# Entropy, Multiproportional, and Quadratic Techniques for Inferring Detailed Migration Patterns from Aggregate Data

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# ENTROPY, MULTIPROPORTIONAL, AND QUADRATIC TECHNIQUES FOR INFERRING DETAILED MIGRATION PATTERNS FROM AGGREGATE DATA. MATHEMATICAL THEORIES, ALGORITHMS, APPLICATIONS, AND COMPUTER PROGRAMS

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## FOREWORD

Interest in human settlement systems and policies has been a central part of urban-related work at IIASA since its inception. From 1975 through 1978 this interest was manifested in the work of the Migration and Settlement Task, which was formally concluded in November 1978. Since then, attention has turned to dissemination of the Task's results and to the conclusion of its comparative study which, under the leadership of Frans Willekens, is carrying out a comparative quantitative assessment of recent migration patterns and spatial population dynamics in all of IIASA's 17 NMO countries.

As part of its work on migration and settlement, IIASA has concluded research on entropy maximization and bi- and multiproportional adjustment techniques to infer detailed migration flows from aggregate data. This paper reports on some of this research and presents a generalized estimation procedure incorporating both maximum likelihood and chi-square estimates. The methods used are then tested with Austrian and Swedish data and the techniques are applied to infer age-specific migration flows for Bulgaria.

Papers summarizing previous work on migration and settlement at IIASA are listed at the back of this paper.

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# ABSTRACT

This paper presents techniques for inferring migration flows by migrant category from some available aggregate data. The data are in the form of marginal totals of migration flow matrices or prior information on certain cell values. A generalized estimation procedure is presented which incorporates both maximum likelihood and $\chi^2$ estimates. The duality results of the optimizing problems rely on the decomposition principle of Rockafellar. We prove the convergence of the general iterative procedure of which the well-known RAS and entropy methods are special cases.

The validity of the methods is tested by comparison of estimates and observations for Austria and Sweden, using $\chi^2$ and absolute percentage deviation test statistics. The techniques are then applied to infer age-specific migration flows for Bulgaria. Algorithms and FORTRAN computer programs are also given.

CONTENTS

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The lack of adequate regional statistics is frequently given as a reason for not endorsing the development of sophisticated regional or multiregional models. In particular, the data problem is severe in the study of internal migration by migrant categories.

Improved modeling of migration and of multiregional demographic phenomena in general can only be fully utilized if additional data requirements are met. When the collection of the necessary data is impossible or too expensive, estimation procedures may be used as substitutes.

This paper presents a particular class of estimation methods which have great potential in migration analysis. They may be used whenever detailed migration flows (e.g., flows by various migrant categories) are required, but only some aggregate data on migration patterns are available. For instance, how can we disaggregate a total origin-destination migration flow matrix in age-specific flows, if the only information available is a national estimate of the age composition of migrants? How would this estimate differ from a situation where we have the age structure of the arrivals and the departures by region, or where we have an initial crude estimate of the age-specific flow matrices? Does additional information increase the values of our estimates significantly? (This is of particular relevance in deciding the coverage of data collection for migration analysis). Questions like these may be answered by applying the techniques proposed in this paper. In addition, these methods are useful to update migration tables since the updating of migration tables is nothing else than estimating migration flows under new conditions (constraints) when initial estimates are available.

The applicability of the methodology is not limited to migration studies, but may be extended to input-output analysis, transportation science (trip distribution, freight flows), and regional economics (e.g., journey-to-work tables): in short, to the analysis of various kinds of interaction tables. In fact, these techniques, in their simplified (two-dimensional) forms, have received considerable attention in the above-mentioned fields.

This paper consists of five major sections. In the first section, the general problem is dealt with in mathematical terms and some special cases, which are of particular practical interest, are given. The second section presents the solutions to the basic problem and its variants. Solutions are obtained by reformulating the original problem in its dual expression. The solution algorithms, which are implemented on the computer follow in the second section. Section 3 presents some numerical illustrations, namely, the estimated values of interregional migration flows by age for Austria and Sweden. For both countries, the estimates are compared with the observed values to investigate the validity of the estimation procedure. The methods are applied to infer migration flows by age for Bulgaria in Section 4. Finally, the computer program MULTENTROPY for estimating migration flows is documented and listed.

## 1. PROBLEM FORMULATION

The estimation procedures for inferring detailed migration patterns from aggregate data, proposed in this paper, have a common feature: the aggregate data appear as marginal sums of two or n-dimensional arrays, the elements of which are unknown and must be estimated. Two main groups of problems may be distinguished: the entropy problem and the quadratic adjustment problem.

(i) The entropy problem: the problem here is to produce a "maximally unbiased" estimation of the elements of an array under the given marginal conditions. In the application of entropy models, two model types may be distinguished: entropy maximizing models and information-divergence (I-divergence) minimizing models.

(a) The entropy maximizing problem: the entropy maximizing problem is to determine the elements of an array that are "most probable" under the given marginal conditions. No initial array is known a priori and the values of the elements of the array are seen as equally likely, apart from the marginal constraints specified. The entropy maximizing method was introduced in regional science by Wilson (1967, 1970). Its two-dimensional case has received considerable attention in the literature and has been elaborated in several ways to recover interregional flow matrices (of people or commodities) from various forms of aggregate data (Chilton and Poet 1973, Nijkamp 1975, Willekens 1977).

(b) The I-divergence minimizing problem: in I-divergence minimizing problems one tries to estimate a "posterior" array which is as "close" as possible to a "prior" array, and which satisfies some given marginal constraints (row and column sums).

The distance function used here to measure the "closeness" of the arrays is the I-divergence or Kullback-Leibler information number (Kullback 1959), also called information for discrimination, information gain, or entropy of a posterior distribution relative to a prior distribution (Renyi 1970). If the known prior array is uniformly distributed, i.e., all elements of the array are equal, then the I-divergence measure is equivalent to the negentropy (entropy with negative sign).

I-divergence with a negative sign was also defined as an entropy measure for the average conditional entropy (Nijkamp and Paelinck 1974a, Theil 1967).

A procedure for minimizing I-divergence in spatial interaction models has been developed by Batty and March (1976). In the two-dimensional case*, the most extensively used method of solving this type of problem is the biproportional adjustment method, better known as the RAS method. It was developed by Stone to update input-output tables and has been studied in detail by Bacharach (1970). The RAS method is equivalent to the Furness method known in traffic models (e.g., Evans 1970, Evans and Kirby 1974). This method was presented by Furness in 1962 at a seminar on the use of computers in traffic planning in London (unpublished paper entitled *Trip Forecasting*).

(ii) The quadratic adjustment problem: the quadratic adjustment problem applies to situations where initial estimates of the entries of an array are available. The estimates, however, do not conform with predefined (measured) row and column totals. The reason may be that different sources yield the estimates and the totals. The sum-constrained adjustment problem is then to find the array which is as close as possible to the initial array but which satisfies the predefined totals.

The distance function, used here to measure the "closeness" of the arrays, is a quadratic-type function. Examples

*A two-dimensional array is a matrix.

can easily be found to show that, unlike the I-divergence minimizing problem, the positivity of the elements of the initial array does not ensure the positivity of the unique solution of the quadratic adjustment problem.

To overcome this weakness of the quadratic adjustment method, we will suggest an adjustment technique which is not quadratic but shows very close relation to the quadratic adjustment method.

## 1.1 Entropy Maximization

The techniques described in this section address problems in which no input matrix is given a priori. Wilson (1967, 1970) proposed to use an index which yields matrix entries which are most probable. The index is the entropy of the matrix.

### a. The Entropy Concept

Suppose that we are given the total number of arrivals $I_j$ and departures $O_i$ by region in a two-region system, and that the problem is to estimate the complete origin-destination migration flow matrix $\underset{\sim}{M}$ with elements $m_{ij}$. For example,

| | | |
|---|---|---|
| $m_{11}$ | $m_{21}$ | $I_1 = m_{.1} = 3$ |
| $m_{12}$ | $m_{22}$ | $I_2 = m_{.2} = 3$ |
| $O_1 = m_{1.} = 4$ | $O_2 = m_{2.} = 2$ | $m_{..} = 6$ |

In contrast to the biproportional adjustment method, no initial estimates of the matrix entries are available. Therefore, our information is limited to the row and column totals of the matrix to be estimated. For given row and column totals of a matrix, there may be a large number of arrangements of entries that satisfy the marginal conditions. For example, if for a two-by-two migration matrix the row sums are three and three, and the column sums are four and two, then there are three possible arrangements of the entries:

$$\underset{\sim}{M}_a = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} \quad \underset{\sim}{M}_b = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \quad \underset{\sim}{M}_c = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

Each arrangement of the entries of $\underset{\sim}{M}$ is called a <u>macrostate</u> of the system.

The true migration flow is represented by one of the three macrostates, $\underset{\sim}{M}_a$, $\underset{\sim}{M}_b$ or $\underset{\sim}{M}_c$. Given the limited information we have about the migration behavior, we don't know which macrostate is the true one. Therefore, we must make a guess. It is here that the entropy method comes in. It selects the macrostate which has the highest probability of occurring. A certain macrostate may be generated by various so-called <u>microstates</u>. A microstate is an assignment of <u>individual</u> migrants to the origin-destination table. In other words, a microstate is a description of the location of every individual in the system, whereas a macrostate gives the number of people in each cell of the table. Consider, for example, the matrix $\underset{\sim}{M}_a$, and denote the individual migrants by m1, m2, m3, m4, m5, and m6. According to $\underset{\sim}{M}_a$, three people migrate from region 1 to region 1, i.e., move within the region. Out of the six migrants, we can select the three in 20 different ways: m1, m2, m3, m4, etc. The possible combinations of three people out of six can easily be computed by the familiar combinatorial formula of statistics: $\frac{6!}{3!(6-3)!}$, where ! denotes the factorial operation, e.g., 6! = 1.2.3.4.5.6.

Once we have made a selection of three people to constitute $m_{11}$, we must select one person out of the remaining three to constitute $m_{12}$. There are only three possible ways of selecting one person out of three, or $\frac{3!}{1!(3-1)!}$. Finally, the two remaining individuals constitute $m_{22}$, since $m_{21} = 0$. Therefore, the total number of ways of selecting three out of six, and one out of the remaining three, and two out of the remaining two, is $\frac{6!}{3!(6-3)!}$ $\frac{3!}{1!(3-1)!}\frac{2!}{2!} = 60$. Each of the 60 ways constitutes a separate microstate, or assignment of individuals. In general, the number of ways in which we can select a particular macrostate from the total number of migrants $m_{..}$ is the combinatorial formula:

$$W = \frac{m_{..}!}{m_{11}!\ m_{12}!\ m_{21}!\ m_{22}!} \qquad (1.1)$$

$$W = \frac{m_{..}!}{\prod_{i,j} m_{ij}!}$$

Applying the above, we get W = 60 for $\underset{\sim}{M}_a$, W = 180 for $\underset{\sim}{M}_b$, and W = 60 for $\underset{\sim}{M}_c$. The value W is the number of microstates which give rise to a particular macrostate, and is called the entropy of the macrostate.

The question of which macrostate to choose as the best estimate of the true migration flow may now be answered. We choose the macrostate with the highest entropy value, i.e., $\underset{\sim}{M}_b$. The use of this selection criterion relies on two critical assumptions:

(i) The probability that a macrostate represents the true migration flow matrix is proportional to the number of microstates of the system which give rise to this macrostate (entropy) and which satisfy the marginal conditions.

(ii) Each microstate is equally probable, i.e., each person in the migrant pool $m_{..}$ has the same probability of moving from i to j.

In the next section, the formal solution to the 2-dimensional entropy maximization problem will be derived.

### b. Solving the Entropy Problem

The first assumption, read in a slightly different way, becomes: the true arrangement of a system is one which maximizes the entropy, i.e., in which the elements tend towards an arrangement which can be organized in as many ways as possible (a maximum "disorder"). This is the second law of thermodynamics. The analogy between the behavior of social and physical systems is not accidental. Several authors have attempted to describe social phenomena by laws from physics (e.g., Isard 1960). This approach is identified as <u>social physics</u>, which is well-known in the early regional science literature. It is, however, unfair to evaluate the application of entropy methods in social sciences only by the physical meaning of the entropy concept (degree of disorder).

The use of the entropy concept in social sciences may also be justified by means of information theory (Jaynes 1957), by means of Bayes's theorem for conditional probabilities (Hyman 1969), or by means of the maximum likelihood estimators (Evans 1971, Batty and MacKie 1972).* In information theory, entropy represents expected information. It indicates the degree of uncertainty about the realization of events in information systems, represented by a probability distribution. Hence, a high entropy value (low uncertainty) is associated with events having a high degree of occurrence. In the maximum-likelihood approach, entropy maximization is equivalent to maximizing the likelihood of a macrostate.

The estimation problem of finding the most probable migration flow matrix which satisfies the row and column sums may now be formulated as follows: find the macrostate with maximum entropy W, subject to the marginal conditions. The solution is given by the solution to the mathematical programming problem:

$$\max W = \frac{m_{..}!}{\prod_{i,j} m_{ij}!} \tag{1.2}$$

subject to

$$\sum_j m_{ij} = m_{i.} = O_i \quad , \qquad \text{for all } i \tag{1.3}$$

*For a comparison, see Wilson (1970:1-10), and Nijkamp (1977:18-20).

$$\sum_i m_{ij} = m_{.j} = I_j \quad , \qquad \text{for all } j \quad . \tag{1.4}$$

Since the maximum of (1.2) coincides with the maximum of any monotonic function of W, we may replace W by the Naperian logarithm of W ($\ell n$ W) in the objective function.

$$\begin{aligned} \ell n\, W &= \ell n\, m_{..}! - \ell n \prod_{i,j} m_{ij}! \\ &= \ell n\, m_{..}! - \sum_i \sum_j \ell n\, m_{ij}! \quad . \end{aligned} \tag{1.5}$$

Function (1.5) is very complex. To make differentiation of (1.5) easier, we replace $\ell n\, m_{ij}!$ by Stirling's approximation: $\ell n\, m_{ij}! = m_{ij}\ \ell n\, m_{ij} - m_{ij}$. Since $\ell n\, m_{..}!$ is a constant, we may write the objective function as

$$\max\ \ell n\, \hat{W} = -\sum_i \sum_j m_{ij}\ \ell n\, m_{ij} - m_{ij} \quad , \tag{1.6}$$

or equivalently,

$$\min\ \ell n\, \hat{W} = \sum_i \sum_j m_{ij}\ \ell n\, m_{ij} - m_{ij} \quad . \tag{1.7}$$

### 1.2 I-Divergence Minimization (Biproportional Adjustment)

The biproportional adjustment method, independently developed by Leontief (1941) and Stone (1963), uses the I-divergence measure as the distance function:

$$I(M||M^{O}) = \sum_i \sum_j m_{ij}\ \ell n \frac{m_{ij}}{m^{o}_{ij}} \quad , \tag{1.8}$$

which is defined for $m_{ij}^{o} \neq o$. The method is better known under Stone's term, RAS.

The basic features of the biproportional adjustment problem are described in this section. Frequently, our information is not limited to row and column sums of a two-dimensional migration matrix; we may also know migration patterns of specific categories of the population (e.g., sex, age). This known information should then be used in adjusting the original estimates. A generalization of the biproportional adjustment process, which enables the consideration of more a priori information, is the multiproportional adjustment process, which will be treated in Section 1.5.

Suppose we are given the total number of arrivals, $I_j$, and departures, $O_i$, by region in a two-region system and that, as before, the problem is to estimate the complete origin-destination migration flow matrix $\underset{\sim}{M}$ with elements $m_{ij}$. For example:

| | | |
|---|---|---|
| $m_{11}$ | $m_{21}$ | $I_1 = m_{.1}$ |
| $m_{12}$ | $m_{22}$ | $I_2 = m_{.2}$ |
| $O_1 = m_{1.}$ | $O_2 = m_{2.}$ | $m_{..}$ |

Suppose we are also given initial estimates of the elements $m_{ij}$, namely $m_{ij}^{o}$, contained in the matrix $\underset{\sim}{M}^{o}$. For example,

$$\underset{\sim}{M}^{o} = \begin{bmatrix} m_{11}^{o} & m_{21}^{o} \\ & \\ m_{12}^{o} & m_{22}^{o} \end{bmatrix} .$$

The initial estimates may be derived from migration tables of previous years, from experts' opinions, or from other sources.* The row sums $m^o_{i.}$ and the column sums $m^o_{.j}$ of $\underset{\sim}{M}^o$ are not equal to the predefined number of departures $O_i = m_{i.}$, and arrivals $I_j = m_{.j}$. Therefore, we have to adjust the elements of $\underset{\sim}{M}^o$ so that they add up to the required totals. Now we may formulate the sum-constrained biproportional adjustment problem in the following form.

Find the p x q matrix $\underset{\sim}{M}$ such that the I-divergence measure is minimized (in migration tables, p = q):

$$\min I(M||M^o) \tag{1.9}$$

subject to

$$m_{i.} = \sum_j m_{ij} = O_i \quad , \qquad i = 1,\ldots,p \tag{1.10}$$

and

$$m_{.j} = \sum_i m_{ij} = I_j \quad , \qquad j = 1,\ldots,q \tag{1.11}$$

Note that since $\sum_i O_i = \sum_j I_j$, the p + q marginal totals are not independent. The number of independent constraints is p + q - 1. Common to the existing methods for constrained adjustment of a two-dimensional array, i.e., of a matrix, are the constraints (1.10) and (1.11).

### 1.3 Entropy Maximization and I-Divergence Minimization: A Comparison

The purpose of this section is to compare the entropy maximizing and I-divergence minimizing technique. Both techniques have the

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*This methodology has been particularly successful for the updating of input-output tables.

same set of constraints.* Differences may therefore be explained by differences in the objective functions.

(i) Note that the entropy objective function to be maximized,

$$\hat{W} = - \sum_{ij} [m_{ij} \ln m_{ij} - m_{ij}] \quad ,$$

is equal to

$$\hat{W} = - \sum_{ij} [m_{ij} \ln m_{ij}] + m_{..} \quad .$$

Since $m_{..}$ is a constant, the maximization of $\hat{W}$ gives the same result as the maximization of the simpler function $\hat{\hat{W}} = - \sum_{ij} m_{ij} \ln m_{ij}$. The objective function $\hat{\hat{W}} = - \sum_i \sum_j m_{ij} \ln m_{ij}$ is in fact more widely used. If $m_{ij}$ is interpreted as a probability by a proper scaling making $\sum_i \sum_j m_{ij} = 1$, then the objective function defines a quantity called <u>statistical entropy</u> or the entropy of the probability distribution. This quantity (multiplied by a constant) has been defined by Shannon as a measure of the uncertainty contained in a probability distribution (Shannon and Weaver 1949).

(ii) However, the biproportional process can still be differentiated by the presence of a term $m_{ij}^o$ containing the initial "guess" of the flow values. Again, if $m_{ij}$ and $m_{ij}^o$ can be interpreted as probabilities, the quantity**

$$W = \sum_{ij} m_{ij} \ln \frac{m_{ij}}{m_{ij}^o}$$

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*This is true only if no cost constraint is considered in the entropy problem.

**When some a priori information exists and is expressed in prior probabilities ($p_i^o$, $i = 1,\ldots,n$), the expected value or information content of a message changing prior information into posterior, is measured by $- \sum_i p_i \ln p_i/p_i^o$ (Jaynes 1957, Theil 1967).

appears as the <u>information divergence</u>.

(iii) Finally, it appears that all those objective functions belong to the same family and are expressing entropy in a different format. But the basic idea is common to the biproportional "minimum deviation" concept and to statistical entropy. In information theory, the biproportional objective function is interpreted as a measure of the "surprise" coming out of the posterior values with respect to the prior ones. Therefore, minimizing the surprise or the deviation from our initial information and finding the most probable flows are the basic concepts present in entropy and biproportional approaches, which both measure such quantities with the same function,

$$\sum_{ij} m_{ij} \ln m_{ij}/m^{o}_{ij} \quad , \tag{1.12}$$

when a priori information is available, and by

$$\sum_{ij} m_{ij} \ln m_{ij} \tag{1.13}$$

when no a priori information exists. Therefore, <u>entropy maximizing problems can generally be formulated as I-divergence minimizing problems</u>:

$$\min d[\underset{\sim}{M},\underset{\sim}{M}^{o}] = \sum_{ij} m_{ij} \ln m_{ij}/m^{o}_{ij} \quad , \tag{1.14}$$

<u>where $m^{o}_{ij}$ can be uniformly set equal to 1</u>.*

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*In general, the I-divergence or multiproportional problem reduces to an entropy problem whenever the elements of the initial array are multiproportional (with the uniform distribution as a special case). Results obtained are therefore independent of the initial array selected. That the RAS solution is the same for different initial biproportional matrices was recently also shown by Friedman (1978), although he did not make the connection to the entropy problem.

## 1.4 Quadratic Adjustment

We may formulate the sum-constrained quadratic adjustment problem in the following way.

Find matrix $\underset{\sim}{M}$ such that a distance norm $d[\underset{\sim}{M},\underset{\sim}{M}^o]$, which is of a quadratic function, is minimized, subject to

$$\sum_j m_{ij} = O_i \quad , \qquad \text{for all } i \quad .$$

$$\sum_i m_{ij} = I_j \quad , \qquad \text{for all } j \quad .$$

The techniques are differentiated by the distance measures $d[M,M^o]$ used.

(i) Least square adjustment: the most obvious distance measure is the euclidean norm, defined as:

$$d[\underset{\sim}{M},\underset{\sim}{M}^o] = \frac{1}{2} \sum_i \sum_j (m_{ij} - m_{ij}^o)^2 \quad . \tag{1.15}$$

(ii) Friedlander adjustment: Friedlander (1961) used a distance norm of the chi-square type to adjust contingency tables:

$$d[\underset{\sim}{M},\underset{\sim}{M}^o] = \frac{1}{2} \sum_i \sum_j \frac{(m_{ij} - m_{ij}^o)^2}{m_{ij}^o} \quad . \tag{1.16}$$

A related method has been developed by Hortensius (1970, *Estimation of the Elements of a Table*, unpublished manuscript in Dutch. The Hague: Central Planning Agency). The distance norm is the weighted deviation:

$$d[\underset{\sim}{M},\underset{\sim}{M}^o] = \sum_i \sum_j \frac{(m_{ij} - m_{ij}^o)^2}{s_{ij}} \quad , \tag{1.17}$$

where $^1/s_{ij}$ is the weight attached to the difference $(m_{ij} - m^o_{ij})$. The weight used is a measure of the uncertainty. The underlying idea is that the more accurate $m^o_{ij}$ is, the less the adjustment is needed. Hence, for accurate estimates of $m^o_{ij}$, the weight $^1/s_{ij}$ should be small. If the elements of $m^o_{ij}$ are estimated from a sample, then not only the estimate or mean $m^o_{ij}$ is known, but also the whole frequency distribution. An appropriate measure for $s_{ij}$ is therefore the variance of the distribution (assuming normal distribution). This measure of uncertainty is being used by Hortensius.

(iii) Modified Friedlander adjustment: an important weakness of the least square and Friedlander approaches is that a strictly positive matrix $\underset{\sim}{M}^o$ does not necessarily yield a strictly positive matrix $\underset{\sim}{M}$ of estimates. Some elements of $\underset{\sim}{M}$ may be negative. To overcome this failure, we suggest the following measure, which is no longer quadratic:

$$d[\underset{\sim}{M},\underset{\sim}{M}^o] = \frac{1}{2} \sum_i \sum_j \frac{(m_{ij} - m^o_{ij})^2}{m_{ij}}$$

## 1.5 Generalization of Flow Estimation Problems to N-Dimensional Arrays

The problem formulation for matrices can easily be extended to more than two dimensions. Suppose that we are given the migration flow matrix of the total population, and that we are interested in the migration patterns of subsets of the population, e.g., sexes, age groups, nationalities, professional categories, etc. Suppose that we also know the number of arrivals and departures of each subset in each region. The migration from i to j by category k is denoted by $m_{ijk}$. The total migration from i to j is $m_{ij.} = c_{ij}$, the number of departures from i by category k is $m_{i.k} = b_{ik}$, and the number of arrivals in j by category k is $m_{.jk} = a_{jk}$. Therefore, the following constraints must be met:

$$\sum_{i=1}^{n} m_{ijk} = a_{jk} \quad , \qquad \text{for } j = 1,2...m \quad , \quad k = 1,2...l \quad .$$

$$\sum_{j=1}^{m} m_{ijk} = b_{ik} \quad , \qquad \text{for } i = 1,2...n \quad , \quad k = 1,2...l \quad .$$

$$\sum_{k=1}^{l} m_{ijk} = c_{ij} \quad , \qquad \text{for } i = 1,2...n \quad , \quad j = 1,2...m \quad .$$

The available information does not always come in this way. In an extreme case, we do not know the flow matrix of the total population but only the total number of arrivals and departures by region, and we know the composition of the migrant categories only at the national level. The constraints therefore take on a different format:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} m_{ijk} = u_k \quad , \qquad \text{for } k = 1,2...l \quad .$$

$$\sum_{i=1} \sum_{k=1} m_{ijk} = v_j \quad , \qquad \text{for } j = 1,2...m \quad .$$

$$\sum_{j=1} \sum_{k=1} m_{ijk} = w_i \quad , \qquad \text{for } i = 1,2...n \quad .$$

where $u_k = m_{..k}$ is the total number of migrants in category k, $v_j = m_{.j.}$ is the total number of arrivals in region j, and $w_1 = m_{i..}$ is the total number of departures from region i.

Various combinations of the bi- and univariate marginal sums are possible: the known total flow matrix with the composition of migrant categories at the national level, number of arrivals and departures by subset only, etc.

Before proceeding to specify various entropy and adjustment problems, we may formulate our problem in general terms. The mathematical formulation of the basic adjustment problem is as follows:

$$\min \sum_{i,j,k} d\left[m_{ijk}, m_{ijk}^{o}\right] , \qquad (1.18)$$

subject to

$$\sum_{i} m_{ijk} = a_{jk} , \quad \text{for all } j \in J, \text{ and } k \in K . \qquad (1.19)$$

$$\sum_{j} m_{ijk} = b_{ik} , \quad \text{for all } i \in I, \text{ and } k \in K . \qquad (1.20)$$

$$\sum_{k} m_{ijk} = c_{ij} , \quad \text{for all } j \in J, \text{ and } i \in K . \qquad (1.21)$$

$$\sum_{i} \sum_{j} m_{ijk} = u_{k} , \quad \text{for all } k \in K . \qquad (1.22)$$

$$\sum_{i} \sum_{k} m_{ijk} = v_{j} , \quad \text{for all } j \in J . \qquad (1.23)$$

$$\sum_{j} \sum_{k} m_{ijk} = w_{i} , \quad \text{for all } i \in I . \qquad (1.24)$$

$$\sum_{k} u_{k} = \sum_{j} v_{j} = \sum_{i} w_{i} = ST . \qquad (1.25)$$

$$m_{ijk} \geq o , \quad \text{for all } i \in I, j \in J \text{ and } k \in K . \qquad (1.26)$$

where

$I = \{1,2...n\}$ ,

$J = \{1,2...m\}$ ,

$K = \{1,2...l\}$ ,

are the index sets, $a_{jk}$, $b_{ik}$, and $c_{ij}$ are bivariate marginal sums, and $u_k$, $v_j$, and $w_i$ are univariate marginal sums.

The elements to be estimated, $m_{ijk}$, may be arranged in a three-dimensional array, $M = [m_{ijk}]$, and the initial estimates constitute the array $M^o = [m^o_{ijk}]$. Both arrays have only nonnegative elements. Some of the elements $m_{ijk}$ may be known exactly. For instance, if intraregional migration is not considered, then the diagonal elements $m_{iik} = m^o_{iik} = 0$. If other migration flows are known a priori (i.e., are fixed to the initial estimate $m^o_{ijk}$), we have to consider the following cell constraints:

$$m_{ijk} = m^o_{ijk} \quad , \qquad (i,j,k) \notin \Gamma \quad , \tag{1.27}$$

where the set $\Gamma$ is defined by setting $\Gamma = \{(i,j,k)\,|\,\text{migration from } i \text{ to } j \text{ by category } k \text{ is possible and not fixed}\}$.

The right hand sides of the constraints are matrices: $\underset{\sim}{A} = [a_{jk}]$, $\underset{\sim}{B} = [b_{ik}]$, and $\underset{\sim}{C} = [c_{ij}]$, and vectors $U = [u_k]$, $V = [v_j]$, and $W = [w_i]$. We shall call the constraints (1.19) to (1.21) <u>face-constraints</u>, because they present given values for the three faces of the "cube" or three-dimensional array M. Analogously, the constraints (1.22) to (1.24) will be labeled <u>edge-constraints</u> because they prescribe values to the edges of the "cube" or three-dimensional array M. If the face-constraints are given, the edge-constraints are redundant since the elements on the edges are the sums of the face elements. Therefore, we shall call the basic problem "three prescribed faces" problem, or shortly, "<u>three faces ( 3F )" problem</u>.

Three special cases of the basic problem are of interest:

(i) Two faces (2F) problem: minimize (1.18) subject to (1.19), (1.20), (1.26), and (1.27).

(ii) One face and one edge (1FE) problem: minimize (1.18) subject to (1.19), (1.24), (1.26), and (1.27).

(iii) Three edges (3E) problem: minimize (1.18) subject to (1.22) to (1.26), and (1.27).

To find the best estimates of the elements $m_{ijk}$, given the constraints and given an array of initial estimates $m^o_{ijk}$, we may generalize the different distance measures:

(i) Three-dimensional I-divergence measure

$$d\left[m_{ijk}, m^o_{ijk}\right] = \sum_{(i,j,k)\in\Gamma} m_{ijk} \ln \frac{m_{ijk}}{m^o_{ijk}} \quad . \qquad (1.28)$$

Note that the entropy function is equivalent when the original array $m^o_{ijk}$ over the index set $\Gamma$ consists of ones. The problem of minimizing the I-divergence measure subject to various marginal sum-constraints is the multiproportional adjustment problem.

(ii) Three-dimensional chi-square measure

$$d\left[m_{ijk}, m^o_{ijk}\right] = \frac{1}{2} \sum_{(i,j,k)\in\Gamma} \frac{(m_{ijk} - m^o_{ijk})^2}{m^o_{ijk}} \quad . \qquad (1.29)$$

The problem of minimizing the $\chi^2$ measure subject to various marginal sum constraints is the multidimensional Friedlander adjustment problem.

(iii) Modified three-dimensional chi-square measure

$$d\left[m_{ijk}, m^o_{ijk}\right] = \frac{1}{2} \sum_{(i,j,k)\in\Gamma} \frac{(m_{ijk} - m^o_{ijk})^2}{m_{ijk}} \quad . \qquad (1.30)$$

It is understood that $\frac{(0 - m^o_{ijk})^2}{0} = 0$ if $m^o_{ijk} = 0$, and $\frac{(0 - m^o_{ijk})^2}{0} = +\infty$ if $m^o_{ijk} > 0$. The problem of minimizing the modified $\chi^2$ measure subject to various marginal sum constraints is the multidimensional modified Freidlander adjustment problem.

In the next section, the basic 3F problem and its variants are transformed in their duality formulation in order to facilitate the derivation of solution algorithms. In establishing the duality results and the solution algorithms for our basic adjustment problem, we will rely on results obtained by Rockafellar (1970) in the field of "perturbation" functions and separable programming. In establishing this result we will assume that there exists a strictly positive feasible solution [$m_{ijk} > 0$ for all $(i,j,k) \in \Gamma$] to the constraints (1.19) - (1.26).

In the case when all migration flows are possible, i.e., when we also estimate the number of people remaining in the same region, or when only migration flows from one region to the same region are impossible and no migration flow is fixed, we will prove the existence of a strictly positive feasible solution (see Appendix A). In the case when no strictly positive feasible solution exists, an asymptotic duality result can be established, and the convergence of an iterative solution procedure for the I-divergence minimizing problems can be proved.

## 2. SOLUTION OF THE MULTIPROPORTIONAL AND OF THE MODIFIED FRIEDLANDER (QUADRATIC) ADJUSTMENT PROBLEMS

In order to solve the multiproportional and the modified multidimensional Friedlander adjustment problems, we first derive their dual formulations. Simple algorithms have been developed for solving these duals. For the case of multiproportional adjustment, a solution algorithm for the primal problem is also given.

### 2.1 Duality Results

Before deriving the dual programs, we formulate a theorem that assures the existence and uniqueness of the optimal solutions. Proofs are given in Appendix A.

Theorem 1

If the solution set defined by the constraints (1.19) - (1.27) contains a feasible solution $M = [M_{ijk}]$ such that

$$m_{ijk} > 0 \quad , \qquad \text{if } (i,j,k)\in\Gamma \quad ,$$

with $\Gamma$ being the index set for which $m^o_{ijk}$ is not fixed, then both the multiproportional and the modified multidimensional Friedlander adjustment programs have unique optimal solutions. Further, the optimal solutions are strictly positive for all indices $(i,j,k)\in\Gamma$.

In order to establish the duality results, we introduce some new notations.

$$I(j,k) = \left\{ i \in I \ (i,j,k)\in\Gamma \right\} \quad \forall j \in J,\ k \in K \quad ,$$

$$J(i,k) = \left\{ j \in J \ (i,j,k)\in\Gamma \right\} \quad \forall i \in I,\ k \in K \quad ,$$

$$K(i,j) = \left\{ k \in K \ (i,j,k)\in\Gamma \right\} \quad \forall i \in I,\ j \in J \quad .$$

Further,

$$\Lambda = (\lambda_{ij})_{i,j=1}^{n,m} \qquad N = (\nu_{ik})_{i,k=1}^{n,l} \qquad \text{and} \quad H = (\zeta_{jk})_{j,k=1}^{m,l}$$

are real valued matrices denoting the Lagrangian multipliers to the face constraints.

(i) <u>Unconstrained dual problem of multiproportional adjustment</u>

Minimize

$$L_1(\Lambda,N,H) = \sum_{(i,j,k)\in\Gamma} m^o_{ijk} \exp\left[-(1+\lambda_{ij}+\nu_{ik}+\zeta_{jk})\right]$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{c}_{ij}\,\lambda_{ij} + \sum_{i=1}^{n}\sum_{k=1}^{l} \bar{b}_{ik}\,\nu_{ik} + \sum_{j=1}^{m}\sum_{k=1}^{l} \bar{a}_{jk}\,\zeta_{jk} \tag{2.1}$$

subject to

$$\lambda_{11} = 1 \quad ,$$

$$\lambda_{ij}, \nu_{ik}, \zeta_{jk} \in R \quad , \qquad \forall i \in I,\ j \in J,\ k \in K \quad ,$$

where

$$\bar{c}_{ij} = c_{ij} - \sum_{k\notin K(i,j)} m^o_{ijk} \quad ,$$

$$\bar{b}_{ik} = b_{ik} - \sum_{j\notin J(i,k)} m^o_{ijk} \quad ,$$

$$\bar{a}_{jk} = a_{jk} - \sum_{i\notin I(j,k)} m^o_{ijk} \quad .$$

Theorem 2

Let the multiproportional adjustment program have an optimal solution $\hat{M} = (\hat{m}_{ijk})$ such that

$$\hat{m}_{ijk} > 0 \quad , \qquad \text{for } (i,j,k,) \in \Gamma \quad .$$

If $(\bar{\Lambda},\bar{N},\bar{H})$ is an optimal solution of the dual program (2.1), then

$$\hat{m}_{ijk} = m^{o}_{ijk} \exp[-1 - \bar{\lambda}_{ij} - \bar{\nu}_{ik} - \bar{\zeta}_{jk}] \quad .$$

(ii) Dual problem of the modified multidimensional Friedlander adjustment program

Minimize

$$L_2(\Lambda,N,H) = -2 \sum_{(i,j,k)\in \Gamma} m^{o}_{ijk} \sqrt{1 + \lambda_{ij} + \nu_{ik} + \zeta_{jk}}$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{c}_{ij} \lambda_{ij} + \sum_{i=1}^{n} \sum_{k=1}^{l} \bar{b}_{ik} \nu_{ik}$$

$$+ \sum_{j=1}^{m} \sum_{k=1}^{l} \bar{a}_{ik} \zeta_{jk} + 2 \, |\Gamma| \qquad (2.2)$$

subject to

$$\left.\begin{aligned} &\lambda_{11} = 1 \quad , \\ &\lambda_{ij}, \nu_{ik}, \zeta_{jk} \in R \quad , \\ &\text{and} \\ &\lambda_{ij} + \nu_{ik} + \zeta_{jk} > -1 \quad , \end{aligned}\right\} \text{for } i \in I, j \in J, k \in K \quad .$$

Theorem 3

Let the modified multidimensional Friedlander adjustment program have an optimal solution $\hat{M} = (\hat{m}_{ijk})$ such that

$$\hat{m}_{ijk} > 0 \quad , \qquad \text{for } (i,j,k) \in \Gamma \quad .$$

If $(\bar{\Lambda},\bar{N},\bar{H})$ is an optimal solution of the dual program (2.2) then

$$\hat{m}_{ijk} = \frac{m^o_{ijk}}{\sqrt{1 + \bar{\lambda}_{ij} + \bar{\nu}_{ik} + \bar{\zeta}_{jk}}} \quad .$$

## 2.2 Solution Algorithms

The algorithms presented here solve the multiproportional and the modified multidimensional Friedlander adjustment programs for the basic three-faces (3F) case. The solution of the special cases (3E), (1FE), and (2F) may be obtained in a similar way.*

### a. Multiproportional Adjustment Problem

(i) Algorithm for solving the unconstrained dual problem (2.1) for the multiproportional adjustment

Step 0

Set

$$S(\text{step}) = 0 \quad ,$$

*If the elements $m^o_{ijk}$ are uniformly distributed, then the special cases of the multiproportional adjustment problem are an entropy problem and this has an analytical solution (Appendix C).

$$\lambda_{ij}^{(0)} = 1 ,$$

$$\nu_{ik}^{(0)} = 1 , \qquad \forall\, i \in I,\ j \in J,\ k \in K .$$

$$\zeta_{jk}^{(0)} = 1 ,$$

Step 1

Set

$$\lambda_{11}^{(S+1)} = \lambda_{11}^{(S)} = 1 ,$$

$$\lambda_{ij}^{(S+1)} = \ln \sum_{k \in K(i,j)} m_{ijk}^{o} \exp - \left(1 + \nu_{ik}^{(S)} + \zeta_{jk}^{(S)}\right) - \ln c_{ij} ,$$

for all $i \in I, j \in J$, except for $i = j = 1$.

Step 2

Set

$$\nu_{ik}^{(S+1)} = \ln \sum_{j \in J(i,k)} m_{ijk}^{o} \exp - \left(1 + \lambda_{ij}^{(S+1)} + \zeta_{jk}^{(S)}\right) - \ln b_{ik}$$

Step 3

Set

$$\zeta_{jk}^{(S+1)} = \ln \sum_{i \in I(j,k)} m_{ijk}^{o} \exp - \left(1 + \lambda_{ij}^{(S+1)} + \nu_{ij}^{(S+1)}\right) - \ln a_{jk}$$

If

$$| \lambda_{ij}^{(S+1)} - \lambda_{ij}^{(S)} | \leq \varepsilon \quad \text{and} \quad | \nu_{ik}^{(S+1)} - \nu_{ik}^{(S)} | \leq \varepsilon \quad \text{and}$$

$$| \zeta_{jk}^{(S+1)} - \zeta_{jk}^{(S)} | \leq \varepsilon \quad \text{for every } i \in I,\ j \in J \text{ and } k \in K \quad ,$$

then STOP else $S \longleftarrow S + 1$, and go to Step 1.

Theorem 4

Let $M = (m_{ijk})$ be a feasible solution of the basic problem such that

$$m_{ijk} > 0 \quad , \qquad \text{for all } (i,j,k) \in \Gamma \quad .$$

Then the above algorithm converges to an optimal solution of the unconstrained dual problem (2.1).

Using the result of Theorem 2, we can easily derive the following direct primal algorithm:

(ii) Algorithm for solving the primal multiproportional adjustment problem

Step 0

Set

$$m_{ijk}^{(o)} = m_{ijk}^{o} \quad , \qquad \forall\, i \in I,\ j \in J,\ k \in K \quad .$$
$$S = 0 \quad ,$$

In the entropy problem, the prior distribution $m_{ijk}^{o}$ is not known and may therefore be set uniformly equal to unity, i.e., $m_{ijk}^{o} = 1$, $\forall$ i,j,k. An improved initial distribution, which gives the same results however, is presented in Appendix C.

Step 1

Set

$$m_{ijk}^{(3S+1)} = m_{ijk}^{(3S)} \frac{c_{ij}}{\sum_{k=1}^{l} m_{ijk}^{(3S)}} ,$$

for all $i \in I, j \in J$, except $i = j = 1$.

Step 2

Set

$$m_{ijk}^{(3S+2)} = m_{ijk}^{(3S+1)} \frac{a_{jk}}{\sum_{i=1}^{m} m_{ijk}^{(3S+1)}} .$$

Step 3

Set

$$m_{ijk}^{(3S+3)} = m_{ijk}^{(3S+2)} \frac{b_{ik}}{\sum_{j=1}^{n} m_{ijk}^{(3S+2)}} .$$

If

$$\left| \frac{m_{ijk}^{(3S+3)}}{m_{ijk}^{(3S+2)}} - 1 \right| \leq \varepsilon , \quad \text{for every } (i,j,k) \in \Gamma ,$$

then STOP else $S \leftarrow S + 1$, and go to Step 1.

b. Modified Multidimensional Friedlander Adjustment Problem

(i) Algorithm for solving the dual problem (2.2)

Step 0 (S = 0)

Set

$$\lambda_{ij}^{(0)} = 0 \quad ,$$

$$\nu_{ik}^{(0)} = 0 \quad , \qquad \forall\ i \in I, j \in J, k \in K \quad .$$

$$\zeta_{jk}^{(0)} = 0 \quad ,$$

Step 1

Let $\lambda_{ij}^{(S+1)}$ be the solution of the equation:

$$\sum_{k \in K(i,j)} \frac{m_{ijk}^{o}}{\sqrt{1 + \lambda_{ij}^{(S+1)} + \nu_{ik}^{(S)} + \zeta_{jk}^{(S)}}} = c_{ij} \quad ,$$

$\forall\ i \in I, j \in J$, except $i = j = 1$ .

(The left-hand side of this equation is a strictly monotone decreasing function of the variable $\lambda_{ij}^{(S+1)}$ in the range $(0, +\infty)$. Therefore, there exists a unique solution $\lambda_{ij}^{(S+1)}$ to it. A possible solution technique is the Newton method).

Step 2

Let $\nu_{ik}^{(S+1)}$ be the solution of the equation, where $\nu_{ik}^{(S)}, \zeta_{jk}^{(S)}$ are considered to be given:

$$\sum_{j \in J(i,k)} \frac{m_{ijk}^{o}}{\sqrt{1 + \lambda_{ij}^{(S+1)} + \nu_{ik}^{(S+1)} + \zeta_{jk}^{(S)}}} = b_{ik} \quad ,$$

$\forall\ i \in I, k \in K$ .

Step 3

Let $\zeta_{jk}^{(S+1)}$ be the solution of the equation:

$$\sum_{i\in I(j,k)} \frac{m^{o}_{ijk}}{\sqrt{1 + \lambda^{(S+1)}_{ij} + \nu^{(S+1)}_{ik} + \zeta^{(S+1)}_{jk}}} = a_{jk} \quad .$$

If

$$\left|\lambda^{(S+1)}_{ij} - \lambda^{(S)}_{ij}\right| \leq \varepsilon \ , \quad \text{and} \ \left|\nu^{(S+1)}_{ik} - \nu^{(S)}_{ik}\right| \leq \varepsilon,$$

and $\left|\zeta^{(S+1)}_{jk} - \zeta^{(S)}_{jk}\right| \leq \varepsilon$ for every $i \in I, j \in J$ ,

and $k \in K$ ,

then STOP else $S \longleftarrow S + 1$, and go to Step 1.

<u>Theorem 5</u>

Let $M = (m_{ijk})$ be a feasible solution of the basic problem (3F) such that

$$m_{ijk} > 0 \ , \quad \text{for all } (i,j,k)\in \Gamma \ ,$$

then the above algorithm for solving the dual program of the modified Friedlander adjustment problem converges to an optimal solution of the dual problem (2.2).

Because of the nature of the modified Friedlander adjustment problem, we could not find a direct primal algorithm. However, using the results of Theorem 3 we can compute the primal solution $\hat{m}_{ijk}$ for $(i,j,k)\in \Gamma$ from the dual solution by the following formula:

$$\hat{m}_{ijk} = \frac{m^{o}_{ijk}}{\sqrt{1 + \bar{\lambda}_{ij} + \bar{\nu}_{ik} + \bar{\zeta}_{jk}}} \quad .$$

## 3. VALIDITY ANALYSIS AND NUMERICAL ILLUSTRATIONS (AUSTRIA, SWEDEN)

The techniques developed in the previous sections may be applied to infer detailed migration patterns from aggregate data. But how accurate are the estimates, and which method gives the best results under given conditions? Although it is difficult at the current state of research to give definite answers, this section attempts to answer these questions numerically (see also Willekens 1977). To test the validity of the estimation procedures, this section applies them to a multi-regional system for which the complete data set exists. Using only the marginal conditions, the detailed migration flows are estimated and the estimates are then compared with the observed flow data. Among the few countries that make detailed migration data available are Austria and Sweden. These data may be aggregated in various ways to yield different sets of marginal totals, which may in turn be used to simulate different levels of data availability. Estimates of the detailed flows are inferred by the entropy maximizing method and the quadratic adjustment methods.

The validity test of the multiproportional and modified Friedlander adjustment methods is basically a comparison between the estimated and observed migration flows and a judgment on the "closeness" of both sets of values. The quality of an estimation procedure is determined by the accuracy with which it can replicate observed data. A key problem in validity analysis is the definition of a composite index that measures the "closeness" of two arrays. In this paper, two such indices are used: the chi-square ($\chi^2$), and the absolute percentage error.

The chi-square statistic measures the relative squared deviation between the estimates and the observed values:

$$\chi^2 = \sum_{i,j,k} \left[m_{ijk} - m^{o}_{ijk}\right]^2 \Big/ m_{ijk} \quad , \qquad \text{for } m_{ijk} \neq 0 \quad . \tag{3.1}$$

The absolute percentage error measures the deviation in absolute terms:

$$APE = \sum_{i,j,k} d_{ijk} \quad ,$$

where

$$d_{ijk} = \frac{|m_{ijk} - m^{o}_{ijk}|}{m^{o}_{ijk}} \quad , \qquad \text{for } m^{o}_{ijk} \neq 0 \quad . \qquad (3.2)$$

The average absolute percentage error, also known as the relative mean deviation or the mean prediction error, is

$$\overline{APE} = \frac{\sum_{i,j,k} |m_{ijk} - m^{o}_{ijk}|}{\sum_{i,j,k} m^{o}_{ijk}} \quad , \qquad \text{for } m^{o}_{ijk} \neq 0 \quad .$$

Both measures will show the same direction, but the chi-square statistic attaches a greater penalty to large deviations.

An important observation of the validity study was that the error is not uniformly distributed among the elements of the array. The relative error, expressed by both the chi-square and the APE, is greatest among the small elements (i.e., minor flows). The reason is the small denominator in (3.1) and (3.2). This observation is consistent with results obtained in input-output analysis (e.g., Hinojosa 1978). In addition, we found that relatively few elements of the array are responsible for most of the error: most elements are in low-error categories.

Because of these observations, and to gain a better idea about the composition of the overall squared or absolute percentage deviation, the error analysis is carried out for subgroups of migration flows. The subgroups are formed on the basis of age (three broad age categories are considered: 0-14, 15-64, 65+) and volume of migration flow (ten size classes are distinguished: 0-199, 200-399, 400-599, ..., 1800-1999, 2000+). Note that the size classes are in terms of observed flows and not in terms of the estimates. The error analysis is also carried out for 12 error categories ( $<2$ percent, 2-4 percent, 4-6 percent,..., 100+ percent).

### 3.1 Entropy Maximization

The Austrian migration data are from the 1971 census and represent the place of residence in 1966, as compared to the census date of 1971. The data were kindly provided by Dr. M. Sauberer, of the Austrian Institute for Regional Planning. Various aggregations of the migration data were made, and techniques presented in the previous sections were applied to test the validity of these methods.

The multidimensional entropy maximization method is a special variant of the multiproportional adjustment method: the initial guesses of the elements of the array to be estimated are set equal to the same scalar value, unity, say: $m^{o}_{ijk} = 1$, for all i,j,k. In other words, it is assumed that no initial guesses of the elements are available.

a. The Three-Faces (3F) Problem

Suppose the problem is to infer origin-destination migration flows by age for Austria (four regions) from the available information on the flow matrix of the total population and on the age composition of the arrivals and departures of each region. The data are in Table 1 and present the three faces of a box (array), the content (elements) of which must be estimated.

The results of the (3F) estimation procedure are shown in Table 2, together with the observed migration flow data. The algorithm took only five iterations to converge (tolerance level $10^{-4}$). The results for an eight-region Swedish system are given in Appendix D. In this case, nine iterations were required to reach the same tolerance level.

The estimates are generally very close to the observed values. The average absolute percentage error is 4.27 percent, a very low figure compared with the one obtained by existing biproportional or two-dimensional entropy methods (Nijkamp and Paelinck

Table 1. Internal migration in Austria, 1966-1971.

a. Migration flow matrix of total population

| to \ from | east | south | north | west | total |
|---|---|---|---|---|---|
| east | 0. | 12564. | 10587. | 3091. | 26242. |
| south | 7460. | 0. | 4532. | 3543. | 15535. |
| north | 11471. | 7715. | 0. | 3629. | 22815. |
| west | 3272. | 7494. | 4158. | 0. | 14924. |
| total | 22203. | 27773. | 19277. | 10263. | 79516. |

East: Wien, Niederösterreich, Burgenland
South: Steiermark, Kärnten
North: Salzburg, Oberösterreich
West: Tirol, Vorarlberg

b. Departures and arrivals by region and age

| | east | | south | | north | | west | | total | |
|---|---|---|---|---|---|---|---|---|---|---|
| age | arrivals | depart. | arrivals | depart. | arrivals | depart. | arrivals | depart. | arrivals | depart. |
| 0 | 1896. | 1783. | 1428. | 1909. | 1733. | 1445. | 985. | 905. | 6042. | 6042. |
| 5 | 1086. | 930. | 734. | 1115. | 981. | 917. | 577. | 416. | 3378. | 3378. |
| 10 | 2264. | 1597. | 1052. | 3662. | 2118. | 1568. | 1961. | 568. | 7395. | 7395. |
| 15 | 7424. | 4172. | 3212. | 8323. | 4801. | 5297. | 4595. | 2240. | 20032. | 20032. |
| 20 | 4582. | 4227. | 2806. | 4625. | 4391. | 3250. | 2558. | 2235. | 14337. | 14337. |
| 25 | 2612. | 2807. | 1883. | 2625. | 2757. | 1885. | 1438. | 1373. | 8690. | 8690. |
| 30 | 1122. | 1123. | 835. | 1062. | 1155. | 924. | 603. | 606. | 3715. | 3715. |
| 35 | 906. | 915. | 622. | 903. | 929. | 750. | 519. | 408. | 2976. | 2976. |
| 40 | 950. | 871. | 540. | 807. | 824. | 688. | 413. | 361. | 2727. | 2727. |
| 45 | 625. | 579. | 374. | 517. | 515. | 447. | 237. | 208. | 1751. | 1751. |
| 50 | 601. | 618. | 441. | 514. | 541. | 478. | 268. | 241. | 1851. | 1851. |
| 55 | 675. | 689. | 458. | 521. | 552. | 489. | 251. | 237. | 1936. | 1936. |
| 60 | 576. | 700. | 434. | 455. | 568. | 439. | 201. | 185. | 1779. | 1779. |
| 65 | 431. | 543. | 331. | 340. | 433. | 328. | 147. | 131. | 1342. | 1342. |
| 70 | 274. | 353. | 213. | 217. | 280. | 209. | 94. | 82. | 861. | 861. |
| 75 | 145. | 194. | 114. | 118. | 156. | 109. | 52. | 46. | 467. | 467. |
| 80 | 49. | 68. | 37. | 39. | 53. | 35. | 17. | 14. | 156. | 156. |
| 85 | 24. | 34. | 21. | 21. | 28. | 19. | 8. | 7. | 81. | 81. |
| total | 26242. | 22203. | 15535. | 27773. | 22815. | 19277. | 14924. | 10263. | 79516. | 79516. |

Table 2. Observed and (3F) estimated migration flows by age, Austria, four regions, 1966-1971.

| | total | migration from east to east | south | north | west |
|---|---|---|---|---|---|
| 0 | 1783. | 0. | 674. | 874. | 234. |
| | | 0- | -670- | -877- | -236- |
| 5 | 930. | 0. | 328. | 482. | 120. |
| | | 0- | -328- | -468- | -134- |
| 10 | 1597. | 0. | 483. | 821. | 293. |
| | | 0- | -537- | -828- | -232- |
| 15 | 4172. | 0. | 1351. | 2029. | 793. |
| | | 0- | -1280- | -2192- | -700- |
| 20 | 4227. | 0. | 1336. | 2246. | 645. |
| | | 0- | -1289- | -2231- | -707- |
| 25 | 2807. | 0. | 939. | 1477. | 392. |
| | | 0- | -910- | -1464- | -433- |
| 30 | 1123. | 0. | 378. | 596. | 149. |
| | | 0- | -368- | -588- | -167- |
| 35 | 915. | 0. | 295. | 493. | 127. |
| | | 0- | -293- | -480- | -142- |
| 40 | 871. | 0. | 280. | 474. | 117. |
| | | 0- | -312- | -438- | -121- |
| 45 | 579. | 0. | 204. | 306. | 69. |
| | | 0- | -222- | -289- | -68- |
| 50 | 618. | 0. | 229. | 314. | 75. |
| | | 0- | -241- | -312- | -65- |
| 55 | 689. | 0. | 261. | 346. | 81. |
| | | 0- | -280- | -331- | -78- |
| 60 | 700. | 0. | 259. | 373. | 67. |
| | | 0- | -269- | -357- | -74- |
| 65 | 543. | 0. | 203. | 290. | 51. |
| | | 0- | -210- | -280- | -53- |
| 70 | 353. | 0. | 131. | 189. | 33. |
| | | 0- | -138- | -182- | -33- |
| 75 | 194. | 0. | 71. | 105. | 18. |
| | | 0- | -74- | -101- | -19- |
| 80 | 68. | 0. | 24. | 37. | 6. |
| | | 0- | -26- | -35- | -7- |
| 85 | 34. | 0. | 13. | 18. | 3. |
| | | 0- | -13- | -18- | -3- |
| total | 22203. | 0. | 7460. | 11471. | 3272. |

| | total | migration from south to east | south | north | west |
|---|---|---|---|---|---|
| 0 | 1909. | 882. | 0. | 556. | 471. |
| | | -853- | 0- | -575- | -481- |
| 5 | 1115. | 493. | 0. | 348. | 274. |
| | | -530- | 0- | -329- | -256- |
| 10 | 3662. | 1371. | 0. | 1075. | 1216. |
| | | -1342- | 0- | -1044- | -1276- |
| 15 | 8323. | 3800. | 0. | 2022. | 2501. |
| | | -3760- | 0- | -1950- | -2613- |
| 20 | 4625. | 2097. | 0. | 1324. | 1203. |
| | | -2081- | 0- | -1381- | -1163- |
| 25 | 2625. | 1187. | 0. | 781. | 656. |
| | | -1225- | 0- | -800- | -600- |
| 30 | 1062. | 465. | 0. | 333. | 263. |
| | | -464- | 0- | -346- | -252- |
| 35 | 903. | 392. | 0. | 281. | 230. |
| | | -408- | 0- | -276- | -219- |
| 40 | 807. | 409. | 0. | 223. | 174. |
| | | -411- | 0- | -240- | -156- |
| 45 | 517. | 277. | 0. | 140. | 100. |
| | | -269- | 0- | -149- | -99- |
| 50 | 514. | 256. | 0. | 147. | 111. |
| | | -263- | 0- | -134- | -117- |
| 55 | 521. | 289. | 0. | 133. | 99. |
| | | -296- | 0- | -132- | -93- |
| 60 | 455. | 246. | 0. | 133. | 76. |
| | | -253- | 0- | -137- | -65- |
| 65 | 340. | 185. | 0. | 100. | 55. |
| | | -190- | 0- | -102- | -48- |
| 70 | 217. | 118. | 0. | 64. | 35. |
| | | -121- | 0- | -65- | -31- |
| 75 | 118. | 63. | 0. | 35. | 19. |
| | | -65- | 0- | -36- | -17- |
| 80 | 39. | 22. | 0. | 11. | 6. |
| | | -22- | 0- | -12- | -5- |
| 85 | 21. | 11. | 0. | 7. | 3. |
| | | -11- | 0- | -7- | -3- |
| total | 27773. | 12564. | 0. | 7715. | 7494. |

- . - : observed flow

Table 2 continued.

| | total | migration from east | south | north to north | west |
|---|---|---|---|---|---|
| 0 | 1445. | 764. | 402. | 0. | 280. |
| | | -814- | -363- | 0- | -268- |
| 5 | 917. | 482. | 251. | 0. | 184. |
| | | -448- | -282- | 0- | -187- |
| 10 | 1568. | 745. | 371. | 0. | 452. |
| | | -771- | -344- | 0- | -453- |
| 15 | 5297. | 2888. | 1107. | 0. | 1302. |
| | | -2892- | -1123- | 0- | -1282- |
| 20 | 3250. | 1806. | 734. | 0. | 710. |
| | | -1861- | -701- | 0- | -688- |
| 25 | 1885. | 1029. | 466. | 0. | 390. |
| | | -998- | -482- | 0- | -405- |
| 30 | 924. | 492. | 241. | 0. | 191. |
| | | -485- | -255- | 0- | -184- |
| 35 | 750. | 402. | 187. | 0. | 162. |
| | | -379- | -213- | 0- | -158- |
| 40 | 688. | 419. | 146. | 0. | 122. |
| | | -411- | -141- | 0- | -136- |
| 45 | 447. | 277. | 101. | 0. | 68. |
| | | -275- | -102- | 0- | -70- |
| 50 | 478. | 273. | 124. | 0. | 81. |
| | | -273- | -119- | 0- | -86- |
| 55 | 489. | 304. | 114. | 0. | 71. |
| | | -295- | -114- | 0- | -80- |
| 60 | 439. | 271. | 110. | 0. | 58. |
| | | -263- | -114- | 0- | -62- |
| 65 | 328. | 204. | 83. | 0. | 41. |
| | | -198- | -84- | 0- | -46- |
| 70 | 209. | 130. | 53. | 0. | 26. |
| | | -125- | -54- | 0- | -30- |
| 75 | 109. | 67. | 27. | 0. | 14. |
| | | -66- | -27- | 0- | -16- |
| 80 | 35. | 22. | 8. | 0. | 4. |
| | | -22- | -8- | 0- | -5- |
| 85 | 19. | 11. | 6. | 0. | 2. |
| | | -11- | -6- | 0- | -2- |
| total | 19277. | 10587. | 4532. | 0. | 4158. |

| | total | migration from east | south | west to north | west |
|---|---|---|---|---|---|
| 0 | 905. | 250. | 352. | 303. | 0. |
| | | -229- | -395- | -281- | 0- |
| 5 | 416. | 111. | 155. | 150. | 0. |
| | | -108- | -124- | -184- | 0- |
| 10 | 568. | 148. | 197. | 222. | 0. |
| | | -151- | -171- | -246- | 0- |
| 15 | 2240. | 736. | 754. | 750. | 0. |
| | | -772- | -809- | -659- | 0- |
| 20 | 2235. | 678. | 736. | 821. | 0. |
| | | -640- | -816- | -779- | 0- |
| 25 | 1373. | 395. | 479. | 499. | 0. |
| | | -389- | -491- | -493- | 0- |
| 30 | 606. | 165. | 216. | 226. | 0. |
| | | -173- | -212- | -221- | 0- |
| 35 | 408. | 113. | 140. | 155. | 0. |
| | | -119- | -116- | -173- | 0- |
| 40 | 361. | 121. | 113. | 127. | 0. |
| | | -128- | -87- | -146- | 0- |
| 45 | 208. | 71. | 69. | 69. | 0. |
| | | -81- | -50- | -77- | 0- |
| 50 | 241. | 73. | 88. | 80. | 0. |
| | | -65- | -81- | -95- | 0- |
| 55 | 237. | 82. | 82. | 72. | 0. |
| | | -84- | -64- | -89- | 0- |
| 60 | 185. | 59. | 64. | 61. | 0. |
| | | -60- | -51- | -74- | 0- |
| 65 | 131. | 42. | 46. | 43. | 0. |
| | | -43- | -37- | -51- | 0- |
| 70 | 82. | 26. | 29. | 27. | 0. |
| | | -28- | -21- | -33- | 0- |
| 75 | 46. | 15. | 16. | 16. | 0. |
| | | -14- | -13- | -19- | 0- |
| 80 | 14. | 5. | 5. | 5. | 0. |
| | | -5- | -3- | -6- | 0- |
| 85 | 7. | 2. | 3. | 2. | 0. |
| | | -2- | -2- | -3- | 0- |
| total | 10263. | 3091. | 3543. | 3629. | 0. |

1974b, Hinojosa 1978). About half of the migration flows have an estimation error of less than four percent, and almost two-thirds of the migration volume is estimated with less than four percent error (Table 3b). About 69 percent of the total absolute percentage error is due to minor migration flows (less than 200 migrants) representing only 11 percent of the flow volume (Table 3a). A similar pattern is obtained if the chi-square statistic is used. The error distribution is, however, more explicit. The minor flows account for 34 percent of the total chi-square value.

The contribution of minor flows to the overall error is further illustrated by the cross-classification of error categories and flow size classes (Table 3c). Minor flows are concentrated in the larger error categories.

The importance of small flow values for the overall error measure raises an additional problem, namely, rounding. Since the most probable estimates are rounded to the nearest integer to represent the number of migrants, error due to rounding may be substantial in the case of minor flows. The error measures in this study do not take into account the effects of rounding. The value of $m_{ijk}$ used in the calculation of the error statistics is the original estimate before rounding.

The effect of age on the error distribution was also investigated. The results are not shown, since no significant difference between the contribution of each age category to the overall error can be observed. This may be the consequence of the uniform age pattern of migrants.

b. The Three-Edges (3E) Problem

In the (3E) problem, it is assumed that the only known information consists of the edges of the box, i.e., the total number of arrivals and departures by region, and the migrant age structure at the national level. The data are shown in the row and column sums of Table 1a and in the last column of Table 1b.

The entropy or most probable estimates of the migration flows by age are calculated directly by equation (C.1). The results are shown in Table 4. The first observation is the nonzero

Table 3. Error analysis of (3F) migration estimates.

a. Analysis by size class (flow volume) and migrant category

| size class | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | cum.abs.% error value | cum.abs.% error -%- | chi-square value | chi-square -%- |
|---|---|---|---|---|---|---|---|---|
| 0- 200 | 112. | 51.85 | 8452. | 10.63 | 1043. | 68.56 | 0.9121e 02 | 33.70 |
| 200- 400 | 45. | 20.83 | 12742. | 16.02 | 241. | 15.81 | 0.5711e 02 | 21.10 |
| 400- 600 | 20. | 9.26 | 9481. | 11.92 | 74. | 4.87 | 0.2255e 02 | 8.33 |
| 600- 800 | 11. | 5.09 | 7687. | 9.67 | 73. | 4.81 | 0.4191e 02 | 15.49 |
| 800-1000 | 9. | 4.17 | 7705. | 9.69 | 36. | 2.36 | 0.1924e 02 | 7.11 |
| 1000-1200 | 3. | 1.39 | 3330. | 4.19 | 8. | 0.52 | 0.2466e 01 | 0.91 |
| 1200-1400 | 7. | 3.24 | 9075. | 11.41 | 25. | 1.63 | 0.1295e 02 | 4.78 |
| 1400-1600 | 1. | 0.46 | 1464. | 1.84 | 1. | 0.06 | 0.1074e 00 | 0.04 |
| 1600-1800 | 0. | 0.00 | 0. | 0.00 | 0. | 0.00 | 0.0000e 00 | 0.00 |
| 1800-2000 | 2. | 0.93 | 3811. | 4.79 | 7. | 0.43 | 0.4201e 01 | 1.55 |
| 2000+ | 6. | 2.78 | 15769. | 19.83 | 14. | 0.95 | 0.1887e 02 | 6.97 |
| total | 216. | 100.00 | 79516. | 100.00 | 1522. | 100.00 | 0.2706e 03 | 100.00 |

b. Analysis by error category

| error category | percentage error | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | average flow |
|---|---|---|---|---|---|---|
| 1 | 0 - 2 | 46. | 21.30 | 24037. | 30.23 | 522.543 |
| 2 | 2 - 4 | 57. | 26.39 | 24756. | 31.13 | 434.316 |
| 3 | 4 - 6 | 31. | 14.35 | 13604. | 17.11 | 438.839 |
| 4 | 6 - 8 | 18. | 8.33 | 6463. | 8.13 | 359.056 |
| 5 | 8 - 10 | 12. | 5.56 | 4026. | 5.06 | 335.500 |
| 6 | 10 - 15 | 28. | 12.96 | 5021. | 6.31 | 179.321 |
| 7 | 15 - 20 | 10. | 4.63 | 798. | 1.00 | 79.800 |
| 8 | 20 - 30 | 10. | 4.63 | 650. | 0.82 | 65.000 |
| 9 | 30 - 40 | 3. | 1.39 | 158. | 0.20 | 52.667 |
| 10 | 40 - 60 | 1. | 0.46 | 3. | 0.00 | 3.000 |
| 11 | 60 - 100 | 0. | 0.00 | 0. | 0.00 | 0.000 |
| 12 | 100 + | 0. | 0.00 | 0. | 0.00 | 0.000 |
| | total | 216. | 100.00 | 79516. | 100.00 | 368.130 |

average absolute percentage error = 4.27
( relative mean deviation )

c. Analysis by size class and error category

| size class | error category 0- 2 | 2- 4 | 4- 6 | 6- 8 | 8-10 | 10-15 | 15-20 | 20-30 | 30-40 | 40-60 | 60-100 | 100+ | total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0- 200 | 21 | 23 | 13 | 8 | 4 | 20 | 10 | 9 | 3 | 1 | 0 | 0 | 112 |
| 200- 400 | 9 | 12 | 10 | 5 | 3 | 5 | 0 | 1 | 0 | 0 | 0 | 0 | 45 |
| 400- 600 | 6 | 9 | 0 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 20 |
| 600- 800 | 1 | 2 | 4 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 11 |
| 800-1000 | 2 | 4 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 |
| 1000-1200 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| 1200-1400 | 1 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 |
| 1400-1600 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1600-1800 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1800-2000 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 2000- | 4 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 |
| total | 46 | 57 | 31 | 18 | 12 | 28 | 10 | 10 | 3 | 1 | 0 | 0 | 216 |

Table 4. Observed and (3E) estimated migration flows by age, Austria, four regions, 1966-1971.

| | total | migration from east | south | east to north | west |
|---|---|---|---|---|---|
| 0 | 1687. | 557. | 330. | 484. | 317. |
| | | 0- | -670- | -877- | -236- |
| 5 | 943. | 311. | 184. | 271. | 177. |
| | | 0- | -328- | -468- | -134- |
| 10 | 2065. | 681. | 403. | 592. | 388. |
| | | 0- | -537- | -828- | -232- |
| 15 | 5593. | 1846. | 1093. | 1605. | 1050. |
| | | 0- | -1280- | -2192- | -700- |
| 20 | 4003. | 1321. | 782. | 1149. | 751. |
| | | 0- | -1289- | -2231- | -707- |
| 25 | 2426. | 801. | 474. | 696. | 455. |
| | | 0- | -910- | -1464- | -433- |
| 30 | 1037. | 342. | 203. | 298. | 195. |
| | | 0- | -368- | -588- | -167- |
| 35 | 831. | 274. | 162. | 238. | 156. |
| | | 0- | -293- | -480- | -142- |
| 40 | 761. | 251. | 149. | 218. | 143. |
| | | 0- | -312- | -438- | -121- |
| 45 | 489. | 161. | 96. | 140. | 92. |
| | | 0- | -222- | -289- | -68- |
| 50 | 517. | 171. | 101. | 148. | 97. |
| | | 0- | -241- | -312- | -65- |
| 55 | 541. | 178. | 106. | 155. | 101. |
| | | 0- | -230- | -331- | -78- |
| 60 | 497. | 164. | 97. | 143. | 93. |
| | | 0- | -269- | -357- | -74- |
| 65 | 375. | 124. | 73. | 108. | 70. |
| | | 0- | -210- | -280- | -53- |
| 70 | 240. | 79. | 47. | 69. | 45. |
| | | 0- | -138- | -182- | -33- |
| 75 | 130. | 43. | 25. | 37. | 24. |
| | | 0- | -74- | -101- | -19- |
| 80 | 44. | 14. | 9. | 12. | 8. |
| | | 0- | -26- | -35- | -7- |
| 85 | 23. | 7. | 4. | 6. | 4. |
| | | 0- | -13- | -18- | -3- |
| total | 22203. | 7327. | 4338. | 6371. | 4167. |

| | total | migration from east | south | south to north | west |
|---|---|---|---|---|---|
| 0 | 2110. | 696. | 412. | 606. | 396. |
| | | -853- | 0- | -575- | -481- |
| 5 | 1180. | 389. | 231. | 339. | 221. |
| | | -530- | 0- | -329- | -256- |
| 10 | 2583. | 852. | 505. | 741. | 485. |
| | | -1342- | 0- | -1044- | -1276- |
| 15 | 6997. | 2309. | 1367. | 2008. | 1313. |
| | | -3760- | 0- | -1950- | -2613- |
| 20 | 5008. | 1653. | 978. | 1437. | 940. |
| | | -2081- | 0- | -1391- | -1163- |
| 25 | 3035. | 1002. | 593. | 871. | 570. |
| | | -1225- | 0- | -800- | -600- |
| 30 | 1298. | 428. | 254. | 372. | 244. |
| | | -464- | 0- | -346- | -252- |
| 35 | 1039. | 343. | 203. | 298. | 195. |
| | | -408- | 0- | -276- | -219- |
| 40 | 952. | 314. | 186. | 273. | 179. |
| | | -411- | 0- | -240- | -156- |
| 45 | 612. | 202. | 119. | 175. | 115. |
| | | -269- | 0- | -149- | -99- |
| 50 | 647. | 213. | 126. | 185. | 121. |
| | | -263- | 0- | -134- | -117- |
| 55 | 676. | 223. | 132. | 194. | 127. |
| | | -296- | 0- | -132- | -93- |
| 60 | 621. | 205. | 121. | 178. | 117. |
| | | -253- | 0- | -137- | -65- |
| 65 | 469. | 155. | 92. | 134. | 88. |
| | | -190- | 0- | -102- | -48- |
| 70 | 301. | 99. | 59. | 86. | 56. |
| | | -121- | 0- | -65- | -31- |
| 75 | 163. | 54. | 32. | 47. | 31. |
| | | -65- | 0- | -36- | -17- |
| 80 | 54. | 18. | 11. | 16. | 10. |
| | | -22- | 0- | -12- | -5- |
| 85 | 28. | 9. | 6. | 8. | 5. |
| | | -11- | 0- | -7- | -3- |
| total | 27773. | 9166. | 5426. | 7969. | 5213. |

| | total | migration from east | south | north to north | west |
|---|---|---|---|---|---|
| 0 | 1465. | 483. | 286. | 420. | 275. |
| | | -814- | -363- | 0- | -268- |
| 5 | 819. | 270. | 160. | 235. | 154. |
| | | -448- | -282- | 0- | -187- |
| 10 | 1793. | 592. | 350. | 514. | 336. |
| | | -771- | -344- | 0- | -453- |
| 15 | 4856. | 1603. | 949. | 1393. | 911. |
| | | -2892- | -1123- | 0- | -1282- |
| 20 | 3476. | 1147. | 679. | 997. | 652. |
| | | -1861- | -701- | 0- | -688- |
| 25 | 2107. | 695. | 412. | 604. | 395. |
| | | -998- | -482- | 0- | -405- |
| 30 | 901. | 297. | 176. | 258. | 169. |
| | | -485- | -255- | 0- | -184- |
| 35 | 721. | 238. | 141. | 207. | 135. |
| | | -379- | -213- | 0- | -158- |
| 40 | 661. | 218. | 129. | 190. | 124. |
| | | -411- | -141- | 0- | -136- |
| 45 | 424. | 140. | 83. | 122. | 80. |
| | | -275- | -102- | 0- | -70- |
| 50 | 449. | 148. | 88. | 129. | 84. |
| | | -273- | -119- | 0- | -86- |
| 55 | 469. | 155. | 92. | 135. | 88. |
| | | -295- | -114- | 0- | -80- |
| 60 | 431. | 142. | 84. | 124. | 81. |
| | | -263- | -114- | 0- | -62- |
| 65 | 325. | 107. | 64. | 93. | 61. |
| | | -198- | -84- | 0- | -46- |
| 70 | 209. | 69. | 41. | 60. | 39. |
| | | -125- | -54- | 0- | -30- |
| 75 | 113. | 37. | 22. | 32. | 21. |
| | | -66- | -27- | 0- | -16- |
| 80 | 38. | 12. | 7. | 11. | 7. |
| | | -22- | -8- | 0- | -5- |
| 85 | 20. | 6. | 4. | 6. | 4. |
| | | -11- | -6- | 0- | -2- |
| total | 19277. | 6362. | 3766. | 5531. | 3618. |

| | total | migration from east | south | west to north | west |
|---|---|---|---|---|---|
| 0 | 780. | 257. | 152. | 224. | 146. |
| | | -229- | -395- | -281- | 0- |
| 5 | 436. | 144. | 85. | 125. | 82. |
| | | -108- | -124- | -184- | 0- |
| 10 | 954. | 315. | 186. | 274. | 179. |
| | | -151- | -171- | -246- | 0- |
| 15 | 2585. | 853. | 505. | 742. | 485. |
| | | -772- | -809- | -659- | 0- |
| 20 | 1850. | 611. | 362. | 531. | 347. |
| | | -640- | -816- | -779- | 0- |
| 25 | 1122. | 370. | 219. | 322. | 211. |
| | | -389- | -491- | -493- | 0- |
| 30 | 479. | 158. | 94. | 138. | 90. |
| | | -173- | -212- | -221- | 0- |
| 35 | 384. | 127. | 75. | 110. | 72. |
| | | -119- | -116- | -173- | 0- |
| 40 | 352. | 116. | 69. | 101. | 66. |
| | | -128- | -87- | -146- | 0- |
| 45 | 226. | 75. | 44. | 65. | 42. |
| | | -81- | -50- | -77- | 0- |
| 50 | 239. | 79. | 47. | 69. | 45. |
| | | -65- | -81- | -95- | 0- |
| 55 | 250. | 82. | 49. | 72. | 47. |
| | | -84- | -64- | -89- | 0- |
| 60 | 230. | 76. | 45. | 66. | 43. |
| | | -60- | -51- | -74- | 0- |
| 65 | 173. | 57. | 34. | 50. | 33. |
| | | -43- | -37- | -51- | 0- |
| 70 | 111. | 37. | 22. | 32. | 21. |
| | | -28- | -21- | -33- | 0- |
| 75 | 60. | 20. | 12. | 17. | 11. |
| | | -14- | -13- | -19- | 0- |
| 80 | 20. | 7. | 4. | 6. | 4. |
| | | -5- | -3- | -6- | 0- |
| 85 | 10. | 3. | 2. | 3. | 2. |
| | | -2- | -2- | -3- | 0- |
| total | 10263. | 3387. | 2005. | 2945. | 1926. |

- . - : observed flow

values for intraregional migration, due to lack of information on the total flow matrix. The (3E) entropy method yields estimates with an average absolute percentage error of 31 percent. The error is not concentrated in the minor flows but is evenly distributed among the flow size classes (Table 5).

c. The One Face-One Edge (1FE) Problem

The (1FE) problem considered here consists of estimating the values of $m_{ijk}$ if the total flow matrix $c_{ij}$ and the age structure of the migrants at the national level $u_k$ are given. The estimates are obtained by (C.2) and are shown in Table 6. By introducing information on the total flow matrix, the average absolute percentage error drops by half, from 31 percent to 16 percent. The contribution of minor flows to the overall error gains in importance, in particular if the squared deviation is used as an error measure (Table 7).

d. The Two-Faces (2F) Problem

Assume that the following two faces are known: the total flow matrix $c_{ij}$ and the age structure of the arrivals by region $a_{jk}$. Applying (C.3) yields the results shown in Table 8. The additional information on the age composition of arrivals reduces the average absolute percentage deviation only slightly, namely, from 16 percent to 12 percent. However, the error distribution changes. Minor flows get a greater share of the total absolute percentage error. Interestingly, however, the chi-square distribution does not follow this pattern of change (Table 9).

Similar results as the ones reported for Austria were obtained for Sweden, where the number of regions was twice as large. The average absolute percentage errors in the (3F), (3E), (1FE), and (2F) problems are respectively 6.32 percent, 34.58 percent, 15.26 percent, and 11.90 percent. The share of the minor flows in the total error was always much higher. Where in the Austrian case the minor flows accounted for 51 to 72 percent of the total percentage absolute deviation, in the Swedish case they amounted to 90 to 94 percent. This may in part be explained by the share of the minor flows in the total number and volume of flows (see Tables 3 and D3).

## Table 5. Error analysis of (3E) migration estimates.

### a. Analysis by size class (flow volume) and migrant category

| size class | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | cum.abs.% error value | cum.abs.% error -%- | chi-square value | chi-square -%- |
|---|---|---|---|---|---|---|---|---|
| 0- 200 | 112. | 51.85 | 8452. | 10.63 | 3454. | 51.40 | 0.1408e 04 | 7.57 |
| 200- 400 | 45. | 20.83 | 12742. | 16.02 | 1583. | 23.55 | 0.4389e 04 | 23.61 |
| 400- 600 | 20. | 9.26 | 9481. | 11.92 | 577. | 8.58 | 0.1940e 04 | 10.44 |
| 600- 800 | 11. | 5.09 | 7687. | 9.67 | 203. | 3.02 | 0.6637e 03 | 3.57 |
| 800-1000 | 9. | 4.17 | 7705. | 9.69 | 313. | 4.65 | 0.1966e 04 | 10.58 |
| 1000-1200 | 3. | 1.39 | 3330. | 4.19 | 64. | 0.95 | 0.2088e 03 | 1.12 |
| 1200-1400 | 7. | 3.24 | 9075. | 11.41 | 204. | 3.03 | 0.2136e 04 | 11.49 |
| 1400-1600 | 1. | 0.46 | 1464. | 1.84 | 52. | 0.78 | 0.8467e 03 | 4.56 |
| 1600-1800 | 0. | 0.00 | 0. | 0.00 | 0. | 0.00 | 0.0000e 00 | 0.00 |
| 1800-2000 | 2. | 0.93 | 3811. | 4.79 | 41. | 0.61 | 0.4460e 03 | 2.40 |
| 2000+ | 6. | 2.78 | 15769. | 19.83 | 229. | 3.41 | 0.4581e 04 | 24.65 |
| total | 216. | 100.00 | 79516. | 100.00 | 6719. | 100.00 | 0.1859e 05 | 100.00 |

### b. Analysis by error category

| error category | percentage error | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | average flow |
|---|---|---|---|---|---|---|
| 1 | 0 - 2 | 3. | 1.39 | 431. | 0.54 | 143.667 |
| 2 | 2 - 4 | 13. | 6.02 | 4221. | 5.31 | 324.692 |
| 3 | 4 - 6 | 7. | 3.24 | 4706. | 5.92 | 672.286 |
| 4 | 6 - 8 | 6. | 2.78 | 1725. | 2.17 | 287.500 |
| 5 | 8 - 10 | 12. | 5.56 | 2220. | 2.79 | 185.000 |
| 6 | 10 - 15 | 16. | 7.41 | 5022. | 6.32 | 313.875 |
| 7 | 15 - 20 | 25. | 11.57 | 7324. | 9.21 | 292.960 |
| 8 | 20 - 30 | 25. | 11.57 | 12173. | 15.31 | 486.920 |
| 9 | 30 - 40 | 42. | 19.44 | 15603. | 19.62 | 371.500 |
| 10 | 40 - 60 | 42. | 19.44 | 21881. | 27.52 | 520.976 |
| 11 | 60 - 100 | 23. | 10.65 | 4054. | 5.10 | 176.261 |
| 12 | 100 + | 2. | 0.93 | 156. | 0.20 | 78.000 |
| | total | 216. | 100.00 | 79516. | 100.00 | 368.130 |

average absolute percentage error = 31.09
( relative mean deviation )

### c. Analysis by size class and error category

| size class | error category 0- 2 | 2- 4 | 4- 6 | 6- 8 | 8-10 | 10-15 | 15-20 | 20-30 | 30-40 | 40-60 | 60-100 | 100+ | total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0- 200 | 2 | 7 | 0 | 3 | 10 | 7 | 17 | 11 | 27 | 11 | 15 | 2 | 112 |
| 200- 400 | 1 | 3 | 1 | 1 | 1 | 5 | 2 | 4 | 5 | 15 | 7 | 0 | 45 |
| 400- 600 | 0 | 1 | 2 | 1 | 0 | 1 | 2 | 4 | 3 | 6 | 0 | 0 | 20 |
| 600- 800 | 0 | 1 | 3 | 1 | 0 | 2 | 0 | 1 | 1 | 2 | 0 | 0 | 11 |
| 800-1000 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 2 | 4 | 0 | 0 | 9 |
| 1000-1200 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 3 |
| 1200-1400 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 2 | 0 | 1 | 0 | 7 |
| 1400-1600 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1600-1800 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1800-2000 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 2 |
| 2000- | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 0 | 0 | 6 |
| total | 3 | 13 | 7 | 6 | 12 | 16 | 25 | 25 | 42 | 42 | 23 | 2 | 216 |

Table 6. Observed and estimated (1FE) migration flows by age, Austria, four regions, 1966-1971.

| | total | migration from east to east | south | north | west |
|---|---|---|---|---|---|
| 0 | 1687. | 0. | 567. | 872. | 249. |
| | | 0- | -670- | -877- | -236- |
| 5 | 943. | 0. | 317. | 487. | 139. |
| | | 0- | -328- | -468- | -134- |
| 10 | 2065. | 0. | 694. | 1067. | 304. |
| | | 0- | -537- | -828- | -232- |
| 15 | 5593. | 0. | 1879. | 2890. | 824. |
| | | 0- | -1280- | -2192- | -700- |
| 20 | 4003. | 0. | 1345. | 2068. | 590. |
| | | 0- | -1289- | -2231- | -707- |
| 25 | 2426. | 0. | 815. | 1254. | 358. |
| | | 0- | -910- | -1464- | -433- |
| 30 | 1037. | 0. | 349. | 536. | 153. |
| | | 0- | -368- | -588- | -167- |
| 35 | 831. | 0. | 279. | 429. | 122. |
| | | 0- | -293- | -480- | -142- |
| 40 | 761. | 0. | 256. | 393. | 112. |
| | | 0- | -312- | -438- | -121- |
| 45 | 489. | 0. | 164. | 253. | 72. |
| | | 0- | -222- | -289- | -68- |
| 50 | 517. | 0. | 174. | 267. | 76. |
| | | 0- | -241- | -312- | -65- |
| 55 | 541. | 0. | 182. | 279. | 80. |
| | | 0- | -280- | -331- | -78- |
| 60 | 497. | 0. | 167. | 257. | 73. |
| | | 0- | -269- | -357- | -74- |
| 65 | 375. | 0. | 126. | 194. | 55. |
| | | 0- | -210- | -280- | -53- |
| 70 | 240. | 0. | 81. | 124. | 35. |
| | | 0- | -138- | -182- | -33- |
| 75 | 130. | 0. | 44. | 67. | 19. |
| | | 0- | -74- | -101- | -19- |
| 80 | 44. | 0. | 15. | 23. | 6. |
| | | 0- | -26- | -35- | -7- |
| 85 | 23. | 0. | 8. | 12. | 3. |
| | | 0- | -13- | -18- | -3- |
| total | 22203. | 0. | 7460. | 11471. | 3272. |

| | total | migration from south to east | south | north | west |
|---|---|---|---|---|---|
| 0 | 2110. | 955. | 0. | 586. | 569. |
| | | -853- | 0- | -575- | -481- |
| 5 | 1180. | 534. | 0. | 328. | 318. |
| | | -530- | 0- | -329- | -256- |
| 10 | 2583. | 1168. | 0. | 717. | 697. |
| | | -1342- | 0- | -1044- | -1276- |
| 15 | 6997. | 3165. | 0. | 1944. | 1888. |
| | | -3760- | 0- | -1950- | -2013- |
| 20 | 5008. | 2265. | 0. | 1391. | 1351. |
| | | -2081- | 0- | -1381- | -1163- |
| 25 | 3035. | 1373. | 0. | 843. | 819. |
| | | -1225- | 0- | -800- | -600- |
| 30 | 1298. | 587. | 0. | 360. | 350. |
| | | -464- | 0- | -346- | -252- |
| 35 | 1039. | 470. | 0. | 289. | 280. |
| | | -408- | 0- | -276- | -219- |
| 40 | 952. | 431. | 0. | 265. | 257. |
| | | -411- | 0- | -240- | -156- |
| 45 | 612. | 277. | 0. | 170. | 165. |
| | | -269- | 0- | -149- | -99- |
| 50 | 647. | 292. | 0. | 180. | 174. |
| | | -263- | 0- | -134- | -117- |
| 55 | 676. | 306. | 0. | 188. | 182. |
| | | -296- | 0- | -132- | -93- |
| 60 | 621. | 281. | 0. | 173. | 168. |
| | | -253- | 0- | -137- | -65- |
| 65 | 469. | 212. | 0. | 130. | 126. |
| | | -190- | 0- | -102- | -48- |
| 70 | 301. | 136. | 0. | 84. | 81. |
| | | -121- | 0- | -65- | -31- |
| 75 | 163. | 74. | 0. | 45. | 44. |
| | | -65- | 0- | -36- | -17- |
| 80 | 54. | 25. | 0. | 15. | 15. |
| | | -22- | 0- | -12- | -5- |
| 85 | 28. | 13. | 0. | 8. | 8. |
| | | -11- | 0- | -7- | -3- |
| total | 27773. | 12564. | 0. | 7715. | 7494. |

| | total | migration from north to east | south | north | west |
|---|---|---|---|---|---|
| 0 | 1465. | 804. | 344. | 0. | 316. |
| | | -814- | -363- | 0- | -268- |
| 5 | 819. | 450. | 193. | 0. | 177. |
| | | -448- | -282- | 0- | -187- |
| 10 | 1793. | 985. | 421. | 0. | 387. |
| | | -771- | -344- | 0- | -453- |
| 15 | 4856. | 2667. | 1142. | 0. | 1048. |
| | | -2892- | -1123- | 0- | -1282- |
| 20 | 3476. | 1909. | 817. | 0. | 750. |
| | | -1861- | -701- | 0- | -688- |
| 25 | 2107. | 1157. | 495. | 0. | 454. |
| | | -998- | -482- | 0- | -405- |
| 30 | 901. | 495. | 212. | 0. | 194. |
| | | -485- | -255- | 0- | -184- |
| 35 | 721. | 396. | 170. | 0. | 156. |
| | | -379- | -213- | 0- | -158- |
| 40 | 661. | 363. | 155. | 0. | 143. |
| | | -411- | -141- | 0- | -136- |
| 45 | 424. | 233. | 100. | 0. | 92. |
| | | -275- | -102- | 0- | -70- |
| 50 | 449. | 246. | 105. | 0. | 97. |
| | | -273- | -119- | 0- | -86- |
| 55 | 469. | 258. | 110. | 0. | 101. |
| | | -295- | -114- | 0- | -80- |
| 60 | 431. | 237. | 101. | 0. | 93. |
| | | -263- | -114- | 0- | -62- |
| 65 | 325. | 179. | 76. | 0. | 70. |
| | | -198- | -84- | 0- | -46- |
| 70 | 209. | 115. | 49. | 0. | 45. |
| | | -125- | -54- | 0- | -30- |
| 75 | 113. | 62. | 27. | 0. | 24. |
| | | -65- | -27- | 0- | -16- |
| 80 | 38. | 21. | 9. | 0. | 8. |
| | | -22- | -8- | 0- | -5- |
| 85 | 20. | 11. | 5. | 0. | 4. |
| | | -11- | -6- | 0- | -2- |
| total | 19277. | 10587. | 4532. | 0. | 4158. |

| | total | migration from west to east | south | north | west |
|---|---|---|---|---|---|
| 0 | 780. | 235. | 269. | 276. | 0. |
| | | -229- | -395- | -281- | 0- |
| 5 | 436. | 131. | 151. | 154. | 0. |
| | | -108- | -124- | -184- | 0- |
| 10 | 954. | 287. | 329. | 337. | 0. |
| | | -151- | -171- | -246- | 0- |
| 15 | 2585. | 779. | 893. | 914. | 0. |
| | | -772- | -809- | -659- | 0- |
| 20 | 1850. | 557. | 639. | 654. | 0. |
| | | -640- | -816- | -779- | 0- |
| 25 | 1122. | 338. | 387. | 397. | 0. |
| | | -389- | -491- | -493- | 0- |
| 30 | 479. | 144. | 166. | 170. | 0. |
| | | -173- | -212- | -221- | 0- |
| 35 | 384. | 116. | 133. | 136. | 0. |
| | | -119- | -116- | -173- | 0- |
| 40 | 352. | 106. | 122. | 124. | 0. |
| | | -128- | -87- | -146- | 0- |
| 45 | 226. | 68. | 78. | 80. | 0. |
| | | -81- | -50- | -77- | 0- |
| 50 | 239. | 72. | 82. | 84. | 0. |
| | | -65- | -81- | -95- | 0- |
| 55 | 250. | 75. | 86. | 88. | 0. |
| | | -84- | -64- | -89- | 0- |
| 60 | 230. | 69. | 79. | 81. | 0. |
| | | -60- | -51- | -74- | 0- |
| 65 | 173. | 52. | 60. | 61. | 0. |
| | | -43- | -37- | -51- | 0- |
| 70 | 111. | 33. | 38. | 39. | 0. |
| | | -28- | -21- | -33- | 0- |
| 75 | 60. | 18. | 21. | 21. | 0. |
| | | -14- | -13- | -19- | 0- |
| 80 | 20. | 6. | 7. | 7. | 0. |
| | | -5- | -3- | -6- | 0- |
| 85 | 10. | 3. | 4. | 4. | 0. |
| | | -2- | -2- | -3- | 0- |
| total | 10263. | 3091. | 3543. | 3629. | 0. |

- . - : observed flow

Table 7. Error analysis of (1FE) migration estimates.

a. Analysis by size class (flow volume) and migrant category

| size class | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | cum.abs.% error value | cum.abs.% error -%- | chi-square value | chi-square -%- |
|---|---|---|---|---|---|---|---|---|
| 0- 200 | 112. | 51.85 | 8452. | 10.63 | 3809. | 70.07 | 0.7952e 03 | 21.72 |
| 200- 400 | 45. | 20.83 | 12742. | 16.02 | 774. | 14.24 | 0.6374e 03 | 17.41 |
| 400- 600 | 20. | 9.26 | 9481. | 11.92 | 232. | 4.27 | 0.1920e 03 | 5.24 |
| 600- 800 | 11. | 5.09 | 7687. | 9.67 | 208. | 3.83 | 0.2945e 03 | 8.04 |
| 800-1000 | 9. | 4.17 | 7705. | 9.69 | 106. | 1.96 | 0.1565e 03 | 4.27 |
| 1000-1200 | 3. | 1.39 | 3330. | 4.19 | 49. | 0.90 | 0.1751e 03 | 4.78 |
| 1200-1400 | 7. | 3.24 | 9075. | 11.41 | 141. | 2.59 | 0.7689e 03 | 21.00 |
| 1400-1600 | 1. | 0.46 | 1464. | 1.84 | 14. | 0.26 | 0.3530e 02 | 0.96 |
| 1600-1800 | 0. | 0.00 | 0. | 0.00 | 0. | 0.00 | 0.0000e 00 | 0.00 |
| 1800-2000 | 2. | 0.93 | 3811. | 4.79 | 3. | 0.05 | 0.1222e 01 | 0.03 |
| 2000+ | 6. | 2.78 | 15769. | 19.83 | 99. | 1.83 | 0.6055e 03 | 16.54 |
| total | 216. | 100.00 | 79516. | 100.00 | 5437. | 100.00 | 0.3662e 04 | 100.00 |

b. Analysis by error category

| error category | percentage error | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | average flow |
|---|---|---|---|---|---|---|
| 1 | 0 - 2 | 19. | 8.80 | 10024. | 12.61 | 527.579 |
| 2 | 2 - 4 | 12. | 5.56 | 4089. | 5.14 | 340.750 |
| 3 | 4 - 6 | 18. | 8.33 | 5945. | 7.48 | 330.278 |
| 4 | 6 - 8 | 4. | 1.85 | 5277. | 6.64 | 1319.250 |
| 5 | 8 - 10 | 12. | 5.56 | 4602. | 5.79 | 383.500 |
| 6 | 10 - 15 | 38. | 17.59 | 13163. | 16.55 | 346.395 |
| 7 | 15 - 20 | 28. | 12.96 | 14785. | 18.59 | 528.036 |
| 8 | 20 - 30 | 31. | 14.35 | 9764. | 12.28 | 314.968 |
| 9 | 30 - 40 | 20. | 9.26 | 7422. | 9.33 | 371.100 |
| 10 | 40 - 60 | 16. | 7.41 | 3523. | 4.43 | 220.188 |
| 11 | 60 - 100 | 10. | 4.63 | 748. | 0.94 | 74.800 |
| 12 | 100 + | 8. | 3.70 | 174. | 0.22 | 21.750 |
| | total | 216. | 100.00 | 79516. | 100.00 | 368.130 |

average absolute percentage error = 16.24
( relative mean deviation )

c. Analysis by size class and error category

| size class | error category 0- 2 | 2- 4 | 4- 6 | 6- 8 | 8-10 | 10-15 | 15-20 | 20-30 | 30-40 | 40-60 | 60-100 | 100+ | total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0- 200 | 7 | 6 | 7 | 2 | 7 | 19 | 10 | 15 | 8 | 13 | 10 | 8 | 112 |
| 200- 400 | 2 | 4 | 7 | 0 | 2 | 7 | 5 | 9 | 8 | 1 | 0 | 0 | 45 |
| 400- 600 | 4 | 1 | 2 | 0 | 1 | 5 | 4 | 3 | 0 | 0 | 0 | 0 | 20 |
| 600- 800 | 1 | 0 | 0 | 0 | 1 | 1 | 5 | 1 | 2 | 0 | 0 | 0 | 11 |
| 800-1000 | 2 | 0 | 1 | 0 | 0 | 3 | 1 | 2 | 0 | 0 | 0 | 0 | 9 |
| 1000-1200 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 3 |
| 1200-1400 | 1 | 0 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 2 | 0 | 0 | 7 |
| 1400-1600 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1600-1800 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1800-2000 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 2000- | 0 | 0 | 0 | 2 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 6 |
| total | 19 | 12 | 18 | 4 | 12 | 38 | 28 | 31 | 20 | 16 | 10 | 8 | 216 |

Table 8. Observed and estimated (2F) migration flows by age, Austria, four regions, 1966-1971.

| | total | migration from east | south | east to north | west |
|---|---|---|---|---|---|
| 0 | 1773. | 0. | 686. | 871. | 216. |
| | | 0- | -670- | -877- | -236- |
| 5 | 972. | 0. | 352. | 493. | 127. |
| | | 0- | -328- | -468- | -134- |
| 10 | 2000. | 0. | 505. | 1065. | 430. |
| | | 0- | -537- | -828- | -232- |
| 15 | 4964. | 0. | 1542. | 2414. | 1007. |
| | | 0- | -1280- | -2192- | -700- |
| 20 | 4116. | 0. | 1347. | 2208. | 561. |
| | | 0- | -1289- | -2231- | -707- |
| 25 | 2606. | 0. | 904. | 1386. | 315. |
| | | 0- | -910- | -1464- | -433- |
| 30 | 1114. | 0. | 401. | 581. | 132. |
| | | 0- | -368- | -588- | -167- |
| 35 | 880. | 0. | 299. | 467. | 114. |
| | | 0- | -293- | -480- | -142- |
| 40 | 764. | 0. | 259. | 414. | 91. |
| | | 0- | -312- | -438- | -121- |
| 45 | 490. | 0. | 180. | 259. | 52. |
| | | 0- | -222- | -289- | -68- |
| 50 | 543. | 0. | 212. | 272. | 59. |
| | | 0- | -241- | -312- | -65- |
| 55 | 553. | 0. | 220. | 278. | 55. |
| | | 0- | -280- | -331- | -78- |
| 60 | 538. | 0. | 208. | 286. | 44. |
| | | 0- | -269- | -357- | -74- |
| 65 | 409. | 0. | 159. | 218. | 32. |
| | | 0- | -210- | -280- | -53- |
| 70 | 264. | 0. | 102. | 141. | 21. |
| | | 0- | -138- | -182- | -33- |
| 75 | 145. | 0. | 55. | 78. | 11. |
| | | 0- | -74- | -101- | -19- |
| 80 | 48. | 0. | 18. | 27. | 4. |
| | | 0- | -26- | -35- | -7- |
| 85 | 26. | 0. | 10. | 14. | 2. |
| | | 0- | -13- | -18- | -3- |
| total | 22203. | 0. | 7460. | 11471. | 3272. |

| | total | migration from east | south | south to north | west |
|---|---|---|---|---|---|
| 0 | 1988. | 908. | 0. | 586. | 495. |
| | | -853- | 0- | -575- | -481- |
| 5 | 1141. | 520. | 0. | 332. | 290. |
| | | -530- | 0- | -329- | -256- |
| 10 | 2785. | 1084. | 0. | 716. | 985. |
| | | -1342- | 0- | -1044- | -1276- |
| 15 | 7485. | 3554. | 0. | 1623. | 2307. |
| | | -3760- | 0- | -1950- | -2613- |
| 20 | 4963. | 2194. | 0. | 1485. | 1284. |
| | | -2081- | 0- | -1381- | -1163- |
| 25 | 2905. | 1251. | 0. | 932. | 722. |
| | | -1225- | 0- | -800- | -600- |
| 30 | 1231. | 537. | 0. | 391. | 303. |
| | | -464- | 0- | -346- | -252- |
| 35 | 1009. | 434. | 0. | 314. | 261. |
| | | -408- | 0- | -276- | -219- |
| 40 | 941. | 455. | 0. | 279. | 207. |
| | | -411- | 0- | -240- | -156- |
| 45 | 592. | 299. | 0. | 174. | 119. |
| | | -269- | 0- | -149- | -99- |
| 50 | 605. | 288. | 0. | 183. | 135. |
| | | -263- | 0- | -134- | -117- |
| 55 | 636. | 323. | 0. | 187. | 126. |
| | | -296- | 0- | -132- | -93- |
| 60 | 569. | 276. | 0. | 192. | 101. |
| | | -253- | 0- | -137- | -65- |
| 65 | 427. | 206. | 0. | 146. | 74. |
| | | -190- | 0- | -102- | -48- |
| 70 | 273. | 131. | 0. | 95. | 47. |
| | | -121- | 0- | -65- | -31- |
| 75 | 148. | 69. | 0. | 53. | 26. |
| | | -65- | 0- | -36- | -17- |
| 80 | 50. | 23. | 0. | 18. | 9. |
| | | -22- | 0- | -12- | -5- |
| 85 | 25. | 11. | 0. | 9. | 4. |
| | | -11- | 0- | -7- | -3- |
| total | 27773. | 12564. | 0. | 7715. | 7494. |

| | total | migration from east | south | north to north | west |
|---|---|---|---|---|---|
| 0 | 1456. | 765. | 417. | 0. | 274. |
| | | -814- | -363- | 0- | -268- |
| 5 | 813. | 438. | 214. | 0. | 161. |
| | | -448- | -282- | 0- | -187- |
| 10 | 1767. | 913. | 307. | 0. | 546. |
| | | -771- | -344- | 0- | -453- |
| 15 | 5212. | 2995. | 937. | 0. | 1280. |
| | | -2892- | -1123- | 0- | -1282- |
| 20 | 3380. | 1849. | 819. | 0. | 713. |
| | | -1861- | -701- | 0- | -688- |
| 25 | 2004. | 1054. | 549. | 0. | 401. |
| | | -998- | -482- | 0- | -405- |
| 30 | 864. | 453. | 244. | 0. | 168. |
| | | -485- | -255- | 0- | -184- |
| 35 | 692. | 366. | 181. | 0. | 145. |
| | | -379- | -213- | 0- | -158- |
| 40 | 656. | 383. | 158. | 0. | 115. |
| | | -411- | -141- | 0- | -136- |
| 45 | 427. | 252. | 109. | 0. | 66. |
| | | -275- | -102- | 0- | -70- |
| 50 | 446. | 242. | 129. | 0. | 75. |
| | | -273- | -119- | 0- | -86- |
| 55 | 476. | 272. | 134. | 0. | 70. |
| | | -295- | -114- | 0- | -80- |
| 60 | 415. | 232. | 127. | 0. | 56. |
| | | -263- | -114- | 0- | -62- |
| 65 | 311. | 174. | 97. | 0. | 41. |
| | | -198- | -84- | 0- | -46- |
| 70 | 199. | 111. | 62. | 0. | 26. |
| | | -125- | -54- | 0- | -30- |
| 75 | 106. | 58. | 33. | 0. | 14. |
| | | -66- | -27- | 0- | -16- |
| 80 | 35. | 20. | 11. | 0. | 5. |
| | | -22- | -8- | 0- | -5- |
| 85 | 18. | 10. | 6. | 0. | 2. |
| | | -11- | -6- | 0- | -2- |
| total | 19277. | 10587. | 4532. | 0. | 4158. |

| | total | migration from east | south | west to north | west |
|---|---|---|---|---|---|
| 0 | 825. | 223. | 326. | 276. | 0. |
| | | -229- | -335- | -281- | 0- |
| 5 | 451. | 128. | 167. | 156. | 0. |
| | | -108- | -124- | -184- | 0- |
| 10 | 843. | 267. | 240. | 337. | 0. |
| | | -151- | -171- | -246- | 0- |
| 15 | 2371. | 874. | 733. | 764. | 0. |
| | | -772- | -809- | -659- | 0- |
| 20 | 1878. | 540. | 640. | 698. | 0. |
| | | -640- | -816- | -779- | 0- |
| 25 | 1176. | 308. | 429. | 439. | 0. |
| | | -389- | -491- | -493- | 0- |
| 30 | 506. | 132. | 190. | 184. | 0. |
| | | -173- | -212- | -221- | 0- |
| 35 | 396. | 107. | 142. | 148. | 0. |
| | | -119- | -116- | -173- | 0- |
| 40 | 366. | 112. | 123. | 131. | 0. |
| | | -128- | -87- | -146- | 0- |
| 45 | 241. | 74. | 85. | 82. | 0. |
| | | -81- | -50- | -77- | 0- |
| 50 | 257. | 71. | 101. | 86. | 0. |
| | | -65- | -81- | -95- | 0- |
| 55 | 272. | 80. | 104. | 88. | 0. |
| | | -84- | -64- | -89- | 0- |
| 60 | 257. | 68. | 99. | 90. | 0. |
| | | -60- | -51- | -74- | 0- |
| 65 | 195. | 51. | 75. | 69. | 0. |
| | | -43- | -37- | -51- | 0- |
| 70 | 125. | 32. | 49. | 45. | 0. |
| | | -28- | -21- | -33- | 0- |
| 75 | 68. | 17. | 26. | 25. | 0. |
| | | -14- | -13- | -19- | 0- |
| 80 | 23. | 6. | 8. | 8. | 0. |
| | | -5- | -3- | -6- | 0- |
| 85 | 12. | 3. | 5. | 4. | 0. |
| | | -2- | -2- | -3- | 0- |
| total | 10263. | 3091. | 3543. | 3629. | 0. |

- . - : observed flow

## Table 9. Error analysis of (2F) migration estimates.

### a. Analysis by size class (flow volume) and migrant category

| size class | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | cum.abs.% error value | cum.abs.% error -%- | chi-square value | chi-square -%- |
|---|---|---|---|---|---|---|---|---|
| 0- 200 | 112. | 51.85 | 8452. | 10.63 | 3336. | 71.96 | 0.5645e 03 | 28.13 |
| 200- 400 | 45. | 20.83 | 12742. | 16.02 | 666. | 14.36 | 0.3940e 03 | 19.64 |
| 400- 600 | 20. | 9.26 | 9481. | 11.92 | 162. | 3.50 | 0.1099e 03 | 5.48 |
| 600- 800 | 11. | 5.09 | 7687. | 9.67 | 181. | 3.91 | 0.2471e 03 | 12.32 |
| 800-1000 | 9. | 4.17 | 7705. | 9.69 | 95. | 2.06 | 0.1374e 03 | 6.85 |
| 1000-1200 | 3. | 1.39 | 3330. | 4.19 | 58. | 1.26 | 0.1984e 03 | 9.89 |
| 1200-1400 | 7. | 3.24 | 9075. | 11.41 | 77. | 1.66 | 0.2026e 03 | 10.10 |
| 1400-1600 | 1. | 0.46 | 1464. | 1.84 | 5. | 0.11 | 0.4370e 01 | 0.22 |
| 1600-1800 | 0. | 0.00 | 0. | 0.00 | 0. | 0.00 | 0.0000e 00 | 0.00 |
| 1800-2000 | 2. | 0.93 | 3811. | 4.79 | 17. | 0.38 | 0.6575e 02 | 3.28 |
| 2000+ | 6. | 2.78 | 15769. | 19.83 | 37. | 0.80 | 0.8236e 02 | 4.10 |
| total | 216. | 100.00 | 79516. | 100.00 | 4636. | 100.00 | 0.2006e 04 | 100.00 |

### b. Analysis by error category

| error category | percentage error | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | average flow |
|---|---|---|---|---|---|---|
| 1 | 0 - 2 | 13. | 6.02 | 10251. | 12.89 | 788.538 |
| 2 | 2 - 4 | 11. | 5.09 | 7766. | 9.77 | 706.000 |
| 3 | 4 - 6 | 14. | 6.48 | 11594. | 14.58 | 828.143 |
| 4 | 6 - 8 | 12. | 5.56 | 5241. | 6.59 | 436.750 |
| 5 | 8 - 10 | 18. | 8.33 | 3656. | 4.60 | 203.111 |
| 6 | 10 - 15 | 41. | 18.98 | 14871. | 18.70 | 362.707 |
| 7 | 15 - 20 | 27. | 12.50 | 11470. | 14.42 | 424.815 |
| 8 | 20 - 30 | 34. | 15.74 | 10291. | 12.94 | 302.676 |
| 9 | 30 - 40 | 16. | 7.41 | 2049. | 2.58 | 128.063 |
| 10 | 40 - 60 | 19. | 8.80 | 1698. | 2.14 | 89.368 |
| 11 | 60 - 100 | 7. | 3.24 | 566. | 0.71 | 80.857 |
| 12 | 100 + | 4. | 1.85 | 63. | 0.08 | 15.750 |
| | total | 216. | 100.00 | 79516. | 100.00 | 368.130 |

average absolute percentage error = 12.08
( relative mean deviation )

### c. Analysis by size class and error category

| size class | 0- 2 | 2- 4 | 4- 6 | 6- 8 | 8-10 | 10-15 | 15-20 | 20-30 | 30-40 | 40-60 | 60-100 | 100+ | total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0- 200 | 1 | 1 | 5 | 4 | 11 | 19 | 11 | 18 | 14 | 18 | 6 | 4 | 112 |
| 200- 400 | 3 | 3 | 1 | 2 | 6 | 13 | 7 | 8 | 1 | 0 | 1 | 0 | 45 |
| 400- 600 | 4 | 3 | 3 | 3 | 0 | 4 | 1 | 2 | 0 | 0 | 0 | 0 | 20 |
| 600- 800 | 0 | 2 | 0 | 0 | 0 | 2 | 4 | 2 | 0 | 1 | 0 | 0 | 11 |
| 800-1000 | 2 | 0 | 1 | 2 | 1 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 9 |
| 1000-1200 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 3 |
| 1200-1400 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 7 |
| 1400-1600 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1600-1800 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1800-2000 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 2 |
| 2000- | 1 | 1 | 2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 |
| total | 13 | 11 | 14 | 12 | 18 | 41 | 27 | 34 | 16 | 19 | 7 | 4 | 216 |

The cases of data availability considered here lead to a firm conclusion: expanding the data set not only reduces the estimation error, but also increases the implicit weight attached to minor flows. This is an inherent problem of the error statistics used: as the deviations between estimates and observations decline, the effects of small denominators become more apparent. The error analysis carried out here may also be used to investigate the marginal value of information on migration. Not each subset of the migration data has the same impact on the quality of the estimates. Hence, in further research the question may be posed: what kind of information on the migration pattern do we need in order to obtain estimates of the detailed flow with an acceptable minimum level of accuracy?

### 3.2 Quadratic (Modified Friedlander) Method

As before, we assume that no a priori information on the elements $m^o_{ijk}$ is known. They are set equal to unity, hence the objective function of the modified Friedlander problem becomes

$$\min d\left[m_{ijk}, m^o_{ijk}\right] = \frac{1}{2} \sum_{i,j,k} \frac{(m_{ijk} - 1)^2}{m_{ijk}} \quad .$$

The modified Friedlander method is applied to the 3F problem only. The estimated values of the migration flows $m_{ijk}$ are shown in Table 10. The algorithm required 60 iterations to converge (tolerance level $10^{-4}$). The convergence is therefore much slower than in the entropy algorithm. In addition to slow convergence, rounding errors may cause problems (all variables are single precision). In the Swedish case, in which an 18 x 8 x 8 array must be estimated containing several small estimates, rounding errors made convergence impossible (although theoretically, a unique solution exists, as proved in Appendix A). Work on an improved solution algorithm is planned. The value of the $\chi^2$ statistic is equal to 1578 (Table 11), and lies between the values obtained by the (2F) and (3F) problems. The $\chi^2$ value is given here for illustrative purposes only. It is not an appropriate goodness-of-fit test for the modified Friedlander method, since it is not independent of the objective function used. As a

Table 10. Observed and estimated (modified Friedlander) migration flows by age, Austria, four regions, 1966-1971.

| | total | migration from east | south | east to north | west |
|---|---|---|---|---|---|
| 0 | 1783. | 0. | 681. | 799. | 304. |
| | | 0- | -670- | -877- | -236- |
| 5 | 930. | 0. | 316. | 428. | 186. |
| | | 0- | -328- | -468- | -134- |
| 10 | 1597. | 0. | 466. | 729. | 402. |
| | | 0- | -537- | -829- | -232- |
| 15 | 4172. | 0. | 1522. | 2156. | 494. |
| | | 0- | -1280- | -2192- | -700- |
| 20 | 4227. | 0. | 1321. | 2411. | 494. |
| | | 0- | -1289- | -2231- | -707- |
| 25 | 2807. | 0. | 915. | 1486. | 407. |
| | | 0- | -910- | -1464- | -433- |
| 30 | 1123. | 0. | 362. | 560. | 201. |
| | | 0- | -368- | -588- | -167- |
| 35 | 915. | 0. | 273. | 465. | 176. |
| | | 0- | -293- | -480- | -142- |
| 40 | 871. | 0. | 252. | 471. | 148. |
| | | 0- | -312- | -438- | -121- |
| 45 | 579. | 0. | 190. | 304. | 85. |
| | | 0- | -222- | -289- | -68- |
| 50 | 618. | 0. | 223. | 300. | 95. |
| | | 0- | -241- | -312- | -65- |
| 55 | 689. | 0. | 257. | 339. | 92. |
| | | 0- | -280- | -331- | -78- |
| 60 | 700. | 0. | 253. | 375. | 72. |
| | | 0- | -269- | -357- | -74- |
| 65 | 543. | 0. | 198. | 292. | 53. |
| | | 0- | -210- | -280- | -53- |
| 70 | 353. | 0. | 128. | 191. | 34. |
| | | 0- | -108- | -182- | -33- |
| 75 | 194. | 0. | 68. | 107. | 19. |
| | | 0- | -74- | -101- | -19- |
| 80 | 68. | 0. | 23. | 38. | 6. |
| | | 0- | -26- | -35- | -7- |
| 85 | 34. | 0. | 13. | 19. | 3. |
| | | 0- | -13- | -18- | -3- |
| total | 22203. | 0. | 7460. | 11471. | 3272. |

| | total | migration from east | south | south to north | west |
|---|---|---|---|---|---|
| 0 | 1909. | 921. | 0. | 628. | 360. |
| | | -853- | 0- | -575- | -481- |
| 5 | 1115. | 503. | 0. | 413. | 199. |
| | | -530- | 0- | -329- | -256- |
| 10 | 3652. | 1426. | 0. | 1194. | 1042. |
| | | -1342- | 0- | -1044- | -1276- |
| 15 | 8323. | 3388. | 0. | 1815. | 3120. |
| | | -3760- | 0- | -1950- | -2613- |
| 20 | 4625. | 2197. | 0. | 1143. | 1285. |
| | | -2081- | 0- | -1381- | -1163- |
| 25 | 2625. | 1268. | 0. | 790. | 567. |
| | | -1225- | 0- | -800- | -600- |
| 30 | 1062. | 470. | 0. | 388. | 204. |
| | | -464- | 0- | -346- | -252- |
| 35 | 903. | 404. | 0. | 325. | 174. |
| | | -408- | 0- | -276- | -219- |
| 40 | 807. | 439. | 0. | 234. | 134. |
| | | -411- | 0- | -240- | -156- |
| 45 | 517. | 298. | 0. | 143. | 76. |
| | | -269- | 0- | -149- | -99- |
| 50 | 514. | 265. | 0. | 162. | 87. |
| | | -263- | 0- | -134- | -117- |
| 55 | 521. | 305. | 0. | 137. | 80. |
| | | -296- | 0- | -132- | -93- |
| 60 | 455. | 258. | 0. | 132. | 65. |
| | | -253- | 0- | -137- | -65- |
| 65 | 340. | 195. | 0. | 98. | 47. |
| | | -190- | 0- | -102- | -48- |
| 70 | 217. | 124. | 0. | 62. | 30. |
| | | -121- | 0- | -65- | -31- |
| 75 | 118. | 67. | 0. | 34. | 17. |
| | | -65- | 0- | -36- | -17- |
| 80 | 39. | 24. | 0. | 10. | 5. |
| | | -22- | 0- | -12- | -5- |
| 85 | 21. | 11. | 0. | 7. | 3. |
| | | -11- | 0- | -7- | -3- |
| total | 27773. | 12564. | 0. | 7715. | 7494. |

| | total | migration from east | South | north to north | west |
|---|---|---|---|---|---|
| 0 | 1445. | 678. | 445. | 0. | 322. |
| | | -814- | -363- | 0- | -268- |
| 5 | 917. | 443. | 283. | 0. | 192. |
| | | -448- | -282- | 0- | -187- |
| 10 | 1568. | 650. | 401. | 0. | 517. |
| | | -771- | -344- | 0- | -453- |
| 15 | 5297. | 3481. | 835. | 0. | 981. |
| | | -2992- | -1123- | 0- | -1282- |
| 20 | 3250. | 1796. | 676. | 0. | 779. |
| | | -1861- | -701- | 0- | -688- |
| 25 | 1885. | 911. | 510. | 0. | 464. |
| | | -958- | -482- | 0- | -405- |
| 30 | 924. | 444. | 282. | 0. | 198. |
| | | -485- | -255- | 0- | -184- |
| 35 | 750. | 362. | 220. | 0. | 168. |
| | | -379- | -213- | 0- | -158- |
| 40 | 688. | 381. | 176. | 0. | 131. |
| | | -411- | -141- | 0- | -136- |
| 45 | 447. | 253. | 119. | 0. | 75. |
| | | -275- | -102- | 0- | -70- |
| 50 | 478. | 251. | 142. | 0. | 86. |
| | | -273- | -119- | 0- | -86- |
| 55 | 489. | 284. | 126. | 0. | 79. |
| | | -295- | -114- | 0- | -80- |
| 60 | 439. | 253. | 122. | 0. | 64. |
| | | -263- | -114- | 0- | -62- |
| 65 | 328. | 190. | 91. | 0. | 47. |
| | | -198- | -84- | 0- | -46- |
| 70 | 209. | 121. | 58. | 0. | 30. |
| | | -125- | -54- | 0- | -30- |
| 75 | 109. | 61. | 31. | 0. | 17. |
| | | -66- | -27- | 0- | -16- |
| 80 | 35. | 20. | 9. | 0. | 5. |
| | | -22- | -8- | 0- | -5- |
| 85 | 19. | 10. | 6. | 0. | 3. |
| | | -11- | -6- | 0- | -2- |
| total | 19277. | 10587. | 4532. | 0. | 4158. |

| | total | migration from east | south | west to north | west |
|---|---|---|---|---|---|
| 0 | 905. | 297. | 302. | 306. | 0. |
| | | -229- | -395- | -281- | 0- |
| 5 | 416. | 140. | 135. | 141. | 0. |
| | | -108- | -124- | -184- | 0- |
| 10 | 563. | 188. | 186. | 194. | 0. |
| | | -151- | -171- | -246- | 0- |
| 15 | 2240. | 535. | 855. | 800. | 0. |
| | | -772- | -809- | -659- | 0- |
| 20 | 2235. | 569. | 809. | 837. | 0. |
| | | -640- | -816- | -779- | 0- |
| 25 | 1373. | 433. | 459. | 482. | 0. |
| | | -389- | -491- | -493- | 0- |
| 30 | 606. | 208. | 191. | 207. | 0. |
| | | -173- | -212- | -221- | 0- |
| 35 | 408. | 141. | 129. | 139. | 0. |
| | | -119- | -116- | -173- | 0- |
| 40 | 361. | 130. | 111. | 120. | 0. |
| | | -128- | -87- | -146- | 0- |
| 45 | 208. | 74. | 66. | 68. | 0. |
| | | -81- | -50- | -77- | 0- |
| 50 | 241. | 85. | 77. | 79. | 0. |
| | | -65- | -81- | -95- | 0- |
| 55 | 237. | 87. | 75. | 76. | 0. |
| | | -84- | -64- | -89- | 0- |
| 60 | 185. | 65. | 59. | 60. | 0. |
| | | -60- | -51- | -74- | 0- |
| 65 | 131. | 46. | 42. | 43. | 0. |
| | | -43- | -37- | -51- | 0- |
| 70 | 82. | 29. | 26. | 27. | 0. |
| | | -28- | -21- | -33- | 0- |
| 75 | 46. | 16. | 15. | 15. | 0. |
| | | -14- | -13- | -19- | 0- |
| 80 | 14. | 5. | 4. | 5. | 0. |
| | | -5- | -3- | -6- | 0- |
| 85 | 7. | 2. | 2. | 2. | 0. |
| | | -2- | -2- | -3- | 0- |
| total | 10263. | 3091. | 3543. | 3629. | 0. |

- . - : observed flow

Table 11. Error analysis of the modified Friedlander migration estimates.

a. Analysis by size class (flow volume) and migrant category

| size class | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | cum.abs.% error value | cum.abs.% error -%- | chi-square value | chi-square -%- |
|---|---|---|---|---|---|---|---|---|
| 0- 200 | 112. | 51.85 | 8452. | 10.63 | 1355. | 54.16 | 0.1903e 03 | 11.79 |
| 200- 400 | 45. | 20.83 | 12742. | 16.02 | 545. | 21.79 | 0.2963e 03 | 18.35 |
| 400- 600 | 20. | 9.26 | 9481. | 11.92 | 153. | 6.11 | 0.9538e 02 | 5.91 |
| 600- 800 | 11. | 5.09 | 7687. | 9.67 | 168. | 6.73 | 0.3418e 03 | 21.17 |
| 800-1000 | 9. | 4.17 | 7705. | 9.69 | 63. | 2.50 | 0.6438e 02 | 3.99 |
| 1000-1200 | 3. | 1.39 | 3330. | 4.19 | 51. | 2.02 | 0.1301e 03 | 8.06 |
| 1200-1400 | 7. | 3.24 | 9075. | 11.41 | 90. | 3.61 | 0.2404e 03 | 14.89 |
| 1400-1600 | 1. | 0.46 | 1464. | 1.84 | 1. | 0.06 | 0.3149e 00 | 0.02 |
| 1600-1800 | 0. | 0.00 | 0. | 0.00 | 0. | 0.00 | 0.0000e 00 | 0.00 |
| 1800-2000 | 2. | 0.93 | 3811. | 4.79 | 10. | 0.42 | 0.1237e 02 | 0.77 |
| 2000+ | 6. | 2.78 | 15769. | 19.83 | 65. | 2.60 | 0.2431e 03 | 15.06 |
| total | 216. | 100.00 | 79516. | 100.00 | 2502. | 100.00 | 0.1614e 04 | 100.00 |

b. Analysis by error category

| error category | percentage error | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | average flow |
|---|---|---|---|---|---|---|
| 1 | 0 - 2 | 22. | 10.19 | 9606. | 12.08 | 436.636 |
| 2 | 2 - 4 | 37. | 17.13 | 10261. | 12.90 | 277.324 |
| 3 | 4 - 6 | 18. | 8.33 | 7370. | 9.27 | 409.444 |
| 4 | 6 - 8 | 30. | 13.89 | 10016. | 12.60 | 333.867 |
| 5 | 8 - 10 | 17. | 7.87 | 10995. | 13.83 | 646.765 |
| 6 | 10 - 15 | 25. | 11.57 | 7359. | 9.25 | 294.360 |
| 7 | 15 - 20 | 26. | 12.04 | 10460. | 13.15 | 402.308 |
| 8 | 20 - 30 | 33. | 15.28 | 12085. | 15.20 | 366.212 |
| 9 | 30 - 40 | 5. | 2.31 | 1064. | 1.34 | 212.800 |
| 10 | 40 - 60 | 2. | 0.93 | 68. | 0.09 | 34.000 |
| 11 | 60 - 100 | 1. | 0.46 | 232. | 0.29 | 232.000 |
| 12 | 100 + | 0. | 0.00 | 0. | 0.00 | 0.000 |
| | total | 216. | 100.00 | 79516. | 100.00 | 368.130 |

average absolute percentage error = 11.03
( relative mean deviation )

c. Analysis by size class and error category

| size class | error category 0- 2 | 2- 4 | 4- 6 | 6- 8 | 8-10 | 10-15 | 15-20 | 20-30 | 30-40 | 40-60 | 60-100 | 100+ | total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0- 200 | 10 | 22 | 7 | 15 | 6 | 13 | 16 | 17 | 4 | 2 | 0 | 0 | 112 |
| 200- 400 | 3 | 9 | 5 | 5 | 4 | 5 | 4 | 9 | 0 | 0 | 1 | 0 | 45 |
| 400- 600 | 3 | 2 | 3 | 5 | 3 | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 20 |
| 600- 800 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 0 | 0 | 11 |
| 800-1000 | 3 | 0 | 1 | 1 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 9 |
| 1000-1200 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 1 | 0 | 0 | 0 | 0 | 3 |
| 1200-1400 | 0 | 2 | 0 | 1 | 0 | 0 | 3 | 1 | 0 | 0 | 0 | 0 | 7 |
| 1400-1600 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1600-1800 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1800-2000 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 2000- | 1 | 0 | 1 | 0 | 2 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 6 |
| total | 22 | 37 | 18 | 30 | 17 | 25 | 26 | 33 | 5 | 2 | 1 | 0 | 216 |

consequence, an objective comparison of the quality of the entropy and the quadratic methods is not straightforward. Further research on the development of appropriate test statistics is needed.

## 4. APPLICATION (BULGARIA)

In this section, the entropy method is applied to infer migration flows by age and by regions of origin and destination for a multiregional system without detailed data. The case of Bulgaria has been selected. It is one of the countries participating in the IIASA Comparative Migration and Settlement Study which lacks some of the migration data required for the multiregional analysis, which this Study intended to perform in the IIASA member countries on a comparative basis.*

Internal migration statistics in Bulgaria are derived from the population register (Philipov 1978:16). Age- and sex-specific data on departures and arrivals are available for each of the 28 districts, and the flow matrix is published for the total population (neither age- nor sex-specific). The year 1975 was retained for the analysis.

To enable the application of the multidimensional entropy method to infer migration flows by age, certain anomalies in the original data had to be taken care of. They are mainly due to the fact that departures are generally underreported and their sum is therefore less than the total number of arrivals, yielding a fictitious net inmigration at the national level. Data preparation for the estimation was carried out by Philipov (1978). The data were also aggregated into seven economic planning regions. Table 12 gives the adjusted values of arrivals and departures and the total flow matrix. They constitute the input data for the (3F) problem. The estimated gross migration flows by age are shown in Table 13. Only eight iterations were needed for convergence (tolerance level $10^{-3}$). Table 14 shows the results using the modified Friedlander method. These results were obtained after 189 iterations for the same tolerance level.

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*The data required were age- and region-specific data of population, fertility, mortality, and migration. The latter had to be known by region of origin and destination (Willekens and Rogers 1978:9).

Table 12. Internal migration in Bulgaria, seven economic regions, 1975.*

## a. Migration flow matrix of total population

| to \ from | n.west | north | n.east | s.west | south | s.east | sofia | total |
|---|---|---|---|---|---|---|---|---|
| n.west | 1896. | 1042. | 411. | 539. | 1261. | 271. | 1673. | 7093. |
| north | 1175. | 4152. | 2764. | 292. | 1427. | 559. | 747. | 11116. |
| n.east | 471. | 1524. | 4642. | 220. | 983. | 994. | 492. | 9326. |
| s.west | 268. | 146. | 122. | 823. | 298. | 67. | 310. | 2034. |
| south | 854. | 1107. | 759. | 813. | 9766. | 2500. | 1039. | 16838. |
| s.east | 110. | 249. | 502. | 103. | 919. | 1685. | 259. | 3827. |
| sofia | 3154. | 1446. | 833. | 1987. | 2264. | 864. | 0. | 10548. |
| total | 7928. | 9666. | 10033. | 4777. | 16918. | 6940. | 4520. | 60782. |

## b. Departures and arrivals by region and age

| | n.west | | north | | n.east | | s.west | | south | | s.east | | sofia | | | total | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| age | arrivals | depart. | arrivals | depart. | arrivals | depart. | arrivals | depart. | arrivals | depart. | arrivals | depart. | arrivals | depart. | age | arrivals | depart. |
| 0 | 649. | 614. | 854. | 669. | 694. | 699. | 101. | 244. | 1119. | 1122. | 333. | 514. | 660. | 548. | 0 | 4410. | 4410. |
| 5 | 451. | 417. | 867. | 715. | 644. | 644. | 84. | 217. | 875. | 875. | 278. | 436. | 410. | 305. | 5 | 3609. | 3609. |
| 10 | 1196. | 1125. | 2130. | 1758. | 1573. | 1660. | 637. | 1110. | 2783. | 2640. | 716. | 1048. | 590. | 284. | 10 | 9625. | 9625. |
| 15 | 2026. | 2712. | 3400. | 2827. | 2918. | 3468. | 731. | 1729. | 5632. | 5712. | 1135. | 2352. | 3237. | 479. | 15 | 19279. | 19279. |
| 20 | 917. | 1366. | 1554. | 1560. | 1489. | 1591. | 185. | 634. | 2519. | 2906. | 516. | 1144. | 2718. | 757. | 20 | 9898. | 9898. |
| 25 | 731. | 740. | 1009. | 876. | 800. | 748. | 129. | 302. | 1467. | 1429. | 320. | 548. | 1004. | 817. | 25 | 5460. | 5460. |
| 30 | 344. | 324. | 444. | 389. | 404. | 400. | 58. | 141. | 661. | 694. | 168. | 312. | 472. | 291. | 30 | 2551. | 2551. |
| 35 | 241. | 177. | 252. | 225. | 272. | 244. | 37. | 119. | 446. | 457. | 116. | 176. | 354. | 320. | 35 | 1718. | 1718. |
| 40 | 185. | 142. | 197. | 177. | 173. | 172. | 31. | 87. | 356. | 338. | 87. | 128. | 246. | 231. | 40 | 1275. | 1275. |
| 45 | 131. | 101. | 109. | 119. | 115. | 109. | 19. | 55. | 251. | 225. | 66. | 90. | 178. | 170. | 45 | 869. | 869. |
| 50 | 98. | 72. | 82. | 84. | 71. | 80. | 8. | 34. | 166. | 144. | 42. | 53. | 109. | 109. | 50 | 576. | 576. |
| 55 | 43. | 42. | 46. | 49. | 51. | 48. | 4. | 21. | 93. | 84. | 16. | 28. | 83. | 64. | 55 | 336. | 336. |
| 60 | 33. | 44. | 48. | 55. | 53. | 59. | 4. | 23. | 84. | 87. | 12. | 30. | 123. | 59. | 60 | 357. | 357. |
| 65 | 21. | 42. | 47. | 44. | 36. | 42. | 4. | 23. | 80. | 82. | 12. | 27. | 117. | 57. | 65 | 317. | 317. |
| 70 | 27. | 70. | 77. | 119. | 33. | 69. | 2. | 38. | 106. | 123. | 10. | 54. | 247. | 29. | 70 | 502. | 502. |
| total | 7093. | 7928. | 11116. | 9666. | 9326. | 10033. | 2034. | 4777. | 16838. | 16918. | 3827. | 6940. | 10548. | 4520. | total | 60782. | 60782. |

Source: Philipov 1978:601.

*Intradistrict migrations are removed. The 28 districts were aggregated into seven economic regions by Philipov (1978:599).

Table 13. Estimated (3F) migration flows by age, Bulgaria, seven economic regions, 1975.

| | | migration from | n.west to | | | | | |
|---|---|---|---|---|---|---|---|---|
| | total | n.west | north | n.east | s.west | south | s.east | sofia |
| 0 | 614. | 166. | 98. | 39. | 14. | 61. | 10. | 226. |
| 5 | 417. | 115. | 85. | 32. | 11. | 44. | 8. | 122. |
| 10 | 1125. | 374. | 241. | 88. | 79. | 156. | 24. | 164. |
| 15 | 2712. | 675. | 401. | 157. | 112. | 326. | 35. | 1005. |
| 20 | 1306. | 205. | 137. | 63. | 22. | 103. | 12. | 765. |
| 25 | 740. | 160. | 105. | 43. | 15. | 74. | 9. | 335. |
| 30 | 324. | 78. | 43. | 19. | 7. | 30. | 4. | 144. |
| 35 | 177. | 35. | 19. | 10. | 3. | 15. | 2. | 92. |
| 40 | 142. | 31. | 16. | 7. | 3. | 14. | 2. | 69. |
| 45 | 101. | 21. | 9. | 5. | 2. | 10. | 2. | 52. |
| 50 | 72. | 18. | 7. | 3. | 1. | 8. | 1. | 34. |
| 55 | 42. | 7. | 4. | 2. | 0. | 4. | 0. | 25. |
| 60 | 44. | 4. | 3. | 2. | 0. | 3. | 0. | 32. |
| 65 | 42. | 2. | 3. | 1. | 0. | 3. | 0. | 32. |
| 70 | 70. | 4. | 4. | 1. | 0. | 4. | 0. | 56. |
| total | 7928. | 1896. | 1175. | 471. | 268. | 854. | 110. | 3153. |

| | | migration from | north to | | | | | |
|---|---|---|---|---|---|---|---|---|
| | total | n.west | north | n.east | s.west | south | s.east | sofia |
| 0 | 669. | 80. | 299. | 109. | 7. | 69. | 20. | 86. |
| 5 | 715. | 76. | 358. | 121. | 7. | 69. | 21. | 64. |
| 10 | 1758. | 212. | 873. | 291. | 45. | 208. | 55. | 73. |
| 15 | 2827. | 320. | 1213. | 434. | 54. | 363. | 68. | 375. |
| 20 | 1560. | 135. | 578. | 242. | 15. | 159. | 32. | 399. |
| 25 | 876. | 89. | 373. | 139. | 9. | 97. | 21. | 147. |
| 30 | 389. | 45. | 159. | 64. | 4. | 41. | 10. | 66. |
| 35 | 225. | 24. | 81. | 40. | 2. | 24. | 6. | 48. |
| 40 | 177. | 20. | 67. | 27. | 2. | 21. | 5. | 35. |
| 45 | 119. | 14. | 38. | 19. | 1. | 16. | 4. | 27. |
| 50 | 84. | 11. | 29. | 12. | 0. | 12. | 3. | 17. |
| 55 | 49. | 4. | 15. | 8. | 0. | 6. | 1. | 13. |
| 60 | 55. | 3. | 16. | 8. | 0. | 6. | 1. | 21. |
| 65 | 44. | 2. | 14. | 5. | 0. | 4. | 1. | 18. |
| 70 | 119. | 6. | 36. | 7. | 0. | 11. | 1. | 58. |
| total | 9666. | 1042. | 4152. | 1524. | 146. | 1107. | 249. | 1446. |

| | | migration from | n.east to | | | | | |
|---|---|---|---|---|---|---|---|---|
| | total | n.west | north | n.east | s.west | south | s.east | sofia |
| 0 | 699. | 31. | 198. | 329. | 5. | 47. | 40. | 49. |
| 5 | 644. | 25. | 200. | 309. | 5. | 39. | 35. | 30. |
| 10 | 1660. | 73. | 513. | 781. | 33. | 125. | 98. | 37. |
| 15 | 3468. | 150. | 968. | 1580. | 53. | 295. | 166. | 257. |
| 20 | 1591. | 53. | 336. | 737. | 12. | 108. | 65. | 228. |
| 25 | 748. | 29. | 205. | 349. | 6. | 54. | 36. | 69. |
| 30 | 400. | 17. | 104. | 191. | 3. | 27. | 20. | 37. |
| 35 | 244. | 9. | 55. | 121. | 2. | 16. | 13. | 28. |
| 40 | 172. | 7. | 43. | 78. | 1. | 14. | 10. | 19. |
| 45 | 109. | 5. | 22. | 50. | 1. | 10. | 7. | 14. |
| 50 | 80. | 4. | 19. | 34. | 0. | 8. | 6. | 9. |
| 55 | 48. | 2. | 10. | 23. | 0. | 4. | 2. | 7. |
| 60 | 59. | 1. | 12. | 26. | 0. | 4. | 2. | 13. |
| 65 | 42. | 1. | 10. | 16. | 0. | 3. | 1. | 11. |
| 70 | 69. | 2. | 18. | 17. | 0. | 6. | 1. | 25. |
| total | 10033. | 411. | 2764. | 4642. | 122. | 759. | 502. | 833. |

Table 13 continued.

| | total | migration from s.west to n.west | north | n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 244. | 32. | 16. | 12. | 26. | 40. | 6. | 110. |
| 5 | 217. | 30. | 19. | 14. | 28. | 39. | 7. | 81. |
| 10 | 1110. | 164. | 92. | 64. | 344. | 231. | 34. | 181. |
| 15 | 1729. | 190. | 98. | 74. | 313. | 309. | 32. | 713. |
| 20 | 634. | 44. | 26. | 22. | 46. | 74. | 8. | 413. |
| 25 | 302. | 30. | 17. | 13. | 28. | 47. | 6. | 160. |
| 30 | 141. | 16. | 8. | 6. | 14. | 20. | 3. | 74. |
| 35 | 119. | 11. | 5. | 5. | 8. | 16. | 2. | 71. |
| 40 | 87. | 9. | 4. | 3. | 7. | 13. | 2. | 49. |
| 45 | 55. | 5. | 2. | 2. | 4. | 9. | 1. | 32. |
| 50 | 34. | 4. | 1. | 1. | 2. | 6. | 1. | 19. |
| 55 | 21. | 1. | 1. | 1. | 1. | 3. | 0. | 14. |
| 60 | 23. | 1. | 1. | 1. | 1. | 2. | 0. | 18. |
| 65 | 22. | 1. | 1. | 0. | 1. | 2. | 0. | 19. |
| 70 | 38. | 1. | 1. | 0. | 0. | 3. | 0. | 32. |
| total | 4777. | 539. | 292. | 220. | 823. | 813. | 103. | 1986. |

| | total | migration from south to n.west | north | n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 1122. | 99. | 106. | 71. | 14. | 622. | 76. | 135. |
| 5 | 875. | 74. | 100. | 63. | 12. | 486. | 62. | 79. |
| 10 | 2640. | 227. | 267. | 166. | 82. | 1619. | 180. | 100. |
| 15 | 5712. | 424. | 459. | 306. | 120. | 3492. | 278. | 633. |
| 20 | 2906. | 177. | 216. | 169. | 32. | 1517. | 129. | 665. |
| 25 | 1429. | 103. | 122. | 85. | 17. | 811. | 75. | 216. |
| 30 | 694. | 56. | 58. | 44. | 9. | 378. | 39. | 108. |
| 35 | 457. | 33. | 32. | 29. | 4. | 245. | 27. | 86. |
| 40 | 338. | 25. | 24. | 18. | 4. | 193. | 20. | 55. |
| 45 | 225. | 16. | 12. | 11. | 2. | 131. | 14. | 39. |
| 50 | 144. | 12. | 9. | 6. | 1. | 86. | 9. | 22. |
| 55 | 84. | 5. | 5. | 5. | 0. | 48. | 3. | 18. |
| 60 | 87. | 4. | 5. | 4. | 0. | 42. | 2. | 29. |
| 65 | 82. | 2. | 5. | 3. | 0. | 40. | 3. | 28. |
| 70 | 123. | 4. | 7. | 3. | 0. | 56. | 2. | 51. |
| total | 16918. | 1261. | 1427. | 983. | 298. | 9766. | 919. | 2264. |

| | total | migration from s.east to n.west | north | n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 514. | 22. | 44. | 76. | 3. | 168. | 146. | 54. |
| 5 | 436. | 18. | 44. | 71. | 3. | 139. | 127. | 34. |
| 10 | 1048. | 46. | 97. | 157. | 17. | 387. | 309. | 36. |
| 15 | 2352. | 95. | 187. | 323. | 28. | 933. | 532. | 253. |
| 20 | 1144. | 37. | 82. | 165. | 7. | 376. | 230. | 247. |
| 25 | 548. | 21. | 45. | 80. | 3. | 194. | 128. | 77. |
| 30 | 312. | 13. | 24. | 47. | 2. | 104. | 77. | 44. |
| 35 | 176. | 6. | 11. | 27. | 1. | 56. | 45. | 30. |
| 40 | 128. | 5. | 9. | 16. | 1. | 45. | 33. | 19. |
| 45 | 90. | 3. | 5. | 11. | 0. | 32. | 25. | 14. |
| 50 | 53. | 2. | 3. | 6. | 0. | 20. | 15. | 8. |
| 55 | 28. | 1. | 2. | 4. | 0. | 10. | 5. | 6. |
| 60 | 30. | 1. | 2. | 4. | 0. | 10. | 4. | 10. |
| 65 | 27. | 0. | 2. | 3. | 0. | 9. | 4. | 9. |
| 70 | 54. | 1. | 3. | 3. | 0. | 18. | 4. | 24. |
| total | 6940. | 271. | 559. | 994. | 67. | 2500. | 1685. | 864. |

Table 13 continued.

| | total | migration from n.west | north | sofia to n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 548. | 219. | 94. | 58. | 32. | 111. | 35. | 0. |
| 5 | 305. | 113. | 60. | 35. | 19. | 59. | 19. | 0. |
| 10 | 284. | 100. | 47. | 26. | 38. | 57. | 16. | 0. |
| 15 | 479. | 172. | 74. | 45. | 51. | 114. | 23. | 0. |
| 20 | 757. | 265. | 129. | 91. | 51. | 182. | 40. | 0. |
| 25 | 817. | 299. | 142. | 90. | 51. | 190. | 45. | 0. |
| 30 | 291. | 117. | 47. | 32. | 19. | 61. | 16. | 0. |
| 35 | 320. | 122. | 48. | 40. | 17. | 73. | 21. | 0. |
| 40 | 231. | 89. | 34. | 23. | 14. | 56. | 15. | 0. |
| 45 | 170. | 67. | 21. | 17. | 9. | 44. | 12. | 0. |
| 50 | 109. | 47. | 14. | 9. | 4. | 28. | 8. | 0. |
| 55 | 64. | 23. | 9. | 8. | 2. | 18. | 3. | 0. |
| 60 | 59. | 19. | 10. | 8. | 2. | 17. | 3. | 0. |
| 65 | 57. | 13. | 12. | 7. | 2. | 19. | 3. | 0. |
| 70 | 29. | 9. | 6. | 2. | 1. | 10. | 1. | 0. |
| total | 4520. | 1673. | 747. | 492. | 310. | 1039. | 259. | 0. |

Table 14. Estimated (modified Friedlander) migration flows by age, Bulgaria, seven regions, 1975.

| | total | migration from n.west | north | n.west to n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 614. | 94. | 151. | 55. | 16. | 113. | 10. | 175. |
| 5 | 417. | 77. | 106. | 51. | 13. | 86. | 10. | 74. |
| 10 | 1125. | 504. | 263. | 57. | 71. | 122. | 10. | 98. |
| 15 | 2712. | 894. | 192. | 56. | 88. | 109. | 10. | 1363. |
| 20 | 1306. | 100. | 143. | 54. | 32. | 102. | 10. | 864. |
| 25 | 740. | 73. | 146. | 55. | 22. | 110. | 10. | 325. |
| 30 | 324. | 54. | 63. | 43. | 9. | 67. | 10. | 79. |
| 35 | 177. | 29. | 30. | 29. | 5. | 35. | 9. | 38. |
| 40 | 142. | 24. | 24. | 22. | 5. | 29. | 8. | 30. |
| 45 | 101. | 17. | 15. | 15. | 3. | 21. | 7. | 22. |
| 50 | 72. | 13. | 11. | 10. | 1. | 16. | 5. | 15. |
| 55 | 42. | 6. | 7. | 7. | 1. | 9. | 2. | 10. |
| 60 | 44. | 5. | 7. | 7. | 1. | 9. | 2. | 14. |
| 65 | 42. | 3. | 7. | 5. | 1. | 8. | 2. | 16. |
| 70 | 70. | 4. | 12. | 5. | 0. | 17. | 1. | 31. |
| total | 7928. | 1896. | 1175. | 471. | 268. | 854. | 110. | 3154. |

| | total | migration from n.west | north | north to n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 669. | 84. | 162. | 142. | 14. | 125. | 28. | 114. |
| 5 | 715. | 86. | 225. | 152. | 12. | 134. | 28. | 78. |
| 10 | 1758. | 198. | 978. | 292. | 27. | 148. | 29. | 84. |
| 15 | 2827. | 327. | 1809. | 281. | 28. | 143. | 29. | 209. |
| 20 | 1560. | 106. | 545. | 271. | 22. | 152. | 29. | 435. |
| 25 | 876. | 72. | 234. | 195. | 18. | 147. | 28. | 183. |
| 30 | 389. | 56. | 71. | 70. | 8. | 80. | 22. | 81. |
| 35 | 225. | 34. | 36. | 40. | 5. | 45. | 17. | 48. |
| 40 | 177. | 27. | 29. | 27. | 4. | 38. | 13. | 39. |
| 45 | 119. | 19. | 16. | 17. | 3. | 26. | 10. | 27. |
| 50 | 84. | 15. | 13. | 11. | 1. | 19. | 6. | 19. |
| 55 | 49. | 7. | 7. | 8. | 1. | 11. | 2. | 13. |
| 60 | 55. | 5. | 7. | 8. | 1. | 11. | 2. | 22. |
| 65 | 44. | 3. | 7. | 5. | 1. | 9. | 2. | 18. |
| 70 | 119. | 4. | 13. | 5. | 0. | 19. | 1. | 76. |
| total | 9666. | 1042. | 4152. | 1524. | 146. | 1107. | 249. | 1446. |

| | total | migration from n.west | north | n.east to n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 699. | 43. | 199. | 207. | 13. | 85. | 64. | 87. |
| 5 | 644. | 43. | 211. | 174. | 11. | 83. | 56. | 65. |
| 10 | 1660. | 48. | 503. | 856. | 20. | 86. | 77. | 69. |
| 15 | 3468. | 49. | 963. | 2169. | 21. | 86. | 77. | 104. |
| 20 | 1591. | 45. | 465. | 786. | 18. | 87. | 74. | 116. |
| 25 | 748. | 41. | 209. | 237. | 15. | 85. | 62. | 99. |
| 30 | 400. | 39. | 84. | 86. | 8. | 73. | 33. | 77. |
| 35 | 244. | 30. | 40. | 47. | 5. | 48. | 21. | 54. |
| 40 | 172. | 24. | 29. | 27. | 4. | 36. | 14. | 37. |
| 45 | 109. | 17. | 16. | 17. | 3. | 23. | 10. | 24. |
| 50 | 80. | 14. | 12. | 11. | 1. | 18. | 7. | 18. |
| 55 | 48. | 7. | 7. | 8. | 1. | 11. | 2. | 13. |
| 60 | 59. | 5. | 7. | 8. | 1. | 11. | 2. | 25. |
| 65 | 42. | 3. | 7. | 5. | 1. | 8. | 2. | 16. |
| 70 | 69. | 4. | 12. | 5. | 0. | 17. | 1. | 30. |
| total | 10033. | 411. | 2764. | 4642. | 122. | 759. | 502. | 833. |

Table 14 continued.

| | total | migration from n.west | north | s.west to n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 244. | 47. | 31. | 23. | 16. | 56. | 10. | 62. |
| 5 | 217. | 43. | 30. | 22. | 13. | 50. | 10. | 48. |
| 10 | 1110. | 148. | 38. | 25. | 376. | 337. | 10. | 176. |
| 15 | 1729. | 93. | 36. | 24. | 337. | 101. | 10. | 1128. |
| 20 | 634. | 68. | 35. | 24. | 34. | 93. | 10. | 370. |
| 25 | 302. | 50. | 33. | 24. | 22. | 70. | 10. | 94. |
| 30 | 141. | 26. | 22. | 18. | 9. | 28. | 9. | 29. |
| 35 | 119. | 21. | 19. | 17. | 5. | 24. | 9. | 25. |
| 40 | 87. | 15. | 14. | 13. | 4. | 16. | 8. | 17. |
| 45 | 55. | 9. | 9. | 9. | 3. | 10. | 6. | 10. |
| 50 | 34. | 6. | 6. | 5. | 1. | 6. | 4. | 6. |
| 55 | 21. | 3. | 4. | 4. | 1. | 4. | 2. | 4. |
| 60 | 23. | 3. | 4. | 4. | 1. | 4. | 2. | 5. |
| 65 | 23. | 3. | 4. | 4. | 1. | 5. | 2. | 5. |
| 70 | 38. | 4. | 8. | 4. | 0. | 9. | 1. | 11. |
| total | 4777. | 539. | 292. | 220. | 823. | 813. | 103. | 1987. |

| | total | migration from n.west | north | south to n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 1122. | 89. | 155. | 114. | 17. | 513. | 100. | 134. |
| 5 | 875. | 87. | 167. | 109. | 13. | 342. | 77. | 80. |
| 10 | 2640. | 204. | 227. | 135. | 77. | 1707. | 203. | 86. |
| 15 | 5712. | 490. | 259. | 139. | 111. | 4244. | 196. | 274. |
| 20 | 2906. | 105. | 204. | 130. | 33. | 1527. | 151. | 755. |
| 25 | 1429. | 72. | 163. | 119. | 22. | 760. | 92. | 201. |
| 30 | 694. | 67. | 92. | 82. | 9. | 266. | 36. | 142. |
| 35 | 457. | 45. | 48. | 59. | 5. | 124. | 22. | 153. |
| 40 | 338. | 36. | 39. | 34. | 5. | 111. | 16. | 98. |
| 45 | 225. | 26. | 20. | 22. | 3. | 67. | 11. | 77. |
| 50 | 144. | 20. | 16. | 13. | 1. | 46. | 7. | 41. |
| 55 | 84. | 8. | 8. | 9. | 1. | 19. | 2. | 37. |
| 60 | 87. | 5. | 8. | 9. | 1. | 12. | 2. | 51. |
| 65 | 82. | 3. | 7. | 6. | 1. | 10. | 2. | 54. |
| 70 | 123. | 4. | 13. | 5. | 0. | 20. | 1. | 80. |
| total | 16918. | 1261. | 1427. | 983. | 298. | 9766. | 919. | 2264. |

| | total | migration from n.west | north | s.east to n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 514. | 28. | 64. | 101. | 7. | 137. | 90. | 88. |
| 5 | 436. | 27. | 62. | 91. | 7. | 116. | 69. | 64. |
| 10 | 1048. | 29. | 74. | 169. | 8. | 333. | 358. | 77. |
| 15 | 2352. | 30. | 76. | 202. | 8. | 1094. | 783. | 159. |
| 20 | 1144. | 29. | 74. | 170. | 8. | 474. | 210. | 179. |
| 25 | 548. | 27. | 65. | 109. | 8. | 150. | 86. | 104. |
| 30 | 312. | 26. | 48. | 59. | 6. | 75. | 33. | 65. |
| 35 | 176. | 21. | 28. | 32. | 5. | 36. | 19. | 36. |
| 40 | 128. | 17. | 21. | 21. | 4. | 26. | 14. | 25. |
| 45 | 90. | 14. | 13. | 14. | 3. | 18. | 10. | 18. |
| 50 | 53. | 9. | 9. | 8. | 1. | 10. | 6. | 10. |
| 55 | 28. | 5. | 5. | 5. | 1. | 5. | 2. | 6. |
| 60 | 30. | 4. | 5. | 5. | 1. | 6. | 2. | 7. |
| 65 | 27. | 3. | 5. | 4. | 1. | 6. | 2. | 7. |
| 70 | 54. | 4. | 11. | 5. | 0. | 14. | 1. | 19. |
| total | 6940. | 271. | 559. | 994. | 67. | 2500. | 1685. | 864. |

Table 14 continued.

| | total | migration from n.west | north | sofia to n.east | s.west | south | s.east | sofia |
|---|---|---|---|---|---|---|---|---|
| 0 | 548. | 265. | 92. | 54. | 17. | 90. | 30. | 0. |
| 5 | 305. | 87. | 66. | 46. | 14. | 64. | 28. | 0. |
| 10 | 284. | 65. | 49. | 39. | 57. | 47. | 27. | 0. |
| 15 | 479. | 143. | 63. | 46. | 138. | 60. | 29. | 0. |
| 20 | 757. | 465. | 89. | 53. | 37. | 82. | 31. | 0. |
| 25 | 817. | 397. | 159. | 62. | 23. | 145. | 31. | 0. |
| 30 | 291. | 77. | 63. | 46. | 9. | 72. | 24. | 0. |
| 35 | 320. | 62. | 52. | 49. | 6. | 134. | 19. | 0. |
| 40 | 231. | 42. | 40. | 31. | 5. | 100. | 14. | 0. |
| 45 | 170. | 29. | 20. | 21. | 3. | 86. | 11. | 0. |
| 50 | 109. | 22. | 16. | 13. | 1. | 51. | 7. | 0. |
| 55 | 64. | 8. | 9. | 10. | 1. | 34. | 2. | 0. |
| 60 | 59. | 6. | 10. | 11. | 1. | 30. | 2. | 0. |
| 65 | 57. | 3. | 10. | 7. | 1. | 34. | 2. | 0. |
| 70 | 29. | 4. | 9. | 5. | 0. | 10. | 1. | 0. |
| total | 4520. | 1673. | 747. | 492. | 310. | 1039. | 259. | 0. |

## 5. COMPUTER PROGRAM FOR INFERRING DETAILED MIGRATION PATTERNS FROM AGGREGATE DATA: A USER'S GUIDE TO MULTENTROPY

A computer program was developed that solves the entropy or multiproportional adjustment problem by the direct primal algorithm, and the modified Friedlander adjustment problem by the dual algorithm.

The program consists of a short main program and a set of subroutines. The package is labeled MULTENTROPY. Each subroutine performs a specific task, such as reading the data, adjusting the array elements, computing the modified Friedlander solution, implementing the error analysis, and so on. The main program coordinates the computations through CALL statements. Information is transmitted from one subroutine to another, as follows:

- arrays: labeled COMMON statements,
- parameters: argument string in the main program.

This section describes the subroutines and explains how the parameters should be defined and the input data prepared. The computer program generates tables in a form ready for publication. In fact, the tables presented in this paper are taken directly from the computer output.

### 5.1 Description of the Subroutines

a. DATENT:

SUBROUTINE DATENT (NA, NR, NY, NDAT, NOP, NRAS, NENTROP, NITER, NTOL, MSIZE, INTF)

*Task:*
- reads data and prints them as they are read in (for details on preparation of data deck, see Section 5.2).
- equates the total flow matrix to face 1 (F1-matrix), the age composition of arrivals to face 2 (F2-matrix), and the age composition of departures to face 3 (F3 matrix).

*Parameters:* see Section 5.2.

*Input:* see Section 5.2.
The data file is read twice; first, to list the file as it is. After rewinding, the file is

read a second time to store the data.

*Output:* data as they are read in. The data are stored in labeled COMMON.

b. PRIDA:

SUBROUTINE PRIDA (NR, NA)

*Task:* prints the data used in the estimation. In the 3F or the modified Friedlander problems, the program prints the arrivals and departures by age and region, and the total flow matrix. They are stored in the arrays AMIG(X,I), OTMIG(X,I), and OMIMA(J,I), respectively.

*Parameters:* see Section 5.2.

*Output:* Tables 1, 12, and C1 of this paper.

c. FRIED and COUN1:

SUBROUTINE FRIED (L1, L2, L3, NITER, NTOL)

*Task:* together with subroutine COUN1, it implements the modified Friedlander method.

*Parameters:* L1 = L2 = NR; L3 = NA.

NITER, NTOL: see Section 5.2.

NB: (i) the algorithm converges slowly;

(ii) if the array is large (e.g., 18 x 8 x 8) and contains several very small elements, rounding errors may cause the algorithm not to converge (all variables are single precision). Ways to get around these problems are being investigated.

d. RAS, COUN, and CHE:

SUBROUTINE RAS (L1, L2, L3, NITER, NTOL)

SUBROUTINE COUN (W, K, N, M, JE, TOL, KE, ITER, L1, L2, L3, CHETOL)

SUBROUTINE CHE (W, N, M, TOL, KE, CHETOL)

*Task:* implements the multiproportional adjustment method. COUN adjusts the array with regard to one face; CHE checks whether convergence is attained.

*Parameters:* L1 = L2 = NR; L3 = NA.

N,M: dimensions of the matrix (face) used in adjusting the array elements.

K: dimension along which they are adjusted. For instance, if K = NA, the age-specific flows are adjusted to add up to the total flow matrix.

JE: refers to the step of the algorithm being implemented. There are three steps and they are numbered exactly as in the description of the algorithm in Section 2. At each step, the array is adjusted with regard to a different face.

TOL: tolerance level.

$$TOL = 10^{-NTOL}$$

KE: parameter indicating whether convergence of the solution algorithm is attained (KE = 1 at convergence).

ITER: the current iteration number.

CHETOL: value of stopping criterion if the number of iterations is the maximum permitted (CHETOL is only given if the algorithm does not converge in the prescribed number of steps).

(W is an array).

e. PRIENT:

SUBROUTINE PRIENT (NA, NR, NDAT, NRAS)

*Task:* prints the results of the estimation process. This subroutine is called to print the results for all cases (3F, 1FE, 3E, 2F, and modified Friedlander).

*Parameters:*
- see Section 5.2.
- in case the detailed migration flows A(I,J,X) are available (NDAT = 1), or initial guesses B(I,J,X) exist (NRAS = 1), these data are printed below the estimates.

*Output:* Tables 1, 2, 4, 6, 8, 10, 12, 13, 14, and D1 and D2.

f. ERROR:

SUBROUTINE ERROR (NA, NR, MSIZE, INTF)

*Task:* error analysis of migration estimates. The subroutine is called only if observed flows are available (NDAT = 1).

a. Error analysis by size class (flow volume) and migrant category.

b. Error analysis by error category.

c. Error analysis by size class and error category.

*Parameters:* see Section 5.2.

*Output:* see Tables 3, 5, 7, 9, and 11 of this paper.

## 5.2 Preparation of the Data Deck

All data are read by the DATENT subroutine in fixed format from unit 5 (the conventional unit for cards in most computers). The fixed (F) format may easily be replaced by free (G) format.

The data decks used to produce the results for Austria and Bulgaria are shown in Tables 15 and 16 respectively. In the first case, detailed flow data were available (NDAT = 1). In the second case, only the age structure of arrivals and departures, and the total flow matrix were known (NDAT = 2).

The card sequence is as follows:

1. Identification card

The first card of the deck is an identification card. It may contain any information for the user. The identification card is read in and saved for page-heading.

2. Parameter card

The parameter card contains instructions to the program concerning characteristics of the data set, and concerning desired computations. The parameter names, their interpretation, required format, and default values are given in Table 17.

3. Names of the regions

In the output, each region is identified by its name. Each name consists of a maximum of eight characters. Any characters

Table 15. Input data file, Austria.

```
                                          austria entropy 30 july 1979 4 regions
18 4 5 1 1 2 2 50 411     200
   east   south   north    west
     0.      0.      0.      0.      0.      0.      0.      0.      0.
     0.      0.      0.      0.      0.      0.      0.      0.      0.
   670.    328.    537.   1280.   1289.    910.    368.    293.    312.
   222.    241.    280.    269.    210.    138.     74.     26.     13.
   877.    468.    828.   2192.   2231.   1464.    588.    480.    438.
   289.    312.    331.    357.    280.    182.    101.     35.     18.
   236.    134.    232.    700.    707.    433.    167.    142.    121.
    68.     65.     78.     74.     53.     33.     19.      7.      3.
   853.    530.   1342.   3760.   2081.   1225.    464.    408.    411.
   269.    263.    296.    253.    190.    121.     65.     22.     11.
     0.      0.      0.      0.      0.      0.      0.      0.      0.
     0.      0.      0.      0.      0.      0.      0.      0.      0.
   575.    329.   1044.   1950.   1381.    800.    346.    276.    240.
   149.    134.    132.    137.    102.     65.     36.     12.      7.
   481.    256.   1276.   2613.   1163.    600.    252.    219.    156.
    99.    117.     93.     65.     48.     31.     17.      5.      3.
   814.    448.    771.   2892.   1861.    998.    485.    379.    411.
   275.    273.    295.    263.    198.    125.     66.     22.     11.
   363.    282.    344.   1123.    701.    482.    255.    213.    141.
   102.    119.    114.    114.     84.     54.     27.      8.      6.
     0.      0.      0.      0.      0.      0.      0.      0.      0.
     0.      0.      0.      0.      0.      0.      0.      0.      0.
   268.    187.    453.   1282.    688.    405.    184.    158.    136.
    70.     86.     80.     62.     46.     30.     16.      5.      2.
   229.    108.    151.    772.    640.    389.    173.    119.    128.
    81.     65.     84.     60.     43.     28.     14.      5.      2.
   395.    124.    171.    809.    816.    491.    212.    116.     87.
    50.     81.     64.     51.     37.     21.     13.      3.      2.
   281.    184.    246.    659.    779.    493.    221.    173.    146.
    77.     95.     89.     74.     51.     33.     19.      6.      3.
     0.      0.      0.      0.      0.      0.      0.      0.      0.
     0.      0.      0.      0.      0.      0.      0.      0.      0.
end
```

1° card

migrations from each region to the various regions

"end" card

Table 16. Input data file, Bulgaria.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | bulgaria | entropy | 7 regions | | | 1° card |
| 15 7 5 2 1 2 1 30 3 0 | | | 0 | | | | | | |
| n.west | north | n.east | s.west | south | s.east | sofia | | | departures and arrivals by region |
| 614. | 417. | 1125. | 2712. | 1306. | 740. | 324. | 177. | 142. | |
| 101. | 72. | 42. | 44. | 42. | 70. | | | | |
| 649. | 451. | 1196. | 2026. | 917. | 731. | 344. | 241. | 185. | |
| 131. | 98. | 43. | 33. | 21. | 27. | | | | |
| 669. | 715. | 1758. | 2827. | 1560. | 876. | 389. | 225. | 177. | |
| 119. | 84. | 49. | 55. | 44. | 119. | | | | |
| 854. | 867. | 2130. | 3400. | 1554. | 1009. | 444. | 252. | 197. | |
| 109. | 82. | 46. | 48. | 47. | 77. | | | | |
| 699. | 644. | 1660. | 3468. | 1591. | 748. | 400. | 244. | 172. | |
| 109. | 80. | 48. | 59. | 42. | 69. | | | | |
| 694. | 644. | 1573. | 2918. | 1489. | 800. | 404. | 272. | 173. | |
| 115. | 71. | 51. | 53. | 36. | 33. | | | | |
| 244. | 217. | 1110. | 1729. | 634. | 302. | 141. | 119. | 87. | |
| 55. | 34. | 21. | 23. | 23. | 38. | | | | |
| 101. | 84. | 637. | 731. | 185. | 129. | 58. | 37. | 31. | |
| 19. | 8. | 4. | 4. | 4. | 2. | | | | |
| 1122. | 875. | 2640. | 5712. | 2906. | 1429. | 694. | 457. | 338. | |
| 225. | 144. | 84. | 87. | 82. | 123. | | | | |
| 1119. | 875. | 2783. | 5832. | 2519. | 1467. | 661. | 446. | 356. | |
| 251. | 166. | 93. | 84. | 80. | 106. | | | | |
| 514. | 436. | 1048. | 2352. | 1144. | 548. | 312. | 176. | 128. | |
| 90. | 53. | 28. | 30. | 27. | 54. | | | | |
| 333. | 278. | 716. | 1135. | 516. | 320. | 168. | 116. | 87. | |
| 66. | 42. | 16. | 12. | 12. | 10. | | | | |
| 548. | 305. | 284. | 479. | 757. | 817. | 291. | 320. | 231. | |
| 170. | 109. | 64. | 59. | 57. | 29. | | | | |
| 660. | 410. | 590. | 3237. | 2718. | 1004. | 472. | 354. | 246. | |
| 178. | 109. | 83. | 123. | 117. | 247. | | | | |
| 1896. | 1042. | 411. | 539. | 1261. | 271. | 1673. | | | flow matrix |
| 1175. | 4152. | 2764. | 292. | 1427. | 559. | 747. | | | |
| 471. | 1524. | 4642. | 220. | 983. | 994. | 492. | | | |
| 268. | 146. | 122. | 823. | 298. | 67. | 310. | | | |
| 854. | 1107. | 759. | 813. | 9766. | 2500. | 1039. | | | |
| 110. | 249. | 502. | 103. | 919. | 1685. | 259. | | | |
| 3154. | 1446. | 833. | 1987. | 2264. | 864. | 0. | | | |
| end | | | | | | | | | "end" card |

Table 17. Parameter specification for MULTENTROPY.

| Columns | Format | Name | Interpretation | Default value |
|---|---|---|---|---|
| 1-2 | I2 | NA | Number of age groups | - |
| 3-4 | I2 | NR | Number of regions | - |
| 5-6 | I2 | NY | Width of age group (e.g., 5 years) | 5 |
| 7-8 | I2 | NDAT | Data availability parameter<br>NDAT = 1: detailed flow data [A(I,J,X)] available; program used for validity test<br>NDAT = 2: only marginal totals available | - |
| 9-10 | I2 | NOP | Selection of adjustment problem<br>NOP = 1; entropy or multiproportional adjustment<br>NOP = 2: modified Friedlander problem | 1 |
| 11-12 | I2 | NRAS | Availability of initial guesses (prior distribution) [B(I,J,X)]<br>NRAS = 1: initial estimates exist<br>NRAS = 2: no initial estimates exist | - |
| 13-14 | I2 | NENTROP | Selection of entropy or multiproportional adjustment problems<br>NENTROP = 1; 3F problem only<br>NENTROP = 2; 3F, 3E, 1FE, 2F problems | 1 |
| 15-17 | I3 | NITER | Maximum number of iterations allowed for the algorithm to converge | 10 |
| 18-19 | I2 | NTOL | Exponent of tolerance level for stopping criterion;<br>. tolerance level = $10^{-NTOLL}$ | 4 |
| 20-21 | I2 | MSIZE | Number of size classes in error analysis | 11 |
| 22-25 | I4 | INTF | Size of each class in error analysis | 200 |

may be used. The names appear in sequence on the same card.

| Cols. | Format | Var. name |
|---|---|---|
| 1-72 | 9 A8 | REG(I),I = 1, NR |

4. Migration data

a. NDAT = 1: detailed flow available. The age structure of migrants from region I to region J is contained on one card, followed by a continuation card. If intraregional migrations are not available, blank cards must be inserted.

The sequence of cards is as follows, with I being the region of origin, J the region of destination, and X the migrant category (in this case, 18 age categories):

| Card # | Cols. | Format | Var. name |
|---|---|---|---|
| For region of origin I: | | | |
| 1a | 1-72 | 9 F8.0 | A(I,1,X),X = 1, 9 |
| 1b | 1-72 | 9 F8.0 | A(I,1,X),X = 10, 18 |
| 2a | 1-72 | 9 F8.0 | A(I,2,X),X = 1, 9 |
| 2b | 1-72 | 9 F8.0 | A(I,2,X),X = 10, 18 |
| . | | | |
| . | | | |
| . | | | |
| . | | | |

b. NDAT = 2: data consist of arrivals and departures by age and the total flow matrix. The card sequence is as follows:

1. Departures and arrivals by region

| Card # | Cols. | Format | Var. name |
|---|---|---|---|
| 1a | 1-72 | 9 F8.0 | OTMIG(X,1),X = 1, 9 |
| 1b | 1-72 | 9 F8.0 | OTMIG(X,1),X = 10,18 |
| 2a | 1-72 | 9 F8.0 | AMIG(X,1),X = 1, 9 |
| 2b | 1-72 | 9 F8.0 | AMIG(X,1),X = 10, 18 |
| 3a | 1-72 | 9 F8.0 | OTMIG(X,2),X = 1, 9 |
| . | | | |
| . | | | |
| . | | | |
| . | | | |

2. Total flow matrix

| Card # | Cols. | Format | Var. name |
|---|---|---|---|
| 1 | 1-72 | 9 F8.0 | OMIMA(1,I),I = 1, NR |
| | (if NR > 9, continuation card needed) | | |
| 2 | 1-72 | 9 F8.0 | OMIMA(2,I),I = 1, NR |
| . | | | |
| . | | | |
| . | | | |
| . | | | |

Note that a typical card in this subset contains the arrivals in region J.

5. The last card of the deck is an "END" card. It may be a colored card to identify the end of the deck to the user.

## 5.3 FORTRAN Listing of MULTENTROPY

### a. Main Program

```
C ------------------------------------------------------------------
C   PROGRAM : MAINENT.FTN   ( THIS VERSION CREATED ON AUGUST 10 1979 )
C
C SUBROUTINES REQUIRED : DATENT, PRIDA, RAS,COUN, CHE, FRIED, PRIENT, ERROR
C TO RUN AT IIASA   : A.OUT 5=INPUT FILE (E.G. ../DATENTROPY/AUSTRIA/AUSDRAS)
C                           6=+OUT
C
C DIMENSION : 12 REGIONS AND 18 MIGRANT CATEGORIES (AGE GROUPS)
C
C ARRAYS USED IN THIS PROGRAM AND RELATED PROGRAMS :
C  SUBROUTINE                 :   RAS        PRIDA        ENTROPNETH.FTN
C                                 ---        ----         -------------
C  DEPARTURES FROM REGION I : F3(I,X) = OTMIG(X,I) = OMA(X,I)
C                             F3(NR,NA)  OTMIG(NA,NR1)  OMA(NA,NR1)
C ARRIVALS IN REGIONS I      : F2(I,X) = AMIG(X,I) = IMA(X,I)
C                             F2(NR,NA)  AMIG(NA,NR1)  IMA(NA,NR1)
C MIGRATION FROM I TO J      : F1(I,J) = OMIMA(J,I)   = ZMIG(3,J,I)
C                             F1(NR,NR)  OMIMA(NR1,NR1)  ZMIG(3,NR1,NR1)
C
C  ARRAY A(I,J,X) AND B(I,J,X) : MIGRATION FROM I TO J BY MIGRANT CATEGORY X
C  ARGUMENTS FOR ARRAYS : X AND IX REFER TO AGE GROUPS
C                         I AND J REFER TO REGION
C
C PARAMETERS : NA = NUMBER OF AGE GROUPS
C              NR = NUMBER OF REGIONS
C              NY = AGE INTERVAL (E.G. 5 YEARS)
C
C              NDAT = OPTION REFERRING TO DATA AVAILABILITY
C                  NDAT=1 DETAILED FLOW DATA AVAILABLE - VALIDITY TEST OF
C                        ADJUSTMENT METHOD (A(I,J,X) KNOWN)
C                  NDAT=2 DETAILES FLOW DATA NOT AVAILABLE - ONLY
C                        MARGINALS ARE READ IN
C
C              NOP = OPTION REFERRING TO ADJUSTMENT PROBLEM SELECTED
C                  NOP=1 ENTROPY OR MULTIPROPORTIONAL ADJUSTMENT
C                  NOP=2 MODIFIED FRIEDLANDER PROBLEM
C
C              NRAS = OPTION REFERRING TO INITIAL GUESSES AVAILABLE
C                  NRAS=1 INITIAL ESTIMATES EXIST ( B(I,J,X) KNOWN )
C                  NRAS=2 NO  "       "       "    ( B(I,J,X) = 1 )
C
C              NENTROP = OPTION REFERRING TO ENTROPY OR MULTIPROPORTIONAL
C                        ADJUSTMENT PROBLEM SELECTED
C                  NENTROP=1 3F PROBLEM ONLY
C                  NENTROP=2 3F, 1FE, 2F, AND 3E PROBLEMS
C
C              NITER = MAXIMUM NUMBER OF ITERATIONS
C                       IF NITER IS REACHED, MESSAGE "WE ARE AT THE LIMIT"
C              NTOL = EXPONENT OF TOLERANCE LEVEL
C                        TOL=10.**(-NTOL)
C              MSIZE = NUMBER OF SIZE CLASSES IN VALIDITY TEST
C              INTF  = SIZE OF EACH CLASS IN VALIDITY TEST
C
C ------------------------------------------------------------------
C
```

```
      COMMON /CRAS/ A(12,12,18),B(12,12,18),F1(12,12),F2(12,18),
     1F3(12,18)
      COMMON /CPRIDA/ AMIG(18,13),OTMIG(18,13),OMIMA(13,13)
      COMMON /CV/ V1(12),V2(12),V3(18),SUMA
      COMMON /CREG/ REG(12)
      COMMON /CNAG/ NAGE(18)
      COMMON /CTIT/ TIT(20)
      INTEGER X
      DOUBLE PRECISION REG
      DATA CAS1/4H 3F /,CAS2/4H 3E /,CAS3/4H1FE /,CAS4/4H 2F /
C
C ------------------------------------------------------------------
C
C  READ MIGRATION DATA
C
      CALL DATENT (NA,NR,NY,NDAT,NOP,NRAS,NENTROP,NITER,NTOL,MSIZE,
     1INTF,L1,L2,L3)
C
C ------------------------------------------------------------------
C
C   PRINT ENTROPY DATA (ARRIVALS AND DEPARTURES AND TOTAL FLOW MATRIX)
C
      CALL PRIDA (NR,NA)
C
C ------------------------------------------------------------------
C
C    ESTIMATE FLOWS BY MODIFIED FRIEDLANDER METHOD
C
C ------------------------------------------------------------
      IF (NOP.EQ.1) GO TO 506
      PRINT 4, (TIT(J),J=1,20)
    4 FORMAT (1H1,50X,20A4)
      PRINT 721
  721 FORMAT (1H0,20X,27HMODIFIED FRIEDLANDER METHOD /21X,27(1H*)/)
        CALL FRIED (L1,L2,L3,NITER,NTOL)
      CALL PRIENT (NA,NR,NDAT,NRAS)
      GO TO 507
C
C --------------------------------------------------------------
C
C  ESTIMATE FLOW BY ENTROPY (3F)
C
C -------------------------------------------------------------
  506 CONTINUE
      PRINT 4, (TIT(J),J=1,20)
  720 FORMAT (1H0,20X,24HMULTIPROPORTIONAL METHOD,3X,A4,2H- ,
     17HPROBLEM/21X,40(1H*)//)
      PRINT 720,CAS1
      CALL RAS (L1,L2,L3,NITER,NTOL)
      CALL PRIENT (NA,NR,NDAT,NRAS)
  507 CONTINUE
C
C ------------------------------------------------------------
C
C  VALIDITY TEST OF ESTIMATION POCEDURE
```

```
C
      IF (NDAT.EQ.1) CALL ERROR (NA,NR,MSIZE,INTF)
C
C -----------------------------------------------------------------
C
C  FIND ANALYTICAL SOLUTION FOR 3E, 1FE AND 2F PROBLEMS
C
C -------------------------------------------------------------------
C
      IF (NOP.EQ.2) GO TO 904
      IF (NENTROP.EQ.1) GO TO 904
      SUMA2=SUMA**2
C THREE EDGES PROBLEM
      DO 471 I=1,NR
      DO 471 J=1,NR
      DO 471 X=1,NA
 471  B(I,J,X)=V3(X)*V2(J)*V1(I)/SUMA2
      PRINT 4, (TIT(J),J=1,20)
      PRINT 720, CAS2
      CALL PRIENT (NA,NR,NDAT,NRAS)
      CALL ERROR (NA,NR,MSIZE,INTF)
C 1FE TOTAL FLOW MATRIX + AGE STRUCTURE AT NATIONAL LEVEL
      DO 477 I=1,NR
      DO 477 J=1,NR
      DO 477 X=1,NA
 477  B(I,J,X)=F1(I,J)*V3(X)/SUMA
      PRINT 4, (TIT(J),J=1,20)
      PRINT 720, CAS3
      CALL PRIENT (NA,NR,NDAT,NRAS)
      CALL ERROR (NA,NR,MSIZE,INTF)
C 2F TOTAL FLOW MATRIX + AGE STRUCTURE OF ARRIVALS
      DO 478 I=1,NR
      DO 478 J=1,NR
      DO 478 X=1,NA
 478  B(I,J,X)=F1(I,J)*F2(J,X)/V2(J)
      PRINT 4, (TIT(J),J=1,20)
      PRINT 720, CAS4
      CALL PRIENT (NA,NR,NDAT,NRAS)
      CALL ERROR (NA,NR,MSIZE,INTF)
C ---------------------------------------------------------------
 904  CONTINUE
      STOP
      END
```

## b. Subroutines

```
      SUBROUTINE DATENT (NA,NR,NY,NDAT,NOP,NRAS,NENTROP,NITER,
     1NTOL,MSIZE,INTF,L1,L2,L3)
      COMMON /CRAS/ A(12,12,18),B(12,12,18),F1(12,12),F2(12,18),
     1F3(12,18)
      COMMON /CPRIDA/ AMIG(18,13),OTMIG(18,13),OMIMA(13,13)
      COMMON /CV/ V1(12),V2(12),V3(18),SUMA
      COMMON /CREG/ REG(12)
      COMMON /CNAG/ NAGE(18)
      COMMON /CTIT/ TIT(20)
      INTEGER X
      DOUBLE PRECISION REG
      DATA END/4HEND /
C
C ------------------------------------------------------------------
C
C  READ AND PRINT INPUT FILE AS IT IS
C  REWIND INPUT FILE
C
C ------------------------------------------------------------------
C
  301 READ (5,1) (TIT(J),J=1,20)
    1 FORMAT (20A4)
      PRINT 356, (TIT(J),J=1,20)
  356 FORMAT (1X,20A4)
      IF (TIT(1).EQ.END) GO TO 302
      GO TO 301
 302  CONTINUE
      PRINT 357
 357  FORMAT (/1X)
      REWIND 5
C
C ------------------------------------------------------------------
C
C  READ TITLE, PARAMETERS, NAMES OF REGIONS AND DEFINE DEFAULT VALUES
C
C ---------------------------------------------------------------------
C
      READ (5,1) (TIT(J),J=1,20)
      READ (5,404) NA,NR,NY,NDAT,NOP,NRAS,NENTROP,NITER,NTOL
     1,MSIZE,INTF
 404  FORMAT (7I2,I3,2I2,I4)
      NR1=NR+1
      READ (5,405) (REG(J),J=1,NR)
 405  FORMAT (9A8)
      L1=NR
      L2=NR
      L3=NA
      IF (NTOL.EQ.0) NTOL=4
      IF (NITER.EQ.0) NITER=10
      IF (NOP.NE.2) NOP=1
      IF (MSIZE.EQ.0) MSIZE=11
      IF (INTF.EQ.0) INTF=200
      IF (NY.EQ.0) NY=5
      IF (NENTROP.NE.2) NENTROP=1
      DO 403 X=1,NA
```

```
 403  NAGE(X)=(X-1)*NY
C
C ---------------------------------------------------------------
C
C  READ MIGRATION DATA
C  NDAT=1  DATA CONSIST OF ARRAY A(I,J,X)
C  NDAT=2  DATA CONSIST OF ARRIVALS AMIG(X,I), DEPARTURES OTMIG(X,I)
C               AND TOTAL FLOW MATRIX OMIMA(J,I)
C                   -- OMIMA(J,I) MIGRATION FROM I TO J --
C  NRAG=1 PRIOR DISTRIBUTION ( B(I,J,X) ) EXISTS
C
C -----------------------------------------------------------------
C
        IF (NDAT.EQ.1) GO TO 501
C
        DO 21 I=1,NR
        READ (5,11) (OTMIG(X,I),X=1,NA)
        READ (5,11) (AMIG(X,I),X=1,NA)
  11  FORMAT (9F8.0)
  21  CONTINUE
      DO 12 J=1,NR
      READ (5,11) (OMIMA(J,I),I=1,NR)
  12  CONTINUE
C
C CHECK DATA FOR CONSISTENCY
C
      DO 507 I=1,NR
      ZX=0.
      DO 509 X=1,NA
  509 ZX=ZX+OTMIG(X,I)
      Z=0.
      DO 508 J=1,NR
 508  Z=Z+OMIMA(J,I)
C       Z = DEPARTURES OF I
      ZZ=ZX-Z
      Z3=ABS(ZZ)
      IF (Z3.LT.2.) GO TO 507
      PRINT 510
 510  FORMAT (1H1,5X,23HDEPARTURES DO NOT MATCH//
     110X,34HIS FLOW MATRIX ENTERED CORRECTLY? //
     110X,21HROW CONTAINS ARRIVALS,2X,25H(MIG. FROM COLUMN TO ROW) )
C PRINT THE DATA BY PRIDA IF INCONSISTENCY IS FOUND
      CALL PRIDA (NR,NA)
      GO TO 904
 507  CONTINUE
C
C EQUATE F3,F2 AND F1 TO OTMIG,AMIG AND OMIMA
C
       DO 13 I=1,NR
       DO 13 X=1,NA
       F3(I,X)=OTMIG(X,I)
  13   F2(I,X)=AMIG(X,I)
       DO 14 I=1,NR
       DO 14 J=1,NR
  14  F1(I,J)=OMIMA(J,I)
```

```
      GO TO 502
C
C ---------------------------------------------------------------
C
C  READ OBSERVED MIGRATIONS BY AGE AND ORIGIN AND DESTINATION
D        AND COMPUTE FACE SUMS
C
 501  CONTINUE
        DO 10 I=1,L1
        DO 10 J=1,L2
        READ (5,11) (A(I,J,K),K=1,NA)
 10     CONTINUE
C
        DO 221 I=1,L1
        DO 221 J=1,L2
        F1(I,J)=0.0
        DO 2 K=1,L3
   2   F1(I,J)=F1(I,J)+A(I,J,K)
 221   OMIMA(J,I)=F1(I,J)
        DO 229 I=1,L1
        DO 229 K=1,L3
        F3(I,K)=0.0
        DO 22 J=1,L2
  22    F3(I,K)=F3(I,K)+A(I,J,K)
 229   OTMIG(K,I)=F3(I,K)
        DO 223 K=1,L2
        DO 223 J=1,L3
        F2(K,J)=0.0
        DO 23 I=1,L1
  23    F2(K,J)=F2(K,J)+A(I,K,J)
 223    AMIG(J,K)=F2(K,J)
 502  CONTINUE
C
C ----------------------------------------------------------------
C
C  COMPUTE EDGES
C
C --------------------------------------------------------------
C
        DO 24 I=1,L1
        V1(I)=0.0
        DO 24 J=1,L2
  24    V1(I)=V1(I)+F1(I,J)
        DO 25 J=1,L2
        V2(J)=0.0
        DO 25 I=1,L1
  25    V2(J)=V2(J)+F1(I,J)
        DO 26 I=1,L3
        V3(I)=0.0
        DO 26 J=1,L2
  26    V3(I)=V3(I)+F2(J,I)
        SUMA=0.0
        DO 915 I=1,L3
  915   SUMA=SUMA+V3(I)
  904  CONTINUE
```

```
C
C ---------------------------------------------------------------
C
C INITIATE THE MATRIX B(I,J,X)
C
C ---------------------------------------------------------------
C
        IF (NRAS.EQ.1) GO TO 503
      DO 222 X=1,L3
      DO 222 I=1,L1
      DO 222 J=1,L2
 222  B(I,J,X)=1.
      GO TO 504
 503  DO 505 I=1,L1
      DO 505 J=1,L2
      READ (5,11) (B(I,J,K),K=1,NA)
  505 CONTINUE
  504 CONTINUE
       RETURN
       END
```

```
      SUBROUTINE PRIDA (NR,NA)
C
C
      DIMENSION OMIT(13),AMIT(13)
C     DIMENSION AMIG(NR1,NA),OTMIG(NR1,NA),OMIMA(NR1,NR1)
      COMMON /CPRIDA/ AMIG(18,13),OTMIG(18,13),OMIMA(13,13)
      COMMON /CREG/ REG(12)
      COMMON /CNAG/ NAGE(18)
      COMMON /CTIT/ TIT(20)
      DOUBLE PRECISION REG,DY
      INTEGER X
      DATA DY/8H   TOTAL/
      NR1=NR+1
      REG(NR1)=DY
C
C ARRIVALS AND DEPARTURES BY MIGRANT CATEGORY
C
      DO 454 X=1,NA
      AMIG(X,NR1)=0.
      OTMIG(X,NR1)=0.
      DO 454 J=1,NR
      AMIG(X,NR1)=AMIG(X,NR1)+AMIG(X,J)
  454 OTMIG(X,NR1)=OTMIG(X,NR1)+OTMIG(X,J)
      DO 455 J=1,NR1
      OMIT(J)=0.
      AMIT(J)=0.
      DO 455 X=1,NA
      OMIT(J)=OMIT(J)+OTMIG(X,J)
  455 AMIT(J)=AMIT(J)+AMIG(X,J)
      NK=7
      NRL=(NR1-1)/NK+1
      IZ1=1
      PRINT 4, (TIT(J),J=1,20)
    4 FORMAT (1H1,50X,20A4)
      PRINT 66
      PRINT 5
    5 FORMAT (1H0,10X,43HARRIVALS AND DEPARTURES BY MIGRANT CATEGORY/
     111X,43(1H*)//)
      ISKIP=3
      DO 480 ILIN=1,NRL
      IF (ISKIP.NE.ILIN) GO TO 60
      PRINT 165
  165 FORMAT (1H1/1X)
      ISKIP=ISKIP+2
   60 CONTINUE
      IZ=NK*ILIN
      IF (NR1.LE.IZ) IZ2=NR1
      IF (NR1.GT.IZ) IZ2=IZ
      PRINT 451, (REG(J),J=IZ1,IZ2)
  451 FORMAT (/5X,7(5X,A8,4X))
      PRINT 452
  452 FORMAT (1X,3HAGE,2X,7(1X,8HARRIVALS,2X,7HDEPART.))
      PRINT 66
   66 FORMAT (1X)
      DO 459 X=1,NA
```

```
  459   PRINT 453, NAGE(X),((AMIG(X,J),OTMIG(X,J)),J=IZ1,IZ2)
  453   FORMAT (1X,I3,2X,14F9.0)
        PRINT 456, ((AMIT(J),OMIT(J)),J=IZ1,IZ2)
  456   FORMAT (/1X,5HTOTAL,14F9.0)
        IZ1=IZ2+1
        PRINT 66
        PRINT 65
  480   CONTINUE
C
C TOTAL FLOW MATRIX
C
        DO 6 J=1,NR1
        OMIMA(J,NR1)=0.
        DO 6 I=1,NR
    6   OMIMA(J,NR1)=OMIMA(J,NR1)+OMIMA(J,I)
        DO 17 J=1,NR1
        OMIMA(NR1,J)=0.
        DO 17 I=1,NR
   17   OMIMA(NR1,J)=OMIMA(NR1,J)+OMIMA(I,J)
        OMIMA(NR1,NR1)=0.
        DO 18 J=1,NR
   18   OMIMA(NR1,NR1)=OMIMA(NR1,NR1)+OMIMA(NR1,J)
        PRINT 4, (TIT(J),J=1,20)
        PRINT 66
        PRINT 65
        PRINT 7
    7   FORMAT (1H0,20X,21HMIGRATION FLOW MATRIX/21X,21(1H*)/)
        NK=12
        NRL=(NR1-1)/NK+1
        IZ1=1
        ISKIP=3
        DO 10 ILIN=1,NRL
        IF (ISKIP.NE.ILIN) GO TO 61
        PRINT 165
        ISKIP=ISKIP+2
   61   CONTINUE
        IZ=NK*ILIN
        IF (NR1.LE.IZ) IZ2=NR1
        IF (NR1.GT.IZ) IZ2=IZ
        PRINT 8, (REG(J),J=IZ1,IZ2)
    8   FORMAT (6X,4HFROM,1X,12(2X,A8))
        PRINT 687
  687   FORMAT (1X,2HTO )
        DO 9 I=1,NR1
        IF (I.EQ.NR1) PRINT 65
    9   PRINT 11, REG(I),(OMIMA(I,J),J=IZ1,IZ2)
   11   FORMAT (1X,A8,2X,12F10.0)
        IZ1=IZ2+1
        PRINT 56
        PRINT 66
   10   CONTINUE
        RETURN
        END
```

```
      SUBROUTINE FRIED(L1,L2,L3,NITER,NTOL)
C
C ----------------------------------------------------------------------
C
C  THIS PROGRAM SOLVES THE MODIFIED FRIEDLANDER PROBLEM BY THE
C  DUAL ALGORITHM (WILLEKENS, POR AND RAQUILLET, 1979)
C
C  PARAMETERS   :   L1=L2=NR
C                   L3=NA
C
C  SUBROUTINE REQUIRED : COUN1
C
C  B(I,J,K) TAKES VALUE OF INITIAL GUESS OR IS EQUAL TO 1.0
C
C  VXV = TOTAL NUMBER OF MIGRANTS
C  VSU = TOTAL NUMBER OF NONZERO CELLS (ELEMENTS) IN PRIOR DISTRIBUTION
C
C ----------------------------------------------------------------------
     COMMON /CRAS/ A(12,12,18),B(12,12,18),F1(12,12),F2(12,18),
    1F3(12,18)
     COMMON /CCOUN/ F(12,18),FX1(12,12),FX2(12,18),FX3(12,18)
     COMMON /CTIT/ TIT(20)
C
     TOL=10.**(-NTOL)
C
C
C  STEP 0 OF ALGORITHM : INITIALIZE DUAL VARIABLES
C
      VXV=0.0
      DO 16 I=1,L1
      DO 16 J=1,L2
      VXV=F1(I,J)+VXV
      FX1(I,J)=0.0
      IF(F1(I,J).NE.0.0) GOTO 16
      DO 17 K=1,L3
 17   B(I,J,K)=0.0
 16   CONTINUE
      DO 18 J=1,L2
      DO 18 K=1,L3
      FX2(J,K)=0.0
      IF(F2(J,K).NE.0.0) GOTO 18
      DO 19 I=1,L1
 19   B(I,J,K)=0.0
 18   CONTINUE
      DO 20 I=1,L1
      DO 20 K=1,L3
      FX3(I,K)=0.0
      IF(F3(I,K).NE.0.0) GOTO 20
      DO 21 J=1,L3
 21   B(I,J,K)=0.0
 20   CONTINUE
      VSU=0.0
      DO 23 I=1,L1
      DO 23 J=1,L2
      DO 23 K=1,L3
```

```
        IF(B(I,J,K).NE.0.0) VSU=VSU+1.0
 23     CONTINUE
        VXV=VXV/VSU
C
C
        DO 1 ITER=1,NITER
        KE=1
C
C  STEP 1 (FACE = TOTAL FLOW MATRIX)
C
        JE=3
        DO 13 I=1,L1
       DO 13 J=1,L2
  13   F(I,J)=F1(I,J)
        CALL COUN1(L3,L1,L2,JE,TOL,KE,ITER,VXV,CHETOL)
C
C  STEP 2 (FACE = ARRIVALS)
C
      JE=2
      DO 14 I=1,L1
      DO 14 J=1,L3
 14   F(I,J)=F2(I,J)
        CALL COUN1(L1,L2,L3,JE,TOL,KE,ITER,VXV,CHETOL)
C
C  STEP 3 (FACE = DEPARTURES)
C
      JE=1
      DO 15 I=1,L1
      DO 15 J=1,L3
 15   F(I,J)=F3(I,J)
        CALL COUN1(L2,L1,L3,JE,TOL,KE,ITER,VXV,CHETOL)
        IF(KE.EQ.1) GO TO 2
 1      CONTINUE
C
C  END OF ITERATION
C
        PRINT 3, NITER,TOL,CHETOL
        GO TO 4
 2      PRINT 5,ITER,TOL
  3   FORMAT (5X,21HWE ARE AT THE LIMIT :,I4,2X,10HITERATIONS,
     16X,17HTOLERANCE LEVEL =,E14.4/48X,17HCONTROL VARIABLE=,E14.4)
  5   FORMAT (5X,20HNUMBER OF ITERATIONS,2H :,I5/
     15X,17HTOLERANCE LEVEL   ,3X,2H :,E14.4)
        DO 37 I=1,L1
        DO 37 J=1,L2
        DO 37 K=1,L3
        IF(B(I,J,K).EQ.0.0) GOTO 37
        B(I,J,K)=VXV*B(I,J,K)*SQRT(1./(1.+FX1(I,J)+FX2(J,K)+FX3(I,K)))
 37     CONTINUE
 4    CONTINUE
      PRINT 65, (TIT(J),J=1,20)
 65   FORMAT (1H1,50X,20A4)
      PRINT 64
 64   FORMAT (1H0,20X,27HMODIFIED FRIEDLANDER METHOD/21X,27(1H*)//)
        RETURN
        END
```

```
      SUBROUTINE COUN1(K,N,M,JE,TOL,KE,ITER,VXV,CHETOL)
C
C ----------------------------------------------------------------------
C
C THIS SUBROUTINE COMPUTES THE MATRIX (FACE)
C IT SOLVES THE NONLINEAR EQUATION BY THE NEWTON METHOD
C JE=1 (DEPART.);  JE=2 (ARRIV);  JE=3 (TOTAL FLOW)
C
C ----------------------------------------------------------------------
C
      COMMON /CRAS/ A(12,12,18),B(12,12,18),F1(12,12),F2(12,18),
     1F3(12,18)
      COMMON /CCOUN/ F(12,18),FX1(12,12),FX2(12,18),FX3(12,18)
C
C
      GU=-1.0
      DO 2 J=1,N
      DO 2 L=1,M
      ISK=0
      ISK1=0
      TV1=F(J,L)
   15 KEE=1
      TV=-F(J,L)
      TD=0.0
      IF(F(J,L).EQ.0.0) GOTO 2
      IF(ISK1.GT.0) GOTO 25
      TS=1.E37
      DO 23 I=1,K
      IF(JE.EQ.1.AND.B(J,I,L).EQ.0.0) GOTO 23
      IF(JE.EQ.2.AND.B(I,J,L).EQ.0.0) GOTO 23
      IF(JE.EQ.3.AND.B(J,L,I).EQ.0.0) GOTO 23
      IF(JE.EQ.1) T=1.+FX1(J,I)+FX2(I,L)+FX3(J,L)
      IF(JE.EQ.2) T=1.+FX1(I,J)+FX2(J,L)+FX3(I,L)
      IF(JE.EQ.3) T=1.+FX1(J,L)+FX2(L,I)+FX3(J,I)
      IF(T.LT.TS) TS=T
   23 CONTINUE
C
C CHECK WHETHER THE SUM OF THE INTITIAL VALUES IS
C SMALLER THAN THE PRESCIBED VALUE F(J,L)
C
C IF NOT, COMPUTE BETTER INITIAL VALUES
C
C
      IF(TS.LE.1.E-32) GOTO 24
      TF=ALOG(VXV)-0.5*ALOG(TS)
      IF(TF.LT.ALOG(F(J,L))) GOTO25
   24 TDG=2.0*(ALOG(VXV)-ALOG(F(J,L)))
      TDG=EXP(TDG)
      TVG=TDG-TS
      ISK1=1
      IF(JE.EQ.3) FX1(J,L)=FX1(J,L)+TVG
      IF(JE.EQ.2) FX2(J,L)=FX2(J,L)+TVG
      IF(JE.EQ.1) FX3(J,L)=FX3(J,L)+TVG
C
C
```

```
  25  CONTINUE
        DO 22 I=1,K
        IF(JE.EQ.1.AND.B(J,I,L).EQ.0.0) GOTO 22
        IF(JE.EQ.2.AND.B(I,J,L).EQ.0.0) GOTO 22
        IF(JE.EQ.3.AND.B(J,L,I).EQ.0.0) GOTO 22
        IF(JE.EQ.1) T=B(J,I,L)*SQRT(1./(1.+FX1(J,I)+FX2(I,L)+FX3(J,L)))
        IF(JE.EQ.2) T=B(I,J,L)*SQRT(1./(1.+FX1(I,J)+FX2(J,L)+FX3(I,L)))
        IF(JE.EQ.3) T=B(J,L,I)*SQRT(1./(1.+FX1(J,L)+FX2(L,I)+FX3(J,I)))
C
C TV : FUNCTION VALUE
C TD : DERIVATIVE
C
        TV=TV+T*VXV
        TD=TD-0.5*VXV*T**3
  22    CONTINUE
C
C NEWTON METHOD
C
        IF(JE.EQ.3) TQ=FX1(J,L)
        IF(JE.EQ.2) TQ=FX2(J,L)
        IF(JE.EQ.1) TQ=FX3(J,L)
        IF(ISK.EQ.0) GU1=TQ
        TX=TQ-TV/TD
C
C  NEW ESTIMATES
C
        IF(JE.EQ.3) FX1(J,L)=TX
        IF(JE.EQ.2) FX2(J,L)=TX
        IF(JE.EQ.1) FX3(J,L)=TX
C
C  COMPUTE STOPPING CRITERION AND CHECK FOR CONVERGENCE
C
        CHETOL=ABS(TV)/TV1
        IF (CHETOL.GT.TOL) KEE=0
        ISK=ISK+1
        IF(ISK.GT.80) PRINT 39,J,L,TV,TQ
  39    FORMAT(2I6,2E11.4)
        IF (ISK.LE.90) GO TO 45
        PRINT 46,ISK
  46  FORMAT (/5X,32HNEWTON METHOD INTERRUPTED AFTER ,I3,
     111H ITERATIONS/)
        KE=1
        GO TO 47
  45  CONTINUE
        IF(KEE.EQ.1) GOTO 29
        KE=0
        GOTO 15
  29    GU1=ABS(GU1-TX)
        IF(GU1.GT.GU) GU = GU1
  2     CONTINUE
        PRINT 38,ITER,GU
  38    FORMAT(5X,I4,2X,4HKESZ,E11.4)
  47  CONTINUE
        RETURN
        END
```

```
      SUBROUTINE PRIENT (NA,NR,NDAT,NRAS)
      DIMENSION BTOT(12)
      DIMENSION IHU(12)
      COMMON /CRAS/ A(12,12,18),B(12,12,18),F1(12,12),F2(12,18),
     1F3(12,18)
      COMMON /CREG/ REG(12)
      COMMON /CNAG/ NAGE(18)
      INTEGER X
      DOUBLE PRECISION REG
C
C ---------------------------------------------------------------
C
C   PRINT ESTIMATES  B(I,J,X) : MIGRATION FROM I TO J BY CATEGORY X
C
C -----------------------------------------------------------------
C
      ICASS=0
 900  CONTINUE
      ICASS=ICASS+1
 450  FORMAT (1H1,1X)
      ISKIP=3
      PRINT 456
      PRINT 456
      PRINT 456
      DO 451 I=1,NR
      IF (I.NE.1.AND.NDAT.EQ.1) GO TO 553
      IF (I.NE.ISKIP) GO TO 452
      ISKIP=ISKIP+2
  553 PRINT 450
  452 CONTINUE
      PRINT 457, REG(I)
 457  FORMAT (20X,15HMIGRATION FROM ,A8,3H TO)
      PRINT 458, (REG(J),J=1,NR)
 458  FORMAT (6X,5X,5HTOTAL,12(1XA8))
      PRINT 456
      ZTOT=0.
      DO 453 X=1,NA
      Z=0.
      DO 582 J=1,NR
 582  Z=Z+B(I,J,X)
      ZTOT=ZTOT+Z
      PRINT 454, NAGE(X),Z,(B(I,J,X),J=1,NR)
 454  FORMAT (1X,I3,2X,F10.0,12F9.0)
      IF (NDAT.NE.1) GO TO 453
      DO 470 J=1,NR
      IZ=INT(A(I,J,X))
 470  IHU(J)=-IZ
      PRINT 455, (IHU(J),J=1,NR)
 455  FORMAT (6X,10X,12(I8,1H-))
 453  CONTINUE
      PRINT 456
      DO 460 J=1,NR
      BTOT(J)=0.
      DO 460 X=1,NA
  460 BTOT(J)=BTOT(J)+B(I,J,X)
```

```
      PRINT 461, ZTOT,(BTOT(J),J=1,NR)
 461  FORMAT (1X,5HTOTAL,F10.0,12F9.0)
      PRINT 455
      PRINT 455
 455  FORMAT (1X)
      IF (I.EQ.1.AND.NDAT.EQ.1) PRINT 39
 451  CONTINUE
  89  FORMAT (//1X,38H- . - : OBSERVED FLOW OR INITIAL GUESS )
      RETURN
      END
```

```
      SUBROUTINE RAS(L1,L2,L3,NITER,NTOL)
C
C ---------------------------------------------------------------
C
C  THIS PROGRAM SOLVES THE MULTIPROPORTIONAL ADJUSTMENT PROBLEM
C  BY THE DIRECT PRIMAL ALGORITHM (WILLEKENS, POR AND RAQUILLET,1979)
C
C  W IS AN ARRAY WITH DIMENSION NR*NR IF JE=3, AND
C                               NR*NA IF JE=1 OR JE=2
C
C  PARAMETERS : L1=L2=NR
C               L3=NA
C
C  REQUIRES SUBROUTINES : COUN, CHE
C
C ------------------------------------------------------------------
C
      DIMENSION W(216)
      COMMON /CRAS/ A(12,12,18),B(12,12,18),F1(12,12),F2(12,18),
     1F3(12,18)
      COMMON /CCOUN/ F(12,18)
C
      TOL=10.**(-NTOL)
C
C  B(I,J,K) TAKES THE VALUE OF THE INITIAL GUESSES
C
      DO 1 ITER=1,NITER
C
C STEP 1 OF ALGORITHM (FACE = TOTAL FLOW MATRIX)
C
        JE=3
        DO 13 I=1,L1
       DO 13 J=1,L2
   13  F(I,J)=F1(I,J)
        CALL COUN(W,L3,L1,L2,JE,TOL,KE,ITER,L1,L2,L3,CHETOL)
        IF(KE.EQ.1) GO TO 2
C
C STEP 2 OF ALGORITHM (FACE = ARRIVALS)
C
      JE=2
      DO 14 I=1,L1
      DO 14 J=1,L3
  14  F(I,J)=F2(I,J)
        CALL COUN(W,L1,L2,L3,JE,TOL,KE,ITER,L1,L2,L3,CHETOL)
        IF(KE.EQ.1) GO TO 2
C
C STEP 3 OF ALGORITHM (FACE = DEPARTURES)
C
      JE=1
      DO 15 I=1,L1
      DO 15 J=1,L3
  15  F(I,J)=F3(I,J)
        CALL COUN(W,L2,L1,L3,JE,TOL,KE,ITER,L1,L2,L3,CHETOL)
        IF(KE.EQ.1) GO TO 2
  1  -  CONTINUE
       PRINT 3, NITER,TOL,CHETOL
  3   FORMAT (5X,21HWE ARE AT THE LIMIT :,I4,2X,10HITERATIONS,
    16X,17HTOLERANCE LEVEL =,E14.4/48X,17HCONTROL VARIABLE=,E14.4)
       GO TO 4
 2     PRINT 5, I,TOL
  5  FORMAT (5X,20HNUMBER OF ITERATIONS,2H :,I5/
    15X,17HTOLERANCE LEVEL   ,3X,2H :,E14.4)
 4     RETURN
       END
```

```
      SUBROUTINE COUN(W,K,N,M,JE,TOL,KE,ITER,L1,L2,L3,CHETOL)
C
C -----------------------------------------------------------------
C
C THIS SUBROUTINE COMPUTES THE MATRIX (FACE)
C IT AGGREGATES OVER THE DIMENSION 'JE'
C JE=1 (DEPART.); JE=2 (ARRIV.); JE=3 (MIGRANT CATEGORIES)
C
C -----------------------------------------------------------------
C
      DIMENSION W(N,M)
      COMMON /CRAS/ A(12,12,18),B(12,12,18),F1(12,12),F2(12,18),
     1F3(12,18)
      COMMON /CCOUN/ F(12,18)
C
C  COMPUTE FACE
C
      DO 1 I=1,N
      DO 1 J=1,M
  1   W(I,J)=0.0
      DO 2 I=1,K
      DO 2 J=1,N
      DO 2 L=1,M
      IF(JE.EQ.1) T=B(J,I,L)
      IF(JE.EQ.2) T=B(I,J,L)
      IF(JE.EQ.3) T=B(J,L,I)
  2   W(J,L)=W(J,L)+T
C
C  COMPUTE STOPPING CRITERION : RATIO OF OBSERVED OVER COMPUTED MARGINALS
C
      DO 3 I=1,N
      DO 3 J=1,M
      IF(F(I,J).GT.1.E-8) GOTO 33
      IF(ITER.EQ.1) GOTO 33
      W(I,J)=1.0
      GOTO 3
  33  CONTINUE
      W(I,J)=F(I,J)/W(I,J)
  3   CONTINUE
C
C CHECK WHETHER MARGINAL CONSTRAINTS ARE MET
C
      CALL CHE(W,N,M,TOL,KE,CHETOL)
      IF(KE.EQ.1) RETURN
C
C COMPUTE IMPROVED ESTIMATE OF DETAILED FLOW
C
      DO 4 I=1,K
      DO 4 J=1,N
      DO 4 L=1,M
      IF(JE.EQ.1) B(J,I,L)=B(J,I,L)*W(J,L)
      IF(JE.EQ.2) B(I,J,L)=B(I,J,L)*W(J,L)
      IF(JE.EQ.3) B(J,L,I)=B(J,L,I)*W(J,L)
  4   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CHE(W,N,M,TOL,KE,CHETOL)
C
C -----------------------------------------------------
C
C     THIS SUBROUTINE CHECKS THE RESULTS
C     IF TOLERANCE LEVEL IS REACHED  KE = 1
C
C -----------------------------------------------------
C
      DIMENSION W(N,M)
      KE=0
      DO 1 I=1,N
      DO 1 J=1,M
      CHETOL=ABS(1.0-W(I,J))
      IF (CHETOL.LT.TOL) GO TO 1
      RETURN
  1   CONTINUE
      KE=1
      RETURN
      END
```

```
      SUBROUTINE ERROR (NA,NR,MSIZE,INTF)
C
C ------------------------------------------------------------
C    PROGRAM ERROR.FTN ( REVISED AND EXTENDED VERSION OF RASERROR.F
C       AND GOODNES.FTN )
C    IT ASSESSES THE PERFORMANCE OF THE ESTIMATION TECHNIQUES BY
C       CALCULATING THE CHI-SQUARE AND THE ABSOLUTE PERCENTAGE
C       DEVIATION BETWEEN ESTIMATED AND OBSERVED FLOWS
C        DEVIATIONS ARE STUDIED WITH REGARD TO VOLUME OF FLOWS
C        (SIZE CATEGORIES), AGE OF MIGRANTS AND ERROR DISTRIBUTION
C
C   PARAMETERS : MSIZE : NUMBER OF SIZE CLASSES OF MIGRATION FLOWS
C                            ( MAXIMUM 10 )
C                INTF  : CLASS SIZE
C                          KS : CLASS TO WHICH A GIVEN FLOW BELONGS
C                NCLASS: NUMBER OF AGE CLASSES CONSIDERED
C                         (NCLASS=3)
C                KA    : AGE CLASSES
C                        E.G  : 0-14,15-64,65+
C                        KA=NCLASS+1  ALL AGES
C
C ------------------------------------------------------------------
C
        DIMENSION ZNUM(4,11),TOTO(4,11),SE(4,11),SEDEV(4,11)
      DIMENSION CHIS(4,11)
      DIMENSION ERCAT(13),TOTCAT(12),ZUMCAT(12)
      DIMENSION IERCA(12),ICROSS(12,13)
      COMMON /CRAS/ A(12,12,18),B(12,12,18),F1(12,12),F2(12,18),
     1F3(12,18)
      COMMON /CREG/ REG(12)
      COMMON /CNAG/ NAGE(18)
      DOUBLE PRECISION REG
      INTEGER X
C
      NCLASS=3
      NCLAS1=NCLASS+1
        DO 10 K=1,NCLAS1
        DO 10 N=1,MSIZE
        ZNUM(K,N)=0
        TOTO(K,N)=0.
        SE(K,N)=0.
        SEDEV(K,N)=0.
        CHIS(K,N)=0.
   10   CONTINUE
C
C
C -------------------------------------------------------------------
C
C  ERROR ANALYSIS BY SIZE CLASS (FLOW VOLUME) AND MIGRANT CATEGORY
C
C -------------------------------------------------------------------
      PRINT 48
   48 FORMAT (1H1,10X,37HERROR ANALYSIS OF MIGRATION ESTIMATES /
     111X,37(1H*)/11X,37(1H*)//)
      PRINT 22
```

```
   22  FORMAT (11X,40HA.- ANALYSIS BY SIZE CLASS (FLOW VOLUME),
      123H AND MIGRANT CATEGORIES/11X,63(1H=)/)
         DO 11 IX=1,NA
         KA=3
         IF(IX.LE.3) KA=1
         IF(3.LT.IX.AND.IX.LE.13) KA=2
         DO 11 I=1,NR
         DO 11 J=1,NR
C        IF (J.EQ.I) GO TO 11
         FLOBS=A(J,I,IX)
C                  A(J,I,IX) MIGRATION FROM J TO I BY CATEGORY IX
C
C IN 3F,1FE,AND 2F PROBLEMS, THE ESTIMATES ARE 0 IF OBSERVATIONS
C   ARE 0, HENCE THEY DO NOT ENTER THE ERROR CALCULATION
C  IN 3E-PROBLEM, FLEST IS NOT 0 IF FLOB IS 0, BUT IT DOES NOT ENTER
C   THE ERROR CALCULATION (ERROR IS INFINITY)
C
        IF (FLOBS.EQ.0.) GO TO 11
         FLEST=B(J,I,IX)
         IMIG=INT(FLOBS)
C
C  DETERMINATION OF SIZE CLASS
C
         KS=1+(IMIG/INTF)
         IF(KS.GT.MSIZE) KS=MSIZE
         ZNUM(KA,KS)=ZNUM(KA,KS)+1
         TOTO(KA,KS)=TOTO(KA,KS)+FLOBS
C
C ABSOLUTE DEVIATION FOR EACH INDIVIDUAL ELEMENT AND
C   TOTAL ABSOLUTE DEVIATION FOR EACH SIZE CLASS ( SE(KA,KS) )
C
         SEE=ABS(FLOBS-FLEST)
       IF (FLOBS.EQ.0.) GO TO 40
       SEM=SEE/FLOBS
         SE(KA,KS)=SE(KA,KS)+SEM
       SEDEV(KA,KS)=SEDEV(KA,KS)+SEE
   40  CONTINUE
C
C  CHI-SQUARE DEVIATION FOR EACH SIZE CLASS ( CHIS(KA,KS) )
C
       IF (B(J,I,IX).EQ.0.) GO TO 11
       ZH1=SEE**2
       ZH2=ZH1/B(J,I,IX)
       CHIS(KA,KS)=CHIS(KA,KS)+ZH2
C
   11  CONTINUE
C
         DO 21 K=1,MSIZE
         DO 21 KA=1,NCLASS
         ZNUM(NCLAS1,K)=ZNUM(NCLAS1,K)+ZNUM(KA,K)
         TOTO(NCLAS1,K)=TOTO(NCLAS1,K)+TOTO(KA,K)
         SE(NCLAS1,K)=SE(NCLAS1,K)+SE(KA,K)
         SEDEV(NCLAS1,K)=SEDEV(NCLAS1,K)+SEDEV(KA,K)
         CHIS(NCLAS1,K)=CHIS(NCLAS1,K)+CHIS(KA,K)
    21   CONTINUE
```

```
C
      ISKIP=3
        DO 30 KA=1,NCLAS1
      IF (KA.NE.ISKIP) GO TO 3
      PRINT 64
      ISKIP=ISKIP+2
    3 CONTINUE
      PRINT 65
   64 FORMAT (1H1,1X)
   65 FORMAT (3X)
      IF (KA.LE.NCLASS) PRINT 1111, KA
      IF (KA.EQ.NCLAS1) PRINT 1115
 1115 FORMAT (/1X,22HALL MIGRANT CATEGORIES/)
 1111  FORMAT (/1X,18HMIGRANT CATEGORY =,I2/)
 1112  FORMAT (1X,10HSIZE CLASS,2X,15HNUMBER OF FLOWS,2X,
     115HVOLUME OF FLOWS,2X,15HCUM.ABS.% ERROR,7X,10HCHI-SQUARE/
     111X,2(5X,5HTOTAL,4X,3H-%-),5X,5HVALUE,4X,3H-%-,9X,
     15HVALUE,4X,3H-%-/)
      PRINT 1112
      ZNUMT=0.
      CTOTO=0.
      SET=0.
      SETDEV=0.
      CHIST=0.
      DO 31 KS=1,MSIZE
      ZNUMT=ZNUMT+ZNUM(KA,KS)
      SET=SET+SE(KA,KS)
      SETDEV=SETDEV+SEDEV(KA,KS)
      CHIST=CHIST+CHIS(KA,KS)
   31 CTOTO=CTOTO+TOTO(KA,KS)
      DO 32 KS=1,MSIZE
      KS1=(KS-1)*INTF
      KS2=KS*INTF
      SE3=100.*SE(KA,KS)
      IF (SET.NE.0.) PER3=SE3/SET
      IF (CTOTO.NE.0.) PER2=100.*TOTO(KA,KS)/CTOTO
      IF (ZNUMT.NE.0.) PER1=100.*ZNUM(KA,KS)/ZNUMT
      IF (CHIST.NE.0.) PER4=100.*CHIS(KA,KS)/CHIST
      IF (KS.LT.MSIZE) PRINT 1000, KS1,KS2,ZNUM(KA,KS),PER1,
     1TOTO(KA,KS),PER2,SE3,PER3,CHIS(KA,KS),PER4
 1000 FORMAT (2X,I4,1H-,I4,2(2X,F8.0,F7.2),2X,F8.0,F7.2,2X,E12.4,F7.2)
      IF (KS.EQ.MSIZE) PRINT 1114, KS1,ZNUM(KA,KS),PER1,TOTO(KA,KS),
     1PER2,SE3,PER3,CHIS(KA,KS),PER4
 1114  FORMAT (2X,I4,1H+,4X,2(2X,F8.0,F7.2),2X,F8.0,F7.2,2X,E12.4,F7.2)
   32 CONTINUE
      PERZ=100.
      SETT=100.*SET
      PRINT 1116, ZNUMT,PERZ,CTOTO,PERZ,SETT,PERZ,CHIST,PERZ
 1116 FORMAT (/4X,5HTOTAL,2X,3(2X,F8.0,F7.2),2X,E12.4,F7.2)
   30   CONTINUE
      ABSER=SETDEV/CTOTO
C
C -------------------------------------------------------------------
C
C  ERROR ANALYSIS BY ERROR CATEGORY
```

```
C    COMPUTE ABSOLUTE PERCENTAGE DEVIATION FOR EACH INDIVIDUAL ELEMENT
C    AND GROUP THE ELEMENTS IN ERROR CATEGORIES (12)
C
C ---------------------------------------------------------------------
C
      PRINT 64
      PRINT 23
   23 FORMAT (//11X,30HB.- ANALYSIS BY ERROR CATEGORY/11X,30(1H=)//)
      DO 46 K=1,12
      ZUMCAT(K)=0.
      TOTCAT(K)=0.
   46 CONTINUE
C ERROR CATEGORIES
      ERCAT(1)=0.
      ERCAT(2)=2.
      ERCAT(3)=4.
      ERCAT(4)=6.
      ERCAT(5)=8.
      ERCAT(6)=10.
      ERCAT(7)=15.
      ERCAT(8)=20.
      ERCAT(9)=30.
      ERCAT(10)=40.
      ERCAT(11)=60.
      ERCAT(12)=100.
      ERCAT(13)=10.**7
      SEEPTOT=0.
      DO 43 IX=1,NA
      DO 43 I=1,NR
      DO 43 J=1,NR
C     IF (J.EQ.I) GO TO 43
      FLOBS=A(J,I,IX)
      FLEST=B(J,I,IX)
      SEE=ABS(FLOBS-FLEST)
      IF (FLOBS.EQ.0.) GO TO 43
      SEEP=100.*SEE/FLOBS
      SEEPTOT=SEEPTOT+SEEP
      DO 44 K=1,12
      IF (SEEP.LT.ERCAT(K).OR.SEEP.GT.ERCAT(K+1)) GO TO 47
      ZUMCAT(K)=ZUMCAT(K)+1.
      TOTCAT(K)=TOTCAT(K)+A(J,I,IX)
   47 CONTINUE
   44 CONTINUE
   43 CONTINUE
      ZUMTOT=0.
      VALTOT=0.
      DO 45 K=1,12
      VALTOT=VALTOT+TOTCAT(K)
   45 ZUMTOT=ZUMTOT+ZUMCAT(K)
C
      PRINT 49
   49 FORMAT (4X,5HERROR,2X,10HPERCENTAGE,4X,15HNUMBER OF FLOWS,4X,
     115HVOLUME OF FLOWS,5X,7HAVERAGE/1X,8HCATEGORY,
     14X,5HERROR,2X,2(7X,5HTOTAL,4X,3H-%-),7X,4HFLOW/)
     - DO 50 K=1,12
```

```
      IERCA(K)=INT(ERCAT(K))
      PER1=100.*ZUMCAT(K)/ZUMTOT
      PER2=100.*TOTCAT(K)/VALTOT
      ZRA=0.
      IF(ZUMCAT(K).NE.0.) ZRA=TOTCAT(K)/ZUMCAT(K)
      IER1=INT(ERCAT(K))
      IF (K.EQ.12) GO TO 56
      IER2=INT(ERCAT(K+1))
      PRINT 51, K,IER1,IER2,ZUMCAT(K),PER1,TOTCAT(K),PER2,ZRA
   51 FORMAT (5X,I3,3X,I3,3H - ,I3,5X,F7.0,F7.2,5X,F7.0,F7.2,F14.3)
      GO TO 50
   56 PRINT 57, K,IER1,ZUMCAT(K),PER1,TOTCAT(K),PER2,ZRA
   57 FORMAT (5X,I3,3X,I3,3H + ,8X,F7.0,F7.2,5X,F7.0,F7.2,F14.3)
   50 CONTINUE
      Z1=100.
      Z2=100.
      ZRA=VALTOT/ZUMTOT
      PRINT 52, ZUMTOT,Z1,VALTOT,Z2,ZRA
   52 FORMAT (/13X,5HTOTAL,7X,F7.0,F7.2,5X,F7.0,F7.2,F14.3)
      PRINT 65
      PRINT 65
      PRINT 65
      ABSERR=100.*ABSER
      PRINT 60, ABSERR
   60 FORMAT (/13X,35HAVERAGE ABSOLUTE PERCENTAGE ERROR =,F8.2
     1/17X,27H( RELATIVE MEAN DEVIATION ) )
C
C -----------------------------------------------------------------
C
C  ERROR ANALYSIS BY SIZE CLASS AND ERROR CATEGORY
C
C -----------------------------------------------------------------
C
       PRINT 64
      PRINT 70
   70 FORMAT (//11X,45HC.- ANALYSIS BY SIZE CLASS AND ERROR CATEGORY/
     111X,45(1H=)//)
      PRINT 71
   71 FORMAT (5X,4HSIZE,45X,14HERROR CATEGORY )
      PRINT 72, (IERCA(J),IERCA(J+1),J=1,11),IERCA(12)
   72 FORMAT (4X,5HCLASS,4X,10(2X,I2,1H-,I2),1X,I2,1H-,I3,3X,I3,1H+,
     12X,5HTOTAL/)
      DO 77 I=1,MSIZE
      DO 77 J=1,13
   77 ICROSS(I,J)=0
      DO 75 IX=1,NA
      DO 75 I=1,NR
      DO 75 J=1,NR
      FLOBS=A(I,J,IX)
      FLEST=B(I,J,IX)
      SEE=ABS(FLOBS-FLEST)
      IF (FLOBS.EQ.0.) GO TO 75
      SEEP=100.*SEE/FLOBS
      IMIG=INT(FLOBS)
      DO 75 K=1,12
```

```
      IF (SEEP.LT.ERCAT(K).OR.SEEP.GT.ERCAT(K+1)) GO TO 76
      KS=1+IMIG/INTF
      IF (KS.GT.MSIZE) KS=MSIZE
      ICROSS(KS,K)=ICROSS(KS,K)+1
76    CONTINUE
75    CONTINUE
      DO 78 KS=1,MSIZE
      ICROSS(KS,13)=0
      DO 78 K=1,12
78    ICROSS(KS,13)=ICROSS(KS,13)+ICROSS(KS,K)
      DO 79 K=1,13
      ICROSS(MSIZE+1,K)=0
      DO 79 KS=1,MSIZE
79    ICROSS(MSIZE+1,K)=ICROSS(MSIZE+1,K)+ICROSS(KS,K)
      DO 74 KS=1,MSIZE
      KS1=(KS-1)*INTF
      KS2=KS*INTF
      IF (KS.LT.MSIZE) PRINT 73, KS1,KS2,(ICROSS(KS,K),K=1,13)
73    FORMAT (2X,I4,1H-,I4,2X,13I7)
      IF (KS.EQ.MSIZE) PRINT 80, KS1,(ICROSS(KS,K),K=1,13)
80    FORMAT (2X,I4,1H-,6X,13I7)
74    CONTINUE
      PRINT 81, (ICROSS(MSIZE+1,K),K=1,13)
81    FORMAT (/4X,5HTOTAL,4X,13I7)
        RETURN
        END
```

## 6. CONCLUSION AND FURTHER RESEARCH

This paper presented and generalized two classes of estimation methods with great potential in migration analysis. It frequently occurs that migration flows by region of origin and destination do not exist for subgroups of the population (such as age, income class, educational level etc.) and that one has to infer these detailed flows from some aggregate information.

The classes and individual techniques may formally be presented as mathematical optimization problems with nonlinear objective functions and linear constraints. The various techniques differ in the objective function selected.

The first class, namely, the bi- and multiproportional adjustment methods, combines the entropy maximization problem and the I-divergence minimizing problem. Both problem formulations have much in common, although historically they were developed independently. The paper presents a generalized formulation integrating both estimation techniques. The entropy method may be considered a special case of the more general I-divergence method. In the entropy problem, no initial guesses of the elements to be estimated are available and they are therefore uniformly set equal to unity. A single solution alogirthm is developed for both problems. Which of the techniques will be used depends on the existing data. The entropy method is more suited to estimate detailed migration flows on the basis of aggregate information only; the I-divergence method lends itself to updating migration flow tables using some recent aggregate information.

In the quadratic adjustment problems, weighted squared deviations between estimates and initial guesses are minimized. The latter may be derived from outdated migration tables or may represent a priori information on the migration pattern. A disadvantage of the quadratic adjustment methods, and among them the Friedlander method, is that the estimates are not always of the appropriate sign. To avoid such anomalies as negative gross outmigration flows, a modified Friedlander method is presented.

The validity of the various estimation procedures is demonstrated using migration data of Austria. The numerical illustration shows that the estimates obtained by the multidimensional entropy method are very close to the observed data in the (3F) case. The goodness-of-fit of the estimates in other cases, where less data were supposed to be available, was less. However a remarkable observation was that the (2F) case yielded estimates that did not deviate much from those obtained in the (3F) case, with much more data to start out with. The reason is the high age-specificity of migration. There seems to be a level of information about the age-composition of migrants that is sufficient to yield good estimates of detailed flows. Having more a priori information on the age structure does not add significantly to the quality of the estimates. This observation may lead to some interesting further research.

A number of topics that need more study has been mentioned in this paper. First, there is the question of how much we need to know in advance to generate acceptable or good estimates of migration flows for subgroups of the population. The answer will depend on the homogeneity of the population structure with regard to variables underlying the classification scheme. For instance, the age-curve of migrants is very homogeneous across populations. Limited information on the age composition may therefore give good estimates of the flows by age.

A related research topic is the development of ways to improve the initial guesses $m^o_{ijk}$. One strategy is to derive the prior distribution $m^o_{ijk}$ from a behavioral migration model in which moves are explained on the basis of push and pull factors, of intervening factors, and of personal factors or migrant characteristics. This approach was applied by Nijkamp (1976) to estimate commuting flows. Another possible strategy may be to introduce a priori expert opinions on detailed migration patterns. Yet another strategy is to identify a structure in the interaction flows to be estimated, to represent this structure by a specific mathematical expression, and to introduce it as additional a priori information to improve the quality of the estimates. In the entropy methods presented in this paper, it is assumed that the estimates $m_{ijk}$ are independent. However, some kind of dependency may be introduced without overcomplicating the estimation procedure. Current research on model migration schedules is of particular relevance in this regard (Castro and Rogers 1979). The increased attention devoted to the analysis of contingency tables or cross-classified data may also be very useful (Bishop et al. 1975; Fienberg, 1977).

A third topic for further research is the extension and application of the techniques presented in this paper to update migration tables. In countries such as the United States, the United Kingdom, Canada, Japan, and France, censuses are the main sources of migration statistics. Changes in migration patterns during the period between censuses are therefore difficult to assess at a detailed level. The multiproportional adjustment method may be relevant to update census migration tables using marginal constraints derived from aggregate data for the years between censuses.

Finally, we need an objective answer to the question How well do the techniques perform? What is needed, therefore, is to develop one or a set of appropriate statistics to measure in objective terms the "goodness-of-fit" of the methods presented. The $\chi^2$ statistic is used most frequently. Related statistics are also used by Nijkamp and Paelinck (1974a) and by Hinojosa (1978), among others. However, this test statistic has some severe problems. Its value is affected heavily by smaller flows (small denominator). (See also Forslund and Schoettner, 1979.)

The research reported in this paper led to some potentially very useful estimation methods for migration analysis. It also led to the formulation of a number of challenging research topics to improve our knowledge of migration patterns. The results presented here and the research priorities identified are not limited, however, to the study of migration. They are applicable to any kind of n-dimensional interaction tables, be they spatial (trip distribution, commuting), sectoral (input-output studies), or of any other nature.

It is not the nature of the data that is important, but their cross-classification. The area of application extends to all cross-classified data or n-dimensional contingency tables.

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## APPENDIX A: PROOFS OF THEOREMS 1 TO 5

### Proof of Theorem 1

Let S denote the set of feasible solutions to constraints (1.19) - (1.27). S is a nonempty, bounded, closed convex set in $R_{+}^{\gamma}$ ($\gamma = |\Gamma|$; the cardinality of set $\Gamma$).

Since the objective functions of both the multiproportional and the modified multidimensional Friedlander adjustment problems are strictly convex over $R_{+}^{\gamma}$, we have a unique optimal solution $s^{*} \in S$ in both cases. $s^{*}$ is a vector containing the elements of the set $\{m_{ijk} | (i,j,k) \in \Gamma\}$.

We will now show that if one element $s_i^{*} = 0$ for some i, then a contradiction ensues. By assumption, we know that there exists $\hat{s} \in S$ such that $\hat{s}_i > 0 \quad 0 \leq i \leq \gamma$.

By definition, the value of the modified three-dimensional chi-square measure (xe: 1.30) is $+\infty$ at point $s^{*}$ and finite at $\hat{s}$, which obviously contradicts the fact that $s^{*}$ is an optimal value for the multidimensional modified Friedlander adjustment problem.

In the case of the multiproportional adjustment problem, the proof is as follows:

Since S is a convex set, we have

$$\lambda\hat{s} + (1-\lambda)s^* \in S \quad , \qquad \forall\ 0 \le \lambda \le 1 \quad ,$$

and

$$\lambda\hat{s} + (1-\lambda)s_i^* > 0 \quad , \qquad \forall\ 0 < \lambda \le 1 \quad .$$

Let

$$f(\lambda) = \sum_{i=1}^{\gamma} \left(\lambda\hat{s}_i + (1-\lambda)s_i^*\right)\ln \frac{\lambda\hat{s}_i + (1-\lambda)s_i^*}{s_i^\circ} \quad 0 < \lambda < 1 \quad ,$$

where the vector $s^\circ$ represents the values of the set $\{m_{ijk}^\circ |$ $(i,j,k)\in\Gamma\}$ in the same order as the vector $s \in S$ represents the values of $\{m_{ijk} | (i,j,k)\in\Gamma\}$.

Consider now the derivate of $f(\lambda)$ for $0 < \lambda < 1$. We have that

$$\frac{\partial f(\lambda)}{\partial\lambda} = \sum_{s_i^*=0} \hat{s}_i \ln \frac{\lambda\hat{s}_i}{s_i^\circ} + \sum_{s_i^*\neq 0} (\hat{s}_i - s_i^*) \ln \frac{\lambda\hat{s}_i + (1-\lambda)s_i^*}{s_i^\circ}$$

$$+ \sum_{i=1}^{\gamma} (\hat{s}_i - s_i^*) \quad .$$

Since

$$\sum_{s_i^*\neq 0} (\hat{s}_i - s_i^*)\ln \frac{\lambda\hat{s}_i + (1-\lambda)s_i^*}{s_i^\circ}$$

is bounded over the interval [0,1] and

$$\lim_{\lambda\to 0} \sum_{s_i^*=0} \hat{s}_i \ln \frac{\lambda\hat{s}_i}{s_i^{(\circ)}} = -\infty \quad ,$$

there exist such $0 < \lambda_0 < 1$ that

$$\frac{\partial f(\lambda)}{\partial \lambda} < 0 \quad , \qquad \forall\ 0 < \lambda < \lambda_0 \quad ,$$

which obviously contradicts the fact that $s_i^*$ is an optimal solution.

Proof of Theorem 2 and Theorem 3

Before starting the proof of Theorems 2 and 3, we first summarize some notions we need from convex analysis, as presented by Rockafellar (1970).

Let f be a closed, proper convex function on $R^n$ (i.e., f is convex and lower semicontinuous and never assumes the value $-\infty$, although $+\infty$ is allowed). The effective domain of f is the convex set dom $f = \{x : f(x) < +\infty\}$. The conjugate function of f(x) is denoted by $f^*(x^*)$ and defined as

$$f^*(x^*) = \sup\{< x, x^* > - f(x) \,|\, x \in \text{ri}(\text{dom } f)\} \quad .$$

The proof to the duality results will rely on the decomposition principle suggested by Rockafellar (1970).

First, we shall consider the problem of minimizing

$$f_1(x_1) + \cdots + f_n(x_n) \qquad x = (x_1, \ldots, x_n) \in R^n \tag{A1}$$

subject to

$$Ax = b \quad ,$$

and

$$x \geq 0 \quad ,$$

where $f_i(x_i)$ is a proper convex function and b is a given vector in $R^n$.

Theorem A1

If the infimum in the above problem is finite and there exists $x \in R^n$ such that

$$Ax = b$$

and

$$x > 0$$

(i.e., all the components of x are strictly positive), then by minimizing the convex function

$$w(\lambda) = f_1^*(- < a^{(n)}, \lambda >) + \cdots + f_n(- < a^{(n)}, \lambda >) + < b, \lambda >$$

(where $f_i^*$ is conjugate to $f_i$ and $a^{(i)}$ represents the ith column of matrix A), and using any optimal solution $\lambda^* \in R^n$ of it, the following subproblems define an optimal solution to the primal problem

$$\min_{x_i} f_i(x_i) + x_i < a^{(i)}, \lambda^* > \qquad (i = 1, \ldots, n) \quad .$$

For the proof of Theorem A1, see Section 28 of Rockafellar (1970).

Both the I-divergence measure (1.28) and the modified three-dimensional chi-square measure can be separated into single-valued convex functions of the variables $m_{ijk}$.

Since the constraints (1.19) - (1.27) are linear equations of variables $m_{ijk}$, we can decompose both the multiproportional and the modified multidimensional Friedlander adjustment problems in the way mentioned above.

The conjugate of the functions

$$f(x) = \begin{cases} x \ln \frac{x}{a} & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ +\infty & \text{if } x < 0 \end{cases} \qquad a > 0$$

and

$$g(x) = \begin{cases} \frac{(x-a)^2}{x} & \text{if } x > 0 \\ +\infty & \text{if } x \leq 0 \end{cases}$$

can be obtained in the following form:

$$f^*(x^*) = a\, e^{x^*-1} \quad , \qquad \forall\, x^* \in R$$

$$g^*(x^*) = \begin{cases} -2a\sqrt{1-x^*} + 2 & \text{if } x^* < 1 \\ +\infty & \text{if } x^* \geq 1 \end{cases} .$$

Now it is easy to see that function w for the multiproportional and for the modified multidimensional Friedlander problem will take the forms of $L_1(\Lambda,N,H)$ and $L_2(\Lambda,N,H)$, respectively.

It is easy to prove that one of the dual variables can be fixed to any constant value in both dual problems without changing the optimal value of the problem. Now, Theorem 2 and Theorem 3 are direct corollaries of Theorem A1.

Proof of Theorem 4 and Theorem 5

Before starting the proof of Theorems 4 and 5, we summarize further notions we need from convex analysis.

The recession function of f is denoted by $f0^+$ and defined as

$$(f0^+)(x) = \lim_{\alpha \downarrow 0} \alpha\, f(\alpha^{-1} x) \qquad x \in \text{dom } f .$$

If, for a nonzero vector, $x \in \text{dom } f$ $(f0^+)(x) \leq 0$, then vector x will be called "direction in which f recedes" or the direction of recession of f.

In what follows, our attention will be focused on the properties of the parameterized nest of level sets

$$\text{lev}_\alpha f = \{x | f(x) \leq \alpha\} , \qquad \alpha \in R$$

belonging to a given proper convex function f.

Let inf f denote the infimum of f(x) as x ranges over $R^n$. For $\alpha = \inf f$, $\text{lev}_\alpha f$ consists of the points x where the infimum of f is attained. We call this level set the minimum set of f.

We shall need the following properties of the level sets.

<u>Lemma 1</u>

Let f be a closed, proper convex function which has no direction of recession. The infimum of f is then finite and attained. Moreover, all the level sets $\text{lev}_\alpha f$ $(\alpha \geq \inf f)$ are nonempty, closed, bounded convex sets.

For the proof of Lemma 1, see Theorem 27.1 of Rockafellar (1970).

On account of the computational value of the algorithms given in Section 2 of the paper, we are going to formulate the procedure for a class of differentiable, closed proper convex functions which have no direction of recession and prove convergence to the minimum set.

Let $f(x_1, x_2, \ldots, x_m)$ be a closed proper convex function on $R^n$. where $x_i \in R^{n_i}$ for all $1 \leq i \leq m$ and $\sum n_i = n$.

For a given $x^\circ \in \text{dom } f$, let us define the sequence of points $x^{(1)}, x^{(2)}, \ldots$ recursively, by letting $x^{(k)}$ be one of the solutions of the minimizing problem

$$\min_{x_l \in R^{n_e}} f\left(x_1^{(s)}, \ldots, x_{l-1}^{(s)}, x_l, x_{l+1}^{(s-1)}, \ldots, x_m^{(s-1)}\right) \qquad \text{(A2)}$$

where

$$k = m\,s + l \qquad\qquad 1 \le l \le m \quad .$$

<u>Lemma 2</u>

Let f be a closed, proper convex function on $R^n$ as defined in the above algorithm. Further, let $x^{(1)}, x^{(2)}, \ldots$ be the sequence generated by the algorithm.

If f is continuously differentiable at every $x \in \text{dom } f$ and has no direction of recession, then

$$\lim_{k \to \infty} f(x^{(k)}) = \inf f$$

and $x^{(1)}, x^{(2)}, \ldots$ is a bounded sequence and all its cluster points belong to the minimum set of f.

<u>Proof</u>

By definition of the sequence $x^{(1)}, x^{(2)}, \ldots$ we have

$$+\infty > f(x^{(0)}) \ge f(x^{(1)}) \ge , \ldots , \ge f(x^{(k-1)}) \ge f(x^{(k)}) \ge \alpha \quad .$$

Hence

$$\lim_{k \to \infty} f(x^{(k)}) = \alpha \ge \inf f \tag{A3}$$

From Lemma 1, it follows that the sequence $x^{(1)}, x^{(2)}, \ldots$ is bounded and $\inf f < +\infty$.

Let $x_c$ denote a cluster point of the sequence $x^{(1)}, x^{(2)}, \ldots$ . From (A3) we get that

$$x_c \in \{x \mid f(x) = \alpha\} \tag{A4}$$

We shall assume that $\alpha > \inf f$ and exhibit a contradiction. Since function f is differentiable over dom f, we have from (A2)

$$\left.\frac{\partial f(x_1,\ldots,x_l,\ldots,x_m)}{\partial x_{l_i}}\right|_{x = x^{(k+1)}} = 0 \qquad \forall 1 \leq i \leq n_l$$

for all l, i and k where k = m s + l and $x_l = (x_{l1},\ldots,x_{l2},\ldots, x_{l_{n_l}}) \in R^{n_i}$ . Hence,

$$\left.\frac{\partial f(x)}{\partial x_j}\right|_{x = x_c} = 0 \qquad \forall 1 \leq i \leq n \quad , \qquad \text{(A5)}$$

which contradicts the fact that $\alpha > \inf f$, because a necessary and sufficient condition for a given point x to belong to the minimum set is exactly (A5).

Lemma 3

Let $L_1(\Lambda,N,H)$ and $L_2(\Lambda,N,H)$ be the same function as they have been defined in (2.1) and (2.2).

If there exists a feasible solution to the constraints (1.19) to (1.27) such that

$$m_{ijk} > 0 \quad , \qquad \text{for all } (i,j,k)\in\Gamma \quad ,$$

then $L_1(\Lambda,N,H)$ and $L_2(\Lambda,N,H)$ are closed proper convex functions and have no direction of recession.

Proof

From the simple form of functions $L_1$ and $L_2$, it is almost trivial to see that the function is a closed proper function over all values of variables $\Lambda,N,M$.

By definition, the recession function of $L_1$ is given as

$$(L_1 0^+)(\Lambda,N,H) = \lim_{\alpha \downarrow 0} \alpha L_1(\alpha^{-1}\Lambda, \alpha^{-1}N, \alpha^{-1}H) \quad .$$

Recall that $\lambda_{11}$ is not treated as a variable, and its value is 1.

If there exist $(i,j,k)\in\Gamma$ such that

$$\lambda_{ij} + \nu_{ik} + \zeta_{jk} < 0 \quad , \qquad \text{if } i \neq 1 \text{ or } j \neq 1 \quad ,$$

$$\nu_{1k} + \zeta_{1k} < 0 \quad , \qquad \text{if } i = j = 1 \quad ,$$

then

$$(L_1 0^+)(\Lambda,N,H) = +\infty \quad .$$

In the case when $\nu_{1k} + \zeta_{1k} \geq 0$ for all $k \in K$, and $\lambda_{ij} + \nu_{ik} + \zeta_{jk} \geq 0$ for all $(i,j,k)\in\Gamma$, we have

$$(L_1 0^+)(\Lambda,N,M) = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}\, \lambda_{ij} + \sum_{i=1}^{n} \sum_{k=1}^{l} b_{ik}\, \nu_{ik} + \sum_{j=1}^{m} \sum_{k=1}^{l} a_{jk}\, \zeta_{jk} \quad .$$

Since there exists a feasible solution to the basic problem such that

$$m_{ijk} > 0 \quad , \qquad (i,j,k)\in\Gamma \quad ,$$

$$m_{ijk} = m^{o}_{ijk} \quad , \qquad (i,j,k)\notin\Gamma \quad ,$$

we have that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}\, \lambda_{ij} + \sum_{i=1}^{n} \sum_{k=1}^{l} b_{ik}\, \nu_{ik} + \sum_{j=1}^{m} \sum_{k=1}^{l} a_{jk}\, \zeta_{jk}$$

$$= \sum_{(i,j,k)\in\Gamma} m_{ijk}(\lambda_{ij} + \nu_{ik} + \zeta_{jk}) \quad .$$

Since

$$\lambda_{11} + \nu_{1k} + \zeta_{1k} > 0 \quad , \qquad \text{for every } k \in K$$

and since there exists $k \in K$ such that

$$(1,1,k) \in \Gamma \quad ,$$

we have

$$(L_1 0^+)(\Lambda,N,M) > 0 \quad .$$

The proof for function $L_2(\Lambda,N,M)$ can be derived in the same way as for function $L_1(\Lambda,N,M)$. It is easy to see that the solution algorithms for our dual problems (2.1) and (2.2) are special cases of our general procedure.

The proof of Theorem 4 and Theorem 5 obviously follows from Lemma 1, Lemma 2, and Lemma 3.

## APPENDIX B: MULTIPROPORTIONAL SOLUTIONS FOR THE SPECIAL CASES (3E), (1FE), AND (2F)

Recall that in the two-dimensional entropy problem without cost factor, the most probable values of the elements of interaction tables can be obtained analytically and are given by the simple expression

$$m_{ij} = \frac{O_i \cdot I_j}{\sum_i I_i} = \frac{m_{i.} \cdot m_{.j}}{m_{..}} \quad .$$

The element $m_{i.}$ denotes the total number of departures out of i, and $m_{.j}/m_{..}$ is the proportion of all migrants that go to j. The latter is independent of the region of origin. The most probable number of migrants from i to j is simply the geometric average of the departures of i and the arrivals of j. In other words, the best estimates are obtained by multiplying the outmigrants by a fixed inmigration profile (multiplicative spatial interaction model).

In the multidimensional case, some analytical solutions to the I-divergence (entropy maximization) method exist, too. A necessary condition for these solutions to exist, however, is that the initial distribution is uniform and that the constraints are in the (3E), (1FE), or (2F) formats.

(i) The Three-edges (3E) Problem

The (3E) problem is to minimize (1.18) subject to (1.22) and (1.26). Let i denote the region of origin, j the region of destination, and k the age group. The solution to the (3E) problem is given by:

$$m_{ijk} = u_k \cdot v_j \cdot w_i / (ST)^2 \quad ,$$

or

$$m_{ijk} = m_{.j.} \cdot m_{..k} \cdot m_{i..} / (m_{...})^2 \quad , \qquad \text{(B1)}$$

where

$$m_{i..} = \sum_{j,k} m_{ijk} \quad ,$$

$$m_{.j.} = \sum_{i,k} m_{ijk} \quad ,$$

$$m_{..k} = \sum_{i,j} m_{ijk} \quad .$$

Note that the solution assumes independence between all three classifications (e.g., number of arrivals is independent of number of departures).

(ii) The One Face-One Edge (1FE) Problem

The (1FE) problem is to minimize (1.18) subject to (1.19), (1.24), and (1.26). Suppose the given face consists of the flow matrix of the total population $c_{ij}$ and the edge represents the age structure of the migrants at the national level. The best estimates of the flow matrices by age are given by:

$$m_{ijk} = c_{ij} \cdot u_k / ST \quad ,$$

or

$$m_{ijk} = m_{ij.} \cdot m_{..k} / m_{...} \quad , \qquad \text{(B2)}$$

where

$$m_{ij.} = \sum_k m_{ijk} \quad .$$

The ratio $m_{..k}/m_{...}$ is the national age composition of the migrants. It is applied to all values of the flow matrix $m_{ij.}$. Hence, the age structure is uniform for all flows. The solution (B2) is therefore a model describing the migration flow under the assumption of a unique age profile of migrants. It may be referred to as the age-profile model. If the given face consists of the age composition of the departures ($m_{i.k}$), and the edge of the number of inmigrants by region ($m_{.j.}$), the model is an inmigration profile model. Finally, if the age structure of arrivals ($m_{.jk}$) is known, together with the total number of outmigrants by region ($m_{i..}$), we have an outmigration profile model. Which of the models would yield better results depends on the homogeneity of the categories considered with regard to the profile adopted. Since the age profile of migrants has a universal pattern, the age-profile model may yield better estimates.

(iii) <u>The Two-faces (2F) Problem</u>

The (2F) problem is to minimize (1.18) subject to (1.19), (1.20), and (1.26). The solution is for given values of $c_{ij}$ and $a_{jk}$.

$$m_{ijk} = c_{ij} \cdot b_{ik} / w_i \quad ,$$

or

$$m_{ijk} = m_{ij.} \cdot m_{i.k} / m_{i..} \quad . \qquad \text{(B3)}$$

The ratios $m_{i.k}/m_{i..}$ are conditional probabilities. The solution therefore implies the assumption of conditional independence between the j (destination) and i (origin) classifications for every k (migrant category, e.g., age group). In (B3), the age structure of migrants is origin-specific. The ratio $m_{i.k}/m_{i..}$ represents the age curve of outmigrants. It is independent of the region of destination.

## APPENDIX C: A SUGGESTION FOR A PRIOR DISTRIBUTION IN ENTROPY MAXIMIZATION

In the entropy problem, an initial or prior distribution of the elements to be estimated does not exist. In other words, no initial guesses $m^o_{ijk}$ are available. The elements $m^o_{ijk}$ may therefore be set uniformly equal to unity or any scalar. In this appendix, some form of initial guess is developed which, however, does not improve the final estimates but speeds up the iterative procedure. It also has the advantage of introducing some logic in the choice of the initial values.

On the basis of two faces only, a most probable distribution of the migration flows may be computed analytically. Letting i and j denote the region of origin and destination respectively, and k the age group, then the best estimates of $m_{ijk}$ for given $m_{ij.}$ and $m_{i.k}$ are given by (B3):

$$m_{ijk} = m_{ij.} \cdot m_{i.k} \;/\; m_{i..} \quad . \tag{B3}$$

The expression gives the best estimate possible if only the total migration flow matrix and the age structure of the departures are known. This estimate may be improved by using the information contained in the third face, namely, $m_{.jk}$ or the age composition of arrivals. Define a correction factor $\mu_{jk}$, expressing the effect of the destination region on the age profile of the migrants, and introduce it to (B3) to improve the estimate obtained:

$$\bar{m}_{ijk} = \mu_{jk} \cdot m_{ij.} \cdot m_{i.k} / m_{i..} \quad . \tag{C1}$$

The correction factor may be derived as follows:

$$\sum_i \bar{m}_{ijk} = m_{.jk} = \mu_{jk} \cdot \sum_i m_{ij.} \cdot m_{i.k} / m_{i..}$$

$$\mu_{jk} = \frac{m_{.jk}}{\sum_i m_{ij.} \cdot m_{i.k} / m_{i..}} \quad . \tag{C2}$$

The correction factor assures that the arrivals or inmigration flows have the required profile. To satisfy the other marginal conditions $m_{ij.}$ and $m_{i.k}$, the multiproportional adjustment method is needed.

In the model (C2), the migration flow $m_{ijk}$ is the product of three effects: a total flow effect, an arrival age-profile effect, and a departure age-profile effect.

## APPENDIX D: MULTIDIMENSIONAL ENTROPY (3F) ESTIMATION OF MIGRATION FLOWS BY AGE (SWEDEN, EIGHT REGIONS, 1974)*

Table D1. Input data.

Table D2. (3F) estimates.

Table D3. Error analysis of (3F) migration estimates.

Legend: Region
1 - Stockholm
2 - East Middle
3 - South Middle
4 - South
5 - West
6 - North Middle
7 - Lower Middle
8 - Upper North

*The data for this analysis were kindly provided by Mr. A. Arvidsson of the Central Bureau of Statistics, Stockholm, and studied, as part of the IIASA Comparative Migration and Settlement Study, by Professor Å. Andersson and Dr. I. Holmberg.

# Table D1. Input data.

## a. Migration flow matrix of total population

| to \ from | region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 | total |
|---|---|---|---|---|---|---|---|---|---|
| region 1 | 0. | 8870. | 2311. | 2870. | 3266. | 3448. | 2148. | 2270. | 25183. |
| region 2 | 10227. | 0. | 2953. | 2155. | 3502. | 4528. | 1349. | 1741. | 26455. |
| region 3 | 2728. | 3005. | 0. | 3209. | 2935. | 650. | 270. | 375. | 13172. |
| region 4 | 3458. | 2885. | 3631. | 0. | 3676. | 732. | 437. | 461. | 15280. |
| region 5 | 3320. | 4484. | 3616. | 3416. | 0. | 2676. | 779. | 947. | 19238. |
| region 6 | 4824. | 5205. | 734. | 871. | 2494. | 0. | 1026. | 795. | 15949. |
| region 7 | 2659. | 1635. | 238. | 369. | 828. | 1128. | 0. | 1225. | 8082. |
| region 8 | 2404. | 1970. | 408. | 524. | 1063. | 873. | 1214. | 0. | 8456. |
| total | 29620. | 28054. | 13891. | 13414. | 17764. | 14035. | 7223. | 7814. | 131815. |

## b. Departures and arrivals by region and age

| age | region 1 arrivals | region 1 depart. | region 2 arrivals | region 2 depart. | region 3 arrivals | region 3 depart. | region 4 arrivals | region 4 depart. | region 5 arrivals | region 5 depart. | region 6 arrivals | region 6 depart. | region 7 arrivals | region 7 depart. | age | region 8 arrivals | region 8 depart. | total arrivals | total depart. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 2122. | 3749. | 3108. | 3222. | 1590. | 1486. | 1546. | 1519. | 2134. | 2088. | 2103. | 1355. | 1031. | 671. | 0 | 1162. | 706. | 14796. | 14796. |
| 5 | 1563. | 2543. | 2127. | 2282. | 1154. | 1029. | 1121. | 1003. | 1418. | 1451. | 1332. | 977. | 695. | 465. | 5 | 777. | 437. | 10187. | 10187. |
| 10 | 930. | 1435. | 1179. | 1290. | 656. | 602. | 745. | 550. | 853. | 833. | 733. | 595. | 421. | 273. | 10 | 349. | 288. | 5866. | 5866. |
| 15 | 3058. | 1801. | 2152. | 2325. | 1049. | 1405. | 1357. | 1008. | 1659. | 1306. | 1139. | 1597. | 515. | 924. | 15 | 543. | 1106. | 11472. | 11472. |
| 20 | 6419. | 4349. | 5246. | 5486. | 2297. | 3276. | 2924. | 2707. | 4073. | 3528. | 2619. | 3359. | 1391. | 1844. | 20 | 1566. | 1986. | 26535. | 26535. |
| 25 | 4602. | 5609. | 4835. | 5300. | 2324. | 2387. | 2514. | 2771. | 3672. | 3284. | 2965. | 2368. | 1513. | 1188. | 25 | 1795. | 1313. | 24220. | 24220. |
| 30 | 2110. | 3097. | 2476. | 2640. | 1327. | 1142. | 1416. | 1397. | 1793. | 1836. | 1523. | 1124. | 830. | 580. | 30 | 941. | 606. | 12416. | 12416. |
| 35 | 1107. | 1663. | 1325. | 1459. | 663. | 645. | 875. | 701. | 981. | 948. | 853. | 641. | 431. | 317. | 35 | 462. | 323. | 6697. | 6697. |
| 40 | 748. | 1044. | 820. | 944. | 427. | 473. | 597. | 415. | 649. | 555. | 538. | 440. | 308. | 196. | 40 | 231. | 251. | 4318. | 4318. |
| 45 | 622. | 899. | 722. | 737. | 378. | 357. | 444. | 317. | 489. | 472. | 447. | 377. | 219. | 164. | 45 | 189. | 187. | 3510. | 3510. |
| 50 | 516. | 806. | 631. | 627. | 336. | 275. | 412. | 298. | 387. | 397. | 441. | 331. | 197. | 151. | 50 | 138. | 173. | 3058. | 3058. |
| 55 | 352. | 704. | 499. | 490. | 250. | 206. | 348. | 204. | 311. | 281. | 335. | 219. | 163. | 120. | 55 | 94. | 128. | 2352. | 2352. |
| 60 | 327. | 820. | 516. | 494. | 282. | 220. | 409. | 178. | 345. | 310. | 377. | 233. | 174. | 134. | 60 | 93. | 134. | 2523. | 2523. |
| 65 | 338. | 603. | 421. | 390. | 220. | 176. | 286. | 155. | 244. | 252. | 311. | 210. | 95. | 92. | 65 | 61. | 98. | 1976. | 1976. |
| 70 | 170. | 251. | 207. | 190. | 120. | 112. | 165. | 90. | 118. | 122. | 122. | 111. | 49. | 60. | 70 | 26. | 41. | 977. | 977. |
| 75 | 111. | 147. | 105. | 113. | 59. | 57. | 69. | 62. | 74. | 58. | 58. | 53. | 33. | 24. | 75 | 22. | 17. | 531. | 531. |
| 80 | 81. | 99. | 85. | 64. | 40. | 42. | 52. | 38. | 38. | 48. | 53. | 44. | 17. | 19. | 80 | 7. | 19. | 373. | 373. |
| 85 | 7. | 1. | 1. | 1. | 0. | 1. | 0. | 1. | 0. | 1. | 0. | 1. | 0. | 1. | 85 | 0. | 1. | 8. | 8. |
| total | 25183. | 29620. | 26455. | 28054. | 13172. | 13891. | 15280. | 13414. | 19238. | 17764. | 15949. | 14035. | 8082. | 7223. | total | 8456. | 7814. | 131815. | 131815. |

# Table D2. (3F) estimates

migration from region 1 to

| | total | region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 3749. | 0. | 1289. | 339. | 362. | 396. | 653. | 353. | 351. |
| | | 0- | -1356- | -330- | -343- | -382- | -657- | -356- | -325- |
| 5 | 2543. | 0. | 884. | 247. | 261. | 263. | 412. | 243. | 233. |
| | | 0- | -920- | -261- | -267- | -257- | -397- | -230- | -211- |
| 10 | 1435. | 0. | 484. | 142. | 174. | 159. | 228. | 145. | 104. |
| | | 0- | -476- | -153- | -176- | -165- | -234- | -155- | -76- |
| 15 | 1801. | 0. | 635. | 184. | 241. | 215. | 280. | 125. | 120. |
| | | 0- | -577- | -190- | -250- | -200- | -265- | -166- | -153- |
| 20 | 4349. | 0. | 1590. | 379. | 507. | 534. | 635. | 355. | 349. |
| | | 0- | -1512- | -354- | -460- | -538- | -650- | -356- | -479- |
| 25 | 5609. | 0. | 1945. | 476. | 587. | 657. | 905. | 510. | 530. |
| | | 0- | -1897- | -459- | -592- | -680- | -918- | -522- | -541- |
| 30 | 3097. | 0. | 1046. | 283. | 347. | 347. | 486. | 296. | 292. |
| | | 0- | -1067- | -271- | -372- | -351- | -498- | -267- | -271- |
| 35 | 1663. | 0. | 556. | 143. | 214. | 187. | 270. | 152. | 141. |
| | | 0- | -593- | -153- | -211- | -177- | -246- | -155- | -128- |
| 40 | 1044. | 0. | 338. | 95. | 143. | 120. | 171. | 106. | 71. |
| | | 0- | -354- | -109- | -128- | -123- | -162- | -109- | -59- |
| 45 | 899. | 0. | 309. | 90. | 113. | 98. | 151. | 79. | 61. |
| | | 0- | -321- | -96- | -115- | -100- | -147- | -74- | -46- |
| 50 | 806. | 0. | 271. | 80. | 110. | 78. | 151. | 71. | 45. |
| | | 0- | -291- | -66- | -108- | -92- | -151- | -66- | -32- |
| 55 | 704. | 0. | 239. | 67. | 104. | 71. | 124. | 66. | 33. |
| | | 0- | -242- | -67- | -135- | -65- | -112- | -62- | -21- |
| 60 | 820. | 0. | 259. | 84. | 130. | 88. | 150. | 74. | 35. |
| | | 0- | -265- | -91- | -135- | -79- | -163- | -65- | -22- |
| 65 | 603. | 0. | 205. | 64. | 89. | 60. | 124. | 40. | 23. |
| | | 0- | -188- | -65- | -94- | -54- | -141- | -35- | -26- |
| 70 | 251. | 0. | 87. | 29. | 43. | 23. | 42. | 18. | 8. |
| | | 0- | -80- | -29- | -37- | -33- | -44- | -19- | -9- |
| 75 | 147. | 0. | 50. | 16. | 20. | 16. | 22. | 14. | 8. |
| | | 0- | -52- | -20- | -19- | -16- | -22- | -15- | -3- |
| 80 | 99. | 0. | 37. | 10. | 14. | 8. | 20. | 7. | 2. |
| | | 0- | -35- | -14- | -16- | -8- | -17- | -7- | -2- |
| 85 | 1. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. |
| | | 0- | -1- | 0- | 0- | 0- | 0- | 0- | 0- |
| total | 29620. | 0. | 10227. | 2728. | 3458. | 3320. | 4824. | 2659. | 2404. |

migration from region 2 to

| | total | region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 3222. | 813. | 0. | 372. | 302. | 528. | 705. | 220. | 282. |
| | | -814- | 0- | -371- | -331- | -524- | -679- | -228- | -275- |
| 5 | 2282. | 614. | 0. | 279. | 224. | 361. | 458. | 153. | 193. |
| | | -613- | 0- | -264- | -238- | -357- | -440- | -161- | -209- |
| 10 | 1290. | 355. | 0. | 157. | 146. | 212. | 247. | 89. | 84. |
| | | -357- | 0- | -135- | -151- | -214- | -252- | -76- | -105- |
| 15 | 2325. | 928. | 0. | 243. | 241. | 343. | 363. | 92. | 116. |
| | | -898- | 0- | -235- | -234- | -336- | -419- | -93- | -110- |
| 20 | 5486. | 2067. | 0. | 521. | 529. | 889. | 857. | 272. | 351. |
| | | -2054- | 0- | -532- | -513- | -920- | -865- | -262- | -340- |
| 25 | 5300. | 1698. | 0. | 522. | 489. | 875. | 977. | 312. | 426. |
| | | -1718- | 0- | -553- | -505- | -849- | -953- | -330- | -392- |
| 30 | 2640. | 781. | 0. | 289. | 269. | 429. | 487. | 168. | 218. |
| | | -795- | 0- | -309- | -244- | -437- | -480- | -173- | -202- |
| 35 | 1459. | 418. | 0. | 151. | 171. | 239. | 281. | 90. | 109. |
| | | -419- | 0- | -144- | -178- | -231- | -285- | -85- | -117- |
| 40 | 944. | 279. | 0. | 100. | 115. | 154. | 178. | 62. | 55. |
| | | -264- | 0- | -93- | -107- | -172- | -183- | -60- | -65- |
| 45 | 737. | 228. | 0. | 86. | 82. | 114. | 142. | 42. | 43. |
| | | -230- | 0- | -83- | -97- | -124- | -128- | -37- | -38- |
| 50 | 627. | 184. | 0. | 74. | 77. | 87. | 137. | 37. | 30. |
| | | -192- | 0- | -79- | -70- | -77- | -133- | -45- | -31- |
| 55 | 490. | 135. | 0. | 57. | 67. | 74. | 105. | 31. | 21. |
| | | -136- | 0- | -55- | -59- | -65- | -121- | -31- | -23- |
| 60 | 494. | 121. | 0. | 62. | 73. | 79. | 109. | 31. | 19. |
| | | -118- | 0- | -57- | -70- | -81- | -105- | -32- | -31- |
| 65 | 390. | 122. | 0. | 47. | 50. | 53. | 90. | 16. | 12. |
| | | -124- | 0- | -49- | -50- | -50- | -89- | -12- | -16- |
| 70 | 190. | 58. | 0. | 26. | 29. | 25. | 37. | 9. | 5. |
| | | -67- | 0- | -29- | -22- | -21- | -39- | -7- | -5- |
| 75 | 113. | 42. | 0. | 13. | 13. | 16. | 18. | 6. | 5. |
| | | -49- | 0- | -11- | -10- | -18- | -13- | -3- | -9- |
| 80 | 64. | 25. | 0. | 7. | 8. | 7. | 14. | 3. | 1. |
| | | -21- | 0- | -6- | -6- | -8- | -21- | 0- | -2- |
| 85 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| | | -1- | 0- | 0- | 0- | 0- | 0- | 0- | 0- |
| total | 28054. | 8870. | 0. | 3005. | 2885. | 4484. | 5205. | 1635. | 1970. |

- . - : observed flow

Table D2 continued.

| | total | migration from region 3 to region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1486. | 196. | 348. | 0. | 359. | 401. | 95. | 31. | 56. |
| | | -200- | -323- | 0- | -350- | -425- | -96- | -36- | -56- |
| 5 | 1029. | 144. | 240. | 0. | 260. | 267. | 60. | 21. | 37. |
| | | -152- | -246- | 0- | -228- | -282- | -78- | -10- | -33- |
| 10 | 602. | 84. | 129. | 0. | 170. | 158. | 33. | 12. | 16. |
| | | -105- | -112- | 0- | -174- | -150- | -32- | -10- | -19- |
| 15 | 1405. | 296. | 272. | 0. | 380. | 345. | 65. | 17. | 30. |
| | | -267- | -304- | 0- | -396- | -344- | -49- | -16- | -29- |
| 20 | 3276. | 636. | 686. | 0. | 804. | 864. | 147. | 49. | 89. |
| | | -617- | -708- | 0- | -858- | -820- | -141- | -55- | -77- |
| 25 | 2387. | 400. | 514. | 0. | 569. | 651. | 129. | 43. | 82. |
| | | -414- | -522- | 0- | -565- | -647- | -107- | -54- | -78- |
| 30 | 1142. | 175. | 244. | 0. | 298. | 303. | 61. | 22. | 40. |
| | | -172- | -241- | 0- | -286- | -307- | -74- | -20- | -42- |
| 35 | 645. | 92. | 133. | 0. | 187. | 167. | 35. | 11. | 20. |
| | | -96- | -113- | 0- | -195- | -176- | -36- | -7- | -22- |
| 40 | 473. | 70. | 92. | 0. | 143. | 122. | 25. | 9. | 11. |
| | | -70- | -81- | 0- | -154- | -120- | -31- | -3- | -14- |
| 45 | 357. | 57. | 75. | 0. | 101. | 89. | 20. | 6. | 9. |
| | | -64- | -75- | 0- | -91- | -83- | -21- | -7- | -16- |
| 50 | 275. | 42. | 58. | 0. | 86. | 62. | 17. | 5. | 6. |
| | | -41- | -55- | 0- | -83- | -59- | -20- | -5- | -12- |
| 55 | 206. | 28. | 43. | 0. | 68. | 48. | 12. | 4. | 4. |
| | | -26- | -42- | 0- | -58- | -66- | -7- | -3- | -4- |
| 60 | 220. | 26. | 42. | 0. | 78. | 54. | 13. | 4. | 3. |
| | | -28- | -50- | 0- | -68- | -54- | -15- | -1- | -4- |
| 65 | 176. | 28. | 36. | 0. | 57. | 39. | 12. | 2. | 2. |
| | | -25- | -37- | 0- | -47- | -48- | -15- | -4- | 0- |
| 70 | 112. | 17. | 23. | 0. | 41. | 23. | 6. | 2. | 1. |
| | | -15- | -24- | 0- | -45- | -16- | -8- | -3- | -1- |
| 75 | 57. | 11. | 11. | 0. | 17. | 13. | 3. | 1. | 1. |
| | | -10- | -11- | 0- | -17- | -12- | -3- | -3- | -1- |
| 80 | 42. | 9. | 9. | 0. | 13. | 7. | 3. | 1. | 0. |
| | | -8- | -9- | 0- | -16- | -7- | -1- | -1- | 0- |
| 85 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| | | -1- | 0- | 0- | 0- | 0- | 0- | 0- | 0- |
| total | 13891. | 2311. | 2953. | 0. | 3631. | 3616. | 734. | 238. | 408. |

| | total | migration from region 4 to region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1519. | 253. | 259. | 385. | 0. | 387. | 114. | 48. | 72. |
| | | -274- | -247- | -372- | 0- | -393- | -123- | -37- | -73- |
| 5 | 1003. | 177. | 169. | 268. | 0. | 245. | 69. | 31. | 45. |
| | | -199- | -160- | -261- | 0- | -232- | -73- | -34- | -44- |
| 10 | 550. | 100. | 89. | 147. | 0. | 141. | 36. | 18. | 19. |
| | | -101- | -88- | -158- | 0- | -123- | -38- | -23- | -19- |
| 15 | 1008. | 276. | 146. | 241. | 0. | 241. | 56. | 19. | 28. |
| | | -222- | -169- | -257- | 0- | -255- | -49- | -27- | -29- |
| 20 | 2707. | 690. | 429. | 579. | 0. | 700. | 149. | 64. | 96. |
| | | -615- | -441- | -658- | 0- | -685- | -147- | -68- | -93- |
| 25 | 2771. | 600. | 445. | 615. | 0. | 730. | 180. | 77. | 123. |
| | | -618- | -442- | -582- | 0- | -758- | -185- | -59- | -127- |
| 30 | 1397. | 278. | 223. | 342. | 0. | 359. | 90. | 42. | 63. |
| | | -305- | -196- | -335- | 0- | -350- | -92- | -48- | -71- |
| 35 | 701. | 138. | 114. | 165. | 0. | 186. | 48. | 21. | 29. |
| | | -155- | -114- | -153- | 0- | -192- | -43- | -22- | -22- |
| 40 | 415. | 84. | 64. | 101. | 0. | 110. | 28. | 13. | 14. |
| | | -98- | -70- | -82- | 0- | -103- | -28- | -19- | -15- |
| 45 | 317. | 66. | 50. | 83. | 0. | 78. | 21. | 9. | 10. |
| | | -61- | -54- | -82- | 0- | -69- | -29- | -10- | -12- |
| 50 | 298. | 61. | 49. | 81. | 0. | 68. | 24. | 9. | 8. |
| | | -69- | -41- | -84- | 0- | -67- | -22- | -7- | -8- |
| 55 | 204. | 38. | 34. | 55. | 0. | 50. | 16. | 6. | 5. |
| | | -46- | -33- | -51- | 0- | -52- | -10- | -5- | -7- |
| 60 | 178. | 30. | 28. | 51. | 0. | 46. | 14. | 5. | 4. |
| | | -24- | -36- | -48- | 0- | -56- | -8- | -3- | -3- |
| 65 | 155. | 34. | 25. | 43. | 0. | 35. | 13. | 3. | 3. |
| | | -35- | -27- | -40- | 0- | -39- | -11- | -2- | -1- |
| 70 | 90. | 19. | 15. | 28. | 0. | 19. | 6. | 2. | 1. |
| | | -14- | -22- | -24- | 0- | -20- | -7- | -3- | 0- |
| 75 | 62. | 16. | 9. | 16. | 0. | 14. | 4. | 2. | 1. |
| | | -17- | -10- | -14- | 0- | -15- | -5- | -1- | 0- |
| 80 | 38. | 10. | 6. | 10. | 0. | 7. | 3. | 1. | 0. |
| | | -16- | -5- | -8- | 0- | -7- | -1- | -1- | 0- |
| 85 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| | | -1- | 0- | 0- | 0- | 0- | 0- | 0- | 0- |
| total | 13414. | 2870. | 2155. | 3209. | 0. | 3416. | 871. | 369. | 524. |

Table D2 continued.

| | total | migration from region 5 to region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 2088. | 299. | 437. | 364. | 386. | 0. | 339. | 111. | 152. |
| | | -314- | -433- | -359- | -374- | 0- | -332- | -120- | -156- |
| 5 | 1451. | 219. | 299. | 265. | 278. | 0. | 213. | 75. | 100. |
| | | -229- | -284- | -265- | -274- | 0- | -221- | -84- | -94- |
| 10 | 833. | 129. | 162. | 151. | 184. | 0. | 117. | 45. | 45. |
| | | -157- | -161- | -142- | -171- | 0- | -106- | -43- | -53- |
| 15 | 1306. | 312. | 236. | 217. | 282. | 0. | 159. | 43. | 57. |
| | | -247- | -242- | -233- | -283- | 0- | -184- | -42- | -75- |
| 20 | 3528. | 799. | 708. | 536. | 710. | 0. | 432. | 145. | 198. |
| | | -705- | -715- | -571- | -748- | 0- | -445- | -155- | -189- |
| 25 | 3284. | 626. | 660. | 512. | 627. | 0. | 470. | 159. | 229. |
| | | -641- | -670- | -515- | -592- | 0- | -496- | -146- | -224- |
| 30 | 1830. | 314. | 358. | 308. | 375. | 0. | 255. | 93. | 127. |
| | | -337- | -366- | -296- | -372- | 0- | -231- | -110- | -118- |
| 35 | 948. | 155. | 183. | 149. | 221. | 0. | 136. | 46. | 59. |
| | | -175- | -194- | -138- | -209- | 0- | -133- | -40- | -59- |
| 40 | 555. | 94. | 101. | 90. | 135. | 0. | 78. | 29. | 27. |
| | | -117- | -88- | -84- | -146- | 0- | -75- | -20- | -25- |
| 45 | 472. | 83. | 90. | 83. | 104. | 0. | 68. | 21. | 23. |
| | | -109- | -82- | -71- | -93- | 0- | -70- | -20- | -27- |
| 50 | 397. | 65. | 74. | 69. | 94. | 0. | 63. | 18. | 15. |
| | | -59- | -77- | -75- | -101- | 0- | -55- | -14- | -16- |
| 55 | 281. | 42. | 53. | 48. | 73. | 0. | 42. | 13. | 9. |
| | | -38- | -59- | -47- | -74- | 0- | -43- | -10- | -10- |
| 60 | 310. | 41. | 54. | 56. | 86. | 0. | 48. | 14. | 9. |
| | | -53- | -47- | -53- | -92- | 0- | -42- | -12- | -11- |
| 65 | 252. | 44. | 45. | 45. | 62. | 0. | 42. | 8. | 6. |
| | | -35- | -48- | -45- | -75- | 0- | -37- | -7- | -5- |
| 70 | 122. | 20. | 22. | 23. | 34. | 0. | 16. | 4. | 3. |
| | | -22- | -20- | -23- | -45- | 0- | -9- | -3- | 0- |
| 75 | 58. | 13. | 10. | 10. | 13. | 0. | 7. | 3. | 2. |
| | | -13- | -9- | -9- | -16- | 0- | -9- | -1- | -1- |
| 80 | 48. | 10. | 9. | 8. | 11. | 0. | 8. | 1. | 1. |
| | | -14- | -7- | -9- | -11- | 0- | -6- | -1- | 0- |
| 85 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| | | -1- | 0- | 0- | 0- | 0- | 0- | 0- | 0- |
| total | 17764. | 3266. | 3502. | 2935. | 3676. | 0. | 2494. | 828. | 1063. |

| | total | migration from region 6 to region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1355. | 256. | 471. | 67. | 63. | 262. | 0. | 129. | 107. |
| | | -237- | -449- | -76- | -67- | -256- | 0- | -135- | -135- |
| 5 | 977. | 195. | 337. | 51. | 48. | 181. | 0. | 91. | 74. |
| | | -175- | -320- | -66- | -59- | -187- | 0- | -85- | -85- |
| 10 | 595. | 123. | 197. | 31. | 34. | 117. | 0. | 58. | 35. |
| | | -108- | -215- | -33- | -32- | -119- | 0- | -52- | -36- |
| 15 | 1597. | 500. | 476. | 75. | 87. | 292. | 0. | 93. | 75. |
| | | -530- | -500- | -54- | -89- | -284- | 0- | -93- | -47- |
| 20 | 3359. | 942. | 1054. | 136. | 161. | 641. | 0. | 232. | 193. |
| | | -1018- | -1081- | -81- | -136- | -673- | 0- | -221- | -149- |
| 25 | 2368. | 569. | 758. | 100. | 109. | 464. | 0. | 196. | 172. |
| | | -581- | -769- | -117- | -101- | -453- | 0- | -181- | -166- |
| 30 | 1124. | 250. | 361. | 53. | 57. | 217. | 0. | 101. | 84. |
| | | -228- | -349- | -53- | -71- | -212- | 0- | -104- | -107- |
| 35 | 641. | 140. | 208. | 29. | 38. | 126. | 0. | 56. | 44. |
| | | -130- | -186- | -41- | -41- | -125- | 0- | -64- | -54- |
| 40 | 440. | 101. | 137. | 21. | 28. | 88. | 0. | 42. | 24. |
| | | -93- | -137- | -33- | -31- | -84- | 0- | -40- | -22- |
| 45 | 377. | 90. | 124. | 19. | 22. | 71. | 0. | 31. | 20. |
| | | -85- | -122- | -24- | -26- | -67- | 0- | -32- | -21- |
| 50 | 331. | 78. | 112. | 18. | 22. | 58. | 0. | 29. | 15. |
| | | -82- | -108- | -13- | -27- | -53- | 0- | -32- | -16- |
| 55 | 219. | 47. | 75. | 11. | 16. | 41. | 0. | 20. | 9. |
| | | -41- | -75- | -15- | -6- | -41- | 0- | -24- | -17- |
| 60 | 233. | 46. | 77. | 14. | 18. | 47. | 0. | 22. | 9. |
| | | -41- | -66- | -21- | -22- | -47- | 0- | -30- | -6- |
| 65 | 210. | 54. | 71. | 12. | 15. | 38. | 0. | 14. | 7. |
| | | -49- | -77- | -11- | -13- | -41- | 0- | -15- | -4- |
| 70 | 111. | 27. | 38. | 7. | 9. | 18. | 0. | 8. | 3. |
| | | -24- | -41- | -7- | -7- | -19- | 0- | -8- | -5- |
| 75 | 53. | 15. | 16. | 3. | 3. | 9. | 0. | 4. | 2. |
| | | -14- | -12- | -3- | -3- | -11- | 0- | -9- | -1- |
| 80 | 44. | 14. | 15. | 2. | 3. | 6. | 0. | 3. | 1. |
| | | -11- | -21- | -2- | -1- | -4- | 0- | -3- | -2- |
| 85 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| | | -1- | 0- | 0- | 0- | 0- | 0- | 0- | 0- |
| total | 14035. | 3448. | 4528. | 650. | 732. | 2676. | 0. | 1128. | 873. |

Table D2 continued.

| | total | migration from region 7 to region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 671. | 150. | 133. | 26. | 36. | 72. | 112. | 0. | 142. |
| | | -149- | -133- | -26- | -44- | -62- | -115- | 0- | -142- |
| 5 | 465. | 111. | 93. | 20. | 26. | 49. | 71. | 0. | 95. |
| | | -111- | -83- | -12- | -28- | -50- | -80- | 0- | -101- |
| 10 | 273. | 70. | 54. | 12. | 19. | 31. | 42. | 0. | 45. |
| | | -55- | -59- | -16- | -17- | -41- | -44- | 0- | -41- |
| 15 | 924. | 344. | 159. | 35. | 58. | 95. | 116. | 0. | 117. |
| | | -363- | -147- | -47- | -53- | -117- | -97- | 0- | -100- |
| 20 | 1844. | 626. | 339. | 61. | 103. | 201. | 224. | 0. | 290. |
| | | -682- | -361- | -48- | -98- | -193- | -223- | 0- | -239- |
| 25 | 1188. | 338. | 218. | 40. | 63. | 130. | 167. | 0. | 232. |
| | | -320- | -228- | -39- | -76- | -113- | -145- | 0- | -267- |
| 30 | 580. | 154. | 107. | 22. | 34. | 63. | 82. | 0. | 117. |
| | | -133- | -116- | -22- | -34- | -58- | -87- | 0- | -130- |
| 35 | 317. | 82. | 59. | 12. | 22. | 35. | 48. | 0. | 59. |
| | | -70- | -53- | -18- | -17- | -42- | -57- | 0- | -60- |
| 40 | 196. | 55. | 36. | 8. | 15. | 23. | 30. | 0. | 30. |
| | | -53- | -28- | -11- | -14- | -24- | -35- | 0- | -31- |
| 45 | 164. | 47. | 32. | 7. | 11. | 18. | 25. | 0. | 24. |
| | | -32- | -27- | -9- | -9- | -28- | -30- | 0- | -29- |
| 50 | 151. | 42. | 30. | 7. | 11. | 15. | 27. | 0. | 19. |
| | | -36- | -27- | -8- | -10- | -15- | -32- | 0- | -23- |
| 55 | 120. | 32. | 25. | 5. | 10. | 13. | 21. | 0. | 13. |
| | | -44- | -15- | -2- | -10- | -14- | -23- | 0- | -12- |
| 60 | 134. | 33. | 27. | 7. | 13. | 16. | 25. | 0. | 14. |
| | | -36- | -23- | -3- | -15- | -13- | -28- | 0- | -16- |
| 65 | 92. | 28. | 18. | 4. | 7. | 9. | 18. | 0. | 8. |
| | | -37- | -22- | -4- | -4- | -5- | -11- | 0- | -9- |
| 70 | 60. | 18. | 12. | 3. | 6. | 6. | 10. | 0. | 5. |
| | | -16- | -16- | -3- | -6- | -2- | -11- | 0- | -6- |
| 75 | 24. | 9. | 4. | 1. | 2. | 2. | 3. | 0. | 3. |
| | | -3- | -7- | -1- | -2- | -1- | -3- | 0- | -7- |
| 80 | 19. | 7. | 4. | 1. | 1. | 1. | 3. | 0. | 1. |
| | | -7- | -4- | -1- | 0- | -1- | -5- | 0- | -1- |
| 85 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| | | -1- | 0- | 0- | 0- | 0- | 0- | 0- | 0- |
| total | 7223. | 2148. | 1349. | 270. | 437. | 779. | 1026. | 0. | 1214. |

| | total | migration from region 8 to region 1 | region 2 | region 3 | region 4 | region 5 | region 6 | region 7 | region 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 706. | 155. | 171. | 37. | 38. | 87. | 86. | 133. | 0. |
| | | -134- | -167- | -56- | -37- | -92- | -101- | -119- | 0- |
| 5 | 437. | 102. | 105. | 24. | 24. | 52. | 49. | 81. | 0. |
| | | -84- | -114- | -25- | -27- | -53- | -43- | -91- | 0- |
| 10 | 288. | 69. | 65. | 16. | 18. | 35. | 30. | 55. | 0. |
| | | -47- | -68- | -19- | -24- | -41- | -27- | -62- | 0- |
| 15 | 1106. | 401. | 228. | 54. | 68. | 128. | 100. | 126. | 0. |
| | | -531- | -213- | -33- | -52- | -123- | -76- | -78- | 0- |
| 20 | 1986. | 658. | 439. | 85. | 110. | 244. | 174. | 275. | 0. |
| | | -728- | -428- | -53- | -111- | -244- | -148- | -274- | 0- |
| 25 | 1313. | 371. | 295. | 59. | 70. | 165. | 136. | 217. | 0. |
| | | -310- | -307- | -59- | -83- | -172- | -161- | -221- | 0- |
| 30 | 606. | 158. | 136. | 30. | 35. | 75. | 63. | 108. | 0. |
| | | -140- | -141- | -41- | -37- | -78- | -61- | -108- | 0- |
| 35 | 323. | 82. | 73. | 15. | 22. | 40. | 35. | 56. | 0. |
| | | -62- | -72- | -16- | -24- | -38- | -53- | -58- | 0- |
| 40 | 251. | 65. | 53. | 12. | 18. | 31. | 27. | 46. | 0. |
| | | -53- | -62- | -15- | -17- | -23- | -24- | -57- | 0- |
| 45 | 187. | 51. | 42. | 10. | 12. | 22. | 20. | 30. | 0. |
| | | -41- | -41- | -13- | -13- | -18- | -22- | -39- | 0- |
| 50 | 173. | 45. | 39. | 9. | 12. | 18. | 21. | 29. | 0. |
| | | -37- | -32- | -11- | -13- | -24- | -28- | -28- | 0- |
| 55 | 128. | 30. | 29. | 7. | 10. | 14. | 15. | 22. | 0. |
| | | -21- | -33- | -13- | -6- | -8- | -19- | -28- | 0- |
| 60 | 134. | 29. | 29. | 8. | 11. | 16. | 17. | 23. | 0. |
| | | -27- | -29- | -9- | -7- | -15- | -16- | -31- | 0- |
| 65 | 98. | 28. | 22. | 6. | 7. | 11. | 13. | 12. | 0. |
| | | -33- | -22- | -6- | -3- | -7- | -7- | -20- | 0- |
| 70 | 41. | 11. | 9. | 3. | 4. | 4. | 5. | 6. | 0. |
| | | -12- | -4- | -5- | -3- | -7- | -4- | -6- | 0- |
| 75 | 17. | 6. | 3. | 1. | 1. | 2. | 2. | 3. | 0. |
| | | -5- | -4- | -1- | -2- | -1- | -3- | -1- | 0- |
| 80 | 19. | 6. | 4. | 1. | 1. | 2. | 2. | 2. | 0. |
| | | -4- | -4- | 0- | -2- | -3- | -2- | -4- | 0- |
| 85 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| | | -1- | 0- | 0- | 0- | 0- | 0- | 0- | 0- |
| total | 7814. | 2270. | 1741. | 375. | 461. | 947. | 795. | 1225. | 0. |

## Table D3. Error analysis of (3F) migration estimates.

### a. Analysis by size class (flow volume) and migrant category

| size class | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | cum.abs.% error value | cum.abs.% error -%- | chi-square value | chi-square -%- |
|---|---|---|---|---|---|---|---|---|
| 0- 200 | 747. | 78.63 | 40783. | 30.94 | 14671. | 93.84 | 0.9148e 03 | 72.48 |
| 200- 400 | 121. | 12.74 | 34331. | 26.04 | 616. | 3.94 | 0.1361e 03 | 10.78 |
| 400- 600 | 41. | 4.32 | 20472. | 15.53 | 192. | 1.23 | 0.1319e 03 | 10.45 |
| 600- 800 | 23. | 2.42 | 15711. | 11.92 | 99. | 0.63 | 0.5235e 02 | 4.15 |
| 800-1000 | 10. | 1.05 | 8815. | 6.69 | 30. | 0.19 | 0.1101e 02 | 0.87 |
| 1000-1200 | 3. | 0.32 | 3166. | 2.40 | 12. | 0.08 | 0.7165e 01 | 0.57 |
| 1200-1400 | 1. | 0.11 | 1356. | 1.03 | 5. | 0.03 | 0.3461e 01 | 0.27 |
| 1400-1600 | 1. | 0.11 | 1512. | 1.15 | 5. | 0.03 | 0.3859e 01 | 0.31 |
| 1600-1800 | 1. | 0.11 | 1718. | 1.30 | 1. | 0.01 | 0.2401e 00 | 0.02 |
| 1800-2000 | 1. | 0.11 | 1897. | 1.44 | 3. | 0.02 | 0.1208e 01 | 0.10 |
| 2000+ | 1. | 0.11 | 2054. | 1.56 | 1. | 0.00 | 0.8493e-01 | 0.01 |
| total | 950. | 100.00 | 131815. | 100.00 | 15634. | 100.00 | 0.1262e 04 | 100.00 |

### b. Analysis by error category

| error category | percentage error | number of flows total | number of flows -%- | volume of flows total | volume of flows -%- | average flow |
|---|---|---|---|---|---|---|
| 1 | 0 - 2 | 125. | 13.16 | 31263. | 23.72 | 250.104 |
| 2 | 2 - 4 | 144. | 15.16 | 35914. | 27.25 | 249.403 |
| 3 | 4 - 6 | 112. | 11.79 | 21996. | 16.69 | 196.393 |
| 4 | 6 - 8 | 82. | 8.63 | 12147. | 9.22 | 148.134 |
| 5 | 8 - 10 | 61. | 6.42 | 6176. | 4.69 | 101.246 |
| 6 | 10 - 15 | 128. | 13.47 | 12580. | 9.54 | 98.281 |
| 7 | 15 - 20 | 90. | 9.47 | 4951. | 3.76 | 55.011 |
| 8 | 20 - 30 | 91. | 9.58 | 4925. | 3.74 | 54.121 |
| 9 | 30 - 40 | 38. | 4.00 | 1042. | 0.79 | 27.421 |
| 10 | 40 - 60 | 34. | 3.58 | 395. | 0.30 | 11.618 |
| 11 | 60 - 100 | 24. | 2.53 | 374. | 0.28 | 15.583 |
| 12 | 100 + | 21. | 2.21 | 52. | 0.04 | 2.476 |
| | total | 950. | 100.00 | 131815. | 100.00 | 138.753 |

average absolute percentage error = 6.32
( relative mean deviation )

### c. Analysis by size class and error category

| size class | error category 0- 2 | 2- 4 | 4- 6 | 6- 8 | 8-10 | 10-15 | 15-20 | 20-30 | 30-40 | 40-60 | 60-100 | 100+ | total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0- 200 | 73 | 82 | 72 | 62 | 52 | 114 | 89 | 86 | 38 | 34 | 24 | 21 | 747 |
| 200- 400 | 27 | 33 | 26 | 16 | 7 | 8 | 1 | 3 | 0 | 0 | 0 | 0 | 121 |
| 400- 600 | 12 | 13 | 9 | 2 | 0 | 3 | 0 | 2 | 0 | 0 | 0 | 0 | 41 |
| 600- 800 | 7 | 9 | 2 | 0 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 23 |
| 800-1000 | 3 | 5 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10 |
| 1000-1200 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| 1200-1400 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1400-1600 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1600-1800 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1800-2000 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 2000- | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| total | 125 | 144 | 112 | 82 | 61 | 128 | 90 | 91 | 38 | 34 | 24 | 21 | 950 |

SELECTED PAPERS ON MIGRATION AND SETTLEMENT AT IIASA

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