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# The Effects of Uncertainty in Generation Expansion Planning - A Review of Methods and Experiences

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THE EFFECTS OF UNCERTAINTY IN GENERATION  
EXPANSION PLANNING--A REVIEW OF METHODS  
AND EXPERIENCES

Kiichiro Tsuji

July 1980  
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## PREFACE

In the workshop "Size and Productive Efficiency--The Wider Implications" which was held at IIASA in June 1979, participants have raised an issue concerned with the effects of future uncertainties on the decisions on size. The issue was pointed out from electricity industry where the recent trend of construction of larger plants have made the lead time longer and longer, making the demand forecast more uncertain than ever. However, the problem is common to all industries which involve high capital investments for a new plant to be installed.

In order to improve our understanding of the effects of uncertainty we have carried out a state-of-the-art review on this subject in electricity generation where the most sophisticated expansion planning models and methodologies are available and where a considerable amount of reported experience on this subject exists. This paper presents the results of this review.

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The Effects of Uncertainty in Generation Expansion  
Planning--A Review of Methods and Experiences

Kiichiro Tsuji

INTRODUCTION

In June 1979, a workshop "Size and Productive Efficiency--The Wider Implications" was held at IIASA, based on the preparatory work by Cantley and Glagolev (1978). The workshop was attended by some 50 scientists representing 13 countries and from various different disciplines and discussed about "problems of scale" in various industries. The workshop, in fact, brought a number of different aspects of problems of scale, not only concerned with the problem of deciding an optimal or an appropriate size of some facilities, but also concerned with the management problems within an organization as well as the implications of large-scale operation or production on national economies.

During the workshop participants have raised an issue concerned with the effects of future uncertainties on the decisions on size. The issue was pointed out from electricity industry where the recent trend of construction of larger plants have made the lead time longer and longer, making the demand forecast more uncertain than ever. However the problem is common to all industries which involve high capital investments for a new plant to be installed.

In order to improve our understanding of the effects of uncertainty we have carried out a review on this subject in electricity generation where the most sophisticated expansion planning models and methodologies are available and where a considerable amount of reported experience on this subject exists.

In this paper we present a comprehensive review of

- i) existing methodologies for electricity generation expansion planning,
- ii) techniques for evaluating the effects of uncertainties, and
- iii) some known consequence of the future uncertainties on generation expansion plans.

Several basic principles for coping with uncertainty are drawn as a result of this survey and they will be presented later in this paper.

#### EFFECTS OF UNCERTAINTY--A SIMPLE EXAMPLE

To describe the sort of "effects" of future uncertainty that we are concerned with, a simple, classical example of selecting unit size in capacity expansion planning is presented in the following:

Suppose that

- the demand grows arithmetically
- there is an economy of scale
- the objective of optimization is to minimize the present worth costs
- the demand must always be met.

An optimal size exists because economies of scale drives the unit size to go larger whereas the discounting tries to split and defer the payments. The result is an optimal cycle of construction, i.e., the units of identical size should be built periodically, and the well-known V-shaped curve (Manne 1967) shown in Figure 1.

An example of the effect of demand uncertainty is illustrated in Figure 2. The three bold phase curves show the cost curves for each different realization of demand increasing rate. Now we assign discrete probability for each demand rate in such a way that the expected value of demand rate is equal to  $D_2$ .

The optimum size when this demand uncertainty is taken into account can be determined by minimizing  $\bar{C}(V)$  with respect to  $V$ , where

$$\bar{C}(V) = v^a + \sum_{i \in I} e^{-\frac{rV}{D_i}} p_i \left\{ \min_V C(V; D_i) \right\} .$$

The cost curve  $\bar{C}(V)$  is shown in Figure 2 (for the numerical values see Appendix), from which the optimum size is smaller than for the deterministic case.

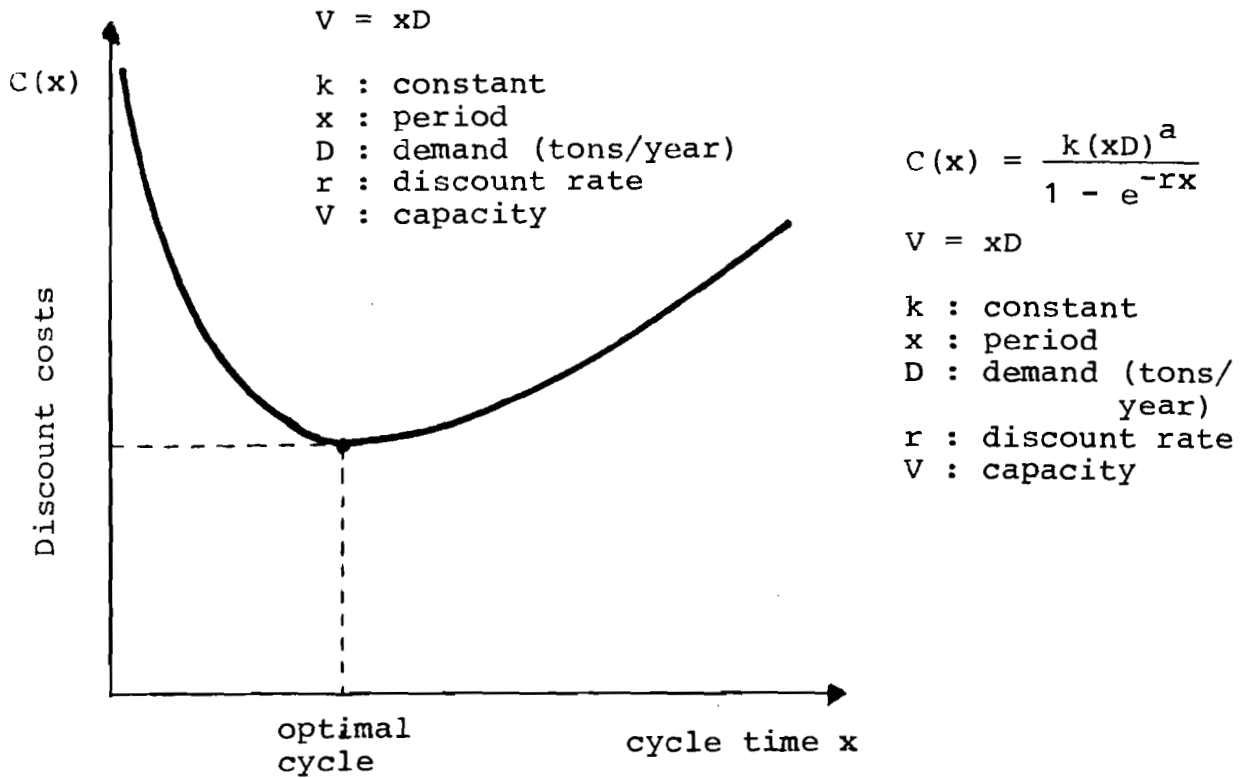


Figure 1. Discounted Cost Function

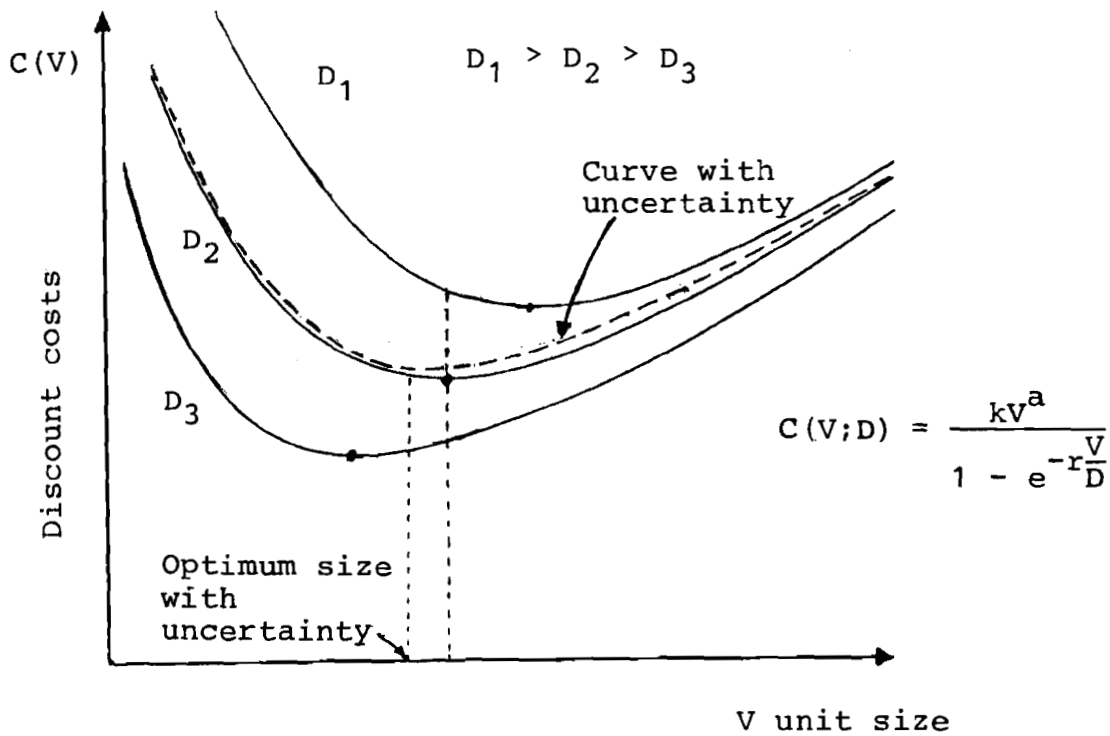


Figure 2. Effect of Demand Uncertainty



Capacity expansion planning in electricity is not as simple as in the example presented here, because there are many types of generating facilities with different capital and operating characteristics. The fact that there is no effective ways of storing electricity requires that the demand which is highly stochastic in nature must be met instantaneously, and this calls for a complicated operating policy for different types of plants and complicates the calculation of operating costs. In addition to this, there are many sources of uncertainty other than the demand such as fuel price, thermal power plant availability, hydro energy availability, construction delay, etc.

These factors prevents the direct application of the results presented in the previous section to generation expansion planning. However there are many methods and models available in generation expansion planning and the effects of uncertainty have been investigated to some extent. These methods and models include LP models, Nonlinear programming model, DP models as well as the computerized (automated) versions of conventional planning methods based on reliability criteria, and some special models which incorporate uncertainties, for example, by representing the demand uncertainties by a probability tree.

Both the conventional and the special models have been used for analyzing the effects of uncertainties. These experience are mostly of numerical nature; examples obtained from a particular system. Nevertheless these examples enhance the nature of the effects of uncertainty in generation expansion planning.

These subjects will be discussed subsequently.

## ELECTRICITY GENERATION EXPANSION PLANNING

Electricity demand grows in most countries year by year. This simply requires that new generation facilities to be added to the existing system sometime in the future. Planning for this expansion involves many aspects, e.g., technical, economic and social, each one of which may call for different type of assessment. However, the whole planning process is normally represented by two major stages; one is generation expansion planning and the other is transmission expansion planning. The former has the characteristic of a more general class of problems referred to as capacity expansion planning and the latter includes more elements from power system analysis such as the load flow and the stability calculations.

In this paper we restrict our attention to generation expansion planning, since our purpose is to investigate the effects of uncertainty in the context of more general capacity expansion planning common to all industries which involves high capital investments. It must be remembered, though, that there are some elements which overlap between generation expansion planning and transmission expansion planning. Hence some elements in transmission expansion planning may very well come

in within the context of generation expansion planning. These two planning stages are not necessarily separable.

### Key Elements of Generation Expansion Planning

Normally, loads (demands) are concentrated at one point in generation expansion planning and all the generating facilities will be connected to this load. The load (demand) is changing hour by hour and generally increasing with time in the long run. There are a number of generating facilities for choice with significantly different capital and operating characteristics. Thus the most fundamental form of generation expansion planning is to choose the type, the capacity and the timing of installation of the plants to be added to a given system, given an appropriate form of demand forecast, over a planning horizon.\* The image of generation expansion planning is shown in Figure 3.

It is clear that this problem is one of the more general class of capacity expansion planning. However the problem is complicated by the fact that the demand which is stochastic in nature must be met instantaneously and thus the estimation of operating costs is complicated.

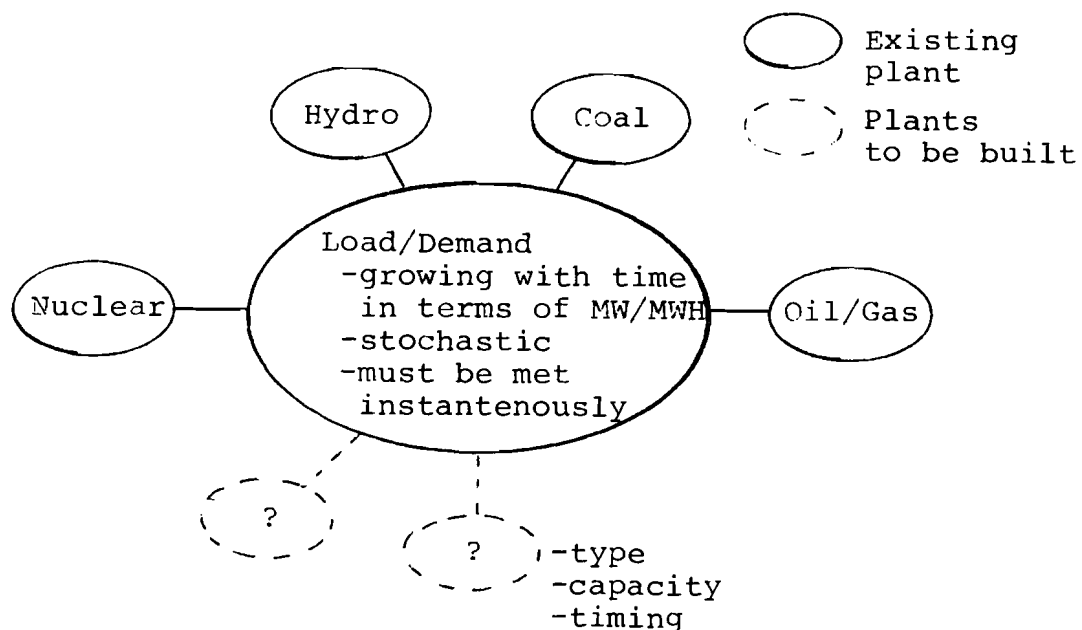


Figure 3. An Image of Generation Expansion Planning

\*Capacity expansion rate will affect the demand through pricing and hence the demand being given exogenously may not be appropriate. Some models takes this price demand interaction into account (see for example, Bergendahl 1978, Manne 1974).

Hour to Hour Load Curve:

Figure 4 shows an example of load curve with respect to time. This curve has the characteristics of a time-varying stochastic process. Assuming no storage devices the demand and the electricity supply must always be met. It may exhibit typical patterns over days (for example weekdays, sundays and saturdays), seasons (winter months, summer months, etc.). It may also have typical patterns for different economic sectors: industrial, commercial, residential, etc. These are also illustrated in Figure 4.

Peak Load/Demand:

By observing the load curve over a certain time period maximum (peak) load (MW) over the period can be defined. This is a key factor in generation expansion planning because the power system must be equipped with enough capacity to meet the peak load.

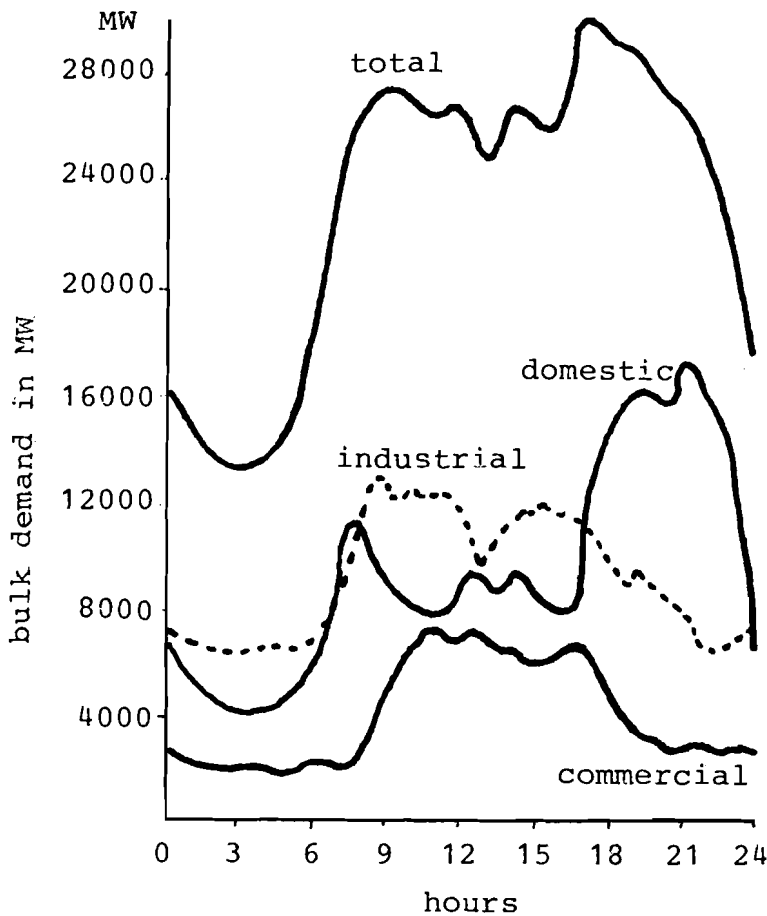


Figure 4. An Example of Load Curve (Source: Berrie 1968)

Load Duration Curve:

The load curve gives the instantaneous power (MW), but often the actual time is irrelevant when the delivered energy (MWH) is to be considered. Load duration curve represents the accumulated period of time over which a given level of load has occurred. Usually it is plotted power vs. hours of the year, and the curve is a nonlinear decreasing function with respect to the accumulated time (see Figure 5).

The area under the load duration curve is the energy (MWH) delivered to the system over  $S_{max}$  hours.  $S_{max}$  is taken to be, for example, one year (8,769 hours), and this curve plays the key role in determining generation mix.

Alternatively, this curve represents a probability distribution function. For example, let  $D(t)$  be a sample function which represents the load curve. Then

$$\text{Prob} \{D(t) \geq s\} = \frac{1}{S_{max}} y^{-1}(s)$$

where  $y^{-1}(\cdot)$  is the inverse function of  $y(\cdot)$  shown in Figure 5.

Plant Types:

There are a number of ways to produce electricity. Typical power plants include hydro run-off, hydro storage, pumped storage, oil fired, coal fired, gas fired and nuclear plants. Each of these has special characteristics which distinguish one plant from another and set out technical constraints on how they are operated. However, it should be noted that the major factor within the context of generation expansion planning which distinguish one type from another is the capital (\$/MW) and the operating (\$/MWH) costs. In this context the difference in the location of plant of the same time can be incorporated into planning if the difference are properly expressed in these costs.

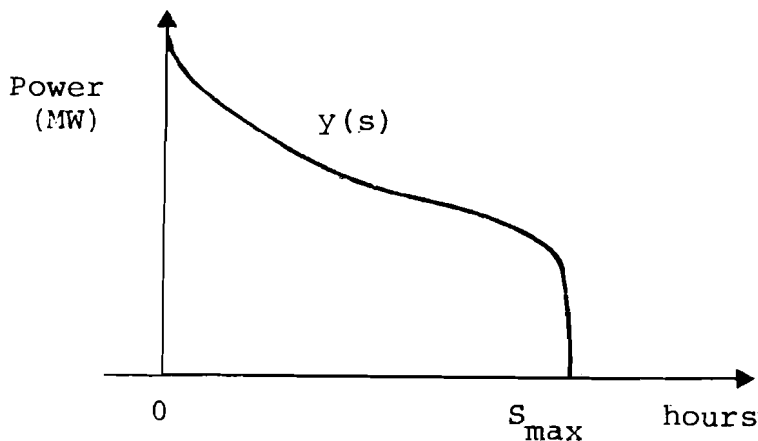


Figure 5. Load Duration Curve

### Reliability:

Each plant is subject to the possibility of out of service due to the scheduled maintenance, forced outages, or in the case of hydro plants, due to the weather. Therefore the amount of actually available capacity at any time is stochastic in nature. With this and the stochastic nature of the load, the total capacity of all the power plants in the system must always be greater than the total demand (reserve margin) in order to ensure the reliable supply of electricity. Evaluation of reliability itself forms a field of active research. Availability, load factor, utilization factor, loss of load probability are among the reliability related factors.

### Other Factors:

There are some other factors to be considered in generation expansion planning. These are, for example, economies of scale, retirement of old plant, construction time, escalation rates on costs, hydro plant storage policy, nuclear fuel cycles, environmental impacts, transmission systems, arrival of new technologies, and a number of possible local restriction factors such as the limited amount of capital, the availability of fuel and the political and social constraints.

### Planning Horizon:

An important factor in generation expansion planning is its planning horizon. It is customary to distinguish three different planning horizons, i.e., short-, medium-, and long-term planning, although the definitions are not very strict. Roughly speaking, short-term planning focuses on the choice of the next couples of plants to be built. In medium-term planning the sequence of plants which should be added to the system over 10-20 years will be considered. In long-term planning (over 20 years) a guideline for the long-term development pattern of the system will be sought. It is clear that for each planning horizon emphasis must be placed on the different groups of factors, for example, in short-term planning the operating policy must be defined more precisely and operating costs evaluation including reliability calculation will have to be performed accurately whereas in long-term planning these factors can be aggregated properly.

### Generation Mix--The Static Case

One of the key objectives of generation expansion planning is the determination of generation mix. All the sophisticated methods which will be reviewed in the later sections are designed to determine an optimal generation mix. These methods take various kinds of factors which affect the optimal generation mix but there is a straightforward way of determining generation mix (Berrie 1968, Phillips et al. 1969).

We consider the case where the demand is given for a single year and every plant are to be built at the same time to satisfy this demand. This is the static case of the generation expansion planning problem. Assume that each plant  $i$  is expressed by the fixed cost  $g_i$  and variable cost  $f_i$ . The cost of each plant can then be represented by a straight line expressed as  $g_i + f_i t$ , where  $t$  is the operating time in hours. In Figure 6 we consider the hypothetical case where there are three plants; nuclear, fossil and gas turbine. These are characterized by high fixed cost and low running cost for nuclear, medium fixed cost and medium running cost for fossil, and low fixed and high running costs for gas turbine.

The bottom of the figure is a load duration curve. Proceeding with the direction of increasing hours of operation, gas turbine gives the least costs until it reaches the intersection with the cost curve of fossil plant, and this procedure is continued to cover the whole hours of operation which gives the bold phased minimal cost curve in Figure 6. The dotted lines starting from two corner points on this curve down to the load duration curve and then reflected on its demand axis give the optimal capacity for each plant.

Although this simple approach is applicable to the static case this gives a rough idea of how the fixed and running costs affect the optimal generation mix. For a much more detailed treatment of this graphical procedure, the reader is referred to Phillips et al. (1969), Buzacott and Tsuji (1980).

## BASIC MODELS, APPROACHES AND EXTENSIONS

Generation expansion planning is the vital part of power system planning and there are extensive literatures in this field. Numerous methods have been proposed and they differ from each other considerably in the degree of detail of the calculation of operating costs and of the constraints to be imposed, and in the degree of mathematical sophistication as well. However, roughly speaking, there are two basic approaches; one is what we might call conventional approaches in which cost calculations are performed in detail but only a number of alternatives for expansion are considered, and the other approach is the use of mathematical programming models.

Here we present a review of a number of basic models and approaches, and to some extent, possible extensions. Although the treatment here will by no means be completely exhaustive, it covers the essential features of the available planning methodologies.

### Approaches Based on Reliability Criterion

A straightforward and pragmatic way of generation expansion planning is first to select a set of alternative plans and second

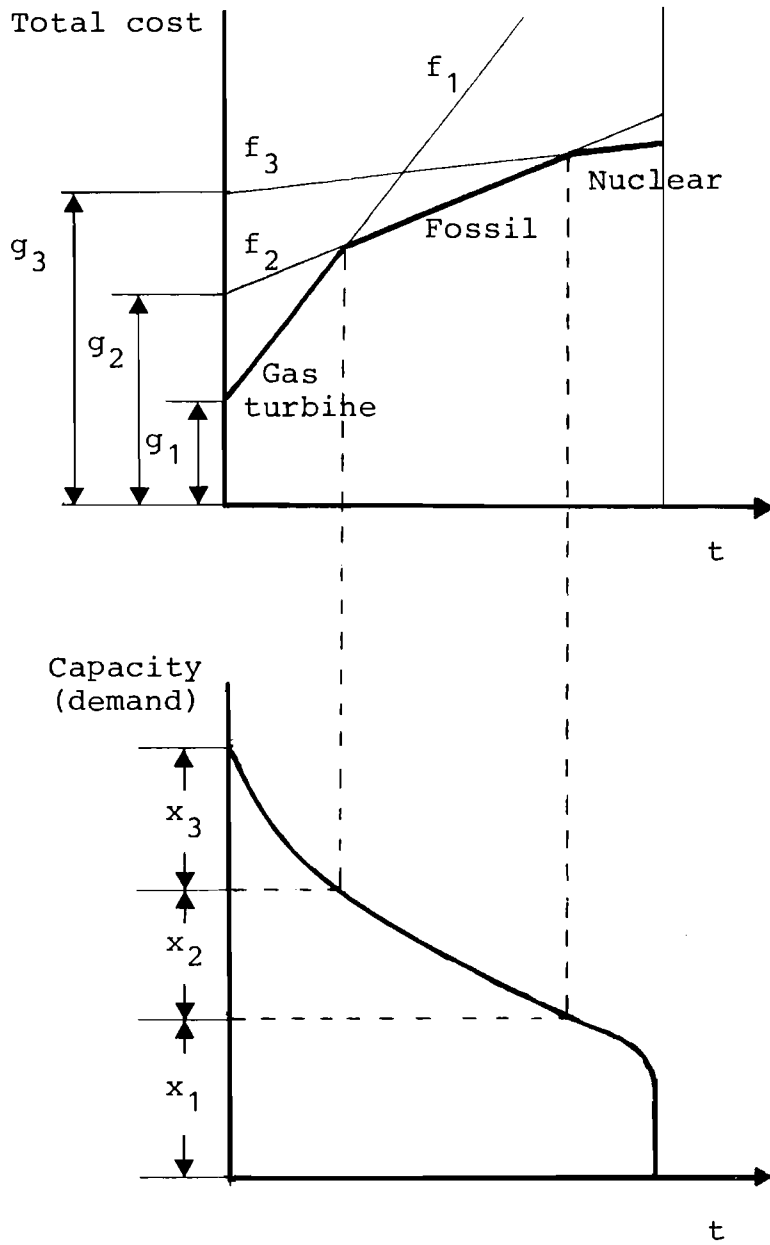


Figure 6. Optimal Generation Mix--The Static Case

to evaluate the reliability of each plan and third to calculate total costs for those plans which meet a certain reliability requirement and finally to choose a plan which results in the minimum total costs. This approach seems to be utilized most commonly in the electricity utilities and there may be as many methods as there are utilities once we get down to the details of their methods. The book by Sullivan (1977) deals extensively with this approach. A simple illustration of this class of approaches is found in the paper by Adamson et al. (1973), from which Figure 7 is taken.

The evaluation of reliability is the most important part of this approach. This is a significant field of study of its own where a vast amount of literatures are available (e.g., Billinton 1970, 1972, Vemuri 1978) and active researches are being undertaken.

The simplest form of reliability evaluation is the use of per cent reserve margin which is derived empirically. More sophisticated methods call for the use of probability theory and the reliability criteria such as Loss of Load Probability (LOLP) are analytically calculated. More detailed calculation is possible by simulating the operation of the system precisely. A glimpse of the methodologies which are actually used in existing utilities can be seen in Billinton (1978).

In principle this approach can be used for any length of planning horizon. However, it is not really suitable for long-term planning, since it involves detailed calculations for each plan and the number of possible alternative plans which are essentially combinatorial is increased tremendously as the planning horizon becomes longer and even the whole process of this approach is fully automated (e.g., Oatman et al. 1973) the amount of calculation can become prohibitive.

#### Linear Programming Models and Extensions

The generation expansion planning problems can be formulated in terms of linear programming and an extensive number of literatures are available. In the following the most basic form of linear programming models is first given and some modifications and extensions will then be discussed.

#### Basic Linear Programming Model (Anderson 1977):

The purpose of the model is to find the type and the capacity of power plants to be installed over a given planning horizon. However in order to calculate the operating costs it is necessary to determine how the existing plants at any time are to be utilized. Thus the variables to be determined are:

$$x_{kn} = \text{Capacity of plant of type } k \text{ to be installed} \\ \text{in year } n, k = 1, \dots, K, n = 1, \dots, T.$$



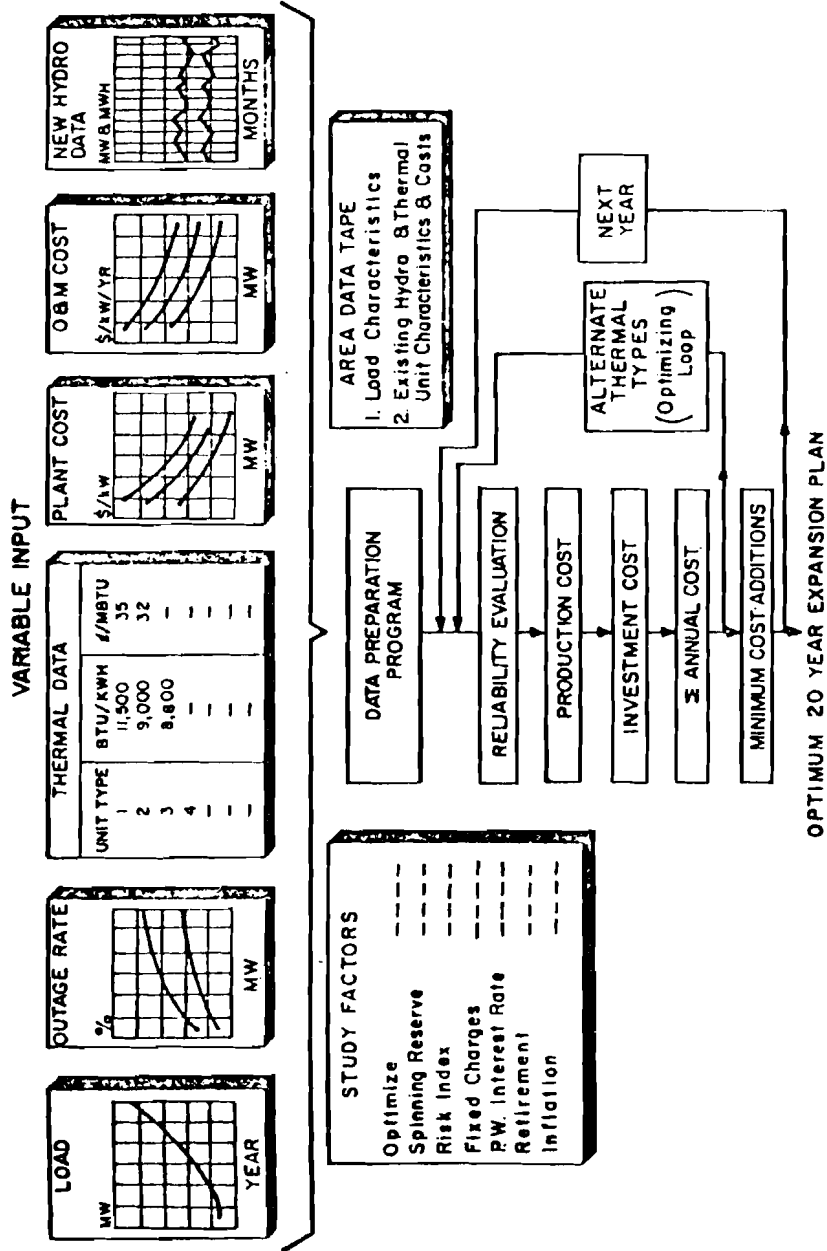


Figure 7. An Example of Approach Base on Reliability Evaluation (Source: Adamson et al. 1973)

$U_{thkn}$  = Capacity of plant of type  $k$  installed in year  $n$  to be actually utilized in the  $h$ -th time interval of the load duration curve in year  $t$ ,  $t = 1, \dots, T$ ,  $h = 1, \dots, H$ ,  $k = 1, \dots, K$ ,  $n = 1, \dots, t$ .

$T$  is the planning horizon and  $n = 0$  implies those plants which are initially in the system. The range of  $n$  depends on  $t$  because at time  $t$  only those plants up to  $t$  can be operated.

Peak demand and the load duration curve are assumed to be given over the years  $n = 1, \dots, T$ . Moreover the load duration curve is approximated by stair case functions where  $\theta_h$ ,  $h = 1, \dots, H$ , and  $D_{th}$ ,  $t = 1, \dots, T$ ,  $h = 1, \dots, H$  are defined in Figure 8.

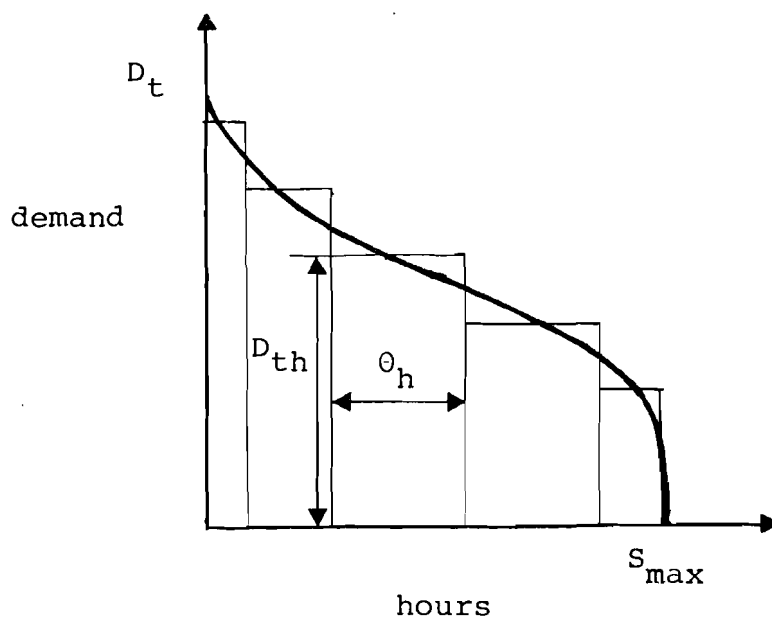


Figure 8. Discrete Approximation of Load Duration Curve

For each plant type  $k = 1, \dots, K$ , the following quantities are assumed to be given:

- $g_{kn}$  : Discounted fixed costs per unit power (\$/MW) for the plant of type  $k$  installed in year  $n$ ,  $n = 1, \dots, T$
- $f_{thkn}$  : Discounted variable costs per unit energy (\$/MWH) for the plant of type  $k$  installed in year  $n$ , over the  $h$ -th segment of the load duration curve for year  $t$ .  $t = 1, \dots, T$ ;  $h = 1, \dots, H$ ;  $n = 0, \dots, t$ .
- $a_{tkn}$  : Availability of the plant of type  $k$  installed in year  $n$ , over the year  $t$ .  $t = 1, \dots, T$ ;  $n = 0, \dots, t$ .
- $x_{k0}$  : Capacity of the plant of type  $k$  initially in the system.
- $m$  : Reserve margin on the total capacity.

The objective function to be minimized is taken to be the total discounted costs which can be expressed as follows:

$$\sum_{k=1}^K \sum_{n=1}^T g_{kn} x_{kn} + \sum_{t=1}^T \sum_{h=1}^H \sum_{k=1}^K \sum_{n=0}^t f_{thkn} U_{thkn} \theta_h \quad [I]$$

The constraints are:

- a) Demand must be met.

$$\sum_{k=1}^K \sum_{n=0}^t U_{thkn} \geq D_{th} \quad , \quad t = 1, \dots, T$$

$$h = 1, \dots, H.$$

- b) Total installed capacity must be larger than the peak demand plus reserve margin.

$$\sum_{k=1}^K \sum_{n=0}^t x_{kn} \geq D_{th} (1 + m) \quad , \quad t = 1, \dots, T$$

$$h = 1.$$

- c) Any plant can be operated up to its available capacity.

$$U_{thkn} \leq a_{tkn} x_{kn} \quad , \quad t = 1, \dots, T$$
$$h = 1, \dots, H$$
$$k = 1, \dots, K$$
$$n = 0, \dots, t \quad .$$

All the variables are nonnegative, and this completes the basic form of linear programming formulation.

It should be noted that the key assumptions which are implicit in the above formulation are

- 1) Discrete unit size is not considered and the program can choose any amount of capacity each year, that is, the decision variables  $x_{kn}$  are continuous.
- 2) The fixed costs are assumed to be proportional to the capacity to be installed. Hence the economy of scale that exists in the installation of power plants is ignored.
- 3) Reliability of the system is considered only by the single value of availability for each year for each plant type, and by the reserve margin.
- 4) Calculation of operating cost is assumed to be approximately modeled by dispatch against an annual load duration curve.
- 5) There are no restrictions on the operation of hydro power plants.

The objective function [I] represents the total discounted costs in which the first term represents the discounted capital costs and the second term represents the discounted operating costs. The operating costs are represented as linear functions of the operating variables which are due to the staircase function approximation of the load duration curve. High accuracy on the evaluation of the operating costs can be achieved only at the expense of increased number of variables and constraints in the above linear programming model, and this is the major disadvantage of the model.

Roughly speaking there are two classes in the way of modifying and extending the above formulation. One class has to do with the modification in calculating operating costs in the objective function and the other class can be termed as various refinements in which more constraints are imposed and/or some of the basic assumptions mentioned above are relaxed. These will be discussed subsequently.

#### Z-Substitutes:

The idea is shown in Beglari and Laughton (1973). Let us define a new operating variables  $Z_{tkn}^i$  instead of  $U_{thkn}$  by

$z_{tkn}^i$  : Capacity of plant of type  $k$  installed in year  $n$  to be actually utilized in the  $i$ -th demand section of the load duration curve for year  $t$ ,  $t = 1, \dots, T$ ;  $i = 1, \dots, H$ ;  $k = 1, \dots, K$ ;  $n = 0, \dots, t$ .

In this case the load duration curve is approximated by the same stair case function as in Figure 8, but interpreted in different way as shown in Figure 9, where the constants  $\tau_i$  and the demand section  $D_t^i$ ,  $i = 1, \dots, H$  are defined.

One more modification is to redefine the variable costs  $f_{thkn}$  by new values  $f_{tkn}^i$  where

$f_{tkn}^i$  : Discounted variable costs per unit energy (\$/MWH) for the plant of type  $k$  installed in year  $n$ , over the demand section  $i$  of the load duration curve for year  $t$ .  $t = 1, \dots, T$ ;  $i = 1, \dots, H$ ;  $n = 0, \dots, t$ .

Having replaced  $U_{thkn}$ ,  $D_{th}$ ,  $\theta_h$ ,  $f_{thkn}$  by  $z_{tkn}^i$ ,  $D_t^i$ ,  $\tau_i$ ,  $f_{tkn}^i$ , we can write down the modified version of linear programming formulation.

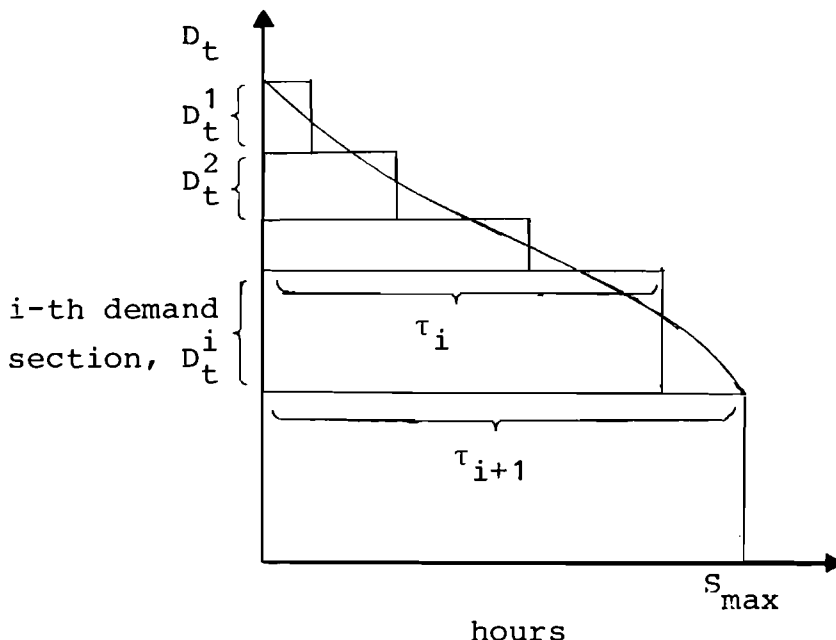


Figure 9. Approximation of Load Duration Curve for Z-substitute Method

The objective function to be minimized is

$$\sum_{k=1}^K \sum_{n=1}^T g_{kn} x_{kn} + \sum_{t=1}^T \sum_{i=1}^H \sum_{k=1}^K \sum_{n=0}^t f_{tkn}^i z_{tkn}^i \tau_i \quad [II]$$

The constraints are:

a') Demand must be met.

$$\sum_{k=1}^K \sum_{n=0}^t z_{tkn}^i \geq D_t^i, \quad t = 1, \dots, T$$

$$i = 1, \dots, H.$$

b') Total installed capacity must be larger than the peak demand plus reserve margin.

$$\sum_{k=1}^K \sum_{n=0}^t x_{kn} \geq \sum_{i=1}^H D_t^i (1 + m), \quad t = 1, \dots, T.$$

c') Any plant can be operated up to its available capacity

$$\sum_{i=1}^H z_{tkn}^i \leq a_{tkn} x_{kn}, \quad t = 1, \dots, T$$

$$k = 1, \dots, K$$

$$n = 0, \dots, t.$$

The advantage of Z-substitute method is that the number of constraints in c) is reduced to 1/H.

#### Quadratic Programming:\*

Now we consider the same set of variables as in the Z-substitute method but this time the load duration curve is approximated by piece-wise linear function (see Figure 10).

Let us assume the merit order of the available plants and the plants  $kn$ ,  $k = 1, \dots, K$ ,  $n = 0, \dots, t$  are renumbered as  $\ell = 1, \dots, \ell_m$ . Thus we will write  $z_{tkn}^i$ ,  $f_{tkn}^i$  as  $z_{t\ell}^i$ ,  $f_{t\ell}^i$ , respectively. Let us consider one block of demand section  $D_t^i$ , as in Figure 11.

In Figure 11 suffix  $t$  is dropped for notational convenience. Note that there is a clear relationship as follows:

\* The description here is a generalization of the development due to Louveaux (1980).

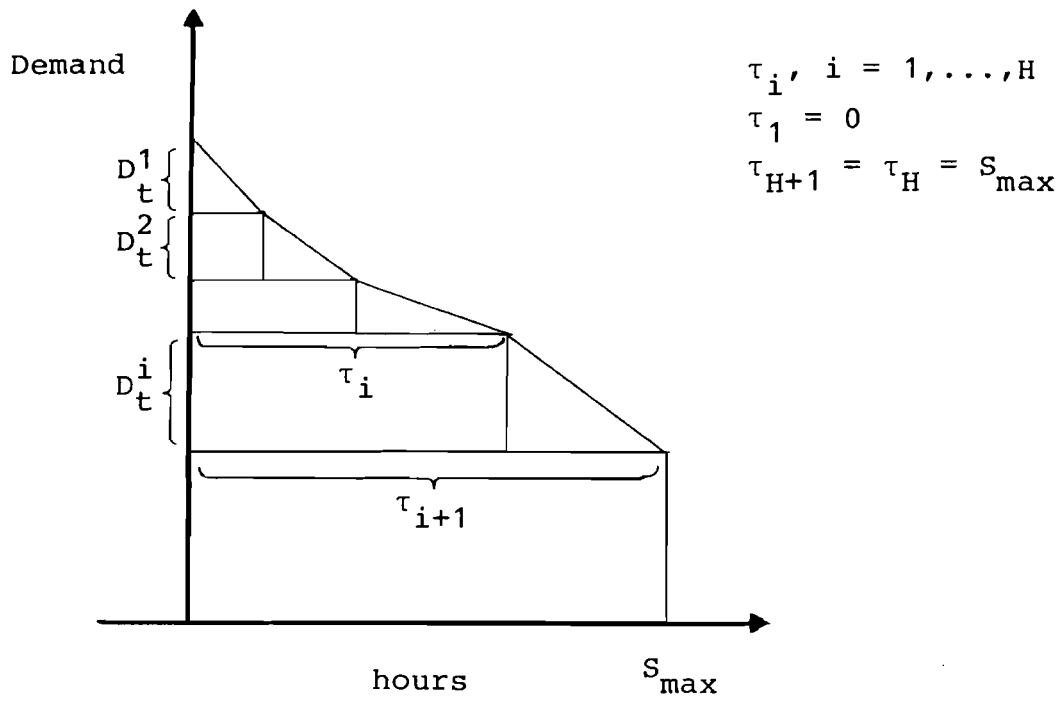


Figure 10. Piecewise Linear Approximation of Load Duration Curve

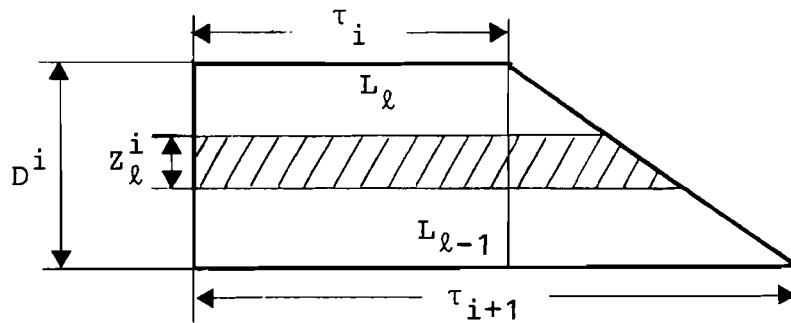


Figure 11. Approximateion of i-th Demand Section

$$\frac{\tau_{i+1} - \tau_i}{D^i} = \frac{L_\ell - \tau_i}{D^i - \sum_{j=1}^{\ell} z_j^i} = \frac{L_{\ell-1} - \tau_i}{D^i - \sum_{j=1}^{\ell-1} z_j^i}, \quad \ell = 1, \dots, m_i,$$

where  $L_{m_i} = \tau_i$  and  $L_0 = \tau_{i+1}$ .

Using this relationship, the energy of the shadowed area is given by

$$z_\ell^i \frac{L_\ell + L_{\ell-1}}{2} = \frac{\tau_{i+1} - \tau_i}{D^i} z_\ell^i \left\{ (D^i - \sum_{j=1}^{\ell-1} z_j^i) + \frac{z_\ell^i}{2} \right\} + \tau_i z_\ell^i$$

Thus the operating costs over one year is

$$\sum_{j=1}^H \sum_{\ell=1}^{m_i} f_\ell^i \left\{ z_\ell^i \tau_i + \frac{\tau_{i+1} - \tau_i}{D^i} z_\ell^i (D^i - \sum_{j=1}^{\ell-1} z_j^i + \frac{1}{2} z_\ell^i) \right\},$$

and the objective function to be minimized can be expressed as follows:

$$\begin{aligned} & \sum_{k=1}^K \sum_{n=1}^T g_{kn} x_{kn} + \sum_{t=1}^T \sum_{i=1}^H \sum_{\ell=1}^{m_i} f_{t\ell}^i z_{t\ell}^i \tau_i \\ & + \sum_{t=1}^T \sum_{i=1}^H \sum_{\ell=1}^{m_i} f_{t\ell}^i z_{t\ell}^i \frac{\tau_{i+1} - \tau_i}{D_t^i} (D_t^i - \sum_{j=1}^{\ell-1} z_{tj}^i + \frac{1}{2} z_{t\ell}^i) \end{aligned} \quad [III]$$

The constraints are exactly the same as a'), b'), and c').

The formulation when  $H = 3$  is given in Louveaux (1980). Due to the piecewise linear approximation, the number of blocks  $H$  may indeed be reduced to 3, thus the number of constraints is reduced significantly at the cost of nonlinearity (quadratic) in the objective junction.

Another way of modifying the form of operating costs is suggested by Beglari et al. (1975). They used the load factor and the utilization factor in order to eliminate the appearance of operating variables in the linear programming formulation.

#### Various Modifications and Extensions:

Various modifications and refinements have been discussed in the literature and some of which are briefly introduced in the following.



Within the content of linear programming formulations discrete unit size can be considered by introducing a set of 0-1 integer decision variables. In this case it is possible to take economies of scale effect into account. Saway et al. (1977) describe a mixed integer linear programming model which allows for the analysis of the tradeoff between the economies of scale achieved by building a large plant and the increased loss in the transmission and also the tradeoff between the construction of a large plant vs. the delayed capital investments afforded by building several smaller plants.

Reliability is normally taken into account in the most aggregated form in the linear programming models. However it is possible to formulate a mixed integer linear programming model in which each plant is assumed to be either up or down with certain probability and LOLP is used as an index of reliability (Sherer et al. 1977).

The possible restrictions on the energy produced by hydro plants, inclusion of replacement, approximate inclusion of transmission lines are all discussed in Anderson (1977). Additional constraints such as the allowance for some plants to be operated only in the base load can be handled (Rutz et al. 1979).

#### Dynamic Programming Models

It is clear that the expansion problem is a sequential decision problem for which the dynamic programming technique is suitably applicable. Both discrete time formulations (e.g., Booth 1972, Petersen 1973) and continuous time formulations (Rogers 1974) have been proposed. Modeling procedure involves defining the state and the state transitions of the system and the objective function to be minimized. In one way the dynamic programming formulation is in the same spirit as in the conventional approaches discussed in the previous section where the creation of alternative plants will be performed much more orderly and effectively, but in other way dynamic programming formulation may become a powerful tool for analyzing the structure of optimal solutions.

With its flexibility of formulation, dynamic programming allows a number of different formulations. Here we give the model by Rogers (1974) for its compact and analytically tractable formulations in order to show the basic idea of this class of models.

Let the state of the system be defined as

$$z = (z_1, z_2, \dots, z_k)^T$$

where  $z_k$ ,  $k = 1, \dots, K$  is the number of plants of type  $k$ .

State transitions occur when a new plant of type  $k$  is added to the system. Define

$z^n \equiv$  the state of the system after  $n$  transitions.

If a plant of type  $k$  is added, then

$$z^{n+1} = z^n + e_k, \quad n = 0, 1, 2, \dots$$

where  $z^0$  is a given initial state, and where  $e_k$  is a unit vector whose  $k$ -th element is 1 and all other elements are zero.

Now define two sequences as follows:

$$S = \{k(1), k(2), \dots\}$$

= the sequence of plant installations where  $k(n)$  is the type of  $n$ -th plant installed.

$$\tau = \{\tau_1, \tau_2, \dots\}$$

= the sequence of timing of plant installations where  $\tau_n$  is the time of  $n$ -th plant installation.

The combined sequence

$$(S, \tau) = \{(k(1), \tau_1), (k(2), \tau_2), \dots\}$$

will give the complete description of what to build and when.  $(S, \tau)$  is called expansion program.

The demand forecast to the future is assumed to be given by the peak demand and the normalized load duration curve.

Let

$D(t)$  = Peak demand at time  $t$

$y(s, t)$  = Normalized load duration curve at time  $t$ ,

and for each plant type  $k$  we assume the following quantities to be given:

- $X_k$  : Peak capacity
- $U_k$  : Annual utilization rate
- $g_k$  : Fixed costs (capital costs +  $\frac{1}{r}$  fixed operating costs)
- $f_k$  : Variable operating costs
- $p_k$  : Prob [plant k fails per unit time]

$$= \frac{\text{time on forced outage}}{\text{time on forced outage} + \text{time available}}$$

Now the objective function to be minimized is taken to be the total discounted costs of the expansion program  $(S, \tau)$ , which can be expressed by

$TC(S, \tau)$  = Total discounted costs of program  $(S, \tau)$

$$= \sum_{n=0}^{\infty} \left\{ \left( \int_{\tau_n}^{\tau_{n+1}} e^{-rt} [L_1(Z^n, t) + L_2(Z^n, t)] dt \right) + e^{r\tau_{n+1}} g_{k(n+1)} \right\} .$$

The first two terms are related to the operation of the plants in the system and the second term is the fixed costs of the plants to be installed. That is

$L_1(Z, t)$  = Costs of outage at time  $t$  when the system state is  $Z$  (annual)

$$= \mu \cdot 8760 \int_{X-D(t)}^X (x - (X - D(t))) dF(x; Z)$$

where

$\mu$  : Costs of forced outage (\$/MWH)

$F(x; Z)$  = Prob [amount of failed capacity  $\leq x; Z$ ]

$X$  = Total maximum capacity

$$= \sum_{k=1}^K Z_k X_k .$$

$$L_2(Z, t) = \text{Operating costs of the system at time } t$$

$$= 8760 \int_0^1 f_{\min}(y(s, t), Z) ds$$

where  $f_{\min}(y, Z)$  is the minimum of the objective function in the following program:

$$\begin{aligned} &\text{minimize} && \sum_{k=1}^K f_k x_k \\ &\text{s.t.} && \sum_{k=1}^K x_k \geq y \\ &&& 0 \leq x_k \leq U_k x_k Z_k, \quad k=1, \dots, K \end{aligned}$$

where  $x_k$  = total power output of all plants of type  $k$ .

Note that  $F(x; Z)$  can be calculated from the  $p_k$ 's. The linear program for calculating  $f_{\min}$  determines an optimal operating policy which ends up with merit order operation. Here it is assumed that each plant can be operated at any energy output per year below annual utilization level of output.

The objective function  $TC(S, \tau)$  is not appropriate for direct optimization. However, it is known (Rogers 1970) that the optimization can be done in the following two steps. Define

$$G_1(Z^{n-1}, k(n), \tau_n) = \int_0^{\tau_n} e^{-rt} [L(Z^{n-1}, t) - L(Z^n, t)] dt + e^{-r\tau_n} g_{k(n)},$$

where  $L(Z, t) = L_1(Z, t) + L_2(Z, t)$ .

First step is to optimize this function with respect to  $\tau_n$ . Let us denote

$$G(Z^{n-1}, k(n)) = \min_{\tau_n} G_1(Z^{n-1}, k(n), \tau_n).$$

Second step is to optimize with respect to the sequence  $S$ , where  $\tau^*$  is an optimal sequence of timing and

$$TC(S, \tau^*) = \sum_{n=1}^N G(Z^{n-1}, k(n)) + G_T(Z^N),$$

where

$$\begin{aligned} G_T(z^N) &= \int_0^{\infty} e^{-rt} L(z^N, t) dt \\ &= \int_0^T e^{-rt} L(z^N, t) dt + \frac{e^{-rT}}{r} L(z^N, T) \end{aligned}$$

In the last expression it is assumed that there is a finite planning horizon  $[0, T]$  and  $D(t) = \text{const.}$  for all  $t \geq T_1$  and this completes the description of the model.

By the nature of dynamic programming the calculation of operating costs can be performed with any degree of detail from rough analytical expression to a detailed simulation, and various factors such as economies of scale on generator units, replacement policy, etc. can be taken into account. The limitation on the degree of detail is due to the amount of computation required to find an optimal policy. Various techniques of reducing the amount of computation has been devised (for example see Booth 1972, Petersen 1973) and there is a well developed computer package which has been used extensively among the power system planners (Covarrubius 1979).

#### Other Models and Remarks

There are other models worth mentioning. Jenkin (1973) has derived a set of differential equations describing the pattern of expansion, motivated by the fact that in the solutions of generation expansion problems using linear/nonlinear programming models every type of generation plants participates in the optimal expansion plan and the transition over the years of planning horizon is rather smooth (see Figure 12). By using calculus of variations he derived an optimal solution which exhibits this property. Schlaepfer (1978) has generalized this approach further by formulating the expansion planning problems as an optimal control problem to which he applied Pontryagin's minimum principle. He showed a necessary condition for an expansion problem to have the property mentioned above. In general, neither participation of every type of alternative plants considered is always the case nor is the smooth transition. That means that any drastic change in, for example, operating costs could change the general picture of the optimal expansion plan. One remark is with regard to the general feature of the various planning models mentioned so far in relation to the length of planning horizon. In principle all the methods are applicable to short-, medium-, and long-term planning. However the linear programming models (such as we described in this section) are not really suitable for representing daily or hourly operation from computational point of view. Dynamic programming models are suitable for a wide range of planning horizon. Conventional methods are not suitable for long-term planning because of the lack of proper optimization mechanisms.

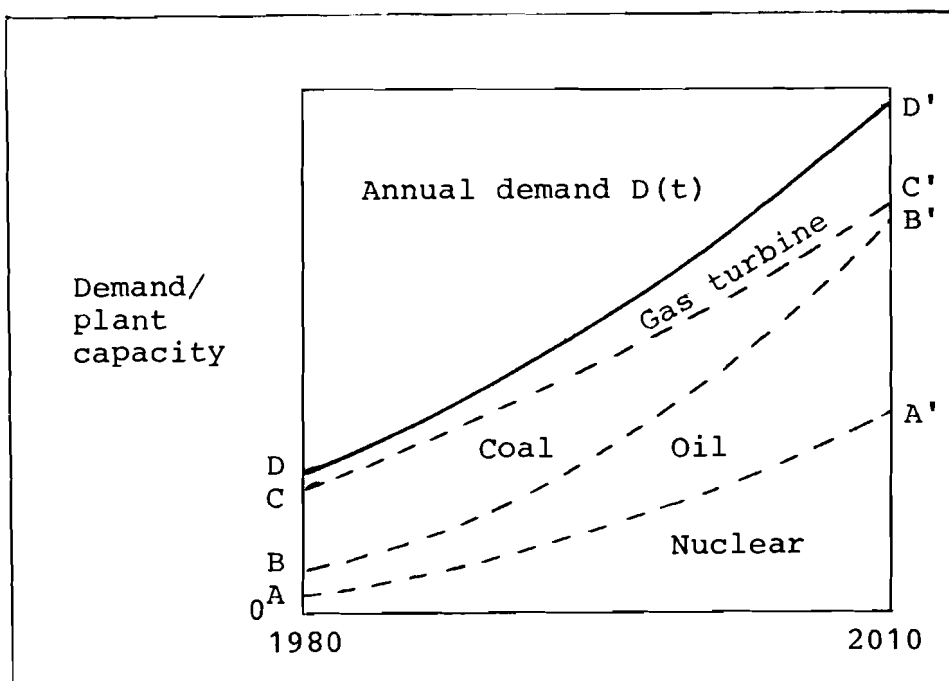


Figure 12. An Example of LP Model Output

These features may be summarized in one picture as shown in Figure 13. The upper and the lower lines define the natural range of detail required for a certain length of planning horizon. At the same time the upper line represents a computational feasibility boundary for each planning method.

#### METHODS FOR TREATING UNCERTAINTY

##### Source of Uncertainty:

Before discussing the variety of methods for treating uncertainties it is worth considering here what are the source of uncertainties and what are the nature of these uncertainties. Table 1 gives a general picture of the sources of uncertainties.

It is important to recognize that there are two categories of uncertainties when we consider the nature of an uncertainty,\* i.e., whether it has the nature of repeated trials (probability can be defined objectively) or it has the nature of a single trial (probability can only be defined subjectively). Table 1 indicates that the uncertainties in demand, hydro energy and thermal plant performance have the nature of both the repeated trials and the single trial, whereas the rest of uncertainties is essentially of the nature of the single trial.

\* In Dhar (1979), randomness and fuzziness are distinguished. The first category corresponds to randomness but the second category considered here is not necessarily the same as fuzziness.

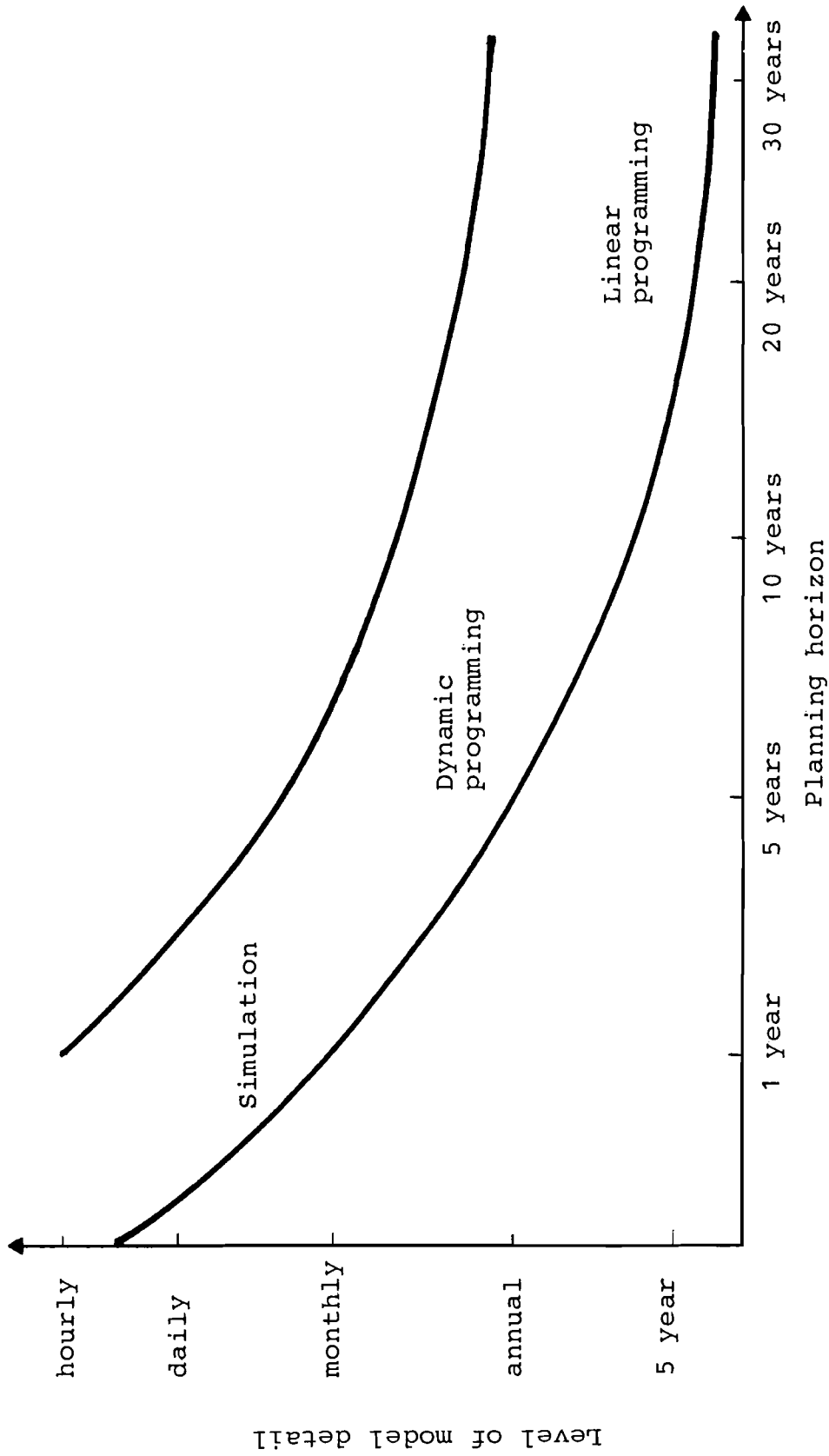


Figure 13. A System of Models for Planning Generation Expansion (Source: Adapted from Rogers 1974)

Table 1. Source of Uncertainties

Uncertainties	Repeated trials	Single events*
- Demand	Distributed around the nominal value	Sudden stop in growth
- Hydro energy		Extremely dry season/year
- Thermal plant performance	Forced outage occurs at some rate	Unexpected low performance
- Fuel price		
- Capital costs		Sudden raise in year x
- Investment/inflation rates		
- New technology		Delay in FBR
- Organizational		Sudden change in organization
- Legislative		An environmental law is put in force
- Energy policy		Restriction on the use of certain type of fuel
- Political		Sudden change

\* This column indicates an example of uncertain events.

For example if the demand curve is observed then it has a daily pattern or a weekly pattern. The distribution around these patterns is caused by a multitude of random activities in the loads in the system and it is possible to obtain this distribution from the historical data. This is the stochastic nature of the demand curve. On the other hand the trend in the demand curve over a long period of time, i.e., the peak demand growth over 10-20 years has the nature of a single trial.

The stochastic nature of the demand, the hydro energy availability and the thermal plant availability has been incorporated extensively in generation expansion planning. All the probabilistic methods in reliability analysis falls into this category. Often load duration curve is modified to account for forced outages, maintenance schedules and random variations in demand (e.g., Vardi 1977).

In the following we will be concerned with the methods for treating the uncertainties which have the nature of single events.



Sensitivity Analysis:

If any of the models and methods described in the previous section is used for a set of given deterministic data, it is possible and is customary to perform sensitivity analysis with respect to the data which have elements of uncertainty. If it is one of the conventional methods using reliability criteria, repeated runs on the same procedure will effectively carry out the sensitivity analysis (e.g., Adamson 1973). If a linear programming model is used, the well developed sensitivity analysis can be carried out in the most efficient manner.

Representations by Event and Associated Probability:

A set of parameter values can be regarded as representing an event or a scenario into the future. When we obtain an optimal expansion plan for each set of parameters, the effects of uncertainty can be investigated by comparing with a reference scenario. It is also possible to assign probability to each event and the expected value of the total discounted costs can be calculated. However, the problem of choosing one policy when several policies are identified in the above procedure is resolved using judgments. Examples of this method are found in Duval (1976), Anderson (1977:Chapter 8) and Garvor et al. (1976).

Representation by Event Tree:

Another way of producing a set of scenario systematically is the use of event tree (probability tree) (see Figure 14).

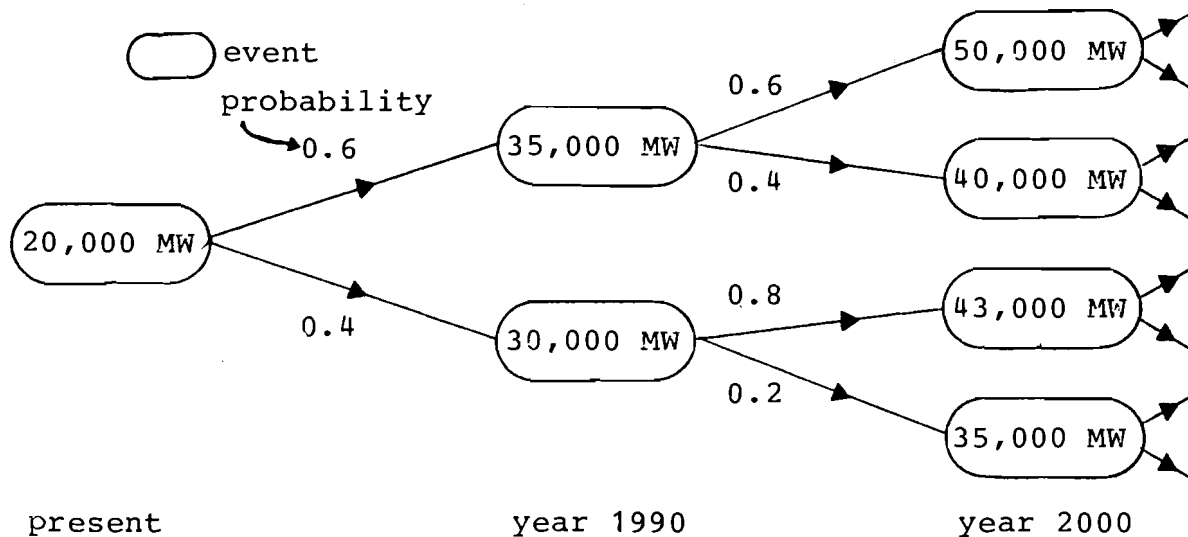


Figure 14. Event/Probability Tree of Demand Growth

By assigning the conditional probabilities to each event sequence, it is possible to calculate probabilities for every path. The effects of uncertainty can be examined by choosing one path as a reference. Also if the mathematical programming models are used then stochastic programming formulation is possible. This type of representation are found in Manne (1974), Cazalet (1980) and Louveaux (1980).

#### Other Methods:

Other set of methodologies has to do with the decision making under uncertainty in which an optimal policy under the presence of uncertainty can be derived directly. These methods include decision analysis and fuzzy set theory. In the application of decision analysis, uncertainties are identified first and a set of alternative plans are considered taking these uncertainties into account. Then a decision tree whose nodes consist of decision nodes at which a decision is taken and probabilistic nodes at which an uncertain event occurs is constructed. In effect a decision tree describes every possible sequences of decisions and outcomes of the uncertain events from which the sequence resulting in the minimum expected total costs is chosen. This method has been applied to generation expansion planning (Sullivan et al. 1977).

An application of fuzzy set theory is reported in Dhar (1979). This paper seems to be the first in the application of fuzzy set theory and its effectiveness in actual planning is yet to be determined.

#### EXPERIENCES ON THE EFFECTS OF UNCERTAINTY

Now our next question is how and to what extent the existence of uncertainties will affect optimal expansion decisions. A number of papers has been published in which some experiences on the effects of uncertainty are described. These experiences are mostly of numerical nature; these are some examples obtained from a particular system. In fact due to the complexity of the generation expansion planning problems and due to the dependence of these numerical results on the particular configuration of the system investigated, generalization of these results are not always possible. Nevertheless these examples enhance the nature of the effects of uncertainty. In the following a summary of those experiences reported in the literature will be given.

#### Demand Uncertainty

Males (1979) summarizes EPRI's experience on the effects of uncertainty on expansion planning by using the figures shown in the following (Figures 15-17). Figure 15 shows the cost penalty as a function of expansion rate. The curve exhibits a non-symmetric character of cost penalty around the optimal expansion rate, i.e., cost penalty is higher when the expansion rate is smaller than optimal.

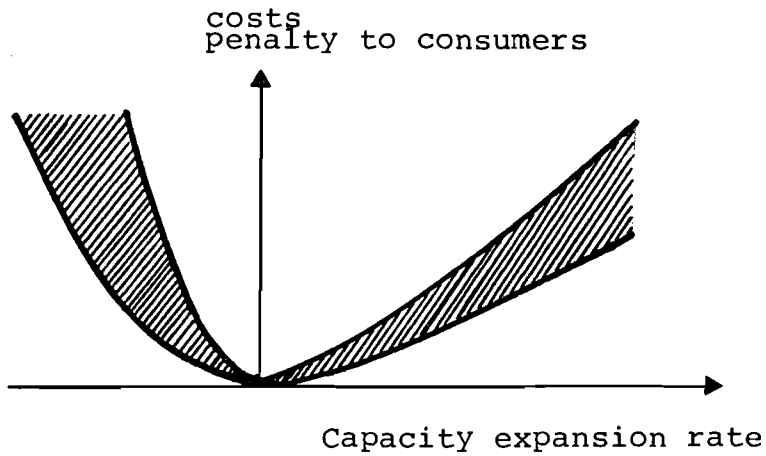


Figure 15. Nonsymmetry of Cost Penalty (Source: Adapted from Males 1979)

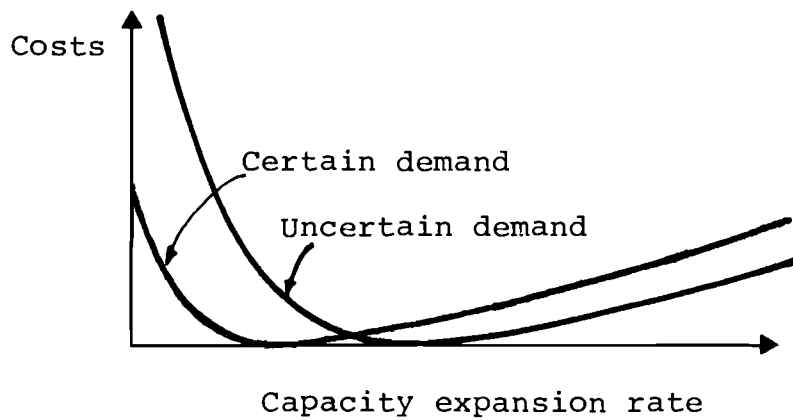


Figure 16. Effect of Uncertainty (Source: Adapted from Males 1979)

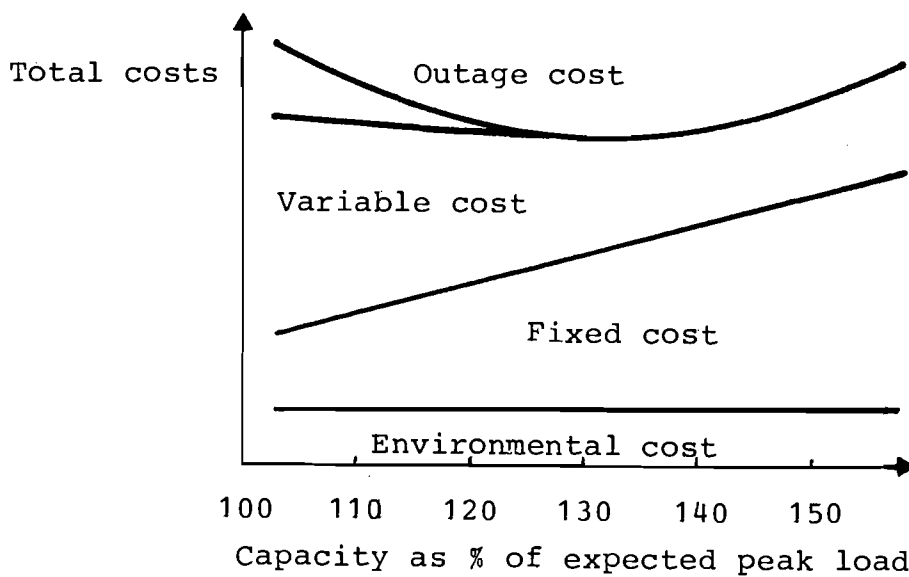


Figure 17. Optimum Total Capacity Under Uncertain Demand (Source: Adapted from Males 1979)

In Figure 16 it is shown that the effects of the existence of uncertainties is to shift the cost penalty curve to the direction of higher expansion rate, which is essentially due to the nonsymmetric characteristic of the cost function. Figure 17 shows the curve of total costs as a function of total capacity where 100% corresponds to an expected peak demand. This curve is due to Cazalet (1980) in which the demand uncertainty is modelled by a probability tree as is discussed in the previous section.

### Effects of Three Major Uncertainties

Duval (1976) has considered the uncertainties with regard to the availability of hydro energy, the availability of thermal plants and the demand. Each uncertainty is assumed to take on two possible outcomes; normal and abnormal and thus creating up to eight different possible outcomes of these uncertainties. Table 2 illustrates this.

The planning model used in this study is of a simulation type in which these possible outcomes are created for each week and the operation of the system is simulated to consider only a subset of events to occur in the simulation model. Thus it is possible to assume for example that the uncertainties with respect to hydro resource does not exist (i.e., only the events No. 3, 5, 7 and 8 in Table 2 can happen) or to assume that no uncertainty exists (i.e., only the event No. 8 in Table 2 happens). In fact, the event No. 8 is the basis of planning and all other outcomes are "hazardous states." The effect of uncertainty is shown in Figure 18.

This example assumes that the only unit type to be considered in the future is nuclear and the curve shows the total cost as a function of the number of nuclear units to be installed. Each curve corresponds to the different situation. For example, the curve denoted by CTH is the results corresponding

Table 2. Possible Outcomes of Uncertainties

	Consumption (Demand)	Hydro resource	Thermal availability	Prob.
1	H	L	L	0.001
2	H	L	H	0.009
3	H	H	L	0.009
4	L	L	L	0.009
5	H	H	H	0.009
6	L	L	H	0.081
7	L	H	L	0.081
8	L	H	H	0.729

H : high, L : low

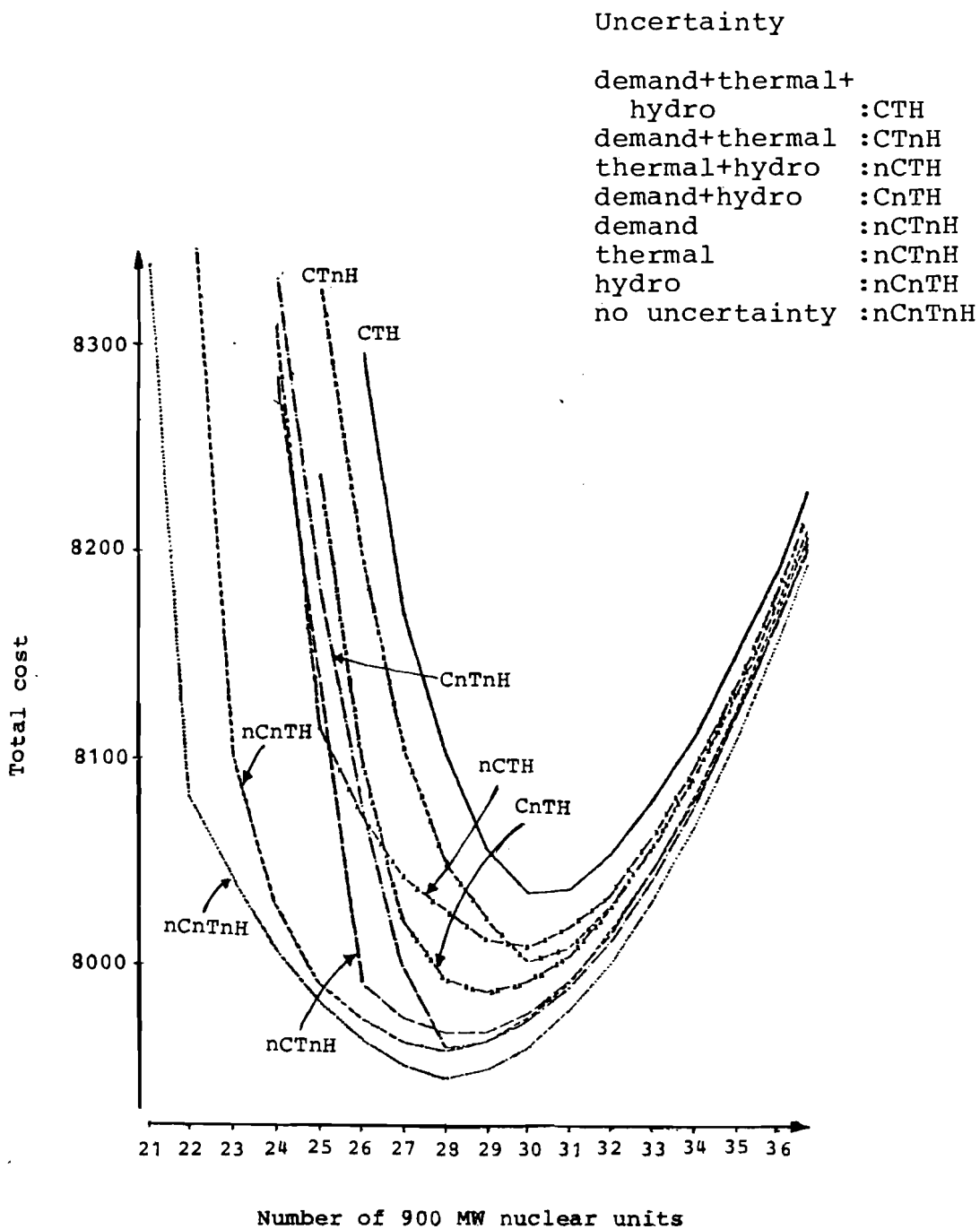


Figure 18. Effects of Uncertainty (Source: Adapted from Duval 1976)

to the probability assignments in Table 2, namely, any of these eight events can happen. Whereas the curve denoted by nCnTnH is the results corresponding to the case where the event No. 8 is the sure event, i.e., no uncertainty exists. Thus each curve between the two above corresponds to the case where various degree of uncertainties is considered.

It can be seen from this result that the existence of uncertainties has the effect of shifting the entire curve up and to the right, implying that under the situation where uncertainties are significant, it is cheaper to have more capacity.

#### Change in Fuel Price and Capital Cost

A change in fuel price will obviously affects the operating costs and therefore the proportion of each generating type in an optimal generation mix will be changed accordingly. The similar arguments hold for the changes in capital costs.

The effects of these changes can be determined by carrying out a sensitivity analysis. The question of how much these changes affect to expansion plans are purely contingent upon the particular system to be investigated. Adamson et al (1973) illustrates some results which are shown in Figure 19 and 20.

Figure 19 illustrates the impact of increasing nuclear capital costs from its nominal value, while keeping capital costs of other generation means constant. It can be observed that the proportion of nuclear power in the optimum generation mix is significantly reduced as nuclear capital costs are increased.

Figure 20 illustrates the impact of changing fossil fuel price. As fuel cost inflation is lowered, the significance of nuclear production cost savings over fossil production costs diminishes. Therefore, more gas turbines and midrange units are included in the generation mix.

#### Impact of Short Term Optimization

Garvor et al. (1976) investigated the impact of two short range strategies, i.e., a strategy which minimizes oil consumption and a strategy which minimizes new investments, by comparing these strategies with the standard strategy of total costs minimization. These short term strategies are considered as typical reactions to the changing environment such as the sudden uprise in the costs of fuel after the oil crisis and the purpose of the study was to examine which strategy is the best for coping with uncertain future.

The method used is the one which represents future possible events by scenario. The strategies used and the scenarios assumed are shown in Figure 21. The first strategy is the usual total costs minimization. The second one is a reaction to the prediction that the oil price will be increased in the future.

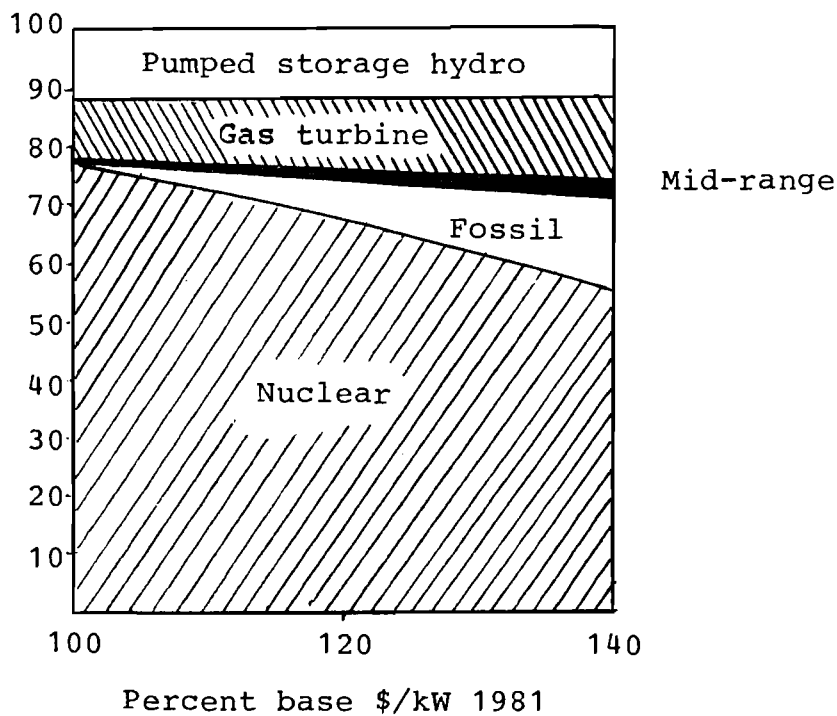


Figure 19. Nuclear Capital Cost Test (Source: Adamson et al. 1973)

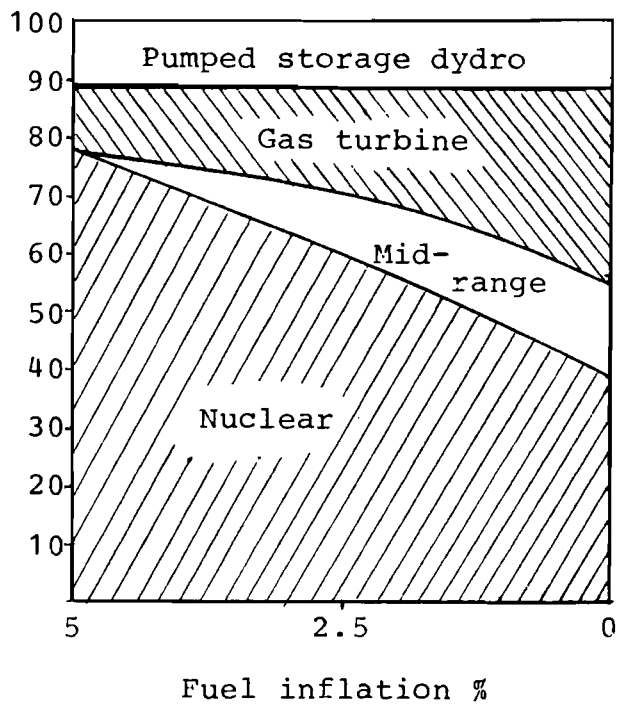


Figure 20. Fuel Inflation Rate Test (Source: Adamson et al. 1973)

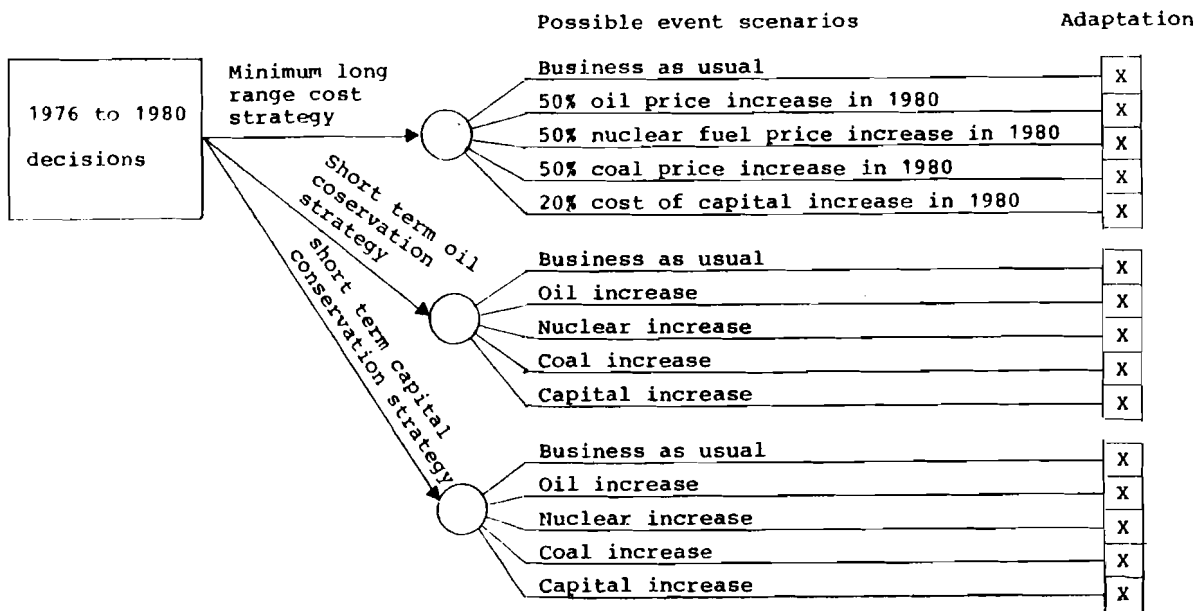


Figure 21. Three Decision Strategies and Five Possible Scenarios Studies to Identify the Costs of Uncertainty (Source: Garvor et al. 1976)

The third one is aimed at reducing the new investments over the immediate future. The latter two strategies are assumed to be implemented over the first five years of the 15 years planning period and then switched back to the first strategy over the rest of the planning period.

A reference expansion plan was projected for an aggregated model of the United States utility systems with the aid of Optimized Generation Planning (Adamson et al. 1973, see Figure 7 in this paper) for each strategy.

The three strategies and five different scenarios generates 15 different cases over which the performance of each strategy were evaluated by several measures; total new financing required, the oil consumption for each expansion, the annual revenue requirements, total revenue requirements over the last 10 years of the 15 year planning period, and finally, the cost of uncertainty.

The last measure is the benefits missed by selecting a strategy that turns out not to be the best under the scenario. For example, if the minimum cost strategy is chosen and the oil price turns out to be higher than expected, then the opportunity loss is defined as the difference between the total revenue



requirements over the last 10 years when the minimum cost strategy is used and when the oil conservation strategy is used. For the last two measures, the expected value is obtained by assigning a probability for each of five different scenarios. Table 3 and Table 4 summarize the last two measures.

It turned out in this particular study that the minimum cost strategy performs best and indicates its robustness with regard to the assumed uncertainties in the future.

#### Effect of Discount Rate

In a way, discount rate can be seen as how much we want to compare the present and the future. Through this mechanism one can weigh the future and artificially reduce the effects of future uncertainties on the expansion plan. How much would the changes in the discount rate affect the expansion plan? Normally, it appears that the power system expansion planning problem is so complex that any solid conclusion based on some analytical model may not be drawn.

Rowse (1978) carried out a numerical investigation of the discount rate sensitivity of the optimal power system expansion plans for a particular electric utility.

A linear programming model which is an extension of the models described elsewhere in this paper is used. A subset of the variables are restricted to be 0 or 1 variables. Planning year is between 1976 through 1990 although the construction lead time precluded any consideration before 1982.

An optimal solution was generated by using 10% discount rate and 8% inflation rate. With this as a reference solution, the program identified five other solutions corresponding to the different values of discount rate (Table 5).

Table 6 displays the sensitivity of each solution to the discount rate. It is notable that the reference optimal solution stayed optimal up to 12% discount rate and near optimal for all the discount rates examined. Further the order of the five alternative plans did not change except the minor alternation of the reference plan and the alternative 1.

This robustness of the optimal solution was demonstrated for two other cases. In one case inflation rates were differentiated between the thermal fuel costs (5%) and all other costs (8%). In the second case, a shorter planning horizon was chosen. In both cases the results were similar to the one shown in Table 6.

In this particular example, the role of hydro power development was apparently dominant. Thus the conclusion is not necessarily generalizable, however the robustness of the optimal solution with respect to the discount rate in this example is remarkable.

Table 3. 1981 Present-worth of the 1981 to 1990 Revenue Requirements (in billions of Dollars)

Scenario	Likelihood	Expansion Strategy		
		Minimum cost	Oil conservation	Capital conservation
Bau	0.500	358	360	360
Oil	0.125	416	411	422
Nuclear	0.125	385	386	377
Coal	0.125	416	424	427
Capital	0.125	406	415	408
Expected value		382	385	384

Source: Garvor et al. 1973

Table 4. Determining the Cost of Uncertainty Associated with The 1981 PWRR in Table 3 (in billions of Dollars)

Scenario	Likelihood	Expansion Strategy		
		Minimum cost	Oil conservation	Capital conservation
Bau	0.500	\$ 0	\$ 2	\$ 2
Oil	0.125	5	0	11
Nuclear	0.125	8	9	0
Coal	0.125	0	8	11
Capital	0.125	0	9	2
Cost of uncertainty		1.6	4.3	4.0

Source: Garvor et al. 1973

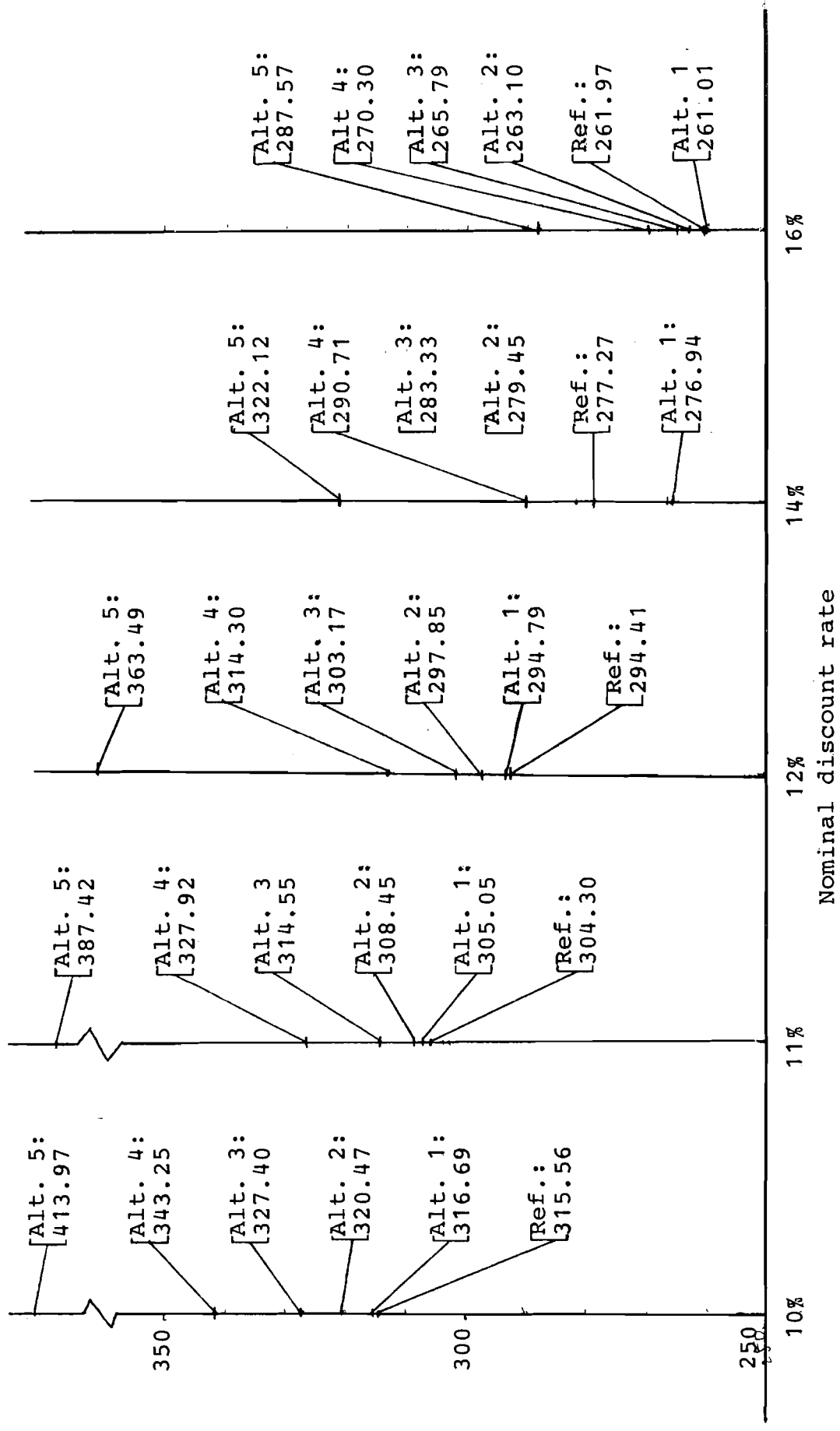
Table 5. Reference and alternative Solutions

Solution	Expansion alternative	Cumulative Power Capacity (in MW)								
		1982	1983	1984	1985	1986	1987	1988	1989	1990
(Power demand)		2061	2192	2331	2449	2574	2737	2907	3064	3228
Reference	Hydro 1	0	0	255	255	510	510	510	765	765
	Hydro 2	285	285	285	285	285	285	285	285	285
	Thermal	0	0	0	0	0	280	280	280	560
Alternative 1	Hydro 1	255	255	255	255	510	510	510	765	765
	Hydro 2	0	0	285	285	285	285	285	285	285
	Thermal	0	0	0	0	0	280	280	280	560
Alternative 2	Hydro 1	255	255	510	765	765	765	765	765	765
	Hydro 2	0	0	0	0	0	0	0	285	285
	Thermal	0	0	0	0	0	280	280	280	560
Alternative 3	Hydro 1	0	0	0	0	255	510	510	765	765
	Hydro 2	285	285	285	285	285	285	285	285	285
	Thermal	0	0	280	280	280	280	280	280	560
Alternative 4	Hydro 1	0	0	0	0	255	255	255	510	510
	Hydro 2	285	285	285	285	285	285	285	285	285
	Thermal	0	0	280	280	280	560	560	560	840
Alternative 5	Hydro 1	0	0	0	0	0	0	0	0	0
	Hydro 2	0	0	0	0	0	0	0	0	0
	Thermal	280	280	560	560	840	1120	1120	1400	1680

Source: Adapted from Rowse 1978

Table 6. Discount Rate Sensitivity of Planning Period Costs (T=9)

(Units are millions of dollars discounted to 1976)



Source: Rowse 1978

## Effect of New Technology

The question as to how a new technology for electricity generation would participate in an optimal generation expansion plan, especially when the date of commercial operation is uncertain, was examined by Manne (1974), using a method called sequential probabilistic linear programming. In this model the decision variables are conditioned by the state-of-world, i.e., the availability of a new technology, in this case a breeder reactor.

A decision tree which expresses the possible changes in the state-of-world is shown in Figure 22. In the diagram  $x_s^t$  denote the decision variables at time  $t$  when the state-of-world is  $s$ , where  $s$  denoted the date of the arrival of breeder reactor. The arrival date is assumed to be either in time 3 or 4 or 10 (in this case the reactor never arrives during the planning horizon) with the probability assigned properly. These probabilities will come in to the objective function which expresses the expected discounted costs.

It was found in this numerical example\* that although the optimal initial decisions are not invariant with the state-of-world the value of information is low. The maximum that could be afforded for a perfect forecast represented by the difference between the expected costs of following an optimal strategy without advance information and the expected costs with this information was found to be small (less than 0.5% of the expected costs). The breeder date seems to affect the timing of installation but the total amount for each type of plants at the end of the period 2 remained virtually the same.

## COPING WITH UNCERTAINTY

Having reviewed the basic expansion planning methodologies as well as methods for dealing with the effects of uncertainty and having observed the results of some numerical experiences reported in the literatures the next question would be to speculate on whether there is anything general to say about how to cope with the future uncertainty.

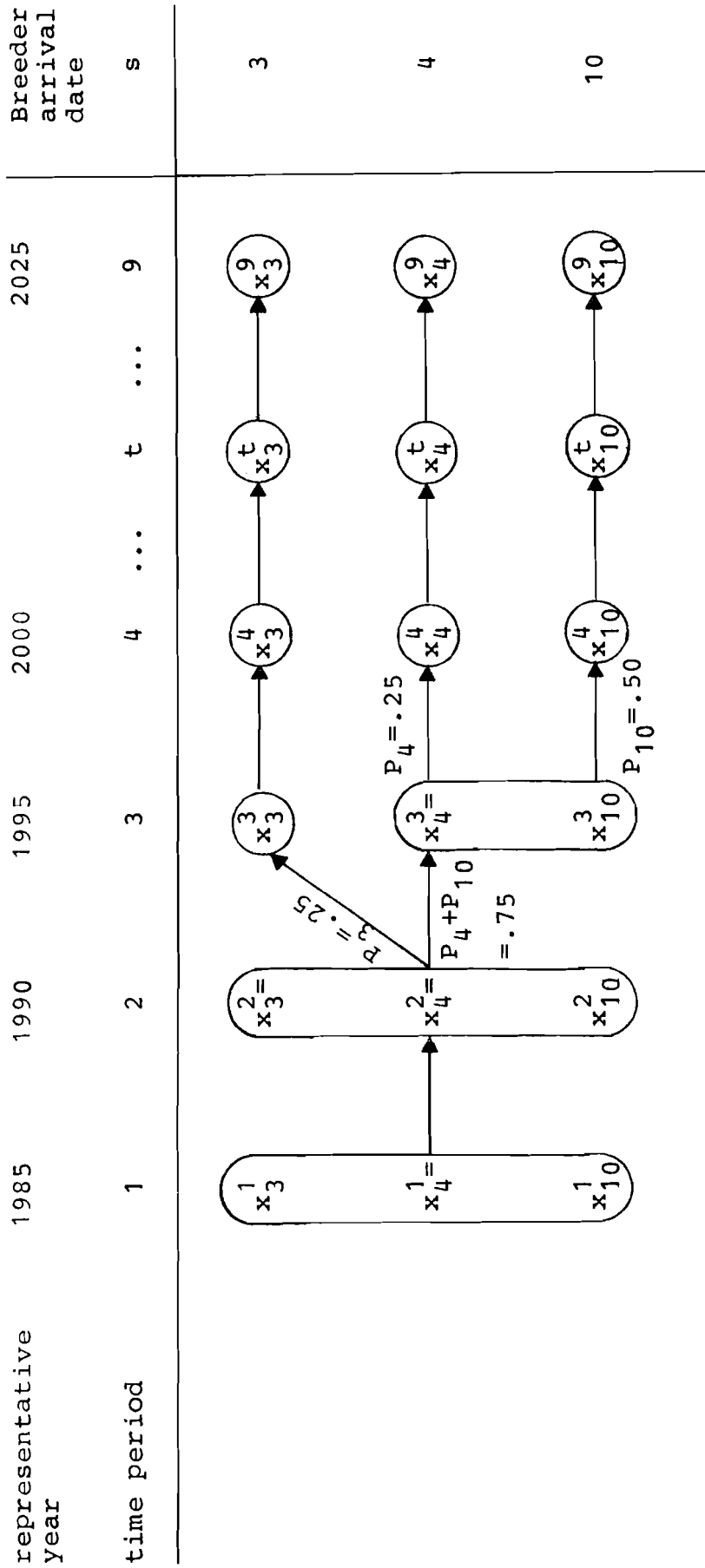
Males (1979) drew the following four basic principles for planning under the face of uncertainty:

- display uncertainty
- value technological options
- supply flexibility
- supply cushion.

Pober (1980) has argued extensively about the source of uncertainty and possible countermeasures through the detailed

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\* In this model, the dependence of demand on price is taken into account.



$$x_3^1 = x_4^1 = x_{10}^1, \quad x_3^2 = x_4^2 = x_{10}^2, \quad x_3^3 = x_4^3 = x_{10}^3$$

$P_s$  = Prib [Breeder arrival date is s]

Figure 21. Structure of Decision Tree (Source: Adapted from Manne 1974)

consideration of a particular utility and has arrived at the general principles of coordination, adaptation and diversification. In fact our review in this paper can give a general support to these principles which will be rephrased in the following.

#### Identify Uncertainty:

It is rather obvious that we need to identify what are the sources of uncertainty. Table 1 lists a number of sources of uncertainty for the electricity generation expansion planning problems but not all of them are described concretely and it is by no means complete. Constant effort for identification and quantification of uncertainties, and inclusion of these uncertainties in the process of expansion planning are very important.

#### Reducing the Degree of Uncertainty:

The degree of uncertainty in demand for example could be reduced by improving the accuracy of its forecast. Demand forecast has been a distinctive area in power system expansion planning in which an enormous amount of effort has been made. Extensive literatures are available (e.g., Hoffman and Wood 1976, Taylor 1975, Sachdeu et al. 1977, Uri 1978a, b). Also modeling in the context of national economy or energy policy (Manne et al. 1979), developing an expansion planning model which takes the price dependence of demand into account (e.g., Bergendahl 1978, Manne 1974), are useful for effectively reducing the degree of uncertainty.

#### Plan for More:

The result shown in the previous sections that the effect of uncertainties in demand, hydraulic energy and thermal plant performance is to shift the entire cost curve up and to the right seems to be quite general. The penalty of not being able to meet the demand (underplanning) is substantially larger than having more capacity (overplanning). Thus as long as there is a possibility of demand growing higher, it is better to prepare for the higher growth case.

#### Diversification:

The fuel price and capital cost uncertainty have the effect of changing the proportion of generation mix. Intuitively it is clear that the amount of impact to a system can be reduced if it has a variety of types of generation plants. The same principle applies to the source of fuel. Dependence on a single resource area would only increase the impact of uncertain events. Setting up interconnections with neighbouring power systems has obviously the effect of diversification.

### Flexibility and Adaptation:

It can be said that the sole cause of uncertainties is the fact that the environment surrounding the expansion problem is changing all the time. New information will be available as time elapses and these information should be taken into account in expansion planning. Constant revision of expansion plan is essential.

### Keep Options for New Technology:

One of the characteristics of generation expansion planning is that a drastic change in the composition of an optimal generation mix is always a possibility. Also technological options can increase flexibility and adaptability of generation expansion planning. Thus it is highly desirable to keep technological options open unless they are definitely determined to be unnecessary.

### CONCLUSION

A review of electricity generation expansion planning is presented. Emphasis was placed on the methods and some known facts about the effects of uncertainty on generation expansion planning.

There are a number of models and techniques for generation expansion planning which enables us to evaluate one way or another the effects of uncertainty and in fact several numerical examples illustrate these effects. The observation that the effect of demand uncertainty is such that a larger reserve margin is preferable seems to be quite general. However other effects cannot be stated generally because of the complexity and the peculiarity of each power system to be considered.

The existence of various sophisticated mathematical programming models as well as the detailed simulation models allows us to investigate in detail the future expansion plans in electricity industry. However, because of the complexity these detailed models may not be helpful to improve our intuitive and basic understanding about the nature of generation expansion planning. Development of a set of simpler models which retain essentials of expansion planning is still desired and the detailed models should be utilized to reinforce basic findings from simpler models. A carefully designed research effort in this direction may prove to be useful.

There seems to be a number of general principles to cope with the inherent existence of uncertainty in generation expansion planning. The principles stated in this paper (in the previous section) are introduced from the general observations on the reviewed literature. More quantitative analysis and justification would be necessary in order to make these statement more substantial and authoritative. The various kinds of models and techniques should be utilized to clarify quantitatively the effects of uncertainty in the light of these general principles.



APPENDIX

A numerical example for the cost function  $C(V;D)$  and

$$\bar{C}(V) = v^a + \sum_{i \in I} e^{-\frac{rV}{D_i}} p_i \{ \min_V C(V;D) \}$$

is given in the following table. Demand uncertainty is described by three possible demand rates ( $D_i$ ,  $i = 1, 2, 3$ ) with discrete probability ( $p_i$ ,  $i = 1, 2, 3$ ) assigned to each demand rate. In this example  $a = 0.7$  and  $r = 0.1$ , and in the table the minimum value for each column is underlined. The optimal capacity for the case where no uncertainty exists is given by 6.754, while when the expected value is taken the optimal capacity is given by 6.6 although the difference in the minimum values is not at all significant.

Table A1. A Numerical Example

V	Values of cost function			Expected cost function
	$D_1=0.8$ $p_1=0.25$	$D_2=1.0$ $p_2=0.5$	$D_3=1.2$ $p_3=0.25$	
5.0	6.636	7.789	8.893	7.777
5.4	<u>6.633</u>	7.775	8.874	7.764
6.5	6.651	7.755	8.833	7.749
6.6	6.654	7.755	8.830	<u>7.748</u>
6.7	6.657	7.755	8.828	7.749
6.754	6.660	<u>7.755</u>	8.826	7.749
6.8	6.661	7.755	8.825	7.749
7.5	6.695	7.760	8.814	7.758
7.6	6.701	7.763	8.813	7.760
8.1	6.734	7.774	<u>8.810</u>	7.773
9.0	6.809	7.808	8.817	7.811

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