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A Mathematical Programming Approach to Land Allocation in Regional Planning

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A MATHEMATICAL PROGRAMMING APPROACH TO LAND ALLOCATION IN REGIONAL PLANNING

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ABSTRACT

This paper deals with the land allocation problem of finding a good locational pattern over time for various activities (such as different types of industries, agriculture, housing, and recreation) within a region. A mathematical programming model is formulated to support long-range regional development studies at IIASA concerning the Malmö area (Sweden) and the Silistra region (Bulgaria). Estimates for the total volume of different activities within the region is assumed to be available (e.g., as econometric forecasts or in the framework of central planning). The problem is then to determine subregional development plans in order to meet the estimated volume for the activities, taking into account the initial situation as well as land available in the subregions. As criteria for evaluating alternative development paths we consider investment and operating costs, transportation and other communication costs, as well as some environmental aspects. While determining the investment and operating costs, economies of scale play an important role for certain activities.

Formally, our model is a dynamic multicriteria optimization problem with integer variables and quadratic objective functions (which may be neither convex nor concave). A solution technique is proposed for this problem. The method, which relies heavily on the network flow structure of the set of constraints, is illustrated using a numerical example. Finally, the implementation of a plan is briefly discussed.

A MATHEMATICAL PROGRAMMING APPROACH TO
LAND ALLOCATION IN REGIONAL PLANNING

Ake E. Andersson and Markuu Kallio

1. Introduction

Many disciplines (e.g., theoretical geography, economics, operations research) have attempted to tackle the problem of finding an efficient allocation of land. Numerous approaches are used and their basic features regarding the treatment of space and time dimensions vary. The alternatives are illustrated in the following table.

Table 1. A classification of approaches to the spatial allocation problem.

	Discrete time	Continuous time
Discrete Space	Most mathematical programming approaches (Andersson and La Bella 1979)	Most optimal control models (Isard et al. 1979)
Continuous Space	New urban economics (Mills 1972) Weber models (Cooper 1967, Nijkamp and Paelinck 1975)	Isard's dynamic transportation-location models (Isard et al. 1979) Beckmann-Puu transportation-location models (Beckmann 1953 and Puu forthcoming)

Approaches involving continuous time and/or continuous space have (up until now) proved to be of limited practical value, except for some qualitative analysis. We have chosen a discrete space and time model, which is able to cope with practical complications in generating policy alternatives for two case studies at IIASA: a long-term development study of the Malmö area in southern Sweden and long-range planning of the Silistra region in Bulgaria. We intend to formulate a regional development program as a (dynamic) mathematical programming problem and provide a procedure for finding an optimal solution for such a problem under various criteria.

We first provide an introductory discussion of regional planning. Thereafter, the problem is formulated as a dynamic (nonconvex) quadratic programming problem with integer variables. We develop a solution procedure for our programming problem, based on the theory of optimization over networks, and illustrate this procedure using a numerical example.

2. Characteristics of regional development

In short, the physical aspect of the regional development planning problem may be stated as a problem of finding a suitable trajectory of the locational pattern of various activities in a region. To elaborate on this statement, the following three considerations are taken into account: (i) the current or initial locational pattern of resources within the region, (ii) future expectations (or plans) for the total volume of different types of activities over time, and (iii) the criteria used to evaluate alternative development of locational patterns for these activities. We shall now discuss each of these considerations in some detail.

The current situation may be described by a map indicating the distribution of resources within different subregions. As an example, the regional subdivision and the main road network for Skåne (including the Malmö area) is given in Figure 1. Resources here are understood in general terms: they include both natural resources and various types of capacity. Examples of resources are capacity for industrial, transportation, farming activities,

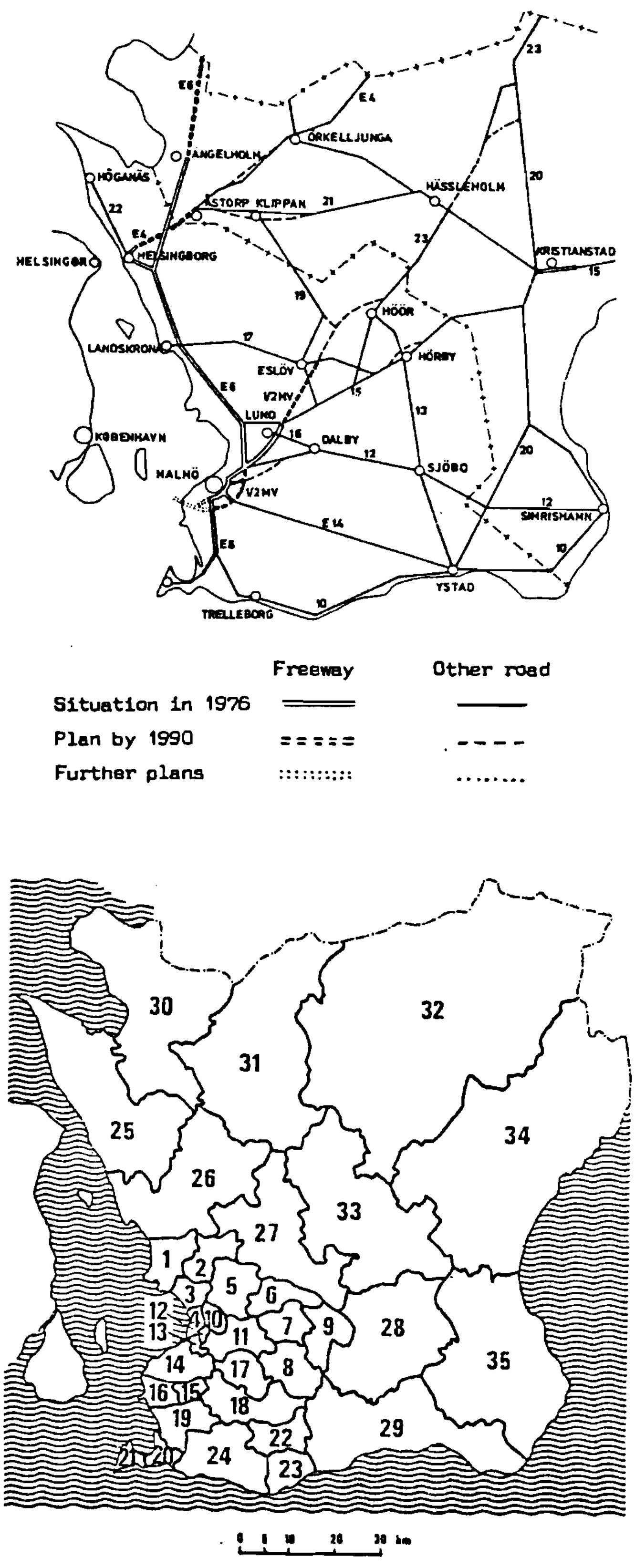

Figure 1. Regional subdivision and the network of main roads in Skåne.

water resources, renewable resources (such as forests), and non-renewable resources (such as mineral deposits).

Resources are used over time for various activities, such as industrial or agricultural production. We shall assume the total volume of such activities is known over the time period under study. Such forecasts may be available in the framework of central planning (as in Bulgaria) or they may be estimated using econometric techniques (as in Sweden), for instance. A feasible locational pattern at a given point of time is one that provides sufficient resources to achieve the estimated activity levels within the region at that time. This may require an increase in some of the resources (such as housing or industrial units) or it may allow a decrease over time (such as use of mineral resources).

In general, there is much freedom in designing feasible patterns: there are alternative locations for most of the activities and, furthermore, certian sites may change activity over time. Then, the following question arises: what are the criteria that we should take into account while comparing alternative feasible locational patterns over time? Clearly a single criterion is insufficient. Over the whole planning horizon, we must simultaneously account for investment costs for changing the capacity for various activities in various subregions over time, the operating cost of the production activities, communication within the region, and environmental problems created by a certain locational pattern. Economies of scale are assumed to play an important role in determining the production costs. Furthermore, the location of a production unit relative to the location of natural resources and other production units may, of course, represent a significant share in operating costs.

3. A planning model

3.1 Feasible allocation of land

Our next task is to formulate the planning problem into a mathematical programming model. We shall first describe the set of alternative location patterns in terms of mathematical relations. Thereafter, we formulate a precise statement of the proposed criteria for evaluating alternative plans.

As indicated above, we adopt a discrete time and discrete space formulation, that is, we consider the planning horizon to be partitioned into T periods ($t = 0,1,2,\ldots, T-1$) and the region partitioned into R subregions ($r = 1,2,\ldots, R$). For instance, each time period t may be five years in length, in which case the planning horizon may consist of three to five such periods. Each region r is associated with a land area L_r and an initial capacity x_{i0}^r for activity i, $i = 1,2,\ldots, I$. Thus, there is an area L_r available for these activities in subregion r during each period t.

We shall denote by x_0 the vector whose components are x_{i0}^r. The allocation of land for different activities i for the first period $t = 0$ is determined by the initial state x_0. For other periods the land use may be altered through investment decisions. Let $y_i^r(t)$ be the increase of capacity i (for activity i) in subregion r during t, let $d_i^r(t)$ be the decrease (demolition), and let $x_i^r(t)$ be the total capacity i in region r at the beginning of period t, for all i, r, and t. In this notation we have, for all t,

$$x(t+1) = x(t) + y(t) - d(t) \quad , \tag{1}$$

where $x(t) \equiv (x_i^r(t))$, $y(t) \equiv (y_i^r(t))$, and $d(t) \equiv (d_i^r(t))$ are nonnegative vectors with $I \times R$ components, and $x(0) = x_0$.

One way of handling economies of scale is to consider a set of indivisible production units only. Assuming that these units correspond to real alternatives, the production cost estimates can be given relatively easily. This approach leads to an integer programming formulation. In particular, for our purposes it is sufficient to consider only one plant size that yields an average production cost close to the minimum possible and yet is a relatively small unit compared with the total capacity increase required. Thus, the vector y(t) indicates that the capacity increases have to be expressed by a nonnegative integer vector.

Notice in equation (1) that no physical deprecation is assumed. Thus the operating cost is assumed to cover the reinvestment cost that is needed to maintain the capacity over period t. For the amount d(t) to be demolished we may have a lower and an upper bound denoted by L(t) and U(t), respectively:

$$L(t) \leq d(t) \leq U(t) \quad . \tag{2}$$

This may be due to initially existing capacity, which ought to be closed down during period t. We shall assume d(t) and x_0 be integer vectors as well, so that x(t) is an integer vector.

Let $z_i(t)$ be the total amount of capacity i at the beginning of period t, and denote $z(t) \equiv (z_i(t))$. Thus we have

$$z_i(t) = \sum_r x_i^r(t) \quad . \tag{3}$$

A minimum requirement for capacities is given by a vector $Z(t) \equiv (Z_i(t))$ corresponding to an estimate of the total volume of activities within the region:

$$z(t) \geq Z(t) \quad . \tag{4}$$

The land availability constraint can approximately be taken into account through the following inequality:

$$\sum_i x_i^r(t) \leq L_r \quad , \quad \text{for all } r \text{ and } t \quad . \tag{5}$$

Although this may seem quite restrictive, it is reasonable to assume in our study that the same amount of land is needed for each unit of various industrial activities. Such a unit is roughly determined by the chosen scale of the production units. For other activities, for which economies of scale are less important, the unit of capacity is determined so that its land requirement is about the same as that for an industrial unit. The purpose of this slightly restrictive assumption is to obtain a network flow formulation, which then greatly simplifies the analysis of our model.

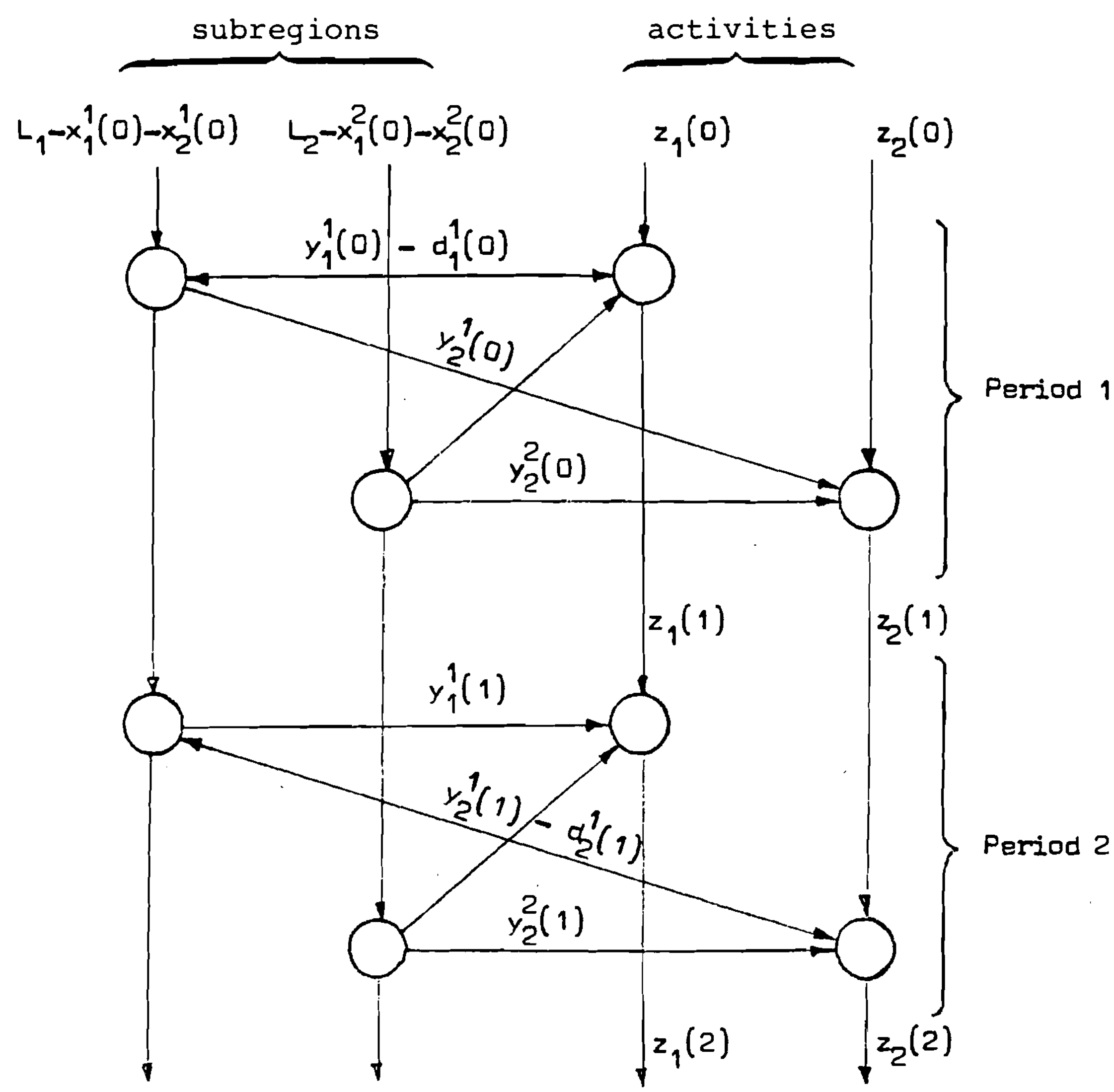

Figure 2. The network structure of a model with $T = I = R = 2$.

Remark: In order to take advantage of the network structure of the model, the subregional capacity levels $x_i^r(t)$ should be suppressed , i.e., one solves $x_i^r(t)$ from (1) and substitutes elsewhere. While doing so, one has to pay special attention to restrict the demolition activities in order to maintain non-negativity for the $x_i^r(t)$ variables. For instance, one may allow demolition only for certain time periods and for some initially existing capacity.

An example of the network structure of our model for a 2-period, 2-region and 2-activity case is given in Figure 2. The nodes on the left refer to the land available and those on the right to the installed capacity. The vertically directed arcs on the left describe unoccupied land and those on the right the capacity carried over from one period to the next (for which we have a lower bound given by equation (4)). The other arcs, which are horizontal and may not be directed, refer to land allocations (for the two activities) or land made available (through demolition). The conservation equations for each node together with the lower bounds, given by equation (4) (for the vertical flows on the right), and the integrality requirements constitute the constraints for a possible land allocation.

3.2 Evaluation criteria

We consider the following decision criteria: (a) investment and demolition costs, operating costs (including transportation of raw material and industrial products), (b) private communication costs (such as commuting, recreation, and leisure time), and environmental considerations including (c) congestion and (d) environmental synergisms. We intend to quantify these considerations as follows.

(a) Investment, demolition, and operating costs. Let $B_i^r(t)$ and $D_i^r(t)$ be the unit cost of investment and demolition, respectively, for capacity i in subregion r during period t. Define $B(t) \equiv (B_i^r(t))$ and $D(t) = (D_i^r(t))$. Then the total investment and demolition cost for period t is given $B(t)y(t) + D(t)d(t)$.

We divide the operating costs into the interaction costs between the activities to be located (such as transportation of goods, communication) and other operating costs (which may also be dependent on the locations of the production units). The costs of interactions between activity i and j located in areas r and s, respectively, will be written as $x_i^r(t)c_{ij}^{rs}(t)x_j^s(t)$, where $x_j^r(t)$ and $x_j^s(t)$ define (as above) the number of units located, and $c_{ij}^{rs}(t)$ is the cost of interaction per unit of activity i on zone r and per unit of activity j on zone s.

Such a formulation of interaction costs was first proposed by Koopmans and Beckmann (1957). Similar formulations have later been developed by Lundquist and Karlquist (1972), Andersson (1974), Snickars (1972), and Los (1978).

We may interpret the interaction cost $c_{ij}^{rs}(t)$ as a potential transportation (communication) cost. The cost $c_{ij}^{rs}(t)$ is then given as a product of the following factors: the frequency of interaction of one unit of activity i with activity j divided by $Z_j(t)$ (the estimated total number of units j), a nondecreasing function of the distance between subregions r and s, and the unit cost of interaction. The product of the first two factors yields an estimate for the number of interactions for one unit of activity i in subregion r with one unit of j in subregion s.

Let $F_i^r(t)$ be the other operating costs for one unit of activity i in subregion r during t. Such a term may, for instance, include interaction costs between the industrial unit and some prelocated sites of interaction (such as a mineral deposit, water supplying area, port of export). If we define a square matrix $C(t) \equiv (c_{ij}^{rs}(t))$ and a vector $F(t) = (F_i^r(t))$, then for period t, the total investment, demolition, and interaction costs, denoted by $I_1(t)$, can be written as

$$I_1(t) = B(t)y(t) + D(t)d(t) + F(t)x(t) + x(t)C(t)x(t) \quad . \tag{6}$$

Alternatively, the interaction costs may be taken through as accessibility concept, which will now be defined. The accessibility A_{ij}^{rs} of a unit j in subregion s for a unit i in subregion r is defined as a product of frequency of interaction of one unit of i with j, and a nonincreasing function of the distance between subregions r and s. Defining a square matrix A as (A_{ij}^{rs}) the total system accessibility is given as $x(t)Ax(t)$. Because a high level of accessibility is desirable, we may replace the interaction cost $x(t)C(t)x(t)$ in equation (6) by the negative of the total system accessibility (possibly multiplied by a positive scalar, since accessibility may not be measured in monetary units).

Both potential transportation (communication) costs and accessibility are of fundamental importance in spatial planning problems. Accessibility has been a dominating concept in the recent development of regional theory. It has been given an axiomatic foundation by Weibull (1976), and our definition above is consistent with his assumptions. Because accessibility adds to the dimensionality of our decision criteria, we shall consider potential transportation costs as a measure of the communication costs.

(b) Private communication costs. We account for private communication costs in a way similar to that of the above. However, a distinction between private and other communication costs is made because these constitute two separate criteria for evaluation in our planning problem.

Private communication costs, denoted by $I_2(t)$, may then be given as

$$I_2(t) = F_p(t)x(t) + x(t)C_p(t)x(t) \quad , \tag{7}$$

where $F_p(t)$ is a vector of unit communication costs between housing and prelocated sites (such as recreation areas, i.e., lakes, rivers, forests, etc.) and $C_p(t)$ is a matrix of potential communication costs of connecting the housing units to other activities to be located. Thus, components of $F_p(t)$, which do not correspond to the housing activities, are defined as equal to zero. Similarly, components of $C_p(t)$ are equal to zero if they do not correspond to interaction with a housing unit.

(c) Congestion. Excessive congestion of activities is the most obvious kind of environmental problem. We measure congestion by capital density allocated by subregions (i.e., congestion at zone r is defined as $\sum_i K_i^r(t)x_i^r(t)/L_r$, where $K_i(t)$ is the capital stock per unit of activity i). Average congestion in a regional system is defined as the weighted sum of the congestion of each subregion. If the ratio of capital stock in subregion r and total capital stock within the region is taken as such weights, then the average congestion, denoted by $I_3(t)$, is written as

$$I_3(t) = \sum_r [\sum_i K_i^r(t) x_i^r(t)]^2 / (K(t) L_r) \qquad (8)$$

$$x(t)G(t)x(t) \quad ,$$

where G(t) is an appropriately defined square matrix and K(t) is the total capital stock.

(d) Environmental synergisms. Environmental problems are normally of a much more complicated and synergistic nature than those described in our congestion cost measure. A unit of heavy industry is, for instance, of little environmental consequence if located together with other heavy industry a considerable distance from housing. On the other hand, if it has to be located close to housing or outdoor recreation, the disturbance can be enormous. Because of the public good nature of pollution, one has to take into account the number of persons affected. An environmental interaction matrix $E = (E_{ij}^{rs})$ would consequently measure the disturbance between different activities i and j located in region r and s, respectively. Naturally, numerical values for the parameters E_{ij}^{rs} may be very difficult to assess. In order to account for the environmental effects at least qualitatively, one might use powers of ten as values for these parameters (e.g., 0.1, 1, 10, 100, etc.). A measure $I_4(t)$ for environmental synergism effects may then be given as

$$I_4(t) = x(t)Ex(t) \quad . \qquad (9)$$

For each of the four criteria c and for each time period t we define a weighting factor $\beta_c(t)$ that accounts for the time preference. Thus, our planning problem becomes a 4-criteria optimization problem, where the criteria I_c are given by

$$I_c = \sum_t \beta_c(t) I_c(t) \quad , \quad \text{for } c = 1,2,3,4 \quad . \qquad (10)$$

We do not propose that a particular multicriteria optimization technique should be used. Rather we suggest that simply nonnegative weights λ_c should be used for the criteria in order to form a linear scalarizing function that would be minimized. In this way a linear approximation for the (negative) utility function g is given as

$$g = \sum_c \lambda_c I_c \quad . \tag{11}$$

Of course, different values for the parameters λ_c may be used in order to generate a set of interesting development alternatives for the region.

3.3 Summary of the model

In summary, the planning problem (P) is to find nonnegative integer vectors x(t), y(t), d(t), and z(t), for all t, to

minimize g in (11)

(P) subject to (1) - (5) and

with the initial state $x(0) = x_0$.

The objective function of this problem is a quadratic form. However, in general this function is not convex. It is easy to see that, for instance, the potential transportation cost matrix C(t) normally is not positive semidefinite. If we have a static 1-activity and 2-zone problem, and the transportation costs are equal to the distances d_{rs} (between subregions r and s), then the potential transportation cost is given as

$$xCx = (x^1, x^2) \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \end{bmatrix} , \tag{12}$$

where the diagonal elements d_{rr} are equal to zero. Clearly, if $d_{rs} > 0$ for $r \neq s$, our matrix C is not positive semidefinite, since for $(x^1, x^2) = (1, -1)$ we have $xCx < 0$. It can be shown that this result holds for multiactivity multizone problems in general (see Snickars 1972). Our planning problem will thus not necessarily have a unique optimum. Instead it is reasonable to expect a number of locational patterns to correspond to local optima, one or more of which are also global optima. This phenomenon is illustrated by a numerical example in Section 5.

4. A solution technique

In this section we consider the network formulation of the problem (P), i.e., we assume that variables $x_i^r(t)$ have been solved from (1), substituted elsewhere, and that their nonnegativity is guaranteed without an explicit consideration. Let x be a vector whose components are our decision variables $y_i^r(t)$, $d_i^r(t)$, and $z_i(t)$, for all i, r and t. Let us denote our objective function in equation (11) by $g = g(x)$ and the set of all nonnegative vectors x satisfying our constraints (1) - (5) by S. In this notation our problem (P) may be restated as finding an integer vector x to

$$(P) \qquad \min_{x \in S} \text{imize } g(x) \quad .$$

Formally, the set S can be described as the set of feasible solutions to a transshipment network as illustrated in Figure 2.

We exploit the fact that every linearized problem (P) (a problem where the objective function of (P) is replaced by a linear function) is a transshipment problem for which very efficient solution techniques exist (see, e.g., Bradley et al. 1977). This is due to the fact that every extreme point of S is an integer solution provided that L_r, $D_j(t)$, and $Z_j(t)$ are integers for all r, j, and t (see, e.g., Dantzig 1963). Thus, while solving the linearized problem, the integrality requirement can be relaxed.

We shall propose the following approach for solving (P):

1° Choose an initial solution $x^0 \in S$, and set the iteration count k to 0.

2° Solve the linearized problem (L):

$$\underset{x \in S}{\text{minimize}} \; \nabla g(x^k) x$$

for an optimal solution $z^k \in S$. Here $\nabla g(x^k)$ denotes the gradient of $g(x)$ at $x = x^k$.

3° Solve the line search problem (Q):
minimize $g(\alpha x^k + (1 - \alpha) z^k)$
for an optimal solution $\alpha^k \in [0,1]$.

4° Stop (i) if $\alpha = 0$, or (ii) if $\min_{i \leq k} g(z^i) - g(\alpha^k x^k + (1-\alpha^k) z^k) < \delta$, where δ is an appropriate tolerance, or (iii) if another appropriate criterion is satisfied (such as computing time). Otherwise replace x^k by $\alpha^k x^k + (1 - \alpha^k) z^k$, k by k + 1, and return to step 2°.

As mentioned above, the linearized problem (L) is a transshipment problem and it can be solved extremely efficiently. For computations, we shall use the code reported in Kallio et al. (1979). The optimal (basic) solution for (L) satisfies the integrality requirements for all varialbes $x_i^r(t)$. Thus, z^k is feasible for (P). We approximate the optimal solution of (P) by the best of the solutions z^k generated by the above procedure. This, of course, may not be an exact solution for (P).

Problem (Q) is a quadratic problem with one variable α and one constraint $0 \leq \alpha \leq 1$. Thus, (Q) is extremely simple. Let (R) be the problem that is obtained by relaxing the integrality requirement on x in (P). Solution x^{k+1} is the best for problem (R) and can be found when moving from x^k in the direction z^k. Thus, the sequence $\{x^k\}$ generated by this procedure is exactly the same as that generated by the Frank-Wolfe method (1956) when applied to problem (R). If x^k converges to an optimal solution x^* for (R) and $\bar{x}$ is optimal for (P), then

$$g(x^*) \leq g(\bar{x}) \leq \min_{i \leq k} g(z^i) \quad , \tag{13}$$

(i.e., $g(x^*)$ is a lower bound on the optimal value of (P)). We may never know $g(x^*)$ but may still be motivated to use the difference of $\min_{i<k} g(z^i)$ and $g(x^k)$ as a stopping criterion, the best feasible values for (P) and (R) found so far. This is illustrated in Figure 3.

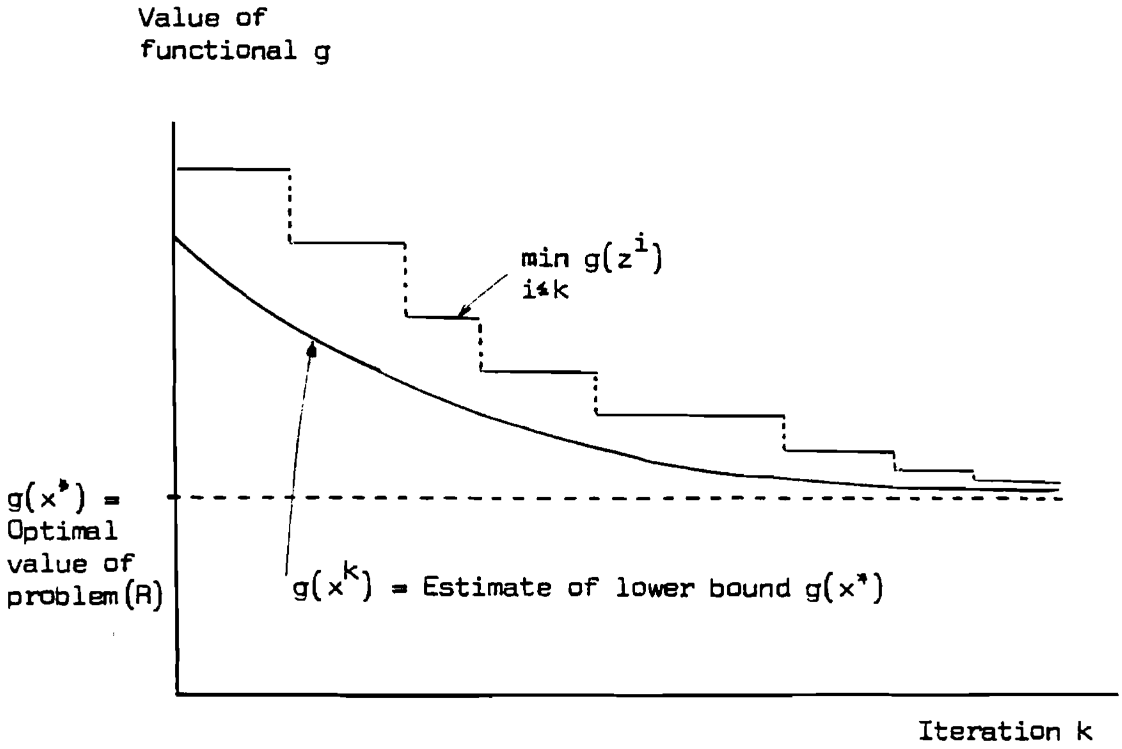

Figure 3. Functional value for Problem (R) (the best one found so far) and for Problem (P) as a function of iteration count k.

If g is a convex function, x^k converges to x^*. Otherwise we may apply the method several times starting with different solutions x^0.

5. A numerical example

As a simple example, we consider a static problem with four regions and four activities. The activities j, their building requirements Z, the regions r, and their land availability L_r are described in Table 2.

Let x_i^r be the number of units i to be located to region r, and denote $x = (x_1^1, x_2^1, x_3^1, \ldots, x_1^4, x_2^4, x_3^4, x_4^4)$. If the investment costs are assumed to be independent of region, they can then be considered as a constant term and thus omitted from further consideration. Our linear term in the objective function shall then consist only of the communication cost between housing and recreation facilities. The linear term is then given as $cx = (0,0,0,54900,0,0,0,45500,0,0,0,32800,0,0,0,39400)x$.

Table 2. An example of land requirements and availability.

j	Activity	Z_j	Region r	L_r
1	agriculture	5	A	1
2	industry	4	B	2
3	service	3	C	5
4	housing	6	D	10

The quadratic term xQx consists of congestion and communication costs, where the matrix $Q = (Q_{ij}^{rs})$ is given as

$$Q_{ij}^{rs} = \begin{cases} \alpha_{ij} d_{rs} + 1/L_r & \text{if } r = s \text{ and } i = j = 4 \,, \\ \alpha_{ij} d_{rs} & \text{otherwise} \,, \end{cases}$$

and

$$(a_{ij}) = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 1 & 5 & 3 & 1 \\ 1 & 4 & 3 & 10 \\ 1 & 4 & 6 & 8 \end{bmatrix} , \qquad (d_{rs}) = \begin{bmatrix} 20 & 30 & 50 & 100 \\ 30 & 30 & 40 & 80 \\ 50 & 40 & 40 & 50 \\ 100 & 80 & 50 & 50 \end{bmatrix} .$$

The objective function appears to be nonconvex. Thus, we ran our solution procedure starting from randomly generated solutions x^0. The procedure was repeated tens of times, each one taking a few seconds in PDP 11 of IIASA. Two local optima were found. Both of these solutions appeared to be equally good, thus yielding to our location problem alternative global optima as conjectured above. The nonzero components of these solutions are given in Table 3.

Table 3. Two local optima of the example.

j\r	A	B	C	D	total
1	1			4	5
2				4	4
3		2		1	3
4			5	1	6
total	1	2	5	10	

j\r	A	B	C	D	total
1	1	2		2	5
2				4	4
3				3	3
4			5	1	6
total	1	2	5	10	

6. Implementation of a plan

A plan generated, for instance, with the aid of our model is of little value if it cannot be implemented. There are essentially four ways of implementing a plan:

a) To leave implementation to the market system without constraints but with charges (rent) for the land use.

b) To use direct central decisions to implement complete investment strategies.

c) To use the planning system to generate zoning constraints for activities and leave the detailed implementation to the market.

d) To use a scheme of negotiations between the allocators of land and the allocators of investment.

Each approach shall now be discussed briefly.

Market implementation. This method consists of determining rental values for land in different zones. Subsequently the decision makers of the sectors would be given a possibility to choose their own preferred location which under the sectorial criteria would yield the desired location pattern. In the following we shall provide some theoretical background on the existence of such rental values.

Consider first a simple case where n activities i are to be located on n available subregions r. Let b_i^r be the net benefit of activity i (excluding the rent for land) given that it has been located on subregion r. Suppose that according to the plan, the locations are determined so that the total net benefit is as large as possible. An optimal plan then results as a solution to the following assignment problem (Dantzig 1963):

$$\text{maximize} \sum_{ir} b_i^r x_i^r$$

$$\text{subject to} \sum_i x_i^r \leq 1 \quad , \tag{14}$$

$$\sum_r x_i^r = 1 \quad , \tag{15}$$

$$x_i \geq 0 \quad .$$

For an optimal basic solution, x_i^r is equal to 1 if activity i is to be located on subregion r and it is zero otherwise.

Let p_r and π_i denote the optimal dual multipliers for constraints (14) and (15), respectively. If according to the (optimal) plan subregion r is assigned to activity i, then $b_i^r - p_r - \pi_i = 0$. We shall interpret p_r as the rent for subregion r. Thus $\pi_i = b_i^r - p_r$ is the profit for activity i. Given the rental values p_r, another location k for i would yield a profit of $\pi_i^k \equiv b_i^k - p_k$. By the optimality condition, $b_i^k - p_k - \pi_i \leq 0$, or $\pi_i^k = b_i^k - p_k \leq \pi_i$, i.e., any other location k for i would yield a profit π_i^k which is no higher than π_i. Thus, profit maximization of each activity separately yields an optimal location pattern under these rental prices.

It is often believed that a decentralized pricing system cannot be used to allocate a resource if there are economies of scale leading to indivisibilities. In fact, it has been shown by Koopmans and Beckmann (1957) for the example above, that decentralized implementation of the optimal solution cannot be achieved in general if the goal function is nonlinear, for example, quadratic. The same is usually true when integrality constraints are superimposed on the system, i.e., when capacity for some activities has to be built in given units of size. In our case, both nonconvexity and the integrality requirement (due to economies of scale) are likely to prevent a market implementation of the plan. The pure market solution to the implementation problem may then be ruled out.

Centralized implementation. Another extreme procedure for implementation is the central decision principle where the plan is enforced by the regional authority. However, this procedure is extremely information-demanding at the level of central planning. A planning model, for instance the one described in this paper, is by numerical necessity of a highly aggregated nature. Such aggregation may rule out a centralized implementation scheme with its requirements of detailed information, i.e., with fine disaggregation into fairly homogeneous branches of industry. One might also argue that it is impossible, or at least uneconomical, to generate very disaggregated technological and administrative data at the central level.

Zoning. One way of using a model for regional planning is as a constraint-generator for more detailed decision making. A compromise between centralized and market implementation is the "zoning principle" according to which central authorities constrain land use for each subregion to fall within an aggregate category of activities leaving all detailed decisions to the market. It is obvious that a planning model can be used to generate such constraints on land use.

Negotiation. Another implementation procedure is a negotiation scheme that also may be seen as a compromise between the pure planning and market approaches. This procedure, however, comes closer to the market implementation. The allocation model may be used to generate a reasonably representative set of pareto optimal locational patterns. These solutions may then be used as reference points in the negotiation between the land allocating authorities and the sectorial decision makers (on investments in new units of production and other activities).

The choice between different implementation approaches cannot be determined objectively but must be decided in an institutional analysis relevant to the region and country.

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