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MIGRATION AGE PATTERNS: MEASUREMENT AND ANALYSIS

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PREFACE

Interest in human settlement systems and policies has been a central part of urban-related work at IIASA since its inception. From 1975 through 1978 this interest was manifested in the work of the Migration and Settlement Task, which was formally concluded in November 1978. Since then, attention has turned to dissemination of the Task's results and to the conclusion of its comparative study, which is carrying out a comparative quantitative assessment of recent migration patterns and spatial population dynamics in all of IIASA's 17 NMO countries.

This paper is a part of the Task's dissemination effort and is the first of several to focus on the age patterns of migration exhibited in the data bank assembled for the comparative study. It focuses on the mathematical description of observed migration schedules, the analysis of their age profiles and the study of how these profiles are influenced by the age composition of the population in the region of origin.

Reports, summarizing previous work on migration and settlement at IIASA, are listed at the back of this paper.

Andrei Rogers
Chairman
Human Settlements
and Services Area

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ABSTRACT

This paper develops support for three principal points. First, the profiles of age-specific gross migration rates all over the world have a fundamental regularity that can be captured and expressed in mathematical form. Second, this mathematical "model" schedule summarizes the empirical regularity in a way that permits analytical examinations to be carried out regarding the fundamental properties of the migration age profiles. Finally, migration rate schedules may be conveniently decomposed to illuminate the influences on migration patterns of migration level, the age composition of migrants, and the age composition of the population in the region of origin.

CONTENTS

MIGRATION AGE PATTERNS: MEASUREMENT AND ANALYSIS

Migration studies have in the past exhibited a curiously ambivalent position with regard to the measurement of migration. This ambivalence is particularly striking because of the contrast it poses with respect to the corresponding studies of mortality and fertility (natality)--literature that is richly endowed with detailed discussions of measurement problems. Haenszel (1967) attributes this paradox to the strong influence of Ravenstein's early contributions to migration analysis:

> Work on migration and population redistribution appears to have been strongly influenced by the early successes of Ravenstein in formulating "laws of migration". Subsequent papers have placed a premium of the development and testing of new hypotheses rather than on descriptions of facts and their collation...This is in contrast to the history of vital statistics. While Graunt more than two centuries before Ravenstein, had made several important generalizations from the study of "bills of mortality" in London, his successors continued to concentrate on descriptions of the forces of mortality and natality by means of rates based on populations at risk (Haenszel, 1967, p. 260).

It is natural to look to the state of mortality and fertility measurement for guidance in developing measures of migration. Like mortality, migration may be described as a process of interstate transfer; however, death can occur but once, whereas migration is a potentially repetitive event. This suggests the adoption of a fertility analog; but migration's definitional dependence on spatial boundaries introduces measurement difficulties that do not arise in fertility analysis.

Migration measurement can usefully apply concepts borrowed from both mortality and fertility analysis, modifying them where necessary to take into account aspects that are peculiar to spatial mobility. From mortality analysis, migration can borrow the notion of the life table, extending it to include increments as well as decrements, in order to reflect the mutual interaction of several regional cohorts (Rogers, 1973a,b, and 1975; Rogers and Ledent, 1976). From fertility analysis, migration can borrow well-developed techniques for graduating age-specific schedules (Rogers, Raquillet, and Castro, 1978). Fundamental to both "borrowings" is a workable definition of migration rate.

Migration Rates and Schedules

During the course of a year, or some such fixed interval of time, a number of individuals living in a particular community change their regular place of residence. Let us call such persons mobiles to distinguish them from those individuals who did not change their place of residence, i.e., the nonmobiles. Some of the mobiles will have moved to a new community of residence; others will simply have transferred their household to another residence within the same community. The former may be called movers, the latter are relocators. A few in each category will have died before the end of the unit time interval.

Assessing the situation with respect to the start and the end of the unit time interval, we may divide movers who survived to the end of the interval into two groups: those living in the same community of residence as at the start of the interval and those living elsewhere. The first group of movers will be referred to as surviving returnees, the second will be called surviving migrants. An analogous division may be made of movers who died before the end of the interval to define nonsurviving returnees and nonsurviving migrants.

A move, then is an event: a separation from a community. A mover is an individual who has made a move at least once during a given interval of time. A migrant (i.e., a surviving or nonsurviving migrant), on the other hand, is an individual who at the end of a given time interval no longer inhabits the same community of residence as at the start of the interval.

(The act of separation from one state is linked to an addition to another). Thus paradoxically, a multiple mover may be a nonmigrant by our definition. This is illustrated by life line C in the multiregional Lexis diagram in Figure 1. Because this particular mover returned to the initial place of residence before the end of the unit time interval, no "migration" took place.*

The focus on migrants instead of on movers reflects the need at some point to calculate probabilities. As Haenszel (1967) has observed:

> the label "migration" had been applied to two related, but different, universes of discourse--a population of "moves" and a population of "people who move". A universe of "moves" can be generated by simultaneous classification of individuals by initial and subsequent place of residence, and the data provide useful descriptions of population redistribution. Such results, however, do not lend themselves to probability statements. Probabilities can be computed only for denumerable populations at risk, whether they be people, telephone poles, or transistors (Haenszel, 1967, p.254).

The simplest and most common measure of migration is the crude migration rate, defined as the ratio of the number of migrants, leaving a particular population located in space and time, to the average number of persons (more exactly, the number of person-years) exposed to the risk of becoming migrants.**

Because migration is highly age selective, with a large fraction of migrants being the young, our understanding of migration patterns and dynamics is aided by computing migration rates for each single year of age. Summing these rates over all ages of life gives the gross migraproduction rate (GMR), the migration analog of fertility's gross reproduction rate.

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* We define migration to be the transition between states experienced by a migrant.

** Because data on nonsurviving migrants are generally unavailable, the numerator in this ratio generally excludes them.

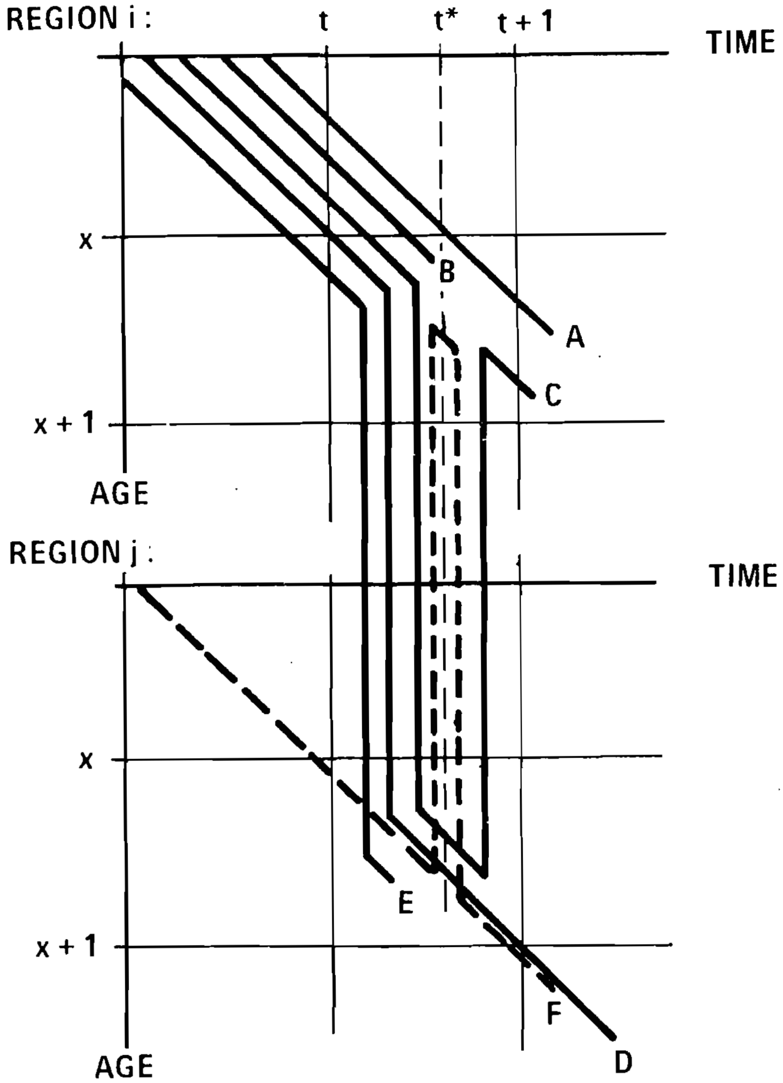


Figure 1. Two-region Lexis diagram.

Figure 2 indicates that age-specific annual rates of residential mobility among whites and blacks in the U.S. during 1966-1971 exhibited a common profile. Mobility rates among infants and young children mirrored the relatively high rates of their parents--young adults in their late twenties. The mobility of adolescents was lower, but exceeded that of young teens, with the latter showing a local low point around age fifteen. Thereafter mobility rates increased, attaining a high peak at about age twenty-two and then declining monotonically with age to the ages of retirement. The mobility levels of both whites and blacks were roughly similar, with whites showing a gross migraproduction rate of about 14 moves and blacks one of approximately 15, over a lifetime undisturbed by mortality before the end of the mobile ages.

Although it has been frequently asserted that migration is strongly sex selective, with males being more mobile than females, recent research indicates that sex selectivity is much less pronounced than age selectivity, and that it is less uniform across time and space. Nevertheless, because most models and studies of population dynamics distinguish between the sexes, most migration measures do also.

Figure 3 illustrates the age profiles of male and female migration schedules in four different countries at about the same point in time between roughly comparable areal units: communes in the Netherlands and Sweden, voivodships in Poland, and counties in the U.S. The migration levels for all but Poland are similar, varying between 3.5 and 5.3 moves per lifetime; and the levels for males and females are roughly the same. The age profiles, however, show a distinct, and consistent, difference. The high peak of the female schedule always precedes that of the male schedule by an amount that appears to approximate the difference between the average ages at marriage of the two sexes.

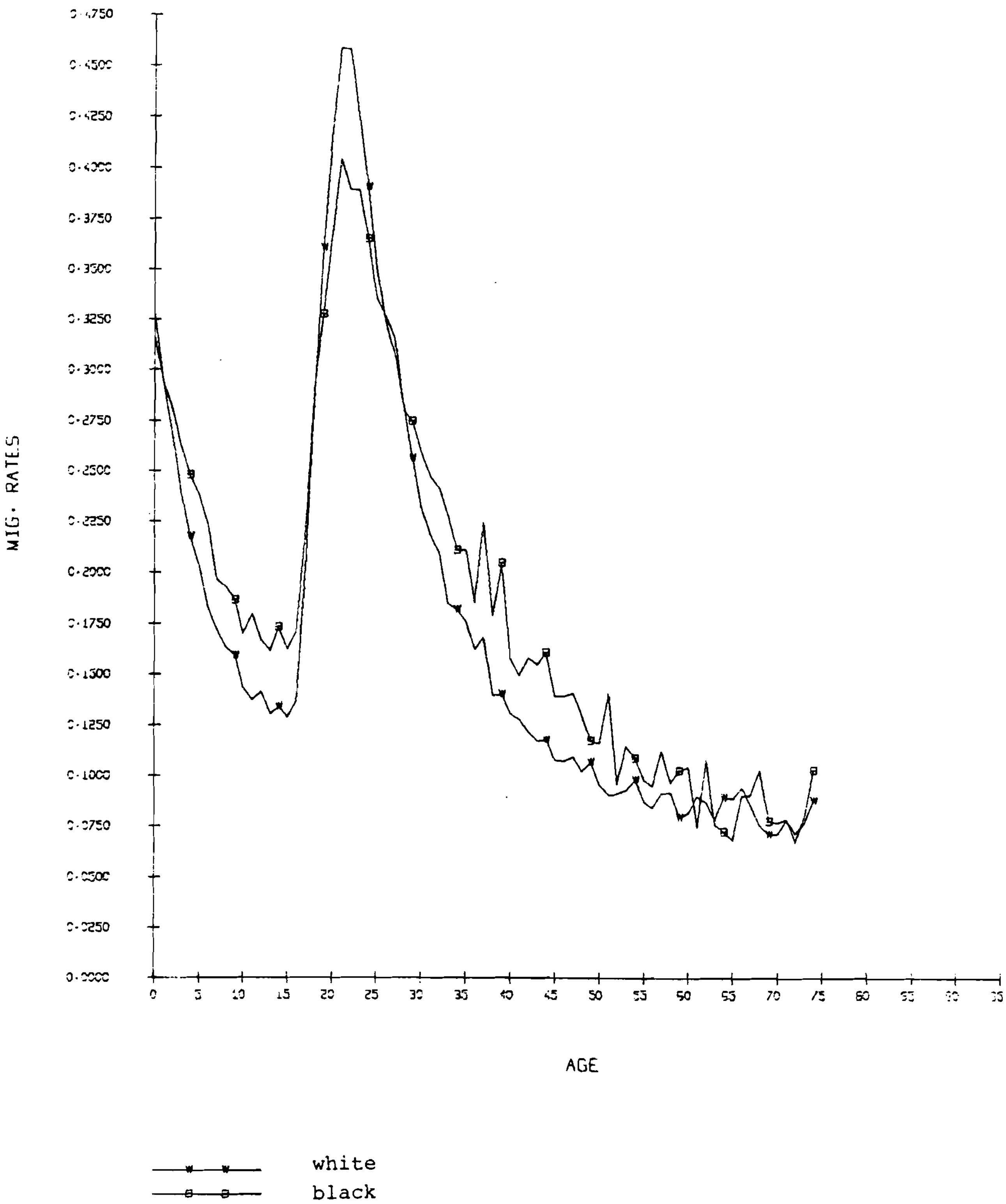
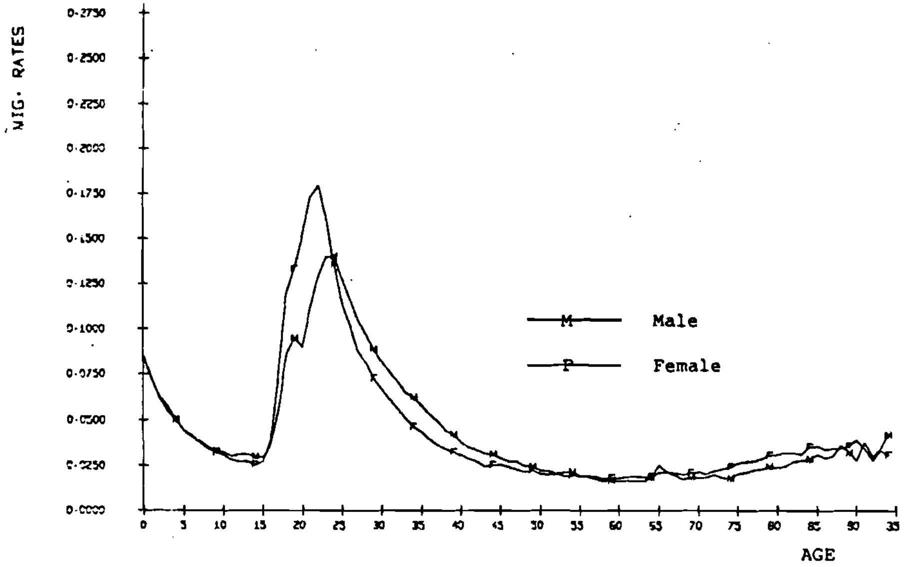
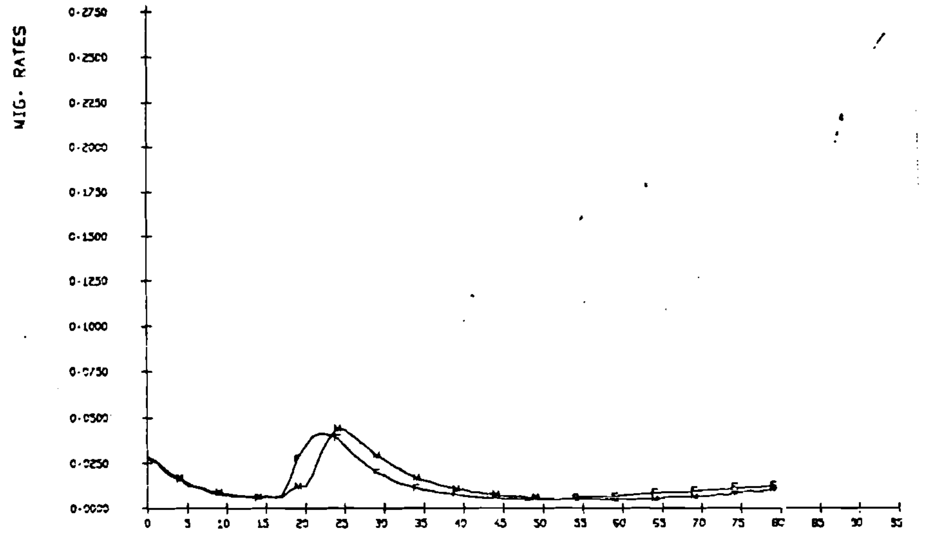


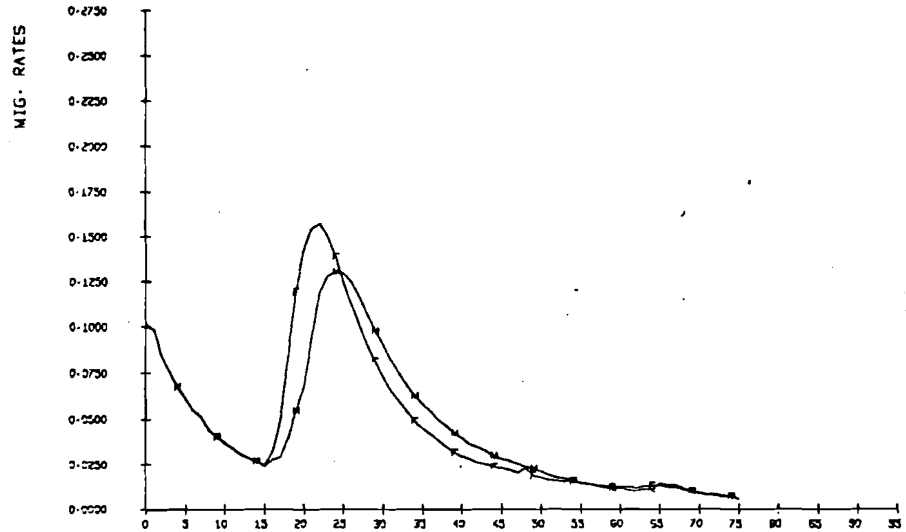
Figure 2. Observed annual migration rates by color and single years of age: The United States, 1966-1971.



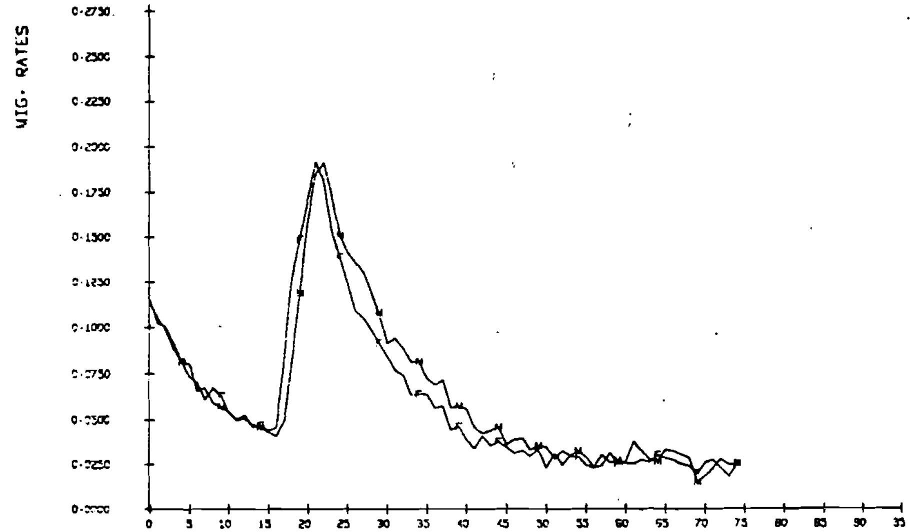
A. The Netherlands, 1972



B. Poland, 1973



C. Sweden, 1968-1973



D. The United States, 1966-1971

Figure 3. Observed annual migration rates by sex and single years of age: the Netherlands, Poland, Sweden, and the United States, around 1970.*

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*Intercommunal migration in the Netherlands and Sweden; intervoivodship migration in Poland; intercounty migration in the United States.

Under normal statistical conditions, point-to-point movements are aggregated into streams between one civil division and another; consequently, the level of interregional migration depends on the size of the areal unit selected. Thus, if the areal unit chosen is a minor civil division such as a county or a commune, a greater proportion of residential location will be included as migration than if the areal unit chosen is a major civil division such as a state or a province.

Figure 4 presents the age profiles of female mobility and migration schedules as measured by different sizes of areal units: 1) all moves from one residence to another, 2) changes of residence within county boundaries, 3) migration between counties, and 4) migration between states. The respective four gross migraproduction rates (GMRs) are 14.3, 9.3, 5.0, and 2.5, respectively. The four age profiles appear to be remarkably similar, indicating that the regularity in age pattern persists across areal delineations of different sizes.

Finally, migration occurs over time as well as across space; therefore, studies of its patterns must trace its occurrence with respect to a time interval, as well as over a system of geographical areas. In general, the longer the time interval, the larger will be the number of return movers and nonsurviving migrants and, hence, the more the count on migrants will understate the number of inter-area movers (and, of course, also of moves). Philip Rees, for example, after examining the ratios of one-year to five-year migrants between the Standard Regions of Great Britain, found that

> the number of migrants recorded over five years in an interregional flow varies from four times to two times the number of migrants recorded over one year. (Rees, 1977, p.247).

A fundamental aspect of migration is its change over time. A time series of age-specific migration rates may be usefully set out in the form of a table with ages for rows and calendar

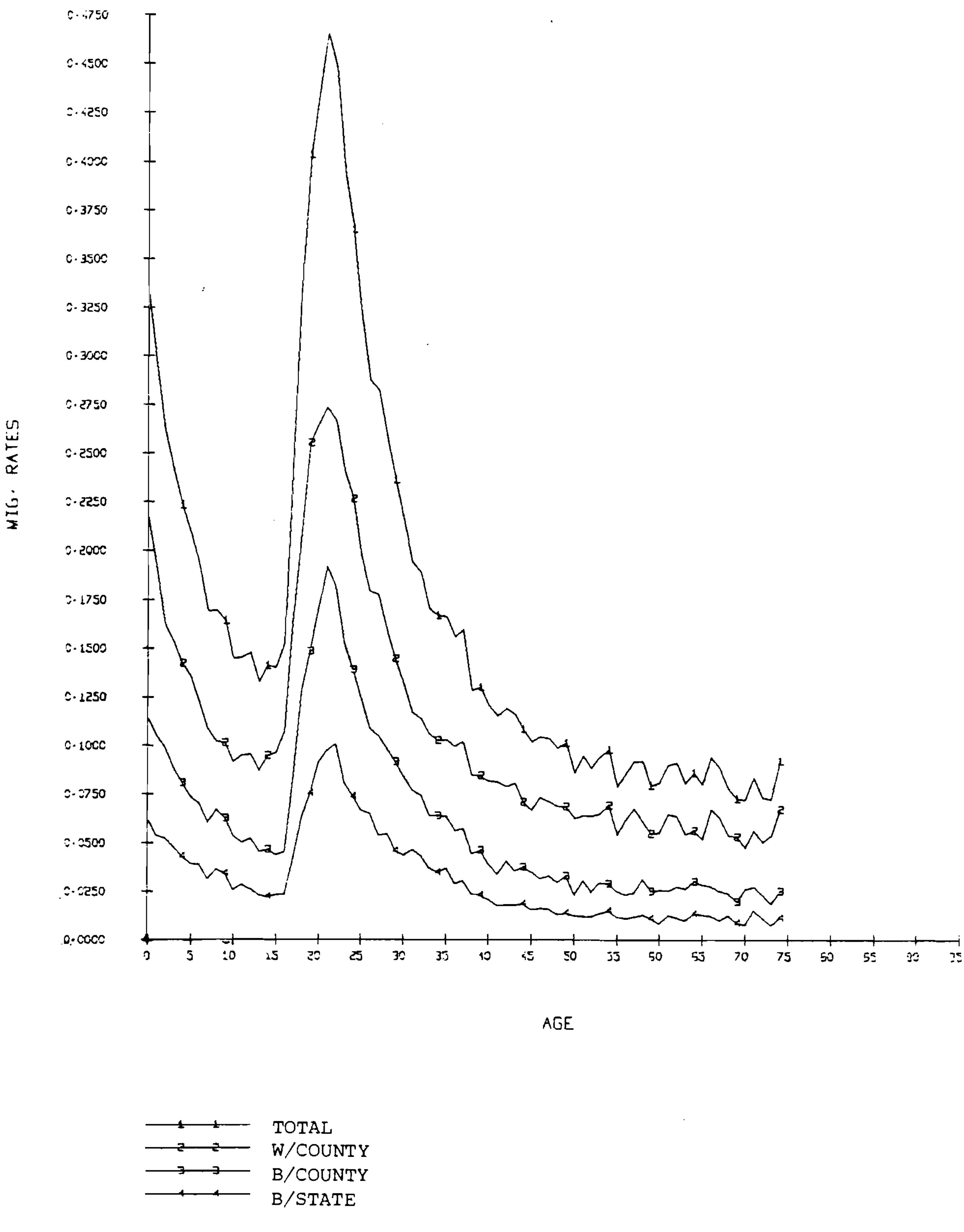


Figure 4. Observed female annual migration rates by levels of areal aggregation and single years of age: the United States, 1966-1971.

years for columns (i.e., paralleling the format of the Lexis diagram in Figure 1). Such a table yields two sets of summary indices of migration. The column sums give a time series of period gross migraproduction rates. Diagonal sums give cohort gross migraproduction rates (those of a cohort of individuals born in the same year). The two series of GMRs normally will differ, with the period series generally fluctuating more than the cohort series.

Table 1 sets out the schedules of female annual migration rates in the Netherlands for five consecutive years. These show a relatively stable level of intercommunal migration of roughly four moves per lifetime (undisturbed by mortality) and a mean age of about 36 years. Both measures exhibit a slight decline during the latter part of the five-year period.

If emigration and immigration may be ignored, then Table 1 also shows the first five years of a cohort's migration, namely, those born in 1972. Thus, that cohort's incomplete gross migraproduction rate calculated over the first five years of age is:

$$GMR_c = .0853 + .0778 + 0.0666 + .0576 + .0507 = .3380 \ .$$

This "cohort" measure may be contrasted with the five comparable period measures:

$$GMR_{72} = .3233 \qquad GMR_{73} = .3456$$

$$GMR_{74} = .3314 \qquad GMR_{75} = .3176$$

$$GMR_{76} = .3047$$

As Ryder (1964) has shown for the case of fertility, period and cohort reproduction rates will differ whenever the age distribution of childbearing varies from one cohort to another. Period gross migraproduction rates become inflated if the mean

Table 1. Observed female annual migration rates by single years of age: the Netherlands, migration between communes, 1972-1976.

AGE x \ TIME t	RATES PER THOUSAND 1972	1973	1974	1975	1976
Ages 0-50					
0	85.3	88.0	78.6	73.6	71.3
1	72.0	77.8	73.8	70.9	65.0
2	61.6	67.3	66.6	63.4	62.4
3	54.7	59.6	59.7	57.6	55.3
4	49.7	52.9	52.7	52.1	50.7
5	44.3	48.5	48.2	45.0	45.8
6	42.0	44.5	46.5	43.7	42.6
7	37.2	39.6	39.8	40.3	38.6
8	34.1	37.5	37.3	36.0	35.4
9	31.9	33.6	34.1	32.8	33.6
10	30.5	32.2	31.9	31.1	31.2
11	27.8	29.6	28.8	28.0	28.5
12	27.1	29.2	28.1	28.2	26.6
13	27.4	28.6	26.8	25.7	26.0
14	25.2	25.8	25.7	24.2	24.2
15	27.1	25.8	25.9	25.0	23.7
16	38.9	37.0	34.4	31.2	26.8
17	76.4	71.8	68.1	64.0	55.3
18	119.3	117.6	119.7	116.9	115.3
19	133.1	131.8	140.1	135.5	136.4
20	150.7	154.5	160.4	154.0	153.2
21	172.6	171.5	176.7	164.0	163.5
22	179.3	174.3	172.4	159.9	152.5
23	160.9	155.8	152.4	140.9	135.2
24	135.5	135.3	133.5	123.2	118.1
25	114.7	116.1	116.3	107.9	102.9
26	102.1	104.4	102.4	96.0	92.7
27	87.3	94.1	92.4	85.8	83.6
28	80.4	83.6	83.3	78.5	73.4
29	72.5	76.7	73.1	69.6	67.4
30	66.4	70.1	67.3	62.0	59.5
31	60.9	62.2	61.6	57.7	55.2
32	55.7	57.9	57.2	53.2	51.4
33	50.7	52.6	53.2	48.8	46.6
34	46.0	48.5	48.2	45.9	43.1
35	43.2	46.9	44.4	42.0	40.6
36	39.0	43.1	41.1	38.1	36.8
37	36.0	37.6	37.8	35.2	34.0
38	33.8	34.7	34.9	32.3	30.5
39	32.2	32.3	31.5	31.0	29.4
40	30.6	30.5	28.5	27.9	25.8
41	28.6	29.4	28.6	25.0	24.9
42	27.1	27.2	25.6	24.4	22.2
43	24.4	26.3	24.1	23.3	21.8
44	24.9	24.7	23.9	22.3	21.0
45	25.4	23.2	24.0	22.1	19.6
46	24.0	22.8	21.4	21.2	19.7
47	22.7	23.4	20.9	19.4	15.5
48	21.3	22.3	21.1	20.2	18.4
49	22.1	20.5	21.5	19.9	17.7
50	20.6	20.5	20.3	20.2	17.8

AGE x \ TIME t	RATES PER THOUSAND 1972	1973	1974	1975	1976
Ages 51-94					
51	20.2	20.7	20.4	19.0	18.9
52	20.1	21.9	20.1	19.4	18.4
53	19.0	20.9	20.8	20.2	16.9
54	19.0	20.3	18.9	18.4	17.3
55	19.1	18.9	18.9	19.0	17.5
56	18.9	19.2	19.0	17.8	17.7
57	18.8	18.8	21.1	19.0	17.6
58	17.6	19.7	18.7	19.3	17.3
59	18.1	18.4	19.7	19.4	18.6
60	18.2	20.0	21.1	19.5	19.0
61	19.0	20.2	21.0	19.1	18.6
62	19.1	19.1	20.0	19.8	17.2
63	18.3	19.4	18.6	18.7	17.8
64	18.7	19.7	20.6	19.3	18.4
65	21.0	22.1	21.1	20.5	19.4
66	21.5	21.4	22.3	19.2	17.3
67	20.8	21.5	20.5	19.3	17.1
68	20.3	19.9	19.7	18.1	16.7
69	21.0	20.7	20.9	18.2	16.6
70	21.6	20.9	22.0	20.6	17.6
71	20.6	21.2	21.5	19.7	16.5
72	22.1	21.0	24.0	20.4	17.8
73	25.2	22.8	22.3	22.1	18.1
74	23.9	23.8	23.3	21.7	18.8
75	26.4	24.9	24.0	22.9	18.9
76	26.9	24.5	25.6	25.1	21.3
77	27.5	27.3	27.9	25.3	20.2
78	28.9	28.0	27.9	25.1	21.6
79	30.7	30.5	28.3	26.6	22.0
80	31.1	29.6	32.3	27.4	25.5
81	32.4	29.9	30.8	31.0	24.0
82	32.0	31.0	29.9	29.7	24.8
83	30.9	32.2	31.0	29.5	25.8
84	34.9	31.6	35.6	30.7	29.2
85	35.6	32.0	34.8	30.4	29.5
86	33.8	34.7	31.1	29.2	28.5
87	34.3	31.2	32.7	27.8	30.9
88	34.6	33.8	34.6	28.2	27.5
89	35.6	33.1	33.4	35.0	27.4
90	39.1	42.0	34.8	32.2	29.0
91	34.1	29.1	35.0	31.2	27.0
92	27.6	34.0	32.5	26.6	33.6
93	33.2	34.2	36.8	28.4	28.1
94	30.7	40.0	26.2	23.4	22.4
GMR	4.16	4.23	4.10	3.93	3.74

age of migration declines monotonically from cohort to cohort. And, if declining economic conditions lead potential migrants to delay their migration, current period indices of migration level may decline only to be followed by a compensatory increase in the future.

The usefulness of a cohort approach in migration as in fertility analysis lies in the importance of historical experience to the explanation of current behavior. As Morrison (1970) points out, migration is induced by transitions from one stage of the life cycle to another, and "chronic" migrants may artificially inflate the migration rates of origin areas heavily populated with migration-prone individuals. Both influences on period migration rates are readily assessed by a cohort analysis.

It is the migration of a period, however, and not that of a cohort, that determines the sudden redistribution of a national population in response to economic fluctuations, and it is period migration rates that are needed to calculate spatial population projections.

Current period migration indices do not distinguish trend from fluctuation and therefore may be distorted; current cohort migration indices are incomplete. Thus it may be useful to draw on Ryder's (1964) translation technique to change from one to the other. As Keyfitz (1977, p.250) observes, the cohort and period moments in Ryder's formulae can "be interpreted, not as child-bearing, but as mortality, marriage, school attendance, income, or some other attribute of individuals". Migration is clearly such an attribute.

The importance of historical experience in interpreting and understanding current migration behavior led Peter Morrison (1970, p.9) to define the notion of staging as being "any linkage between a prior sequence and subsequent migration behavior". Morrison recognizes four kinds of staging: geographic, life-cycle, socioeconomic, and experiential. Geographical staging refers to return migration and to what is conventionally understood to mean "stage migration", i.e., the idea that migrants tend to move to places not very dissimilar from those they left behind. Life-cycle staging views migration as arising out of breaks in an individual's or household's life cycle, such as entry into the labor force, marriage, child rearing, retirement. Socio-economic staging sees migration sequences as being conditioned by sociostructural factors such as occupation, educational attainment, and income level. Finally, experiential staging refers to movement experience in terms of number of previous moves and duration since the last move. It is the "parity" dimension of migration analysis and will be referred to as "migrativity".

Calculations of migration rates of increasing specificity seek to unconfound the "true" migration rates from weights that reflect the arithmetical influence of the past. This process of measuring migration "at different levels of specificity of occurrence and exposure yields products which draw ever finer distinctions between current behavior and the residue of past behavior reflected in the exposure distribution at any time" (Ryder, 1975, p.10).

Such products may be weighted and aggregated to produce the "crude" rates of higher levels of aggregation. For example, the age-sex specific migration rate is a weighted aggregation with respect to the migration "parity-duration" distribution just as the crude migration rate is a weighted aggregation with respect to the age-sex distribution.

Model Migration Schedules

It appears that the most prominent regularity found in empirical schedules of age-specific migration rates is the selectivity of migration with respect to age. Young adults in their early twenties generally show the highest migration rates and young teenagers the lowest. The migration rates of children mirror those of their parents; hence the migration rates of infants exceed those of adolescents. Finally, migration streams directed toward regions with warmer climates and into or out of large cities with relatively high levels of social services and cultural amenities often exhibit a "retirement peak" at ages in the mid-sixties or beyond.

Figure 5 illustrates a typical observed age-specific migration schedule (the jagged outline) and its graduation by a model schedule (the superimposed smooth outline) defined as the sum of four components:

1) a single negative exponential curve of the pre-labor force ages, with its rate of descent, α_1;
2) a left-skewed unimodal curve of the labor force ages with its rates of ascent and descent, λ_2 and α_2, respectively;
3) an almost bell-shaped curve of the post-labor force ages with its rates of ascent and descent, λ_3 and α_3, respectively; and
4) a constant curve c, the inclusion of which improves the quality of fit provided by the mathematical expression of the schedule.

The decomposition described above suggests the following simple sum of four curves (Rogers, Raquillet, and Castro, 1978)*:

$$
\begin{aligned}
M(x) = {} & a_1 e^{-\alpha_1 x} \\
& + a_2 e^{-\alpha_2 (x-\mu_2) - e^{-\lambda_2 (x-\mu_2)}} \\
& + a_3 e^{-\alpha_3 (x-\mu_3) - e^{-\lambda_3 (x-\mu_3)}} \\
& + c \qquad\qquad , \; x = 0,1,2,\ldots,z \qquad (1)
\end{aligned}
$$

* Both the labor force and the post-labor force components in equation (1) are described by the "double exponential" curve formulated by Coale and McNeil (1972) for their studies of nuptiality and fertility.

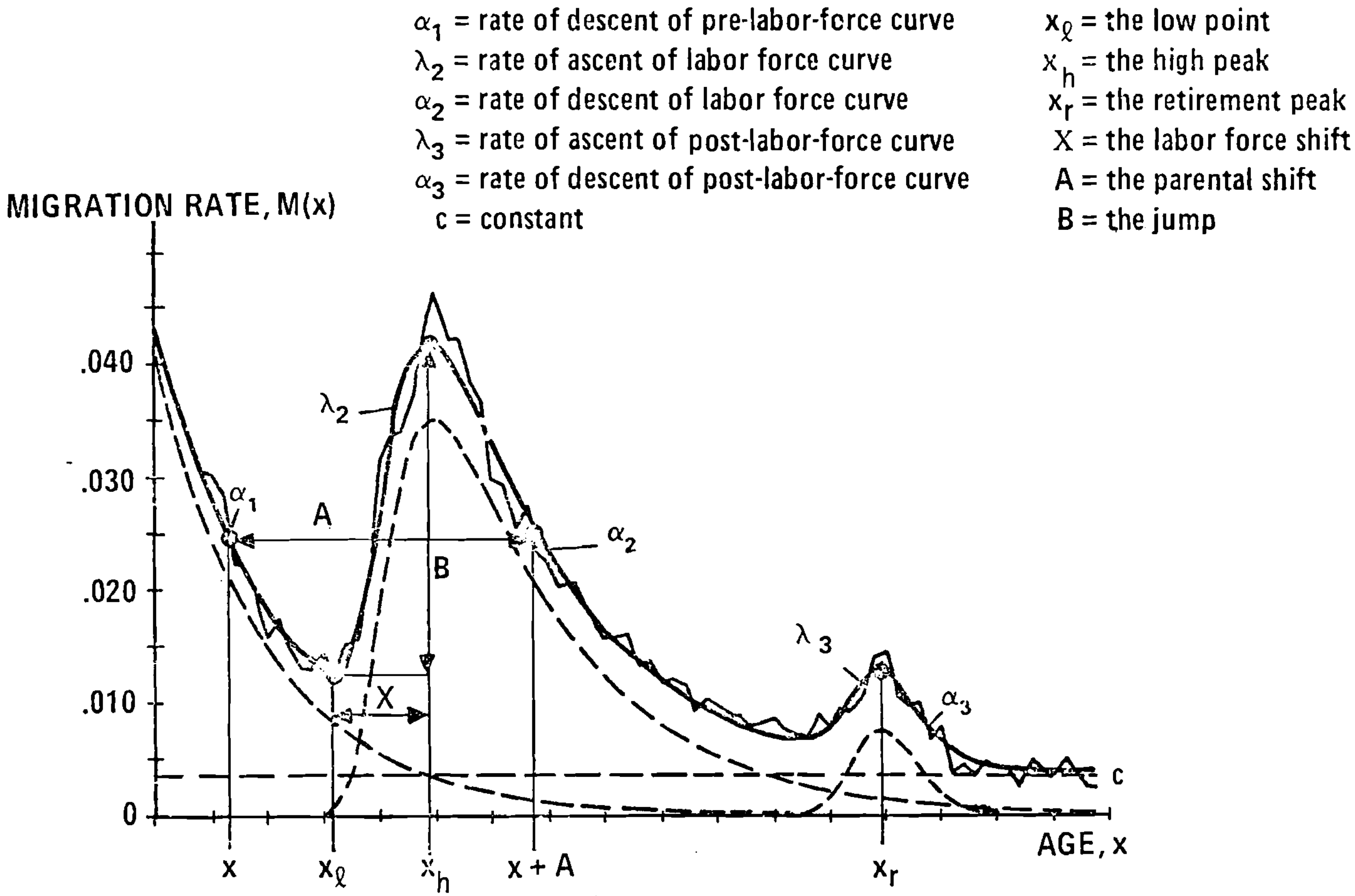


Figure 5. The model migration schedule.

The "full" model schedule in equation (1) has eleven parameters: a_1, α_1, a_2, μ_2, α_2, λ_2, a_3, μ_3, α_3, λ_3, and c. The profile of the full model schedule is defined by seven of the eleven parameters: α_1, μ_2, α_2, λ_2, μ_3, α_3, and λ_3. Its level is determined by the remaining four parameters: a_1, a_2, a_3, and c. A change in the value of the gross migraproduction rate of a particular model schedule alters proportionally the values of the latter but does not affect the former. However, as we shall see in the next section, certain aspects of the profile also depend on the allocation of the schedule's level among the labor, pre-labor, and post-labor force age components, and on the share of the total level accounted for by the constant term, c. Finally, migration schedules without a retirement peak may be represented by a "reduced" model with seven parameters, because in such instances the third component of equation (1) is omitted.

Table 2 sets out illustrative values of the basic and derived measures presented in Figure 5. The data refer to 1974 migration schedules for an eight-region disaggregation of Sweden (see Appendix A). The method chosen for fitting the model schedule to the data was a functional-minimization procedure known as the modified Levenberg-Marquardt algorithm.* Minimum chi-square estimators were used instead of least squares estimators. The differences between the two parametric estimates tend to be small, and because the former give more weight to age groups with smaller rates of migration, we use minimum chi-square estimators in the remainder of the paper.

To assess the quality of fit that the model schedule provides when it is applied to observed data, we calculated the "mean absolute error as a percentage of the observed mean":

$$E = \frac{\frac{1}{n}\sum_{x} \left| \hat{M}(x) - M(x) \right|}{\frac{1}{n}\sum_{x} M(x)} \cdot 100 \quad .$$

This measure indicates that the fit of the model to the Swedish data is reasonably good, the eight indices of goodness-of-fit being 6.87, 6.41, 12.15, 11.01, 9.31, 10.77, 11.74, and 14.82, for males and 7.30, 7.23, 10.71, 8.78, 9.31, 11.61, 11.38, and 13.28 for females. Figures 6 and 7 illustrate graphically

*See Appendix C and Brown and Dennis (1972), Levenberg (1944), and Marquardt (1963).

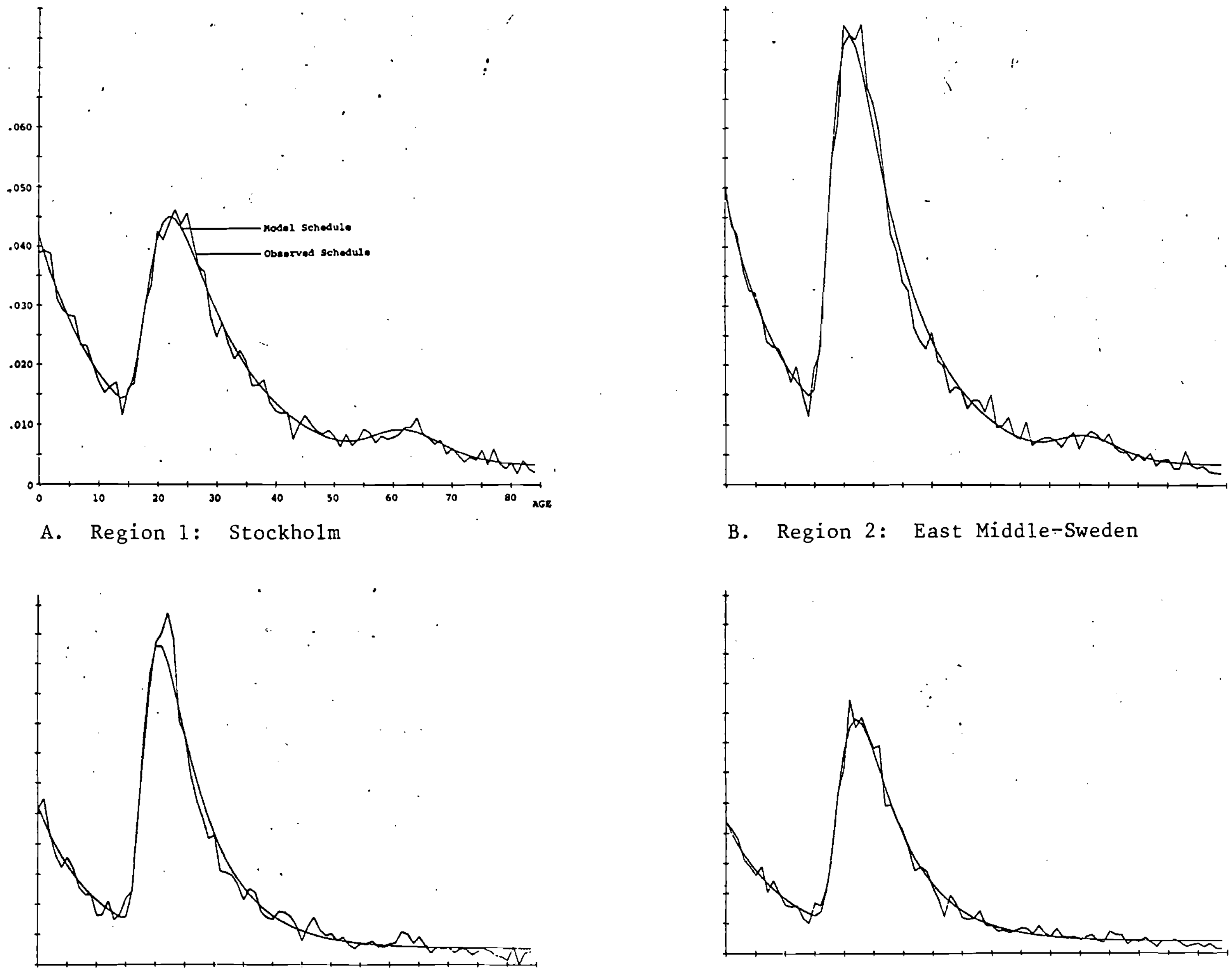
Table 2. Parameters and variables defining observed model migration schedules: Swedish regions, 1974.

Parameters and Variables	1. Stockholm M	1. Stockholm F	2. East Middle-Sweden M	2. East Middle-Sweden F	3. South Middle-Sweden M	3. South Middle-Sweden F	4. South M	4. South F
GMR*	1.49	1.47	1.77	1.85	1.02	1.09	0.89	0.86
a_1	.042	.041	.044	.048	.024	.025	.025	.021
α_1	.098	.091	.081	.099	.098	.109	.117	.104
a_2	.059	.067	.095	.114	.068	.084	.066	.067
μ_2	20.81	19.27	20.19	18.40	19.86	18.46	21.13	19.88
α_2	.077	.092	.086	.103	.105	.128	.114	.129
λ_2	.374	.374	.406	.500	.408	.568	.270	.442
a_3	.000	.000	.000	.000	---	---	---	---
μ_3	77.12	89.55	74.64	85.49	---	---	---	---
α_3	.796	.428	1.404	.517	---	---	---	---
λ_3	.142	.068	.206	.079	---	---	---	---
c	.003	.003	.002	.003	.002	.003	.002	.002
$\bar{n}$	31.08	29.64	28.40	27.83	28.45	28.11	28.20	28.12
%(0-14)	25.58	25.89	23.19	22.86	21.31	20.57	22.78	21.93
%(15-64)	64.44	65.08	71.06	70.51	72.30	71.59	70.80	70.77
%(65+)	9.98	9.02	5.76	6.63	6.39	7.83	6.41	7.29
δ_{1c}	13.39	13.46	21.16	14.77	11.72	8.64	13.43	9.93
δ_{12}	.715	.612	.462	.421	.352	.293	.379	.311
δ_{32}	.002	.000	.000	.000	---	---	---	---
β_{12}	1.26	.987	.947	.960	.940	.847	1.02	.809
σ_2	4.83	4.06	4.73	4.84	3.90	4.43	2.37	3.43
σ_3	.178	.160	.147	.153	---	---	---	---
x_l	16.39	14.83	15.88	14.79	15.38	15.08	14.53	15.61
x_h	24.69	22.68	23.77	21.41	23.06	21.01	24.15	22.58
x_r	64.80	61.74	60.03	60.77	---	---	---	---
X	8.30	7.85	7.89	6.62	7.68	5.93	9.62	6.97
A	27.87	25.47	29.86	27.21	29.91	27.26	29.89	27.87
B	.030	.031	.049	.061	.034	.046	.026	.033

*The GMR, its percentage distribution across the three major age categories (i.e., 0-14, 15-64, 65+), and $\bar{n}$ all are calculated with a model schedule spanning an age range of 95 years.

Table 2. Parameters and variables defining observed model migration schedules: Swedish regions, 1974, (continued).

Parameters and Variables	5. West		6. North Middle-Sweden		7. Lower North-Sweden		8. Upper North-Sweden	
	M	F	M	F	M	F	M	F
GMR	0.82	0.84	1.25	1.37	1.38	1.52	1.08	1.29
a_1	.021	.022	.031	.028	.034	.031	.024	.023
α_1	.089	.106	.104	.100	.123	.119	.136	.126
a_2	.046	.055	.084	.114	.109	.141	.079	.115
μ_2	20.36	19.36	19.75	18.13	19.62	17.93	19.48	17.60
α_2	.091	.114	.103	.136	.118	.148	.114	.142
λ_2	.416	.442	.437	.572	.427	.701	.448	.720
c	.001	.002	.002	.004	.003	.004	.003	.004
$\bar{n}$	28.46	28.36	28.11	27.98	28.27	27.97	29.94	28.88
%(0-14)	23.55	23.19	21.52	19.48	19.83	18.25	18.28	16.45
%(15-64)	70.38	69.06	72.49	72.66	73.57	73.60	73.43	74.67
%(65+)	6.07	7.75	5.99	7.85	6.59	8.14	8.29	8.88
δ_{1c}	14.50	10.10	13.40	7.64	11.39	7.40	8.24	5.99
δ_{12}	.458	.395	.369	.241	.309	.219	.304	.200
β_{12}	.979	.926	1.00	.730	1.04	.801	1.19	.889
σ_2	4.56	3.88	4.23	4.20	3.63	4.74	3.93	5.08
x_l	16.11	15.24	15.56	14.74	15.19	15.07	15.20	14.79
x_h	23.80	22.29	22.93	20.58	22.56	20.12	22.47	19.83
X	7.69	7.05	7.37	5.84	7.37	5.05	7.27	5.04
A	29.57	27.42	29.92	27.09	30.15	26.94	31.61	28.36
B	.024	.028	.044	.061	.054	.080	.041	.067



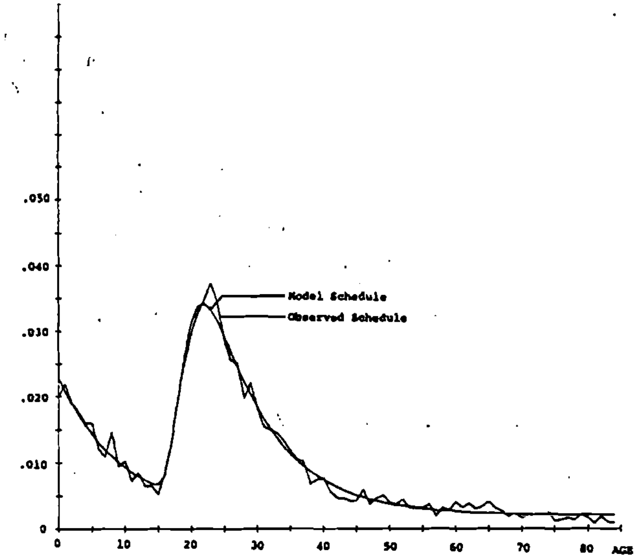
A. Region 1: Stockholm

B. Region 2: East Middle-Sweden

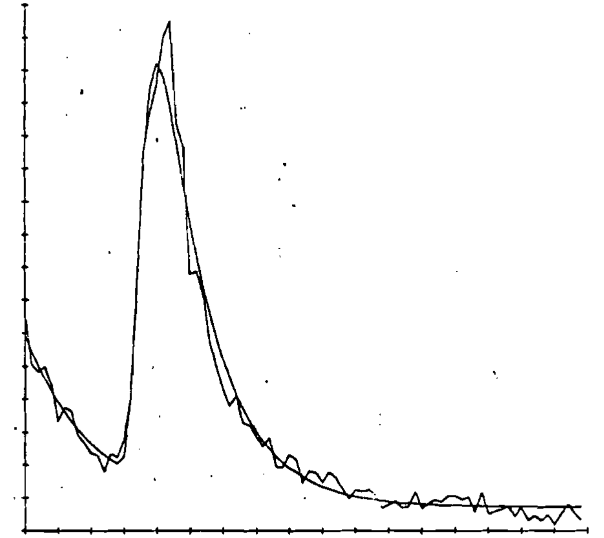
C. Region 3: South Middle-Sweden

D. Region 4: South

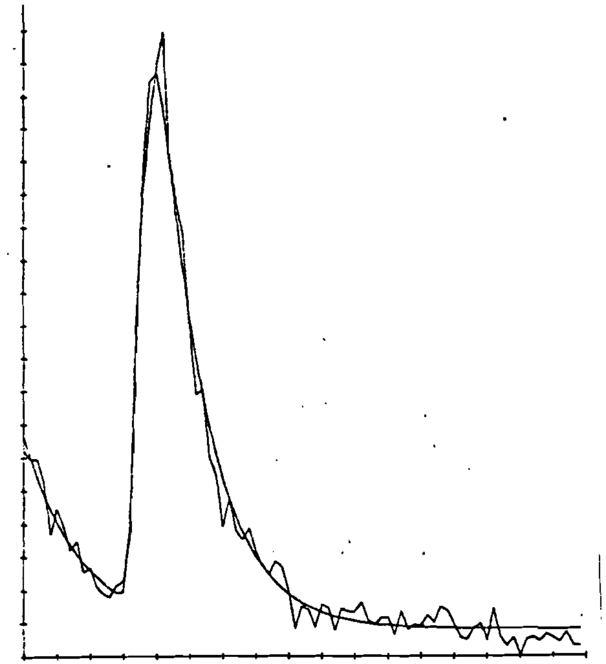
Figure 6. Observed and Model Migration Schedules: Females, Swedish Regions, 1974.



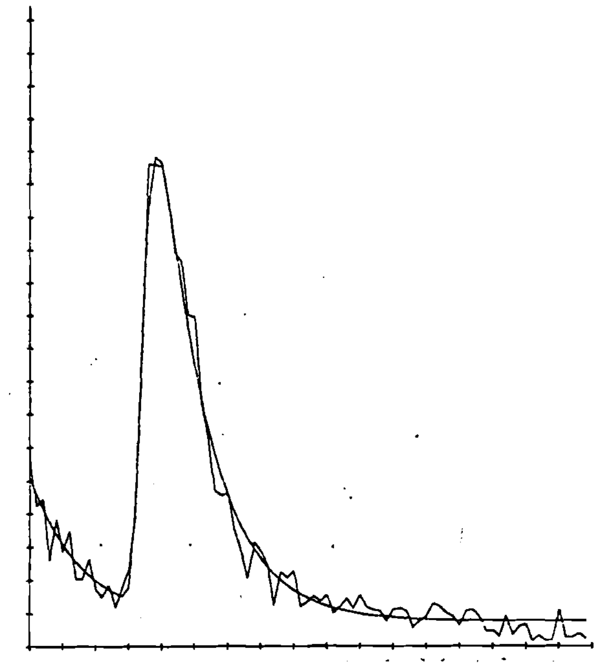
A. Region 5: West



B. Region 6: North-Middle Sweden



C. Region 7: Lower-North Sweden



D. Region 8: Upper-North Sweden

Figure 7. Observed and Model Migration Schedules:
Females, Swedish Regions, 1974.

this goodness-of-fit of the model schedule to the observed regional migration data for Swedish females.

Model migration schedules of the form specified in equation (1) may be classified into families according to the ranges of values taken on by their principal parameters. For example, we may order schedules according to their migration levels as defined by the values of the four level parameters in equation (1), i.e., a_1, a_2, a_3, and c (or by their associated gross migraproduction rates). Alternatively, we may distinguish schedules with a retirement peak from those without one, or we may refer to schedules with relatively low or high values for the rate of ascent λ_2 or the mean age $\bar{n}$. In many applications, it is also meaningful to characterize migration schedules in terms of several of the fundamental measures illustrated in Figure 5, such as the low point, x_l, the high peak, x_h, and the retirement peak, x_r. Associated with the first pair of points is the labor force shift, X, which is defined to be the difference in years between the ages of the high peak and the low point, i.e., $X = x_h - x_l$. The increase in the migration rate of individuals aged x_h over those aged x_l will be called the jump, B.

The close correspondence between the migration rates of children and those of their parents suggests another important shift in observed migration schedules. If, for each point x on the post-high-peak part of the migration curve, we obtain (where it exists) by interpolation the age, $x - A_x$ say, with the identical rate of migration on the pre-low-point part of the migration curve, then the average of the values of A_x, calculated incrementally for the number of years between zero and the low-point x_l, will be defined to be the observed parental shift, A.

An observed (graduated) age-specific migration schedule may be described in a number of useful ways. For example, references may be made to the heights at particular ages, to locations of important peaks or troughs, to slopes along the schedule's age profile, to ratios between particular heights or slopes, to areas under parts of the curve, and to both horizontal and vertical distances between important heights and locations. The various descriptive measures characterizing an age-specific model migration schedule may be conveniently grouped into the following categories and sub-categories:

Basic measures (the 11 fundamental parameters and their ratios)

heights : a_1, a_2, a_3, c

locations: μ_2, μ_3

slopes : $\alpha_1, \alpha_2, \lambda_2, \alpha_3, \lambda_3$

ratios : $\delta_{1c} = a_1/c$, $\delta_{12} = a_1/a_2$, $\delta_{32} = a_3/a_2$,
$\beta_{12} = \alpha_1/\alpha_2$, $\sigma_2 = \lambda_2/\alpha_2$, $\sigma_3 = \lambda_3/\alpha_3$

Derived measures (properties of the model schedule)

areas : GMR, %(0-14), %(15-64), %(65+)

locations: $\bar{n}$, x_l, x_h, x_r

distances: X, A, B

A convenient approach for characterizing an observed model migration schedule (i.e., an empirical schedule graduated by equation (1)) is to begin with the central labor force curve and then to "add-on" the pre-labor and post-labor force components, and the constant component. This approach is represented graphically in Figure 8.

One can imagine describing a decomposition of the model migration schedule along the vertical and horizontal dimensions, e.g., allocating a fraction of its level to the constant component and then dividing the remainder among the other three (or two) components. The ratio $\delta_{1c} = a_1/c$ measures the former allocation, and $\delta_{12} = a_1/a_2$ and $\delta_{32} = a_3/a_2$ reflect the latter division.

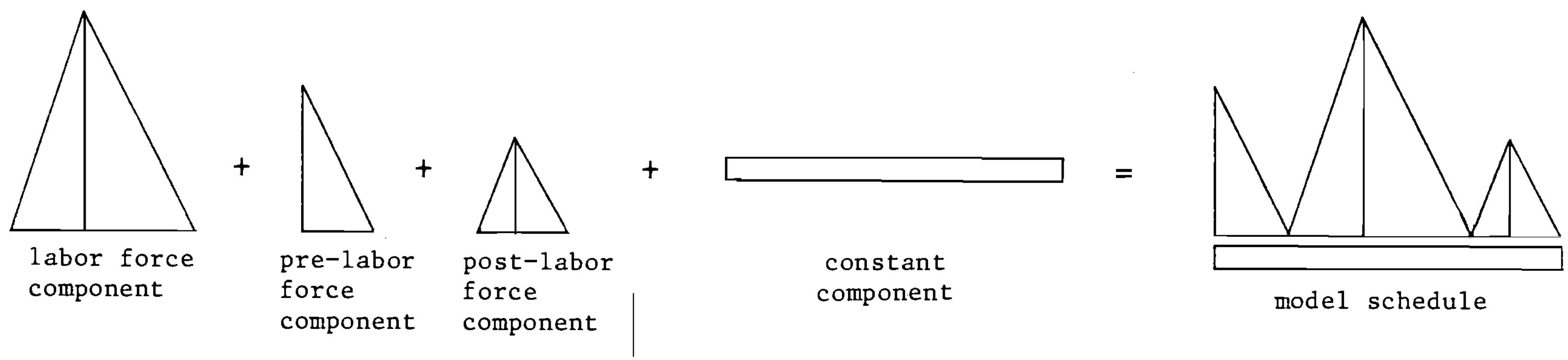
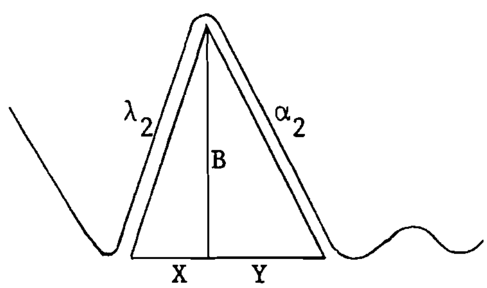


Figure 8. Schematic diagram of the fundamental components of the model migration schedule.

The heights of the labor force and pre-labor force components are reflected in the parameters a_2 and a_1, respectively, therefore the ratio a_2/a_1 indicates the degree of "labor dominance", and its reciprocal, $\delta_{12} = a_1/a_2$, the index of child dependency, measures the level at which children migrate with their parents. Thus the lower the value of δ_{12}, the lower is the degree of child dependency exhibited by a migration schedule and, correspondingly, the greater is its labor dominance. This suggests a dichotomous classification of migration schedules into child dependent and labor dominant categories.

An analogous argument applies to the post-labor force curve, and $\delta_{32} = a_3/a_2$ suggests itself as the appropriate index. However it will be sufficient for our purposes to rely simply on the value taken on by the parameter λ_3, with positive values pointing out the presence of a retirement peak and a zero value indicating its absence. High values of λ_3 will be interpreted as identifying retirement dominance.

Labor dominance reflects the relative migration levels of those in the working ages relative to those of children and pensioners. Labor asymmetry refers to the shape of the skewed bell-shaped curve describing the profile of labor-force-age migration. Imagine that a perpendicular line, connecting the high peak with the base of the bell-shaped curve (i.e., the jump, B), divides the base into two segments X and Y as, for example, in the schematic diagram:



Clearly, the ratio Y/X is an indicator of the degree of asymmetry of the curve. A more convenient index, using only two parameters of the model schedule is the ratio $\sigma_2 = \lambda_2/\alpha_2$, the index of labor asymmetry. Its movement is highly correlated with that of Y/X, because of the approximate relation:

$$\sigma_2 = \frac{\lambda_2}{\alpha_2} \sim \frac{B}{X} \div \frac{B}{Y} = \frac{Y}{X} \quad ,$$

where $\sim$ denotes proportionality. Thus σ_2 may be used to classify migration schedules according to their degree of labor asymmetry.

Again, an analogous argument applies to the post-labor force curve, and $\sigma_3 = \lambda_3/\alpha_3$ may be defined to be the index of retirement asymmetry.

When "adding-on" a pre-labor force curve of a given level, to the labor force component, it is also important to indicate something of its shape. For example, if the migration rates of children mirror those of their parents, then α_1 should be approximately equal to α_2, and $\beta_{12} = \alpha_1/\alpha_2$, the index of parental-shift regularity, should be close to unity.

The Swedish regional migration patterns described in Figures 6 and 7, and in Table 2, may be characterized in terms of the various basic and derived measures defined above. We begin with the observation that the outmigration levels in all of the regions are similar, ranging from a low of 0.82 for males in Region 5 to a high of 1.85 for females in Region 2. This similarity permits a reasonably accurate visual assessment and characterization of the profiles in Figures 6 and 7.

Large differences in gross migraproduction rates give rise to slopes and vertical relationships among schedules that are non-comparable when examined visually. Recourse then must be made to a standardization of the areas under the migration curves, for example, a general re-scaling to a GMR of unity. Note that this difficulty does not arise in the numerical data in Table 2, because, as we pointed out earlier, the principal slope and location parameters and ratios used to characterize the schedules are not affected by changes in levels. Only heights, areas, and vertical distances, such as the jump, are level-dependent measures.

Among the eight regions examined, only the first two exhibit a definite retirement peak, the male peak being the more dominant one in each case. The index of child dependency is highest in Region 1 and lowest in Region 8, distinguishing the latter region's labor dominant profile from Stockholm's child dependent outmigration pattern. The index of labor asymmetry varies from a low of 2.37, in the case of males in Region 4 to a high of 5.08 for the female outmigration profile of Region 8. Finally, with the possible exception of males in Region 1 and females in Region 6, the migration rates of children in Sweden do indeed seem to mirror those of their parents. The index of parental-shift regularity is 1.26 in the former case and .730 in the latter; for most of the other schedules it is close to unity.

Table 2 describes interregional migration flows between Swedish regions, Tables I, II, and III in Appendix B provide comparable descriptions for the migration schedules previously illustrated in Figures 2,3, and 4. They present the necessary basic and derived measures with which to carry out a comparative analysis of the differences in levels and age profiles exhibited by those schedules, an analysis that is beyond the scope of this paper.

Properties of the Model Migration Schedule

The age profiles of model migration schedules without a retirement peak are determined by the four parameters α_1, μ_2, α_2, λ_2 and the ratio $\delta_{12} = a_1/a_2$. To simplify our analysis of the properties of such schedules we shall assume that their index of parental-shift regularity is sufficiently close to unity for us to set $\alpha_1 = \alpha_2$. Consequently, "pure" profile measures such as x_l, x_h, and A, will vary only as a function of the four parameters:

μ_2, α_2, λ_2, and δ_{12}.* The first locates the labor force curve on the age axis; the second and third define its rates of descent and ascent, respectively (and, therefore, also its labor asymmetry $\sigma_2 = \lambda_2/\alpha_2$); and the last relates the height of the pre-labor force curve to that of the labor force component. The rate of descent of the pre-labor force curve is fixed by the assumption that $\beta_{12} = \alpha_1/\alpha_2 = 1$.

The observed model schedules presented in Table 2, and in Tables I, II, and III of Appendix B, exhibit the following ranges of values for the four parameters of interest:

$$17 \le \mu_2 \le 25$$

$$.06 \le \alpha_2 \le .3$$

$$.2 \le \lambda_2 \le .8$$

$$.2 \le \delta_{12} \le .8 \quad .$$

Together with the observed range of 2.6 to 22 for the ratio δ_{1c}, the 2^5 alternative combinations of extreme values for the five parameters generate 32 hypothetical "synthetic" model migration schedules. Figure 9 illustrates 24 of these schedules, all of which have been scaled to a unit GMR to permit a more accurate visual comparison.

The six schedules in Figure 9A delineate two families of <u>labor dominant</u> ($\delta_{12} = .2$) profiles, with δ_{1c} fixed at 22. The tallest three exhibit a steep rate of descent $\alpha_2 = .3$; the shortest three show a much more moderate slope of $\alpha_2 = .06$. Within each family of three curves, one finds variations in μ_2 and in the rate of ascent, λ_2. Increasing the former shifts the curve to the right along the horizontal axis; increasing the latter parameter raises the relative height of the high peak.

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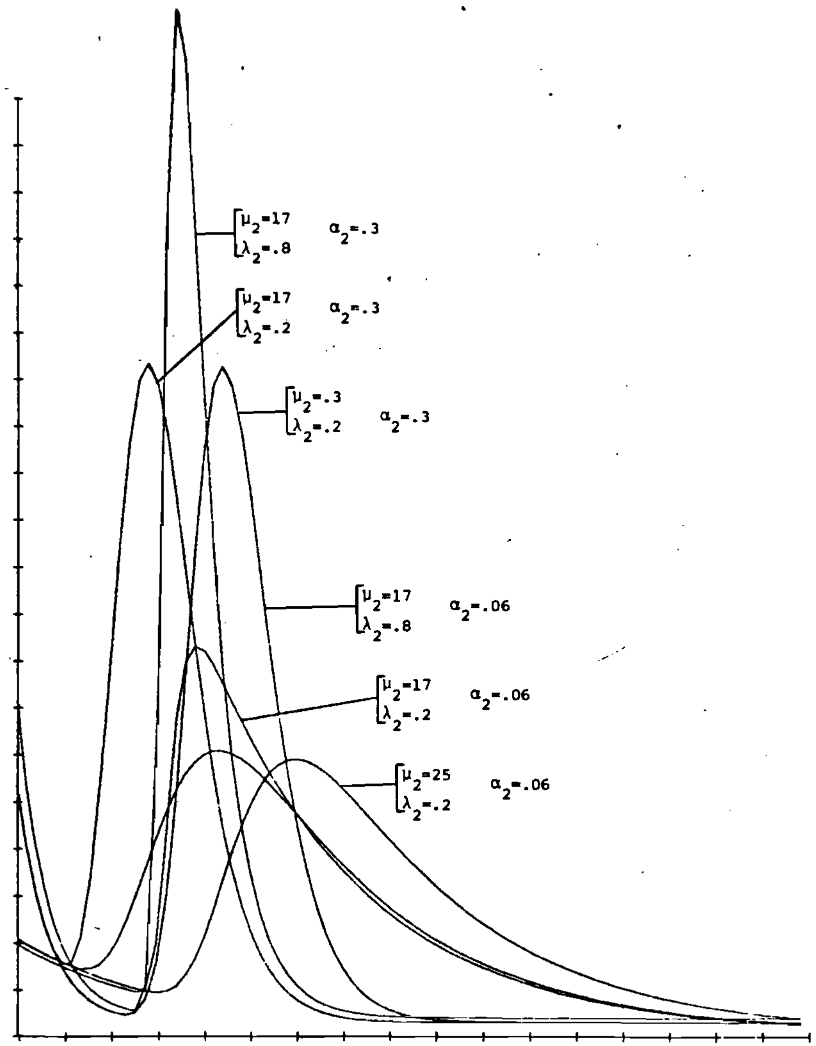
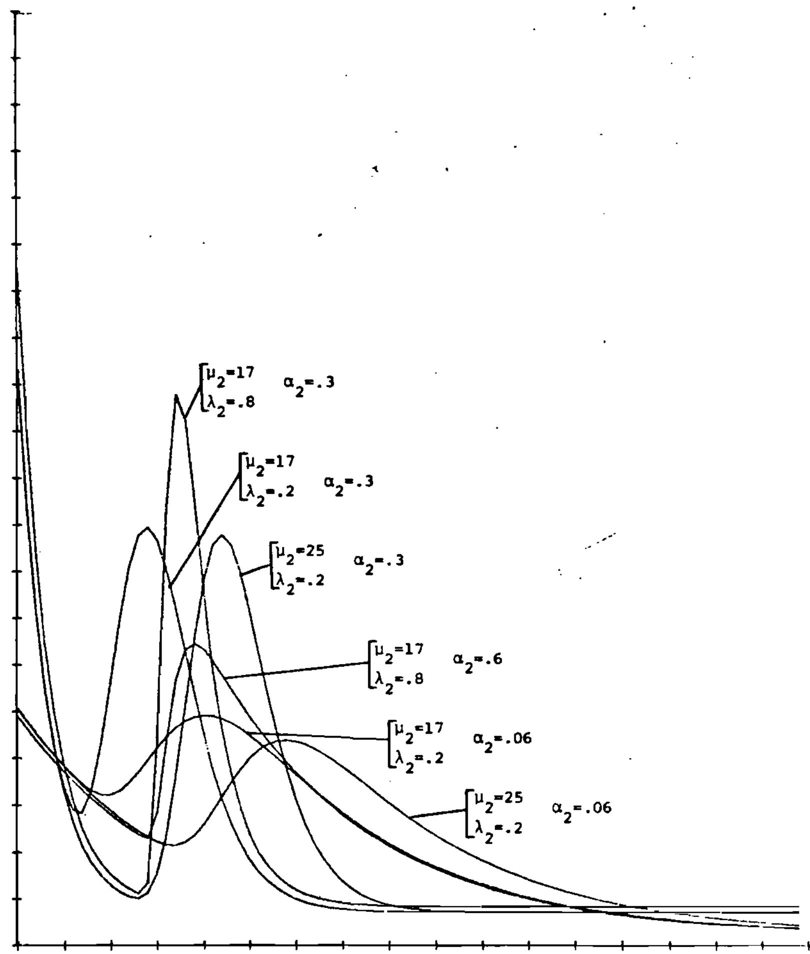
*Measures influenced by the relative share of the total migration level accorded to the constant component, i.e., "impure" profile measures such as $\bar{n}$ and %(0-14), will, in addition, also depend on δ_{1c}.

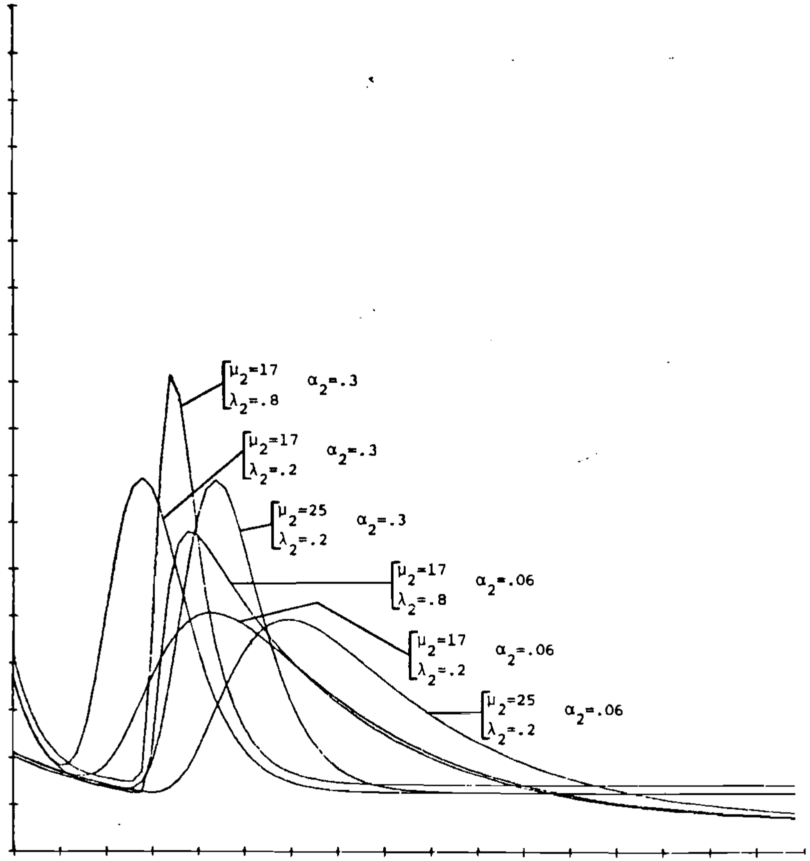


A. Labor dominant schedule with δ_{1c} = 22.0 and δ_{12} = .2

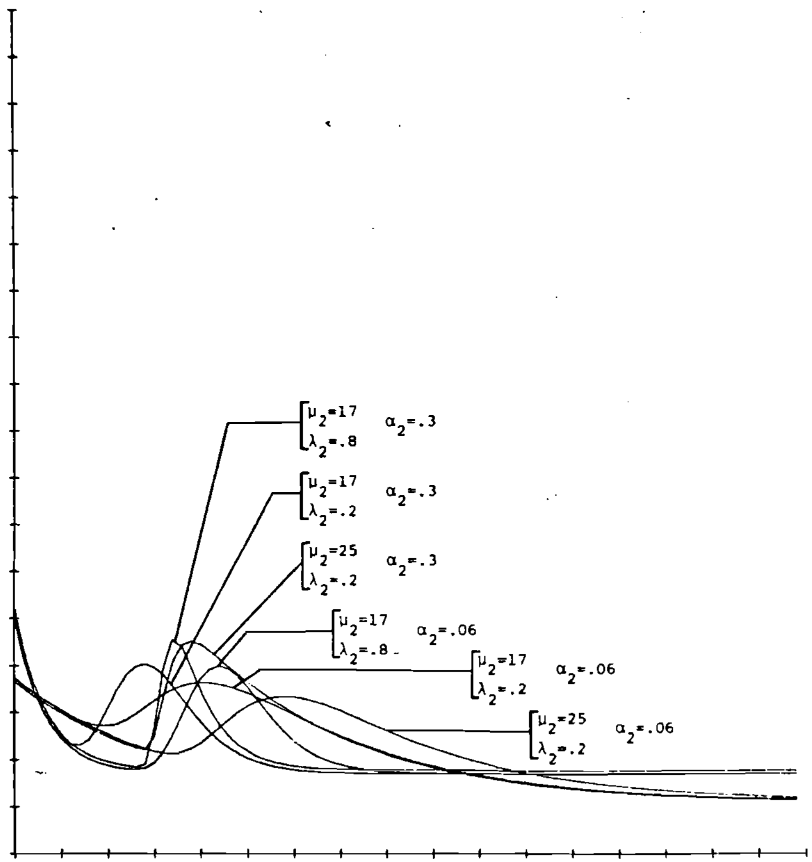


B. Child dependent schedule with δ_{1c} = 22.0 and δ_{12} = .8

Figure 9. Hypothetical model migration schedules with unit gross migraproduction rates and different parameter combinations.



C. Labor dominant schedule with δ_{1c} = 2.6 and δ_{12} = .2



D. Child dependent schedule with δ_{1c} = 2.6 and δ_{12} = .8

Figure 9. Hypothetical model migration schedules with unit gross migraproduction rates and different parameter combinations (continued).

The six schedules in Figure 9B depict the corresponding two families of child dependent (δ_{12} = .8) profiles. The results are generally similar to those in Figure 9A, with the exception that the relative importance of migration in the pre-labor force age groups is increased considerably. The principal effects of the change in δ_{12} are: (1) a raising of the intercept $a_1 + c$ along the vertical axis, and (2) a simultaneous reduction in the height of the labor force component in order to maintain a constant area of unity under each curve.

Finally, the dozen schedules in Figures 9C and 9D describe similar families of migration curves, but in these profiles the relative contribution of the constant component to the unit GMR has been increased significantly (i.e., δ_{1c} = 2.6). It is important to note that such "pure" measures of profile as x_l, x_h, X, and A remain unaffected by this change, whereas "impure" profile measures such as $\bar{n}$ and %(0-14) now take on a different set of values.

It is difficult to examine, in Figure 9, how changes in the values of the fundamental four parameters affect profile measures such as A, x_l, and x_h. Figure 10 illustrates, therefore, how the parental shift, for example, varies as a function of μ_2, α_2, λ_2, and δ_{12}. Its variation appears to be directly correlated with the variation of μ_2, inversely associated with changes in α_2 and δ_{12}, and very weakly influenced by values of λ_2. This pattern of relationships may be illuminated by an analytical argument.

For ages immediately following the high peak x_h, the labor force component of the model migration schedule is closely approximated by the function

$$a_2 e^{-\alpha_2(x_2-\mu_2)} .$$

Recalling that the pre-labor force curve is given by

$$a_1 e^{-\alpha_2 x_1}$$

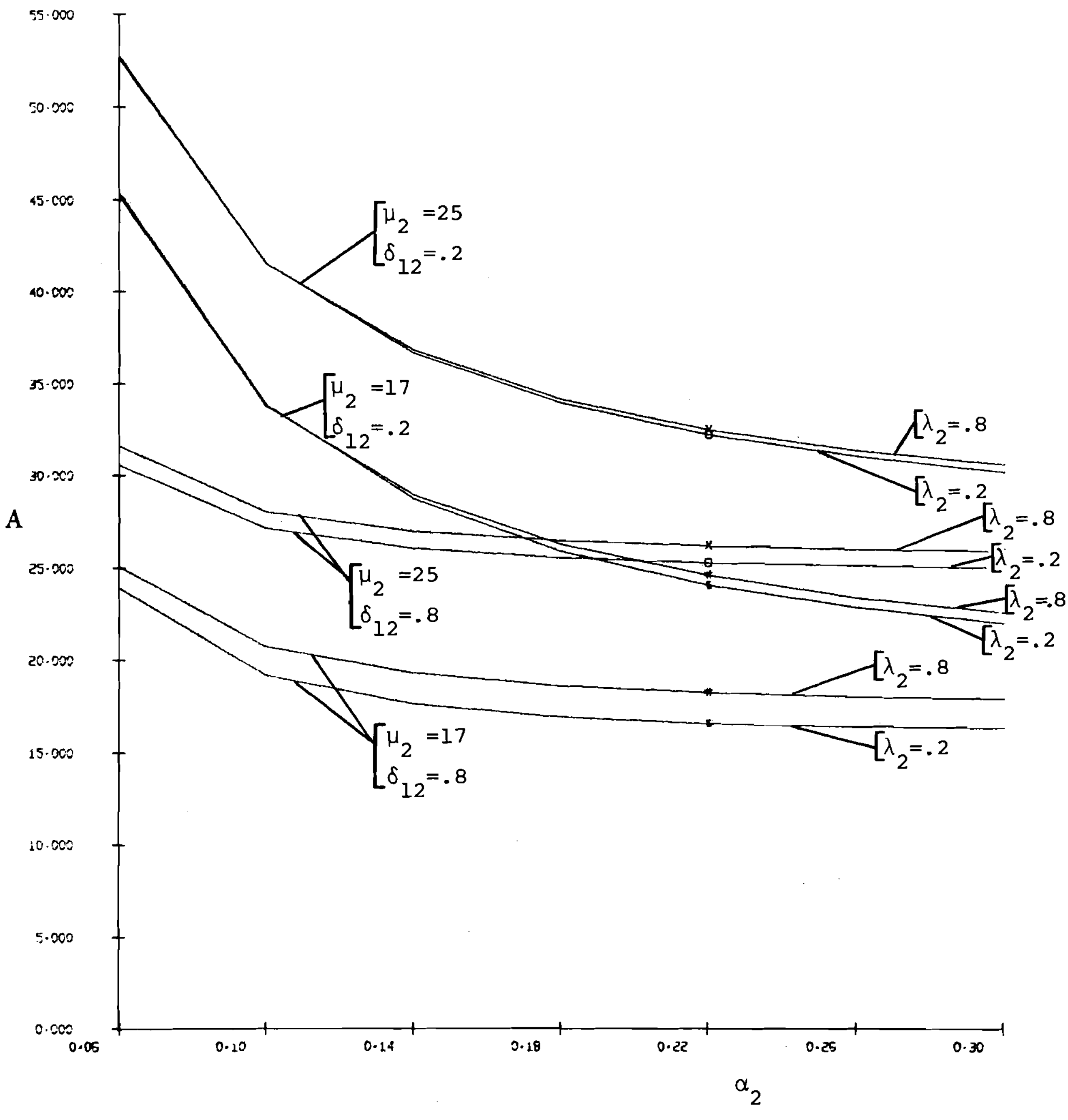


Figure 10. Variation of the parental shift as a function of α_2, for alternative fixed values of μ_2, λ_2, and δ_{12}.

when $\alpha_1 = \alpha_2$, we may equate the two functions to solve for the difference in ages that we have called the parental shift, i.e.,

$$A = x_2 - x_1 = \mu_2 + \frac{1}{\alpha_2} \ln \frac{1}{\delta_{12}} \quad . \tag{2}$$

Table 3 compares the values of this analytically defined "theoretical" parental shift with the corresponding observed parental shifts presented earlier in Table 2 for Swedish males and females. The two definitions appear to produce similar numerical values, but the analytical definition has the advantage of being simpler to calculate and analyze.

Equation (2) shows that the parental shift will increase with increasing values of μ_2 and will decrease with increasing values of α_2 and δ_{12}. If three parameters assume values within the ranges set out above, then A should vary between a low of

$$17 + \frac{1}{.3} \ln \frac{1}{.8} = 17 + 3.33(.22)$$

and a high of

$$25 + \frac{1}{.06} \ln \frac{1}{.2} = 25 + 16.67(1.61)$$

i.e.,

$$17.7 \leq A \leq 51.8 \quad .$$

However, because of the patterns of joint variation among μ_2, α_2, and δ_{12}, the parental shift varies within a much narrower range of values in the observed data set out in this paper:*

$$25 \leq A \leq 33 \quad .$$

---

*Stoto (1977) has suggested that the parental shift may be closely approximated by the mean age of childbearing.

Table 3. Observed and theoretical values of the parental shift: Swedish regions, 1974.

The Parental Shift	1. Stockholm	2. East Middle-Sweden	3. South Middle-Sweden	4. South	5. West	6. North Middle-Sweden	7. Lower North-Sweden	8. Upper North-Sweden
	Regions of Sweden							
Observed,[a] males	27.87	29.86	29.91	29.89	29.57	29.92	30.15	31.61
Theoretical,[b] males	26.67	28.97	29.63	29.74	28.84	29.43	29.74	30.59
Observed,[a] females	25.47	27.21	27.26	27.87	27.42	27.09	26.94	28.36
Theoretical,[b] females	24.49	26.33	27.51	28.21	27.19	27.69	27.53	28.59

[a] Source: Table 2.

[b] Source: Rogers, Raquillet, and Castro [1978], p. 497.

In addition to the parental shift, three other measures usefully characterize the profile of a "regular" (i.e., $\alpha_1 = \alpha_2$) model migration schedule without a retirement peak. They are: the low point, x_l, the high peak, x_h, and the horizontal distance between them, the labor force shift, X. Figure 11 shows how the first two measures vary as a function of α_2, for alternative fixed values of μ_2, λ_2, and δ_{12}. The diagrams indicate that the influence of α_2 and δ_{12} on the location of the low point and the high peak is negligible, for high values of λ_2, and that in such instances x_l and x_h may be expressed as simple functions of μ_2:

$$x_l \doteq \mu_2 - 2.7$$

$$x_h \doteq \mu_2 + 2.0 \quad .$$

Thus for $\lambda_2 = .8$, X $\doteq$ 5 years, declining slightly as α_2 increases.

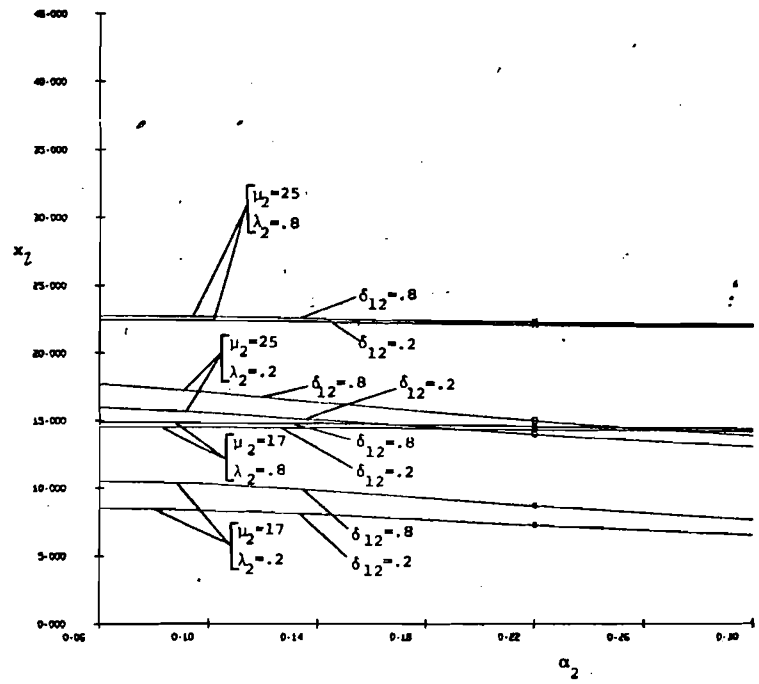
For relatively low values of λ_2, both α_2 and δ_{12} influence the location of the low point and the high peak. However, the impact of a changing δ_{12} on x_l never exceeds a range of two years, in our 32 schedules, and its influence on the high peak is negligible. Indeed, for $\lambda_2 = .2$, we may adopt the approximation

$$x_h \doteq \mu_2 + 5 - 20\alpha_2 \quad .$$

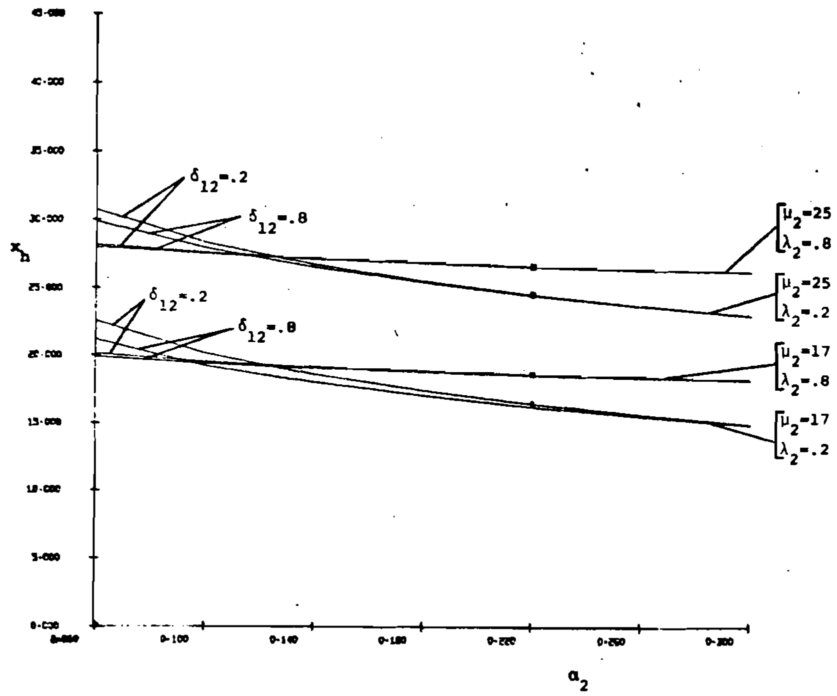
An analogous, but somewhat cruder, approximation also may be adopted for the low point:

$$x_l \doteq \mu_2 - 8 - 15\alpha_2 \quad .$$

Thus for $\lambda_2 = .2$, $X \doteq 13 - 5\alpha_2$.



A. Variation of the low point



B. Variation of the high peak

Figure 11. Variation of the low point and high peak as a function of α_2, for alternative fixed values of μ_2, λ_2, and δ_{12}.

Migration Proportions and Schedules

The age profile of a schedule of migration rates reflects the influences of two age distributions: the age composition of migrants and that of the population of which they were a part (Rogers, 1976). This can be easily demonstrated by decomposing the numerator and denominator of the fraction that defines an age-specific migration rate, M(x).

If O(x) denotes the number of outmigrants of age x, leaving a region with a population of K(x) at that age, then

$$M(x) = \frac{O(x)}{K(x)} = \frac{O \cdot N(x)}{K \cdot C(x)} = o \cdot \frac{N(x)}{C(x)} \quad , \qquad (3)$$

where

O = total number of outmigrants;

N(x) = proportion of migrants aged x years at the time of migration;

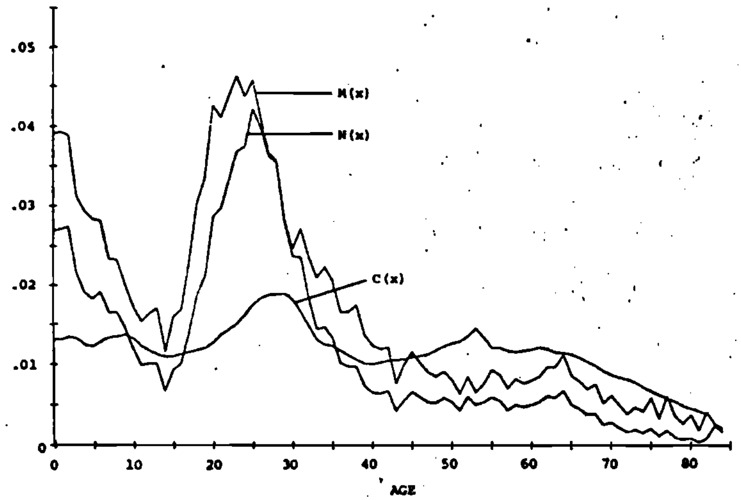
K = total population;

C(x) = proportion of total population aged x years at mid-year;

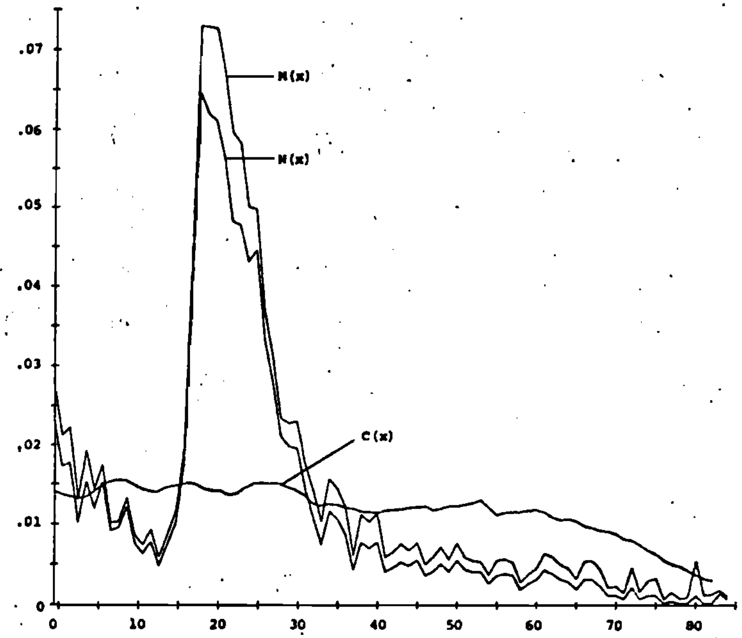
o = crude outmigration rate.

We define the collection of N(x) values to be the migration proportion schedule (MPS) that is associated with a migration rate schedule (MRS) of M(x) values. Figure 12A illustrates both schedules for Swedish female outmigration from Stockholm in 1974 (highest δ_{12} value). Figure 12B presents comparable data for Upper North-Sweden (lowest δ_{12} value). Both figures also show the age composition of the origin region's population.

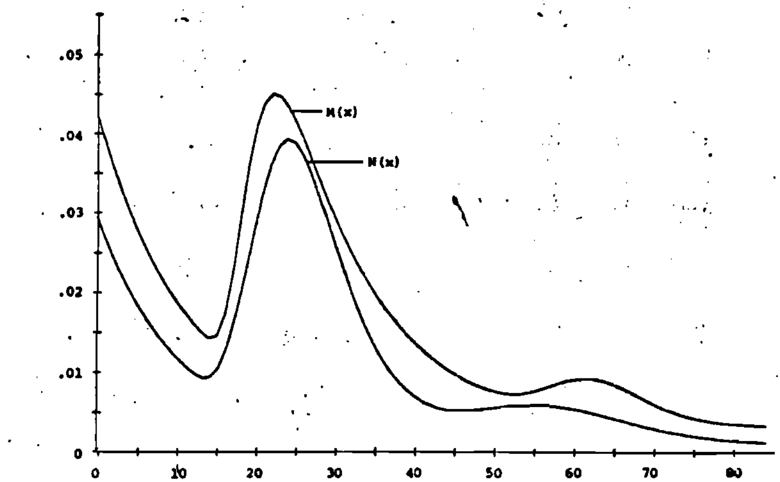
A number of observations follow from the decomposition set out in equation (3). First, it is clear that different population age compositions, in a region of origin, will give rise to different age-specific migration rate schedules for the same



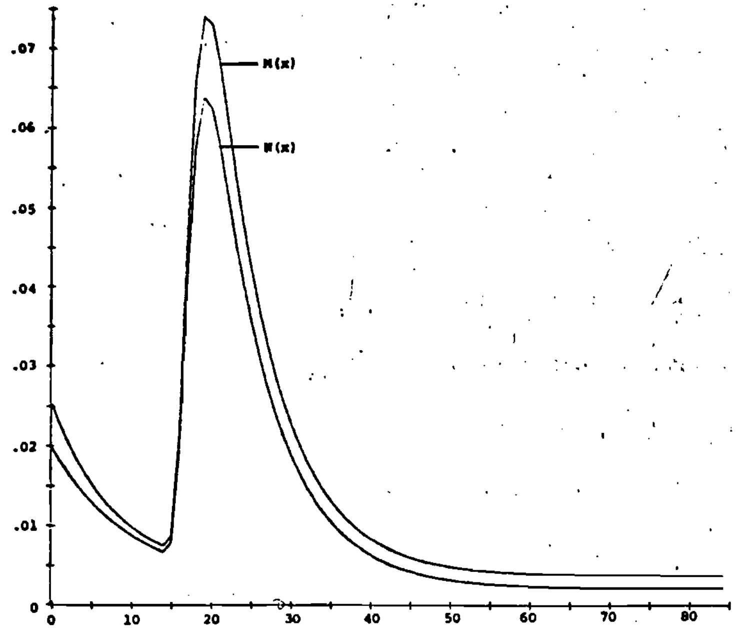
A. Region 1: Stockholm, Observed



B. Region 8: Upper North-Sweden, Observed



C. Region 1: Stockholm, Model



D. Region 8: Upper North-Sweden, Model

Figure 12. Migration Proportions, Migration Rates, and Age Compositions: Females, Swedish Regions, 1974.

migration proportion schedule. The level of the former determines whether it is situated above or below the latter. Since the GMRs of the migration rate schedules in Figure 12A and 12B exceed unity, both M(x) profiles lie above the N(x) profiles. Finally, for the same N(x) schedule, an "old" population age composition (e.g., that of a stationary population) will produce relatively high pre-labor force migration, whereas a "young" age composition will instead generate relatively high post-labor force migration.

The observed migration proportion schedules in Figures 12A and 12B appear to have the same fundamental profile as the model migration schedule defined by equation (1). Figures 12C and 12D illustrate that this indeed is the case, and Table 4 presents the estimated parameters and descriptive measures that result from fits of that model schedule to the eight-region data for Sweden. A comparison of the values in Table 4 with corresponding values in Table 2 is instructive, and reveals that the ranges of variation are similar for most profile measures.

Having shown that the age composition of migrants may be described by the model migration schedule in equation (1), we now shall consider how to describe the age composition of the population of which they were a part. This will then permit us to examine how these two age distributions combine to influence the age profile of a migration rate schedule.

The age composition of a stable population with an intrinsic rate of growth, r, and survivorship function $l(x)$ is given by

$$C(x) = \frac{e^{-rx} l(x)}{\int_0^\infty e^{-ra} l(a) da} = b e^{-rx} l(x) \quad . \tag{4}$$

Substituting this expression into equation (3), with N(x) taking on the profile of the model migration schedule defined by equation (1) gives an analytical decomposition of the migration rate schedule that permits an exploration of how changes in N(x) and

Table 4. Parameters and variables defining observed model migration proportion schedules: Swedish regions, 1974.

Parameters and Variables	1. Stockholm		2. East Middle-Sweden		3. South Middle-Sweden		4. South	
	M	F	M	F	M	F	M	F
GMR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
a_1	.032	.030	.026	.028	.024	.023	.029	.024
α_1	.093	.093	.066	.096	.088	.100	.106	.102
a_2	.075	.095	.063	.096	.082	.090	.091	.102
μ_2	23.26	25.04	20.95	20.57	20.57	18.86	22.26	21.15
α_2	.124	.196	.093	.155	.129	.147	.135	.161
λ_2	.278	.190	.390	.306	.370	.491	.256	.349
a_3	.000	.000	.000	.000	--	--	--	--
μ_3	90.74	93.27	84.71	90.26	--	--	--	--
α_3	.361	.243	.575	.289	--	--	--	--
λ_3	.058	.046	.100	.050	--	--	--	--
c	.000	.001	-.000	.001	.002	.002	.002	.002
$\bar{n}$	27.38	27.36	26.03	25.91	26.90	27.01	26.66	27.38
%(0-14)	26.46	25.29	24.29	23.75	22.58	21.50	23.14	21.73
%(15-64)	69.59	69.40	72.89	72.14	72.12	71.47	72.26	71.66
%(65+)	3.96	5.31	2.82	4.12	5.30	7.03	4.61	6.61
δ_{1c}	77.38	32.82	-59.43	32.56	13.88	9.78	18.20	10.71
δ_{12}	.427	.312	.418	.291	.294	.258	.305	.236
δ_{32}	.000	.001	.001	.001	---	---	---	---
β_{12}	.753	.471	.712	.621	.686	.680	.780	.630
σ_2	2.25	.970	4.18	1.98	2.88	3.34	1.89	2.16
σ_3	.162	.188	.173	.174	--	--	--	--
x_l	16.81	14.77	16.41	14.45	15.53	14.93	15.02	15.57
x_h	25.96	24.68	24.37	22.66	23.29	21.25	24.60	23.28
x_r	58.09	55.27	50.03	53.13	--	--	--	--
X	9.15	9.91	7.96	8.21	7.76	6.32	9.58	7.71
A	28.47	26.55	29.64	26.18	28.23	26.20	29.40	27.49
B	.030	.030	.032	.038	.038	.045	.035	.043

Table 4. Parameters and variables defining observed model migration proportion schedules: Swedish regions, 1974 (continued).

Parameters and Variables	5. West M	5. West F	6. North Middle-Sweden M	6. North Middle-Sweden F	7. Lower North-Sweden M	7. Lower North-Sweden F	8. Upper North-Sweden M	8. Upper North-Sweden F
GMR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
a_1	.026	.027	.022	.016	.022	.019	.021	.018
α_1	.087	.103	.085	.029	.107	.102	.107	.098
a_2	.074	.088	.081	.110	.095	.100	.088	.095
μ_2	21.36	20.68	20.34	19.37	20.08	17.90	19.75	17.54
α_2	.118	.148	.127	.211	.139	.154	.127	.141
λ_2	.369	.343	.400	.433	.389	.722	.421	.730
c	.002	.002	.002	.000	.002	.003	.002	.002
$\bar{n}$	26.90	27.28	27.48	26.43	27.61	27.50	27.82	26.76
%(0-14)	24.30	23.77	21.18	20.27	19.21	18.53	18.77	17.25
%(15-64)	70.94	69.23	73.14	74.33	74.68	73.72	75.30	76.37
%(65+)	4.76	7.00	5.67	5.40	6.11	7.76	5.93	6.38
δ_{1c}	17.55	11.25	11.74	64.69	10.58	7.25	10.88	8.30
δ_{12}	.359	.304	.269	.151	.229	.199	.243	.189
β_{12}	.736	.693	.670	.139	.771	.661	.844	.691
σ_2	3.14	2.32	3.16	2.05	2.80	4.68	3.32	5.17
x_l	16.40	15.20	15.62	14.56	15.10	15.11	15.18	14.79
x_h	24.30	23.01	23.10	20.97	22.65	20.01	22.53	19.75
X	7.90	7.81	7.48	6.41	7.55	4.90	7.35	4.96
A	28.71	26.67	28.79	25.01	29.18	26.18	30.00	27.56
B	.035	.037	.039	.046	.045	.056	.044	.057

C(x) combine to induce corresponding changes in M(x). Numerical computations using this analytical decomposition require values for the survivorship function $l(x)$.

During the past decade increasing use has been made of model life tables and stable populations to estimate the characteristics of populations having inadequate birth and death statistics and inaccurate or incomplete data on age and sex distributions. The extreme flexibility of the Brass logit system (Brass, 1971) in computer applications makes it particularly attractive for our purposes. This scheme subjects the survivorship function of a "standard" life table to the so-called logit transformation, and then considers life tables with logits that are linearly related to the logits of the standard table. The fundamental linear relation is

$$Y(x) = \alpha + \beta Y_s(x) \quad ,$$

where

$$Y_s(x) = \tfrac{1}{2} ln \frac{1 - l_s(x)}{l_s(x)} \quad ,$$

and $l_s(x)$ is the survivorship probability of the standard life table.

The parameters α and β of the Brass system should not be confused with our same two terms that describe characteristics of the model migration schedule. Variation of α here refers to changes in mortality levels, and variation in β reflects changes in the relation between childhood and adult mortality. Adopting the $Y_s(x)$ values of the standard life table set out in Hill and Trussell (1977), and setting $\alpha = -1.5$ and $\beta = 1.0$, gives a model life table with an expectation of life at birth of 78.9 years (Brass, 1971, p. 109). The $l(x)$ values may be calculated using

$$l(x) = 1 \Big/ 1 + e^{2\alpha + \beta Y_s(x)} \quad , \quad 1 \leq x \leq 99 \quad ,$$

with $l(0)$ set equal to 1 and $l(100)$ to zero. Figure 13 illustrates the age compositions of stable populations with this mortality regime and four different rates of growth, $r = -.01$, 0, .01, and .02, respectively.

In 1974 the crude female outmigration rates for the Stockholm and the Upper-North regions of Sweden were .0191 and .0168, respectively. Their model migration proportion schedules were defined in Table 4 and illustrated in Figures 12C and 12D. Combining these two components of equation (3) with the stable age compositions in Figure 13 gives the hypothetical migration rate schedules set out in Figures 14A and 14B below.

The schedules in Figure 14 confirm our earlier observation that an old population age composition produces a relatively high pre-labor force migration, whereas a young age composition generates a relatively high post-labor force migration. Indeed the rates of the latter tend to "curl" upwards at the older ages to what appear to be unrealistically high levels.

To examine numerically the patterns of variation induced by changes in age composition, the model migration schedule of equation (1) was fitted to the hypothetical schedules in Figure 14.* The results are set out in Table 5, and suggest several observations.

First, it is apparent that increasing the rate of growth shifts the low point of the schedule toward younger ages, while simultaneously moving the high point toward older ages. Consequently, the labor force shift, X, increases as the population becomes younger. The same pattern of variation is exhibited by the parental shift and the mean age, i.e., younger populations have a larger value of A and of $\bar{n}$. Since all of these measures are directly associated with μ_2, it is not surprising to find that the latter parameter also increases as r increases. Finally, a move toward younger age compositions produces lower values of

*To reduce the influence of the very old age groups, only the migration rates of those under 75 years of age were used in the fitting process.

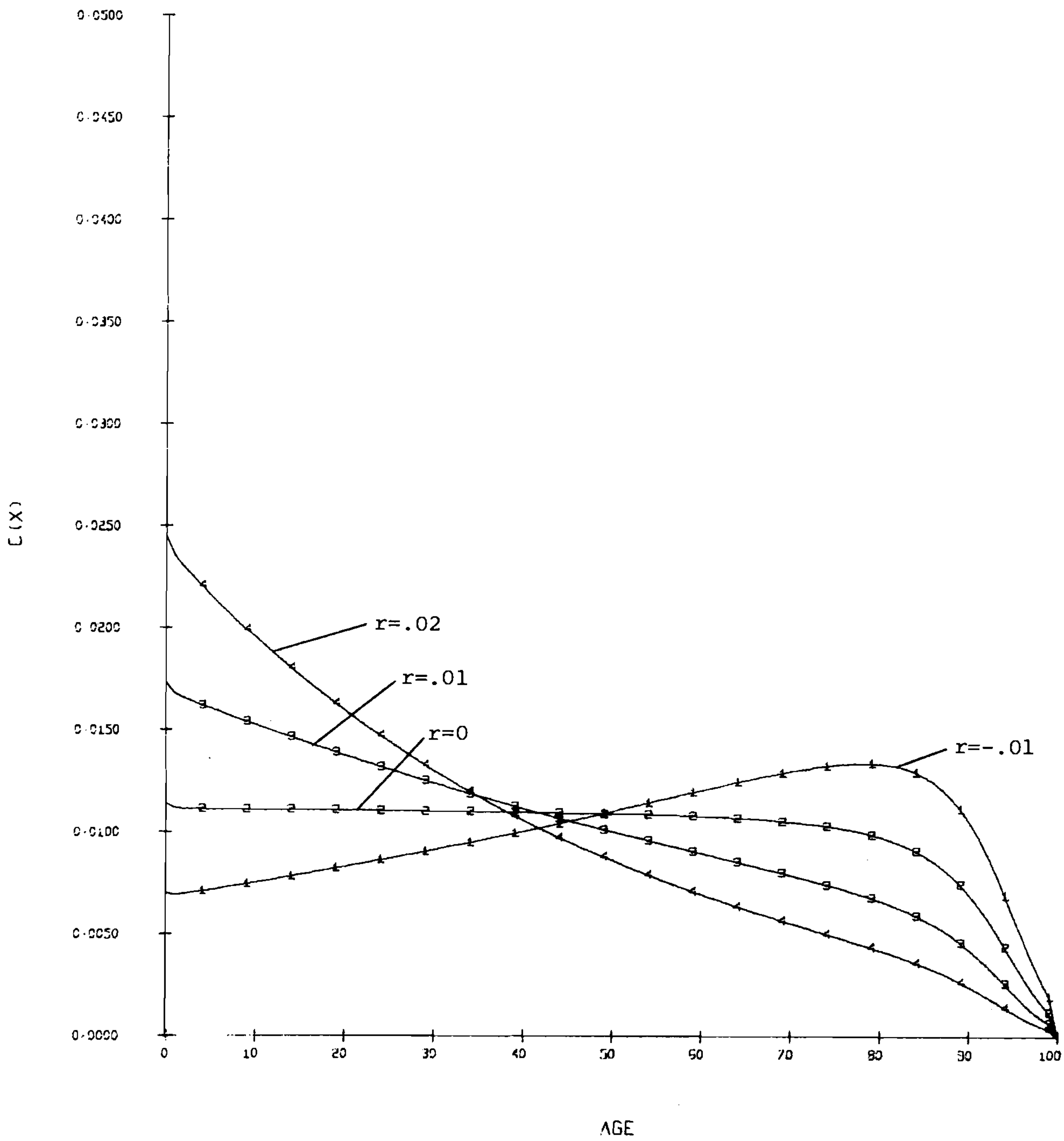
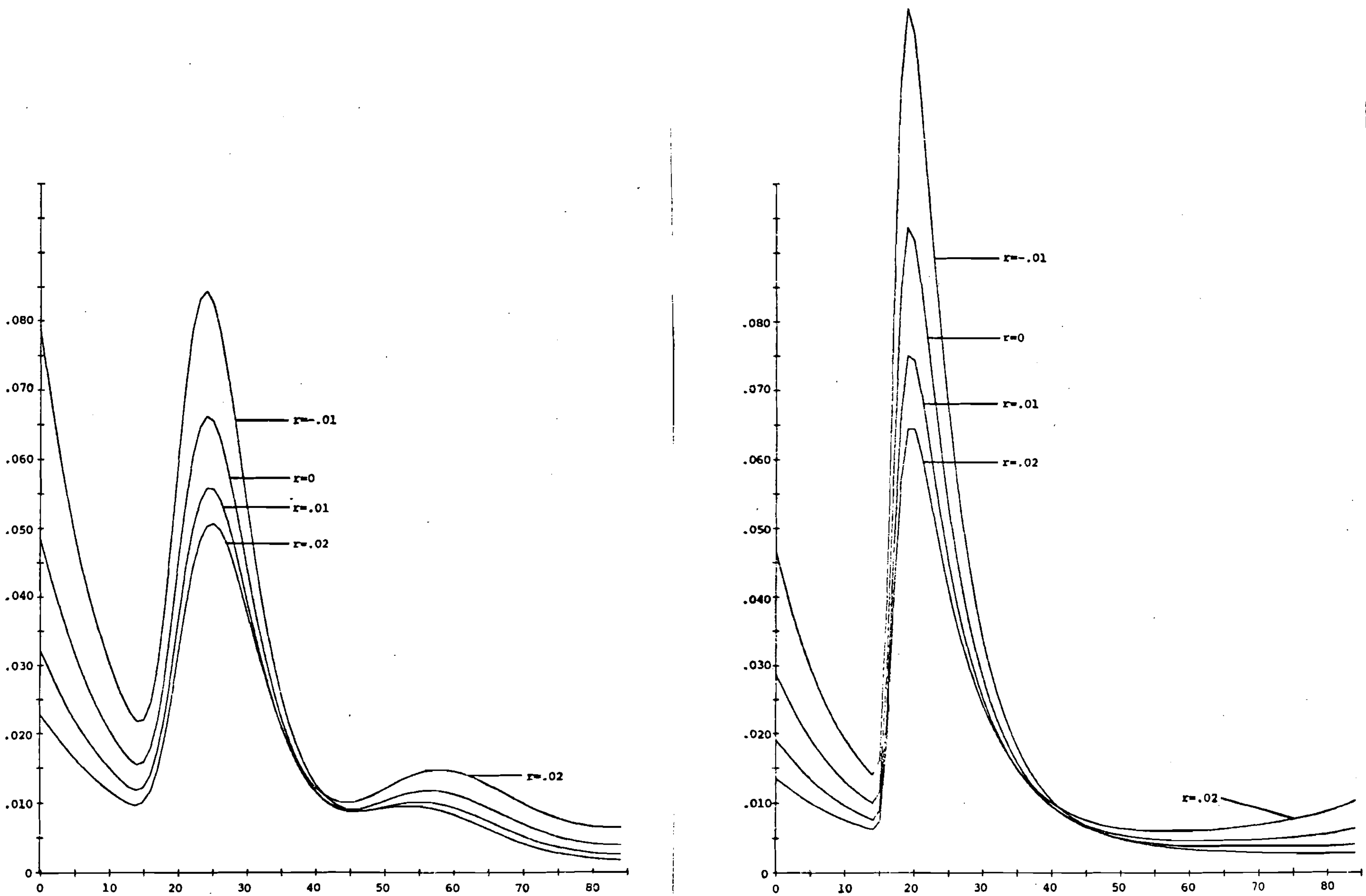


Figure 13. Age compositions of stable populations with an expectation of life at birth of 78.9 years and different rates of increase.



A. Migration Proportions and Crude Rate of Stockholm

B. Migration Proportions and Crude Rate of Upper North-Sweden

Figure 14. Hypothetical Model Migration Schedules for Swedish Females.

Table 5. Parameters and variables defining hypothetical model migration schedules for Swedish females with an expectation of life at birth of 78.9 years.

Parameters and Variables	Migration Proportions and Crude Rate of							
	1. Stockholm				8. Upper North-Sweden			
	r=-.01	r=0	r=.01	r=.02	r=-.01	r=0	r=.01	r=.02
GMR	2.16	1.72	1.52	1.51	1.95	1.53	1.34	1.28
a_1	.082	.050	.031	.021	.046	.027	.016	.009
α_1	.099	.092	.088	.088	.098	.098	.108	.165
a_2	.206	.161	.136	.123	.194	.145	.117	.103
μ_2	24.70	24.80	24.99	25.30	17.50	17.55	17.63	17.76
α_2	.199	.192	.187	.185	.147	.142	.139	.140
λ_2	.196	.195	.192	.189	.745	.726	.700	.670
a_3	.001	.001	.001	.001	---	---	---	---
μ_3	85.32	84.83	85.76	89.81	---	---	---	---
α_3	.224	.216	.213	.210	---	---	---	---
λ_3	.050	.051	.051	.048	---	---	---	---
c	.001	.002	.003	.004	.003	.003	.004	.006
$\bar{n}$	24.38	27.73	31.69	36.14	24.34	27.05	30.34	34.19
%(0-14)	30.53	24.97	19.72	15.04	20.66	17.06	13.73	10.77
%(15-64)	65.89	69.23	71.18	71.50	79.91	76.25	76.44	75.36
%(65+)	3.57	5.80	9.10	13.46	4.44	6.69	9.84	13.87
δ_{1c}	65.62	28.64	12.14	5.75	15.90	7.81	3.62	1.50
δ_{12}	.398	.307	.232	.168	.236	.185	.136	.087
δ_{32}	.004	.008	.011	.011	---	---	---	---
β_{12}	.496	.477	.468	.474	.669	.690	.780	1.18
σ_2	.982	1.01	1.03	1.02	5.06	5.12	5.04	4.77
σ_3	.225	.237	.240	.229	---	---	---	---
x_l	14.92	14.80	14.66	14.47	14.84	14.78	14.68	14.52
x_h	24.37	24.66	24.97	25.30	19.63	19.76	19.92	20.09
x_r	53.89	55.44	56.94	58.55	---	---	---	---
X	9.45	9.86	10.31	10.83	4.79	4.98	5.24	5.57
A	24.71	26.63	28.73	33.54	25.44	27.65	30.66	36.51
B	.063	.051	.044	.041	.112	.084	.068	.059

of λ_2 and δ_{12}. The decline in the latter reflects the dampening effect on pre-labor force migration rates that a younger population induces if the migration proportion schedule is held fixed.

Conclusion

This paper develops support for three principal points. First, the profiles of age-specific gross migration rates all over the world have a fundamental regularity that can be captured and expressed in mathematical form. Second, this mathematical model schedule summarizes the empirical regularity in a way that permits analytical examinations to be carried out regarding the fundamental properties of the migration age profiles. Finally, migration rate schedules may be conveniently decomposed to illuminate the influences on migration patterns of migration level, the age composition of migrants, and the age composition of the population in the region of origin.

In a subsequent paper we plan to examine further the regularities exhibited by a large number of migration schedules in order to develop a family of model schedules for use in situations where observed migration data are scarce, inadequate, or inaccurate. The use of the model migration schedule to study cause- and status-specific age profiles will also be explored.

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APPENDIX A

AGGREGATION OF SWEDISH COUNTIES INTO REGIONS

Region (viksområden)	Counties (län)
1. Stockholm	Stockholm
2. Östra mellansverige (East Middle-Sweden)	Uppsala Södermanland Östergötland Örebro Västmanland
3. Småland och öarna (South Middle-Sweden)	Jönköping Kronoberg Kalmar Gotland
4. Sydsverige (South)	Blekinge Kristianstad Malmöhus
5. Västsverige (West)	Halland Göteborg och Bohus Älvsborg Skaraborg
6. Norra mellansverige (North Middle-Sweden)	Värmland Kopparberg Gävleborg
7. Mellersta norrland (Lower North-Sweden)	Västernorrland Jämtland
8. Ovre norrland (Upper North-Sweden)	Västerbotten Norrbotten

APPENDIX B

Table I. Parameters and variables defining observed model migration schedules: the Netherlands, Poland, Sweden, and the United States, around 1974.*

	Netherlands, 1972		Poland, 1973		Sweden, 1968-73		United States, 1966-71	
	M	F	M	F	M	F	M	F
GMR	4.10	4.19	1.10	1.13	3.52	3.62	5.28	4.99
a_1	.065	.068	.028	.026	.101	.101	.095	.094
α_1	.205	.247	.253	.327	.116	.124	.110	.105
a_2	.286	.339	.090	.092	.210	.236	.228	.233
μ_2	23.08	19.77	24.31	21.94	21.27	19.05	19.37	18.44
α_2	.179	.202	.212	.272	.100	.115	.103	.119
λ_2	.212	.399	.306	.359	.371	.535	.657	.654
c	.024	.026	.006	.008	.008	.010	.025	.024
$\bar{n}$	36.74	36.51	36.47	38.12	29.41	28.71	34.20	33.38
%(0-14)	16.18	15.57	18.49	17.35	23.85	22.99	20.22	21.41
%(15-64)	66.26	66.15	64.10	62.02	68.62	68.53	65.36	63.97
%(65+)	17.56	18.28	17.42	20.63	7.53	8.48	14.41	14.62
δ_{1c}	2.69	2.67	4.32	3.36	12.55	10.08	3.83	3.89
δ_{12}	.226	.202	.311	.286	.482	.426	.416	.402
β_{12}	1.14	1.22	1.19	1.20	1.15	1.07	1.07	.887
σ_2	1.18	1.98	1.44	1.32	3.70	4.63	6.40	5.50
x_l	13.29	14.32	17.00	15.42	16.44	15.57	16.50	15.57
x_h	23.85	21.49	25.51	22.73	24.65	21.83	22.12	20.96
X	10.56	7.17	8.51	7.31	8.21	6.26	5.62	5.39
A	32.12	29.41	31.22	28.13	30.06	27.38	28.95	25.82
B	.102	.143	.034	.035	.102	.128	.138	.134

*Intercommunal migration in the Netherlands and Sweden; intervoivodship migration in Poland; intercounty migration in the United States.

Table II. Parameters and variables defining observed model migration schedules: the United States, 1966-1971.

Parameters and Variables	RESIDENTIAL MOBILITY		FEMALE MOBILITY OR MIGRATION			
	white	black	residen-tial	within county	between county	between states
GMR	14.37	15.40	14.27	9.30	4.99	2.53
a_1	.251	.249	.248	.156	.094	.052
α_1	.124	.074	.118	.130	.105	.099
a_2	.594	.390	.583	.357	.233	.127
μ_2	18.86	18.34	18.30	18.23	18.44	18.62
α_2	.115	.067	.122	.126	.119	.113
λ_2	.533	.553	.587	.528	.654	.602
c	.080	.070	.082	.058	.024	.010
$\bar{n}$	35.56	35.14	35.44	36.61	33.38	31.69
%(0-14)	20.26	21.40	20.83	20.43	21.41	22.26
%(15-64)	62.80	63.43	61.85	60.70	63.97	65.39
%(65+)	16.94	15.17	17.32	18.87	14.62	12.35
δ_{1c}	3.13	3.55	3.02	2.67	3.89	5.10
δ_{12}	.423	.637	.425	.437	.402	.414
β_{12}	1.08	1.10	.967	1.03	.887	.871
σ_2	4.64	8.20	4.83	4.19	5.50	5.31
x_l	15.37	15.28	15.13	14.72	15.57	15.55
x_h	21.65	21.32	20.89	20.86	20.96	21.29
X	6.28	6.54	5.76	6.14	5.39	5.74
A	27.33	27.87	25.52	25.39	25.82	26.12
B	.322	.234	.320	.186	.134	.071

Table III. Parameters and variables defining observed model migration schedules: the Netherlands, 1972-1976.

Parameters and Variables	MALES 1972	MALES 1973	MALES 1974	MALES 1975	MALES 1976	FEMALES 1972	FEMALES 1973	FEMALES 1974	FEMALES 1975	FEMALES 1976
GMR	4.10	4.21	4.15	3.85	3.71	4.19	4.26	4.21	3.95	3.76
a_1	.065	.070	.063	.060	.057	.068	.073	.066	.063	.059
α_1	.205	.192	.174	.161	.155	.247	.225	.208	.191	.172
a_2	.286	.284	.279	.255	.246	.339	.322	.320	.286	.267
μ_2	23.08	23.10	22.19	21.78	21.24	19.77	19.62	19.40	19.11	18.91
α_2	.179	.174	.168	.162	.159	.202	.188	.190	.179	.172
λ_2	.212	.217	.248	.260	.292	.399	.422	.470	.515	.601
c	.024	.025	.024	.022	.021	.026	.026	.025	.023	.021
$\bar{n}$	36.74	36.62	36.59	36.12	35.91	36.51	36.23	36.22	35.85	35.33
%(0-14)	16.18	17.00	16.82	17.32	17.36	15.57	16.31	16.20	16.56	16.92
%(15-64)	66.26	65.53	65.72	65.71	65.89	66.15	65.71	65.81	65.87	66.12
%(65+)	17.56	17.47	17.46	16.96	16.75	18.28	17.98	17.99	17.57	16.96
δ_{1c}	2.69	2.87	2.60	2.76	2.77	2.67	2.85	2.61	2.70	2.78
δ_{12}	.226	.248	.226	.236	.233	.202	.226	.206	.218	.222
β_{12}	1.14	1.10	1.03	.997	.979	1.22	1.19	1.09	1.07	1.00
σ_2	1.18	1.25	1.47	1.61	1.84	1.98	2.24	2.47	2.88	3.50
x_l	13.29	13.71	14.01	14.12	14.42	14.32	14.58	14.87	15.04	15.45
x_h	23.85	24.10	23.71	23.56	23.28	21.49	21.52	21.32	21.16	20.98
X	10.56	10.39	9.70	9.44	8.86	7.17	6.94	6.45	6.12	5.53
A	32.12	31.54	31.12	30.58	30.24	29.41	29.01	28.47	28.25	27.78
B	.102	.102	.104	.097	.098	.143	.141	.146	.138	.138

APPENDIX C

NONLINEAR PARAMETER ESTIMATION IN MODEL MIGRATION SCHEDULES

This appendix will briefly illustrate the mathematical programming procedure used to estimate the parameters of the model migration schedule. The nonlinear estimation problem may be defined as the search for the "best" parameter values for the function:

$$
\begin{aligned}
M(x) &= a_1 e^{-\alpha_1 x} \\
&+ a_2 e^{-\alpha_2 (x-\mu_2) - e^{-\lambda_2 (x-\mu_2)}} \\
&+ a_3 e^{-\alpha_3 (x-\mu_3) - e^{-\lambda_3 (x-\mu_3)}} \\
&+ c \quad ; \qquad (1)
\end{aligned}
$$

best in the sense that a pre-defined objective function is minimized when the parameters take on these values.

This problem is the classical one of nonlinear parameter estimation in unconstrained optimization. All of the available methods start with a set of given initial conditions, or initial guesses of the parameter values, from which they begin a search for better estimates following specific convergence criteria. The iterative sequence ends after a finite number of iterations, and the solution is accepted as giving the "best" estimates for the parameters.

The problem of selecting a "good" method has been usefully summarized by Bard (1974, p.84) as follows:

> ...no single method has emerged which is best for the solution of all nonlinear programming problems. One cannot even hope that a "best" method will ever be found, since problems vary so much in size and

nature. For parameter estimation problems we must seek methods which are particularly suitable to the special nature of these problems which may be characterized as follows:

1. A relatively small number of unknowns, rarely exceeding a dozen or so
2. A highly nonlinear (though continuous and differentiable) objective function, whose computation is often very time consuming
3. A relatively small number (sometimes zero) of inequality constraints. (Those are usually of a very simple nature, e.g., upper and lower bounds.)
4. No equality constraints, except in the case of exact structural models (where, incidentally, the number of unknowns is large)

For computational convenience, we have chosen the Marquardt method (Levenberg, 1944; Marquardt, 1963). This method seeks out a parameter vector P* that minimizes the following objective function:

$$\phi(P) = ||f_P||_2^2 \tag{A1}$$

where f_P is the residual vector and $||\cdot||_2^2$ represents the known Euclidean vector norm. For the case of a model schedule with a retirement peak, vector P has the following elements:

$$P^T = [\, a_1, \alpha_1, a_2, \alpha_2, \mu_2, \lambda_2, a_3, \alpha_3, \mu_3, \lambda_3, c] \quad . \tag{A2}$$

The elements of the vector f_P can be computed by either of the following two expressions:

$$f_P(x) = (M(x) - \hat{M}_P(x)) \tag{A3}$$

or

$$f_P(x) = (M(x) - \hat{M}_P(x)) \Big/ \hat{M}_P^2(x) \tag{A4}$$

where M(x) is the observed value at age x and $\hat{M}_P(x)$ is the estimated value using equation (1) and a given vector P of parameter estimates.

By introducing equation (A3) in the objective function set out in equation (A1), the sum of squares is minimized; if, on the other hand, equation (A4) is introduced instead, the chi-square statistic is minimized.

In matrix notation, the Levenberg-Marquadt method follows the next iterative sequence:

$$P_{q+1} = P_q - \{J_q^T J_q + \lambda_q D_q\}^{-1} J_q^T f P_q \quad ,$$

where λ is a non-negative parameter adjusted to ensure that at each iteration the value of function (A1) is reduced, J_q denotes the Jacobian matrix of $\phi(P)$ evaluated at the q^{th}-iteration, and D is a diagonal matrix equal to the diagonal of $J^T J$.

The principal difficulty in nonlinear parameter estimation is that of convergence, and this method is no exception. The algorithm starts out by assuming a set of initial parameters, and then a new vector P is estimated according to the value of λ, which in turn is also modified following gradient criteria. Once given stopping values are achieved, vector P* is assumed to be the optimum. However, in most cases, this P* reflects local minima that may be improved with better initial conditions and a different set of gradient criteria.

Using the data described in this paper, several experiments were carried out to examine the variation in parameter estimates that can result from different initial conditions (assuming Newton's gradient criteria).* Among the cases studied, the most significant differences were found for the vector P with eleven parameters, principally among the parameters of the retirement component. For schedules without the retirement peak, the vector P* shows no variation in most cases.

The impact of the gradient criteria on the optimal vector P* was also analyzed, using the Newton and the Steepest Descent methods. The effects of these two alternatives were reflected in

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*For a complete description of gradient methods, see Fiacco (1968) and Bard (1974).

the computing times but not in the values of the vector P*. Nevertheless, Bard (1974) has suggested that both methods can create problems in the estimation, and therefore they should be used with caution, in order to avoid unrealistic parameter estimates. It appears that the initial parameter values may be improved by means of an interactive approach suggested by Benson (1979).

RELATED PAPERS ON MIGRATION AND SETTLEMENT

Rogers, A., ed. (1978) Migration and Settlement: Selected Essays. RR-78-6. Laxenburg, Austria: International Institute for Applied Systems Analysis.

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