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Energy Resources: Economy Development Models

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INTRODUCTION

This paper is an attempt to extend methodological research of complex systems development at IIASA. The most typical and probably most urgent example is analysis and planning of long-range development of energy systems. During the last decade interest in energy problems has considerably increased all over the world and we are now witnesses of significant progress in the field [1 - 12]. This concerns however mostly the particular implementation of different energy models. As for methodological mathematical analysis of the problem we deal here, at initial stages, with inevitable lags, though first attempts have already been made in this direction (see, for example, [13,14].

Meanwhile, in analyzing outputs of energy models implemented in different ways, many methodological problems are arising: for example, how to link energy supply, resources and economy models into a whole system? what is the world ("global") energy model: is that game-theoretical, optimization or simulation? how does our uncertainty in the "future" input data influence our "certainty" of present decisions? etc. These questions do not only relate to energy models but are also of concern for any problems of long range development of a complex system [15,16] (for example, analysis of interaction of manpower-economy development in the long run [17]).

IIASA seems to be an unique place for stimulating such kind of methodological work. Different approaches, different opinions, different models, which are under permanent discussion or investigation at IIASA -- all of this eventually and inevitably becomes a point of view, a starting point of any methodology.

This paper, as mentioned above, should be considered as an initial attempt in this direction. To start with we describe three basic dynamic optimization models: energy supply, resources and economy development models. These models are formalized in the framework of dynamic linear programming [18,19].

In describing these models we try to draw out the typical features of different models, omitting the details of particular implementation in order to obtain three basic formalized models -- energy - resources - economy which could be useful for subsequent mathematical analysis. Therefore the structure of the paper directly follows the goals of the paper. In each of three sections we first consider a basic model and then some real models, which relate to the basic model viewed as modifications of this basic model.

The models are considered independently on a national (regional) level. The linkage of models (e.g. energy-economy) is discussed in the fourth section.

1. Energy Supply Models

We start with Energy Supply Systems (ESS) for the reason that it plays a central role in any energy resources studies.

The main purpose of the ESS model is to study major energy options over the next 25-50 years and longer thus determining the optimal-feasible transition from the mix of technologies for energy production currently used (fossil), to a more progressive and, in some sense, optimal, future mixture of technologies (nuclear, coal, solar, etc.) for a given region (country).

Considering ESS models we will basically follow the Häfele-Manne model [3]. Then different versions and modifications of the ESS models will be discussed.

In formulating DLP problems, it is useful to single out [19]:

(i) *state equations* of the systems with the distinct separation of *state* and *control* variables;

(ii) *constraints* imposed on these variables;

(iii) *planning period* T - the number of time periods during which the system is considered and the *length* of each time period;

(iv) *performance index* (objective function) which quantifies the quality of a program.

We will consider these four stages separately as applied to the ESS model.

1.1. Basic Model

a. State Equations

The ESS model is broken down into two subsystems: energy production and resource consumption subsystems. Hence two sets of state equations are needed.

Energy Production and Conversion Subsystem. The subsystem consists of a certain number of technologies for energy production (fossil, nuclear, solar, etc.). The state of the subsystem at each time period t is described by the values of capacities in that period t for all energy production technologies.

Let

$y_i(t)$ be the value of the ith energy production capacity ($i = 1,\ldots,n$) in time period t;

n be the total number of different technologies for energy production to be considered in the model;

$v_i(t)$ be the increase of the ith capacity in time period t ($i = 1,\ldots,$).

It is assumed that a life-time of each capacity is limited and constitutes τ_i for the ith capacity.

Thus the state equations, which describe the development of the energy production and conversion subsystem will be the following:

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t-\tau_i) \quad (i=1,\ldots,n \quad t=0,1,\ldots,T-1) \tag{1.1}$$

with the given initial conditions

$$y_i(0) = y_i^0 \quad (i = 1,\ldots,n) \quad . \tag{1.2}$$

The increase of the new capacities $v_i(t)$ in preplanning period ($t < 0$) is also assumed to be known:

$$v_i(-\tau_i) = v_i^0(-\tau_i),\ldots,v_i(-1) = v_i^0(-1) \quad (i = 1,\ldots,n) \tag{1.3}$$

where $\{v_i^0(-\tau_i),\ldots,v_i^0(-1)\}$ are given numbers.

Equations (1.1) can be rewritten in a vector form

$$y(t+1) = y(t) + v(t) - v(t-\tau) \tag{1.1a}$$

$$y(0) = y^0 \tag{1.2a}$$

Here

$y(t) = \{y_i(t)\}$ $(i = 1,\ldots,n)$ is a *state vector* of the subsystem in time period t; it describes the state of the energy production and conversion subsystem in this period, and

$v(t) = \{v_i(t)\}$ $(i = 1,\ldots,n)$ is a *control vector*; it describes control actions in time period t; $\tau = \{\tau_i\}$.

Resources Consumption Subsystem. State equations of this subsystem describe the dynamics of cumulative amounts of extracted primary energy resources.

Let

$z_j(t)$ be the cumulative amount of the jth resource extracted by the beginning of time period (year) t, $(j = 1,\ldots,m)$;

m be the total number of different primary resources under consideration.

$q_{ji}(t)$ be the ratio of the amount of the jth resource (primary energy input) required for loading the ith energy production capacity (secondary energy output) in time period t $(i = 1,\ldots,n;\ j = 1,\ldots,m)$; $q_{ji}(t)$ is the conversion process $i \to j$.

Generally, some capacities may not be completely loaded; therefore we introduce a new variable $u_i(t)$ which is the intensity of production for the ith capacity $(i = 1,\ldots,n)$ in time period t;

Evidently,

$$u_i(t) \leq y_i(t) \qquad (i = 1,\ldots,n) \tag{1.4}$$

or

$$u(t) \leq y(t) \quad . \tag{1.4a}$$

Supposing the primary energy resource extraction in time period t is proportional to the value of intensities of energy production in this period we can write the state equations in the form

$$z_j(t+1) = z_j(t) + \sum_{i=1}^{n} q_{ji}(t)\, u_i(t) \tag{1.5}$$

with intial conditions

$$z_j(0) = z_j^0 \qquad (j = 1,\ldots,m) \quad , \tag{1.6}$$

or in matrix form

$$z(t+1) = z(t) + Q(t)u(t) \quad , \tag{1.5a}$$

$$z(0) = z^0 \quad . \tag{1.6a}$$

Here $z(t)$ is a state vector, $u(t)$ is a control vector. The linkage of the subsystems (1.1) and (1.5) is carried out by means of inequalities (1.4).

In some cases it is necessary to introduce stocks of the extracted primary resources (inventory resources). Let $\tilde{z}_j(t)$ be such a variable for the jth resource and $w_j(t)$ is the annual extraction of this resource; then the state equation for the inventory subsystem will be the following:

$$\tilde{z}(t+1) = \tilde{z}(t) + w(t) - Q(t)u(t) \quad .$$

In the above case $\tilde{z}(t) = 0$ for all t and $w(t) = Q(t)u(t)$. This is a reasonable assumption because one can neglect the accumulation of stocks of resources for long-range considerations.

It should be noted that the real equations of resource consumption subsystem are more complex (see references and discussion at the end of this section).

b. Constraints

The state equations (1.1) and (1.5) determine dynamic constraints on variables. We also have static constraints on variables for each time period t.

Nonnegativity Constraints. Evidently, all variables introduced into the state equations (1.1) and (1.5) cannot be negative:

$$v(t) \geq 0 \quad , \quad y(t) \geq 0 \quad , \quad u(t) \geq 0 \quad , \quad z(t) \geq 0 \ . \qquad (1.7)$$

Availability Constraints. First, the upper bounds should be imposed on the annual construction rates

$$v_i(t) \leq \bar{v}_i(t) \qquad (i = 1,\ldots,n) \quad , \qquad (1.8)$$

where $\bar{v}_i(t)$ are the given numbers.

In a more general form these constraints can be written as

$$F(t)v(t) \leq f(t) \qquad (1.9)$$

where f(t) is the vector of non-energy inputs which are needed for the energy production subsystem. The matrix F(t) denotes the amounts of these resources required for the construction of a unit of the ith capacity in time period t. Bounds on new technology introduction rates can also be written in the form (1.9). More general cases where the time lags between investment decisions and actual capacities increases are taken into account are considered in Section 3.1. In this case we can directly link the ESS model with the economic model described in Section 3.

The constraints on the availability of the primary energy resources can be given in the form:

$$z(t) \leq \bar{z}(t) \qquad (1.10)$$

where $\bar{z}(t)$ is the vector of all available energy resources (resources in the ground) in time period t.

The constraints on the availability of the secondary energy production capacities are given by (1.4).

Demand Constraints. The intermediate and final demands of energy are supposed to be given for all planning periods. Hence the demand constraints can be written as

$$\sum_{i=1}^{n} d_{ki}(t)\, u_i(t) \geq d_k(t) \tag{1.11}$$

or

$$D(t)\, u(t) \geq d(t) \tag{1.11a}$$

$d(t) = \{d_k(t)\}$ is the given vector for all $t = 0,1,\ldots,T-1$ of energy demand, both intermediate and final (e.g., electricity and nonelectric energy for final demand);

$D(t) = \{d_{ki}(t)\}$ is the matrix with the components $d_{ki}(t)$, defining either intermediate consumption of the secondary energy k per unit of the secondary energy production or conversion efficiency of capacity i to produce a unit of the secondary energy k.

c. Planning Period

The planning period is broken down into T steps where T is given exogeneously. Each step contains a certain number of years (e.g. one, three, five). In [3] the planning period equals 75 years and each step corresponds to three years, thus T = 25. Since information on the coefficients of the model becomes more inaccurate with the increasing number of steps it is useful to consider steps which have different length. For example, in [20] the planning period is 100 years and T is equal to 10 periods (five periods six years each, the next three periods ten years each and the last two periods twenty years each.)

d. Objective Function

The choice of the objective function is one of the important stages in model building. Discussion of economic aspects of ESS modelling objectives comes out of the framework of this paper. Here we would like specifically to underline only two points: 1) in many cases the objective functions can be expressed as linear functions of state and control variables, thus making it possible to use LP techniques. 2) The optimization procedure should not be viewed as a final one in the planning process (yielding an "unique" optimal solution), but only as a tool for analyzing the connection between policy alternatives and system performance. Thus in practical applications the policy analysis with different objective functions is required. For our purpose it is sufficient however to limit ourselves by some typical examples of objectives.

Below we consider the objective function which expresses the total capital costs both for operation and construction, discounted over time:

$$J = \sum_{t=0}^{T-1} \beta(t) \left[\sum_{i=1}^{n} c_i^u(t)\, u_i(t) + \sum_{i=1}^{n} c_i^v(t)\, v_i(t) \right] , \qquad (1.12)$$

where

$c_i^u(t)$ are the operating and maintenance costs for the ith capacity in time period t;

$c_i^v(t)$ are the investment costs for the ith capacity in time period t;

$\beta(t)$ is the discount rate.

In vector form,

$$J = \sum_{t=0}^{T-1} \beta(t) \left[(c^u(t), u(t) + (c^v(t), v(t)) \right] . \qquad (1.12a)$$

It should be noted that the term $(c^u(t), u(t))$ expresses not only direct operating and maintenance costs at step t but also may indirectly include the cost for primary resources consumed at this step. In a more explicit way this cost can be written as $(c^u(t),Q(t)u(t))$, where $c^u(t)$ should increase with the cumulative amount of resources being consumed. This leads to a nonlinear objective function. A reasonable approximation in this case is a step-wise function for $c^u(t)$. Thus, $c^u(t)$ in (1.12) can be a step-wise function with values on each step depending on the values of cumulative extraction resources $z(t)$ (or on the difference $\bar{z}(t) - z(t)$).

e. Statement of the Problem

First we introduce definitions.

A sequence of vectors

$$v = \{v(0),\ldots,v(T-1)\} \quad , \quad u = \{u(0),\ldots,u(T-1)\}$$

are *controls* of the system.

A sequence of vectors

$$y = \{y(0),\ldots,y(T)\}$$

determined by (1.1,1.2) is a *(capacities) trajectory* of the system; a sequence of vectors

$$z = \{z(0),\ldots,z(T)\} \quad ,$$

determined by (1.5,1.6) is a *(cumulative resources) trajectory* of the system.

Sequences of vectors $\{v,u,y,z\}$, which satisfy all constraints of the problem (e.g. (1.1 - 1.11) in the case) are *feasible*. Choosing a feasible controls v and u one can obtain by (1.1-1.3) and (1.5,1.6) feasible trajectories y and z and compute the value of objective function (1.12). Thus,

$$J = J(y(0),z(0),v,u) = J(v,u) \quad . \tag{1.13}$$

A feasible control $\{v^*,u^*\}$, which minimizes the (1.12) or (1.13), we will call an *optimal control*.

Now we can formulate the optimization problem for the energy supply system.

Problem 1.1. Given the state equations

$$y(t+1) = y(t) + v(t) - v(t-\tau) \qquad (1.1a)$$

$$z(t+1) = z(t) + Q(t)u(t) \qquad (1.5a)$$

with initial conditions

$$y(0) = y^0 \qquad (1.2a)$$

$$z(0) = z^0 \qquad (1.6a)$$

and known parameters

$$v(-\tau) = v^0(-\tau), \dots, v(-1) = v^0(-1) \quad . \qquad (1.3)$$

Find controls $\{v,u\}$ and corresponding trajectories $\{y,z\}$, which satisfy the constraints $v(t) \geq 0$; $u(t) \geq 0$; $y(t) \geq 0$; $z(t) \geq 0$

$$F(t)v(t) \leq f(t) \qquad (1.9)$$

$$u(t) \leq y(t) \qquad (1.4)$$

$$z(t) \leq \bar{z}(t) \qquad (1.10)$$

$$D(t)u(t) \geq d(t) \qquad (1.11a)$$

and minimize the objective function

$$J(v,u) = \sum_{t=0}^{T-1} \beta(t) \left[(c^u(t), u(t)) + (c^v(t), v(t)) \right] \qquad (1.12a)$$

Verbally, the policy analysis in the energy supply system model, which is formalized as Problem 1.1, can be stated as follows.

At the beginning of the planning period energy production capacities (1.2a) are known, they are broken down to several "homogeneous" technologies (fossil, nuclear, solar, etc.). There are different options of developing these initial energy production capacities in the system during the considered period. These options are subject to constraints on primary energy resources availability (1.5a,1.6a,1.10) and constraints on non-energy resources (1.9), required for the construction of new energy production capacities. Each of these options has its own advantages and disadvantages. The problem is to find an optimal mix of these options, which

- is balanced over all advantages and disadvantages of each individual option and phased over time;
- meets the given demand in secondary energy (1.11a);
- minimizes the total operational and construction expenditures (1.12a).

There are two important vector-parameters in the model, which are given exogenously: non-energy resources f(t), available within the planning period and the demand for secondary energy d(t). These values determine mainly the interaction of the energy supply system with the economy development system (see Section 4).

1.2. Discussion

Above a simplified version of the energy supply system (ESS) model was considered, which reveals however the major features of real systems. The particular implementation of the ESS models is naturally more detailed and complicated, and depends to a great extent on the general approach selected for the whole ESS model and on energy and economic assumptions used for building its separate submodels. We will not, however, concern the physical peculiarities of particular ESS models but try to underline below the methodological specifics of the ESS models and their relations to Problem 1.1. It should be noted that notations are changed below compared to the original versions of the models in order to facilitate analysis and comparison of the models.

a. Häfele-Manne Model [3,21]. First of all, there is no division between old and new capacities in the model described above. All capacities are divided into two groups: new technologies for which additional capacities are being constructed during planning horizon and "old" ones. We denote by $y(t) = \{y_i(t)\}$ $(i = 1,\ldots,n)$ and $y_0(t) = \{y_{0_i}(t)\}$ $(i = 1,\ldots,n_0)$ the vectors of new and old capacities. Secondly, a total loading of capacities is assumed in [3]; that is

$$u(t) = y(t) \quad . \tag{1.13}$$

Thus, in this case, the state equations for the energy resources consumption subsystem have the form

$$z(t+1) = z(t) + A[y(t) + y_0(t)] \tag{1.14}$$

for coal and petrogas, and

$$\begin{aligned} z(t+1) = z(t) &+ A_1y(t+1) + Ay(t) + B_1v(t+2) \\ &+ B_2v(t+1) + B_3v(t) - B_4v(t-\tau) \end{aligned} \tag{1.15}$$

for natural uranium and plutonium.

Demand constraints in [3] are written in the form

$$Dy(t) + D_0y_0(t) \geq d(t) \tag{1.16}$$

for final demand and

$$D_1y(t) + D_2[v(t+1) - v(t-\tau)] \geq 0 \tag{1.17}$$

for intermediate demand.

In [3] the objective function is considered in a linear form, similar to (1.12) (model societies 1 and 2) and in non-linear form:

$$J = \sum_{t=0}^{T-1} [a_1(t)d_1^{b_1}(t) + a_2(t)d_2^{b_2}(t)] \quad (1.18)$$

(model society 3). In the latter case it is supposed that demands ($d_1(t)$ for electric and $d_2(t)$ for nonelectric energy) are responsive to price and hence endogenously determined in the model.

b. *ETA Model* [22,23]. The ETA: a model for energy technology assessment is closely related to energy supply system models considered above. The model was developed by A. Manne and represents further development of the nonlinear version (society 3) of the Häfele-Manne model [3]. ETA is a medium-size nonlinear programming model (with linear constraints). It contains for 15 stages planning horizon (each 5 years long) altogether 300 rows, 700 columns and 2500 nonzero matrix elements and requires on IBM 370/168 70 seconds to solve one case and 30 seconds for each subsequent case. The problems were solved by a reduced gradient algorithm by B. Murtagh and M. Saunders.

Formally, ETA model constraints have the form of (1.1-1.3), (1.13 - 1.17). The objective function may be viewed in either of two equivalent ways: maximizing the sum of consumers' plus producers' surplus, or minimizing the sum of the costs of conservations plus interfuel substitution plus the costs of energy supply. In the latter case it is a combination of (1.12) and (1.18). The result of this objective function is that ETA automatically allows for price-induced conservation and also for interfuel substitution.

c. *MESSAGE* [24,25]. The models considered above (Problem 1.1) are formalized as DLP models of general type (one-index models). By introducing energy flows (from supply points to demand points) we come to a dynamical LP model of the transportation type (two-indices). MESSAGE and DESOM energy models can be written in this form.

MESSAGE was developed by A. Voss, M. Agnew and L. Schrattenholzer at IIASA as an extension of the W. Häfele - A. Manne model. The model differs from its predecessors [3,21] by inclusion of all allocated secondary energy to end users; an increased number of supply technologies; distinction between different price categories of natural resources and by adding costs of resources extracted to the objective function.

Below we consider a simplified version of the MESSAGE model.

Let

$x_{ji}(t)$ be the energy flows in time period t from supply category j to demand category i.

Then, as usual in the transportation problem, we can define: the supply of energy of type i, that is the intensity $u_i(t)$ of the ith production capacity in time period t

$$u_i(t) = \sum_j \alpha_{ji} x_{ji}(t) \qquad (1.19)$$

where

α_{ji} specifies the production of secondary energy type i per unit of primary energy resource j.

The consumption $w_j(t)$ of primary energy resource j in time period t is

$$w_j(t) = \sum_i \beta_{ji} x_{ji}(t) \qquad (1.20)$$

where

β_{ji} specifies the technical efficiency of a conversion technology for energy flow from primary resource j to secondary energy i.

The dynamics of the secondary energy production subsystem and primary energy (resource) consumption subsystem are described in a conventional way (cf. (1.1) and (1.5)):

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t-\tau_i) \tag{1.21}$$

$$z_j(t+1) = z_j(t) + w_j(t) \tag{1.22}$$

where we use the same notations as in (1.1) and (1.5), annual consumption $\tilde{w}_j(t)$ is defined from (1.20), and

$$u_i(t) \leq y_i(t) \tag{1.23}$$

where $u_i(t)$ is defined from (1.19).

In addition we have demand constraints

$$\sum_j x_{ji}(t) \geq d_i(t) \tag{1.24}$$

where

$d_i(t)$ is the given exogenously demand for secondary energy i in time period t.

Taking into account some additional constraints on variables which are of the same form as for Problem 1.1, we can finally formulate the following DLP problem.

Problem 1.2. Given the state equations

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t-\tau_i) \qquad (i = 1,\ldots,n)$$

$$z_j(t+1) = z_j(t) + w_j(t) \qquad (j = 1,\ldots,m)$$

with initial conditions

$$y_i(0) = y_i^0 \;;\quad z_j(0) = z_j^0 \;;\quad v_i(t-\tau_i) = v_i^0(t-\tau_i)$$
$$(i = 1,\ldots,n\,;\; j = 1,\ldots,m\,;\; t < \tau_i) \;.$$

Find controls $\{x_{ji}(t)\}$, $\{v_i(t)\}$ and corresponding state $\{y_i(t)\}$, $\{z_j(t)\}$ variables which satisfy the conditions

$$u_i(t) = \sum_j \alpha_{ji}\, x_{ji}(t) \leq y_i(t)$$

$$w_i(t) = \sum_j \beta_{ji}(t)\, x_{ji}(t) \quad ; \quad z_j(t) \le \bar{z}_j$$

$$\sum_j x_{ji}(t) \ge d_i(t)$$

$$v_i(t) \le \bar{v}_i(t)$$

$$v_i(t) \ge 0 \quad ; \quad x_{ji}(t) \ge 0 \quad ; \quad y_i(t) \ge 0 \quad ; \quad z_j(t) \ge 0$$

and minimize the objective function

$$J = \sum_{t=0}^{T-1} \beta(t) \left[\sum_{i=1}^{n} c_i^1 u_i(t) + \sum_{i=1}^{n} c_i^2 v_i(t) + \sum_{j=1}^{m} c_j^3 w_j(t) \right] . \quad (1.25)$$

It should be noted that Problem 1.2 is only a simplified version of MESSAGE. The real model includes different price categories m for primary energy resources:

$$z_i(t) = \sum_m z_{im}(t) \quad ,$$

distinction between primary and secondary conversion processes:

$$y_i^{(1)}(t) \quad ; \quad v_i^{(1)}(t) \quad \text{and} \quad y_i^{(2)}(t) \quad , \quad v_i^{(2)}(t)$$

and other conditions.

The typical dimension of the MESSAGE model is the following. Planning period T is equal to 13 time periods (65 years divided by 5 years time periods). The number of demand constraints is $7 \times T$, the number of resources constraints is $5 \times T$, the number of total resources availability constraints 17×1, resources extraction intensity constraints $2 \times T$, capacity loading constraints $35 \times T$, the number of equations for capital stocks $35 \times T$ and the number of capacity loading constraints $5 \times T$. All this gives us in terms of conventional LP problems about 1097 rows and 1202 columns, with some 90 constraints for each time period.

d. *DESOM* [20]. DESOM (Dynamic Energy System Optimization Model) was developed in the Brookhaven National Laboratory and is an extension of the Brookhaven Energy System Optimization Model (BESOM) which was a static, single period LP model. In DESOM the demand sector has been disaggregated into technology related end uses (22 mutually exclusive end uses as defined by their energy conversion processes). The general structure of DESOM is similar to Problem 1.2.

Let us consider the state equations for capacities development in the form

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t-\tau_i) - v_{0i}(t) \qquad (1.26)$$

where the meaning of control $v_i(t)$ and state $y_i(t)$ variables is the same as in (1.1); $v_{0i}(t)$ is the exogenously given decrease of existing (old) capacities in time period t.

In [20] a scenario variable $\alpha(t)$ is introduced which restricts the growth rate of capacities:

$$y_i(t+1) \leq \alpha(t) y_i(t) \quad . \qquad (1.27)$$

Generally $\alpha(t)$ are greater than 1, which implies that installed capacity may expand in time period t; if $\alpha(t) < 1$ the capacity will decrease in time period t.

Using (1.26) one can rewrite the inequality (1.27) in the following form which is similar to the inequality given in [20]:

$$y_{0i}(t+1) + \sum_{g=t-\tau_i}^{t} v_i(g) \leq \alpha(t) \left[y_{0i}(t) + \sum_{g=t-1-\tau_i}^{t-1} v_i(g) \right]$$

where

$$y_{0i}(t) = y_i(0) - \sum_{g=0}^{t-1} v_{0i}(g)$$

is the inherited capacity (capital stock of old capacities) for conversion process i in time period t (given exogenously).

To link the production subsystem with the resources consumption subsystem, demand and other constraints on intermediate energy flows are introduced in [20]. Let

$x_{k\ell}(t)$ be the amount of energy flow in time period t from supply category k to meet energy demand ℓ.

Then we can define

$$u_i(t) = \frac{1}{\Delta} \sum_{(k,\ell) \in \Omega(i)} \frac{x_{k\ell}(t)}{r_{k\ell}} \qquad (1.28)$$

where

$r_{k\ell}$ is the load factor for intermediate energy flow from supply category k to demand category ℓ;

Δ is the length of time period; generally, $\Delta = \Delta(t)$;

$\Omega(i)$ is the set of indices (k,ℓ), which defines the path of intermediate energy flow from supply k to demand ℓ;

$u_i(t)$ is the amount of installed capacity for conversion process i required in time period t to deliver $x_{k\ell}(t)$; that is, $u_i(t)$ being the intensity of conversion process i in time period t.

Evidently, the amount of installed capacity to be available in time period t must be sufficient to produce intermediate energy flows utilizing capacity for conversion process i in time period t:

$$\frac{1}{\Delta} \sum_{(k,\ell) \in \Omega(i)} \frac{x_{k\ell}(t)}{r_{k\ell}} \leq y_i(t) \qquad (1.29)$$

which is similar to (1.4).

Off-peak electrical intermediate energy flows that use capacity installed for peak requirements are not included in (1.29). In this case

$$\frac{1}{\Delta} \sum_{(k,\ell) \in \Omega(i)} x_{k\ell}(t) \leq q_i y_i(t) \qquad (1.30)$$

where q_i is an overall load factor applied to all electrical capacity, saying that a conversion facility of type i can only operate q_i proportion of the time.

Introducing intermediate energy flows variables allows to write down demand and resource constraints.

The amount of energy from intermediate energy flows $x_{k\ell}(t)$ must be sufficient to meet demands $d_\ell(t)$:

$$\sum_k x_{k\ell}(t) = d_\ell(t)$$

for each demand category ℓ.

On the other hand, intermediate energy flows $x_{k\ell}(t)$ in time period t define a demand for primary energy resource j:

$$\sum_{k,\ell} s_{jk\ell} x_{k\ell}(t) = w_j(t) \quad , \qquad (1.31)$$

where

$s_{jk\ell}$ are supply coefficients representing the technical efficiency of conversion technology for intermediate energy flow of resource j from supply k to demand ℓ;

$w_j(t)$ is the amount of resource j to be used in time period t.

Introducing the cumulative amount $z_j(t)$ of resource j extracted till the end of time period t, one can write the state equation for the resource consumption subsystem in the form

$$z_j(t+1) = z_j(t) + w_j(t) \quad ; \quad z_j(0) = z_j^0 \qquad (1.32)$$

which is similar to (1.5).

Evidently,

$$z_j(t+1) = z_j(0) + \sum_{g=0}^{t} w_j(g) \quad .$$

In [20] there are upper and lower limits on cumulative resources:

$$\underline{z}_j \leq z_j(t) \leq \overline{z}_j \quad . \tag{1.33}$$

$\overline{z}_j$ is associated with the real world resource j availability; the lower limit $\underline{z}_j$ assures some minimum consumption.

In addition to (1.33) in [20] there is a restriction on the growth rate of resources extraction, that is the amount of resource j to be extracted in time period t+1 is no greater than $\beta_j(t)$ times the amount of resource j to be extracted in time period t:

$$w_j(t+1) \leq \beta_j(t)\, w_j(t) \quad . \tag{1.34}$$

Generally $\beta_j(t) > 1$; to phase out a resource over time one can set $\beta_j(t) \leq 1$ for t in later time periods.

Like in other models there are environmental constraints in [20]. They are written in the form

$$\sum_{k,\ell} e_{k\ell m}\, x_{k\ell}(t) \leq E_m(t) \tag{1.35}$$

where

$e_{k\ell m}$ is the amount of emission of type m for intermediate energy flow from k to ℓ;

$E_m(t)$ is the maximum permissible quantity of emissions of type m in time period t.

The objective function of the problem is to minimize the total discounted cost, i.e.

$$J = \sum_{t=0}^{T-1} \gamma(t) \left[\sum_{k\ell} c^1_{k\ell}(t)\, x_{k\ell}(t) + \sum_i c^2_i(t)\, v_i(t) + \sum_j c^3_j(t) w_j(t) \right] \tag{1.36}$$

where

$c^1_{k\ell}(t)$ is the cost for intermediate energy flows (undiscounted);

$c^2_i(t)$ is the cost per year for capacity to be built in time period t for conversion process i;

$c^3_j(t)$ is the cost for resource j in time period t.

Provision for recapturing the remaining life for the variables $v_i(t)$ in the last time period is incorporated in the model but is not shown in (1.36).

Thus the optimization problem for the DESOM model can be formulated as follows:

Problem 1.3. Given the state equations

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t-\tau_i) - v_{0i}(t)$$

$$z_j(t+1) = z_j(t) + w_j(t)$$

with initial states

$$y_i(0) = y_i^0$$

$$z_j(0) = z_j^0$$

and known parameters

$$v(-\tau_i) = v^0(-\tau_i), \ldots, v(-1) = v^0(-1) \quad ;$$

$$v_{0i}(t),\ t = 0,1,\ldots,T-1 \quad ,$$

find controls $\{v_i(t)\}$, $\{w_j(t)\}$, $\{x_{k\ell}(t)\}$, and corresponding trajectories $\{y_i(t)\}$, $\{z_j(t)\}$, which satisfy the constraints

$$v_i(t) \geq 0 \quad , \quad x_{k\ell}(t) \geq 0 \quad , \quad y_i(t) \geq 0 \quad ; \quad z_j(t) \geq 0$$

$$\sum_k x_{k\ell}(t) = d_\ell(t)$$

$$\sum_{k,\ell} s_{jk\ell}\, x_{k\ell}(t) = w_j(t)$$

$$\frac{1}{\Delta} \sum_{k,\ell} \frac{x_{k\ell}(t)}{r_{k\ell}} \leq y_i(t) \quad ; \qquad i \in I^1$$

$$\frac{1}{\Delta} \sum_{k,\ell} x_{k\ell}(t) \leq q_i y_i(t) \quad , \qquad i \in I^2$$

$$\underline{z}_j(t) \leq z_j(t) \leq \overline{z}_j(t)$$

$$y_i(t+1) \leq \alpha(t)\, y_i(t)$$

$$w_j(t+1) \leq \beta_j(t)\, w_j(t)$$

and minimize the objective function

$$J = \sum_{t=0}^{T-1} \gamma(t) \left[\sum_{k\ell} c^1_{k\ell}(t)\, x_{k\ell}(t) + \sum_i c^2_i(t)\, v_i(t) + \sum_j c^3_j(t)\, w_j(t) \right] .$$

Considering Problem 1.3, one can see that it is very close to those considered above (excluding the special way of introducing the intermediate flows $x_{k\ell}(t)$).

As reported in [20] the model without environmental constraints had 130 rows constraints and 750 variables per time period. The first version of the model contains four periods optimization problem and it takes about 30 minutes on the IBM 370/155 to solve it. The standard base case is being developed. This case will cover a 100-year period from 1973 to 2073. It will consist of six five-year periods providing considerable details to the turn of the century; three ten-year periods to permit large-scale introduction of fusion and solar technologies and two twenty-year periods to reduce truncation effects.

e. *SPI Model.* The model has been developed in the Siberian Power Institute (SPI) of the Siberian Department of the USSR Academy of Sciences to analyze possible energy development strategies and comparison of tendencies in science and technology. The model is part of the system of models for long-term energy development forecasting (for 30 to 40 years). As this system of models was described in a few references (see, for example, [1,2,8 - 10]), we discuss here only some specific features of the SPI ESS model.

The SPI model has a specific block structure with detailed description for each region k and year t of production connections of energy conversion at all stages ranging from extraction of primary energy (different kinds of fuel and nuclear fuel, water, solar and geothermal energy) via production and distribution of secondary energy (liquid, solid and gaseous fuels, secondary nuclear fuel, electric energy, steam, hot water) to the production of final energy utilized in industry, transport, agriculture, municipal and service sectors. For each time period t the model consists of oil, coal, gas, nuclear and electro energy blocks and for each region k of fuel and electro energy supply blocks. Each block can be generated, introduced to a computer and updated independently.

The balance equations for each region k and year t of production and distribution are the following:

<u>of primary energy α</u>

$$\sum_{j(\alpha)} a^k_{\alpha j}(t)\, x^k_{\alpha j}(t) + \sum_{k'} x^{kk'}_{\alpha}(t) = \sum_{j(\beta)} b^k_{\alpha j}(t) x^k_{\beta j}(t)$$

$$+ \sum_{k'} b^{kk'}_{\alpha}(t)\, x^{kk'}_{\alpha}(t) + d^k_{\alpha}(t)$$

<u>of secondary energy β</u>

$$\sum_{j(\beta)} a^k_{\beta j}(t)\, x^k_{\beta j}(t) + \sum_{k'} x^{kk'}_{\beta}(t) = \sum_{j(\gamma)} b^k_{\beta j}(t)\, x^k_{\gamma j}(t)$$

$$+ \sum_{k'} b^{kk'}_{\beta}(t)\, x^{kk'}_{\beta}(t) + d^k_{\beta}(t)$$

of final energy γ

$$\sum_{j(\gamma)} a^k_{\gamma j}(t)\, x^k_{\gamma j}(t) = d^k_\gamma(t) \quad .$$

Here

$x^k_{\alpha j}(t)$; $x^k_{\beta j}(t)$ and $x^k_{\gamma j}(t)$ are the intensities of production of primary α, secondary β and final γ energy with technology j for region k and year t;

$x^{kk'}_\alpha(t)$ and $x^{kk'}_\beta(t)$ are the unknown scales of transportation of primary α and secondary β energy between regions k and k' at year t;

$a^k_{\alpha j}(t)$, $a^k_{\beta j}(t)$, $a^k_{\gamma j}(t)$ are the technological coefficients of output of energy in the process of its conversion.

$b^k_{\alpha j}(t)$; $b^k_{\beta j}(t)$ are similar coefficients of energy consumption;

$b^{kk'}_\alpha(t)$, $b^{kk'}_\beta(t)$ specify energy losses during transportation;

$d^k_\alpha(t)$, $d^k_\beta(t)$, $d^k_\gamma(t)$ are demands for primary α, secondary β and final energy γ in region k and year t.

The nonenergy resources (WELMM factors [26]) constraints which are similar to (1.9) are written in the form

$$\sum_{\alpha,k,j} f^k_{i\alpha j}(t)\, x^k_{\alpha j}(t) + \sum_{\beta,k,j} f^k_{i\beta j}(t)\, x^k_{\beta j}(t) + \sum_{\gamma,k,j} f^k_{i\gamma j}(t)\, x^k_{\gamma j}(t) \le f_i(t) \quad .$$

For each non-renewable kind of primary energy α we have constraints

$$\sum_{j,k,t} x^k_{\alpha j}(t) \le \bar{z}_\alpha$$

which are similar to (1.31) - (1.33).

One can see that these conditions, though having a much more detailed form are of the same structure as the constraints of the models mentioned above. The description of the dynamics of system development differs however in some relations in the SPI model.

The equations linking blocks t and t+1 have the following form in the SPI model [9]:

$$\sum_{j \in J_0} x_{ij}(t) + \sum_{j \in J_1} x_{ij}(t) = \tilde{y}_i(t+1) = \sum_{j \in J_0} x_{ij}(t+1) + x_i(t+1)$$

where

i is the index of any energy unit (plant, station, etc.) ;

j is the index of the type of the conversion process.

The set of indices J_0 is associated with conversion (or production) capacities which exist in time period t ("old") and the set of indices J_1 is associated with capacities which were built till the end of year t ("new"); thus $\tilde{y}_i(t+1)$ is the production capacity of type i at the end of year t (or at the beginning of year t+1); $x_i(t+1)$ is the capacity of type i which is dismantled in year t+1.

The above equations can be rewritten in a form closer to state equations (1.1):

$$\sum_{j \in J_0} x_{ij}(t+1) = \sum_{j \in J_0} x_{ij}(t) + \sum_{j \in J_1} x_{ij}(t) - x_i(t+1) \quad .$$

Evidently, $\sum_{j \in J_0} x_{ij}(t)$ may be associated with $y_i(t)$, whereas $\sum_{j \in J_1} x_{ij}(t) - x_i(t+1)$ with $v_i(t) - v_i(t-\tau_i)$.

The other peculiarity of the model is the objective function. The minimization of the total discounted cost was considered not being quite adequate in view of uncertainty in prices. Therefore, the objective function of the model is given in the form of discounted consumption of total expenditures for different material resources and manpower (WELMM factors):

$$J = \sum_{t=0}^{T-1} \sum_{i} \beta(t) E_i(t) f_i(t) ,$$

where coefficient $E_i(t)$ matches the value of the ith resource $f_i(t)$ with the remaining resources, $\beta(t)$ is a discounting factor.

The dimension of the model is 500-600 constraints and 4000-5000 variables for the long-range planning variant and 1200-1300 constraints and 6000-7000 variables for the five-year planning problem. To solve these optimization problems a special program package has been developed which gives a 3-4 fold reduction of the computation time in comparison with the conventional approach.

2. Resources Model

The resources model is aimed at the evaluation of long-term resources exploration and extraction strategies. It also provides inputs for the energy supply model (see Section 1), essentially by establishing relations between available quantities of given natural resources and their possible cost of production (extraction) [26 - 39].

We consider production of natural resources over a given planning horizon at a regional (national) level. The lengths of a time step and of the whole planning horizon correspond to that in the energy supply model. Availability of resources are expressed in physical units and costs are measured in monetary units.

The model's structure is similar to the energy supply model in the sense that it is a DLP model in which the optimal mix of technologies for exploration and extraction of natural energy resources is determined.

2.1. Basic Model

a. State Equations

The model consists of two subsystems: resources accounting subsystem and capital stocks subsystem. The first subsystem describes a shift of resources from speculative to hypothetical categories and from hypothetical to identified categories. Here we use definitions given in [32 - 36]. Both renewable and nonrenewable resources are considered. The second subsystem describes the accumulation and depletion of capacities (capital stocks) for exploration and extraction of both renewable and nonrenewable resources.

In the resources extraction we consider various mixes of extraction technologies.

Before describing the resource model, let us consider a simple example, which illustrates how the dynamic of the process is to be described. Let $x(t)$ be the total amount of nonrenewable resource at time period t. Applying given extraction technologies one can extract only a certain portion of the whole amount of this resource in the ground. We denote the extractable (recoverable) resource by x(t). It is convenient to refer to $\hat{x}(t)$ as a net value and to x(t) as a gross value. Between gross and net values of the resource the following relation holds

$$x(t) = \hat{x}(t)/\delta$$

where δ, $0 \leq \delta \leq 1$ is the recoverability factor of the resource (for a fixed technology) at time t.

Bearing this in mind we can use three types of description of the process in gross values, in net values and mixed type. Let u(t) be the (gross) amount of the resource, extracted in time period t, and $\tilde{u}(t)$ be the (gross) amount of the resource, shifted from the hypothetical to the identified category. Then the balance equation is

$$x(t+1) = x(t) - u(t) + \tilde{u}(t) \qquad (t = 0,1,\ldots,T-1)$$

Evidently, that

$$x(t) \geq 0 \qquad \text{for all } t,$$

which is equivalent to

$$\sum_{\tau=0}^{t} u(\tau) \leq x(0) + \sum_{\tau=0}^{t} \tilde{u}(\tau) \qquad (t=1,\ldots,T) \quad .$$

To obtain a description in 'net' units all the variables must be multiplied by δ. Due to linearity of the relations:

$$\hat{x}(t+1) = \hat{x}(t) - \hat{u}(t) + \hat{\tilde{u}}(t)$$

(We refer to the variable with a roof as to net values.)

In practice a mixed description is generally used:

$$x(t+1) = x(t) - \hat{u}(t)/\delta(t) + \tilde{u}(t) \quad .$$

In this case the condition

$$x(t) \geq 0$$

is equivalent to

$$\sum_{\tau=0}^{t} u(\tau) \leq \delta\left[x(0) + \sum_{\tau=0}^{t} \tilde{u}(\tau)\right] \quad .$$

The value

$$x(0) + \sum_{\tau=0}^{t}\left[\tilde{u}(\tau) - \hat{u}(\tau)\right] \geq (1-\delta)\left[x(0) + \sum_{\tau=0}^{t} \tilde{u}(\tau)\right]$$

denotes the (gross) amount of the resource remaining in the ground after t periods of extraction.

Further we will use the mixed description, omitting the roof for variable $\hat{u}(t)$ for simplicity [37,38].

Nonrenewable Resources. Let

$x_i^1(t)$ be the (gross) amount (stock) of an identified nonrenewable resource of category i at time period t;

$u_{mi}^1(t)$ be the (net) amount of resource of category i extracted by technology m in time period t (extraction intensity);

M_i^1 be the total number of extraction technologies which can be applied to nonrenewable resource i;

$u_{ki}^2(t)$ be the (gross) amount of resources of category i shifted from the hypothetical to the identified category by the exploration technology k;

K_i^1 be the total number of exploration technologies which can be applied to nonrenewable resource i.

Then the dynamics in total amounts of identified nonrenewable resources will be the following:

$$x_i^1(t+1) = x_i^1(t) - \sum_{m\in M_i^1} u_{mi}^1(t)/\delta_{mi}^1(t) + \sum_{k\in K_i^1} u_{ki}^2(t). \quad (2.1)$$

Here $\delta_{mi}^1(t)$ is the recoverability of resource i by technology m at time t.

For hypothetical resources we introduce, in a similar way, (all variables are "gross" values):

$x_i^2(t)$ is the total amount of hypothetical resources of category i in time period t;

$u_i^3(t)$ is the total amount of resource i shifted from the speculative to the hypothetical category as a result of exploration activity in time period t;

The state equations for this group of nonrenewable resources will be the following:

$$x_i^2(t+1) = x_i^2(t) - \sum_{k\in K_i^1} u_{ki}^2(t) + u_i^3(t) \quad (2.2)$$

Similarly, for the speculative category of nonrenewable resources:

$$x_i^3(t+1) = x_i^3(t) - u_i^3(t) + u_i^4(t) \quad , \quad (2.3)$$

where

$x_i^3(t)$ is the total estimate of speculative resources of category i at step t;

$u_i^4(t)$ is the change of estimate of hypothetical category i as a result of improved scientific knowledge.

In (2.1) - (2.3) $i=1,\ldots,N_1$, where N_1 is the total number of considered categories of nonrenewable resources.

In the state equations (2.1) - (2.3)

$$\left\{x_i^1(t) \quad , \quad x_i^2(t) \quad , \quad x_i^3(t)\right\} \quad (i = 1,\ldots,N_1)$$

are state variables for the nonrenewable resources subsystem;

$$\left\{u^1_{mi}(t),\quad u^2_{ki}(t),\quad u^3_i(t),\quad u^4_i(t)\right\} \quad (m\in M^1_i\ ,\ k\in K^1_i\ ,\ i = 1,\ldots,N_1)$$

are control variables.

Renewable Resources. In a similar way we can write the state equations for renewable resources:

$$y^1_i(t+1) = y^1_i(t) + \sum_{k\in K^2_i} v^2_{ki}(t) \tag{2.4}$$

$$y^2_i(t+1) = y^2_i(t) - \sum_{k\in K^2_i} v^2_{ki}(t) + v^3_i(t) \tag{2.5}$$

$$y^3_i(t+1) = y^3_i(t) - v^3_i(t) + v^4_i(t),\ i = 1,\ldots,N_2 \tag{2.6}$$

where

$y^1_i(t)$ is the total flow of the available resource of category i in time period t;

$y^2_i(t)$ is the total flow of the hypothetical resource i in time period t;

$y^3_i(t)$ is the total flow of the speculative resource i in time period t;

$v^2_{ki}(t)$ is the intensity of exploration technology k applied to resource i in time period t;

K^2_i is the total number of exploration technologies for resource i;

N_2 is the total number of considered categories of renewable resources.

In the renewable resources subsystem (2.4) - (2.6)

$$\{y^1_i(t)\ ;\ y^2_i(t)\ ,\ y^3_i(t)\} \quad (i = 1,\ldots,N_2)$$

are the state variables;

$$\{v^2_{ki}(t)\ ,\ v^3_i(t)\ ,\ v^4_i(t)\} \quad (k = 1,\ldots,K^2_i\ ;\ i = 1,\ldots,N_2)$$

are the control variables.

Initial conditions are supposed to be given for all resource categories

$$
\begin{aligned}
x_i^1(0) &= x_i^{1,0} , \quad i = 1,\ldots,N_1 \\
y_i^1(0) &= y_i^{1,0} , \quad i = 1,\ldots,N_2 \\
x_i^2(0) &= x_i^{2,0} , \quad i = 1,\ldots,N_1 \\
y_i^2(0) &= y_i^{2,0} , \quad i = 1,\ldots,N_2 \\
x_i^3(0) &= x_i^{3,0} , \quad i = 1,\ldots,N_1 \\
y_i^3(0) &= y_i^{3,0} , \quad i = 1,\ldots,N_2 \;.
\end{aligned}
\qquad (2.7)
$$

Capacities' Dynamics. Alongside with subsystems which describe resources extraction and exploration, it is necessary to introduce a subsystem of resource extraction and exploration capacities development. It is described by equations which are similar to (1.1). Let for extraction subsystem:

$z_m(t)$ be the extraction capacities of type m in time period t;

$w_m(t)$ be the increase of the mth extraction capacities in time period t;

τ_m be the service time of the mth capacity.

Then the state equations for this submodel will be the following:

$$z_m(t+1) = z_m(t) + w_m(t) - w_m(t-\tau_m) \qquad (2.8)$$

where in the general case $m \in M_1 \cup M_2$ - the union of two sets:

M_1 (the total set of technologies for extracting nonrenewable resources), and

M_2 (the total set of technologies for extracting renewable resources.)

Initial conditions are given:

$$z_m(0) = z_m^0 \quad . \tag{2.9a}$$

$$w_m(t-\tau_m) = w_m^0(t-\tau_m) \quad ; \qquad 0 \le t \le \tau_m - 1 \quad . \tag{2.9b}$$

The dynamics of exploration capacities can be described in a similar way. For simplicity, these equations are omitted here.

b. Constraints

Exploration and extraction of natural resources are subject to a number of constraints.

Physical Constraints. All variables in the model are non-negative by their physical sense:

$$\begin{aligned} x_i^1(t) &\ge 0 , & u_{mi}^1(t) &\ge 0 , \\ x_i^2(t) &\ge 0 , & u_{ki}^2(t) &\ge 0 , \\ x_i^3(t) &\ge 0 , & u_i^3(t) &\ge 0 , \\ & & u_i^4(t) &\ge 0 , \end{aligned} \tag{2.10}$$

$$i = 1,\ldots,N_1 \; ;$$

$$m = 1,\ldots,M_1 \; ;$$

$$k = 1,\ldots,K_1 \; .$$

$$\begin{aligned} y_i^1(t) &\ge 0 , & v_{mi}^1(t) &\ge 0 , \\ y_i^2(t) &\ge 0 , & v_{ki}^2(t) &\ge 0 , \\ y_i^3(t) &\ge 0 , & v_i^3(t) &\ge 0 , \\ & & v_i^4(t) &\ge 0 , \end{aligned} \tag{2.11}$$

$$i = 1,\ldots,N_2 \; ;$$

$$m = 1,\ldots,M_2 \; ;$$

$$k = 1,\ldots,K_2 \; .$$

$$z_m(t) \geq 0 \; , \quad w_m(t) \geq 0 \; , \quad m \in M_1 \cup M_2 \; .$$

Recoverability Constraints. The recoverability of resources is assumed to be associated with the type of resources and technology used for their extraction.

As mentioned in the introduction of this section they can be written for nonrenewable resources as

$$x_i^1(t) \geq 0 \tag{2.12}$$

which is equivalent (due to (2.1) and (2.7)) to:

$$\sum_{\tau=0}^{t} \sum_{m \in M_i^1} u_{mi}^1(\tau) / \delta_{mi}^1(\tau) \leq x_i^{1,0} + \sum_{\tau=0}^{t} \sum_{k \in K_i^1} u_{ki}^2(\tau) \; . \tag{2.12a}$$

$$(i = 1,\ldots,N_1)$$

For renewable resources such constraints can be written as:

$$\sum_{m \in M_i^2} v_{mi}^1(t)/\delta_{mi}^2(t) \leq y_i^1(t) \quad (i = 1,\ldots,N_2) \tag{2.13}$$

Here $v_{mi}^1(t)$ is the amount of the renewable resource i extracted by technology $m \in M_i^2$ in time period t (extraction intensity). Opposite to (2.1) this variable does not enter the equation (2.4) for renewable resources.

Due to (2.4) and (2.7) the condition (2.13) is equivalent to:

$$\sum_{m \in M_i^2} v_{mi}^1(t)/\delta_{mi}^2(t) \leq y_i^{1,0} + \sum_{\tau=0}^{t-1} \sum_{k \in K_i^2} v_{ki}^2(\tau) \tag{2.14}$$

Availability Constraints. The simplest form of these constraints can be expressed as upper bounds on control variables:

$$
\begin{aligned}
u^2_{ki}(t) &\le \bar{u}^2_{ki}(t) \\
u^3_i(t) &\le \bar{u}^3_i(t) \\
u^4_i(t) &\le \bar{u}^4_i(t) \quad .
\end{aligned}
\tag{2.15}
$$

and

$$
\begin{aligned}
v^1_{mi}(t) &\le \bar{v}^1_{mi}(t) \\
v^2_{ki}(t) &\le \bar{v}^2_{ki}(t) \\
v^3_i(t) &\le \bar{v}^3_i(t) \\
v^4_i(t) &\le \bar{v}^4_i(t) \quad .
\end{aligned}
\tag{2.16}
$$

These constraints are similar to (1.8) and may express very roughly the availability of various technologies for exploration and extraction over time.

The development of a resource system can require some other resources (such as land, manpower, etc.) which are external in respect to the system (WELMM factors, see [26]). These constraints can be written in the form similar to (1.9):

$$\sum_{s,i} r^{\nu\ell u}_{si}(t)\, u^{\nu}_{si}(t) \le R^{\nu\ell u}(t) \tag{2.17}$$

$$\sum_{q,i} r^{\nu\ell v}_{qi}(t)\, v^{\nu}_{qi}(t) \le R^{\nu\ell v}(t) \qquad \begin{pmatrix} \ell = 1,\ldots,L \\ \nu = 1,2,3,4 \end{pmatrix} \tag{2.18}$$

where

$R^{\nu\ell u}(t)$ and $R^{\nu\ell v}(t)$ are the amounts of external resource ℓ (WELMM factor ℓ), available in time period t for nonrenewable and renewable resources subsystems ($\ell = 1,\ldots,L$) and for each group of exploration activities ν.

L is the total number of WELMM factors considered as external to the model;

$r_{si}^{\nu\ell u}(t)$, $r_{qi}^{\nu\ell v}$ are the (normative) consumptions of WELMM factor ℓ per unit of the production output;

$$s \in M_i^1 \;, \text{ if } \nu = 1, \qquad s \in K_i^1 \;, \text{ if } \nu = 2$$

and

$$q \in M_i^2 \;, \text{ if } \nu = 1, \qquad q \in K_i^2 \;, \text{ if } \nu = 2 \;;$$

the first subindices should be dropped on the left sides (2.17), (2.18), if $\nu = 3,4$.

Practically, coefficients $r_{si}^{\nu\ell u}(t)$ and $r_{qi}^{\nu\ell v}(t)$ are negligibly small for $\nu = 2,3,4$.

The other important type of availability constraints is connected with the linkage of resource extraction and production capacities. In fact, extraction of resources is limited at each time period by available production capacities:

$$\sum_i u_{mi}^1(t) \leq z_m(t) \;, \qquad m \in M_1 \tag{2.19}$$

$$\sum_i v_{mi}^1(t) \leq z_m(t) \;, \qquad m \in M_2 \tag{2.20}$$

where $z_m(t)$, $m \in M_1$, $m \in M_2$ are defined from equation (2.7).

In its turn, the development of extracting capacities subsystem (2.7) may be limited by resources, available for construction of new capacities. In this case, control variables $w_m(t)$ in (2.7) are subject to constraints, which are similar to (2.17) and (2.18).

Demand Constraints. Demands are exogeneous for the resource model. These constraints can be written in the form:

$$\sum_{m \in M_i^1} u_{mi}^1(t) \geq d_i^u(t) \qquad i = 1,\ldots,N_1 \tag{2.21}$$

for nonrenewable resources, and

$$\sum_{m \in M_i^2} v_{mi}^1(t) \geq d_i^v(t) \qquad i = 1,\ldots,N_2 \qquad (2.22)$$

for renewable resources, where $d_i^u(t)$ and $d_i^v(t)$ are the demands of resource of type i in time period t.

It should be noted that accurate estimation of demands $\{d_i^u(t), d_i^v(t)\}$ is very important in the resource model, because of the strong influence of these parameters on the timing and corresponding costs to put into operation new extraction technologies and intensity of exploration activities, that is, on the optimal solution.

c. Objective Function

There can be different objective functions for the resource system development. Following the ESS model we determine the objective function as to minimize the total discounted costs required to implement a resource development strategy:

$$J(u^1,u^2,u^3,u^4,v^1,v^2,v^3,v^4,w) =$$

$$= \sum_{t=0}^{T-1} \beta(t) \Bigg\{ \Bigg[\sum_{m,i} c_{mi}^{1u} u_{mi}^1(t) + \sum_{k,i} c_{ki}^{2u} u_{ki}^2(t) +$$

$$+ \sum_i c_i^{3u} u_i^3(t) + \sum_i c_i^{4u} u_i^4(t) \Bigg] + \Bigg[\sum_{m,i} c_{mi}^{1v} v_{mi}^1(t) +$$

$$+ \sum_{k,i} c_{ki}^{2v} v_{ki}^2(t) + \sum_i c_i^{3v} v_i^3(t) + \sum_i c_i^{4v} v_i^4(t) \Bigg] +$$

$$+ \sum_m c_m^z z_m(t) + \sum_m c_m^w w_m(t) +$$

$$+ \Bigg[\sum_{\ell,m,i} c_{mi}^{1\ell u} r_{mi}^{1\ell u} u_{mi}^1(t) + \sum_{\ell,k,i} c_{ki}^{2\ell u} r_{ki}^{2\ell u} u_{ki}^2(t) +$$

$$+ \sum_{\ell,i} c_i^{3\ell u} r_i^{3\ell u} u_i^3(t) + \sum_{\ell,i} c_i^{4\ell u} r_i^{4\ell u} u_i^4(t) \Bigg] +$$

$$+ \left[\sum_{\ell,m,i} c_{mi}^{1\ell v} r_{mi}^{1\ell v} v_{mi}^{1}(t) + \sum_{\ell,k,i} c_{ki}^{2\ell v} r_{ki}^{2\ell v} v_{ki}^{2}(t) + \right.$$

$$\left. + \sum_{\ell,i} c_{i}^{3\ell v} r_{i}^{3\ell v} v_{i}^{3}(t) + \sum_{\ell,i} c_{i}^{4\ell v} r_{i}^{4\ell v} v_{i}^{4}(t) \right] \Bigg\} \quad .$$

Here

c_{mi}^{1u}, c_{ki}^{2u}, c_{i}^{3u}, c_{i}^{4u}, c_{mi}^{1v}, c_{ki}^{2v}, c_{i}^{3v}, c_{i}^{4v} are exploratory costs for non- and renewable resources,

c_m^z are operational costs,

c_m^w are capital investment costs,

$c_{mi}^{1\ell q}$, $c_{mi}^{1\ell q}$,.... are costs of WELMM factors (external resources).

Transportation costs can also be included in the model.

d. *Statement of the Problem*

Finally we can formulate the problem of optimal development of the resource system as follows:

Problem 2.1. Given the state equations for nonrenewable resources subsystem ($i = 1,\ldots,N_1$):

$$x_i^1(t+1) = x_i^1(t) - \sum_{m \in M_i^1} u_{mi}^1(t)/\delta_{mi}^1(t) + \sum_{k \in K_i^1} u_{ki}^2(t) \; ; \; x_i^1(0) = x_i^{1,0}$$

$$x_i^2(t+1) = x_i^2(t) - \sum_{k \in K_i^1} u_{ki}^2(t) + u_i^3(t) \quad ; \qquad x_i^2(0) = x_i^{2,0}$$

$$x_i^3(t+1) = x_i^3(t) - u_i^3(t) + u_i^4(t) \quad ; \qquad x_i^3(0) = x_i^{3,0}$$

for renewable resources subsystem ($i = 1,\ldots,N_2$):

$$y_i^1(t+1) = y_i^1(t) + \sum_{k \in K_i^2} v_{ki}^2(t) \quad ; \qquad y_i^1(0) = y_i^{1,0}$$

$$y_i^2(t+1) = y_i^2(t) - \sum_{k\in K_i^2} v_{ki}^2(t) + v_i^3(t) \quad ; \qquad y_i^2(0) = y_i^{2,0}$$

$$y_i^3(t+1) = y_i^3(t) - v_i^3(t) + v_i^4(t) \quad ; \qquad y_i^3(0) = y_i^{3,0}$$

for extracting capacities subsystem ($m \in \{1,\dots,M_1\}$ and $m \in \{1,\dots,M_2\}$):

$$z_m(t+1) = z_m(t) + w_m(t) - w_m(t-\tau_m) \; ; \; z_m(0) = z_m^0 \quad ;$$

$$w_m(t-\tau_m) = w_m^0(t-\tau_m) \qquad 0 \le \tau \le \tau_m - 1 \quad .$$

Find controls $\left\{u_{mi}^1(t)\, ,\, u_{ki}^2(t)\, ,\, u_i^3(t)\, ,\, u_i^4(t)\right\}$, $\left\{v_{mi}^1(t),\, v_{ki}^2(t)\, ,\, v_i^3(t)\, ,\, v_i^4(t)\right\}$, $\left\{w_m(t)\right\}$ and corresponding trajectories $\left\{x_i^1(t)\, ;\, x_i^2(t)\, ,\, x_i^3(t)\right\}$, $\left\{y_i^1(t)\, ;\, y_i^2(t)\, ,\, y_i^3(t)\right\}$ and $\left\{z_m(t)\right\}$, which satisfy the constraints:

a) nonnegative constraints

$u_{mi}^1(t) \ge 0$	$x_i^1(t) \ge 0$	$v_{mi}^1(t) \ge 0$	$y_i^1(t) \ge 0$
$u_{ki}^2(t) \ge 0$	$x_i^2(t) \ge 0$	$v_{ki}^2(t) \ge 0$	$y_i^2(t) \ge 0$
$u_i^3(t) \ge 0$	$x_i^3(t) \ge 0$	$v_i^3(t) \ge 0$	$y_i^3(t) \ge 0$
$u_i^4(t) \ge 0$		$v_i^4(t) \ge 0$	

b) recoverability constraints

$$x_i^1(t) \ge 0 \qquad (i = 1,\dots,N_1)$$

$$\sum_m v_{mi}^1(t)/\delta_{mi}^2(t) \le y_i^1(t) \qquad (i = 1,\dots,N_2)$$

c) external resources availability constraints ($\ell = 1,\ldots,L$; $\nu = 1,\ldots,4$):

$$\sum_{s,i} r_{si}^{\nu\ell u}(t)\, u_{si}^{\nu}(t) \leq R^{\nu\ell u}(t)$$

$$\sum_{q,i} r_{qi}^{\nu\ell v}(t)\, v_{qi}^{\nu}(t) \leq R^{\nu\ell v}(t)$$

d) production capacities availability constraints:

$$\sum_{i} u_{mi}^{1}(t) \leq z_m(t) \quad , \qquad m \in M_1$$

$$\sum_{i} v_{mi}^{1}(t) \leq z_m(t) \quad ; \qquad m \in M_2$$

e) demand constraints

$$\sum_{m} u_{mi}^{1}(t) \geq d_i^{u}(t) \qquad (i = 1,\ldots,N_1)$$

$$\sum_{m} v_{mi}^{1}(t) \geq d_i^{v}(t) \qquad (i = 1,\ldots,N_2)$$

and minimize the objective function ($\nu = 1,2,3,4$):

$$J(u,v,w) = \sum_{t=0}^{T-1} \beta(t) \Big[\sum_{\nu,s,i} c_{si}^{\nu u}\, u_{si}^{\nu}(t) + \sum_{\nu,s,i} c_{si}^{\nu v}\, v_{si}^{\nu}(t) + \sum_{m} c_m^{z}\, z_m(t) + \sum_{m} c_m^{w}\, w_m(t) + \sum_{\nu,\ell,s,i} c_{si}^{\nu\ell u}\, r_{si}^{\nu\ell u}\, u_{si}^{\nu}(t) + \sum_{\nu,\ell,s,i} c_{si}^{\nu\ell v}\, r_{si}^{\nu\ell v}\, v_{si}^{\nu}(t) \Big] .$$

2.2. Discussion

Problem 2.1 is general enough and allows different modifications. They give a possibility to carry out policy analysis of extraction and/or exploration activities for one resource or for a group of resources, for a region or a country; to determine optimal balance of these activities for nonrenewable and renewable resources. Below we consider some examples of these modifications and particular cases of Problem 2.1.

a. *Extraction and Exploration Model*

First we consider the case for analyzing interrelations between extraction and exploration activities for a nonrenewable energy resource (e.g. coal, oil, etc.)

The problem consists of the following. For a given region (country) there are known initial values of identified and hypothetical stocks of the resource, classified on n different categories (e.g. on-shore crude oil, natural gas and off-shore crude oil.)

There are also M different extraction and K different exploration technologies. The intensities of these technologies depend at some time period on the extraction and exploration capacities available at this time period.

The problem is to determine the optimal mix of extraction and exploration activities in a given planning horizon (extraction and exploration plans), which is balanced with the development of the capacity subsystem and yields the maximum output for this planning horizon.

Using the conditions of Problem 2.1, this problem can be formalized as follows:

Problem 2.2. Let be given initial stocks of identified and hypothetical resources

$$x_i^1(0) = x_i^{1,0} \quad ; \quad x_i^2(0) = x_i^{2,0} \qquad (2.23)$$

with the state equations for extraction activities:

$$x_i^1(t+1) = x_i^1(t) - \sum_{m\in M} u_{mi}^1(t)/\delta_{mi}^1(t) + \sum_{k\in K} u_{ki}^2(t) \qquad (2.24)$$

and exploration activities:

$$x_i^2(t+1) = x_i^2(t) - \sum_{k\in K} u_{ki}^2(t) + \tilde{u}_i^2(t) \qquad (2.25)$$

where $\tilde{u}_i^2(t)$ is the increase of the hypothetical resource of category i at time t (discovery rate).

Let also be given the initial values of extraction and exploration capacities

$$z_m^1(0) = z_m^{1,0} \quad ; \quad z_k^2(0) = z_k^{2,0} \qquad (2.26)$$

with the state equations

$$z_m^1(t+1) = z_m^1(t) + w_m^1(t) - w_m^1(t-\tau^1) \qquad (2.27)$$

$$z_k^2(t+1) = z_k^2(t) + w_k^2(t) - w_k^2(t-\tau^2) \quad . \qquad (2.28)$$

The intensities of extraction and exploration activities $u_{mi}^1(t)$, $u_{ki}^2(t)$, as well as intensities of new extraction and exploration capacities construction $w_i^1(t)$, $w_i^2(t)$ are subject to budget and other resources constraints

$$\sum_{m,i} r_{mi\ell}^{1u}(t) u_{mi}^1(t) + \sum_{k,i} r_{ki\ell}^{2u}(t) u_{ki}^2(t) +$$

$$+ \sum_i r_{i\ell}^{1w}(t) w_i^1(t) + \sum_i r_{i\ell}^{2w}(t) w_i^2(t) \leq R_\ell(t) \qquad (2.29)$$

$$\sum_i u_{mi}^1(t) \leq z_m^1(t) \quad ; \quad \sum_k u_{ki}^2(t) \leq z_m^2(t) \qquad (2.30)$$

$$x_i^1(t) \geq 0 \quad . \qquad (2.31)$$

Find nonnegative control sequences

$\{u^1_{mi}(t)\}$, $\{u^2_{ki}(t)\}$, $\{w^1_i(t)\}$ and $\{w^2_i(t)\}$

and corresponding nonnegative state variables

$\{x^1_i(t)\}$, $\{x^2_i(t)\}$ and $\{y^1_i(t)\}$, $\{y^2_i(t)\}$,

which yield a maximum total output of the resource

$$J = \sum_{t=0}^{T-1} \sum_{m,i} \kappa_i u^1_{mi}(t) \quad , \qquad (2.32)$$

where κ_i is the energy conversion factor for the resource of category i. Here $\tilde{u}^2_i(t)$ (discovery rate) is considered as a scenario variable.

b. Extraction Model

If the increase $\{\tilde{u}_i(t)\}$ of the identified resource is considered as a scenario variable (but not as a result of controllable exploration activities), then the state equations for the extraction system will be simplified:

$$x_i(t+1) = x_i(t) - u_i(t)/\delta_i(t) + \tilde{u}_i(t); \quad x_i(0) = x^0_i \qquad (2.33)$$

where $\tilde{u}_i(t)$ is the value of the identified resource of category i shifted from the hypothetical category i at time period t, $u_i(t)$ is the total extraction of resource i at time period t (we do not single out different extraction technologies in this example).

The development of the extraction capacities subsystem is described by the state equation similar to (2.27):

$$z_i(t+1) = z_i(t) + w_i(t) - w_i(t-\tau_i); \quad z_i(0) = z^0_i \qquad (2.34)$$

with constraints:

$$u_i(t) \le z_i(t) \quad ; \quad w_i(t) \ge 0 \quad ; \quad z_i(t) \ge 0 \quad ; \qquad (2.35)$$

$$\sum_i r^w_{i\ell}(t)\, w_i(t) + \sum r^u_{i\ell}(t)\, u_i(t) \leq R_\ell(t); \quad w_i(t) \geq 0 \qquad (2.36)$$

$$x_i(t) \geq 0 \quad . \qquad (2.36a)$$

The problem is to determine the extraction policy of the given identified resource, which is subject to constraints in the extraction capacity (2.35), availability of external resources (2.36) and recoverability of this resource (2.36a). It gives the total maximal output during the planning period.

The objective function can be written again as (2.32), or, if we introduce the cumulative amount of the resource extracted,

$$\xi(t+1) = \xi(t) + \sum_i \kappa_i u_i(t) \quad , \qquad \xi(0) = 0 \qquad (2.37)$$

as maximization of $\xi(T)$.

c. Exploration Model.

This model allows to determine exploration policies which yield a maximum shift of the hypothetical category of resources to the identified category. This system is a counterpart of the extraction system and is described by the equations:

$$x_i(t+1) = x_i(t) - u_i(t) + \tilde{u}_i(t) \quad ; \qquad x_i(0) = x_i^0 \qquad (2.38)$$

$$z_i(t+1) = z_i(t) + w_i(t) - w_i(t-\tau_i); \qquad z_i(0) = z_i^0 \qquad (2.39)$$

$$u_i(t) \leq z_i(t) \quad ; \qquad u_i(t) \geq 0 \quad , \qquad z_i(t) \geq 0 \qquad (2.40)$$

$$\sum_i r_{i\ell}(t)\, w_i(t) \leq R_\ell(t) \quad ; \qquad w_i(t) \geq 0 \qquad (2.41)$$

$$x_i(t) \geq 0 \qquad (2.42)$$

$$J = \sum_{t=0}^{T-1} \sum_i u_i(t) \rightarrow \max \quad . \qquad (2.43)$$

d. Cost Minimization

In the above examples the objective was to obtain a maximum output from the extraction and/or exploration systems. For many practical purposes it is necessary to obtain dependence between optimal cost J* and the cumulative availability of a resource (e.g. for calculating cost coefficients in the objective function (1.12) of the energy supply system model). It can be done by a simple optimization model

$$x_i(t+1) = x_i(t) - u_i(t)/\delta_i(t) + \tilde{u}_i(t); \quad x_i(0) = x_i^0$$

$$z_i(t+1) = z_i(t) + w_i(t) - w_i(t-\tau_i) \quad ; \quad z_i(0) = z_i^0$$

$$\xi(t+1) = \xi(t) + \sum_i \kappa_i(t)\, u_i(t) \quad ; \quad \xi(0) = 0$$

$$\sum_i \kappa_i u_i(t) \geq d(t) \quad ; \quad u_i(t) \geq 0 \quad ; \tag{2.44}$$

$$u_i(t) \leq z_i(t) \quad ; \quad z_i(t) \geq 0$$

$$x_i(t) \geq 0$$

$$J = \sum_t \sum_i \left[c_i^u(t)\, u_i(t) + c_i^w(t)\, w_i(t)\right] \to \min \quad . \tag{2.45}$$

This model differs from the extraction model by the objective function (2.45) and by the inclusion of demand constraints (2.44). Resources constraints (2.36) are omitted here as they are taken into account by cost coefficients $c_i^u(t)$ and $c_i^w(t)$ in the objective function (2.45).

Clearly, in this simple model

$$\sum_i \kappa_i u_i^*(t) = d(t)$$

for optimal $u_i^*(t)$. Hence

$$\xi(t+1) = \xi(t) + d(t) \quad ; \quad \xi(0) = 0$$

and

$$\xi(T) = \sum_{t=0}^{T-1} d(t) \quad . \tag{2.47}$$

The problem is to calculate cost-supply curves

$$J^* = J(u^*(z(T)) = \varphi(z(T)) \quad .$$

It should be noted that the behavior of these curves strongly depends on the behavior of the demand curve $d(t)$.

e. Dimension of the Models

Finally, we calculate a typical dimension of the resources model. Let

M be the total number of different countries in a region,

L be the number of resource provinces within a country,

K be the number of basins within a province,

T be the length of a planning horizon,

ℓ be the number of different resource categories in a basin,

m be the number of different technologies which can be used in exploration and extraction

k be the number of WELMM factors limiting extraction.

One can see that the model will have

$(3\ell+m) \cdot K \cdot L \cdot M$	state equations,
$(2\ell+k+m) \cdot K \cdot L \cdot M$	constraints (nonnegativity constraints are not counted), and
$3 \cdot \ell \cdot m \cdot K \cdot L \cdot M$	control variables for each time period.

For example, consider a region consisting of only one country with two resource provinces. Assuming that the average number of basins in a province is equal to 3, the average number of different resource categories is 2 (for instance, crude oil and natural gas), the number of different technologies is 2 and the limiting WELMM factors are 2, we calculate that we would have for each time period 48 state equations, 48 constraints and the number of constraints equal to 78. Thus, for a quite realistic size the resources model is manageable and can be solved even by standard LP codes.

f. Resource Models under Uncertainty Conditions

One of the intrinsic features of the resources model is uncertainty in parameters, especially for speculative and hypothetical categories of resources. The conventional way in handling this difficulty is to consider these parameters as scenario variables (e.g. $\tilde{u}_i^2(t)$ in (2.25), or $\tilde{u}_i(t)$ in (2.33)) carrying out numerous computer runs for different hypotheses on these variables.

A more sophisticated approach is to consider maxmin problems associated with the given model. The maxmin approach allows to evaluate upper and lower bounds of the objective function for optimization problems with uncertainty conditions and to elaborate extraction and exploration policies which guarantee the required results within a given range of uncertain parameters. Methods for solving maxmin DLP problems are considered in [40].

Another approach to treat uncertainty conditions in resource models is the statement of the problem in a multistage stochastic programming framework [41].

3. Economy Development Models

In this section we present a model simulating optimal behavior of the entire economy of a region for different objectives. Interest in optimal economy models has been increasing in recent years. The reason is that they allow one to take into account some "optimal" mix of the dynamics of such important economic indicators as production levels, capital investment, intermediate and final consumption of goods to be produced. Different optimization models of economy development are considered, for example, in [15,16,42-45]. However, we shall not analyze all these models here, but restrict ourselves by describing a multibranch industrial model INTERLINK [46-48], which is conceptionally based on its predecessor-π-model developed at the Computer Center of the USSR Academy of Sciences [44-45]. The model presented below can be seen as a simplified version of the original π-model.

3.1. Basic Model

a. *State Equations*

The system under consideration is broken down into two subsystems: production and capacities development (or capital stock accumulation)

Production subsystem. The operation of industry is described in terms of n producing sectors. Let

$x_i(t)$ be the stock of production in sector i (i=1,...,n) accumulated up to a time period t,

$u_i(t)$ be the gross output (production level) of sector i in time period t,

$v_i(t)$ be the additional capital stock constructed in time period t, and

$a_{ij}(t)$ be the input-output coefficients.

We assume also, that

τ_j is the time (number of time periods) required to construct and to put into operation additional capacity in sector j;

$b_{ij}(t)$ are capital coefficients;

$w_i(t)$ is the final consumption of sector i products, and

$s_i(t)$ is the net export.

Then the state equations describing the production subsystem can be written as follows:

$$x_i(t+1) = x_i(t) + u_i(t) - \sum_{j=1}^{n} a_{ij}(t)\, u_j(t) - \sum_{j=1}^{n} \sum_{\tau=0}^{\tau_j} b_{ij}(t-\tau)\, v_j(t-\tau) - w_i(t) - s_i(t) \tag{3.1}$$

$$(i = 1,2,\ldots,n \,;\, t = 0,1,\ldots,T-1) \quad .$$

Initial inventories and preplanning controls are assumed to be given:

$$x_i(0) = x_i^0 \quad , \tag{3.2}$$

$$v_i(t-\tau_i) = v_i^0(t-\tau_i) \quad , \tag{3.3}$$

$$(i = 1,\ldots,n \,;\, t = 0,\ldots,\tau_i-1) \quad .$$

Assuming $\tau_j = \bar{\tau}$ for all sectors $j = 1,\ldots,n$, equation (3.1) can be rewritten in a matrix form:

$$x(t+1) = x(t) + (I - A(t))u(t) - \sum_{\tau=0}^{\bar{\tau}} B(t-\tau)\, v(t-\tau) - w(t) - s(t) \quad , \tag{3.1a}$$

where

$x(t) = \{x_i(t)\}$ is a state vector, $u(t) = \{u_i(t)\}$,

$v(t) = \{v_i(t)\}$, $w(t) = \{w_i(t)\}$ are control vectors, and

$s(t) = \{s_i(t)\}$ is considered here as exogenous vectors $(i = 1,\ldots,n)$.

For some problems export/import must be considered as control (decision) variables. In these cases the net export $s(t)$ is better represented as follows:

$$s(t) = s^+(t) - s^-(t) \quad , \quad s^+(t) \geq 0 \quad , \quad s^-(t) \geq 0$$

where $s^+(t)$ is the vector of import, $s^-(t)$ is the vector of export.

Capacities Development Subsystem. Let

$y_i(t)$ be the value of the production capacities of type i

and $d_i(t)$ be the depreciation factor in sector i at time period t $(i = 1,\ldots,n)$.

Then the dynamics of production capacities is written as follows:

$$y_i(t+1) = (1 - d_i(t))\, y_i(t) + v_i(t - \tau_i) \quad (i=1,\ldots,n) \; . \qquad (3.4)$$

The initial capital stocks are given

$$y_i(0) = y_i^0 \; . \qquad (3.5)$$

Assuming again for simplicity that

$$\tau_i = \bar{\tau} \quad \text{for all } i = 1,\ldots,n \quad ,$$

we can rewrite (3.4) in a matrix form:

$$y(t+1) = (I - D(t))y(t) + v(t - \bar{\tau}) \quad , \qquad (3.4a)$$

where $D(t)$ is a diagonal matrix with $d_i(t)$ on the main diagonal; $y(t) = \{y_i(t)\}$ $(i = 1,\ldots,n)$ is a state vector for the production capacities subsystem.

b. Constraints

Evidently any economic system is operating within certain constraints which imply physical, economic, institutional and other limits to the choice of controls.

Physical Constraints. All state and control variables are nonnegative:

$$x_i(t) \geq 0 \quad , \qquad u_i(t) \geq 0 \quad ,$$

$$y_i(t) \geq 0 \quad , \qquad v_i(t) \geq 0 \quad , \qquad (3.6)$$

$$w_i(t) \geq 0 \quad ,$$

$$(i = 1,\dots,n \,;\, t = 0,1,\dots,T-1) \quad .$$

Resources Availability Constraints. The production system requires certain external resource inputs for its operation. First of all, these are labor and primary resources. Both constraints can be written in a similar way:

for labor resources:

$$\sum_{j=1}^{n} \ell_{kj}(t)\, u_j(t) \leq \ell_k(t) \quad , \qquad (k = 1,\dots,K) \qquad (3.7)$$

where

$\ell_k(t)$ is the total labor of category k (k=1,...,K) available in time period t;

$\ell_{kj}(t)$ are the labor output ratios for sector j;

and for other resources (WELMM factors):

$$\sum_{j=1}^{n} r_{mj}(t)\, u_j(t) \leq r_m(t) \quad , \quad m = 1,\dots,M \qquad (3.8)$$

where

$r_m(t)$ is the total amount of resource category m, (WELMM factor m) available in time period t;

$r_{mj}(t)$ are specific resource requirements per unit of sector i production (resource - output ratios) in time period t.

In the matrix form equations (3.7) and (3.8) are

$$L(t)\, u(t) \leq \ell(t) \qquad (3.7a)$$

$$R(t)\, u(t) \leq r(t) \quad . \qquad (3.8a)$$

Production Capacities Constraints. The gross output of each sector is limited by the available production capacity

$$u_i(t) \leq y_i(t) \qquad (i = 1,\ldots,n) \tag{3.9}$$

or, in vector form

$$u(t) \leq y(t) \quad . \tag{3.9a}$$

Inventory Constraints. These constraints relate to the possibility of accumulating limited amounts of good's stocks. For storable goods:

$$0 \leq x_i(t) \leq \bar{x}_i(t) \quad , \tag{3.10}$$

where

$\bar{x}_i(t)$ are the given stock capacities, and

$x_i(t)$ are calculated from (3.1).

For nonstorable goods we have instead of (3.10):

$$x_i(t+1) \geq x_i(t) \quad , \tag{3.11}$$

which is similar to the following inequality:

$$u_i(t) - \sum_{j=1}^{n} a_{ij}(t)\, u_j(t) - \sum_{j=1}^{n} \sum_{\tau=0}^{\tau_j} b_{ij}(t-\tau)\, v_j(t-\tau) - w_i(t) - s_i(t) \geq 0 \tag{3.12}$$

or

$$(I - A(t))\, u(t) - \sum_{\tau=0}^{\bar{\tau}} B(t-\tau)\, v(t-\tau) - w(t) - s(t) \geq 0 \quad . \tag{3.12a}$$

It should be stressed that in many practical cases, the accumulation of goods stocks in large amounts is unreasonable or too expensive. Hence, $\{x_i(t)\}$ are small in comparison to the outputs of the system. Therefore we can consider the balance equation (bill of goods) in the form of inequality (which is equivalent to (3.12a)):

$$(I - A(t)\, u(t) \geq \sum_{\tau=0}^{\bar{\tau}} B(t-\tau)\, v(t-\tau) + w(t) + s(t) \qquad (3.13)$$

or equality

$$(I - A(t))\, u(t) = \sum_{\tau=0}^{\bar{\tau}} B(t-\tau)\, v(t-\tau) + w(t) + s(t) \qquad (3.13a)$$

both for storable and nonstorable goods.

Consumption Constraints. Final consumption is usually bounded for each sector i. In many cases it can be represented in the form:

$$w_i(t) \geq g_i(t)\, \omega(t) \quad , \qquad (3.14)$$

where

$\omega(t)$ = total final consumption,

$g_i(t)$ = share of total consumption provided by sector i;

Exogenously given vector $g(t) = (g_i(t),\ldots,g_n(t))$ predefines the profile of a final consumption over time. The introduction of a consumption profile allows one to use a scalar control $\omega(t)$ instead of control vector $w(t)$:

$$w(t) \geq g(t)\, \omega(t) \quad . \qquad (3.14a)$$

c. Objective Function

Above, $\{u,v,w\} = \{u_i(t), v_j(t), w_i(t)\}$ are control variables, $\{x,y\} = \{x_i(t), y_i(t)\}$ are state variables. The choice of optimal control depends on the choice of the objective function of a problem. In the following we consider typical examples of the objective functions.

Maximization of the Cumulative Discounted Goods Supply. In this case the objective function is

$$J = \sum_{t=0}^{T-1} \beta(t)\, \omega(t) \qquad (3.15)$$

where $\beta(t)$ is the discounting factor.

If we consider only the last step, then the objective function will be

$$J = \sum_{i=1}^{n} h_i^w(T)\, w_i(T) \qquad (3.16)$$

where $h_i(t)$ are weight coefficients for different products.

Maximization of the Final Stock of Goods

$$J = \sum_{i=1}^{n} h_i^x(T)\, x_i(T) \qquad (3.17)$$

$h_i^x(T)$ are weight coefficients ("costs") for $x_i(T)$.

Maximization of the Terminal Values of Production Capacities

$$J = \sum_{i=1}^{n} h_i^y(T)\, y_i(T) \qquad (3.18)$$

$h_i^y(T)$ are weight coefficients for $y_i(T)$.

Minimization of Total Expenses. This criterion is similar to the objective functions, considered in Sections 1 and 2:

$$J = \sum_{t=0}^{T-1} \beta(t) \left[(c^u(t), u(t)) + (c^v(t), v(t)) + (c^y(t), y(t)) \right] \qquad (3.19)$$

where

$c^u(t)$, $c^y(t)$ are operating and maintenance costs,

$c^v(t)$ is the investment cost,

$\beta(t)$ is the discounting factor.

Other objective functions are also possible [42-47]. It should be noted that goals of control can also be expressed by additional constraints, such as

$$\omega(T) \geq \overline{\omega}(T) \tag{3.20}$$

$$x(T) \geq \overline{x}(T) \tag{3.21}$$

$$y(T) \geq \overline{y}(T) \quad . \tag{3.22}$$

For example, one wishes to minimize the total expenses (3.19) under the given level of final consumption (3.20).

d. Statement of the Problem

For reference purposes we are writing below a typical optimization problem that occurs in economy models.

Problem 3.1. Given the state equations of the production subsystem:

$$x(t+1) = x(t) + (I - A(t))u(t) - \sum_{\tau=0}^{\overline{\tau}} B(t-\tau)\, v(t-\tau) - w(t) - s(t) \tag{3.1a}$$

and of the capital stock subsystem:

$$y(t+1) = [I - D(t)]\, y(t) + v(t-\overline{\tau}) \tag{3.4a}$$

with initial conditions

$$x(0) = x^0 \tag{3.2a}$$

$$v(t-\overline{\tau}) = v^0(t-\overline{\tau}) \quad ; \qquad 0 \leq t \leq \overline{\tau} - 1 \tag{3.3a}$$

$$y(0) = y^0 \quad . \tag{3.5a}$$

Find controls $u = \{u(0),\ldots,u(T-1)\}$, $v = \{v(0),\ldots,v(T-\overline{\tau}-1)\}$, $w = \{w(0),\ldots,w(T-1)\}$ and corresponding trajectories $x = \{x(0), \ldots,x(T)\}$, $y = \{y(0),\ldots,y(T)\}$, which satisfy the conditions:

nonnegativity constraints:

$$\begin{aligned} x(t) &\geq 0 \qquad & u(t) &\geq 0 \\ y(t) &\geq 0 \qquad & v(t) &\geq 0 \\ z(t) &\geq 0 \qquad & w(t) &\geq 0 \end{aligned} \tag{3.6a}$$

labor availability constraints:

$$L(t)\,u(t) \leq \ell(t) \tag{3.7a}$$

resources availability constraints:

$$R(t)\,u(t) \leq r(t) \tag{3.8a}$$

production capacity constraints:

$$u(t) \leq y(t) \tag{3.9a}$$

inventory constraints for storable goods:

$$x(t) \leq \bar{x}(t) \tag{3.10a}$$

for nonstorable goods:

$$(I - A(t))u(t) \geq \sum_{\tau=0}^{\bar{\tau}} B(t-\tau)\,v(t-\tau) + w(t) + s(t) \tag{3.13a}$$

consumption constraints

$$w(t) \geq g(t)\,\omega(t) \tag{3.14a}$$

and maximize the objective function

$$J = \sum_{t=0}^{T-1} \beta(t)\,\omega(t) \quad . \tag{3.15}$$

3.2. Discussion

Below we consider some modifications and extensions of Problem 3.1.

a. *Conversion Model* [44-45]. In many practical cases it is necessary to take into account the process of reconstruction (conversion) of production. In this case the above conditions should be replaced:

state equation (3.1) by:

$$x_i(t+1) = x_i(t) + u_i(t) - \sum_{j=1}^{n} a_{ij}(t)\, u_j(t) -$$

$$- \sum_{j=1}^{n} \sum_{\tau=0}^{\tau_j} b_{ij}(t-\tau)\, v_j(t-\tau) - \sum_{j,s=1}^{n} \sum_{\tau=0}^{\tau_j} b_{ij}^{s}(t-\tau^{s}) \cdot$$

$$\cdot\, v_j^{s}(t-\tau^{s}) - w_i(t) - s_i(t) \quad . \qquad (3.23)$$

Here

$v_j^s(t)$ is the additional production capacity in sector j, obtained from conversion of sector s into j, started at step t;

$b_{ij}^s(t)$ are the capital coefficients of conversion $s \to j$.

τ_j^s is the number of steps, required for conversion $s \to j$.

The state equation (3.4) is replaced by:

$$y_i(t+1) = (1 - d_i(t))\, y_i(t) + v_i(t-\tau_i) - \sum_{s=1}^{n} v_s^{i}(t-\tau_i^{s}) +$$

$$+ \sum_{s=1}^{n} k_i^{s}(t-\tau_i^{s})\, v_i^{s}(t-\tau_i^{s}) \quad , \qquad (3.24)$$

where $k_i^s(t)$ is the conversion coefficient, which shows the increase of the production capacity in sector i per unit of conversion activity $s \to i$.

The capacity constraints (3.9) in the case of conversion are replaced by:

$$u_i(t) \leq x_i(t) + \sum_{\tau=0}^{\tau_i-1} c_i(t-\tau)\, v_i(t-\tau) -$$

$$- \sum_{s=1}^{n} \sum_{\tau=0}^{\tau_i^{s}-1} v_i^{s}(t-\tau) + \sum_{s=1}^{n} \sum_{\tau=0}^{\tau_i^{s}-1} c_i^{s}(t-\tau)\, v_i^{s}(t-\tau) \quad . \qquad (3.25)$$

b. Capital Stock Subsystem. In some cases it is more convenient to describe the development of the production subsystem in terms of capital stock rather than in terms of production capacities. In these cases, instead of state equations (3.4) or (3.24) we have to introduce the state equations:

$$z_i(t+1) = (1 - \tilde{d}_i(t))z_i(t) + v_i(t - \tau_i) - \\ - \sum_{s=1}^{n} v_s^i(t - \tau_i^s) + \sum_{s=1}^{n} v_i^s(t - \tau_i^s) \qquad (3.26)$$

where

$z_i(t)$ is the capital stock in sector i at time period t;

$\tilde{d}_i(t)$ is the depreciation factor.

If there is no conversion in the system, then the last right term in (3.26) must be omitted.

Besides, the production capacity constraints (3.9) are replaced by

$$\beta_i(t)\, u_i(t) \leq z_i(t) \qquad (i = 1,\ldots,n) \quad , \qquad (3.27)$$

where $\beta_i(t)$ is the capital-output ratio.

c. Simplified Model. Here we describe a simplified version of Problem 3.1, which may be of interest for more long-range and aggregate considerations, for example, for the case of linking energy-economy submodels.

We assume that the time period is such that time lags can be excluded from consideration and, further we neglect a possibility to stock goods. The conversion processes are not considered in the model either. Thus the problem can be formulated as follows:

Problem 3.1a. Given the state equations for capital stock subsystem in the form

$$y(t+1) = (I - D(t))y(t) + v(t)$$

with initial state

$$y(0) = y^0 \quad ,$$

subject to constraints

bill of goods:

$$(I - A(t))u(t) = B(t)v(t) + w(t) + s(t)$$

resources availability constraints:

$$L(t)\,u(t) \leq \ell(t)$$

$$R(t)\,u(t) \leq r(t)$$

production constraints:

$$u(t) \leq y(t)$$

consumption constraints:

$$w(t) \geq g(t)\,\omega(t) \quad ,$$

find controls $\{v(t), u(t), \omega(t)\}$ and corresponding trajectory $\{y(t)\}$, which maximize the objective function

$$J = \sum_{t=0}^{T-1} \beta(t)\,\omega(t) \quad .$$

d. *INTERLINK Model*. The model was developed at IIASA by I. Zimin for modelling economy development of a region (country) in the IIASA system of energy development models. It represents a version of the dynamic multibranch π-model [44,45]; its structure is close to Problem 3.1 and its description is given in [46-48].

The typical dimension of the INTERLINK model is the following: number of state equations (sectors of economy) equals 17, number of constraints - 41 for each time period. Each time period corresponds to 5 years; the planning horizon is equal to 50 years, hence the total number of time periods is 10.

The total dimension of the corresponding LP problem: the number of rows is about 600, the number of columns is also about 600.

4. Linkage of the Models

Above three models were considered - energy supply system, primary resources system and economy development system which were presented by Problems 1-3, respectively. Each of these models can be used individually for energy resources and technology assessment.

However, this approach of separate analysis is limited in its possibilities because many important features of the systems are missing due to their interactions. Thus one should build models of the whole system of energy-resources-economy interaction and hence we ought to investigate ways of linkages of individual models into a whole system. This new stage of energy policy modelling has started just recently, only 2-3 years ago [1,2,8-12,23,49-51]. Two basic approaches can be singled out here. First, to integrate separate models into a single optimization problem with the corresponding objective function [50 - 53]. The second approach is to investigate linkage of submodels, considering these submodels on some independent basis each with its own objective function [1,9,11,12,23,45].*)

Both approaches naturally have their own advantages and drawbacks. The major advantage of the first, "machine" approach, is that it allows to take into account all the constraints and interactions between many factors influencing the decision and combine them in some optimal way. However, building an integrated model evidently leads to a very large optimization problem which, although sometimes possible to solve, is always very difficult to interpret.

The "manual" approach -- information obtained from one submodel is interpreted by an analyst and provided as input to another submodel -- is more attractive but is much more time consuming and sometimes may lead to uncertainty whether the "true optimal" solution for a whole system has been obtained.

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*) We don't consider "non-optimization" approaches which come out from the framework of this paper (see [49]).

Thus, we need to combine both approaches and therefore we consider both of them below starting with the integrated model.

4.1. Integrated Model

Considering the ESS and economy models we can see that there are two main linkages between them: final demand for energy which is the output of the economy model and nonenergy resources supply for which the requirements are outputs of the ESS model. We shall combine the ESS model (Problem 1.1) and the economy model (Problem 3.1) into a whole system, using subscript E for the energy sector and NE for the nonenergy sectors.

For a uniform representation we assume that both the industrial processes of economy and energy sectors are described in terms of physical flows. Besides, in the model below we omit, for simplicity, time lags in construction and put into operation production capacities, that is, we will use simplified versions of the ESS and Economy models.

a. State Equations

Production Subsystem. This is a combination of state equations (1.1a) and (3.4a) for energy and nonenergy sectors respectively in their simplified form (we describe depreciation of the capacities in the same way for both equations):

$$y_E(t+1) = (I - \Delta_E(t))y_E(t) + v_E(t) \tag{4.1}$$

$$y_{NE}(t+1) = (I - \Delta_{NE}(t))y_{NE}(t) + v_{NE}(t) \tag{4.2}$$

with intial states

$$y_E(0) = y_E^0 \tag{4.3}$$

$$y_{NE}(0) = y_{NE}^0 \quad . \tag{4.4}$$

Here $y_E(t)$ and $y_{NE}(t)$ are vectors of production capacities for energy and nonenergy sectors, $v_E(t)$ and $v_{NE}(t)$ are the increases of these capacities in time period t.

Energy Resources Consumption Subsystem. To describe the accumulation consumption of primary energy resources we will first use the equation (1.5a) (instead of the detailed version of Problem 2.1:

$$z_E(t+1) = z_E(t) + Q_E(t)\, u_E(t) \quad ; \qquad (4.5)$$

$$z_E(0) = z_E^0 \qquad (4.6)$$

$$0 \leq z_E(t) \leq \bar{z}_E(t) \qquad (4.7)$$

where

$z_E(t)$ is the vector of cumulative amounts of primary energy resources extracted at the beginning of time period t;

$u_E(t)$ is the vector of activities in the energy sector,

upper limits $\bar{z}_E(t)$ may be estimated from the resource model (Section 2).

b. Constraints

The most important constraint in the model is the balance between the production of goods and their consumption (Bill-of-Goods Balance). Like in the simplified version of the economy model (Problem 3a) we neglect the possibility to stock goods, thus considering the static form of these conditions:

for energy output (upper index "E" for matrices):

$$-A_{NE}^{E}(t)u_{NE}(t) + (I - A_E^E(t))u_E(t) =$$

$$= B_{NE}^{E}(t)v_{NE}(t) + B_E^E(t)v_E(t) + w_E(t) + s_E(t) \qquad (4.8)$$

for nonenergy products (upper index "NE" for matrices):

$$(I - A_{NE}^{NE}(t))u_{NE}(t) - A_E^{NE}\, u_E(t) =$$

$$= B_{NE}^{NE}(t)v_{NE}(t) + B_E^{NE}(t)v_E(t) + w_{NE}(t) + s_{NE}(t) \qquad (4.9)$$

We also have production capacity constraints for energy sectors:

$$u_E(t) \leq y_E(t) \tag{4.10}$$

and for nonenergy sectors

$$u_{NE}(t) \leq y_{NE}(t) \tag{4.11}$$

(see (1.4) and (3.9), respectively).

Labor availability constraints (3.7) are written in the form:

$$L_{NE}(t)\, u_{NE}(t) + L_E(t)\, u_E(t) \leq \ell(t) \tag{4.12}$$

and for WELMM factors (cf. (3.8)):

$$R_{NE}(t)\, u_{NE}(t) + R_E(t)\, u_E(t) \leq r(t) \quad . \tag{4.13}$$

Final consumption constraints (3.14) can be written as

$$w_E(t) \geq g_E(t)\, \omega(t) \tag{4.14}$$

$$w_{NE}(t) \geq g_{NE}(t)\, \omega(t) \quad . \tag{4.15}$$

where the given vectors $g_{NE}(t)$ and $g_E(t)$ specify profiles of final consumption for nonenergy and energy products, respectively.

Evidently, all the variables are nonnegative:

$$u_{NE}(t) \geq 0 \;\; ; \;\; u_E(t) \geq 0 \;\; ; \;\; v_{NE}(t) \geq 0 \;\; ; \;\; v_E(t) \geq 0$$

$$y_{NE}(t) \geq 0 \;\; ; \;\; y_E(t) \geq 0 \;\; ; \;\; \omega(t) \geq 0 \quad . \tag{4.16}$$

c. Statement of the Problem

Finally, we obtain the following optimization problem:

Problem 4.1. Given the state equations

$$y_E(t+1) = (I - \Delta_E(t))y_E(t) + v_E(t) \tag{4.1}$$

$$y_{NE}(t+1) = (I - \Delta_{NE}(t))y_{NE}(t) + v_{NE}(t) \tag{4.2}$$

with initial states

$$y_E(0) = y_E^0 \tag{4.3}$$

$$y_{NE}(0) = y_{NE}^0 \quad . \tag{4.4}$$

Find controls $\{v_E(t)\},\{v_{NE}(t)\}$ and $\{u_E(t)\},\{u_{NE}(t)\}$ and corresponding trajectories $\{y_E(t),y_{NE}(t)\}$, which satisfy constraints:

- bill-of-goods constraints:

$$(I - A_E^E(t))u_E(t) - A_{NE}^E(t)u_{NE}(t) = B_E^E(t)v_E(t) + B_{NE}^E(t)v_{NE}(t) + + w_E(t) + s_E(t); \tag{4.8}$$

$$-A_E^{NE}(t)u_E(t) + (I - A_{NE}^{NE}(t))u_{NE}(t) = = B_E^{NE}(t)v_E(t) + B_{NE}^{NE}(t)v_{NE}(t) + w_{NE}(t) + s_{NE}(t) \tag{4.9}$$

- production capacity constraints:

$$u_E(t) \le y_E(t) \tag{4.10}$$

$$u_{NE}(t) \le y_{NE}(t) \tag{4.11}$$

- primary energy resources constraints:

$$z_E(t+1) = z_E(t) + Q_E^T(t)u_E(t) \tag{4.5}$$

$$z_E(0) = z_E^0 \tag{4.6}$$

$$z_E(t) \le \overline{z}(t) \tag{4.7}$$

- labor constraints:

$$L_E(t)u_E(t) + L_{NE}(t)u_{NE}(t) \leq \ell(t) \qquad (4.12)$$

- WELMM constraints:

$$R_E(t)u_E(t) + R_{NE}(t)u_{NE}(t) \leq r(t) \qquad (4.13)$$

- final consumption constraints:

$$w_E(t) \geq g_E(t)\omega(t) \qquad (4.14)$$

$$w_{NE}(t) \geq g_{NE}(t)\omega(t) \qquad (4.15)$$

- nonnegativity constraints:

$$u_E(t) \geq 0 \quad u_{NE}(t) \geq 0 \quad v_E(t) \geq 0 \quad v_{NE}(t) \geq 0$$

$$y_E(t) \geq 0 \quad y_{NE}(t) \geq 0 \quad \omega(t) \geq 0 \qquad (4.16)$$

which maximize the objective function*)

$$J = \sum_{t=0}^{T-1} \beta(t)\,\omega(t) \quad . \qquad (4.17)$$

Problem 4.1 again is a DLP model. Its solution gives, in principle, possibilities to investigate the interactions between (a detailed) energy and nonenergy sectors of an economy. As mentioned above, we can solve Problem 4.1 as a whole DLP problem, or in an iterative mode, giving special attention to the links between ESS and economy parts of the integrated model.

*) This objective function is chosen only for illustrative purposes. Many other objectives are of interest for this integrated model.

Clearly, in the same way the resources model (Problem 2.1) can be included in the integrated model (instead of the simplified equations (4.5 - 4.7)). We will not, however, consider it here.

In the integrated model there is an important feature which cannot be seen explicitly from the matrix notations of Problem 4.1. Practically, all the individual models which are to be integrated into a system may have different levels of aggregation. In fact, when we investigate the influence of ESS on economy development, the ESS model should be presented much more in detail than the economy model. In this case, a special model is to be developed which determines the influence (impact) of energy development upon the economy as a whole (see below).

Therefore having in view the linkage of the energy-resources-economy models, one has to take into account first, the means of linkage (machine or man-machine), and secondly, the level of aggregation and specifics of each individual model.

4.2. Iterative Approach

Now we consider the iterative interaction between ESS and economy model. The general scheme is the following.

Considering the integrated model described above (Problem 4.1), one can see that it is basically the economy model (Problem 3.1), partitioned on energy (E) and nonenergy (NE) sectors. On the other hand, it includes as a part the ESS model. In fact, equations (4.1), (4.3), (4.5) - (4.7), (4.11) and (4.14) are the same as in the Problem 1.1 formulation.

We denote by

$$d_E(t) = A^E_{NE}(t)u_{NE}(t) + B^E_{NE}(t)v_{NE}(t) + w_E(t) + s_E(t) \quad (4.18)$$

the demand for secondary energy, and by

$$D_E(t) = (I - A^E_E(t)) \quad . \quad (4.19)$$

Then one can rewrite (4.9) as

$$D_E(t)u_E(t) = d_E(t) + B_E^E(t)v_E(t) \qquad (4.20)$$

which is similar to the demand constraints (1.11) of the ESS model because of the smallness of the last right term.

Let's denote the requirements of ESS in non-energy products by

$$f_E^{NE}(t) = B_E^{NE}(t)v_E(t) + A_E^{NE}(t)u_E(t) \quad . \qquad (4.21)$$

Taking into account that the requirements of nonenergy products for operation and maintenance of energy production (the second term on the right side of (4.21)) are small in comparison with the requirements for construction (the first term), one can see from (4.21) and (1.9), that

$$F(t) = B_E^{NE}(t) \quad . \qquad (4.22)$$

Let us have some initial estimate of the energy demand $\bar{d}_E(t)$ for the given planning period $0 \le t \le T-1$. Solving the ESS model (Problem 1.1) for this demand, we can calculate the required increase in capacities $\bar{v}_E(t)$ of ESS and the corresponding values of production capacities $\bar{y}_E(t)$ and its output (intensities) $\bar{u}_E(t) \le \bar{y}_E(t)$.

The requirement of ESS in non-energy resources $\bar{f}_E^{NE}(t)$ is calculated from (4.21). Now we can solve the economy model (Problem 3.1) or the integrated model (Problem 4.1) with fixed $\bar{u}_E(t)$, $\bar{v}_E(t)$, $\bar{y}_E(t)$, subject to a certain set of assumptions about future development of the entire economy.

This solution yields intensities (gross outputs) $\bar{u}_{NE}(t)$ and additional capital investments $\bar{v}_{NE}(t)$ in nonenergy sectors as well as a new value of the corresponding demand for energy $\bar{\bar{d}}_E(t)$ (calculated from (4.18)). If old $\bar{d}_E(t)$ and new $\bar{\bar{d}}_E(t)$ values of energy demand coincide, the procedure terminates, in the opposite case we have to repeat the iteration with a modified demand.

Generally speaking, the solution obtained in such a way is not an optimal solution for Problem 4.1, but a satisfactory one because it satisfies all constraints of the problem and optimizes (separately) two objectives ((1.12) and (3.15), for example) for energy and nonenergy sectors.

To obtain an optimal solution of the whole Problem 4.1 in such an iterative way, one may use different decomposition methods. In this case the dual variables (marginal estimates), obtained from the solution of the economy model, define the corresponding objective function for the ESS model (instead of (1.12)). Convergences properties depend on the particular implementation of the procedure. It should also be noted that for the implementation of this procedure the economy model should be sufficiently disaggregated in order to provide the ESS model with the shadow prices of sufficient detalization.

But in practice a single optimal solution of Problem 4.1 is not very valuable -- no matter whether it has been obtained "automatically" by the simplex method, applied to Problem 4.1, or in some iterative way. Clearly, such a complex system requires a man-machine iterative procedure with an energy-economic analysis of separate iterations. Let us now look at the points where "man" interference is appropriate. They are:

- Changing the objective function in the whole Problem 4.1 and in the ESS model (Problem 1.1). (In fact, this is a vector-optimization problem [13]).

- Determining energy demand $d_E(t)$ not from the equation (4.18), but from a special energy demand model (e.g. [54]).

- Determining requirements in non-energy resources $f_E^{NE}(t)$ for ESS by a special model [55].

- Changing the parameters of the model (especially associated with assumption on technology innovation and profile of consumption).

Many of these interferences may be considered as attempts to take into account nonlinearities of the system.

4.3. Discussion

a. *PILOT Model* [50-53]. This model has been developed by G. Dantzig and S. Parikh at the Stanford University. This is a DLP model on a pilot scale that describes in physical terms technological interactions within the sectors of the U.S. economy including a detailed energy sector.

The structure of the model is quite similar to the model of Problem 4.1. Dynamic equations include capacity balance constraints, manpower skill adjustment limit constraints and those related to raw energy reserves, cumulative exploration and production and intermediate energy stocks.

The capacity balance is equivalent to (4.1) and (4.2). Manpower skill adjustment constraints specify the educational and training capacities and are written in the form (cf. (1.27) and (1.34) in DESOM model):

$$L(t+1) \leq \beta L(t) \quad ,$$

where the manpower vector L(t) is partitioned on skill groups.

The resources constraints are similar to constraints (2.24) and (2.25) and are intended to keep an accurate record of the energy reserves, cumulative exploration (and production) and stocks.

The (static) constraints represent energy demand requirements, energy processing and operating capacity limitations, and environmental aspects. The linkage between energy and nonenergy sectors is given by the bill-of-goods constraints (4.8), (4.9).

The objective of the model maximizes the discounted vector bill-of-goods received per person, summed over time. It can be expressed as

$$J = \sum_{t=1}^{T} \lambda(t) \, (M(t),p(t)) \quad ,$$

where the vector M(t) represents the consumption levels and vector p(t) is the population distribution over different income levels.

The detailed model will include 87 - sector input-output matrix, the energy sector may be modelled by approximately 150 equations per period. Thus, an order of magnitude for the number of constraints for each time period in an integrated model, with a reasonable level of details, may be about 400: 87 for industrial activity, 2 times 87 for capacity constraints and about 150 for detailed energy sector. A 20-25 period model (e.g. a 75 - year triannial model) would therefore have approximately 8,000 to 10,000 constraints.

As noted in [53] these LP models would be among the largest models built to date. Therefore as a first step, a much smaller model which "incorporates many, if not all, of the essential features of its larger counterpart" [53] has been attempted. This pilot model is expected to have about 125 equations per period. For a 30-year triannial model, there will be about 1.250 to 1.400 equations. Initially, the model will be solved using straight simplex method of the MPS/370 system.

b. *IMPACT Model*. This model is an extension of the model developed by Yu. Kononov and V. Tkachenko at the Siberian Power Institute [55-56]. The model serves to investigate the influence of long-term changes in technology, structure and rates of energy development upon other branches of the national economy.

The model is described by the following equations (for details, see [56]).

Direct requirements of ESS in nonenergy products:

$$f_E^{NE}(t) = A_E^{NE}(t)u_E(t) + \sum_{\tau=0}^{\bar{\tau}} B_E^{NE}(t-\tau)v_E(t-\tau) \quad . \qquad (4.23)$$

If we neglect by time lags $\bar{\tau}$ in construction, then the equation

(4.21) is obtained. In the original version of the IMPACT model [56] a carried forward presentation is used, that is

$$f_E^{NE}(t) = A_E^{NE}(t)u_E(t) + \sum_{\tau=t}^{t+\bar{\tau}} \tilde{B}_E^{NE}(\tau-t)v_E(\tau) \quad (4.23a)$$

where the matrix $\tilde{B}_E^{NE}(\tau-t)$ denotes the contribution for the construction of additional capacities to be put into operation at time period $\bar{\tau}$, where $t \leq \tau \leq t + \bar{\tau}$.

Total (direct and indirect) products (material, equipment, etc.) requirements are derived from equation (4.9):

$$(I - A_{NE}^{NE}(t))u_{NE}(t) = B_{NE}^{NE}(t)v_{NE}(t) + f_E^{NE}(t) + w_{NE}(t) + s_{NE}(t) \quad (4.24)$$

Using $v_{NE}(t)$ and $v_E(t)$ one can also calculate the total direct and indirect capital investments. The model includes also several equations for evaluating direct and indirect expenses of WELMM resources.

The model operates in the following way. Problem 1.1 for the given demand $\bar{d}_E(t)$ of secondary energy is solved. Initially, nonenergy resources constraints (1.9) are not taken into account. The solution of the problem gives the values $\bar{u}_E(t)$ and $\bar{v}_E(t)$, which are drivers for the IMPACT model. Using (4.23) one can calculate $\bar{f}_E^{NE}(t)$ for given $\bar{u}_E(t)$ and $\bar{v}_E(t)$. Substituting $\bar{f}_E^{NE}(t)$ into the equation (4.24) and solving linear equations (4.24) with additional conditions [56]:

$$v_{NE}(t) = \max\{\min_{\tau<t} (u_{NE}(t) - u_{NE}(\tau));0\} \quad (4.25)$$

one can find the indirect investment $v_{NE}(t)$ into economy which ESS needs to meet the given demand $\bar{d}_E(t)$.

We are only describing the general scheme of the IMPACT model. The particular implementation of this model depends on the specifics of the ESS and economy models to be linked.

c. SPI Model [9]. The interactions between energy and nonenergy sectors of the national economy were also analyzed at the Siberian Power Institute of the Sibirian Branch of the USSR Academy of Sciences. For this analysis a special multisector model has been developed. The model describes the interactions of the energy (E) sector with those nonenergy (NE) sectors which directly or indirectly influence the energy sector. There are 8 such nonenergy sectors with 31 types of products.

The mathematical formulation of the model is close to Problem 4.1 (below we change the notations in comparison with the original version in [9]).

The development of the production subsystem is described by the state equations which are similar to (4.1) and (4.2)

$$\sum_e y_{ieE}(t+1) = \sum_e \delta_i(t)\, y_{ieE}(t) + \sum_e v_{ie}(t)$$

$$y_{iNE}(t+1) = \delta_i(t) y_{iNE}(t) + v_{iNE}(t) \quad .$$

The balance of goods is written in the dynamic form (cf. (3.1) and (4.8), (4.9)):

for nonenergy sectors:

$$z_{iNE}(t+1) = z_{iNE}(t) + a_{iNE}(t) y_{iNE}(t) - \sum_j a_{ijNE}(t)\, y_{jNE}(t) -$$

$$- \sum_j \sum_\tau b_{ijNE}(t+\tau)\, v_{jNE}(t+\tau) - w_{iNE}(t) - s_{iNE}(t)$$

for energy sectors:

$$z_{iE}(t+1) = z_{iE}(t) + \sum_e a_{ieE}(t)\, y_{ieE}(t) - \sum_j a_{ijE}(t)\, y_{jNE}(t)$$

$$- w_{iE}(t) - s_{iE}(t) \quad .$$

(For the energy sector the stocks are fuels.)

Here $z_{iNE}(t)$, $z_{iE}(t)$ are the stocks of production for non-energy and energy sectors respectively at time period t; $a_{iNE}(t)$ and $a_{ieE}(t)$ are loading coefficients of production capacities, hence

$$u_{iNE}(t) = a_{iNE}(t)y_{iNE}(t)$$

$$u_{ieE}(t) = a_{ieE}(t)y_{ieE}(t)$$

where $u_{iNE}(t)$, $u_{ieE}(t)$ are the production levels (gross outputs) at time t:

The subindex e for the energy sector corresponds to the different technology of energy production. Thus the energy sector is represented in a more disaggregated form in comparison to the nonenergy sectors of the model.

As in the IMPACT model there is used time forward ($\tau > 0$) presentation of the requirements for construction.

Constraints on labor and other limited resources are given in a similar way as (4.12) and (4.13):

$$\sum_i \sum_e r^E_{\nu ie}(t)y_{ieE}(t) + \sum_e \sum_\tau r^E_{\nu ie}(t,\tau)\, v_{ieE}(\tau) +$$

$$+ r^{NE}_{\nu i}(t)y_{iNE}(t) + \sum_\tau r^{NE}_{\nu i}(t,\tau)\, v_{iNE}(\tau)] \leq r_\nu(t) \quad .$$

The model is solved in an iterative mode.

5. DLP Canonical Form

Considering the models described above, one can see that all of them can be reduced to a single canonical form [18,19].

Problem 5.1. Given the state equations

$$x(t+1) = A(t)x(t) + \sum_{\tau=0}^{t} B(t-\tau)\, u(t-\tau) \quad , \tag{5.1}$$

with initial conditions

$$x(0) = x^0 \quad ; \quad u(t-\tau) = u^0(t-\tau) \qquad (0 \le \tau \le t-1) \tag{5.2}$$

and constraints

$$G(t)x(t) + D(t)u(t) \le f(t) \tag{5.3}$$

$$x(t) \ge 0 \quad ; \qquad u(t) \ge 0 \quad . \tag{5.4}$$

Find control $u = \{u(0),\ldots,u(T-1-\tau)\}$ and the corresponding trajectory $x = \{x(0),\ldots,x(T)\}$, which maximize the objective function

$$J(u) = (a(T),x(T)) + \sum_{t=0}^{T-1} [(a(t),x(t) + (b(t),u(t))] . \tag{5.5}$$

Here $\{u(t)\}$ are control variables, $\{x(t)\}$ are state variables.

One can see that all the models considered in the previous sections can be either reduced to this canonical DLP problem, or the methods developed for the canonical problem can be directly applied to these models. Problem 5.1. represents a DLP problem in a canonical form and can be viewed either as some staircase linear programming problem or as optimal control theory problem. Hence, both methods -- linear programming and control theory -- can be applied to the solution of Problem 5.1. A survey of these methods is given in [18,19].

CONCLUSION

Different individual energy-resources-economy models and their linkages into a whole system have been discussed in the preceding sections. It has been shown that all these models are reduced to a canonical form of the DLP problem. Therefore, a unified methodological approach can be developed to analyze and solve these models. Below we discuss briefly several further directions of methodological analysis of energy models.

a. Energo-Economic Analysis. In this paper we analyzed common mathematical features of the models. Of great interest is the analysis of the models -- objective functions, constraints, level of aggregation, uniform data bank, etc. -- from the economy and energy technology point of view.

b. Vector Optimization Methods. Clearly, a single objective function is not a realistic modelling of the energy systems. A discussion of this problem can be found, for example, in [57].

c. Duality Theory. The shadow prices which are the solution of the dual problem provide a valuable tool for a marginal analysis of the model. Duality theory for the canonical DLP Problem 5.1 is given in [58]. The application of this theory to energy models, discussed in this paper, would be useful in many respects.

d. Numerical Solution Methods. As mentioned above, Problem 5.1 is an LP problem. Hence, standard LP codes can be (and have been) applied for the solution of energy models. Special methods which take into account the specifics of DLP problems have also been developed ([59 - 64], see also references in [19]).

Experimental codes of these algorithms show good results in comparison to the standard simplex methods [60,61].

e. Post-optimal Analysis. Of great practical interest are methods of post-optimal analysis of solution including parametric

DLP methods, sensitivity and stability analysis. A general theory of linear and quadratic parametric programming has been developed recently in [65].

f. Implementation of the Solution. The implementation of the optimal solution has no less importance than the finding of this solution. We have to mention here the questions of realization of optimal solution as a program (that is, as a time sequence of controlling actions) or as a feedback control (that is, the current control action is determined by the current state of the system).

g. Linkage of the Models. The development of linkage methods of individual models is becoming probably the most important issue for the time being. Here we can single out:

- relations between long-, medium- and short-term energy models (for example, how the optimal solution of an aggregated long-term model relates to the solution of a more detailed short-term model);

- methods of linkage of energy-resources-economy individual models into an integrated energy model for a nation or a region (some of these methodological questions were discussed in Section 4 of the paper);

- methods of linkage of national energy models into a world model.

Discussion of some methodological and computer implementation methods of models linkage can be found for example in [1,9,10,11, 12,23,66 - 68].

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