility function $Z(x,y)$ holds across time and space. This implies that the parameter set $\{X,Y,\alpha,\beta\}$ should not change over time and space. Some justification will be given for this.

(1) The ideal levels, X, at which patients should be admitted.

X is a measure of the morbidity in the community. Morbidity is related to the demographic structure of society. Table 2 gives the proportions of the population in the five AHAs in the following important care groups:

a. children
b. women of child bearing age
c. women (in need of gynaecological care)
d. elderly

Although there is some variation across the region with regard to the proportion of elderly people, overall, Table 2 suggests it is not unreasonable to say that from a demographic point of view, the potential calls on the health care system are likely to be the same for each AHA.

The need for hospital in-patient services is not a function of age and sex alone. Many other factors are known to play a part: social, occupational, hereditary environmental, etc. However, it is very difficult to quantify their effect. Frequently used proxy indicators of morbidity [DHSS (1976)] are mortality statistics. Table 3 gives some of the mortality rates for the five AHA's.

Table 2. Estimates of percentage of 1977 "catchment" population in important care groups.

Care group	Avon	Cornwall	Devon	Gloucestershire	Somerset
Children (age 0-14)	21.7	20.4	20.1	22.7	21.7
Women of child bearing age (age 15-44)	20.2	18.8	18.2	19.6	19.9
All women	52.0	51.9	51.3	51.2	51.2
Elderly (age 65+)	14.9	17.5	18.5	14.6	16.3

SOURCE: SWRHA (1979).

Table 3. Selected mortality rates in the South West Health Region.

	Avon	Cornwall	Devon	Gloucestershire	Somerset
Perinatal mortality per 1000 live and still births (1977)	15.0	15.0	17.1	18.5	15.5
Neonatal mortality per 1000 live births (1977)	8.1	7.4	7.8	9.0	10.1
Mortality rate ages 15-64 per 1000 (1976)	4.4	4.4	4.0	3.9	4.4
Mortality rate ages 65-74 per 1000 (1976)	40	35	32	33	37
Mortality rate ages 75+ per 1000 (1976)	132	108	106	110	124

SOURCE: SWRHA (1979), Office of Population and Census Studies (SD25 for 1976).

Table 3 suggests again that it would not be unreasonable to say that the potential calls on the health care system are likely to be approximately the same for each AHA.

The changes in the various factors (e.g. age, sex, social, environmental, etc.) which affect the need for hospital in-patient services are likely to be small over a period of about five years. Hence it can be assumed that the parameter X does not change over the time period under consideration.

(2) The ideal levels of resources, Y, which patients should receive.

Styles of clinical management differ. Much variation in lengths of stay is likely to be unrelated to resource supply. In the absence of a detailed study, it is assumed in this paper that the aggregate behavior of clinicians in terms of ideal standards of care is the same for each AHA.

Over time, however, lengths of stay in hospital are known to be declining. A study carried out in England [DHSS (1978)] showed that the mean duration of length of stay for all acute specialities was 12.5 days in 1967 and 9.6 days 10 years later. This represents an average annual reduction of 2-7% per year assuming the same case mix. Such a decrease can be thought of as an "improvement in technology" or an "improvement in productivity". Two implications follow from this result. Firstly, the ideal levels of hospital resources which patients should receive decrease over time. Secondly, the levels of hospital resources must be "discounted over time". Accordingly, it has been assumed that 100 bed-days in 1975 is equivalent to 97.5 bed-days in 1976, and 100 doctor-days in 1975 is equivalent to 97.5 doctor-days in 1976, and so on for other years*.

Table 4 gives the percentage changes in hospital admissions and resource supply from 1975 to 1976 and from 1976 to 1977 for all the seven patient categories for the South West Health Region.

Table 4. Percentage changes in hospital admissions and resource supply for the seven patient categories for the South West Health Region.

	1975 1976	1976 1977
Admissions	+4.3%	1.5%
Available bed supply (for seven patient categories)	-4.1%	-1.5%
Consultant supply (for seven patient categories)	+1.8%	-1.2%

The changes from 1976 to 1977 are consistent with an approximate 2.5% improvement in "productivity"/ "technical" development. However the changes from 1975 to 1976, suggest an increase in "productivity" of about 8% with regard to bed supply, and 2.5% with regard to consultant supply. Note that in 1975, there was a strike of the junior hospital doctors in England.

*In this study it has been assumed that lengths of stay (and hence available bed-days per patient and doctor-days per patient) are decreasing at the same rate for all patient categories. This is an approximation since it is known that rates of decrease of length of stay vary with patient category. The London Planning Consortium (1979) studied this issue in some detail. From an analysis of 1962-1975 data from England and Wales they predicted the rate of decrease of annual length of stay for 1976 for a variety of specialities. The following estimates are taken from their report - general surgery 1.1%, general medicine 2.6%, gynaecology 3.4%, paediatrics 3.8%, ophthalmology 1.9%.

Some of the difference in "productivity" between 1975 and 1976 may be because of this strike. If this is the case then it will lead to the introduction of some error into the DRAM parameter estimation process.

(3) The power parameters (α,β).
These parameters reflect the relative importance that the Health Care System gives to achieving the ideal levels X and Y. It has been assumed that this aggregate behavior will not change over the time period under consideration.

In summary it has been assumed that the same utility function holds over time and space, except that the ideal levels of care per patient change over time.

The parameter estimation process was carried out in three stages. Models were calibrated for bed supply and doctor supply separately. Then a two resource (beds and doctors) model was calibrated. This process is described next. Before doing this, it is necessary to extend further the notation of section 2. The model parameters will be estimated from 10 data points. The actual data for data point i (i = 1,2,N) will be represented thus - $x_j(i)$, $y_{j\ell}(i)$ with the mode subscript k removed as there is only one mode. Thus the amount of resource type ℓ used at data point i is

$$\sum_j x_j(i) y_{j\ell}(i) = R_\ell(i)$$

Further, let $\hat{x}_j(i)$ and $\hat{y}_{j\ell}$ be the predicted levels using DRAM given a particular parameter set (X,Y,α,β) and resource availabilities $R_\ell(i)$ at data point i. The following measures of goodness-of-fit can then be defined

$$SS\hat{x}_j = \sum_i \left(\frac{x_j(i) - \hat{x}_j(i)}{w_j} \right)^2$$

$$SS\hat{y}_{j\ell} = \sum_i \left(\frac{y_{j\ell}(i) - \hat{y}_{j\ell}(i)}{v_{j\ell}} \right)^2$$

where w_j is weighted average of $x_j(i)$ and $v_{j\ell}$ is a weighted average of $y_{j\ell}(i)$. As an indication of the goodness-of-fit of DRAM, it is useful to make the following comparisons

$$SS\hat{x}_j \quad \text{with} \quad SS\hat{x}_j = \sum_i \left(\frac{x_j(i) - w_j}{w_j} \right)^2$$

$$SS\hat{y}_{j\ell} \quad \text{with} \quad SS\hat{y}_{j\ell} = \sum_i \left(\frac{y_{j\ell}(i) - v_{j\ell}}{v_{j\ell}} \right)^2$$

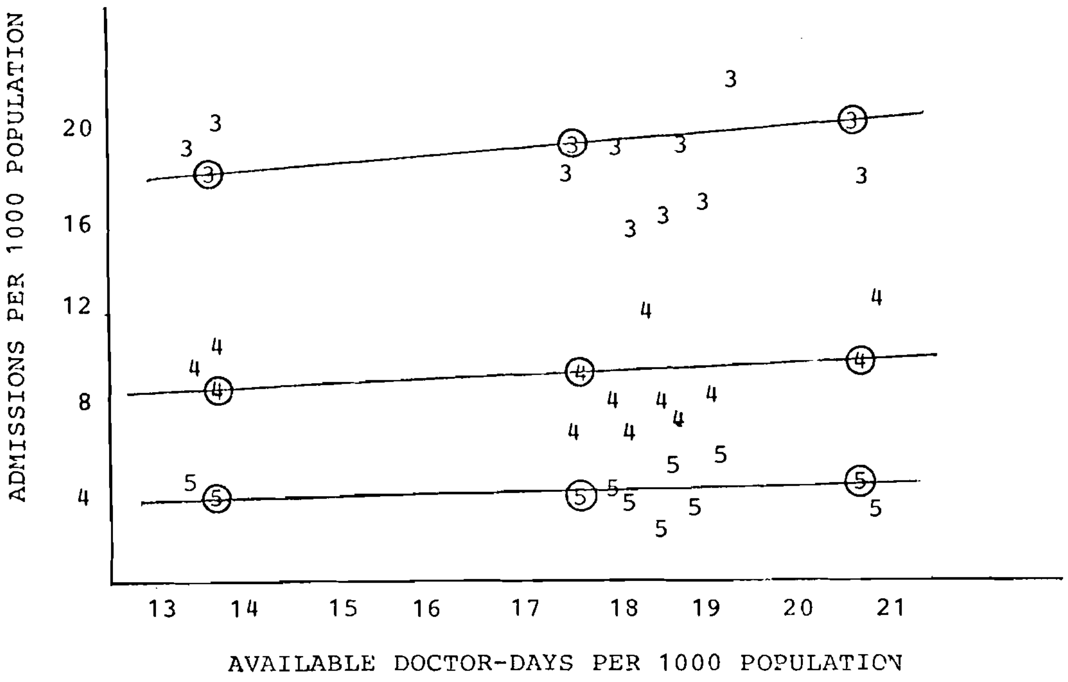
4.2. Parameter Estimation for DRAM with One Resource - Hospital Doctors

DRAM was parameterized firstly for one resource - hospital doctors. The parameters for this model were estimated using the techniques described in Appendix A. They are given in Table 5. Figures 1 and 2 give the admission rates and supply levels per patient, both actual and from the model (using the parameters in Table 5), plotted against total doctor-days per 1000 population for each of the 10 data points. Total doctor-days (per 1000 population) is defined to be the number of doctor-days actually available for all seven patient categories.

Figure 1 shows that admission rates for general medicine increase as total doctor supply increases. General surgery admission shows a similar tendency except that the rate of increase is slower. Further the figure suggests that the admission rate for general medicine is more elastic to total doctor supply than the admission rate for general surgery. This is an indication that the α for general medicine is less than the α for general surgery. Table 5 shows that the estimated α for general medicine is indeed less than the estimate for general surgery.

Table 5. One-resource (hospital doctors) DRAM parameter estimates for South West Health Region in-patient care.

Treatment category	Admission rates X_j	α_j	$SS\hat{x}_j$	$SS\bar{x}_j$	Supply levels: doctors Y_{j1}	β_{j1}	$SS\hat{y}_{j1}$	$SS\bar{y}_{j1}$
General Surgery	28	.21	.010	.026	.32	.001	.100	.212
General Medicine	23	.001	.203	.395	.27	4.0	.062	.078
Obstetrics & Gynaecology	24	1.0	.087	.058	.19	.001	.156	.293
Traumatic & Orthopaedic Surgery	12	1.0	.547	.517	.46	.33	.080	.096
Otorhinolaryngology	52	1.0	.406	.390	.49	1.0	.715	.756
Paediatrics	10	.001	.228	.307	.31	1.0	.265	.372
Ophthalmology	3.8	.001	.330	.234	1.1	.059	.785	.843



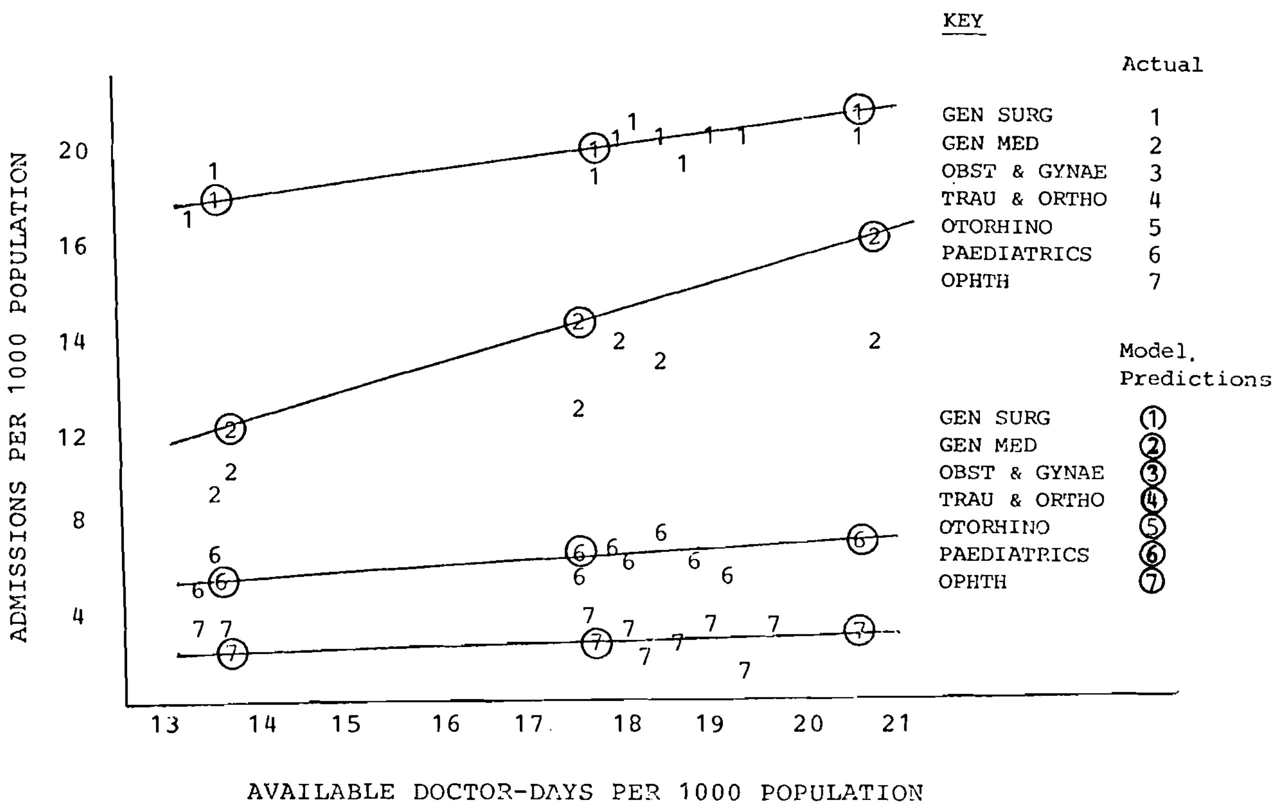


Figure 1. Admission rates.

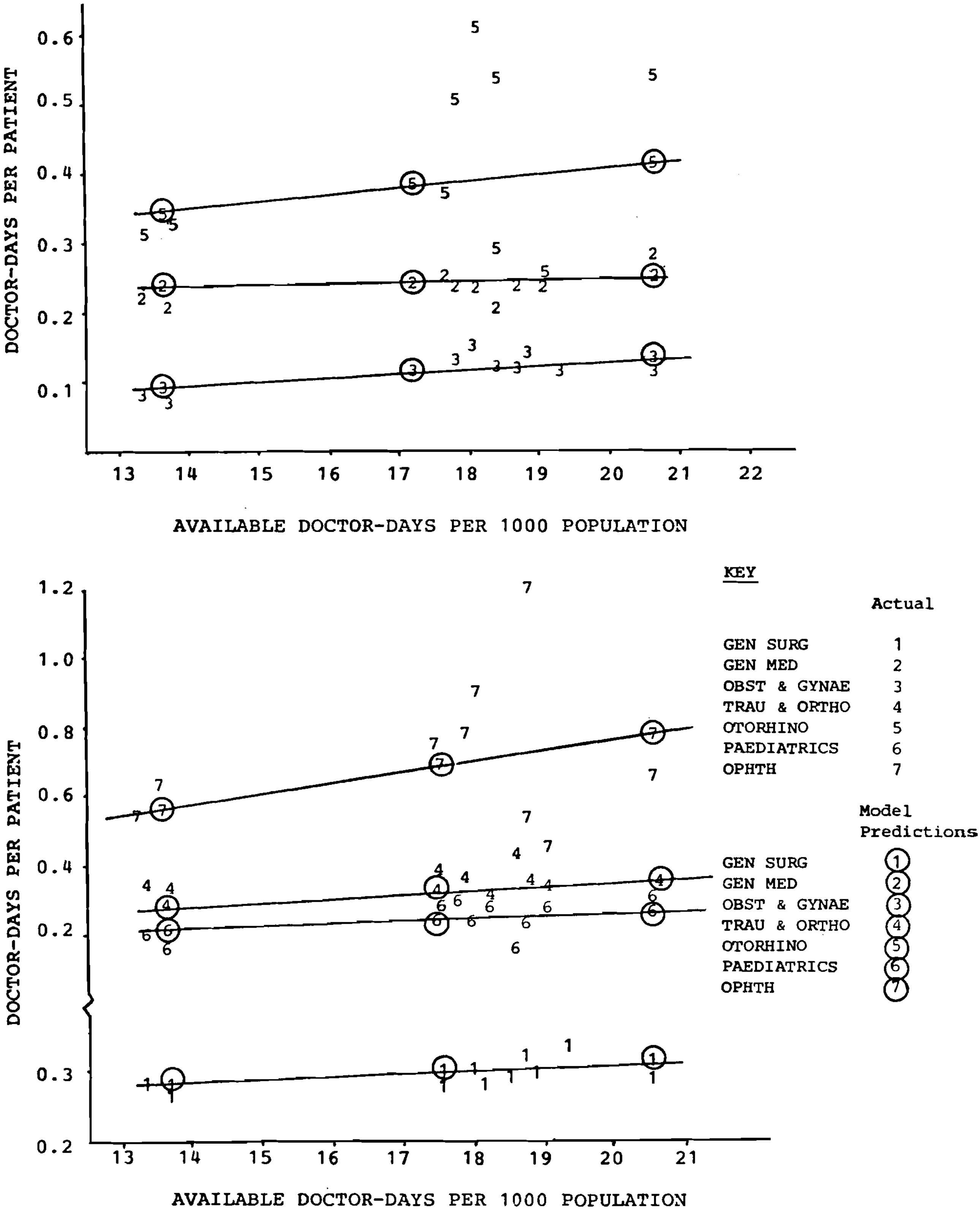


Figure 2. Supply levels (doctors).

Figure 2 also shows that doctor-days per patient for both general surgery and obstetrics and gyneacology increase as the total doctor supply increases, i.e., the supply levels for these two categories are elastic to the total doctor supply. Table 5 shows that low β's have been estimated for these two categories.

In the one-resource version of DRAM, the model assumptions imply that for each patient category, the admission rates and supply levels per patient should monotonically increase. Figures 1 and 2 show that for some categories DRAM fails to reproduce much of the observed results. A comparison between $SS\hat{x}_j$ and $SS\overline{x}_j$, and $SS\hat{y}_{j1}$ and $SS\overline{y}_{j1}$ indicates how well the model reproduces the observed results. In the parameterization process we are seeking parameters $\{X,Y,\alpha,\beta\}$ which simultaneously make the ratios $SS\hat{x}_j/SS\overline{x}_j$ and $SS\overline{y}_{j2}$ small (i.e., < 1). If, for instance, $SS\hat{x}_j/SS\overline{x}_j$ is approximately one, then the model does not reproduce the actual results any better than taking the mean of all the actual x_j. Table 5 indicates that DRAM has been most successful in reproducing actual admission rates for general surgery, general medicine and paediatrics, and reproducing actual supply levels per patient for general surgery, obstetrics and gynaecology, and paediatrics.

4.3. Parameter Estimation for DRAM with One Resource - Hospital Beds

The parameter estimates when the resource is hospital beds are given in Table 6. Figures 3 and 4 give the admission rates and supply levels per patient, both actual and from the model (using the parameters in Table 6) plotted against total bed-days per 1000 population for each of the 10 data points. Total bed-days (per 1000 population) is defined to be the number of bed-days available for all the seven categories.

Examination of the ratios $SS\hat{x}_j/SS\overline{x}_j$ and $SS\hat{y}_{j1}/SS\overline{y}_{j1}$ in Table 6, suggests that this version of DRAM has been most successful in reproducing actual admission rates for paediatrics and ophthalmology and reproducing actual supply levels per patient for general surgery, general medicine, and otorhinolaryngology.

Table 6. One-resource (hospital beds) DRAM parameter estimates for South West Health Region in-patient care.

Treatment category	Admission rates				Supply levels: beds			
	X_j	α_j	$SS\hat{x}_j$	$SS\bar{x}_j$	Y_{j1}	β_{j1}	$SS\hat{y}_{j1}$	$SS\bar{y}_{j1}$
General Surgery	21	50	.030	.026	16	.28	.096	.138
General Medicine	24	.062	.405	.395	22	.12	.034	.079
Obstetrics & Gynaecology	25	1.5	.046	.058	8.3	17	.093	.081
Traumatic & Orthopaedic Surgery	14	.40	.524	.517	23	1.1	.190	.194
Otorhinolaryngology	5.4	1.0	.339	.390	16	.001	.196	.285
Paediatrics	12	.001	.229	.307	12	50	.199	.197
Ophthalmology	4.2	.15	.169	.234	16	1	.253	.203

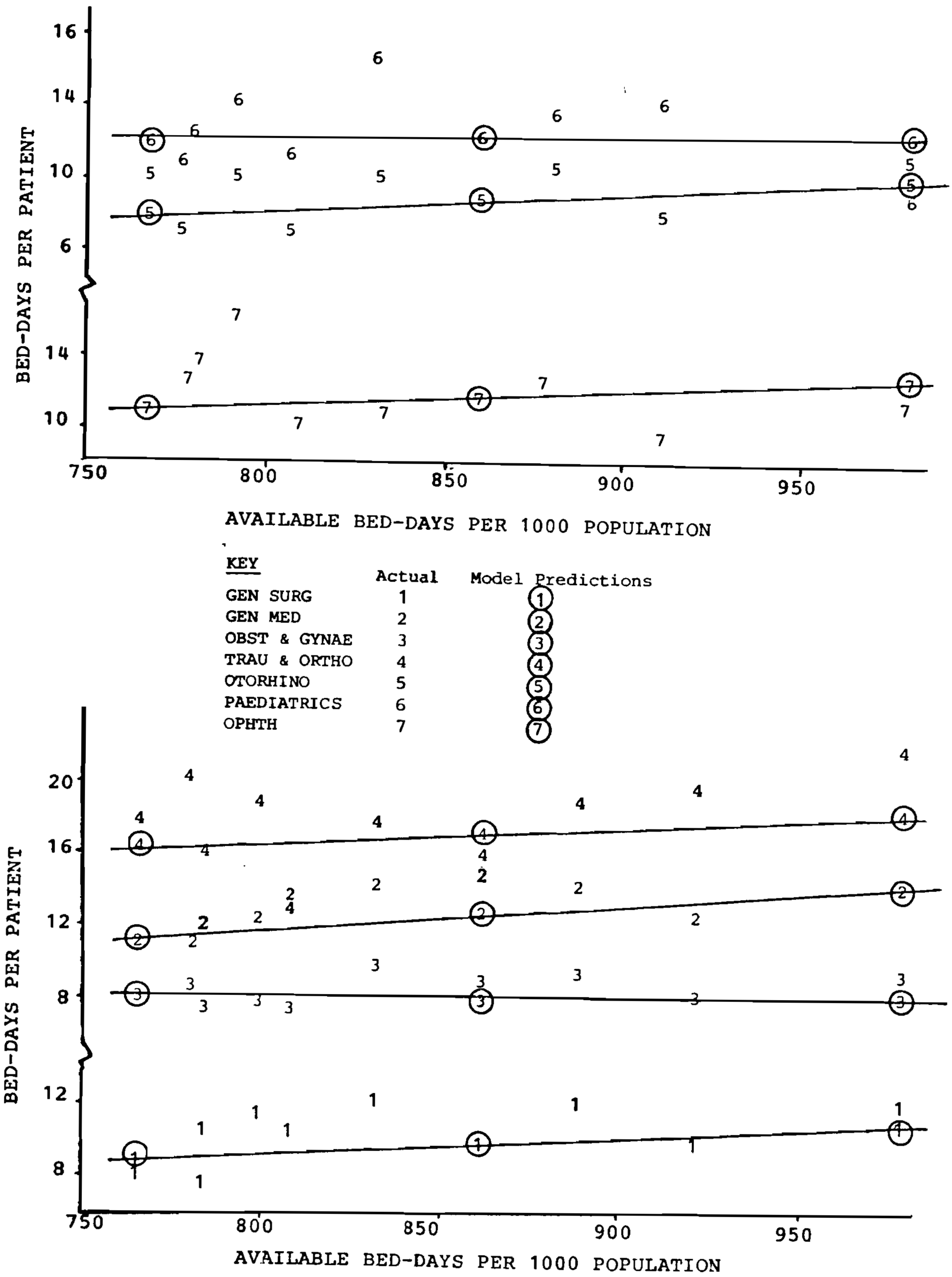


Figure 3. Supply levels (beds).

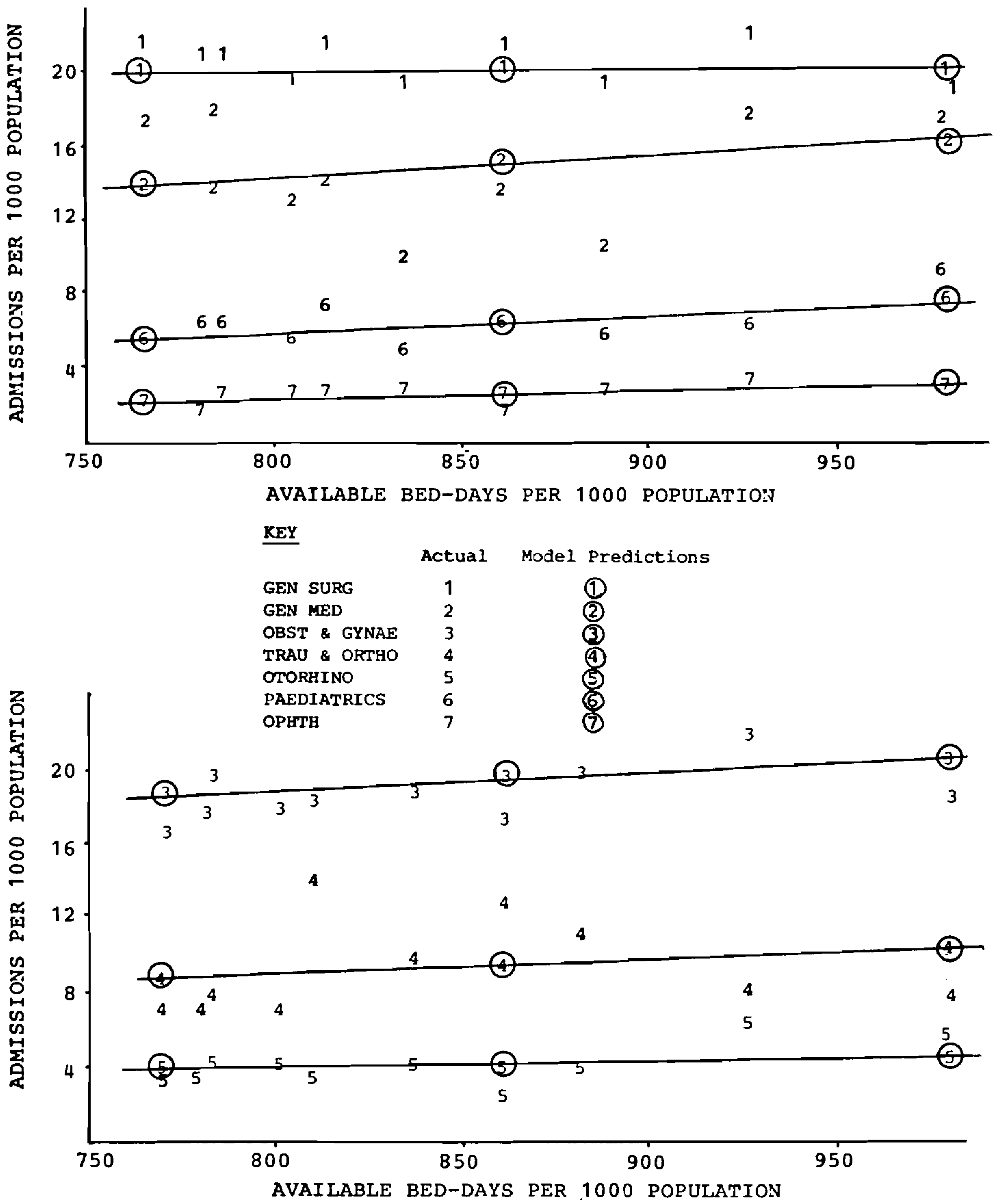


Figure 4. Admission rates.

As mentioned earlier, the model assumptions for a one-resource version of DRAM imply that for each patient category, the admission rates and supply levels per patient should monotonically increase as total resource supply increases. Some of the actual variation in Figures 1- 4 do not follow this pattern. However, failure of the actual data to follow this pattern does not imply that a two-resource DRAM will not reproduce the actual data as these are likely to be interactions between the two resources not indicated in the figures.

In this section and the previous one, two one-resource versions of DRAM have been parameterized for seven patient categories. The extent to which the two models can reproduce actual data do not completely overlap. In a two-resource model, it is hoped that the good parts of the individual models will be retained, and further that the interaction between the availabilities of the resources will be able to reproduce more of the observed behavior.

4.4. Parameter Estimation for DRAM with Two Resources - Hospital Beds and Hospital Doctors.

To calculate the parameters for DRAM for two resources - hospital doctors and hospital beds - it is necessary to estimate the ratio of the marginal costs of these resouces (C_ℓ) when all needs for health care are met. Further, since the following analysis uses average costs, it has been assumed that the above ratio equals the ratio of the average costs for the two resources at the current resource levels.

From Health Service Costing Returns for 1977 (SWRHA 1977b) the average net total cost (1976/77) per in-patient day for (acute) hospitals with over 100 beds for the South West Health Region is as follows:

	£	
Medical Staff	3.75	(I_1)
Nursing Staff	11.53	(I_2)
Medical and Surgical Supplies	3.78	(I_3)
Diagnostic and Paramedical Support Services	2.97	(I_4)
General Services (Administration, Catering, Domestic, Estate Management)	13.51	(I_5)
	35.51	(TOT)

On average for the seven patient categories under consideration, there is approximately one doctor-day for every 50 bed-days. The problem is now to allocate the above cost headings between the two resource types. The measure of hospital doctor resource has been taken as the number of consultants. Consultants generally had a team of junior doctors and support staff. The measure of this resource type can therefore be regarded as the number of "consultant teams". Thus the cost of this resource type must include the cost of employing a consultant and his team (i.e. cost I_1). It can be argued that the use of some or all of the medical and surgical supplies is directly proportional to the number of consultant teams. Thus some or all of this cost should be attributed to the hospital doctor resource type. A similar argument holds for Diagnostic and Paramedical Support Services. Three possible allocations of costs to the hospital doctor resource type were considered.

(1) Only medical staff costs contribute

The cost ratio of one doctor-day to one bed-day is taken to be

$$\frac{50\ I_1}{TOT-I_1} = 5.85$$

(2) Medical staff and diagnostic and paramedical support services costs contribute

The cost ratio of one doctor-day to one bed-day is taken to be

$$\frac{50(I_1+I_4)}{TOT-I_1-I_4} = 11.61$$

(3) Medical staff, diagnostic and paramedical support services, and medical and surgical supplies costs contribute

The cost ratio of one doctor-day to one bed-day is taken to be

$$\frac{50(I_1+I_3+I_4)}{TOT-I_1-I_3-I_4} = 20.91$$

In the remainder of the paper, for parameter estimation purposes, the marginal cost ratio (when all demands for health care are satisfied) of one doctor-day to one bed-day has been taken as ten. The above analysis has been somewhat crude, and a more detailed analysis of the ratio of the costs of the two resources may be worthwhile. However, empirical evidence shows that parameter estimates may not be sensitive to changes in this ratio.

Using the above ratio, the parameters for the two-resource version of DRAM were estimated. They are given in Table 7. A comparison of the ratios $SS\hat{x}_j/SS\bar{x}_j$ and $SS\hat{y}_{j\ell}/SS\bar{y}_{j\ell}$, between the two-resource model and the two individual resource models given in Table 8. Overall the results of the two-resource model are similar to the results of the two individual models (taking the better results were appropriate). For the two-resource model, a reduction in the reproducibility of actual results for admission rates is balanced by an improvement for supply levels.

If the patient categories are ordered by the measure

$$\frac{SS\hat{x}_j}{SS\bar{x}_j} + \frac{SS\hat{y}_{j1}}{SS\bar{y}_{j1}} + \frac{SS\hat{y}_{j2}}{SS\bar{y}_{j2}}$$

then the following ranking is produced (lowest first):

-- General Medicine
-- Paediatrics
-- General Surgery
-- Otorhinolaryngology
-- Obstetrics and Gynaecology
-- Traumatic and Orthopaedic Surgery
-- Ophthalmology

Aspden and Ruznak (1980) parameterized a two-resource (hospital beds and doctors) DRAM for Czechoslovakian hospital in-patient care. Ordering the same seven treatment categories

Table 7. Two-resource (hospital beds and doctors) DRAM parameter estimates for South West Health Region in-patient care.

Treatment category	Admission rates X_j	α_j	$SS\hat{x}_j$	$SS\bar{x}_j$	Supply levels: beds Y_{j1}	β_{j1}	$SS\hat{y}_{j1}$	$SS\bar{y}_{j1}$	Supply levels: doctors Y_{j2}	β_{j2}	$SS\hat{y}_{j2}$	$SS\bar{y}_{j2}$
General Surgery	23	5.7	.025	.026	15	.51	.056	.138	.25	3.7	.135	.212
General Medicine	28	.001	.239	.395	19	.53	.030	.079	.28	9.5	.047	.078
Obstetrics and Gynaecology	24	2.7	.057	.058	8.6	8.3	.086	.081	.22	.87	.146	.293
Traumatic and Orthopaedic Surgery	12	2.2	.535	.517	21	1.9	.179	.194	.39	5.1	.085	.096
Otorhinolaryngology	5.9	1.0	.385	.390	16	.001	.119	.285	.70	1.0	.587	.756
Paediatrics	12	.001	.170	.307	17	1.0	.205	.197	.37	1.7	.184	.372
Ophthalmology	2.6	5.0	.237	.234	16	1.0	.284	.203	1.1	1.8	.711	.843

Table 8. Comparison of $SS\hat{x}_j/SS\bar{x}_j$ and $SS\hat{y}_{j\ell}/SS\bar{y}_{j\ell}$.

Treatment category	Two individual resource model Admissions Min $\{SS\hat{x}_j/SS\bar{x}_j\}$ from Table 5 & 6	Supply levels:doctors $SS\hat{y}_{j1}/SS\bar{y}_{j1}$ from Table 5	Supply levels:beds $SS\hat{y}_{j1}/SS\bar{y}_{j1}$ from Table 5	Two resource model Admissions $SS\hat{x}_j/SS\bar{x}_j$ from Table 7	Supply levels:doctors $SS\hat{y}_{j1}/SS\bar{y}_{j1}$ from Table 7	Supply levels:beds $SS\hat{y}_{j2}/SS\bar{y}_{j2}$ from Table 6
General Surgery	.38	.47	.69	.96	.63	.41
General Medicine	.51	.79	.43	.61	.60	.38
Obstetrics & Gynaecology	.79	.53	1.15	.98	.50	1.06
Traumatic & orthopaedic Surgery	1.01	.83	.98	1.03	.89	.97
Otorhinolaryngology	.87	.95	.69	.99	.78	.42
Paediatrics	.74	.71	1.01	.55	.49	1.04
Ophthalmology	.72	.93	1.25	1.01	.84	1.40

by the same measure, produces the following list (lowest first):

-- General Medicine
-- General Surgery
-- Ophthalmology
-- Obstetrics and Gynaecology
-- Otorhinolaryngology
-- Traumatic and Orthopaedic Surgery
-- Paediatrics

The lists are very similar, apart from the switching of paediatrics and ophthalmology, and suggest the hypothesis that the levels of hospital beds and doctors are more important for forecasting admission rates and supply levels per patient for general surgery and general medicine than for traumatic and orthopaedic surgery.

As Table 7 indicates, the two-resource DRAM is able to reproduce the observed results better for some treatment categories than for others. Ideally, for planning purposes, a better model would be desirable. To achieve this further analysis is necessary, requiring access to health care planners within the SWRHA, and access to more detailed data. For instance, it would be interesting to consider whether

- Any improvements could be gained by disaggregating the patient categories
- There are better resource measures of the resource types hospital beds and hospital doctors; for instance, using the "total number of hospital doctors of all grades within a speciality", rather than "the total number of consultants within a speciality"
- Cross-area flows of patients vary with patient category
- Other resource types should be introduced into the model (e.g. diagnostic services, aneasthetists, nurses, operating theaters) to improve its explanatory power

In the remainder of this paper, only the patient categories - general surgery, general medicine, and obstetrics and gynaecology, will be considered. This is because a "consultant-year" is probably too coarse a measure of the hospital doctor resource type, and for this reason the patient categories - otorhinolaryngology, paediatrics, and ophthalmology - have been excluded. Traumatic and orthopaedic surgery has been excluded (pending further analysis) because so far there is no indication that for this category total available hospital beds and doctors are related to admission rates and supply levels per patient. In terms of resources, the three chosen patient categories utilize 64% of the bed-days and 55% of the bed-days of the 7 patient categories.

A two-resource DRAM was parameterized for the general surgery, general medicine, and obstetrics and gynaecology treatment categories, using 1975 and 1976 data. The parameters are given in Table 9. In the next section this model is "tested" by using it to predict the outcomes for 1977 and by comparing these predictions with the observed data.

4.5. Predicting the Allocation of Health Care Resources in 1977 Using a Model Developed from 1975 and 1976 Data

Section 4.1. considered how the DRAM parameter set $\{X,Y,\alpha,\beta\}$ changed over time. It was considered that for the time scales under consideration, only the parameter Y changed with time. It was assumed that the actual supply levels per patient where reduced annually by 2½%. The parameters in Table 9 were calculated from 1975 and 1976 data, with the 1976 data standardized to 1975 (as indicated in Section 4.1.). Thus the Y parameters in Table 9 relate to 1975. By a process of trial and error, it was found that the Y parameters should be reduced by 6.8% to produce an average reduction of 5% in the supply levels per patient. The adjusted parameter set was used to predict the allocation of resources for 1977, given the total available resources for the three patient categories actually allocated in 1977. The predictions and actual results are given in Table 10.

The question arises: Are the differences between observed and actual values consistent with the predictive accuracy of the

Table 9. Two-resource (hospital beds and doctors) DRAM parameter estimates for South West Health Region in-patient care.

Treatment category	Admission rates				Supply levels: beds				Supply levels: doctors			
	x_j	α_j	$SS\hat{x}_j$	$SS\bar{x}_j$	γ_{j1}	β_{j1}	$SS\hat{y}_{j1}$	$SS\bar{y}_{j1}$	γ_{j2}	β_{j2}	$SS\hat{y}_{j2}$	$SS\bar{y}_{j2}$
General Surgery	26	4.2	.014	0.026	11	2.2	.058	.138	.24	11	.128	.212
General Medicine	45	.001	.131	0.395	15	2.0	.052	.079	.27	23	.061	.078
Obstetrics and Gynaecology	20	200	.069	.058	8.6	6.5	.061	.081	.19	3.8	.118	.293

Table 10. 1977 Allocation of resources - model predictions and actual results.

	AVON		CORNWALL		DEVON		GLOS		SOMERSET	
	Prediction	Actual	Prediction	Actual	Prediction	Actual	Prediction	Actual	Prediction	Actual
General Surgery										
Admission rate per 1000 pop	21.6	21.8	20.8	20.5	20.8	20.8	19.0	19.7	20.7	21.5
Bed-days per patient	9.6	9.9	8.3	7.5	8.4	9.4	10.6	10.1	8.2	8.6
Doctor-days per patient	0.19	0.22	0.19	0.16	0.19	0.19	0.17	0.14	0.19	0.18
General Medicine										
Admission rate per 1000 pop	19.8	17.9	15.9	18.6	15.9	14.1	9.9	10.3	15.7	12.4
Bed-days per patient	12.6	12.2	10.8	10.3	10.9	11.4	14.0	13.8	10.6	12.6
Doctor-days per patient	0.23	0.20	0.23	0.21	0.23	0.25	0.22	0.27	0.23	0.31
Obstetrics and Gynaecology										
Admission rate per 1000 pop	19.6	23.1	19.6	18.0	19.6	19.4	19.6	20.3	19.6	19.0
Bed-days per patient	7.7	7.5	7.2	7.8	7.3	6.8	8.0	8.6	7.2	7.1
Doctor-days per patient	0.11	0.11	0.11	0.14	0.11	0.12	0.09	0.08	0.12	0.11

model? To answer this question, it will be assumed that the error term associated with the prediction of x_j is normal, with mean 0 and variance σ_j^2; and the error term associated with the prediction of $y_{j\ell}$ is normal with mean 0 and variance $w^2_{j\ell}$. σ_j and $w_{j\ell}$ will be estimated as follows

$$\sigma_j = \left[\frac{\sum_{i=1}^{N} (\hat{x}_j(i) - x_j(i))^2}{N-4} \right]^{\frac{1}{2}} \qquad \left(= w_j \left(\frac{SS\hat{x}_j}{N-4}\right)^{\frac{1}{2}} \text{ from Table 9}\right)$$

$$w_{j\ell} = \left[\frac{\sum_{i=1}^{N} (\hat{y}_{j\ell}(i) - y_{j\ell}(i))^2}{N-4} \right]^{\frac{1}{2}} \qquad \left(= v_{j\ell} \left(\frac{SS\hat{y}_{j\ell}}{N-4}\right)^{\frac{1}{2}} \text{ from Table 9}\right)$$

where in this instance N = 10
(using the divisor (N-4) in the above expressions is discussed in Appendix C).

Thus if the difference between the observed and the actual value is divided by the appropriate standard deviation, the resulting standardized error variable is distributed normally with mean 0 and variance 1. There are 45 such variables, and a comparison between these standardized error variables (calculated from Table 10) and the expectation given a normal distribution of mean 0 and variance 1 is given in Table 11. The table indicates that there is no statistical difference between the actual and theoretical deviations. This suggests that the model has successfully predicted the resource allocation in 1977 given the assumed predictive accuracy of the model.

Table 11. Distribution of standardized error variable.

	-0.85	-0.84 -0.26	-0.25 +0.25	0.26 0.84	0.85
Actual	8	9	10	11	7
Expected assuming normal distribution mean 0 variance 1	9	9	9	9	9

5. ILLUSTRATIVE EXAMPLES OF THE USE OF THE TWO-RESOURCE DRAM FOR SOUTH WEST HEALTH REGION IN-PATIENT HOSPITAL CARE

In the previous section, the estimation of the parameters for a three-patient-category model of the South West Health Region in-patient hospital care was described. The model parameters were derived from 1975 and 1976 data, and it successfully predicted the results for 1977. Normally, the next stage would be to derive the DRAM parameters using all three years data, then use the new model to make predictions for different allocations of resources. However, 1975 was the year of the junior hospital doctors' strike referred to earlier. Its effect on the utilization of available health care resources is uncertain. In view of this, it was decided to estimate the parameters of the three-patient-category model using 1976 and 1977 data only. The 1977 data was adjusted to be compatible with the 1976 data using the approach given in Section 4.1. The (1976) parameter estimates are given in Table 12. Examination of the ratios $SS\hat{x}_j/SS\bar{x}_j$ and $SS\hat{y}_{j\ell}/SS\bar{y}_{j\ell}$ for Tables 9 and 12 indicates that the model derived from 1976 and 1977 data has reproduced the actual results better than the model derived from 1975 and 1976 data.

Table 12. Two-resource (hospital beds and doctors) DRAM parameter estimates for South West Health Region in-patient care.

Treatment category	Admission rates				Supply levels: beds				Supply levels: doctors			
	X_j	α_j	$SS\hat{x}_j$	$SS\bar{x}_j$	Y_{j1}	β_{j1}	$SS\hat{y}_{j1}$	$SS\bar{y}_{j1}$	Y_{j2}	β_{j2}	$SS\hat{y}_{j2}$	$SS\bar{y}_{j2}$
General Surgery	26	2.4	.004	.013	13	.98	.053	.126	.28	2.8	.111	.258
General Medicine	32	.015	.155	.418	15	2.3	.038	.083	.25	5.0	.193	.188
Obstetrics and Gynaecology	33	.43	.040	.071	9.0	3.3	.037	.072	.19	2.0	.099	.322

5.1. Predicting the Allocation of Health Care Resources in the Wessex Health Region Using a Model Developed from South West Health Region Data

Section 4.5. described how a three-patient-category DRAM made successful predictions across time in the same health region. This section describes how a three-patient-category model was used to make predictions across space for the same year. The three-category model (parameters given in Table 12) derived from South West Health Region data for 1976 and 1977 was used to make predictions in the allocation of health care resources in the Wessex Health Region in 1976, given the actual resources available in the region for 1976. The Wessex health region adjoins the South West Health Region, and consists of four Area Health Authorities:

Dorset (estimated resident population in 1976, 576,000)

Hampshire (1,459,000)

Wiltshire (515,000)

Isle of Wight (113,000)

The Isle of Wight is an island off the coast of Hampshire and it only has a population of just over 110,000. This is a rather small population for prediction purposes, and so the Hampshire and the Isle of Wight areas were combined. The population of Wiltshire was augmented by 174,000 to take into account the fact that the Wiltshire Area Authority also provides hospital care for some people living in the South West Health Region. (In the earlier calculations for the South West Health Region, the appropriate population figures were reduced by the corresponding amount.) In making the predictions for Wessex, it was assumed that the same model was applicable to both Health Regions. This is implicitly assuming that morbidity patterns and aggregate behavior of the Health Care Systems are the same for both regions. Table 13 gives both the predicted and actual allocations of heatlh care resources for Wessex in 1976.

Table 13. Allocation of Resources in the Wessex Health Region.

	DORSET		HAMPSHIRE AND ISLE OF WIGHT		WILTSHIRE	
	Predicted	Actual	Predicted	Actual	Predicted	Actual
General Surgery						
Admission rate per 1000 pop	20.9	19.8	20.3	18.7	21.5	22.4
Bed-days per patient	9.7	10.9	9.5	9.4	9.8	9.8
Doctor-days per patient	.19	.20	.18	.17	.21	.19
General Medicine						
Admission rate per 1000 pop	15.8	13.5	14.0	13.8	17.3	14.6
Bed-days per patient	12.5	13.2	12.3	13.1	12.6	12.9
Doctor-days per patient	.24	.27	.24	.23	.24	.28
Obstetrics and Gynaecology						
Admission rate per 1000 pop	20.7	16.7	19.3	18.7	21.8	22.5
Bed-days per patient	7.9	10.3	7.8	8.5	8.0	8.7
Doctor-days per patient	.11	.14	.10	.14	.12	.13

Here again it is necessary to ask whether the differences between the observed and actual values are consistent with the predictive accuracy of the model. Using the methods of section 4.5., the standardized error variables were calculated. A comparison between the distribution of these variables and the expectation is given in Table 14. A χ^2 test carried out on the data in Table 14 revealed that the two sets of data were not statistically different (χ^2_4 statistic not significant at the 25% level). This suggests that the model has successfully predicted the resource allocation in Wessex for 1976, given the assumed predictive accuracy of the model.

Table 14. Distribution of standardized error variables.

		-0.84	-0.25	0.84	0.85
	-0.85	-0.26	+0.25	0.84	
Actual	8	7	5	2	5
Expected assuming normal distribution mean 0 variance 1	5.4	5.4	5.4	5.4	5.4

5.2. Using the Two-Resource DRAM to Predict the Consequences of Changes in Resource Mix in the South West Health Region

In the introduction of this paper, the point was made that an issue facing the SWRHA Health Care Planners was the consequence of increasing the number of hospital doctors and reducing the number of hospital beds. Having indicated in earlier sections that within the assumed predictive accuracy the three-category DRAM can successfully predict across space and time, we are now in a position to assist with the above problem for the general surgery (and urology), general medicine (and cardiology), and obstetrics and gynaecology patient categories.

In the first approach to the problem, changes in resource mix will be considered which do not involve any overall increase in marginal cost. It will be assumed that the marginal costs of the resources (at existing levels of utilization) are such that 1 doctor-day = 10 bed-days. In 1976, for the three patient categories used in the model the resource allocation for the whole of the SWHR was 530 bed-days per 1000 population and 10.0 doctor days per 1000 populations. Predictions will now be considered of what would have happened in 1976 had the resource mix been

A 520 bed-days, 11 doctor-days per 1000 population

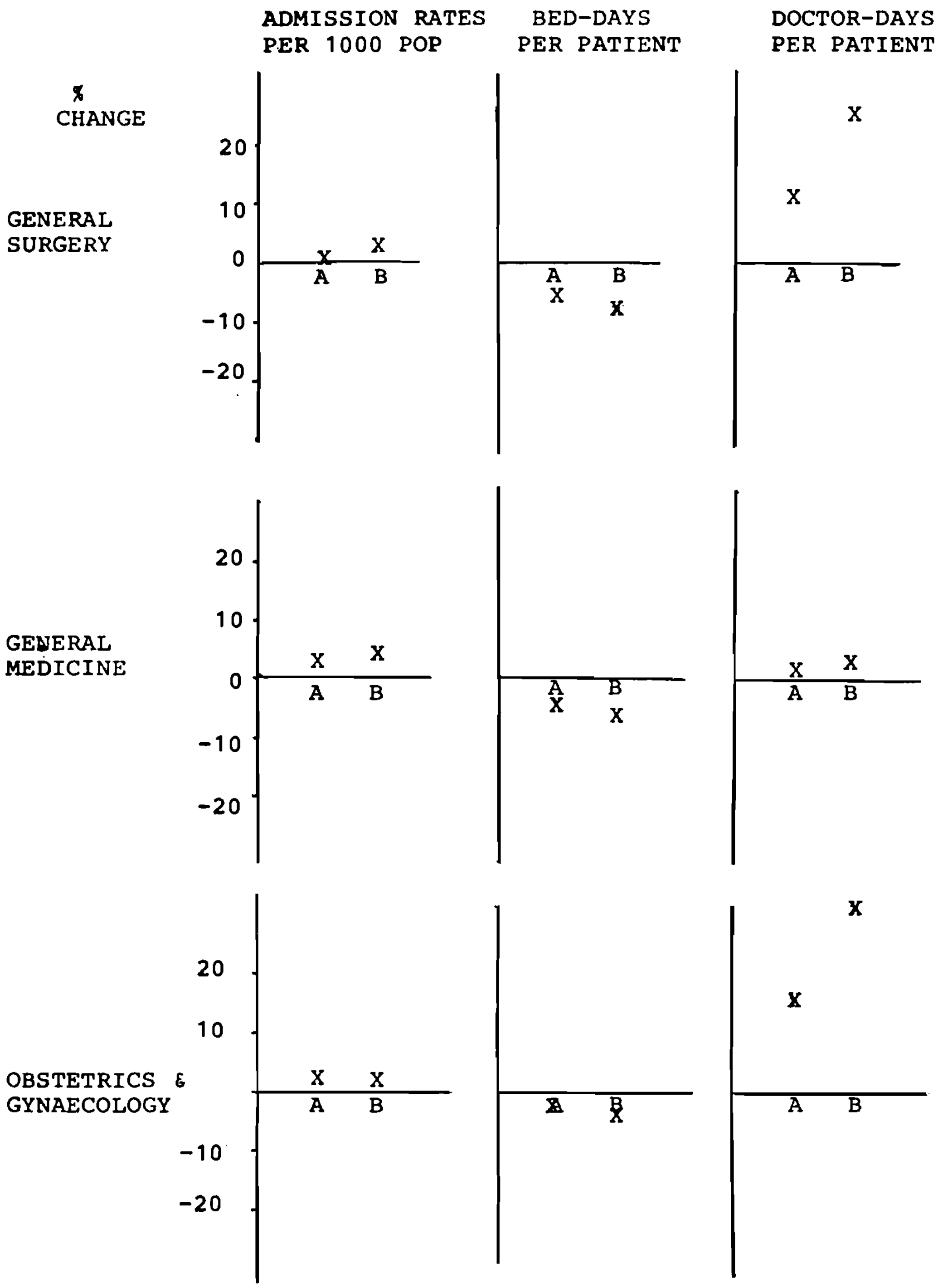
B 510 bed-days, 12 doctor-days per 1000 population

Before doing this, it must be demonstrated that the model reproduces quite closely the actual allocation of resource for 1976. Table 15 shows that the actual allocations and the model predictions are quite close.

Table 15. 1976 allocation of resources in the South West Health Region.

	Admission rates per 1000 pop.		Bed-days per patient		Doctor-days per patient	
	Predicted	Actual	Predicted	Actual	Predicted	Actual
General Surgery	20.7	20.5	9.2	9.6	.19	.20
General Medicine	15.2	14.7	12.2	12.2	.24	.24
Obstetrics & Gynaecology	20.0	20.0	7.8	7.7	.12	.12

Figure 5 gives the percentage changes in the model predictions for 1976 resource levels predicted for the resource mixes A and B. Figure 5 indicates that changes in resource mix would



Predicted for resource allocations
(A) 520 bed-days, 11 doctor-days per 1000 pop
(B) 510 bed-days, 12 doctor-days per 1000 pop

Figure 5. Percentage changes in model predictions using 1976 data.

have given rise in 1976 to only a small increase in admission rates. Bed-days per patient would have been slightly reduced and doctor-days per patient would have been increased by a greater amount.

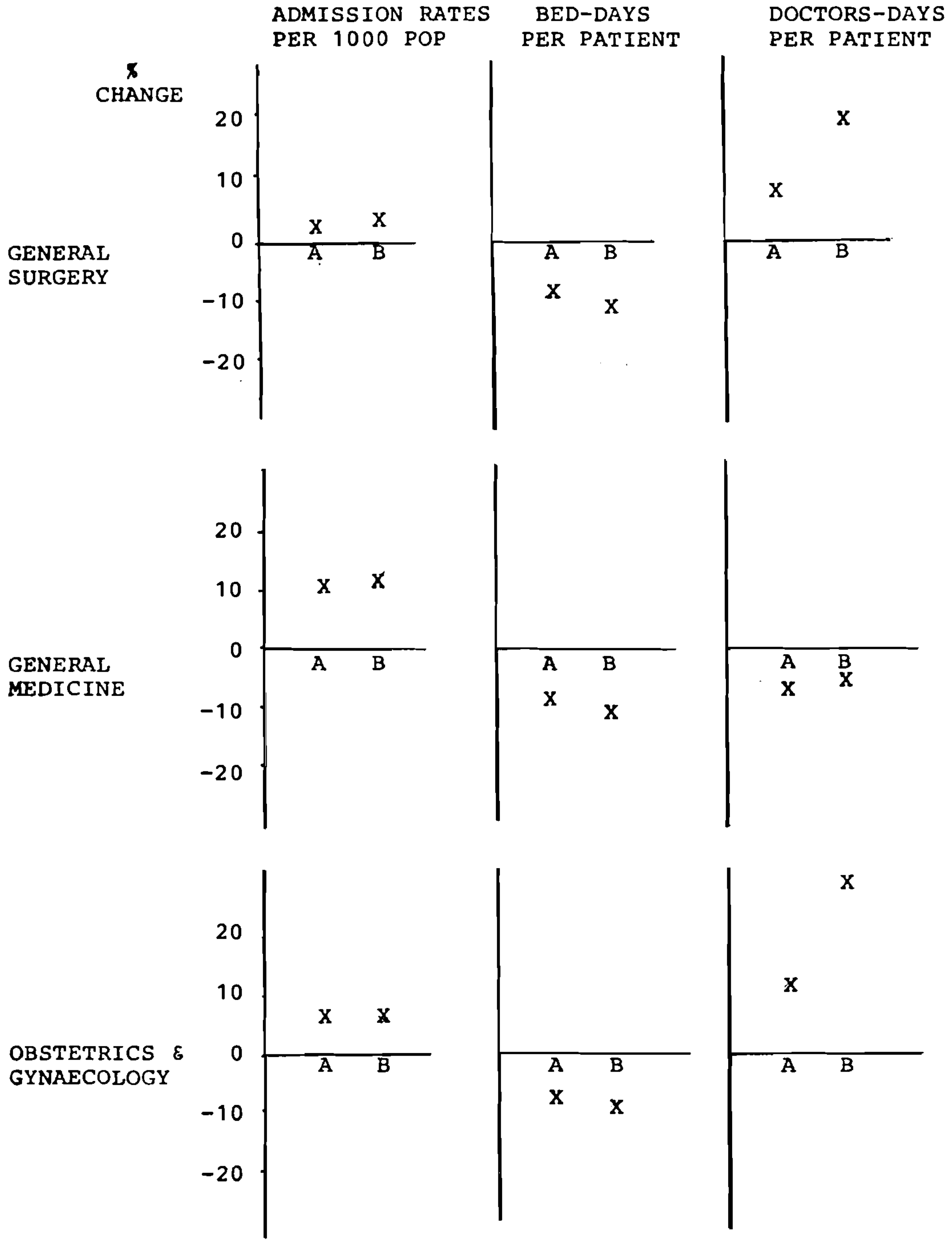
The above analysis has considered the consequences of changing the mix of resource in 1976. In a real application of the model, the parameters would be estimated for a given year (or group of years) and then used to make estimates for some future years. This we shall now do. The model calibrated from 1976 and 1977 data will be used to make predictions for 1978. In section 4.1. there was a discussion of how the parameter set $\{X, Y, \alpha, \beta\}$ was likely to change over time. It was decided that it was only necessary to change the ideal supply levels per patient. It was assumed that the average supply levels per patient declined by 2½ per annum. To make the predictions for 1978, the parameters $\{Y\}$ were changed so as to give an average reduction in supply levels per patient of 5% at 530 bed-days per 1000 population and 10.0 doctor-days per patient (the parameters in Table 12 were calculated for the year 1976). Figure 6 gives the percentage changes in model predictions (using the 1976 model) for the 1976 mix of resources, predicted for the two resource mixes (below) using the 1978 model

A 520 bed-days, 11 doctor-days per 1000 population
B 510 bed-days, 12 doctor-days per 1000 population

The percentage changes given in Figure 6, thus arise from two sources

1 changes in the resource mix
2 changes in $\{Y\}$ because of improvements in "productivity"

After comparing Figures 5 and 6, it appears that 2 is likely to have a greater effect in increasing admission rates than 1. The percentage changes in supply levels per patient in Figure 6 are less than those in Figure 5, as would be expected since the ideal



For 1976 resource allocation (530 bed-days, 10 doctor-days per 1000 pop)

Predicted for resource allocation using 1978 model parameters -
(A) 520 bed-days, 11 doctor-days per 1000 pop
(B) 510 bed-days, 12 doctor-days per 1000 pop

Figure 6. Percentages changes in model predictions using 1976 parameters.

levels of supply {Y} have been reduced.

In the above analysis, the consequences in the SWHR of changing the resource mix have been considered in the case where there is no overall increases in marginal cost. This constraint will now be relaxed and more general changes considered. Suppose it is required to compare the consequences in 1978 of two resource mixes

C 530 bed-days, 10 doctor-days per 1000 population
D 580 bed-days, 11 doctor-days per 1000 population

Table 16 gives the model predictions for resource mixes C and D.

The model predicts that more patients will be treated for resource mix D. Further, the model indicates the differential rates of increase. For instance, it is estimated that the admission rate for general medicine is 10% higher for D than for C (α_j is relatively small for general medicine - Table 12). Whereas, it is estimated that the admission rates for general surgery is only 2.6% higher for D than for C (α_j is relatively high for general surgery). The model also predicts larger supply levels per patient for resource mix D.

An alternative way of using the model would be to estimate the admission rates and supply levels per patient in 1978, for a range of total resource levels, for example for all combinations of 480,530,580 bed-days per 1000 population and 9,10,11,12 doctor-days per 1000 population. Having done this, one would take each patient category and see how admission rates and supply levels per patient vary with total resource levels. Figure 7 gives a possible way of illustrating the results for general surgery. In this graph, the axes are the resource availabilities. The figure indicates resource mixes where the model predicts the same admission rate ("contour lines"). Similar contour lines are given for the supply levels per patient. Thus the health care planner can see how predictions of admission rates and supply levels per patient vary with resource mix.

Table 16. Predicted allocation of health care resources in SWHR.

	Treatment category	Model Prediction 530 bed-days per 1000 pop 10 doctor-days per 1000 pop	Model Prediction 580 bed-days per 1000 pop 11 doctor-days per 1000 pop
ADMISSION RATES PER 1000 POP	Gen.Surg.	21.2	21.8
	Gen.Med.	16.5	18.2
	Obst.& Gynae.	21.1	22.4
BED-DAYS PER PATIENT	Gen.Surg.	8.8	9.2
	Gen.Med.	11.4	11.7
	Obst.& Gynae.	7.3	7.4
DOCTOR-DAYS PER PATIENT	Gen.Surg.	0.19	0.20
	Gen.Med.	0.22	0.22
	Obst.& Gynae.	0.11	0.12

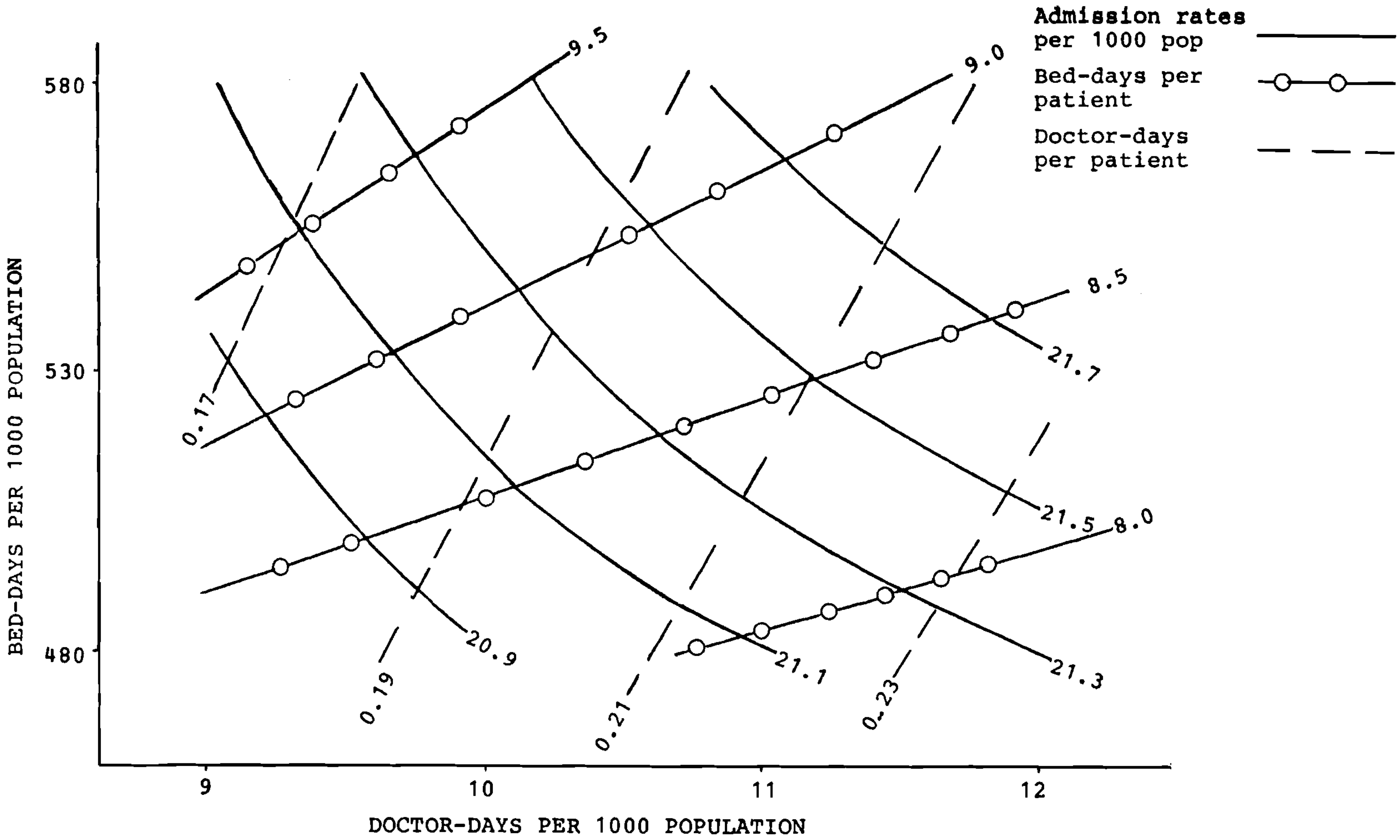


Figure 7. Two-resource DRAM model (using 1978 parameters) predictions for General Surgery in the South West Health Region.

6. CONCLUSION

This working paper began by pointing out that a particular problem facing the health care planners in the South West Health Region, UK, is the consequence (in terms of changes in the admission rates and supply levels per patient of hospital resources) of increasing the number of hospital doctors and decreasing the number of hospital beds for hospital in-patient care.

The above problem has been considered within the context of the Disaggregated Resource Allocation Model (DRAM). Firstly, parameters were estimated for a model with seven patient categories

-- General surgery
-- General medicine
-- Obstetrics and gynaecology
-- Traumatic and orthopaedic surgery
-- Otorhinolaryngology
-- Paediatrics
-- Ophthalmology

and two resource types

-- Hospital doctors
-- Hospital beds

from 1975 and 1976 data from the South West Health Region. The ability with which this model was able to reproduce the actual 1975 and 1976 resource allocations for the seven patient categories had similarities with another model parameterized by Aspden and Ruznak (1980) for the same patient categories and the same resource types. For example, both models reproduced the actual resource allocations best for general surgery and general medicine. The actual resource allocations for traumatic and orthopaedic surgery were relatively poor in both models.

In the remainder of the paper, models with three patient categories were considered. Traumatic and orthopaedic was excluded because neither the SWHR model nor the Czechoslovakian model reproduced well the actual behavior for this category. Otorhinolaryngology, paediatrics and ophthalmology were all excluded because the measure of the hospital doctor supply was considered to be probably too coarse to be consistent with the DRAM assumptions.

The parameters for a three-patient-category/two-resource DRAM were estimated from 1975 and 1976 data. Within the assumed predictive accuracy of the model, DRAM successfully predicted the resource allocations for 1977 in the South West Health Region. Similarly, the parameters for a three-patient-category/two-resource DRAM were estimated from 1976 and 1977 data from the South West Health Region. Within the assumed predictive accuracy of this model, DRAM successfully predicted the resource allocations for 1976 in the Wessex Health Region, the region adjacent to the South West Health Region. Thus the three-category DRAM successfully predicted across time and space.

The second of the three-patient-category/two-resource models was then used to explore the consequences of changing the resource mix in the South West Health Region. For increases in the supply of doctors and decreases in the supply of beds which involved no overall increases in marginal cost, the model predicted that a small increase in admission rates would occur. Further runs of the model indicated that these increases would be smaller than the increases in admission rates arising because of improvements in "productivity" in the health care system (provided total resource levels were kept fixed). The consequences of more general changes in the mix of hospital doctors and beds were also considered.

All the above analyses have been carried out using aggregated data from the SWHR and with little contact with officials of SWRHA. To a certain extent, therefore, the analyses described in this paper are of an indicative nature. They indicate how DRAM could be used to explore the consequences for hospital inpatient care of changes in the mix of hospital resources. With access to health care planners within the region and more detailed data, a version of DRAM could be produced which gives a more comprehensive version of hospital in-patient care and is more capable of reproducing actual allocations of health care resources. In particular, improvements could be achieved by using a finer measure of hospital doctor supply, e.g. all hospital doctors within a speciality rather than just consultants. Improved accuracy may be achieved by assuming cross area flows of patients vary with patient category. This may be one of the reasons why the model is unable to reproduce actual allocations of resources for traumatic and orthopaedic surgery. Another reason could be that hospital beds and hospital doctors are not the important resources for this category of patient. In general, it would be worthwhile considering the inclusion of other resource types (e.g., operating theaters, nurses) into the model of hospital in-patient care.

APPENDIX A: PARAMETER ESTIMATION FOR DRAM

1. Introduction

To estimate the DRAM parameters (X,Y,α,β) for South West Health Region hospital in-patient care, the approach of Hughes (1978c) was followed. The approach is described here in largely qualitative terms. The technical details can be found in Hughes (1978c).

It is assumed the utility function Z (equation 1), is applicable both to the whole of the South West Health Region and also to each of the individual areas within the region. Further, by making some adjustments (see section 4.1.) the same utility function can be used for each area for successive years. Thus each area provides an independent data point for each year to estimate (X,Y,α,β). The available data points are split into two approximately equal groups. Initial estimates of (X,Y) are provided, and the (α,β) are estimated using the first data set (details given below). Given these estimates of (α,β), new (X,Y) are then estimated from the second data set (details also given below). Given these new (X,Y) further (α,β) are then estimated using the first data set and so on until successive estimates of (X,Y,α,β) only change by a small amount.

Before discussing these two estimation procedures, it is necessary to introduce additional notation. Following the notation introduced in section 4.1., the N data points are defined as

$x_{jk}(i)$, $y_{jk\ell}(i)$, $R_\ell(i)$ $\quad i = 1...N \quad$ λ_ℓ is the Lagrange

multiplier associated with each resource constraint

$$\sum_j \sum_k x_{jk} y_{jk\ell} = R_\ell$$

2. Estimates of (α,β) given (X,Y)

To start the estimation process, λ_ℓ must be provided externally for each resource type. The same λ_ℓ is used for all data points. More will be given later about the choice of λ_ℓ.

Hughes (1978c) has indicated that in a certain sense unbiased estimates of (α,β) can be determined by solving iteratively the following set of equations given (X,Y):

$$\ln\left(x_{jk}(i)\right) = a^x_{jk} + \left(\frac{1}{\alpha_j+1}\right)\sum_\ell A_{jk\ell} \ln\left(R_\ell(i)\right) + \varepsilon^x_{jk}(i) \;\forall_{j,k,i}$$

$$\ln\left(y_{jk\ell}(i)\right) = a^y_{jk\ell} + \frac{1}{(\beta_{jk\ell}+1)} \sum_m B_{\ell m} \ln\left(R_m(i)\right) + \varepsilon^y_{jk\ell}(i) \;\forall\; j,k,\ell,i$$

where

a^x_{jk}, a^y_{jk} are unknown constants

$A_{jk\ell}$, $B_{\ell m}$ are known functions of α and β given X,Y, and λ.

and

$\varepsilon^x{}_{jk}$, $\varepsilon^y{}_{jk\ell}$ are random uncorrelated error terms with zero means.

Within the above iteration process, there is a mechanism to maintain the non-negativity conditions on (α,β). If at the end of an iteration an α or β is estimated to be negative, then the parameter is set to 0.001 if the prediction error for the parameter is small, otherwise it is reset to some arbitrary level (normally 1 or 5). The estimation of (α,β) is depicted in Figure A1.

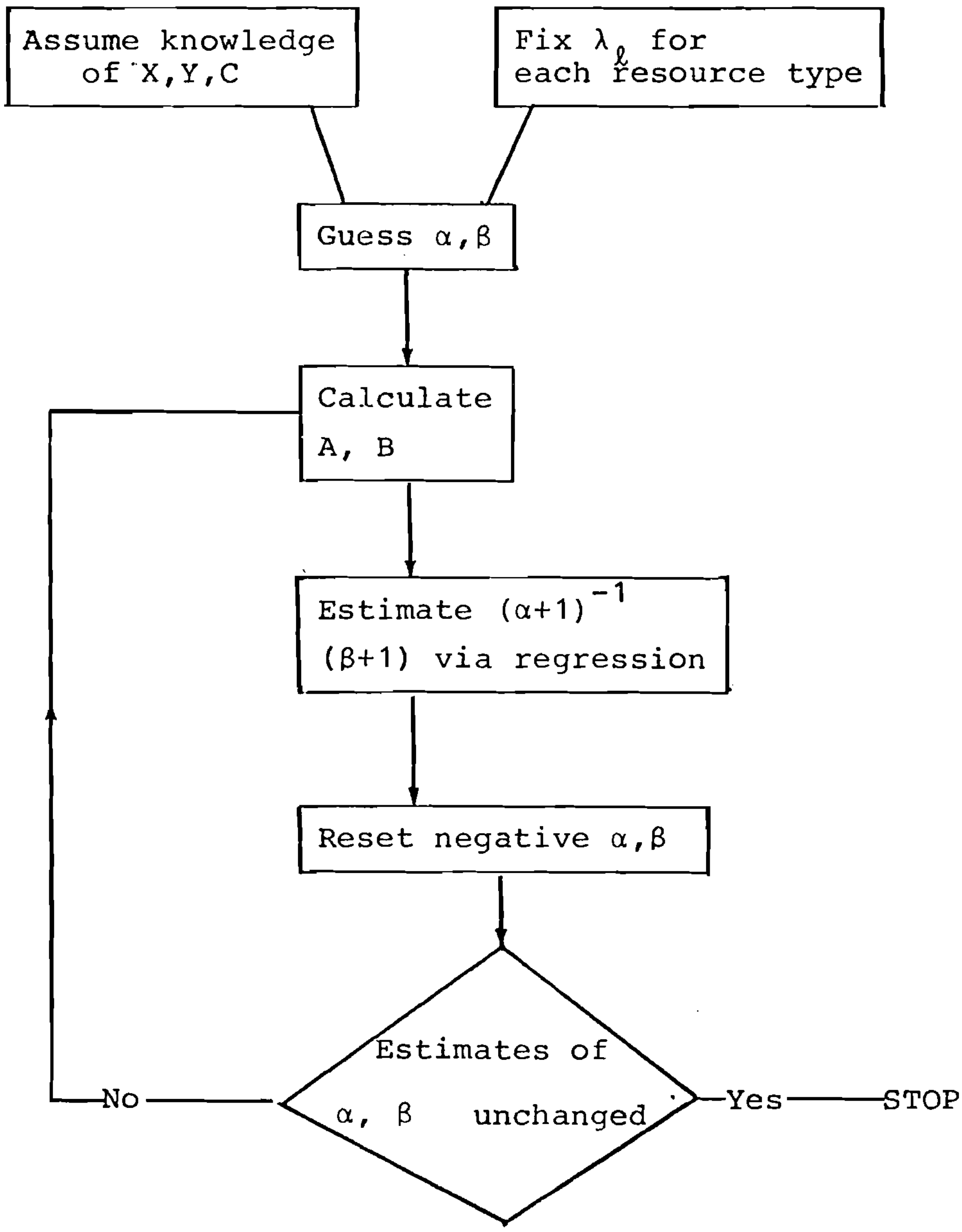


Figure A1. Estimation of $\{\alpha,\beta\}$.

3. Estimates of (X,Y) given (α,β)

Hughes (1978c) shows that

$$X_{jk} = x_{jk}(\mu_{jk})^{\frac{1}{\alpha_j+1}} \qquad \text{from equation 3}$$

$$Y_{jk} = y_{jk\ell}(\lambda_\ell)^{\frac{1}{\beta_{jk\ell}+1}} \qquad \text{from equation 2}$$

where μ_{jk} is a function of $\alpha,\beta,Y,$ and λ. Thus given (α,β), (X,Y) can be estimated iteratively if θ_ℓ is known. Huhges shows that if we can specify Θ_ℓ the ratio of type ℓ resources at ideal levels to current usage, i.e.

$$\sum_j \sum_k X_{jk} Y_{jk\ell} = \Theta_\ell \sum_j \sum_k x_{jk} y_{jk\ell} \qquad \forall \ell$$

then λ_ℓ can be determined.

The above is the procedure for the first data point. For the second (and succeeding) data points the value of the ideal resource needs (i.e. $\sum_j \sum_k X_{jk} Y_{jk\ell}$) specified for the first data point is used similarly to determine λ_ℓ for the second (and succeeding) data points. Thus the specification of Θ_ℓ at the first data point is used to fix λ_ℓ for each of N data points. Each data point provides an estimate of (X,Y). A weighted average of the N estimates of (X,Y) is then produced. The estimation of (X,Y) is depicted in Figure A2.

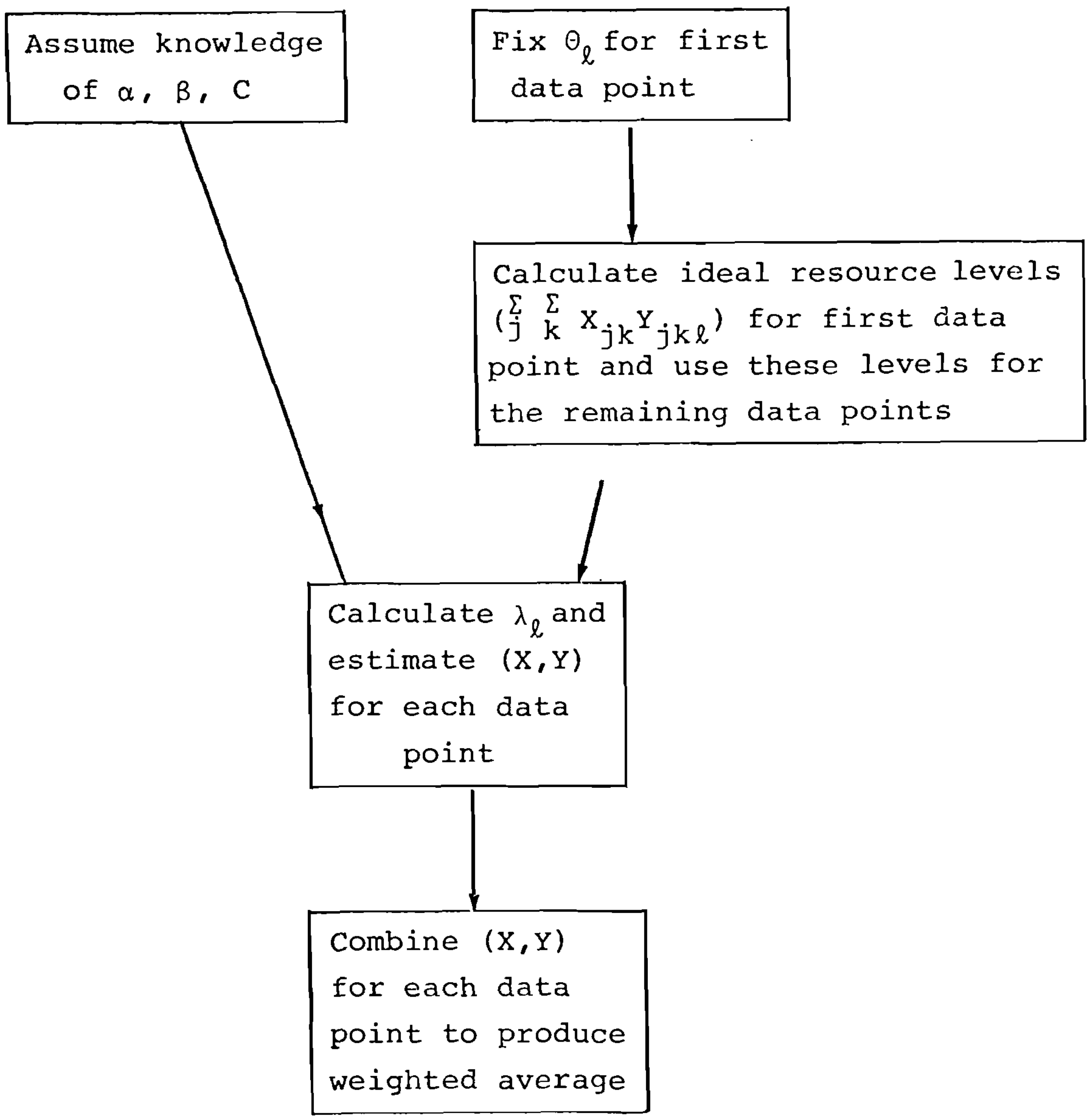


Figure A2. Estimation of ideal levels.

4. The Linkage Between the Estimation Procedures

The two estimation procedures are linked in the following manner:

(1) The estimates of (X,Y) are used as input for the other procedure. This is similar for (α,β)

(2) Both estimation procedures require the input of values for λ_ℓ. These should be consistent in the following sense. Consider parameter estimation when there is one resource type and ten data points (five data points for each procedure). In (X,Y) estimation, setting Θ_1 means that λ_1 is fixed for the five data points, e.g.

		R_1	λ_1
Data point	1	600	1.5
"	2	540	1.8
"	3	520	2.0
"	4	510	2.2
"	5	480	2.6

If data points 6-10 have an average resource level of 535, then λ for the (α,β) estimation should satisfy $1.8<\lambda<2.0$.

Arising from the second of the two linkage mechanisms, is the fact that Θ_ℓ must be provided externally. Θ_ℓ is the ratio of type ℓ resources at ideal levels to current usage at a particular data point. Health care planners should be able to provide an approximate estimate of this ratio. The complete parameter estimation process is given in Figure A3.

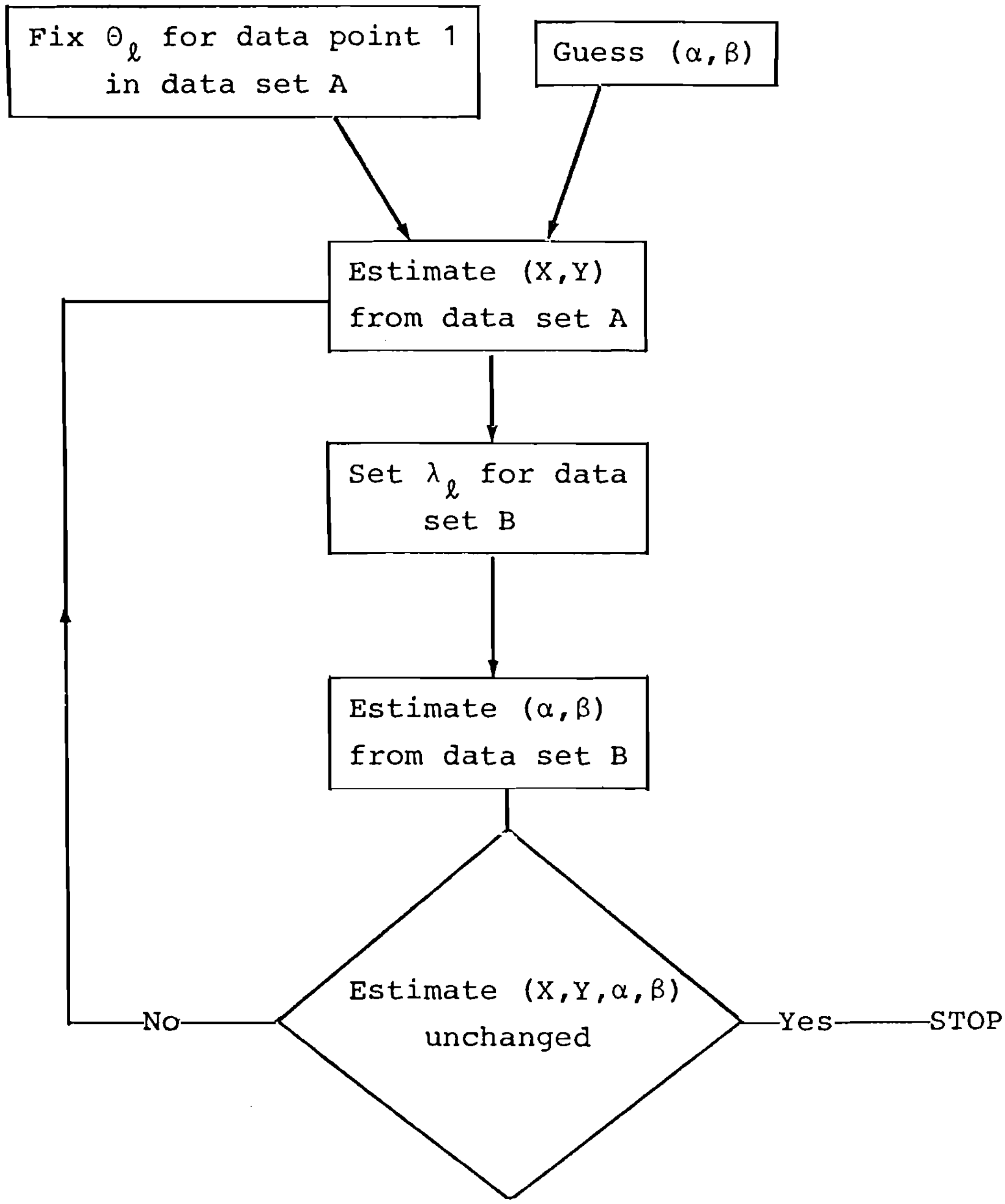


Figure A3. The parameter estimation process.

5. Measure of Goodness-of-Fit

In addition to the parameter estimation mentioned above, it is also useful to have some way of deciding whether successive sets of (X,Y,α,β) are "better". In addition, it is useful to consider whether different values of Θ_ℓ give rise to "better" parameter sets. Lastly, it is interesting to see if certain parameters from the set (X,Y,α,β) are fixed exogenously, whether the estimation procedure produces "improved" parameter sets.

The following measure of goodness-of-fit has been used to compare parameter sets:

$$SS = \sum_j \sum_i \left(\frac{x_j(i)-\hat{x}_j(i)}{w_j} \right)^2 + \sum_j \sum_\ell \sum_i \left(\frac{y_{j\ell}(i)-\hat{y}_{j\ell}(i)}{v_{j\ell}} \right)^2$$

where

(1) $x_j(i)$, $y_{j\ell}(i)$ $(i=1...N)$ are the actual data points and

$\hat{x}_j(i)$, $\hat{y}_{j\ell}(i)$ $(i=1...N)$ are the predicted levels from DRAM given a particular parameter set and resource availabilities at data point i are

$R_\ell(i) = \sum_j x_j(i) y_{j\ell}(i)$;

(2) w_j and $v_{j\ell}$ are scaling factors, set as follows -

w_j is an average (possibly weighted) of $x_j(i)$, $i=1...N$

$v_{j\ell}$ is an average (possibly weighted) of $y_{j\ell}(i)$, $i=1...N$;

(3) the modal subscript has been omitted.

In practice it is useful to split this measure into the following sections:

$$SS\hat{x}_j = \sum_i \left(\frac{x_j(i)-\hat{x}_j(i)}{w_j} \right)^2$$

$$SS\hat{y}_{j\ell} = \sum_i \left(\frac{y_{j\ell}(i)-\hat{y}_{j\ell}(i)}{v_{j\ell}} \right)^2$$

Thus

$$SS = \sum_j SS\hat{x}_j + \sum_j \sum_\ell SS\hat{y}_{j\ell}$$

6. Computational Procedure

Experience has shown that the parameter estimation procedure given in Figure A3 converges about half the time within 6 to 9 iterations. Convergence is assumed when the change in parameter estimates is about 4 %.

If there is no sign of convergence after seven iterations, the process should be stopped. Frequently, in such cases parameter estimates are oscillating. Often this arises when the actual admission rates (or resource supply levels per patient) exhibit great variation independent of total resource availability.

Whether the estimation procedure converges or not the function SS should be calculated and

$$SS\hat{x}_j \text{ compared with } SS\bar{x}_j = \sum_i \left(\frac{x_j(i)-w_j}{w_j} \right)^2 \qquad \forall j$$

$$SS\hat{y}_{j\ell} \text{ compared with } SS\bar{y}_{j\ell} = \sum_i \left(\frac{y_{j\ell}(i)-v_{j\ell}}{v_{j\ell}} \right)^2 \qquad \forall j\ell$$

If $SS\hat{x}_p > SS\bar{x}_p$ then w_p is a better predictor of the actual results than $\hat{x}_p(i)$. In a one-resource model, this normally arises when $x_p(i)$ is independent of total resource supply. In such circumstances a better model fit (i.e. smaller SS) is normally achieved if X_p is fixed at w_p and α_p is set to a large number in the parameter estimation process.

A similar approach should be adopted if

$$SS\hat{y}_{j\ell} > SS\bar{y}_{j\ell} \quad \text{for a particular } j\ell.$$

As a result of the above comparison there are four options:

(1) Parameter estimation procedure converged and no (X,Y,α,β) fixed. The (X,Y,α,β) should be regarded as the best estimates the method can produce.

(2) Parameter estimation procedure converged and some (X,Y,α,β) fixed. The parameter estimation procedure given in Figure A3 should be run again. Convergence should occur again and after calculating SS, no further (X,Y,α,β) should be fixed. The second set of (X,Y,α,β)

should be regarded as the best estimate the method can produce.

(3) Parameter estimation procedure did not converge and no (X,Y,α,β) fixed. This seems an unlikely event. In such cases perhaps the data points should be reallocated to the two groups, and the parameter estimation process started again.

(4) Parameter estimation procedure did not converge and some (X,Y,α,β) fixed. The parameter estimation procedure given in Figure A3 should be run again and SS calculated. Further $SS\hat{x}_j$ and $SS\hat{y}_{j\ell}$ comparisons should be carried out and more (X,Y,α,β) fixed if necessary, and so on.

Normally a maximum of two runs of the procedure given in Figure A3, should produce usuable (X,Y,α,β).

7. Illustrative Example

In this section the methods described in the previous sections will be illustrated. Parameters will be estimated from a set of data points x_j and y_{j1} generated from the following model,

$$
\begin{array}{lll}
X_1 = 100 & X_2 = 100 & \\
Y_{11} = 10 & Y_{21} = 10 & \text{Ideal level of resources} = 2000 \text{ units} \\
\alpha_1 = 0.1 & \alpha_2 = 10 & \\
\beta_{11} = 0.1 & \beta_{21} = 10 &
\end{array}
$$

The data points are given in Table A1. In estimating the parameters it will be assumed that the ideal level of resources is known, i.e. θ_1 is known. The data points were randomly split into the two groups for estimation purposes as follows.

Data set A - Estimation of (X,Y) data points 1,4,5,6,8

Data set B - Estimation of (α,β) data points 2,3,7,9,10

The average resource levels of data set B is 1120. So using the linkage procedure described in section 4 of this Appendix, the value of λ, in the (α,β) estimation should be set approximately

Table A1. Data points.

Data point i	$x_1(i)$	$x_2(i)$	$y_{11}(i)$	$y_{21}(i)$	$x_1(i)y_{11}(i)+ x_2(i)y_{21}(i)=R(i)$
1	25.4	77.1	.63	7.58	600
2	31.3	81.9	1.20	8.09	700
3	37.4	85.6	1.95	8.49	800
4	43.5	88.5	2.79	8.80	900
5	49.5	90.7	3.64	9.04	1000
6	55.2	92.5	4.47	9.23	1100
7	60.7	93.9	5.26	9.38	1200
8	66.1	95.1	6.00	9.50	1300
9	71.3	96.1	6.69	9.61	1400
10	76.3	97.0	7.34	9.70	1500

equal to the value of λ at data point 6 arising from the (X,Y) estimation.

Starting with the following initial values $\alpha_1 = 1$, $\alpha_2 = 1$, $\beta_{11} = 1$, $\beta_{21} = 1$, the estimation procedure gave the following results

Iteration 1

$X_1 = 65.7$ $X_2 = 124.7$

$Y_{11} = 5.30$ $Y_{21} = 13.25$

λ at data point 6 is 1.89

Iteration 2

λ set at 1.98

$\alpha_1 = 0.710$ $\alpha_2 = 12.1$

$\beta_{11} = 0.273$ $\beta_{21} = 11.7$

Iteration 3

$X_1 = 86.9$ $X_2 = 100.8$

$Y_{11} = 11.27$ $Y_{21} = 10.13$

λ at data point 6 is 3.13

Iteration 4

λ set at 3.00

$\alpha_1 = 0.401$ $\alpha_2 = 11.6$

$\beta_{11} = 0.247$ $\beta_{21} = 11.4$

Iteration 5 — $X_1 = 95.0$ $X_2 = 100.3$

$Y_{11} = 10.45$ $Y_{21} = 10.05$

λ at data point 6 is 2.85

Iteration 6 — λ set at 2.80

$\alpha_1 = 0.278$ $\alpha_2 = 10.7$

$\beta_{11} = 0.161$ $\beta_{21} = 10.6$

Iteration 7 — $X_1 = 95.3$ $X_2 = 100.3$

$Y_{11} = 10.42$ $Y_{21} = 10.04$

λ at data point 6 is 2.64

Iteration 8 — λ set at 2.64

$\alpha_1 = 0.205$ $\alpha_2 = 10.1$

$\beta_{11} = 0.102$ $\beta_{21} = 10.0$

Iteration 9 — $X_1 = 95.1$ $X_2 = 100.3$

$Y_1 = 10.43$ $Y_2 = 10.0$

The above indicates that the (X,Y) parameter estimates are quicker to stabilize at their true values than the (α,β) parameter estimates. Other empirical evidence confirms this view that the (X,Y) parameter estimates stabilize quicker than the (α,β) parameter estimates. Fortunately the sensitivity analysis (described in Appendix B) indicates that less accurate estimates are required for the (α,β) parameters than for the (X,Y) parameters.

Further empirical work has shown that even with the correct (X,Y) and the above data points, there will always be a small error in the estimates of (α,β). This is because λ varies as the resource level changes and in the (α,β) estimation process it is assumed that the same λ is valid for all data points. The error arising from this assumption in this particular case can be estimated by using the parameter values at Iteration 9 to reproduce the original data, and measuring the error as the average of $\frac{100 \times |\text{actual} - \text{estimated}|}{\text{actual}}$ for all data points for x_1, x_2 etc.

Calculated on this basis the average error is

for	x_1	is	2.5%
	x_2	is	0.1%
	y_{11}	is	1.3%
	y_{12}	is	0.2%

This error can be regarded as small.

8. Fixing the Value of θ_ℓ

In parameterizing DRAM for Czechoslovakia hospital in-patient care [Aspden and Ruznak (1980)], values of θ_ℓ were chosen so that actual resource levels were approximately 40-75% of ideal levels. Examination of the differences between actual results and model prediction indicated that the chosen values of θ_ℓ did not introduce bias.

In this section the problem will be approached a little differently. The effect of using incorrect θ_ℓ will be considered by estimating the parameter set (X,Y,α,β) from the data in Table A1 assuming (i) the ideal level of resource is 2500 units and (ii) the ideal level of resource is 1600 units. Table A2 gives the parameter estimates and the average absolute percentage errors (as defined earlier) for these estimates. The table indicates that errors in θ_ℓ do not introduce any further error when considering the difference between actual results and model predictions. However, uncertainty about θ_ℓ implies uncertainty about the estimates (X,Y,α,β). In particular, interpreting the estimates of (X,Y) as prediction of "ideal levels" of care must be done with some caution.

Table A2. Parameter estimates.

	Ideal level=2500		Ideal level=1600	
Parameter estimates	α_1 =0.027	α_2 =9.48	α_1 =0.647	α_2 =12.20
	β_{11}=0.049	β_{21}=9.46	β_{11}=0.292	β_{21}=11.88
	X_1 =113.3	X_2 =102.4	X_1 =78.6	X_2 =97.9
	Y_{11}= 12.8	Y_{21}= 10.3	Y_{11}= 8.2	Y_{21}= 9.8
x_1	2.4		2.2	
x_2	0.2		0.0	
y_{11}	1.3		1.2	
y_{21}	0.2		0.1	

APPENDIX B: SENSITIVITY ANALYSIS

This appendix gives the results of a sensitivity analysis of the parameters of a one-resource DRAM. The aim of the analysis is to see how sensitive the model solution is of changes in the parameter set $\{X,Y,\alpha,\beta\}$. This is important for two reasons. Firstly, such an analysis gives some indication on the relative accuracy with which the parameters should be estimated. Secondly, it indicates the effect of changes in the health care systems, e.g. changing morbidity patterns (changes in X), reduction in length of stay (changes in Y).

The basic model on which the sensitivity analysis was carried out is given in Table B1. The model solution for this parameter set is given in Table B2. In the sensitivity analysis, one parameter is perturbed (changes to .5 or 1.5 of its original value) and a new model solution calculated. The parameters perturbed in this are marked by an (*) in Table B1. For each of the four types of parameters $\{X,Y,\alpha,\beta\}$ two parameter values are changed, one has a high numerical value and the other a low numerical value. For instance, for the parameter X_j, X_2 and X_4 have been perturbed, X_2 is the largest X_j and X_4 is the next to the smallest. The results of the sensitivity analysis are given in Table B3.

Table B1. Basic parameter set.

Patient category j	X_j (Ideal number of patients to be treated)	α_j	Y_{j1} (Ideal number of units of resource per patient)	β_{j1}
1	69.3	.205*	16.6	316.88
2	69.5*	.001	23.1	6.03*
3	45.5	5.711*	12.2*	3.06
4	6.9*	.969	39.4	1.00
5	13.2	.870	19.4	1.00*
6	18.0	2.608	48.2*	.001
7	8.6	.001	34.1	1.00

Available resource R = 2119 units

*parameter varied in sensitivity analysis

Table B2. Model solution for basic parameter set.

Patient category j	x_j	x_j/X_j	y_{j1}	y_{j1}/Y_{j1}
1	32.5	.47	16.6	1.00
2	29.1	.42	20.3	.88
3	40.1	.88	9.8	.80
4	4.7	.68	24.9	.63
5	8.7	.66	12.2	.63
6	15.0	.84	19.3	.40
7	4.0	.46	21.6	.63

Table B3. Results of sensitivity analysis.

Parameter (P_j) changed		Percentage change in x_j	Percentage change in y_{j1}	Max percentage change in other x_j	Max percentage change in other y_{j1}
X_2	+ 50%	+31.4	- 2.0	-11.4	-13.5
	- 50%	-34.5	+ 2.1	+12.9	+15.8
X_4	+ 50%	+49.0	- 1.6	- 2.8	- 3.1
	- 50%	-49.5	+ 1.6	+ 3.0	+ 3.2
α_1	+ 50%	+ 4.2	0	- 1.7	- 1.8
	- 50%	- 5.2	0	+ 1.7	- 1.9
α_3	+ 50%	+ 3.7	- 0.2	- 0.8	- 0.8
	- 50%	- 8.5	+ 0.5	+ 1.7	+ 1.8
Y_3	+ 50%	- 1.3	+46.1	- 9.3	-10.1
	- 50%	+ 1.4	-48.7	+10.0	+10.9
Y_6	+ 50%	- 1.0	+39.1	- 6.5	- 3.6
	- 50%	+ 1.3	-45.8	+ 7.8	+ 4.2
β_2	+ 50%	- 2.1	+ 3.9	- 0.6	- 0.9
	- 50%	+ 5.0	- 8.8	+ 1.7	+ 2.0
β_5	+ 50%	- 1.6	+ 9.4	- 0.4	- 0.5
	- 50%	+ 2.6	-13.8	+ 0.6	+ 0.6

The results of the sensitivity analysis (see Table B3), indicate that changes in X_j and α_j have the greatest effect on x_j, and changes in Y_{j1} and β_{j1}, on y_{j1}. Further, changes in α_j and Y_{j1} give rise to greater perturbations of the basic solution, than do the same percentage changes in α_j and β_{j1}, i.e. the model solution is more sensitive to changes in ideal levels of care, than to changes in the parameters respresenting the relative importance of achieving these ideal levels.

The results given in the above paragraph indicate that greater accuracy is required in estimating the parameters $\{X,Y\}$ than the parameters $\{\alpha,\beta\}$.

APPENDIX C: PREDICTION ERRORS FOR x_{jk} and y_{jk} ESTIMATED BY DRAM

Suppose the DRAM parameters (X,Y,α,β) have been estimated from N data points, $x_{jk}(i)$, $y_{jk\ell}(i)$ $i=1...N$. If DRAM with this parameter set is now used to estimate x_{jk} and $y_{jk\ell}$ for given levels of resource R_ℓ, what confidence can be placed in these estimates? Can we estimate the variance of the difference between the prediction and an observed value? DRAM is a non-linear model and to produce an analytically exact solution to these problems would be very difficult. Instead, a simplified approach has been adopted.

Let $\hat{x}_{jk}(i)$ and $\hat{y}_{jk\ell}(i)$ be the predicted levels using DRAM given the estimated parameter set (X,Y,α,β) and resource availabilities $R_\ell(i)$ at data point i $(i=1...N)$.

It will be assumed that the variance $(\sigma^2_{\hat{jk}})$ of the prediction $\hat{x}_{jk}$ (a similiar argument holds for $\hat{y}_{jk\ell}$) can be estimated from $\sum_i \left(\hat{x}_{jk}(i)-x_{jk}(i)\right)^2$. What divisor should be used? Suppose there are J patient categories, K treatment modes, and L types of resources. DRAM predicts JK x_{jk}'s and JKL y_{jk}'s - in total JK(L+1) predictions. DRAM requires J(1+k+2KL) parameters, i.e. J(1+K+2KL) degrees of freedom can be considered lost. Further NL degrees of freedom are lost because there are L resource constraints at each data point. Thus the number of degrees of freedom considered lost per prediction is $\frac{J(1+K+2KL)+NL}{JK(L+1)}$.

When J=3, K=1, L=2, N=10, this ratio is approximately 4. Thus 4 degrees of freedom can be considered lost from $\sum_i \left(\hat{x}_{jk}(i) - x_{jk}(i)\right)^2$ and it is assumed σ^2_{jk} could be estimated by

$$\frac{\sum_i \left(\hat{x}_{jk}(i) - x_{jk}(i)\right)^2}{N-4}$$

This is a somewhat approximate approach and further analysis may prove worthwhile.

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