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PREFACE

The aim of the IIASA Health Care Systems Modelling Task is to build a family of submodels for the National Health Care System (HCS), as an aid to Health Service planners. The modelling work is proceeding along the lines proposed in earlier papers. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

This paper is concerned with the prevalence of chronic illnesses in Finland for the years 1964 and 1968. Three different methods of analysis of variance are used to establish quantitative relationships between the prevalence of chronic illnesses and socio-economic factors such as income of family head, quantity of medical supplies, and distance to the nearest physician.

Recent publications in the IIASA Health Care Systems Modelling Task are listed at the end of this paper.

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ABSTRACT

The purpose of this paper is to present empirical evidence of the importance of socio-economic conditions for health. Simulation models of health care systems should take into account social and economic factors in order to represent reality more adequately.

This paper is concerned with the prevalence of chronic illnessess in Finland for the years 1964 and 1968. Three different methods of analysis of variance are used to establish quantitative relationships between the prevalence of chronic illnesses and socio-economic factors such as income of family head, quantity of medical supplies, and distance to the nearest physician. A strong inverse relationship between the prevalence of chronic illnesses and income was found to be about ten times higher than the relationship between chronic illnesses and medical supply variables.

CHRONIC ILLNESSES AND SOCIO-ECONOMIC CONDITIONS: THE FINLAND CASE 1964 AND 1968

INTRODUCTION

Over the last hundred years a considerable change in the causes of death has taken place simultaneously with the formation of industrialized countries. Infectious diseases, the most dangerous killers of the last century, nowadays account for less than five percent of the causes of death in developed countries. Life expectancy has nearly doubled during the industrialization process. The causes of death have shifted towards cardiovascular diseases and the various forms of cancer, accounting for about half of all deaths. Generally speaking, the main causes of death today are of the chronic and degenerative type.

The present paper deals with the prevalence of chronic illnesses and its connection with economic and social factors. Finnish data (Purola, 1974) are analyzed in more detail, with U.S. survey results (Lawrence, 1964) presented for comparison. Both case studies share the same essential features:

1. A high proportion of the population is chronically ill.
2. Chronic illnesses vary greatly with age.
3. There is a strong inverse relationship between chronic illness and income.
4. The health care system did not produce a uniform distribution of chronic illnesses.

CHRONICALLY ILL BY AGE AND SEX

The results of two surveys performed in Finland in 1964 and 1968 show that an average of about 30 percent of the population are chronically ill (Table 1). The data also show that there is no noticeable difference of the percentage of chronic illnesses according to sex and that the frequency of illnesses does not change over time very much. Understandably, however, there is a distinct increase in the frequency of chronic illnesses within the older age groups.

The U.S. survey divided the chronically ill into ill persons with restrictions in their usual activities and those chronically ill but without restrictions (Table 2). For chronically ill with restrictions the U.S. figures show an even higher increase of illness with age than the Finnish figures show. The existing differences between the U.S. and Finnish results could be explained

Table 1. Prevalence index of chronic illness among adults in Finland (all age groups = 100).

Age group	Female		Male	
	1964	1968	1964	1968
15-34	32.6	32.3	33.0	28.0
35-44	91.6	85.0	106.0	89.9
45-54	126.6	131.3	146.3	158.1
55-64	182.3	191.8	189.7	210.8
65-74	214.4	205.0	217.7	221.3
75-	291.2	211.3	223.3	222.0
All age groups	100	100	100	100
Percentage of chronically ill adults (15 years and over)	33.4	31.9	30	29.6

Source: Purola, 1974, p. 75.

Table 2. Prevalence index of chronically ill (U.S. 1962/63) with and without restrictions in usual activities (all age groups = 100).

Age group	with restrictions	without restrictions	Total
-14	16.1	54.5	43.8
15-44	64.5	118.1	103.1
45-64	166.9	135.8	144.5
65-	394.4	100.6	182.5
All age groups	100	100	100
Percentage of chronically ill	12.4	32.1	44.5

Source: Lawrence, 1976, p. 94.

partly by differences in the definition of what is called "chronic illness" and partly by differences in the population surveyed. The Finnish questionnaire asked, "Have you any handicaps or injury that reduces your general capacity to work, or any longstanding illness?" This corresponds more or less with the definition for "chronic illnesses with restrictions of usual activities" of the U.S. study. The authors of the U.S. survey define these restrictions as the inability to perform the usual activities of the corresponding sex group, such as working, attending school, or spending leisure time. While in addition, the Finnish study included only adults in its survey, the U.S. study referred to the total population. So the U.S. study should, *ceteris paribus*, show lower percentage values of chronically ill than the Finnish study.

CHRONIC ILLNESSES BY RESIDENTIAL AREA AND SEX

The Finnish survey not only determined the chronically ill persons but also the frequency of chronic diseases. On the average, there was a rate of about 1.5 illnesses per one sick person. There were, of course, much higher rates for elderly patients due

to the well-known "multimorbidity" of older age groups. The most frequent diseases were diseases of the circulatory system closely followed by diseases of the locomotive organs. These two categories accounted for nearly half of the chronic diseases (22.1 percent points of 47.8 total).

The Finnish survey also provided information concerning the place of residence and the sex of the people interviewed. In order to evaluate the effects of these two conditions on chronic morbidity, a simple analysis of variance was performed. For "all chronic diseases" a significant influence (on the 10% level) of the residential area (urban/rural) could be shown. It was found that 56.3% (= 46.72% + 9.57%) of people living in rural areas were chronically ill, but 37.2% (= 46.72% - 9.57%) of people living in urban areas were chronically ill. Table 3 shows the differences in the frequency of some chronic diseases by sex and residential area in Finland. Each single disease category shows a higher computed value for rural residence than for urban. Usually, women show essentially higher rates of chronic illness than men except for diseases of the respiratory system, injuries, poisoning, and diseases of the nervous system and sensory organs. This could be attributed to the more risky life-style of males and their higher exposure to accidents.

CHRONICALLY ILL ADULTS BY INCOME, RESIDENCE, AND MEDICAL SUPPLY

For the analysis of chronically ill persons in connection with certain social and economic factors, three different methods of analysis of variance were applied to the Finnish data. The different methods are described in detail in Appendix. Three sets of *a priori* assumptions with respect to approaches were used with increasing degrees of complexity:

1. Ordinary Least Squares (OLS) under linear restrictions with equal variances of the disturbance terms
2. Generalized Least Squares (GENLSQ) with different and unknown variances of the disturbance terms
3. LOGIT transformation of data and binomially distributed observations in combined with a maximum likelihood approach

Table 3. Rate of chronic diseases (Finland, 1968) per 100 adults by sex and residence (deviances from mean).

Main disease category	Grand Mean	Female/ Male	Rural/ Urban	R^2**
Circulatory organs	11.25	±1.90	±2.10	.934
Locomotive organs	10.30	±1.95	±2.95	.972
Digestive organs	3.95	∓0.05	±1.00*	.990*
Inaccurately defined disorders	3.80	±0.85	±1.10*	.988
Nervous system and sensory organs of perception	3.48	∓0.57	±0.52	.975
Injuries and poisoning	3.23	∓1.77	±0.52	.912
Allergies, endocrinological, metabolic and nutritional disorders	3.23	±0.73*	±0.02	.989
Respiratory organs	1.55	∓1.00	±0.40	.904
Other	5.95	±1.20	±0.95	.963
All chronic diseases	46.72	±3.22	±9.57*	.997*

*significant on a 10% level.
**squared correlation coefficient.

Data source: Purola, 1974, p. 84.

The empirical basis of this analysis is a set of about 17 000 interviews with adults in 1964 and 1968. They were asked about the existence of chronic illnesses and about their social and economic background. Some of the results of this survey can be seen in Tables 4 and 5. The tables show the number of chronically ill and healthy persons, respectively, by annual income of family head and by a variable to characterize the medical supply. Chronically ill persons categorized by distance to the nearest physician and/or by type of community are given in Table 4; in Table 5, the local density of physicans is used instead of the distance to the nearest physician to characterize medical supply. Table 6 shows the computed percentage rates of the chronically ill. The high impact of the variable "family income" becomes evident. The percentage rate of the lowest income bracket is as high as approximately 50%, for family incomes higher than 10 000 Finnish Marks per year only about 20%. Although it is not clearly understood if people are chronically ill because they are poor, or if they are poor because they are ill, there is some evidence in favor of the first argument. Not the individual income is considered but the family income. If a housewife, therefore, becomes chronically ill, no effect on the family income could be noted. Housewives account for more than 50% of female population. Their rate of chronic illness is in the ages of high morbidity in Finland and the Scandinavian countries higher than that of men (see Table 1 and Karisto, 1978, p. 85), so their relative weight in chronic illness rates seems to be high. Nevertheless, a very strong correlation of chronic illness with family income can be found.

Table 4. Number of interviewed adults classified by chronic illness, type of community, distance to the nearest physician, annual income of family head (Finland 1964 and 1968).

Survey Year 1964

Distance to the nearest doctor		Annual Income of Family Head of Preceding Year in Finnish Marks of 1967				
		-2500	2501-5000	5001-7500	7501-10 000	10 000
Urban	ill	247	233	359	360	474
	healthy	226	341	915	1219	1960
Rural:						
-3 km	ill	152	137	135	83	78
	healthy	146	220	376	345	339
4-9 km	ill	330	282	187	122	85
	healthy	385	418	486	298	241
10-19 km	ill	481	301	216	80	71
	healthy	555	558	421	244	261
20- km	ill	400	260	94	53	46
	healthy	464	425	223	136	108

Survey Year 1968

Distance to the nearest doctor		Annual Income of Family Head of Preceding Year in Finnish Marks of 1967				
		-2500	2501-5000	5001-7500	7501-10 000	10 000
Urban	ill	261	320	351	397	636
	healthy	363	489	864	1370	2836
Rural:						
-3 km	ill	174	154	121	92	126
	healthy	192	238	300	336	580
4-9 km	ill	231	191	143	100	106
	healthy	265	275	310	312	308
10-19 km	ill	442	297	208	140	78
	healthy	437	445	419	322	260
20- km	ill	414	278	121	79	50
	healthy	442	433	250	180	171

Source: Purola, 1974, p. 271.

Table 5. Number of interviewed adults classified by chronic illness, type of community, number of physicians in community per 10 000 inhabitants, and annual income (Finland, 1964 and 1968).

Survey Year 1964

Number of doctors per 10 000 inhabitants		Annual Income of Family Head of Preceding Year in Finnish Marks of 1967				
		-2500	2501-5000	5001-7500	7501-10 000	10 000
Urban	ill	247	233	359	360	474
	healthy	226	341	915	1219	1960
Rural:						
11- doctors	ill	14	23	18	12	9
	healthy	24	18	52	45	24
10-4 doctors	ill	147	114	81	45	39
	healthy	164	196	224	153	118
3-2 doctors	ill	644	460	312	165	145
	healthy	704	732	680	434	508
-1 doctors	ill	558	383	221	116	87
	healthy	658	675	550	391	299

Survey Year 1968

Number of doctors per 10 000 inhabitants		Annual Income of Family Head of Preceding Year in Finnish Marks of 1967				
		-2500	2501-5000	5001-7500	7501-10 000	10 000
Urban	ill	261	320	351	397	636
	healthy	363	489	864	1370	2830
Rural:						
11- doctors	ill	22	9	13	7	18
	healthy	23	24	45	35	76
10-4 doctors	ill	129	120	75	57	42
	healthy	145	229	151	146	213
3-2 doctors	ill	610	417	267	173	143
	healthy	641	589	605	516	526
-1 doctors	ill	500	374	238	174	157
	healthy	527	549	478	453	504

Source: Purola, 1974, p. 272.

Table 6. Percentage of chronically ill adults by social and economic categories (Finland, 1964 and 1968).

DIS64, DIS68 respectively

Type of community, distance to the nearest physician	Year	Annual Income of Family Head of Preceding Year in Finnish Marks of 1967					Average of All Income Brackets
		-2500	2501-5000	5001-7500	7501-10 000	10 001	
Urban	1964	52.2	40.6	28.2	22.8	19.5	26.4
	1968	41.8	39.6	28.9	22.5	18.3	25.0
Rural Communities	1964	51.0	86.2	26.4	19.3	18.7	29.0
-3 km	1968	47.6	39.2	28.8	21.5	17.8	28.8
4-9 km	1964	46.2	40.3	27.8	29.0	26.0	35.5
	1968	46.6	40.9	31.6	24.3	25.6	34.4
10-19 km	1964	46.4	35.0	34.0	24.7	21.4	36.0
	1968	50.3	40.0	33.2	30.3	23.0	38.2
20- km	1964	46.3	38.0	29.7	28.0	29.9	38.7
	1968	48.4	39.0	32.6	30.5	22.7	39.0
All rural communities	1964	46.8	37.7	29.6	24.9	22.8	35.0
	1968	48.6	39.9	31.7	26.3	21.5	35.4

DENS64, DENS68 respectively

Type of community, number of physicians per 10 000 inhabitants	Year	Annual Income of Family Head of Preceding Year in Finnish Marks of 1967					Average of All Income Brackets
		-2500	2501-5000	5001-7500	7501-10 000	10 001	
Urban	1964	52.2	40.6	28.2	22.8	19.5	26.4
	1968	41.8	39.6	28.9	22.5	18.3	24.9
Rural Communities	1964	36.8	56.1	25.7	21.1	27.3	31.8
11- physicians	1968	48.9	27.3	22.4	16.7	19.1	25.4
10-4 physicians	1964	47.3	36.8	26.6	22.7	24.8	33.3
	1968	47.1	34.4	33.2	28.1	16.5	32.4
3-2 physicians	1964	47.8	38.6	31.5	27.5	22.2	36.1
	1968	48.8	41.5	30.6	25.1	21.4	35.9
-1 physicians	1964	45.9	36.2	28.7	22.9	22.5	34.7
	1968	48.7	40.5	33.2	27.8	23.8	36.5
All rural communities	1964	46.8	36.5	29.6	24.8	22.8	35.1
	1968	48.6	39.8	31.7	26.3	21.4	35.4

Source: Compiled on the basis of Purola, 1974, p. 271-272.

THE RESULTS OF ANALYSIS OF VARIANCE

As described in detail in Appendix 1, three methods of analysis of variance were applied to the data in Tables 4 to 6. Each of the four data sets shows the percentage of chronically ill adults by family income and by a variable indicating the medical supply. The four data sets are given abbreviated names according to the special variable used measuring the medical supply (Table 6):

- -- DIS 64 refers to the data set including doctor's distances in 1964
- -- DIS68 refers to the data set including doctor's distances in 1968
- -- DENS64 refers to the data set including density of doctors in 1964
- -- DENS68 refers to the data set including density of doctors in 1968.

The general model

$$y_t = \mu + \alpha_i + \gamma_j + u_t \quad , \qquad \begin{aligned} i &= 1...m \\ j &= 1...n \\ t &= n(i-1) + j \end{aligned}$$

is applicable to each of the four data sets. Only the meaning of α_i is different for DIS64/68 and DENS64/68, as follows:

α_i	For DIS64/68	For DENS64/68
α_1	urban area with short distance to the nearest doctor	urban area with high density of doctors
α_2	rural area, distance up to 3 km	rural area, 11 or more doctors per 10 000 inhabitants
α_3	rural area, distance 4 to 9 km	rural area, 4 to 10 doctors per 10 000 inhabitants
α_4	rural area, distance 10 to 19 km	rural rea, 3 to 2 doctors per 10 000 inhabitants
α_5	rural area, distance more than 20 km	rural area, less than 2 doctors per 10 000 inhabitants

Table 7. Parameter estimates and corresponding t-values by Ordinary Least Squares (OLS) and Generalized Least Squares (GENLSQ).

Data Set	DIS64		DIS68		DENS64		DENS68	
Parameter	(OLS)	(GENLSQ)	(OLS)	(GENLSQ)	(OLS)	(GENLSQ)	(OLS)	(GENLSQ)
μ	32.8	31.9	33.0	32.8	32.4	31.9	31.4	31.6
	(46.4)	(82.6)	(76.9)	(84.8)	(30.4)	(46.9)	(46.1)	(51.6)
α_1	-0.1	-0.7	-2.8	-2.7	0.2	-0.6	-1.2	-1.3
	(-0.1)	(-1.2)	(-3.3)	(-4.6)	(0.1)	(-0.7)	(-0.9)	(-1.8)
α_2	-2.0	-1.9	-2.0	-2.4	0.9	0.6	-4.6	-3.5
	(-1.4)	(-2.2)	(-2.4)	(-2.9)	(0.4)	(0.3)	(-3.3)	(-1.7)
α_3	1.1	1.5	0.8	1.1	-0.9	-0.5	0.4	-0.3
	(0.8)	(1.9)	(0.9)	(1.3)	(-0.4)	(-0.4)	(0.3)	(-0.3)
α_4	-0.5	0.2	2.4	2.5	1.0	1.4	2.0	1.9
	(-0.4)	(0.2)	(2.8)	(3.2)	(0.5)	(1.7)	(1.5)	(2.4)
α_5	1.6	1.0	1.6	1.4	-1.3	-0.9	3.4	3.2
	(1.1)	(1.1)	(1.9)	(1.6)	(-0.6)	(-1.0)	(2.5)	(3.8)
γ_1	15.6	15.3	13.9	14.0	13.5	15.5	15.6	14.2
	(11.1)	(19.4)	(16.2)	(17.7)	(6.3)	(20.0)	(11.4)	(18.1)
γ_2	5.7	6.1	6.8	6.9	9.2	6.2	5.2	7.0
	(4.0)	(7.8)	(7.9)	(8.9)	(4.3)	(8.1)	(3.8)	(9.1)
γ_3	-3.6	-2.7	-2.0	-1.7	-4.4	-2.8	-1.8	-1.7
	(-2.5)	(-3.9)	(-2.3)	(-2.3)	(-2.0)	(-4.0)	(-1.3)	(-2.3)
γ_4	-8.0	-7.8	-7.2	-7.5	-9.1	-7.9	-7.4	-7.6
	(-5.7)	(-10.9)	(-8.4)	(-10.9)	(-4.3)	(-11.1)	(-5.4)	(-11.1)
γ_5	-9.7	-10.8	-11.5	-11.7	-9.2	-11.0	-11.6	-12.0
	(-6.9)	(-16.2)	(-13.4)	(-19.7)	(-4.3)	(-16.5)	(-8.5)	(-20.4)

from the mean value μ according to different levels of medical supply. A short distance to the nearest physician seems to be a rather small advantage. In these areas there are about 2 to 3 percent points less chronically ill adults. There are some differences in the magnitude of effects of distance but the direction remains basically the same between 1964 and 1968. With one exception, all the estimated values for α_3 and α_5 are positive, every estimator of α_1 and α_2 is negative. One could conclude that living in an urban area or near a doctor is connected with a slight reduction of chronic illness, but that living farther away from a doctor's practice increases the probability of having a chronic disease. However, if one uses the LOGIT-approach (see Appendix) and tests the overall influence of discourse on chronic illness by the F-test (Table 8), no significant influence can be proved. The values of α, γ, and μ in Table 8 cannot be compared with the results of Table 7 because of different transformations applied to y_t.

The measured influence of the density of doctors for 1964 is weaker than the influence of the distance to doctors. Even the sign of the estimated parameter varies with the method applied. No conclusion can be drawn on the direction of the influence of the density of doctors. Only for 1968 there is some evidence that a lower density corresponds to a higher rate of chronic illness, but once again the F-test does not indicate any significant influence.

The picture changes considerably if one studies the influence of family income. Here one finds a very stable situation. Not only the sign of the estimated parameters remains constant over time and method used, but the magnitude of the coefficients remains about the same. The probability of the adult member of a family earning less than 2500 Fmk to be chronically ill is (45.5%, minimal value) nearly double the probability of rich families (23.2%, maximum value) being chronically ill. The F-values for the influence of family income are rather high and are significant in each case. It is interesting to note that all the F-values increase over time. At this stage of investigation, it cannot be clarified if this is due to real changes or due to the learning processes of the interviewers of the survey. Additionally, it can be seen that there is a very stable monotonic decrease of the percentage of

Table 8. Parameter estimates for the LOGIT approach and χ^2 values.

Data set	DIS64	DIS68	DEN64	DEN68
Parameter				
μ	-.797	-.755	-.797	-.818
α_1	-.020	-.129	-.017	-.063
α_2	-.073	-.096	.032	-.190
α_3	.061	.050	-.030	-.003
α_4	.001	.115	.062	.104
α_5	.030	.061	-.047	.152
γ_1	.687	.626	.695	.634
γ_2	.307	.335	.313	.341
γ_3	-.094	-.041	-.096	-.040
γ_4	-.360	-.331	-.365	-.336
γ_5	-.539	-.589	-.548	-.600
$\chi^2(\alpha = 0)$	5.02	32.65	6.60	29.77
$\chi^2(\gamma = 0)$	563.00	627.18	602.17	669.37
χ^2(res.)	48.99	20.97	26.78	18.10
$F_\alpha(4,16)$	0.1	1.6	0.2	1.6
$F_\gamma(4,16)$	11.5	29.9	22.5	37.0

chronically ill according to family income. For each of the five income brackets, the lower the average family income the higher the probability of chronic illnesses.

One could assume the results presented here are unique for Finland, because Finland is known as having one of the highest male mortality rates of the developed countries, but the high influence of the family income remains the same if one compares U.S. statistics (Table 9), although the prevalence rate is somewhat lower.

Table 9. Percentage of chronically ill with restrictions in their usual activities classified by age and family income (USA 1962/63).

Annual Family Income ($)	-1999	2000-3999	4000-6999	7000+
Age				
15-44	13.4	10.6	7.4	6.1
45-64	41.2	26.2	17.5	13.8
65+	58.1	47.8	43.3	39.7
All ages (including children)	28.6	16.0	8.9	7.9

Source: Lawrence, 1976, p. 96.

The above results indicate stringent inequalities in the health status of the Finnish and U.S. populations. The inequalities are not distributed at random but correspond to family income levels. The measured influence extended by medical supply indicators is rather low. The results of many other empirical studies point in the same direction: environmental, social, and economic conditions can explain the level and distribution of illnesses to a high degree. The medical supply itself cannot explain the problem of diseases and mortality sufficiently.

If one applies this result to the model building activity of the health care system, one ends up with the conclusion that health care system models of the resource allocation type show one side of the coin. Simulation models of the health care system should also include the other side of the coin--social and economic variables--in order to reflect the real world more adequately.

APPENDIX: THREE APPROACHES OF STATISTICAL ANALYSIS

For the statistical analysis of data on chronic diseases and environmental factors, three different approaches to the analysis of variance method were used. In this Appendix, the basic model and the methodological background are described in detail. A general linear model was chosen as the basic model. Linear restrictions were added due to the necessities of parameter identification. For the three approaches computer programs were written:

Approach 1: Ordinary Least Squares (OLS)[1] under linear restrictions, the simplest and most direct method

Approach 2: An iterative method of Generalized Least Squares (GENLSQ)[1] with unknown diagonal covariance matrix of the disturbances

Approach 3: LOGIT-transformation of data and maximum likelihood method for binomially distributed observations (LOGIT)[1]

The basic model applied is a linear one. The dependent variable --in this case the proportion of chronically ill--is explained by a linear combination of exogenous variables plus an error term

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[1] The block letters in brackets refer to the names of the corresponding computer programs.

(irrespective of whether the endogenous variable is transformed as in Approach 3 or not):

$$y = X\beta + u$$

where

y is a T-vector of observations,

β is a K-vector of parameters to be estimated, and

u is a random T-vector with expected value of zero and with a covariance-matrix $\sum$.

$$E[u] = 0, \qquad E[uu'] = \sum \quad .$$

The T x K-matrix X consists of zeros and ones only. Its shape depends on the experimental set-up. Given a typical two-dimensional analysis of variance model

$$y_t = \mu + \alpha_i + \gamma_j + u_t \quad , \qquad \begin{aligned} i &= 1...m \\ j &= 1...n \\ t &= n \cdot (i - 1) + j \quad , \end{aligned}$$

the T x K-matrix X has the following shape

$$X = \begin{bmatrix} e & E_1 & I \\ e & E_2 & I \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ e & E_m & I \end{bmatrix}$$

with

$$T = m \cdot n$$

and

$$K = m + n + 1$$

where

e is the n-unit vector,

E_i is a n x m-matrix X with ones in the i-th column and zeros otherwise, and

I the n x n-identity matrix.

In the above notation the K-vector β can be written as

$$\beta = \begin{bmatrix} \mu \\ \alpha \\ \vdots \\ \alpha_m \\ \gamma_1 \\ \vdots \\ \gamma_n \end{bmatrix} \qquad K = m + n + 1 \quad .$$

As can be easily seen, the rank of X is less than K, precisely K - 2[1] for the two-dimensional case.

If one applies the least squares principle in the case of a singular X'X-matrix, β cannot be determined uniquely. To overcome this problem, one adds linear restrictions on β:

$$L\beta = z$$

where

L is a (K - r) x K-matrix and

z is a (K - r)-vector.

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[1] The first column of X is simply the sum of the columns 2 to m + 1, and the sum of the columns m + 2 to K.

For the special case of two-dimensional analysis of variance, L was chosen as follows

$$L = \begin{bmatrix} 0 & 1 & 1 & \ldots & 1 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & 1 & 1 & \ldots & 1 & 1 \end{bmatrix}$$

and

$$z = 0 \quad .$$

This means there is no restriction on μ, and the sum of the α_i's and of the γ_j's equals zero

$$\sum_{i=1}^{m} \alpha_j = 0 \quad , \qquad \sum_{j=1}^{n} \gamma_j = 0$$

APPROACH 1: ORDINARY LEAST SQUARES (OLS)

Now the extended model can be formally written as

$$\begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} X \\ L \end{pmatrix} \beta + \begin{pmatrix} u \\ o \end{pmatrix} \quad , \qquad \sum = \sigma^2 I \quad ;$$

the best unbiased estimator of β is

$$\hat{\beta} = (X'X + L'L)^{-1}(X'y + L'z) \quad ;$$

and the covariance-matrix of $\hat{\beta}$ is determined by

$$E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)'] = E[Z^{-1}X'uu'XZ^{-1}] = \sigma^2 Z^{-1}X'XZ^{-1}$$

where

$$Z = (X'X + L'L) \quad .$$

An estimator of the disturbance-term u can be given by

$$\hat{u} = y - X\hat{\beta} = y - XZ^{-1}(X'y - L'z) \quad .$$

As can be shown for complementary matrices X and L (Rao, 1971, p. 118)

$$XZ^{-1}L = 0 \quad , \qquad XZ^{-1}X'X = X \quad ,$$

u can be expressed simply by

$$\hat{u} = (I - XZ^{-1}X')u \quad .$$

Since $(I - XZ^{-1}X)$ is idempotent, an estimator of σ^2 can be given by

$$\hat{u}'\hat{u} = u'(I - XZ^{-1}X')u \quad .$$

The expected value of $\hat{u}'\hat{u}$ results in

$$\begin{aligned} E[\hat{u}'\hat{u}] &= E[u'u] - E[u'XZ^{-1}X'u] \\ &= T\sigma^2 - E[\text{trace } u'XZ^{-1}X'u] \\ &= T\sigma^2 - \sigma^2 \text{ trace } XZ^{-1}X' \\ &= (T - K + 2)\sigma^2 \end{aligned}$$

since $XZ^{-1}X'$ is idempotent and therefore

$$\begin{aligned} \text{trace}[XZ^{-1}X'] &= \text{trace}[Z^{-1}X'X] = \text{rank}[Z^{-1}X'X] \\ &= \text{rank}[X'X] = K - 2 \quad .^{1)} \end{aligned}$$

An unbiased estimator of σ^2 can now be given by[1)]

$$\sigma^2 = \frac{\hat{u}'\hat{u}}{T - K + 2} \quad .$$

[1)] For the two-dimensional case.

If one assumes normally distributed error terms, $\hat{\beta}_i$ can be tested against the hypothesis

$$H_o: \quad \beta_i = 0$$

by the term

$$t = \frac{\hat{\beta}_i}{\hat{\sigma}_{\hat{\beta}_i}} \quad .$$

where t is distributed by Student's t-Distribution with T - K + 2 degrees of freedom.

If one wants to test the combined hypothesis $\beta_2 = \beta_2^*$ where β_2 is one part of the parameter vector β of dimension n, β^*_2 any predetermined vector, and

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} ,$$

the model becomes[1)]

$$\begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} X \\ L \end{pmatrix} \beta + \begin{pmatrix} u \\ o \end{pmatrix} = \left(\begin{array}{c|c} X_1 & X_2 \\ L_1 & L_2 \end{array}\right) \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} u \\ o \end{pmatrix} \quad .$$

One gets the F-distributed variable by means of

$$F = \frac{u^{*\prime} u^* - \hat{u}'\hat{u}}{\hat{u}'\hat{u}} \cdot \frac{T - r}{r - r_1}$$

where

$$u^* = y - X_2\beta_2^* - X_1\hat{\beta}_1, \ \hat{\beta}_1 \text{ estimated under given } \beta_2^*,$$

[1)] In the two-dimensional case, β_2 corresponds to γ.

$$r = \text{rank } X'X,$$

$$r_1 = \text{rank } X_1'X_1,$$

$$r_2 = \text{rank } X_2'X_2,$$

with $(r - r_1)$ and $(T - r)$ degrees of freedom.

APPROACH 2: GENERALIZED LEAST SQUARES (GENLSQ)

In this application the dependent variable is of a special type: its value is always a fraction of one, and it is binomially distributed. To take this heteroscedastic situation into account, one should apply generalized least squares. The model assumptions must be changed. The covariance-matrix of the disturbances is no longer assumed to be proportional to the identity matrix. It is, instead, assumed to be a diagonal matrix in which elements may differ because of the different precisions of the samples:

$$E[u] = 0$$

$$\sum_{uu} = \begin{bmatrix} \sigma_1^2 & . & . & . & 0 \\ 0 & \sigma_2^2 & . & . & . \\ . & & . & & . \\ . & & & . & . \\ 0 & . & . & . & \sigma_T^2 \end{bmatrix} = V \quad .$$

An iterative procedure was programmed:

1. Approximation of V-matrix

$$V = \text{diag } \frac{y_t(1 - y_t)}{n_t} = \text{diag } \sigma_t^2$$

where y_t are the observed percentage values and n_t the sample size. The elements of V represent approximations

of the variances of binomially distributed random variables.

2. Computation of β by generalized least squares method under the assumption of a known V-matrix.
3. Computation of $\hat{y} = X\hat{\beta}$.
4. Computation of a new matrix V.

$$V = \text{diag}\ \frac{\hat{y}_t(1 - \hat{y}_t)}{n_t} = \text{diag}\ \hat{\sigma}^2_t \quad .$$

After this, one can start at Step 2 and proceed up to a stop-rule.[1)]

For the derivation of an estimator for β by the maximum likelihood principle, normally distributed disturbances were assumed. The initial value of their variances was taken from the corresponding binomial distribution. The likelihood function can be written as follows

$$L(\beta,\sigma_1^2,\ldots\sigma_T^2) = \frac{1}{(2\pi)^{\frac{T}{2}}} \prod_{i=1}^{T} (\sigma_i^2)^{-\frac{1}{2}} \exp - \{\tfrac{1}{2}(y - X\beta)'V^{-1}(y - X\beta)\} .$$

The linear restrictions on β are added in the logarithmic form by means of Lagrangian multipliers λ

$$\ln L = -\frac{T}{2}\ln 2\pi - \frac{1}{2}\sum_{i=1}^{T} \ln\sigma_i^2 - \frac{1}{2}(y - X\beta)'V^{-1}(y - X\beta) + \lambda'(z - L\beta) .$$

Partial derivatives to β and σ_i^2 result in

$$\frac{\partial \ln L}{\partial \beta} = X'V_y^{-1} - X'V^{-1}X\beta - L'\lambda = 0$$

and

---

1) The program stops if the $\hat{\beta}_i$'s maximum percentage change in succeeding steps is less than 0.001.

$$\frac{\partial \ln L}{\partial \sigma_i^2} = -\frac{1}{2\sigma_i^2} + \frac{1}{2}(y_i - \sum_j x_{ij}\beta_j)^2 \frac{1}{\sigma_i^4} = 0 \quad .$$

In matrix notation one gets the normal equations in the following partitioned form

$$\begin{bmatrix} X'V^{-1}X & \vdots & L' \\ L & \vdots & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \lambda \end{bmatrix} = \begin{bmatrix} X'V^{-1}y \\ z \end{bmatrix} \quad .$$

For abbreviation, the left-hand side matrix is called A. The inverted matrix A^{-1} is partitioned analogously:

$$A = \begin{bmatrix} X'V^{-1}X & L' \\ L & 0 \end{bmatrix} \quad , \qquad A^{-1} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad ,$$

Since A is symmetric, A^{-1} is symmetric too, and $S_{21}' = S_{12}$.
With these abbreviations the following equations hold

$$X'V^{-1}XS_{11} + L'S_{21} = I_K \qquad X'V^{-1}XS_{12} + L'S_{22} = 0$$

$$LS_{11} = 0 \qquad LS_{12} = I_{K-r}$$

$$S_{11}X'V^{-1}X + S_{12}L = I_K \qquad S_{11}L' = 0$$

$$S_{21}X'V^{-1}X + S_{22}L = 0 \qquad S_{21}L' = I_{K-r} \quad .$$

$\hat{\beta}$, a conditionally unbiased estimator of β, can be expressed by

$$\hat{\beta} = S_{11}X'V^{-1}y + S_{12}z \quad .$$

Unbiasedness of $\hat{\beta}$ is proved by

$$E[\hat{\beta}] = E[S_{11}X'V^{-1}(X\beta + u)] = E[(I_K - S_{12}L)\beta + S_{11}S'V^{-1}u]$$

$$= E[\beta - S_{12}z + S_{12}z + S_{11}X'V^{-1}u] = \beta$$

The covariance-matrix of β is given by

$$E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)'] = E[S_{11}X'V^{-1}uu'V^{-1}XS_{11}]$$

$$= S_{11}X'V^{-1}VV^{-1}XS_{11}$$

$$= S_{11}X'V^{-1}XS_{11}$$

$$= (I_K - S_{12}L)S_{11}$$

$$= S_{11} \quad .$$

It can easily be shown that $\hat{\beta}$ fulfills the linear restrictions $L\hat{\beta} = z$.

Unfortunately, the estimate of the covariance matrix of the disturbances in general is biased. This can be shown as follows. As

$$\hat{u} = (I - XS_{11}X'V^{-1})u \quad ,$$

$$E[\hat{u}\hat{u}'] = E[(I - XS_{11}X'V^{-1})uu'(I - XS_{11}X'V^{-1})']$$

$$= (I - XS_{11}S'V^{-1})V(I - V^{-1}XS_{11}X')$$

$$= V - 2XS_{11}X' + XS_{11}X'V^{-1}XS_{11}X'$$

$$= V - 2XS_{11}X' + X(I_K - S_{12}L)S_{11}X'$$

$$= V - XS_{11}X' - XS_{12}LS_{11}X'$$

$$= V - XS_{11}X' \neq V \quad .$$

If T is large, the distribution of $\hat{\beta}_i$ can be approximated by a normal distribution. By means of

$$N = \frac{\hat{\beta}_i - \hat{\beta}_i^*}{\sqrt{s_{ii}}}$$

the hypothesis H_o: $\beta_i = \beta_i^*$ can be tested, where S_{ii} is the i-th diagonal element of S_{11} and β_i^* a predetermined value.

APPROACH 3: LOGIT-TRANSFORMATION[1] (LOGIT)

If--as in this applicaton--the dependent variable varies between zero and one only, then one usually should not expect the relationship between the endogenous and the exogenous variable to be linear. With certain values of the exogenous variables, the interval between 0 and 1 of the endogenous variable could be exceeded, which is contradictory to percentage figures. In addition to this fact, the distribution of the dependent variable is not of the normal type (although it can be approximated by it) but obeys the laws of the binomial distribution

$$\phi(a) = \frac{N!}{a!(N - a)!} \pi^a (1 - \pi)^{N-a} \quad .$$

If there is a given probablity π for the occurrence of a certain event, $\phi(a)$ represents the probability of the occurrence of the event after N trials. The parameters of the distribution are given by

$$E[a] = N\pi \quad ; \qquad V[a] = N\pi(1 - \pi) \quad .$$

If one does not express this probability for the absolute values but, instead, for proportions $p = a/N$, one gets

$$E[p] = \pi \quad ; \qquad V[p] = \frac{\pi(1 - \pi)}{N} \quad .$$

---

[1] Here the author follows a book by A. Linder and W. Berchtold (1976).

To get rid of the restriction on the range, one can transform the dependent variable. In this case the LOGIT transformation was chosen

$$z = \ln \frac{\pi}{1 - \pi}$$

which has the following advantages:

-- While π varies between zero and one, z varies on the real numbers. Therefore, linear models can be more approximately applied to z,

$$E[z] = X\beta$$

-- The LOGIT transformation is invariant against biased data. Biases occurring in differences of π are ruled out in the domain of z (Linder, 1976, pp. 25-27)
-- Changes in extreme values of z will bring about minor changes in π rather than changes in central values of z. This property could be helpful, for extreme values could more likely correspond to errors in the measurement

One derives the estimator for β on the basis of a likelihood function as a product of binomial distributions

$$L = \prod_{t=1}^{T} \binom{n_t}{a_t} \pi_t^{a_t} (1 - \pi_t)^{n_t - a_t}$$

or, in logarithmic terms,

$$\ln L = \sum_{t=1}^{T} a_t \ln \pi_t + \sum_{t=1}^{T} (n_t - a_t) \ln(1 - \pi_t) + \text{const} \quad .$$

Here, π_t is dependent via the inverse LOGIT transformation on z_t

$$\pi_t = \frac{\exp z}{1 + \exp z}$$

and the T-vector z is the dependent variable of the linear model

$$E[z] = X\beta \quad .$$

The maximum of the likelihood-function is found by computing partial derivatives of ln L and by setting them to zero.

$$\frac{\partial \ln L}{\partial \beta_j} = \sum_{t=1}^{T} \frac{\partial \ln L}{\partial \pi_t} \frac{\partial \pi_t}{\partial z_t} \frac{dz_t}{d\beta_j} \qquad j = 1 \ldots k \quad ,$$

$$= \sum_{t=1}^{T} \underbrace{\left[\frac{a_t}{\pi_t} - \frac{n_t - a_t}{1 - \pi_t} \right] \frac{\partial \pi_t}{\partial z_t}}_{d_t} \cdot \frac{dz_t}{d\beta_j}$$

$$= \sum_{t=1}^{T} d_t \cdot \frac{dz_t}{d\beta_j} = \sum d_t x_{tj} = 0$$

or, in matrix notation,

$$\frac{\partial \ln L}{\partial \beta} = X'd = 0$$

where

$\frac{\partial \ln L}{\partial \beta}$ is a K-column vector,

d a T-column vector, and

X a T x K-matrix.

This system of equations is non-linear and cannot be solved explicitly. Therefore, an iterative solution procedure is applied --the Newton-Raphson Method. One starts with an approximation of the vector β, b_o and computes the Taylor approximation near b_o

$$\left.\frac{\partial \ln L}{\partial \beta}\right|_{\beta = b_o + \partial\beta} = \left.\frac{\partial \ln L}{\partial \beta}\right|_{\beta = b_o} + \left.\frac{\partial}{\partial \beta}\left(\frac{\partial \ln L}{\partial \beta}\right)\right|_{\beta = b_o} \partial\beta = 0 \quad .$$

In using expected values

$$E\left[\frac{a_t}{n_t}\right] = \pi_t$$

for the second term, one gets

$$E\left[\frac{\partial^2 \ln L}{\partial\beta_j \partial\beta_k}\right] = -\sum_{t=1}^{T} \frac{n_t}{\pi_t(1-\pi_t)}\left(\frac{\partial\pi_t}{\partial z_t}\right)^2 \frac{\partial z_t}{\partial\beta_j}\cdot\frac{\partial z_t}{\partial\beta_k} = -I_{jk} \cdot$$

By using the abbreviations

$$W_t = \frac{u_t}{\pi_t(1-\pi_t)}\left(\frac{\partial\pi_t}{\partial z_t}\right)^2$$

and

$$W = \operatorname{diag} W_t \quad ,$$

the matrix J of the I_{jk} can be written as

$$J = X'WX$$

and

$$\partial\beta = J_o^{-1}\left.\frac{\partial \ln L}{\partial\beta}\right|_{\beta=b_o} = J_o^{-1}X'd_o \quad .$$

The iteration procedure thus results in

$$b_{l+1} = b_l + \partial\beta\Big|_{\beta=b_l}$$

One can use $|\partial\beta_l| < \varepsilon$ as a stopping rule. The elements of J^{-1} can be used as an approximation of the variances and covariances of the elements of β.

For the case of LOGIT transform, W_t can be specified by

$$W_t = \frac{n_t}{\pi_t(1-\pi_t)} \cdot \pi_t^2(1-\pi_t)^2 = n_t \cdot \pi_t(1-\pi_t) \quad ,$$

which corresponds to the elements of the matrix V^{-1} in the second approach ($\frac{\partial\pi_t}{\partial z_t} = 1$ in that case).

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