International Institute for
Applied Systems Analysis
IIASA www.iiasa.ac.at

Uncertainty and the Choice of Policy Instruments for Meeting an Environmental Quality Standard

Anderson Jr., R.J.

IIASA Working Paper

WP-80-163

November 1980

Anderson Jr., R.J. (1980) Uncertainty and the Choice of Policy Instruments for Meeting an Environmental Quality Standard. IIASA Working Paper. WP-80-163 Copyright © 1980 by the author(s). http://pure.iiasa.ac.at/1306/

UNCERTAINTY AND THE CHOICE OF POLICY INSTRUMENTS FOR MEETING AN ENVIRONMENTAL QUALITY STANDARD

Robert J. Anderson, Jr.

November 1980
WP-80-163

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

Broadly stated, there are three steps in environmental quality management. These are (1) selection of environmental quality standards, (2) determination of feasible programs for reducing discharges to the environment or for increasing its assimilative capacities, and (3) implementation of a policy to bring about discharge reductions and/or expansion of assimilative capacity.

Much of REN's research program has been devoted to state-of-the-art analysis of the effects of discharges on environmental systems and modifications in environmental systems that could be made to enhance systems' abilities to assimilate discharges. For example, case studies of Lake Balaton in Hungary and Attersee and Neusiedlersee in Austria have advanced the state of the art of modeling water quality and are providing a menu of feasible programs for managing the quality of these lakes.

Subsequent REN studies will build on these modeling efforts to evaluate alternative management plans and policies. This working paper is the first of a series of reports that are planned to result from these efforts. It considers the problem of choosing a policy instrument (i.e. a means to implement environmental quality standards) that is both effective (i.e. will result in the environmental quality standard being met) and efficient (i.e. will meet standard at least cost). Three alternative policy instruments are considered. These are (1) emission standards, which prescribe maximum allowable rates of discharges of pollutants to the environment, (2) emission charges, which prescribe payments that emitters must make per unit of pollution discharged to the environment, and (3) transferable emission permits, which entitle their holder to a face value quantity of emissions, and which may be transferred among emitters.

REN's analyses of specific environmental management problems dramatically illustrate that uncertainty pervades the modeling and management process. Until recently, this fact of the modeling and management problem (i.e. uncertainty) was largely ignored in studies of the policy instrument question. This paper builds on the recent literature to show how, in the presence of uncertainty, policy might best be fashioned to meet an environmental quality standard efficiently.

ACKNOWLEDGEMENTS

Helpful comments and suggestions from Carson E. Agnew, Jesse Ausubel, Janusz Kindler, and Paul Portney are gratefully acknowledged. None is to be held responsible to any errors, ambiguities, or other faults that may remain.

TABLE OF CONTENTS

UNCERTAINTY AND THE CHOICE OF POLICY INSTRUMENTS FOR MEETING AN ENVIRONMENTAL QUALITY STANDARD

Robert J. Anderson, Jr.

1. INTRODUCTION

Several papers have examined the effect of uncertainty on the choice of pollution control policy instruments.* These papers have shown that the optimal policy instrument (i.e. the policy instrument--including emission taxes, emission standards, or transferable emission permits--that maximizes expected net social benefits) depends upon the specific circumstances at hand. In particular, it has been shown that when the marginal costs and marginal benefits of pollution control are uncertain, the type of policy instrument that maximizes expected net social benefits depends upon the parameters of the marginal cost and marginal benefit functions, and the form in which randomness enters the model.

In practice, the process by which most environmental quality targets are set does not involve an attempt to maximize expected social benefits from use of environmental resources. Rather, quality standards are set based upon other criteria such as the protection of the health of the most susceptible segments of the population, with an adequate margin of safety.

This paper examines the effect of uncertainty on the choice of policy instruments in cases in which the objective is to find

---

* See Adar and Griffin (1976), Fishelson (1976), and Yohe (1976).

a policy that minimizes the expected cost of meeting a given environmental quality standard. These cases, as noted above, typify the policy problem when quality standards are based, at least in part, on non-economic criteria.

Our analysis will show that rather definite conclusions can be reached concerning optimal policy instruments for minimizing expected costs of meeting an environmental quality standard. In particular, we will demonstrate that two policy instruments--emission charges and transferable emission permits--will always meet quality standards at lower expected total cost than will an emission standards policy. We will also demonstrate that implementation of environmental quality standards via emission standards or transferable emission permits provides greater certainty than does implementation via an emission charges policy that environmental quality standards will be met. Thus, if one wishes to adopt a policy that minimizes expected costs of meeting a quality standard with a relatively high degree of certitude, transferable emission permits are the best of the three policy instruments examined.

The plan of the paper is as follows. In Section 2 we examine a simple diagrammatic model that illustrates the factors involved in a comparison of policy instruments to meet environmental quality standards when there is uncertainty. Section 3 generalizes this analysis by deriving essentially the same results in a more general mathematical framework. Section 4 offers some concluding comments.

2. A SIMPLE DIAGRAMMATIC ANALYSIS

To begin, let us examine a simple case in which there are two pollution sources (i.e. two "emitters") and one point at which an environmental quality standard must be met (i.e. one "receptor"). We assume that the pollution control authority is uncertain about one of the emitter's (which we shall call "Emitter 1" below) costs of controlling emissions. The authority is assumed to know exactly (i.e. without uncertainty) the other emitter's (i.e., "Emitter 2") control costs and the diffusion relationships that relate units of emissions from both of the emitters to the concentration of pollution at the single receptor site.

This situation is depicted in Figure 1 below, where we have shown Emitter 1's marginal cost of emission control curves. In drawing this figure, we have assumed that Emitter 1's marginal cost of control curve may take on one of two values, shown respectively by the curves $C(u_1)$ and $C(u_2)$, where u is a random variable.* The dotted curve C'-C represents Emitter 2's marginal cost curve. We have denoted the optimum (i.e., least cost) levels of emissions from Emitter 1 that are consistent with attainment of the environmental quality standard by the vertical lines $\bar{e}_1$ and $\bar{e}_2$ in the figure. The former, $\bar{e}_1$, represents the optimum level of Emitter 1's emissions if the random variable u takes on the value u_1, and $\bar{e}_2$ represents the emitter's optimum level of emissions if u takes on the value u_2.**

The pollution control authority does not know in advance which value u will take on so it cannot set emission standards, a quantity of marketable permits, or emission charges that will be exactly correct, except by accident. Rather, the best it can hope to do is to find a policy that is best in some average sense. This indeed is a sensible objective for it to pursue. We shall assume that regardless of the particular policy instrument chosen by the authority, its goal is to minimize expected (in a mathematical sense) costs subject to the constraint that expected concentrations of pollution in the environment does not exceed the environmental quality standards.

If this is the objective pursued by the agency, and if it chooses regulation for its policy instrument, then it can be shown that it will set an emission standard, $\hat{e}$, inside the range of values of optimum emissions levels under alternative possible cost conditions (i.e., between $\bar{e}_1$ and $\bar{e}_2$). The exact levels of the emission standards that will minimize expected costs subjects to the environmental quality standard depends upon the probability distribution of the uncertain marginal control cost function, the environmental

---

*The linearity of the marginal cost of control curves is the result of the elimination of second order terms from an approximation of the unknown total cost relationship. The assumption is that the slope of the marginal cost of control curve is known; uncertainty involves only the intercept. See also Sections 3 and 4.

**That is, $\bar{e}_1$ is the level of Emitter 1's emissions that equates the marginal costs of pollution reduction from Emitters 1 and 2 if $u = u_1$; $\bar{e}_2$ is the level that achieves this if $u = u_2$. Emitter 2's emissions are adjusted correspondingly to insure that its marginal cost of pollution reduction is equal to Emitter 1's marginal cost of pollution reduction, and that the pollution standard is met.

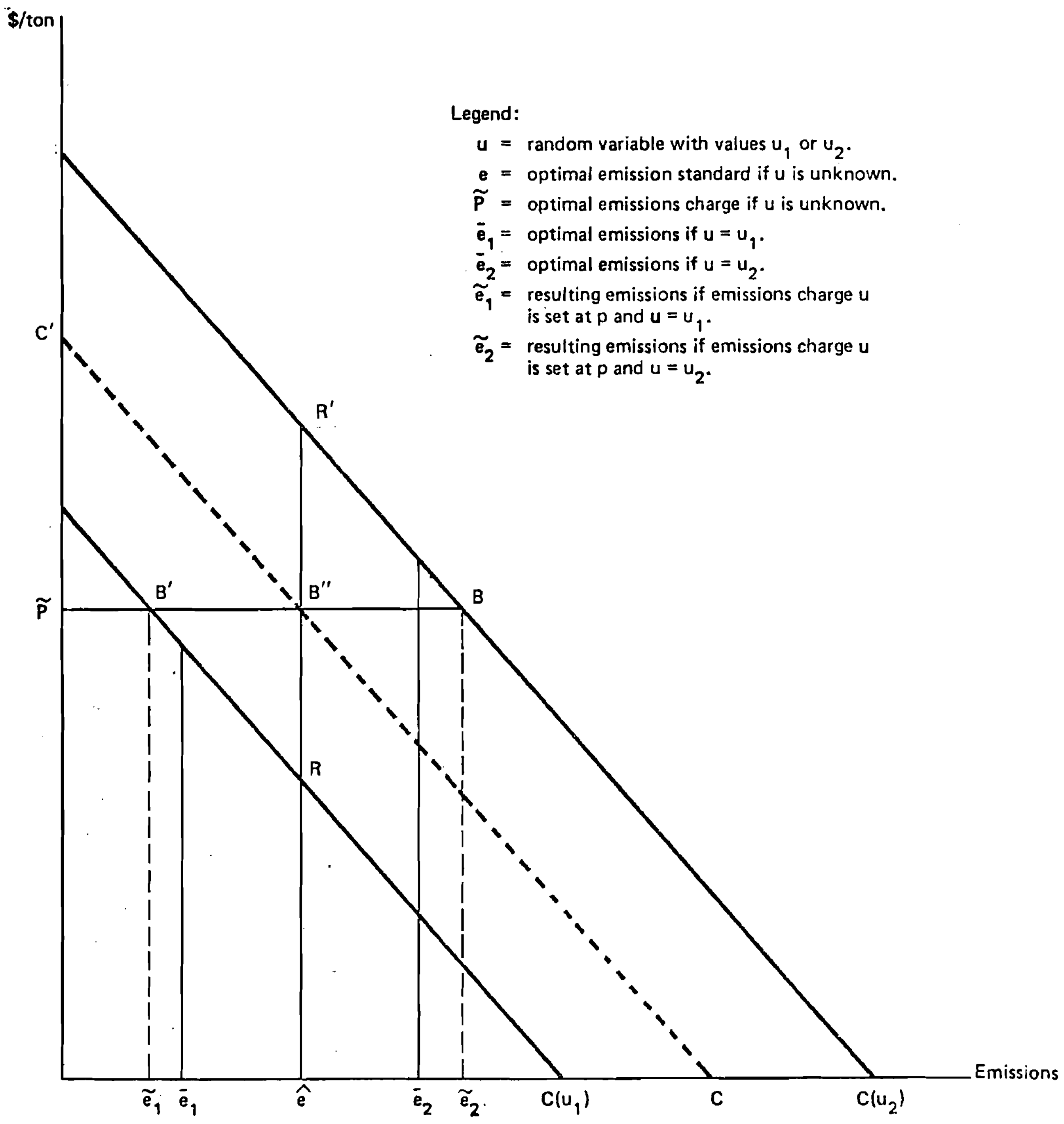

Figure 1. Diagrammatic Analysis of the Effect of Uncertainty on Policy Instrument Choice.

quality standard, and the nature of the diffusion relationships that relate emissions to environmental concentrations of pollutants.

We have drawn $\hat{e}$ in Figure 1 as though it fell precisely midway between $\bar{e}_1$ and $\bar{e}_2$. This would occur in fact in the situation shown in which Probability $(u=u_1)$ = Probability $(u=u_2) = 0.5$, if the diffusion parameters translating the emissions of each source into ambient concentrations at the assumed receptor were equal, and if the marginal cost function of Emitter 2 (shown by the dotted line C'-C" in Figure 1) fell exactly halfway between the two marginal cost functions shown for Emitter 1. In this case, the emissions standard for Emitter 2 would also be set at the level $\hat{e}$.

If the authority decides to implement the environmental quality standard via an emission standard, it obviously will be certain about the resulting level of emissions.* Emitter 1 (and Emitter 2) will emit $\hat{e}$. If diffusion relationships are known with certainty (as we have assumed here), this also implies that the authority, if it resorted to use of emission standards, would be absolutely certain that the ambient standard would be met.

This certainty about emissions and environmental quality comes at a cost. Inspection of Figure 1 shows that while the authority would be certain about the costs of control incurred by Emitter 2, Emitter 1's costs could either be an amount given by the triangle $\hat{e}$ R $C(u_1)$, or a somewhat larger amount given by the triangle $\hat{e}$ R'$C(u_2)$. Which cost in fact is incurred will depend upon the unknown (to the authority) value taken on by the random variable u. The cost of the certainty about emissions and environmental quality guaranteed by resort to the emission standards policy instrument is thus that the costs incurred for control are uncertain, and may turn out to be very large.

As an alternative to the imposition of emission standards, the authority might seek to set charges on emissions that would minimize expected cost and would achieve expected ambient concentrations equal to the standard level. Such a charge is depicted in Figure 1 by the horizontal line, $\tilde{p}$. Under this charge, Emitter 1's emissions are uncertain. They will either be $\tilde{e}_2$ or $\tilde{e}_1$, depending upon the

---

*We assume costless monitoring.

value taken on by u.* This means that the resulting environmental quality also is uncertain. If u takes on the value u_2, then resulting pollution concentrations will exceed the environmental standard. If, however, it takes on the value u_1, pollution concentrations will be lower than the standard level. The charge level is set so that if we took the expected (average) value of the marginal cost functions for Emitter 1 (which is equal to C'-C), the resulting emissions would satisfy the ambient constraint.

While a charge policy results, as shown above, in uncertainty about environmental pollution concentrations, it results in perfect certainty about costs. This can be seen by examining total costs under a charge policy as shown in Figure 1. If u takes on the value u_2, then total costs are given by the area of the triangle $\tilde{e}_2\,B\,C(u_2)$. If u takes on the value u_1, then total costs are given by the area of the triangle $\tilde{e}_1\,B'\,C(u_1)$. These two triangles, it can be shown, have precisely the same area. Hence, we conclude that no matter which value u takes, total costs will be the same amount.** Thus there is no uncertainty about total control cost.

To this point we have considered two policy instruments, emission standards and emission charges, which achieve expected environmental pollution concentrations equal to an environmental quality standard. We have shown that use of the emission standard policy results in certainty about emissions and ambient concentrations, and uncertainty about costs. Use of the emission charges policy instrument, in contrast, results in uncertainty about emissions and ambient concentrations, and certainty about costs.

The final step of our comparison of these two policy instruments is to examine their expected costs. To do this using our diagram, we must make use of the specific assumptions we have made for this example. Expected costs under the emission charges approach

---

*Emitter 2's emissions at a charge of $\tilde{p}$, in the case depicted in Figure 1, would be $\hat{e}$.

**Section 3 explains the mathematical assumptions about uncertainty in cost that lead to this result.

are relatively easy to compute since, under our assumptions, the charge policy removes uncertainty about costs. Emitter 1's expected costs under the charge policy are simply $\tilde{e}_2 B\ C(u_2)$, or $\tilde{e}_1\ B'C(u_1)$ (which are equal). These are equal to $\hat{e}\ B''C$, which is the area we shall use to compare the expected costs of a charge policy with those of a regulatory policy. Emitter 2's expected cost of control, in the case depicted, is also $\hat{e}B''C$.

Emitter 1's expected costs under the emission standards approach are given by one-half (i.e. the probability that u takes on the value u_1) times the area of the triangle $\hat{e}R\ C(u_1)$ (i.e. total cost when u takes on the value u_2). Emitter 2's costs are $\hat{e}B''C$.

Let the distance between the points R and R', and $C(u_1)$ and $C(u_2)$, be denoted by d. Further, let the distance between $\hat{e}$ and R be denoted by h and the distance between $\hat{e}$ and $C(u_1)$ be denoted by b. Let us also neglect Emitter 2's costs since they are the same amount (i.e. $\hat{e}B''C$) under both policies. Then, we know, using the specifics behind the construction of Figure 1 that the area of the triangle representing the expected costs under emission charges (ignoring Emitter 2's costs) is

Expected Cost Under Emission Charge =

$$\text{Area } (\hat{e}\ B''\ C) = \frac{1}{2}(b+\frac{1}{2}d)(h+\frac{1}{2}d) = \frac{1}{2}bh + \frac{1}{4}dh+\frac{1}{4}db+\frac{1}{8}d^2$$

The expected cost under emission standards (again ignoring Emitter 2's costs) is given by the weighted (by the probabilities associated with alternative values of u) areas of the two triangles ($\hat{e}\ RC(u_1)$ and ($\hat{e}\ R'C(u_2)$)) described above. This expected value is

Expected Cost Under Emission Standards =

$$0.5\ (\frac{1}{2}bh) + 0.5\ (\frac{1}{2}(b+d)(h+d)) = \frac{1}{2}bh + \frac{1}{4}dh + \frac{1}{4}db + \frac{1}{4}d^2$$

A comparison of the expected cost under emission charges with that under emission standards shows that the former is smaller by the amount $\frac{1}{8}d^2$ (i.e., $\frac{1}{4}d^2 - \frac{1}{8}d^2$).

That is, we have shown (albeit under special circumstances which we will generalize in the next section) that the expected cost of implementing on environmental standard via an emission charges approach is less than that of implementing it via an emission standards approach. As noted above, however, uncertainty about resulting emissions and ambient concentrations is greater under emission charges than it is under emission standards.

To complete our analysis, let us consider possible outcomes of a transferable emission permits approach. Our assumption in conducting this analysis is that Emitters 1 and 2 know their costs of control with certainty, and reveal them in the process of buying and selling transferable permits. That is, the schedule of bids submitted by Emitter 1, under a transferable permit policy would be either $C(u_1)$ or $C(u_2)$, depending upon the value actually taken by u, while Emitter 2 would submit the schedule of bids C'C. Note that if the bidding process operates as we have assumed, at the time the authority must allocate permits to bidders, all uncertainty has been removed. In particular, it now knows what Emitter 1's costs are with certainty. This has two consequences. First, it can be absolutely certain that the allocation of permits chosen in fact will minimize costs. Moreover, <u>before</u> bids are received, the authority knows that an <u>ex post</u> optimal decision will be made. Thus, <u>ex ante</u> (i.e., before bids are received) expected costs under a transferable permit approach will be precisely the same as those under an emission charge approach.

The second consequence of knowing Emitter 1's costs with certainty and allocating permits accordingly is that uncertainty about resulting emissions and ambient concentrations is also removed. The authority can be certain of each emitter's emission levels.

Our simple diagrammatic analysis thus leads us to conclude that implementation of an environmental quality standard via the transferable permits or emissions charge policy instruments results in lower expected control costs than does implementation via emission standards. Moreover, the transferable permits instrument offers as great certainty about resulting emissions and ambient environmental quality as does the emission standards instrument, and both result in greater certainty about environmental quality than does the emission charges instrument.

3. A MATHEMATICAL ANALYSIS

We can demonstrate these conclusions more rigorously and extend them a bit by means of a relatively simple analysis. This analysis parallels Weitzman's (1974) analysis of economic planning.

3.1 Formulation

For this purpose, it is helpful to introduce a modest amount of notation, and some simplifying assumptions. Our basic assumptions are these:

(a) We confine our attention to the case of a single receptor.

(b) Let $C_i(e_i,u_i)$ represent the i^{th} emitter's cost function, where e_i represents its quantity of pollutant emissions, and u_i is a random variable representing the control authority's uncertainty about the emitter's costs. $C_i(e_i,u_i)$ is assumed to be twice continuous differentiable, and its derivatives are assumed to possess the following properties: $\partial C_i(e_i,u_i)/\partial e_i = C_i' < 0$; $\partial C_i(e_i,u_i)/\partial u_i > 0$; ${}^2C_i(e_i,u_i)\partial e_i \partial u_i > 0$; $\partial^2 C_i(u_i)/\partial e_i{}^2 = C_i'' > 0$.

(c) Let $d_i(v_i)$ represent the uncertain coefficient of proportion relating units of pollutant emissions from the i^{th} emitter to ambient pollution concentrations at a receptor. "v_i" is a random variable representing the agency's uncertainty about the pollution diffusion relationship.

(d) Let "s" represent an ambient pollution standard.

(e) $E(v_i,u_j) = E(v_i)E(u_j) = 0$, where E is the mathematical expectation operator; that is, the random variables u_i and v_j are statistically independent for all i and j.

(f) In view of uncertainty in the diffusion relationship, the authority seeks control policies that achieve expected ambient pollution concentrations equal to the ambient pollution standard.

(g) The emitter knows its control costs with certainty.

These assumptions are stronger than strictly are required to derive the results obtained below. For example, we do not need to assume (as we do in (g) above) that emitters are certain about their costs; it is sufficient to assume that they are less uncertain than the pollution control authority is.

Our objective, recall, is to compare three different policy approaches for implementing an environmental quality standard. Under the first approach, which involves the establishment of emission standards for sources, the authority seeks a set of emission standards that minimizes expected costs subject to achieving expected ambient concentrations equal to the ambient standard. We may formulate this problem using the notation set forth above as

$$(1) \qquad E\,\{\sum_{i=1}^{N_s} C_i(\hat{e}_i, u_i)\} = \underset{e_i}{\text{minimum}}\; E\,\{\sum_{i=1}^{N_s} C_i(e_i, u_i)\}$$

$$\text{subject to } E\,\{\sum_{i=1}^{N_s} d_i(v_i)e_i\} = s$$

where N_s, is the number of emitters. The solutions $\hat{e}_i$ are the emission standards that minimize expected costs subject to the environmental quality constraint, and

$$E\,\{\sum_{i=1}^{N_s} C_i(\hat{e}_i, u_i)\}$$

is the expected cost under this set of regulations.

The second policy approach we examine involves the setting of emission charges so as to minimize expected cost while attaining expected ambient pollution concentrations equal to the environmental quality standard. Let $h_i(p_i, u_i)$ be the function which relates i^{th} emitter's emissions to the charge levied on its emissions. The random variable u_i is included reflecting the fact that the pollution control authority, since it does not know emitters' cost functions, cannot be certain about emitters responses to any given set of emission charges. Then using the notation established above, the authority's problem is to find the $\tilde{p}_i$ such that

$$(2) \qquad E\,\{\sum_{i=1}^{N_s} C_i(h_i(\tilde{p}_i,u_i)\} = \underset{p_i}{\text{minimum}}\; E\,\{\sum_{i=1}^{N_s} C_i(h_i(p_i,u_i),u_i)$$

$$\text{subject to } E\,\{\sum_{i=1}^{N_s} d_i(v_i)h_i(p_i,u_i)\} = s$$

The third policy approach we examine requires emitters to bid to purchase permits from the pollution control authority. The authority is to sell no more permits to emitters than would result in expected ambient pollution concentrations just equal to the environmental quality standard. We will assume that each emitter bids the maximum amount it is willing to pay for permits. Each submits its bids in the form of a demand schedule for permits.

The marginal cost savings to, say, Emitter 1 from purchase of an incremental permit is simply $-C_1(e_1,u_1)$, the derivative of its cost function with respect to emissions. This then is the amount that it would be willing to pay for the last permit unit purchased, and represents Emitter 1's demand function for permits. Assuming that the pollution control authority does not wish to charge a monopoly price for permits, but rather to maximize the net value of (i.e., willingness to pay for) permits issued subject to the constraint that the environmental quality standard be met on an expected value basis, it can be shown that the authority's problem is to choose the number of permits to issue to each emitter, $\bar{e}_i$, such that

$$(3) \quad E\,\{\sum_{i=1}^{N_s} C_i(0,u_i) - C(\bar{e}_i,u_i)\} = \underset{e_i}{\text{maximum}}\; E\,\{\sum_{i=1}^{N_s} C_i(0,u_i) - C(e_i,u_i)\}$$

$$\text{subject to } E\,\{\sum d_i(v_i)e_i\} = s$$

which is precisely equivalent apart from the constant terms $E\{C(0,u_i)\}$, to

$$(4)\quad E\{\sum_{i=1}^{N_s} C_i(\bar{e}_i,u_i)\} = \underset{e_i}{\text{minimum}}\ E\{\sum_{i=1}^{N_s} C_i(e_i,u_i)\}$$

$$\text{subject to } E\{\sum d_i(v_i)e_i\} = s$$

where $E\{\sum C(\bar{e}_i,u_i)\}$ represents expected total costs at the optimal permit allocation.

The most important thing to note about equation (4) is its similarity to equation (1). There is, however, one important difference. Equation (4) represents the problem faced by the pollution control authority <u>before</u> it has received bid schedules from emitters. In fact, the problem that the authority will solve begins <u>after</u> it has received bid schedules. This means that, under our assumption that emitters are perfectly certain about their costs and base their bids upon them, the pollution authority will be perfectly certain about control costs at the time it makes its permit allocation. Therefore it can set an allocation that <u>exactly</u> minimizes costs.

3.2 Analysis

Our analysis compares the results of the policy approaches described above in terms of two criteria. The first criterion is the expected cost to which each approach leads. Other things being equal, we should prefer a policy approach which leads to lower expected cost to one which leads to higher expected cost. The second criterion is the variance of expected ambient pollution concentrations. We have constrained all policies to result in expected ambient pollution concentrations equal to the given environmental quality standard. This being the case, other things being equal, we would prefer policies that result in a relatively tight distribution of ambient pollution concentrations around the environmental quality standard.

The starting point for our analysis of the three policy instruments described above is a quadratic approximation of emitters' cost functions. This is done because it is far easier to work with quadratic forms than with many more general forms, and because in many instances quadratic forms provide quite good approximations. We will approximate the cost functions about the point $\hat{e}_i$ by the functions

$$(5)\quad C_i(e_i,u_i) \doteq a_i(u_i) + (C_i' + \alpha_i(u_i))(e_i - \hat{e}_i) + \frac{1}{2}C_i''(e_i - \hat{e}_i)^2$$

where the $a_i(u_i)$, and $\alpha_i(u_i)$ are random variables, and where the C_i' and C_i'' are constants. The sign $\doteq$ means "approximately equal".

We shall assume that the $\alpha_i(u_i)$ have been standardized so that $E\{\alpha_i(u_i)\} = 0$ for all i. Note also that since u_i and v_i are independent for all i and j by assumption, $E\{\alpha_i(u_i)d_i(v_i)\} = E\{\alpha_i(u_i)\} \cdot E\{d_i(v_i)\} = 0$.

The basic approximation given in equation (5) and assumptions about the random errors imply several other approximations that will be useful in the analysis we shall develop below. The most important of these are the following:

$$(6)\qquad \frac{dC_i}{de_i} = C_i'(e_i,u_i) \doteq (C_i' + \alpha_i(u_i)) + C_i''(e_i - \hat{e}_i)$$

$$(7)\qquad E\{C_i'(\hat{e}_i,u_i)\} \doteq C_i'$$

$$(8)\qquad E\{C_i''(e_i,u_i)\} \doteq C_i''$$

Note that equations (7) and (8) provide us with an interpretation of each of the fixed coefficients appearing in equation (5).

We now have all of the basic ingredients and relationships required to analyze and compare the three policy approaches described in Section 3.1. There is a fair amount of manipulation and substitution involved in our analysis, so perhaps it will be useful to outline our analytical strategy. We will begin by comparing an emission standards policy [as described in equation (1) above] to an emission charges policy [as described in equation (2) above). Our first step in this comparison will be to derive an explicit approximate expression for the functions $h_i(p_i,u_i)$, which give the distributions of emissions levels (as perceived by the pollution control authority) which would result from any given set of changes, p_i. The levels of emissions that actually would arise depend, of course, on the values taken by the u_i, which are known under our assumptions only by the emitters.

The second step of our analysis is to substitute the expressions we derive for $h_i(p_i,u_i)$ into the equations representing the problem of finding a price to minimize expected cost [equation (2) above], and to find an expression for the values of the p_i, which we have denoted as $\tilde{p}_i$, which solve this problem.

The third step is to take the resulting expression for $\tilde{p}_i$, substitute it in our approximation expression for costs [i.e., equation (5) above] , and evaluate expected costs. This yields an estimate of expected costs which we will compare with a similar expression for expected costs evaluated at $\hat{e}_i$. This comparison will show that the expected total cost of the emissions charge policy is lower than the expected total cost of the emissions standard policy.

The final step in our comparison of emissions standards and emissions charges is to compare the variances of the resulting distributions of ambient air quality. Our analysis will show that the emissions standard policy would result in a smaller variance in ambient concentrations than would an emission charges policy. The size of the difference in the variances is related to the same factors that give an emission charges policy an expected cost advantage. That is, the larger the expected cost advantage that an emission charges policy has over an emission standards policy, the greater the dispersion of the distribution of ambient pollution concentrations about the environmental quality standard level.

Our comparison of a transferable emissions permit policy to emissions standards and emissions charges follows a similar sequence of steps to that outlined above. Our analysis will show, as noted above, that the expected cost of a transferable permits policy is precisely equivalent to that of an emission charges policy, and hence is less than that of an emission standards policy. We will also show that the dispersion of the distribution of ambient pollution concentrations resulting from a transferable emission permits policy is the same as that resulting from an emission standards policy and less than that resulting from an emission charges policy.

Proceeding according to the strategy outlined above, we begin by deriving an approximate expression for the functions $h_i(.)$ which relate sources' resulting emissions levels to charge levels. We know that for the i^{th} emitter to minimize costs when faced with an emission charge p_i and state of nature u_i, it will adjust emissions to the point where

$$(9) \qquad -C_i'(e_i,u_i) = -C_i'(h_i(p_i,u_i),u_i) = p_i$$

That is, it will adjust emissions to the point where its incremental cost saving from increasing emissions is just equal to the incremental charge liability it incurs by increasing emissions.

Substituting our approximate expression for the derivative of the cost function (equation (6)) into equation (9), we obtain

$$-(C_i' + \alpha_i(u_i)) - C''(h_i(p_i,u_i) - \hat{e}_i) \doteq p_i$$

which after rearrangement yields the following approximate expression for the functions $h_i(.)$.

$$(10) \qquad \tilde{e}_i(u_i) = h_i(p_i,u_i) \doteq \hat{e}_i - \frac{p_i+C_i'+\alpha_i(u_i)}{C_i''} \quad i = 1,\ldots,N_s$$

These functions give the cost minimizing levels of emissions that will be chosen by each emitter at any charge level. The random term $\alpha_i(u_i)$ which appears in equation (10) reflects the fact that the agency does not know with certainty what level of emissions each emitter will pick because it does not know each emitter's costs with certainty.

The next step is to substitute equation (10) into our approximate expression for costs (i.e. equation (5)), and to evaluate approximate expected total costs. When this is done, we obtain

$$E\{\sum_{i=1}^{N_s} C_i(h_i(p_i,u_i),u_i)\} \doteq E\{\sum_{i=1}^{N_s} \Big[a_i(u_i) + (C_i'+\alpha_i(u_i)) - \left(\frac{p_i+C_i'+\alpha_i(u_i)}{C_i''}\right)$$

$$(11) \qquad + \frac{C_2''}{2}\left(\frac{p_i+C_i'+\alpha_i(u_i)}{C_i''}\right)^2\Big]\}$$

$$= \sum_{i=1}^{N_s} \left[E\{C_i(\hat{e}_i,u_i)\} + \frac{1}{2}\,\frac{p_i^{\,2} - C_i'^{\,2}}{C_i''} - \frac{1}{2}\,\frac{\sigma_i^{\,2}}{C_i''}\right]$$

where $\sigma_i^{\,2} = E\{\alpha_i(u_i)\alpha_i(u_i)\}$.

Using the approximate expression for expected total costs given in equation (11) above, we proceed to find an expression for the set of prices which minimizes this expression and attains expected ambient concentrations approximately equal to the environmental quality standard. This is done by minimizing (11) with respect to the p_i, subject to the environmental quality constraint. The first order necessary conditions for this are

$$\tilde{p}_i = \lambda\check{d}_i \qquad\qquad i = 1,\ldots,N_s$$

$$(12) \qquad \sum_{i=1}^{N_s} \check{d}_i\, E\left\{\hat{e}_i - \frac{\tilde{p}_i+C_i'+\alpha_i(u_i)}{C_i''}\right\} = s$$

where $\check{d}_i = E\{d_i(v_i)\}$. Note that since we require expected ambient pollution concentrations under the emissions standard policy to be equal to the environmental quality standard level, the constraint equation presented in (12) implies that $\tilde{p}_i = -C_i'$. This, in turn, implies (by substituting back into equation (10)) that at the optimal emissions charge rates, the distribution of emissions rates perceived by the agency is

$$(13) \qquad \tilde{e}_i = \hat{e}_i - \frac{\alpha_i(u_i)}{C_i''} \qquad i = 1,\ldots,N_s$$

and that expected costs under the emissions charge policy as given by equation (11) evaluated at $p_i = - C_i'$ are

$$(14) \qquad \sum_{i=1}^{N_s} \left[E\{C_i(\hat{e}_i, u_i)\} - \frac{1}{2} \frac{\sigma_i^2}{C_i''} \right]$$

3.3 Results

Inspection of equation (14) reveals immediately that the expected costs of the emission charges policy is less than the expected cost that would result from the emission standards policy. This can be seen by noting that the expected cost of the emission charges policy is equal to the expected cost of the emission standards policy less a term which depends upon the variances of the intercepts of the marginal cost functions. The difference between the expected cost of an emission standards policy and the expected cost of an emission charges policy is

$$(15) \qquad \Delta \doteq \frac{1}{2} \sum_{i=1}^{N_s} \frac{\sigma_i^2}{C_i''}$$

The expected cost advantage of the optimal charge policy comes at a price, however. The variance of ambient pollution concentration under the emission standards policy is simply

$$(16) \qquad V_e = \sum_{i=1}^{N_s} \check{d}_i \, \hat{e}_i^2$$

where $\check{d}_i$ is the variance of $d_i(v_i)$. In contrast, the variance of ambient pollution concentrations under the emission charges policy can be shown (by substituting the expression for the $\tilde{e}_i$ contained in equation (13) into the constraint equation and evaluating the variance of the resulting expression) to be

$$(17) \qquad V_i = \sum_{i=1}^{N_s} \check{\check{d}}_i \hat{e}_i^{\,2} + \check{\check{d}}_i \frac{\sigma_i^{\,2}}{C_i''^{2}}$$

This is clearly larger than the variance of ambient pollution concentrations under emission standards, and the difference between the two increases with increasing uncertainty about emitters' costs.

Our comparison of emission standards to emission charges thus leads us to the conclusion that an emission charges policy leads to lower expected cost, and less certainty about resulting ambient pollution concentration, than does an emission standards policy. The more uncertain are emitters' cost functions, the larger the cost advantage of an emission charges policy relative to an emission standards policy, and the greater the environmental quality certainty advantage of an emission standards policy relative to an emission charges policy. Any choice between these policies thus rests upon weighing the benefits of cost savings against the benefits of greater certainty concerning environmental quality. This trade-off analysis would have to be made in order to decide which of the two policies--emission standards or emission charges--is better.

It is a relatively simple matter to extend the analysis presented above to consideration of the expected cost and variance of ambient pollution concentrations which would result from a transferable emission permits policy. Under this type of policy, given our assumptions that emitters always know exactly their cost functions and reveal them in bidding for permits, the agency allocates an amount of permits to each emitter, $\bar{e}_i$, such that*

$$(18) \qquad C_i'(\bar{e}_i, u_i) = \lambda \check{d}_i$$

$$\sum_{i=1}^{N_2} d_i \bar{e}_i = s \quad .$$

---

*Equation (18) follows directly from the first order necessary conditions for the problem stated in equation (4) above.

Setting our approximate expression for incremental costs in equation (6) above equal to the right-hand side of equation (18) above and solving for $\bar{e}_i$, we obtain

$$(19) \qquad \bar{e}_i \doteq \hat{e}_i - \frac{\lambda \check{d}_i + C'_i + \alpha_i(u_i)}{C''_i}$$

Note the striking similarity between equations (19) and (10). Indeed, they are precisely the same equation, and a full analysis of equations (18) and (19) paralleling the analysis of equation (12) above leads to the conclusion that ex ante, the optimal allocation $\bar{e}_i$ of permits under the optimal marketable permit system is

$$(20) \qquad \bar{e}_i = \hat{e}_i - \frac{\alpha_i(u_i)}{C''_i}$$

If follows immediately, by substitution back into equation (5) and taking expectations, that the transferable emission permits policy enjoys the same expected cost advantage that is enjoyed by the emission charges policy. That is, expected cost under transferable permits is less than expected cost under emission standards by the amount given in equation (15) above.

In computing the variance of the distribution of ambient pollution concentrations resulting from a transferable emission permit policy we proceed as we did above for the emissions charge policy, with one very important exception. Under the transferable permit policy, the pollution control authority learns the $\alpha_i(u_i)$ from emitters through the bids submitted prior to making its decisions. The $\alpha_i(u_i)$ are thus not random variables at the time the decision is made, and the variance of ambient concentrations is given approximately by equation (16)--the equation which gives the variance of ambient concentrations under the emission standards policy.

This is a most interesting result. If we accept the assumptions upon which it rests, we conclude that both a transferable emission permits and an emission charges policy have cost advantages over an emission standards policy. We also conclude that the variance of ambient pollution concentrations under a transferable permits policy is equal to that under an emission standards policy. Based on our two criteria, expected cost and the precision with which a policy meets environmental quality standard levels, our analysis leads to the conclusion that a transferable permits policy is the best policy instrument to use in order to implement an environmental quality standard.

4. CONCLUDING COMMENTS

The conclusions reached above admittedly depend upon the assumption that control cost functions may be approximated satisfactorily by a quadratic of the form of equation (5). Malcomson (1978) has pointed out that this approximation is not necessarily valid, and that when it is not, conclusions concerning the relative magnitudes of expected costs under different allocations policies may be affected.

This criticism, while its importance is not to be minimized, is perhaps less serious in the context we are considering here. Under the assumption that emitters know and reveal their actual costs in the process of offering to buy and sell emissions permits, we may be assured that whatever the nature of randomness in emitters' cost functions, costs of meeting (in expectation) an environmental quality standard will be minimized. The conclusion that a transferable emission permits policy would lead to the lowest expected cost of the policies considered thus appears to hold, Malcomson's cautions notwithstanding.

Our results concerning the relative expected costs of an emission standards policy and an emission charges policy are not robust with respect to alternative specifications of the way in which random errors enter cost functions. Examples can be constructed in which an emission standards policy would result in a lower expected cost than would an emission charges policy. While one may quibble about the reasonableness of formulations that lead to the conclusion that an emission standards policy would result in lower expected control costs, the logical possibility of such a result cannot (and should not given present uncertainty about pollution control costs) be denied.*

Somewhat less technical--and more telling in the arena of public debate--arguments have been advanced against the use of emission charges or transferable emission permits. Most have concerned practical "difficulties" with the design and administration of a workable transferable permits system. Tietenberg (1979) has reviewed and analyzed these arguments and found them to be, for the most part, without merit.

To be sure, there are a number of practical issues concerning the choice of optimal policy instruments for meeting an environmental quality standard that are not addressed by our formal analysis in Sections 2 and 3 and that also were not at issue in Tietenberg's analysis. Interestingly, consideration of these issues tends to reinforce the conclusion that a system of transferable permits is the best policy instrument for meeting an environmental quality standard.

For example, a particularly difficult problem that cannot be avoided is the accommodation of policy to changing circumstances. Over time the price level changes (i.e. inflation), relative prices change, emitters expand and contract their operations, new emitters seek to enter the area, and technology changes, And, with the passage of time, uncertainty becomes pervasive and potentially disruptive. A transferable emission permits policy deals with this type of problem in a particularly effective and natural way. Simply put, changed circumstances are accommodated by changes in the prices at which permits are transferred. If, for example, there is marked economic growth in an area, this will be reflected in increased demands for emission permits in the area, and other things being equal, an increase in their price. A transferable permits policy thus adjusts automatically to changing circumstances. In contrast, an emission standards policy or an emission changes policy require that the pollution control authority act to adjust the policy in the face of changed conditions.

It is also relatively easy to deal with the additional uncertainty that the time dimension introduces. The easiest way to

---

*Weitzman (1978) makes the case for quibbling.

do this is to allow forward transactions in permits. In this way, emitters can shield themselves from uncertainty in much the same fashion that companies engaging in international trade shield themselves from uncertainty associated with exchange rate fluctuations by buying and selling currencies in forward markets. This provides an automatic way for emitters to cope with uncertainty.

In sum, the transferable emissions permit policy has many desirable features to recommend it. It promises both efficiency and effectiveness. It accommodates change and uncertainty. The practical arguments that have been advanced against this policy for the most part are groundless.

REFERENCES

Adar, Zvi and James M. Griffin, (1976). *Uncertainty and the Choice of Pollution Control Instruments.* Journal of Environmental Economics and Management. 3. pp 178-188.

Anderson, R.J., Reid, R.O., and E.P. Seskin, (1979). *An Analysis of Alternative Policies for Attaining and Maintaining A Short-term* NO_2 *standard.* MATHETECH Inc. September 1979.

Fishelson, G. (1976). *Economic Control Policies under Uncertainty,* Journal of Environmental Economics and Management, 3, pp. 189-197.

Malcomson, J.M., (1978). *Prices vs. Quantities: A Critical Note on the Use of Approximations.* Review of Economic Studies. February, 1978, pp. 203-207.

Tietenberg, T.H. (1979). *Transferable Discharge Permits and the Control of Stationary Sources of Air Pollution: A Survey and Synthesis.* Paper delivered at the International Institute for Environment and Society, Berlin, West Germany, September 27, 1979.

Weitzman, M.L., (1974). *Prices vs. Quantities.* Review of Economic Studies. October 1974, pp. 477-491.

Weitzman, M.L. (1978) *Reply to 'Prices vs. Quantities: A Critical Note on the Use of Approximation'* by James M. Malcomson. Review of Economic Studies, February, 1978, pp. 209-210.

Yohe, G.W. (1976). *Substitution and the Control of Pollution.* Journal of Environmental Economics and Management. 3, pp. 312-324.