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The Probability Distribution of Water Inputs and the Economic Benefits of Supplementary Irrigation

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THE PROBABILITY DISTRIBUTION OF WATER INPUTS AND THE ECONOMIC BENEFITS OF SUPPLEMENTARY IRRIGATION

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PREFACE

Several of REN's studies have examined the effect of risk and uncertainty on the status and management of environmental resources. This paper examines some of the effects of risk and uncertainty on the economics of production processes that depend upon randomly-varying environmental resource inputs. Work on this topic is continuing in the context of REN's studies in its Regional Water Management, Ecological Modeling, and Climate tasks.

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THE PROBABILITY DISTRIBUTION OF WATER INPUTS AND THE ECONOMIC BENEFITS OF SUPPLEMENTARY IRRIGATION

Robert J. Anderson, Jr.

1. INTRODUCTION

The production of a multitude of goods and services in the world's economies are dependent upon randomly-varying environmental resource inputs. Crop production depends upon precipitation, solar radiation, nutrient availability, and a host of other environmental factors. The production of air transportation depends upon precipitation, wind speed, wind velocity, wind variability, visibility, temperature, etc., commercial and sport fishery depend upon randomly varying fish populations. In many production processes, thus, environmental factors are important determinants of the productivity of other man-made inputs, and the economic viability of these activities.

Some controllable inputs into production processes are valuable precisely because they substitute for randomly varying environmental inputs. For example, microwave landing systems reduce the importance of visibility in air transport operations. Supplementary irrigation reduces the variability in water input to crop production.

In recent studies of the demand for supplementary irrigation in Skåne, my colleague Susan Arthur and I have investigated the effects of supplementary irrigation on the probability distributions of total water inputs to crop production over the irrigation season.* Arthur's investigation shows that supplementary irrigation, as it currently is being practiced in Skåne, increases the mean and reduces the variance of the distribution of total seasonal water inputs. My results, based on an economic model of irrigation demand, imply similar effects on the probability distribution of water inputs.

This paper examines the relationship between changes in the probability distribution of water inputs induced by supplementary irrigation and the economic benefits--as measured by increases in expected farm income--derived from supplementary irrigation. The analysis will show that the contribution of supplementary irrigation to expected farm income may be factored approximately into two components. The first component depends upon the effect of supplementary irrigation on the mean of the water input distribution. In general, increases in the mean of the distribution of water inputs result in increases in expected farm income.

The second component depends upon the effect of supplementary irrigation on the variance of the probability distribution of the water input. It will be shown, in particular, that reduced variability of the distribution of water inputs confers economic benefits. Moreover, this conclusion will be shown to hold even if the effect of supplementary irrigation on the mean of the water input distribution is negligible. It will also be shown that

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*Arthur (1980) and Anderson (1980).

this conclusion does not require any special assumptions about farmers' aversion to risk. We will show that farmers who seek to maximize the expected value of farm income (i.e. who are risk neutral) derive benefits (i.e. prefer) probability distributions of water inputs that have relatively low variability to ones that have relatively high variability, other things being equal.

The plan of the paper is as follows. In Section 2, a model is developed which relates the probability distribution of water inputs to the expected (i.e. average) level of farm income. As noted above, this model approximates the relationship between water inputs and farm income in terms of the mean and the variance of the probability distribution of water inputs. Using this representation, we will show that, if yield is a concave function of water input, expected farm income varies inversely with the variance of the probability distribution of water input.

Section 3 shows how the approximating relationship developed in Section 2 can be applied to estimate the effect of supplementary irrigation (or, for that matter, any activity--e.g. climate modification, improved climate forecasting--that reduces the variability of the water input) on farm income. Two illustrations are reported. In the first illustration, the approximation is used to evaluate the effects on farm income of changes in the distribution of water inputs implied by my model of irrigation demand in Skåne.* In this particular illustration, the approximation yields exact results since the production function used relating water inputs to crop yields is quadratic. The second illustration is based on results reported in Arthur (1980) on the effect of supplementary irrigation on water input probability distributions.

*Anderson (1980).

Section 4 offers some concluding remarks and comments about the application of the model developed in Section 2 to other problems. One interesting and immediate application is the resolution of a difficulty in Anderson (1980). In that paper, the estimated contribution of irrigation to expected farm income was overstated since the model assumed that irrigation decisions are made with perfect foreknowledge of the amount of water that would be supplied by precipitation. The approximate calculations of contributions to income based on Arthur's results presented in Section 3 do not depend upon this assumption. They therefore provide a check on the results in Anderson (1980). Section 4 also notes briefly the possible application of the approximation developed in Section 2 to the study of costs and benefits of changes in climate.

2. A MODEL

Let us begin our analysis by adopting the following notation and assumptions:

x = n x 1 vector of inputs controlled by the farmer (e.g. land area planted, fertilizer applied, irrigation water applied, etc.);

c = n x 1 vector of input prices corresponding to x;

$\tilde{w}$ = m x 1 random vector of inputs not controlled by the farmer (e.g. temperature, total water input, solar radiation);

$F(x,\tilde{w})$ = production function for an agricultural commodity, assumed to be such that $F_x = \partial F/\partial x$ and $F_w = \partial F/\partial w$ are positive, $F(\cdot)$ is strictly concave, and $F(\cdot)$ possesses continuous partial derivatives up to the third order;

$\tilde{p}$ = price of the agricultural commodity under consideration, assumed to be a random variable;

$\tilde{\pi}$ = farm income.

The production function, $F(x,\tilde{w})$, and the random vector of productive inputs, $\tilde{w}$, are assumed to be defined over some specific period of time corresponding to some or all of the crop season. As is well known, crop yields depend upon both total quantities of inputs and the intraseasonal distribution of their application. A production function that relates seasonal input totals to crop yield is thus an approximation. An analytical derivation of this approximation is offered in Anderson (1980).

We also assume that farmers choose the inputs under their control (i.e. the vector x) so as to maximize expected farm income. In terms of the notation set forth above, a representative farmer is assumed to select the levels of inputs under his control so as to solve the following problem:

$$\text{Maximize } E\{\tilde{\pi}\} = E\{\tilde{p}F(x,\tilde{w}) - c'x\} \tag{1}$$

where $E\{\cdot\}$ is the mathematical expectation operator.

Direct evaluation of the mathematical expectation required by equation (1) may be difficult or even impossible, depending upon the form of the joint distribution of $(\tilde{p},\tilde{w})$ and the form of the function, $F(\cdot)$. Let us therefore expand the function $\tilde{\pi}$ about the point $\hat{x}$ such that

$$\bar{p}F_x(\hat{x},\bar{w}) - c = o$$

where $\bar{w} = E\{w\}$ and $\bar{p} = E\{\tilde{p}\}$. When this is done, we obtain the following approximate expression for $\tilde{\pi}$,

$$\begin{aligned}\tilde{\pi} \doteq\ & \bar{p}F - c'\hat{x} + (\bar{p}F_x - c)'(x-\hat{x}) + F(p-\bar{p}) + \bar{p}F_w'(w-\bar{w}) \\ & + \tfrac{1}{2}\bar{p}(w-\bar{w})'F_{ww}(w-\bar{w}) + \tfrac{1}{2}\bar{p}(x-\hat{x})'F_{xx}(x-\hat{x}) + \bar{p}(x-\hat{x})'F_{xw}(w-\bar{w}) \\ & + (p-\bar{p})F_w'(w-\bar{w}) + (p-\bar{p})F_x'(x-\hat{x}) \quad . \qquad (2)\end{aligned}$$

where F_{xx} is the matrix whose (i,j) element is $\frac{\partial^2 F}{\partial x_i \partial x_j}$, and similarly for other terms. The function $F(\cdot)$ and its derivatives appearing in equation (2) are all evaluated at the point $(\hat{x},\bar{w})$.

Taking the mathematical expectation of equation (2), we obtain the following approximate expression for expected farm income,

$$\begin{aligned}\bar{\pi} = E\{\tilde{\pi}\} \doteq\ & \bar{p}F - c'\hat{x} + \tfrac{1}{2}\bar{p}\,\mathrm{tr}\{F_{ww}V_{ww}\} + \tfrac{1}{2}\bar{p}(x-\hat{x})'F_{xx}(x-\hat{x}) \\ & + F_w'V_{wp} \qquad (3)\end{aligned}$$

where $V_{ww} = E\{(w-\bar{w})(w-\bar{w})'\}$, $V_{wp} = E\{(p-\bar{p})(w-\bar{w})\}$, and $\mathrm{tr}\{\cdot\}$ is the trace operator.*

The choice of x that maximizes the approximation in equation (3) is obvious by inspection since, by assumption, F_{xx} is negative definite.** Clearly we must set $x = \hat{x}$. At this value of x, we obtain the result that the expected value of farm income at the optimum input levels is given approximately by the following expression.

$$\bar{\pi} \doteq \bar{p}F - c'\hat{x} + \tfrac{1}{2}\bar{p}\ \mathrm{tr}\{F_{ww}V_{ww}\} + F_w'V_{wp} \quad . \qquad (4)$$

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*We have used the fact that $\bar{p}F_x - c = 0$ to eliminate a term from equation (3).

**This follows from the assumption that $F(\cdot)$ is strictly concave.

Examination of equation (4) reveals that our approximation of expected farm income involves terms in the means, variances, and covariances of the random variables in the model. The effect of changes in these parameters on expected farm income may be seen most easily by examination of the total differential of equation (4), which appears below in equation (5)

$$d\bar{\pi} = \bar{p}dF + F\,d\bar{p} - c'd\hat{x} - \hat{x}'dc + \tfrac{1}{2}\bar{p}\,\mathrm{tr}\{F_{ww}dV_{ww}\} + \tfrac{1}{2}\,\mathrm{tr}\{F_{ww}V_{ww}\}d\bar{p} + F_w'dV_{wp} + V_{wp}'\,dF_w \qquad (5)$$

Noting that

$$dF = F_x'd\hat{x} + F_w'd\bar{w}$$

and

$$dF_w = F_{wx}d\hat{x} + F_{ww}d\bar{w}$$

and assuming for the moment that $d\bar{p} = dc = 0$, $dV_{ww} = 0$, and $dV_{wp} = 0$, we have

$$d\pi = \bar{p}F_w'd\bar{w} + V_{wp}'(F_{wx}d\hat{x} + F_{ww}d\bar{w})$$

Note further that $d\hat{x} = -\,F_{xx}^{-1}F_{xw}\,d\bar{w}$

which implies

$$d\pi = \bar{p}F_w'd\bar{w} + V_{wp}'(F_{ww} - F_{wx}F_{xx}^{-1}F_{xw})d\bar{w} \qquad (6)$$

Equation (6) takes the sign of $d\bar{w}$ since F_w is positive and F is concave by assumption.* In other words, provided that $d\bar{p}$ is zero, the effect of increasing $\bar{w}$ is to increase expected profits, $\bar{\pi}$.

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*Concavity of F implies that $F_{ww}-F_{wx}F_{xx}^{-1}F_{xw}$ is negative definite.

Now let us suppose that $d\bar{w}$ is zero and, to be specific about the manner in which V_{ww} and V_{wp} change, that $dV_{ww} = -\alpha V_{ww}$ and $dV_{wp} = -\beta V_{wp}$, where α and β are constants between 0 and 1. The inspection of equation (5) reveal that the change in expected farm income associated with this change is

$$d\bar{\pi} = -\alpha\frac{1}{2}\bar{p}\,tr\{F_{ww}V_{ww}\} - \beta F_w' V_{wp} \tag{7}$$

Since F_{ww} is negative definite by assumption and V_{ww} is positive definite, the first term on the right-hand side of equation (7) is unambiguously positive. In general, the elements of the vector V_{wp} will be non-positive. That is, high values of the vector $\tilde{w}$ will tend to be associated with low values of $\tilde{p}$. Recalling our convention that higher values of $\tilde{w}$ correspond to greater outputs (i.e. the elements of $\tilde{w}$ are inputs with positive marginal products), non-positive covariance between $\tilde{w}$ and $\tilde{p}$ reflects the fact that many times fluctuations in random inputs are large enough to cause large fluctuations in total output, thus affecting market price. In the likely event that V_{wp} is non-positive, the second term on the right-hand side of equation (7) is non-negative. We conclude that the effect of reducing V_{ww} and increasing V_{wp} is, unambiguously, to increase expected farm income.

Equations (4)-(7) summarize an approximate relationship between parameters of the probability distribution of randomly varying inputs and expected farm income. These relationships establish that measures which increase the mean value of randomly varying inputs, or decrease their variances result, in general, in increases in expected farm income.

The derivation of the results and equations presented above depends on the validity of the second order approximation contained in equation (2). Interestingly, the qualitative conclusions presented above concerning the effects of changes in the mean and variance may be shown to hold under a variety of more general assumptions.* We do not explore these more general formulations here since our main interest is the derivation and application of quantitative results based on equations (4) through (7).

3. SOME ILLUSTRATIVE ESTIMATES

Application of the results derived in Section 2 may be readily illustrated using data reported in Arthur (1980) and Anderson (1980) on the demand for supplementary irrigation in Skåne. We begin with an illustration based on the data reported in Anderson (1980).

From equation (4), we see that two kinds of parameters are required to estimate expected farm income. These are selected parameters of the production function relating water inputs to crop yields, and the mean and variance of the water input probability distribution. We shall assume in our calculations that the covariance between water input and crop price (i.e. V_{wp} in equations (4)-(7)) is zero.

Estimates of the parameters of production functions relating the yield per hectare of table potatoes and sugar beets are reported in Table 1. The rationale for this form of production function and the estimation of the parameters are described in Anderson (1980).

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*An excellent summary of more general results is contained in McCall (1971).

Table 1. Estimates of Production Function Parameters

$$y = a_1(I + Br) - a_2(I + Br)^2$$

	a_1	a_2	B	F_{ww}
Potatoes	3.4826	0.0087	0.65	-0.0164
Sugar Beets	4.1707	0.0091	0.75	-0.0182

Notes: I = irrigation
r = precipitation
B = precipitation efficiency parameter

Source: Anderson (1980), Table 3.

Estimates of the means and variances of the probability distributions of water inputs with and without supplementary irrigation are reported in Table 2. These are calculated by taking total and partial expectations with respect to the complete and truncated Weibull densities that characterize water inputs with and without supplementary irrigation, as reported in Anderson (1980).

Table 2. Estimates of Mean and Variance of Probability Distribution of Water Inputs with and without Supplementary Irrigation

	Potatoes		Sugar Beets	
	Mean	Variance	Mean	Variance
Without	110.2	952.6	127.9	1159.8
With	197.2	44.3	214.6	144.9

Source: Computed from data in Anderson (1980), Tables 2 and 6.

All of the data needed to compute expected farm income according to equation (4) are available in Tables 1 and 2. The computations of expected farm income with and without supplementary irrigation according to equation (4) are reported in Table 3. Lines (1) through (4) of the table report various components of the calculation of expected farm income per hectare in the absence of irrigation. Line (1) reports crop yields evaluated at the means (adjusted for relative efficiency) of precipitation. Thus, crop yields evaluated at the mean are 278.1 decitons per hectare and 384.6 decitons per hectare for table potatoes and sugar beets, respectively.

Line (2) reports the effects of variability in the water inputs (adjusted precipitation in this case) on expected yields. As can be seen, the effect of variability in both cases is to reduce mean yields below the levels of yields evaluated at the means shown in line (1). Expected yields are equal to the sums of the figures reported in lines (1) and (2). Line (3) reports the products of expected yields (i.e. the sum of lines (1) and (2)) times prices. The 1978 net farm prices of sugar beets and potatoes in the Skåne region of Sweden were used in this computation. The figures calculated according to this procedure, which represent roughly the contribution of the water input to expected farm income without irrigation, are repeated in line (4).

Lines (5) through (9) report components of the calculation of expected farm income when supplementary irrigation is practiced as described in Anderson (1980). Line (5) reports crop yields evaluated at the means of the total water inputs. As can be seen by comparing the data in line (1) to the data in line (5), supplementary irrigation substantially increases yields evaluated at the means.

Table 3. Calculation of Supplementary Irrigation Contribution to Farm Income based on Anderson (1980)

	Potatoes	Sugar Beets	
(1) F	278.1	384.6	Without Irrigation
(2) $\frac{1}{2}tr\{F_{ww}V_{ww}\}$	- 7.8	- 10.6	
(3) $\bar{p}[(1)+(2)]$	21623.1	5610.7	
(4) $\bar{\pi}_{w/o}$	21623.1	5610.7	
(5) F	348.4	475.9	With Supplementary Irrigation
(6) $\frac{1}{2}tr\{F_{ww}V_{ww}\}$	- 0.4	- 1.3	
(7) $\bar{p}[(5)+(6)]$	27840.0	7118.7	
(8) $c'x$	348.0	346.8	
(9) $\bar{\pi}_w$	27492.0	6771.9	
(10) $\Delta\bar{\Pi}$	5868.9	1161.2	Differences
(11) $\bar{p}[(3)-(6)]$	592.0	139.5	
(12) (11)/(10)x100	10.1	12.0	

Line (6) shows the effect of variability in the total water inputs on expected yields. These effects are still negative, although a comparison of lines (2) and (6) shows that supplementary irrigation reduces the negative effects of variability by reducing the variability of the water inputs.

Line (7) reports expected yields times net farm crop prices. Again 1978 net farm prices were used in calculating these totals.

Line (8) reports the expected variable costs of supplementary irrigation. Unit variable costs are estimated at 4 skr per mm per hectare, as described in Anderson (1980).

Line (9) of the table reports expected farm incomes net of variable costs of irrigation. As can be seen by comparison of columns (4) and (9), the results of supplementary irrigation are substantial increases in expected farm income.

Line (10) reports the increments in expected farm income due to supplementary irrigation. The estimated increments are 5868.9 skr per hectare per year and 1161.2 skr per hectare per year for table potatoes and sugar beets, respectively. These estimates agree closely with the estimates presented in Anderson (1980) of incremental expected incomes of 5942.3 skr per hectare per year and 1186.2 skr per hectare per year, respectively, for table potatoes and sugar beets. Indeed, since the production function relating crop yield to water inputs used in Anderson (1980) is quadratic, the two should coincide exactly. The difference between them is a result of rounding errors in different calculation procedures.

Line (11) reports the contributions to expected farm income associated specifically with reduced variability in the water inputs. Note that these contributions are substantial, amounting to over 10 percent of the total increments to expected farm income, as is reported in Line (12).

The estimates reported in Line (11) also can be interpreted as the increases in expected farm income that would occur if the variances of the water inputs were decreased as described in Table 2, even if mean water inputs were unchanged. Or put it another way, these are the increments to farm income that could be expected if one were somehow to reduce the variability of precipitation to the levels reflected in Table 2.

The use of the approximation described in Section 2 also can be illustrated instructively using results reported in Arthur (1980) on supplementary irrigation in Skåne. Although Arthur's calculations do not assume that farmers behave as if to maximize expected profits and our basic formulae--equations (4) through (7) in Section 2--assume they do, we can still use Arthur's results. The effect of non-optimal input levels (i.e. levels of inputs that do not maximize expected farm income) is to make equation (3) (instead of equation (4)) the appropriate expression to use to evaluate expected farm income. The difference between the two expressions is

$$1/2\ \bar{p}\ (x-\hat{x})'F_{xx}(x-\hat{x}) + (\bar{p}F_x - c)'(x-\hat{x})$$

Since this term is negative in general (recall our assumption that the production function is a concave function of the vector of inputs), use of equation (4) to evaluate expected income when input levels are non-optimal tends to result in an overstatement of the expected income actually realized. The size of the over-statement is given by the above expression in the deviations of input levels from optimal levels. This in turn means that our calculations of absolute levels of expected farm income under supplementary will be biased upward, although there is little reason to suppose that the bias is very large or that the contribution of supplementary irrigation to expected farm income would be seriously overestimated.

Arthur's results on the effects of supplementary irrigation on water inputs are summarized in Table 4. Column (1) of Table 4 reports Arthur's estimates of the mean and variance of the probability distributions of water inputs to potatoes and sugar beets in Skåne, assuming that no irrigation is undertaken. For example, Arthur found that mean precipitation over the period relevant to potato growing was 176 mm and the variance of precipitation over this same period was 3136 mm^2.

Column (2) reports Arthur's estimates of the mean and variance of the probability distribution of water inputs assuming that supplementary irrigation is practiced according to a set of rules described in her paper. The water input for which these statistics are reported, is the sum of precipitation plus irrigation water applied.

Table 4. Summary of Means and Variances of Probability Distribution of Water Inputs

	No Irrigation	Irrigation	Difference
Potatoes			
Mean Water Input	176	274	98
Variance of Water Input	3136	1089	- 2047
Sugar Beets			
Mean Water Input	174	259	85
Variance of Water Input	2601	1156	- 1445

Source: Arthur (1980), p. 13, Table 4.

As can be seen by comparison of the results reported in Columns (1) and (2), the irrigation rules simulated by Arthur increased the means and reduced the variances of the water inputs. Column (3) of Table 4 which reports the differences between corresponding figures in columns (2) and (1), shows this clearly. For example, irrigation of potatoes increased the mean water input by 98 mm per season, and reduced the variance of the water input to potatoes by 2047 mm^2 per season.

Two "adjustments" must be made to put Arthur's results into a farm suitable for application of the formulae developed in Section 2. First, Arthur does not present any data on the effects of irrigation on crop yields. Since estimates of certain parameters of the water input - crop yield relationship are required to perform the calculations explained in Section 2, we shall perform these calculations using the relationships reported in Table 2 above.

Second, the water inputs used to estimate the production parameters reported in Table 2 were calculated by adjusting precipitation by relative efficiency parameters whose values are less than one, and adding adjusted precipitation to irrigation.* Arthur's results pertain to the sum of irrigation plus unadjusted precipitation.

Translation of Arthur's results on the means of the water input distributions into a form compatible with the production relationship in Table 2 is straightforward. This can be accomplished by multiplying the respective efficiency parameters times the mean precipitations reported in Table 4 and adding mean irrigations reported in Table 4.

Obtaining the appropriate estimates of variances is slightly more complex. Table 4 reports σ_r^2, the variances of precipitation, which are proportional to the variances of the water inputs in the "Without Irrigation" cases. The constants of proportionality are the squares of the efficiency parameters.

In the "With Irrigation" case, the variances of the probability distributions of water inputs should be computed according to

$$\sigma_W^2 = \sigma_I^2 + 2B\sigma_{Ir} + B^2\sigma_r^2 \quad ,$$

where σ_W^2 is the variance of the water input when supplementary irrigation is practiced, σ_I^2 is the variance of the quantity

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*The precise calculation was $W = I + Br$, where I is irrigation, r is precipitation, and B is the relative efficiency parameter. This parameter was estimated to be 0.65 for table potatoes, and 0.75 for sugar beets.

of irrigation water added, σ_r^2 is the variance of precipitation, and σ_{Ir} is the covariance between precipitation and irrigation. The data in Table 4 report instead the following quantity for the variances of the water input distributions under supplementary irrigation

$$\sigma_I^2 + 2\sigma_{Ir} + \sigma_r^2 \quad .$$

While it is not possible from the data reported in Arthur (1980) to make an exact calculation of the quantities required to compute the variances of the water input distribution under irrigation, it is possible to obtain lower and upper bounds on the variances of these distributions.

As a lower bound for the variances under irrigation, we propose a level of zero. As extreme as this proposal may seem, it actually may be quite a good approximation. In particular, if an irrigation rule of the form

$$\begin{aligned} I &= T - Br \quad \text{if} \quad Br \leq T \\ &= 0 \qquad\quad\;\, \text{Otherwise} \end{aligned}$$

(where T is some target quantity of water input) is applied, and if the probability that $Br \leq T$ is negligible, then the variance of the water input will, in fact, be approximately zero.*

An approximate upper bound on the variance of the water input under supplementary irrigation may be obtained by subtracting the variance with irrigation from the variance without

*See Anderson (1980) for a discussion of a model in which irrigation quantities are determined as described above.

irrigation (both as reported in Table 4), multiplying the result by B, subtracting $(1-B)B^2\sigma_r^2 + B^2\sigma_r^2$ from this result, and multiplying through by -1. This calculation yields

$$\bar{\Sigma} = B\sigma_I^2 + 2B\sigma_{Ir} + (1-B)B^2\sigma_r^2 + B^2\sigma_r^2 \quad .$$

The rationale for this calculation also is based on the approximation of irrigation demand as the difference between a target level of water input and adjusted precipitation (see above). In particular, given this approximation we know that

$$\sigma_I^2 = \int_0^T (I-\mu_I)^2 h(I)dI = B^2 \int_0^{T/B} (r-\mu_r)^2 g(r)dr \leq B^2 \int_0^{\infty} (r-\mu_r)^2 g(r)dr$$
$$= B^2\sigma_r^2$$

This implies that

$$\sigma_w^2 = \sigma_I^2 + 2B\sigma_{Ir} + B^2\sigma_r^2 \leq \bar{\Sigma}$$

as was to be established.

Table 5 reports calculations based on these relationships, the theory presented in Section 2, and the data in Tables 2 and 4. For ease of comparison, the same quantities are reported in the various lines of Table 5 as reported in the corresponding lines of Table 3. However, some lines have been subdivided to allow for presentation of upper and lower bounds on results that, for reasons explained above, cannot be calculated exactly.

Perhaps the most important conclusion to emerge from examination of the results in Table 5 is the importance of variance reduction as a source of economic benefits. As is shown in the table, reduction in the variance of the probability distributions of water inputs accounts for roughly 10 to 20 percent of the increment to expected farm income associated with supplementary irrigation.

Table 5. Calculation of Supplementary Irrigation Contribution to Farm Income based on Arthur (1980)

		Potatoes	Sugar Beets
(1) F		284.5	389.3
(2) $\frac{1}{2}tr\{F_{ww}V_{ww}\}$		- 10.9	- 13.3
(3) $\bar{p}[(1)+(2)]$		22760.0	5639.8
(4) $\bar{\pi}_w$		22760.0	5639.8
(5) F		347.2	476.2
(6) $\frac{1}{2}tr\{F_{ww}V_{ww}\}$	$\underline{L}$	0	0
	$\underline{U}$	- 3.8	- 6.8
(7) $\bar{p}[(5)+(6)]$	$\underline{L}$	274755.0	7041.3
	$\underline{U}$	27776.0	7143.0
(8) $c'x$		392.0	340.0
(9) $\bar{\pi}_w$	$\underline{L}$	27033.5	6701.3
	$\underline{U}$	27384.0	6803.0
(10) $\Delta\Pi$	$\underline{L}$	4373.5	1061.5
	$\underline{U}$	4624.0	1163.2
(11) $\bar{p}[(5)-(6)]$	$\underline{L}$	568.0	97.5
	$\underline{U}$	872.0	191.5
(12) (11)/(10)x100	$\underline{L}$	13.1	9.2
	$\underline{U}$	18.9	17.2

4. CONCLUDING COMMENTS

The analysis developed in Section 2 and illustrated in Section 3 has a number of interesting applications. For example, in my earlier analysis of supplementary irrigation in Skåne (and in the first illustration presented in Section 3), the contribution of expected farm income was calculated based on the assumption that farmers were perfectly certain about the quantity of water input that would be supplied by precipitation. The calculations presented in Section 3 based on Arthur's results do not rest explicitly on this assumption.*

Interestingly, the results of these calculations are not terribly dissimilar. Estimates of contribution of supplementary irrigation to farm income based on Arthur's results generally imply smaller benefits than do those based on my earlier analysis. This is to be expected since Arthur's analysis did not assume that farmers behave as if to maximize farm income.

Another interesting application of the analysis developed here is to the study of economic effects of climate change. Virtually all analyses of the economic effects of changing climates have concentrated on effects induced by changes in the mean values of various climatic parameters such as precipitation, temperature, and solar radiation. Yet the possibility exists that climate changes may also involve higher moments of the

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*The approximation of irrigation demand as the difference between a target level of water input and precipitation <u>may</u> implicitly introduce some presupposition of foreknowledge <u>into</u> the analysis. I am not sure. It should be noted however that this assumption is not a crucial part of the analysis developed in Section 2. It is necessitated by the fact that Arthur (1980) does not report exactly the quantities required by the analysis described in Section 2, and this assumption provides a basis for approximating the required magnitudes from the data available.

probability distributions of climatic outcomes. The analysis presented above implies that, depending upon the direction of change of these parameters, benefits or disbenefits may follow, even if the means of the distributions were unchanged. For example, the analysis presented above implied that a reduction in the variance of precipitation, other things being equal, would result in an increase in expected crop yields, and an increase in expected farm income. Moreover, such increases could be quite large.

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