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SOCIALLY OPTIMAL AGRICULTURAL EROSION-SEDIMENTATION CONTROL CONSIDERING BOTH SOIL CONSERVATION AND WATER QUALITY

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ABSTRACT

Social concern about erosion and sedimentation arises principally from two factors. One factor is the future social costs in the form of reduced productivity that arise from erosion, while the second is the current and to some extent future social costs resulting from sediment pollution. This paper presents a dynamic non-linear optimization model that can be used to determine the socially optimal level of soil conservation when both of the above factors are considered. The objective function in the model is the present value of consumers' plus producers' surplus less off-site sediment damages, over a long planning horizon.

The model is applied to a watershed that is fairly representative of the Corn Belt. Results indicate that substantially more soil conservation than presently occurs is justified from society's viewpoint.

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INTRODUCTION

Soil erosion is an important social problem for two primary reasons. One reason is the increasing need for food, which calls for erosion control because erosion affects the long-run productivity of our soil resource. The second reason that erosion is of concern is its relationship to environmental quality. By reducing soil producitvity, erosion results in future social costs, while pollution related to erosion results in current and to some extent future social costs.

Taking the United States as an example, it has been estimated that the nationwide average rate of erosion on cropland is 168 times that of commercial forests and that of grassland is 11 times higher than forests. On a total basis, cropland contributes about 50 percent of the sediment delivered to streams and lakes in the U.S. (U.S.E.P.A.). This sediment includes a large but undefined amount of toxic pesticides and plant nutrients. Estimations by the regional offices of the Environmental Protection Agency indicate that approximately 35 percent of the nation's waterways had water quality standard violations and that approximately 40 percent of these problems were attributable to nonpoint sources, primarily agriculture and forestry (Pisano, 1976a).

The erosion-sedimentation problem may become more acute with time. In view of a world-wide rise in demand for agricultural products, cultivation of land will intensify and the nonpoint source problem will increase in geographical extent and magnitude. However, as Davis (1977) points out, this expansion could take place without seriously damaging natural resources if land users would install adequate soil and water management systems as conversion takes place. Davis also estimates that currently 42 percent of the nation's cropland has no conservation treatment.

Historically, soil conservation policy for the United States has consisted of setting erosion control "guidelines" in the form of soil loss tolerance levels for individual soil types. Adherence to these guidelines has been purely voluntary. Typically, soil loss tolerance levels have been based on a "physical" notion of conservation without reference to projected economic conditions facing society. Tolerance levels most often represent subjective judgements about the amount of erosion that can occur annually without reducing crop production in the foreseeable future with current production technology.

Only recently has the environmental quality aspect of erosion come to the forefront in formulation of erosion control policies. Pollution laws in the U.S. now call for the definition and implementation of "best management practices" for erosion control. It appears that in most situations, best management practices are being defined as practices that will not result in erosion levels exceeding the "tolerance levels". This is an unfortunate definition for two reasons. First, *environmental* guidelines are based on a physical notion of *conservation*. Although environmental quality and conservation are closely related, the relationship is not one-to-one. A second reason the definition is an unfortunate one is that it does not consider economic factors.

This paper presents a formal approach for determining optimal erosion rates from a societal viewpoint. The approach incorporates the economics of both the environmental quality and soil conservation aspects of the erosion-sedimentation issue. Such a formulation requires explicit statements of perhaps otherwise latent assumptions regarding relationships among variables believed to be relevant in determining optimum soil loss--with such a formulation, the implied "best management practices" or "soil loss tolerance levels" achieve a balance between:

1. the net social benefits of current and future food production, and

2. the current and future net social costs of erosion related pollution and reduced land productivity. The model is a partial equilibrium model in the sense that only the agricultural sector will be considered.

For empirical determinization of socially optimal soil loss, a dynamic nonlinear optimization model is formulated. The criterion function is the sum of discounted producers' and consumers' surpluses minus sediment damages. As an illustration, the framework is applied to a watershed located in Illinois.

CONCEPTUAL FRAMEWORK

It is important to note at the outset of this paper that there is a technical distinction between erosion and sediment. Erosion is the movement of soil away from a particular plot or site, while eroded soil particles become sediment when they are deposited at another site. Given this distinction, it can be seen that erosion is of major concern is conservation, while sediment is of major concern in pollution.

Naturally sediment is functionally related to erosion. The exact functional relationship or transport mechanism is not presently well known. As a first approximation, sediment is typically assumed to be proportional to erosion, with the proportionality constant called the "delivery ratio".

It is appropriate to view the soil as a resource that provides physical support for the plant and its roots, and also as a store house for water, plant nutrients, and organisms that directly or indirectly affect plant growth. Viewed in this way, it can be seen that biomass production can be increased by increasing the root penetrability of the soil, by increasing the water storage capacity of the soil, and by other means. Conversely, biomass production can decline if nutrients and other factors are stripped from the soil by certain agricultural practices. Erosion affects future biomass production by carrying away plant nutrients and organic matter, and, by reducing topsoil and thereby forcing plants to obtain nutrients from the less penetrable and less productive subsoil.

From a technical perspective, soil "conservation" could be considered as being directed toward two goals:

1. Maintaining the productive services the soil provides.
2. Abating sediment and hence pollution[1].

Given the view of "soil" as expressed previously, the first goal does not necessarily mean that absolutely no soil is eroded as it is conceivable that maintaining the productive services at a certain level could be accomplished with erosion. In this case, plant nutrients and other factors could be manipulated to maintain productivity. Also formation of topsoil from subsoil could replace the eroded soil.

Let us consider now the economic rationale for soil conservation. From a societal point of view, the economically optimal level of soil conservation (or optimal level of husbanding of soil) can be defined as the soil use and management practices that maximize the well being of the people who are directly or indirectly affected by soil and that are produced by soil. Intuitively, it can be seen that this level of soil management may mean a decrease in the productivity of soil, an increase in productivity accomplished by improving the soil or no change in the productivity of the soil, depending on future as well as current human values. Thus, this economic concept of "conservation" may differ from the technical notion of conservation. In the remainder of this paper we are concerned with determining the socially optimal degree of conservation; that is, we are concerned with the "economic" concept of conservation[2].

If one uses the concept of economic surplus as an approximate measure of social well-being (for the moment assume that there are no pollution externalities), the optimal level of soil conservation is obtained by the maximization of the present value of economic surpluses summed for consumers and producers. And conservation can be seen as affecting the future supply functions and thus affecting future producers' surplus. But because the supply curve determines price and thus consumers' surplus, conservation can be seen to affect future consumers as well.

Because conservation affects future as well as current generations the issue must be investigated in a dynamic setting. An infinite time horizon appears appropriate for such an investigation. For hydrological reasons, the watershed is the appropriate unit of analysis for erosion-sedimentation studies. A country's land resources can appropriately be viewed as comprised of many "small" watersheds, linked by the downstream movement of soil and also linked by economic interdependencies. Thus an ideal national model for analyzing the economics of erosion-sedimentation would be one based on many small watersheds, with each watershed having its own sub-model tied with other sub-models through common demand, transportation costs, and input factors which are not fixed at the watershed level.

To adequately reflect erosion-sedimentation facotrs in a model, it is imperative to divide the land base according to soil type-slope-erosion capability classes. The number of such soil classes to include in an empirical model is largely determined by data availability and computational considerations.

THE FORMAL MODEL

Based on the above conceptual view of the erosion sedimentation issue, a model appropriate for determining the socially optimal level of erosion can be formally stated as a dynamic nonlinear optimization problem. The objective function for this model is:

$$(1) \quad \underset{A,N}{\text{MAX}}\ J = \sum_{t=1}^{T} \beta^{t} \left\{ \sum_{j} \sum_{r} \int_{o}^{Q^{*}_{jtr}} H_{jtr}(Q_{jtr})\,dQ - \sum_{j} \sum_{i} \sum_{l} \sum_{m} C_{tjilm} \times A_{tjilm} - \sum_{j} \sum_{k} \sum_{\eta} V_{thkj} \times W_{thkj} - D_{t}(S_{ti}, N_{tjilm}) \right\}$$

where

β = social discount factor

t = time index

T = social planning horizon (may be infinite)

j = commodity index

r = consuming region index

H_{jtr} = compensated demand curve for commodity j in consuming regions

Q^*_{jtr} = market equilibrium quantity of commodity j in region r

i = watershed index (single or multiple watersheds may comprise a consuming region

l = soil class index

m = conservation practice-tillage system combination index

C_{tjilm} = per acre variable production costs (excluding external costs)

A_{tjilm} = planted acreage

N_{tjilm} = fertilizer input rate

V_{thkj} = per unit cost of transporting commodity j from the h-th consuming region to the k-th consuming region

W_{thkj} = units of commodity j transported from region h to region k

S_{ti} = sediment load in the i-th watershed

D_t = external costs associated with sediment and/or fertilizer.

The socially optimal resource use policy for each period of the planning horizon can be found by maximizing equation (1) subject to a set of economic, resource, and technological constraints and relationships. These are:

Demand-Supply Identity

$$(2) \quad Q_{jtr} = \sum_{i \epsilon r} \sum_{l} \sum_{m} Y_{tjilm} A_{tjilm} + \sum_{h} T_{thrj} - \sum_{k} T_{trkj} \quad \forall\ j,t,r$$

where

Y_{tjilm} = yield per planted acre

Production Functions

(3) $Y_{tjilm} = f(N_{tjilm}, E_{(t-1)jilm} \cdots E_{ojilm}, \bar{Y}_{ojilm})$

where

E_{tjilm} = erosion rate

$\bar{Y}_{ojilm}$ = measure of initial soil productivity

Erosion relationship

(4) $E_{tjilm} = f(A_{tjilm})$

Sediment Load Relationship

(5) $S_{ti} = d(E_{tjilm}, S_{ti})$ (i = watersheds upstream to watershed i)

Land Constraint

(6) $\sum_j \sum_m A_{tjilm} \leq L_{til}$ i,t,l

where

L_{til} = total available acreage of soil class l in watershed i in period t

Variable Production Cost

(7) $C_{tjilm} = f(N_{tjilm}, Y_{tjilm}, R_t)$

where

R_t = per unit cost of fertilizer.

The first term in the criterion function (1) is the area under all demand curves, while the second term is total variable production costs excluding external costs. The third term is total costs of transporting commodities between consuming regions. External costs attributable to sediment and fertilizer pollution are reflected in the last term in equation (1).

Constraint (2) shows that the total quantity of a commodity consumed in each consuming region must equal production in that region plus net transportation of that commodity into the region. Equation (e) reflects the influence of both erosion and fertilization on per-acre yield and accounts for the possibility of substituting fertilizer for eroded topsoil. The relationship between crop acerages and erosion is represented by equation (4), while the relationship between sediment load in the i-th watershed and erosion in that watershed as well as sediment in upstream watersheds is given in (5). Availability of land is reflected in equation (6). Per acre variable production costs as related to fertilization rate and yeild is represented by equation (7).

AN EMPIRICAL CASE STUDY

Although the above theoretical model is not the most elaborate that can be envisioned, it is nevertheless most ambitious from both computational and data requirement standpoints. To empirically implement the model for a large country (say the U.S.) with a reasonable degree of accuracy and detail, would require in our opinion, delineation of many watersheds, soil classes and production practices. With ten commodities, 100 watersheds, and 10 soil classes in each watershed, which would perhaps be a reasonable amount of detail, one would have a control vector of dimension 20,000 (Acreage and fertilizer rate) for each time period of a long time horizon. Since the model is non-linear this would obviously be a most ambitious computation undertaking.

To reduce the computational burden, one must decide between a small watershed model or an extremely aggregated national model. We selected the small watershed model approach because we thought that it would give better insight into the issue than would a very aggregated national model.

With a very small land base, like a small watershed, demand for products from that area can be assumed perfectly elastic. This assumption, while appropriate, does not allow one to gain insight into the price and consumer aspects of the erosion-sedimentation issue. To simultaneously gain insight into the consumer as well as producer (and thus social) effects and make the model more representative of a detailed national model, it was assumed that the production area takes up a fixed proportion of demand nationwide. This, in turn, implies that at any price level the price elasticities of demand for the watershed and the nation as a whole are equal. It also implies that the percentage change in production in the watershed will also occur outside the watershed.

Study Watershed

The Big Blue watershed in the northeastern Pike Country, Illinois, was selected as an area for which there was adequate data for an analysis and which was reasonably representative of the U.S. Corn Belt in terms of erosion potential. The watershed covers 1757.6 acres, of which about 58 percent was in corn in 1973, 28 in soybeans, 5 percent in small grains, and 8 in hay. While this acreage distribution is more representative of the Corn Belt than the whole U.S., the erosion rates and management practices are fairly representative of the U.S.

A large portion of the soils in the watershed are moderately thick loess. Except for an area of prairie soils in the northern part of the area the soils developed under timber vegetation. All together seventeen different soil types are included in this study. Their slope classes range from A (0 to 2 percent) up to E (12 to 18 percent). In order to keep the computational burden at a reasonable level, total acreage considered in the model was subdivided into only two groups. One comprises all the land with a slope class A or B and the other includes those acres in the class C, D, or E. Henceforth, the groups of soils will be called soil group 1 and soil group 2. Table 1 shows the acreage, distance-adjusted delivery ratio, and initial topsoil level for each soil group.

Crop Production Activities

Crop production activities in the model are framed in terms of crop rotation. This reflects the fact that soil erosion not only depends on the current crop but also on the cropping patterns of the past. All cropping patterns considered are currently practiced in the area and include the major crops. Crop rotations included are:

1. Continuous corn
2. corn/soybeans
3. corn/corn/soybeans/oats
4. corn/wheat/meadow/meadow
5. pasture.

These rotations are permitted on all soils, with the exception that pasture is not an alternative on land with slopes less than 4 percent (soil group 1).

Three tillage systems are included in the model: fall plow (conventional tillage), plow-plant, and chisel plow. These tillage systems may be used for any of the crops except pasture. Permanent pasture or renovation of an existing stand is assumed to utilize only a conventional tillage system.

Three conservation practices are included in the model: straight row cultivation, contouring and terracing. Straight row cultivation is an alternative on any soil, while contouring is permitted only on lands having a slope class of A or B (soil

Table 1. Soil grouping data.

Soil Group	Slope Classes	Acreage	Percent of group acreage with slope: A	B	C	D	E	Distance Adjusted Delivery Ratio	Initial Top Soil Level (inches)
1	A, B	842.3	29.8	70.2	-	-	-	0.158	8.532
2	D, C, E	915.3	-	-	47.7	34.1	18.2	0.179	4.713

group 1) and terracing only on land with slope class C and higher (soil group 2). Combining the three tillage systems with the three conservation practices gives nine management alternatives from which to choose.

Soil Loss Coefficients

The soil loss coefficients computed for this study reflect only sheet and rill erosion, which by far accounts for most of the total erosion. Soil losses for each crop production alternative on each soil group were calculated with the Universal Soil Loss Equation (Wischmeier and Smith). This equation is:

$$A = R \cdot K \cdot LS \cdot C \cdot P \tag{1}$$

where

A = average annual soil loss in tons per acre,
R = rainfall erositivity index,
K = soil erodibility factor,
LS = factor that reflects the combined effect of length, steepness, and shape of the field slope,
C = cropping system and management factor, and
P = supporting (e.g., conservation) practice factor.

Numerical values for these factors and sources of data are given by Frohberg.

Yield Functions

Crop yield, in addition to depending on soil type and crop rotation, is assumed to depend on nitrogen fertilization rate and topsoil thickness. Since the task of this study was to find the socially optimal soil erosion level over time, an accurate relationship between yield level and topsoil thickness is very important. This relationship is as difficult to quantify as it is important. Soil scientists have not addressed this problem recently. Some investigations for Illinois were done in the late forties and early fifties which relate yield to topsoil thickness (Odell-Odell and Oschwald; Rust). However, it is believed that there are some interactions between yeild response to nitrogen and to thickness of topsoil (Engelstad et. al.; Engelstad and Schrader). In a study done on Marshall and Monona Silt Loam in south-western Iowa, Engelstad, et. al., measured the effect of yield of both nitrogen applied and average organic carbon content in the zero to twelve inch layer. Their results indicate that for various levels of organic carbon content the yield response to nitrogen shifts in a non-parallel fashion. In other words, there is interaction between nitrogen and organic carbon, which can be used as a proxy for topsoil thickness.

Combining the data in Engelstad et. al. and Engelstad and Schrader with data by Welch the corn yield functions in Table 2 and the wheat and oats yield function in Table 3 were synthesized. Since little nitrogen is applied to soybeans, pasture and alfalfa, these crops have a basic yield level which is varied only by changes in the topsoil thickness. Yield functions for these crops are given in Table 4. The methodology that was used to obtain the functions in Table 2, 3, and 4 is discussed in detail by Frohberg.

The yield functions given in Table 2, 3, and 4 are for a conventional tillage system and straight row cultivation. In some cases, alternative tillage systems and conservation practices influence yield. Based on information in Seitz, et. al., yields under a chisel plow tillage system were assumed to be reduced by 5 percent. A plow-plant system was assumed to not change yield.

Production Cost

Crop production cost depends on the crop rotation, tillage system, conservation practice and soil group. Using data from many sources (see Frohberg) the production cost coefficients given in Tables 5, 6 and 7 were estimated.

Demand Functions

The watershed demand functions used in this study are given in Table 8. These were derived from national demand functions on the assumption that price elasticities of demand at a given price level were equal for the watershed and the nation. The national demand functions were estimated using common econometric procedures. The procedures and national demand functions are discussed by Frohberg.

Sediment Damage Function

Sediment damage may be defined as a reduction of benefits or an increase in the costs of other activities inside or outside the watershed. (Lee, et. al.). Hence we are concerned only with what is referred to as offsite damages, as on-site erosion damages in the form of reduced productivity have been accounted for in prededing sections. A procedure developed by Lee, et. al. was used to estimate off-site sediment damages in the Big Blue watershed. This procedure accounts for the following categories of damage:

1. Increase in annual reservoir cost.

   In the absence of sediment deposition, a reservoir (in the watershed or downstream) would function indefinitely and its construction cost would be amortized in perpetuity. However, sediment reduces the useful life of a reservoir and thus increases its annual amortized cost. The increase in annual cost resulting from sediment deposition can be expressed as the difference between:

Table 2. Yield functions for corn.

Soil Group	Rotation	Parameters of the yield function*: h_o	h_1	h_2	h_6	h_7	h_8
1	1	36.59	19.39	1.74	0.7224	0.00178	0.0855
	2	38.54	20.37	1.83	0.8706	0.00284	0.1030
	3	39.51	20.89	1.88	0.7249	0.00190	0.0857
	4	70.68	33.13	2.97	0.3884	0.00177	0.0459
2	1	33.04	17.14	1.57	0.7269	0.00179	0.0860
	2	35.38	18.70	1.68	0.9173	0.00316	0.1086
	3	35.76	18.91	1.69	0.7179	0.00190	0.0849
	4	62.77	31.07	2.78	0.3721	0.00176	0.0440

*The functions are of the form:

$$Y = h_o + h_1 \log D - h_2 (\log D)^2 + h_6 N - h_7 N^2 - h_8 N \log D$$

where

Y is yield of corn in bushels per acre, N is nitrogen applied in pounds per acre and D is topsoil depth in inches.

Table 3. Yield functions for wheat and oats.

Soil Group	Rotation	Crop	Parameters of the yield function*: h_3	h_4	h_5	h_6	h_7
1	3	wheat	24.22	.6259	.0269	.5768	.00466
	4	oats	26.87	.6943	.0298	.9469	.00657
2	3	wheat	20.50	.5332	.1597	.5856	.00478
	4	oats	23.08	.6002	.1798	.0409	.00657

*The functions are of the form:

$$Y = h_3 + h_4 D - h_5 D^2 + h_6 N - h_7 N^2$$

where

Y is yield in bushels per acre, N is per-acre nitrogen application rate, and D is topsoil thickness in inches.

Table 4. Yield functions for soybeans, alfalfa and pasture.

Soil Group	Crop	Parameters of the yield function* h_3	h_4	h_5
1	soybeans	32.63	.8430	.0362
	alfalfa	4.08	.1053	.0045
2	soybeans	26.11	.6789	.2033
	alfalfa	3.46	.0899	.0269

*The functions are of the form:

$$Y = h_3 + h_4D - h_5D^2$$

where

Y is yield of soybeans in bushels per acre of hay in tons per acre, and D is topsoil thickness in inches.

Table 5. Production cost coefficients for each crop and rotation as indicated (in 1976 dollars per acre).

		Costs not proportional to yield:			Costs
Crop	Rotation	Preharvest non-labor	Harvest non-labor	labor	proportional to yield
Corn	1	78.90	29.40	21.80	0.219
Soybeans	2	72.13	15.20	24.04	0.102
Wheat	4	53.00	14.70	12.96	0.105
Oats	3	62.22	14.70	9.56	0.094
Alfalfa	4	77.72	70.45	4.00	9.73
Pasture	5	15.30*	89.91*	--	--

*Includes labor costs.

Table 6. Changes in preharvest costs of corn for rotations not given in Table 5.

Rotation	Change in preharvest cost (dollars per acre)
2	-5.50
3	-2.70
4	-5.50

Table 7. Cost adjustment coefficients for alternative tillage system and conservation practices.

Tillage System	Conservation Practice: Straight Row	Contouring	Terracing
Conventional (Fall Plow)	--	0.90	5.40
Plow-plant	1.26	2.16	6.66
Chisel plow	-1.20	-0.30	4.20

Table 8. Demand functions at the watershed level for five crops*.

Crop	η	γ	ν
Corn	-10.303	0.000168	0.4270
Soybeans	-152.208	-0.004718	3.0946
Wheat	32.400	0.018163	0.0060
Oats	8.273	0.001124	-0.0844
Hay	183.860	0.135670	-1.0866

*The functions are of the form:

$$P = \eta - \gamma Q + \nu T$$

where

P is price, Q is quantity demanded, and T is a time variable (T = year - 1900).

a) the annuity corresponding to the total cost for the estimated life of the reservoir taking sedimentation into account; and b) the annuity corresponding to an infinite life in the absence of sediment.

2. Increase in flood damage after the useful life of the reservoir.

   There are no flood preventation benefits after the reservoir's useful economic life has ended. The assumption is made that flood damage returns to its level prior to the construction of the reservoir. Included in this damage estimation are: a) the annual flood damage that occurs prior to the end of the sediment pool capacity; b) the increase in flood damage following the end of the estimated life of the total reservoir capacity; and c) the increasing level of flood damages occuring between a) and b).

3. Increase in upstream drainage ditch maintenance cost.

   Not all of the soil that is eroded reaches a reservoir. Some is trapped in the drainage network of the watershed. Sedimentation of the drainage network imposes damage in the form of cleaning or dredging to maintain its viability.

4. Sediment damage as part of downstream damage.

   Some of the total downstream flood damage is due to sediment being deposited on flooded streets, homes, businesses, etc. This component of total sediment damage reflects that fraction of total flood damage.

5. Increase in water supply costs after the end of the reservoir's economic life.

   The municipal and industrial water supply benefit of the reservoir ceases at the end of the economic life of the reservoir. The reduction in water supply benefit due to sedimentation of the reservoir constitutes a part of total sediment damage and includes a) the increase in water supply costs following the end of the sediment pools life, but prior to the end of the life of the water supply pool.

6. Increase in water treatment cost due to sedimentation of a water supply reservoir.

   Suspended sediment and some attached elements must be removed from municipal and industrial water. The costs of this treatment are reflected in this component of total sediment damage.

Off-site sediment damages in Big Blue watershed as a function of soil erosion are shown in Table 8 for various interest (or social preference) rates. Damages were estimated for various erosion levels and, for a given interest rate, were found to be approximately proportional to the erosion level. That is, the marginal external costs of sediment is constant over the relevant range. Computational formulae and data used to obtain the values in Table 9 are given by Frohberg.

Time Horizon and Penalty Function

The model is set up to effectively cover a time span of 48 years. Succeeding years were explicitly included in the model by using a penalty function which represents the loss in economic surplus in the 49th and all following years due to erosion during the first 48 years. This penalty function method was used to reduce the number of time periods (and thus variables and constraints) in the optimization model, yet approximate an infinite decision horizon.

The penalty function is dependent on the depletion of the topsoil during the first 48 years, the demand schedule, yeild levels, and production costs after the 48th year. Another factor influencing the penalty is the rate of soil depletion which very likely is influenced by the depth of topsoil in year 49. The rate of soil depletion after the 48th year is not an endogenous variable in the primary model.

To estimate the penalty function, some simplifying assumptions had to be made. These are:

1. The process of soil erosion ceases to continue after year 48[3]
2. Demand, yield and production cost for all years after period 49 remain at the level forcast for year 49.
3. Only those tillage systems and conservation practices that are the most effective in reducing erosion are used after year 48.

Under these assumptions, a static optimization model for year 49 was set up and solved for different levels of topsoil, the largest being the initial topsoil level. Then, the differences in economic surplus were regressed on topsoil thickness. The resulting function is:

$$F(D_{49,1},D_{49,2}) = 40872.6 + 311235/D^3_{49,1} + 14608/D^3_{49,2} - 1049 \cdot D_{49,1}D_{49,2} \tag{2}$$

Table 9. Estimated sediment damage functions* for various interest rates.

Damages in dollars per ton at annual interest rate (percent) of:										
2	3	4	5	6	7	8	9	10	11	12
3.177	2.970	2.809	2.676	2.564	2.467	2.384	2.312	2.250	2.195	2.148

*The damage function as of the form

$S = d\overline{M}$

where

S is total damages, $\overline{M}$ is the total tons of sediment in the watershed, and $\overline{d}_r$ is obtained from this table for the r-th interest rate.

where

F = annual loss in economic surplus

$D_{49,1}$ = topsoil depth in year 49 for soil group 1.

It was assumed that F would be sustained over an infinite period. Hence, the present value of F into perpetuity was used in the model as a penalty term.

Mathematical Presentation of the Empirical Model

Based on the discussion in the preceding subsections the problem can be written as follows:

$$(3) \quad \underset{X}{\mathrm{MAX}} \sum_{t=1}^{T} \beta^{t} \left\{ \sum_{j=1}^{5} [\alpha_{tj} Q_{tj} - \frac{\gamma j}{2} Q_{tj}^{2} - C_{tj}] - S_{t} \right\} - \left(\frac{\beta^{T+1}}{r} \right) F(D_{49,1}, D_{49,2})$$

$$\text{or } \underset{X}{\mathrm{MAX}} \sum_{t=1}^{T} \beta^{t} \left\{ \sum_{j=1}^{5} [\alpha_{tj} \sum_{i=1}^{2} \sum_{k=1}^{6} \sum_{m=1}^{9} W_{jk} Y_{tijkm} X_{tikm} \right.$$

$$- \frac{\gamma_j}{2} \left(\sum_{i=1}^{2} \sum_{k=1}^{6} \sum_{m=1}^{9} W_{jk} Y_{tijkm} X_{tikm} \right)^{2}$$

$$- \sum_{i=1}^{2} \sum_{k=1}^{6} \sum_{m=1}^{9} \left(c_{tjkm} + d_{j} Y_{tijkm} + P_{tn} N_{tijk}^{*} \right) W_{jk} X_{tikm}$$

$$\left. - \bar{d}_{r} \sum_{i=1}^{2} d_{i}^{*} b_{i}^{*} \sum_{k=1}^{6} \sum_{m=1}^{9} M_{tikm} X_{tikm} \right\}$$

$$- \left(\frac{\beta^{T+1}}{r} \right) F(D_{49,1}, D_{49,2})$$

subject to constraints

$$(4) \quad g_{ti}^{*}(X) = \sum_{k=1}^{6} \sum_{m=1}^{9} X_{tikm} - L_{i} \leq 0 \qquad t,i$$

$$(5) \quad g_{tikm}(X) = -X_{tikm} \leq 0 \qquad t,i,k,m$$

and subject to the state equation

$$(6) \quad D_{(t+1)i} = D_{ti} - \sum_{k=1}^{6} \sum_{m=1}^{9} M^{*}_{tikm} X_{tikm}$$

and the yield functions

$$(7) \quad Y_{tijkm} = [h_{oijk} \cdot \delta_{m} + h_{1ijk} \log D_{ti} - h_{2ijk} (\log D_{ti})^{2}]$$
$$+ [h_{3ijk} + h_{4ijk} D_{ti} - h_{5ijk} D^{2}_{ti}] + h_{6ijk} N^{*}_{tijk}$$
$$- h_{tijk} (N^{*}_{tijk})^{2} - h_{8ijk} (\log D_{ti}) N^{*}_{tijk}$$

and the optimal nitrogen level given by

$$(8) \quad N^{*}_{tijk} = \frac{h_{6ijk} - h_{8ijk} \log D_{ti} - P_{tn}/P^{*}_{tj}}{2h_{7ijk}}$$

where

t = index for time periods, with one t representing 16 years ($t=1,2,3$).

i = index for soil groups ($i=1,2$).

j = index for crops ($j=1,2,\ldots,6$).

k = index for crop rotation ($k=1,2,\ldots,6$) where rotation 6 represents idle land.

m = index for combinations of tillage systems and conservation practices ($m=1,2,\ldots,9$).

β = discount factor set equal to the geometric mean of annual discount factors over a 16 year period.

r = social time preference rate.

Y_{tijkm} = per-acre yield of crop j in rotation k with tillage system-conservation practice combination m on soil group i in period t.

X_{tikm} = acreage of rotation k with tillage system-conservation practice combination m on soil group i in period t.

N^*_{tikj} = optimal rate of nitrogen fertilizer applied to crop j in rotation k on soil group i in period t.

M_{tikm} = annual gross soil loss in inches per acre on soil group i.

M^*_{tikm} = annual gross soil loss in inches occuring as an average over the total acreage of soil group i when using rotation k with a tillage system-conservation practice combination m in period t; that is,

$$M^*_{tikm} = \frac{M_{tikm}}{L_i} .$$

C_{tj} = production cost of crop j in year t.

S_t = sediment damage in year t.

D_{ti} = depth of topsoil of soil group i in period t, measured in inches.

L_i = total acreage of soil group i available for crop production in any time period.

α_{tj}, γ_j = coefficients of the demand function for commodity j in time period t.

W_{jk} = weighting factor for crop j in rotation k.

c_{tjkm} = total production cost nonproportional to the yield of crop j grown in rotation k with tillage system-conservation practice combination m in period t.

d_j = total production cost proportional to the yield of crop j.

P_{tn} = price of nitrogen fertilizer in period t.

P^*_{tj} = price of crop j in period t set prior to solving the model[4].

$\bar{d}_r$ = sediment damage per ton of soil delivered to the drainage system and reservoir for a given time preference rate, measured in dollars.

d^*_i = distance adjusted sediment delivery ratio for soil group i.

b^*_i = bulk density of soil group i.

h_{vijkm} = the v-th coefficient of the yield function for crop j in rotation k on soil group i with tillage system-conservation practice combination m.

δ_m = factor to adjust yield for the tillage system-conservation practice m.

The optimal nitrogen application rate, N^*_{tijk}, is among other variables a function of topsoil depth (for corn) and of the respective commodity prices. Both of these variables are endogenous to the model. However, the optimization model structure would be substantially more complex if the commodities which are endogenous to the model were used to compute N^*_{tijk}. For this reason, it was decided to compute N^*_{tijk} with prices set prior to the solution of the model. As long as the prices obtained from the optimization model are close to the prices used to compute N^*_{tijk}, *a priori*, the bias should be small, yet the computational burden much lower.

Model results for a social time preference rate (STPR) of two percent are shown in Tables 10 through 12, and results for a STPR of eight percent are shown in Tables 13 through 15. All results are based on the assumption that no technical progress occurs in the future. To reflect expected population and income increases, demand was shifted outward over the 48 year time period[7]. This explains the rapid increase in commodity prices that was obtained in the solution for either STPR (see Table 10 or 13).

Tillage systems currently used in the watershed are primarily fall plowing with a limited amount of plow-planting and chisel plowing. However, the model shows that with either a 2 or 8 percent STPR it is to society's advantage to use mostly a plow-plant system, with some of the acreage tilled with a chisel plow, and none of the acreage fall plowed (Table 12 and 15). Furthermore, the model shows that most of the land should be contoured or terraced, while most of the farmers in the watershed presently use straight row cultivation.

Third period soil losses with a 2 percent STPR are about the same as first period losses. However, with an 8 percent STPR, losses in the third period are about 25 percent greater than first period losses. In terms of inches of topsoil lost, the differences in soil loss over the 48 year period are not great. Because of the more than doubling of the price of crops that are intensive users of nitrogen fertilizer, the fertilization rate increased over time. Although soil erosion decreases yields overtime, ceteris paribus, the increased fertilization resulted in a very slight net increase in yields in the final period.

It sould be noted that these results depend in a critical way on the projections of future demand. With lower future demand, less soil conservation would be expected in the socially optimal solution.

IMPLEMENTATION

The model results shown above indicate that substantially more soil erosion control than presently occurs is justified from society's viewpoint. Because of the dual problem of conserving soil for furure generations and abating annual environmental pollution, designing a policy which would lead to the socially optimal land use is difficult indeed. If farmers had an infinite decision horizon, treated the social time preference rate as their private discount rate, and maximized the present value of profit, then a policy which only internalized the annual sediment damages would lead to the socially desired result. However, there are many indications that most farmers do not have an extremely long decision horizon, and some do not even come close to selecting production activities that maximize profit. Also, they may discount the future at a rate different from the social time preference rate. Thus, a policy that would lead to the socially optimal level of erosion controls must be directed toward the decision process of farmers as well as directed toward internalizing the pollution externalities. The model presented in this paper can be modified to reflect farmers' decision criteria and the impacts of various policies on erosion controls implemented by individual farmers. We suggest that future research be directed toward this end.

Table 10. Commodity prices, quantities produced, production costs, economic, consumers' and producers' surplus, land rent, and sediment damage for each period with a 2 percent social time preference rate.

Item		Period 1	2	3
Prices (in 1967 dollars):	Corn	2.26	3.61	5.02
	Soybeans	6.37	11.44	16.51
	Wheat	3.69	11.97	21.00
	Hay	58.28	61.21	79.73
Quantities[*]:	Corn	122,472	122,560	122,265
	Soybeans	17,288	18,313	19,336
	Wheat	1,606	1,152	655
	Hay	311	225	127
Production Costs[#]:	Corn	1.04	1.07	1.11
	Soybeans	1.86	1.88	1.89
	Wheat	1.12	1.14	1.18
	Hay	22.12	21.55	21.99
Economic Surplus[†]		1,929.181	1,915,343	1,873,922
Consumers' Surplus[†]		1,719,772	1,537,120	1,372,706
Producers' Surplus[†]		209,408	378,223	501,216
Land rent (dollars per acre)		109.14	215.19	285.17
Sediment damage[†]		2,246.77	1,876.34	1,671.73

[*] Hay is measured in tons, while all other crops are in bushels.

[#] Per bushel or ton, in 1967 dollars, indiscounted.

[†] The present value of surpluses or damages is taken from an "average" year within a 16 year perid

Table 11. Topsoil level at the beginning of each period, and annual soil loss for both groups with a 2 percent social time preference rate.

	At the beginning or during:							
	Period 1		Period 2		Period 3		Period beyond model horizon	
	Soil Group:		Soil Group:		Soil Group		Soil Group:	
Item	1	2	1	2	1	2	1	2
Topsoil level (inches)	8.532	4.713	8.081	4.227	7.622	3.771	7.134	3.312
Soil loss (inches)	0.028	0.030	0.029	0.028	0.031	0.029		
Soil loss (ton/acre)	4.21	4.89	4.27	4.58	4.55	4.62		
Soil loss relative to period 1*	1.0	1.0	1.015	0.937	1.083	0.945		
Area weighted average of values of line above	1.0		0.977		1.011			

*Calculated by using soil loss in tons.

Table 12. Acreage allocation with regard to tillage methods and conservation practices with a 2 percent social time preference rate.

Combination of tillage methods and/or conservation practices	Period:		
	1	2	3
Plow plant and straight row cultivation	111.64	70.02	50.21
Chisel plow and straight row cultivation	108.40	0.15	44.05
Plow plant and contouring	758.87	772.55	799.13
Chisel plow and contouring	0.01	0.01	0.00
Plow and terracing	735.78	914.73	806.60
Chisel plow and terracing	42.96	0.12	57.70
Total plow plant	1,606.28	1757.30	1655.93
Total chisel plow	151.37	0.27	101.75
Total straight row	220.04	70.17	94.25
Total contouring	758.88	772.56	799.13
Total terracing	778.74	914.85	864.30

Table 13. Commodity prices, quantities produced, production cost, economic, consumers' and producers' surplus, land rent, and sediment damage for each period with an 8 percent social time preference rate.

Item		Period: 1	2	3
Prices (1967 dollars):	Corn	2.26	3.64	5.05
	Soybeans	6.61	11.63	16.68
	Wheat	3.89	11.51	20.91
	Hay	58.93	67.94	79.56
Quantities*:	Corn	122,460	122,365	122,143
	Soybeans	17,236	18,271	19,301
	Wheat	1,595	1,177	660
	Hay	306	227	128
Production costs#:	Corn	1.03	1.07	1.10
	Soybeans	1.84	1.87	1.89
	Wheat	1.14	1.16	1.18
	Hay	22.56	22.20	21.91
Economic Surplus†		1,257,128	813,371	518,593
Consumers' Surplus†		1,117,979	650,592	378,919
Producers' Surplus†		139,149	162,779	139,674
Land rent (dollars per acre)		79.17	92.61	79.47
Sediment damage†		1,141.53	634.79	460.60

*Hay is measured in tons, while all other crops are in bushels.

#Per bushel or ton, in 1967 dollars, indiscounted.

†Present value over the 16 year period.

Table 14. Topsoil level at the beginning of each period, and annual soil loss for both soil groups with an 8 percent social time preference rate.

	At the beginning or during:							
	Period 1		Period 2		Period 3		Period beyond model horizon	
	Soil Group:		Soil Group:		Soil Group:		Soil Group:	
Item	1	2	1	2	1	2	1	2
Topsoil level (inches)	8.532	4.713	8.071	4.202	7.597	3.709	7.100	2.985
Soil loss (inches)	0.029	0.032	0.030	0.031	0.031	0.045		
Soil loss (ton/acre)	4.30	5.14	4.41	4.96	4.63	7.27		
Soil loss relative to period 1*	1.0	1.0	1.025	0.966	1.077	1.415		
Area weighted average of values in line above	1.0		0.996		1.253			

*Calculated by using soil loss in tons.

Table 15. Acreage allocation with regard to tillage methods and conservation practices with an 8 percent social time preference rate.

Combination of tillage methods and/or conservation practices	Period 1	2	3
Plow plant and straight row cultivation	115.96	92.40	149.80
Chisel plow and straight row cultivation	173.24	55.83	87.18
Plow plant and contouring	775.90	779.95	787.14
Chisel plow and contouring	0.00	0.00	0.00
Plow plant and terracing	673.48	816.32	640.79
Chisel plow and contouring	55.01	18.09	92.71
Total plow plant	1529.35	1683.66	1577.73
Total chisel plow	228.25	73.92	179.89
Total straight row	289.21	148.23	236.97
Total contouring	779.90	774.95	787.14
Total terracing	692.49	834.41	733.51

NOTES

1. This definition of conservation from a technical viewpoint is implied in most of the conservation literature. Although other definitions could be proposed, this one will suffice here to distinguish between conservation from a technical viewpoint and from an economic viewpoint.

2. It is interesting to note that the erosion-sedimentation controls proposed by the Environmental Protection Agency for enactment under the amended 1972 Federal Water Pollution Control Act (U.S. Congress) are based on a technical definition of the issue. (See for example, Pisano, 1976b).

3. This is a very strong assumption. Alternatively, one could assume that the rate of erosion continues at a rate which is determined by the most soil conserving technology. This, however, would complicate the model's structure further.

4. This is discussed in the section of the text dealing with the solution algorithm.

5. For a detailed discussion of the multiplier method, see Bertsekas (1976a and 1976b).

6. For most widely used rotations, it can be shown that plow-plant and chisel plow had an absolute advantage over fall plow. Therefore, fall plow was not considered as a viable alternative except for pasture. (See Frohberg).

7. See Frohberg for a discussion of these demand shifts.

REFERENCES

Davis, R.M. (1977) Soil conservation on agricultural land: the challenge ahead. Journal of Soil and Water Conservation 32:5-8.

Engelstad, O.P., W.D. Schrader, and L.C. Dumenil (1961) The effect of surface soil thickness on corn yields: I. As determined by a series of field experiments in farmer-operated fields. Soil Science of America Proceedings 25:494-497.

Engelstad, O.P., and W.D. Schrader (1961) The effect of surface thickness on corn yields: II. As determined by an experiment using normal surface soil and artificially exposed subsoil. Soil Science of America Proceedings 25:497-499.

Frohberg, K.K. (1977) Optimal soil loss over time from a societal viewpoint. Ph.D. Dissertation, Dept. of Agricultural Economics, University of Illinois.

Lee, M.T., A.S. Narayanan, K. Gunterman, and E.R. Swanson (1974) Economic analysis of erosion and sedimentation: Hambough-Martin watershed. AERR-127, Department of Agricultural Economics, Agricultural Experiment Station, University of Illinois at Urbana-Champaign.

Odell, R.T. (1950) Relationship between corn yield and depth of surface soil on TAMA silt loam in 1946 and 1947. Department of Agronomy, University of Illinois at Urbana-Champaign.

Odell, R.T., and W.R. Oschwald (1966) Productivity of Illinois soils. Circular 1016, College of Agriculture, Cooperative Extension Service, University of Illinois at Urbana-Champaign.

Pisano, M.A. (1976a) Nonpoint sources of pollution: a federal perspective. Journal of the Environmental Engineering Division 15.

Pisano, M.A. (1976b) Nonpoint pollution: an EPA view of areawide water quality management. Journal of Soil and Water Conservation 31.

Rust, R.H. (1950) Relationship between corn yield and depth of surface soil on Elliot silt loam in 1949 and 1950. Department of Agronomy, University of Illinois at Urbana-Champaign.

Seitz, W.D., M.B. Sands, and R.G.F. Spitze (1975) Evaluation of agricultural policy alternatives to control sedimentation. Research Report 99, Water Resources Center, University of Illinois at Urbana-Champaign.

Welch, L.F. (1976a) Data set about yield response for various soil types and rotations. Unpublished Working Paper, Department of Agronomy, University of Illinois at Urbana-Champaign.

U.S.E.P.A. (Oct. 1973) Methods for identifying and evaluating the nature and extent of nonpoint sources of pollution. EPA report 430/9-73-014.

Wischmeier, W.H., and D.D. Smith (1962) Predicting rainfall-erosion losses from cropland east of the Rocky Mountains. Agriculture Handbook 282, U.S.D.A.