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## PREFACE

Agricultural development policies influence patterns of internal migration and population distribution directly and indirectly. The reverse is also true. Hence, a nation's agricultural development policy should be consistent with its population distribution policy. Accordingly, the policy field to be investigated must be extended to include side effects and secondary consequences in these interdependent spheres.

Methodological research and a number of case studies are envisaged to deal with this problem in the Food and Agriculture Program, in the Human Settlements and Services Area, and in the Regional Development Task. This paper is among the first of such studies.

## CONTENTS

# 1 INTRODUCTION

A growing demand exists for computer-operated, normative models that deal with regional planning problems. (A model is called normative if it enables its user to find a strategy or policy that maximizes a given objective (utility) function.) In Poland, for example, the planned economy requires optimal decisions regarding the allocation of production factors (labor, land, water, capital, etc.) to be made at each regional level. A decentralized regional management and planning system exists for that purpose. Its smallest administrative unit (*gmina*) usually consists of an urban center and its agricultural periphery.

Regional planners are concerned with the allocation of local production factors in order to achieve full employment and the fastest possible rate of regional growth within a given planning interval (in Poland usually 5, 10, or 15 years). Their planning, however, must use factor and output prices that planners at the national level agree on. Also, the production structure must satisfy demand requirements. For that purpose, regional planning units cooperate with higher-level planning organizations and an exchange of information between planning units is necessary.

Employment is the key factor in regional development. It is influenced to some extent by demographic factors and depends a great deal on urbanization processes and migration from rural to urban areas; it also includes a large group of commuters.

The main questions that local planners and decision makers try to answer can be summarized as follows: What is the best regional specialization? Should local resources be used to develop the agricultural sector or the urban sector? What is the best employment and migration pattern? How should one deal with environmental pollution?

In order to construct a model that considers these questions, several extensions of the classical model of a regional economy are needed. In particular, the regional model should be normative and dynamic. Modern economic development is characterized by a continuous process in which an increasing number of small or obsolete farms and factories lose their self-sufficiency while modern farms and factories grow continuously in different locations. Agglomerations create scale economies but congestion and environmental pollution create diseconomies. In order to achieve the maximum economic efficiency and consumer welfare, the optimal allocation of production factors should change over time and over space.

In order to describe the scale economies, as will be shown in section 3, it is necessary to introduce a nonconcave production function and impose nonlinear constraints on the amounts of regional resources. That, in turn, requires a new optimization theory, which is described in section 3. The theory is suited to the existing microeconomic and macroeconomic planning in which planners compare the discounted costs of inputs and outputs for each production unit in order to find the best adjustment of technologies to regional characteristics. The macroeconomic strategy for development may, generally speaking, not coincide with the microeconomic strategies, or with what the consumers think is the best way of development. Sometimes a question is raised: What is the best instrument for allocation of production factors? The market may not be an efficient allocation mechanism. Koopmans and Beckmann (1957) have shown that when technical conditions require land to be used in indivisible amounts for production, a competitive land market is likely to cause an inefficient land use pattern. In planned economies, competitive markets generally do not exist for production factors; planners are responsible for the optimal allocation of production factors. This creates a new problem: What should be done when the planner's strategy for allocation of factors of production, most importantly labor, is not in agreement with what the consumers would like it to be? In other words, is it possible to find a policy dealing with allocation of labor (by migration) that would satisfy the planner's long-term objectives as well as the residential short-term utility?

It is generally believed that administrative restrictions (labor or residence permits) are not the most effective policy instruments in this respect. In many countries, there are social benefit programs for migrants which can be used to influence the migrants' choice of places to live. For example, in Poland the government and local authorities help migrants with housing. The main problem is to find the best migration policy, one that would satisfy both general development objectives and the preferences of migrants. In this paper, such a policy is derived using the cost-benefit approach.

The next important requirement for dynamic normative models is that they should be based on past data, and all exogenous processes should be checked for accuracy with historical runs. The models should be used before making policy, however, to predict the impact of different policies of future regional development.

Equilibrium concepts of classical economics (such as the Arrow-Debreau framework, which requires that a set of local markets exists and that the future trades exist in the present) are of little use in constructing dynamic normative models. Such an approach requires the supply and demand side of the model to be projected forward to the end of the planning horizon. The forecasts are subject to random errors which may cause, generally speaking, random losses in the objective function. The losses represent the price the planner has to pay for the lack of complete information. In a similar way, a retailer may suffer losses when he is planning to set up a shop in a market economy.

This paper should be regarded as a methodological study to determine whether a computerized regional model can be constructed to improve the planning of rural-urban development, employment, and migration policy. The paper may also be regarded as an extension of the theory presented in Kulikowski (1976). In this paper, the production functions are assumed to have an increasing return to scale, while the input resources are constrained by nonlinear integral relations.

In the next section the effect of agglomeration patterns on transportation costs, and production input and output is considered. Optimal strategies for allocation of production factors, with labor exogenous, are derived. The third section deals with allocation strategy in a single sector regional economy, and the fourth section with allocation strategy in a multisector model. The last sections are concerned with optimization of regional development and rural–urban migration policy, assuming that labor supplies in rural and urban areas are exogenous, while migration of labor is endogenous.

## 2 THE EFFECT OF AGGLOMERATION PATTERN ON TRANSPORTATION COST AND PRODUCTION INPUTS AND OUTPUTS

It is well known (see, e.g., Artle and Varaiya 1975, Mills 1972, Rogers 1977) that technological development requires that production factors such as labor, capital, minerals, land, water, etc., be nearby. Thus, the existence of regional characteristics favorable for the allocation of production factors helps to create industrial agglomeration and scale economies. The benefits resulting from scale economies are usually nullified by an increase in input costs connected with growing transportation expenses and an increase in environmental costs from overcrowding. For a planner, it is important to learn how production costs and output depend on the existing spatial allocation of labor and other production factors, and how decisions regarding the future allocation of production factors over space and within the planning interval will affect the production cost and expected benefits.

Since transport cost is one of the main factors affecting inputs, and consequently production costs, we shall start our analysis with typical models of spatial labor allocation that include commuters to a given central business district (CBD).

Consider a circular city with radius $R_1$ and a centrally located CBD. The spatial allocation of labor is described by a density function of the type

$$D(x) = Dx^{-\alpha} , \qquad R_0 \leqslant x \leqslant R_1 , \tag{1}$$

where

$x$ is the distance to the CBD center,
$D, \alpha, R$ are given positive quantities,* $\alpha \neq 2, 3$ .

The number of laborers, $L_1$, commuting to the CBD (with $R_0 = 0$) is

$$L_1 = 2\pi \int_0^{R_1} D(x)x \, dx = \frac{2\pi D}{2-\alpha} R_1^{\,2-\alpha} \tag{2}$$

If we assume that the transportation cost is $c_1$ (per kilometer per person) the total cost $C_1$ of transportation of $L_1$ is

$$C_1 = 2\pi \int_0^{R_1} c_1 x^2 D(x) \, dx = c_1 \frac{2\pi D}{3-\alpha} R_1^{\,3-\alpha} \tag{3}$$

*The first extensive study of population density functions of the exponential type was that by Clark (1951); see also Mills (1972). The parameters of Clark's function were estimated for a large number of cities and a variety of years, showing the steady decrease of gradients of $D$ over time according to the intensities of suburbanization processes.

Eliminating $R_1$ from (2) and (3), one gets

$$C_1 = A_1 L_1^{\gamma_1} , \tag{4}$$

where

$$A_1 = \frac{2\pi c_1 D}{3-\alpha}\left(\frac{2-\alpha}{2\pi D}\right)^{\gamma_1} , \qquad \gamma_1 = \frac{3-\alpha}{2-\alpha} .$$

As the next example, consider a linear city (i.e., a city along a river or road with the CBD located in the center of the segment $[-R_2, R_2]$) and labor density function $D(x)$.

In that case instead of (2) and (3) one obtains for total labor

$$L_2 = 2\int_0^{R_2} D(x)\, dx ,$$

and for labor transportation cost

$$C_2 = 2c_2 \int_0^{R_2} xD(x)\, dx ,$$

where $c_2$ is the transportation per person per kilometer.

Assuming that $D(x)$ is described by (1) with $\alpha \neq 1, 2$, one gets

$$C_2 = A_2 L_2^{\gamma_2} , \tag{5}$$

where

$$A_2 = \frac{2c_2 D}{2-\alpha}\left(\frac{1-\alpha}{2D}\right)^{\gamma_2} , \qquad \gamma_2 = \frac{2-\alpha}{1-\alpha} .$$

Using (4) and (5) it is also possible to construct the relation $C(L)$ for a circular city with $2n$ radial segments each of length $R_2$ km. The suburban transportation system brings the segment commuters to the city borders, where they change to the urban transportation system. The resulting cost function becomes

$$C(L) = C_1(L) + nC_2(L) . \tag{6}$$

Assuming $\alpha = 1/2$, one can see that for $L \leqslant L_1$, $C(L)$ increases along with $L$ according to the function

$$C(L) = A_1{}^{5/3} \ . \tag{7}$$

In other words, one assumes here that the workers start to commute by segment lines when the labor supply within the city is exhausted. Starting from $L = L_1$,

$$C(L) = C_1(L_1) + nA_2L^3 \ , \qquad L \geqslant L_1 \ . \tag{8}$$

When $\alpha \to 0$ (which corresponds to a complete suburbanization), $C(L) = A_1L^{3/2}$ for $L \leqslant L_1$, and $C(L) = A_1(L_1) + nA_2L^2$ for $L \geqslant L_1$.

The main result of this analysis is that the transportation cost in typical monocentric, radially symmetric cities is a convex function of the total labor or amount of goods transported.* It is possible (at least in theory) to derive the cost function for a more complicated urban structure (e.g., the polycentric city with dominant CBD and satellite subcenters) if the population and employment allocation patterns and the transport system are known.

It should also be observed that, knowing the changes over time of $D(x)$ and the transport system parameters, we can derive the cost function $C(L)$ as a function of time.

The next important consideration is the effect of transport cost on CBD labor cost. In the simple model above, one can assume that there is an exogenous (outside of the city) salary $\bar{\omega}_1$. The worker outside of the city will choose to work in the CBD if he is offered a salary $\omega_1$ that is greater than $\bar{\omega}_1$ by at least an amount that will compensate for his traveling expenses. In other words, the CBD labor cost of $X_1$ employees can be assumed to be

$$Y_1 = \omega_1(X_1)X_1 = C_1(X_1) + \bar{\omega}_1X_1 \ . \tag{9}$$

One can derive the numerical values of $\omega(X_1)X_1$ if the salary $\bar{\omega}$, and $C_1(X_1)$ are known.

Now we can turn our attention to the effect of transportation on the factor and production costs. In particular, capital (in the form of equipment) is subject to transportation costs. If a quantity $X_2$ is rented (at the exogenous rent of $\bar{\omega}_2$) in the CBD and delivered to different

*If there is congestion, $C(L)$ increases at an even faster rate than that assumed in (8).

locations with an intensity described by a function similar to (1), we obtain for capital cost, $Y_2$, a relation similar to (9):

$$Y_2 = \omega_2(X_2)X_2 = C_2(X_2) + \bar{\omega}_2 X_2 \ .$$

In a similar way, one can describe the cost of services (e.g., medical, educational, etc.) to the population in the CBD spread over the city with density (1).

The cost of water ($Y_3$) increases along with the amount of water supplied, $X_3$. As shown by O'Luoghaire and Himmelblau (1974), the operational cost of water delivered from a pumping station (assumed to be in the CBD), to users (including industry, agriculture, and households) spread over an area, increases along with $X_3$ faster than a linear function.* Then $Y_3(X_3)$ can be described by a function similar to (9), with $C_3(X_3)$ growing faster than a linear function.

The cost of land, $Y_4(X_4)$, as a function of land area $X_4$, is, however, growing slower than a linear function. In order to show this, it is necessary to recall that the theoretical models closest to von Thünen's approach (see Stevens 1968) imply that the land rent, $r(x)$, must decrease with the distance from the CBD, $x$. If a single commodity is produced in a city, with Cobb-Douglas production function, and if the transportation cost is proportional to $x$, it is possible to show that $r(x) = ax^{-b}$, where $a$ and $b$ are positive numbers. The cost of land, $Y_4$, within the circular area $X_4 = \pi R^2$ then becomes

$$Y_4(X_4) = 2\pi \int_0^R xr(x)\, dx = \frac{2\alpha\pi}{2-b} R^{2-b} = AX_4^{\gamma_4} \ , \qquad (10)$$

where

$$A = \frac{2a\pi^{b/2}}{2-b}, \qquad \gamma_4 = 1 - b/2 \ .$$

Now we can turn to the transportation cost impact on the output, $X$. When the output is produced in the CBD and sold at local shops with intensity proportional to the population density (1), the transportation

*Strictly speaking, the water production system requires capital as an input and water cost includes the rent of the capital component. It is possible to show, however, that the capital cost increases along with the investment size (i.e., the reservoirs, pipe system, etc.) faster than a linear function.

cost, $C(X)$, can be easily derived by using formulae (1)–(8). For the net output one then gets

$$Y(X) = \bar{p}X - C(X)\,, \tag{11}$$

where $\bar{p}$ is regarded here as exogenous. It can be observed that $C(X)$ decreases the output value.

Another factor that decreases output is the pollution treatment cost. Generally speaking, environmental protection costs can be assigned to producers as well as to consumers. For example, the disposal of solid waste generated by the residential sector creates transport costs determined by the population density pattern, (1), which the city assigns to the household owners. The cost of liquid waste disposal has a form similar to the clean water supply cost function already discussed, and is assigned to the polluters. The cost of an abatement policy in air pollution is partly assigned to the consumers (when they are required to install antismog devices in the exhaust systems of their cars) and partly to the producers (when they are required to install antismog filters). That policy is equivalent to the increases in transportation cost in (3) and (5), and increases in the $C(X)$ function in (11). For given production processes and a given pollution level, it is possible to derive the corresponding cost functions. These functions, e.g., (4), increase with output, $X$, but generally faster than a linear function.

It should be mentioned that this paper does not discuss the effect of production costs and output factors connected with agglomeration and congestion (e.g., sociological or environmental). Rather, the transformation of the spatial allocation pattern of production factors into nonlinear cost functions, depending on the level of utilization of these factors, is shown. As shown in the simple model of the monocentric radially symmetric city, the transformation can easily be derived by excluding the space variable $R_1$ from (2), (3), etc. For a given city, it is also possible to approximate the $Y_i(X_i)$ functions using statistical data. As an example, consider land cost (10) which can be written in the form

$$\log[Y_4/\bar{\omega}_4] = \log b_4 + \gamma_4 \log X_4\,, \tag{12}$$

where

$\bar{\omega}_4 = aR_1^{-b}$ (the rent at the city limits $R_1$ kilometers from the CBD),

$$b_4 = \frac{2R_1^{b}\pi^{b/2}}{2-b}\,.$$

We can drop the assumption that the city is monocentric and radially symmetric and regard (12) as an approximation of $Y_4(X_4)$. Then, assuming that we have data $Y_4^t$, $\bar{\omega}_4^t$, $X_4^t$ for past observations $t = 0,-1,-2,\ldots$, we can use linear regression to estimate coefficients $b_4$ and $\gamma_4$.

A similar method can be used to estimate the cost function $[Y_\nu(X_\nu)]$ parameters for the remaining factors. It is necessary to approximate (perhaps by using regression) $Y_\nu(X_\nu)$ by

$$Y_\nu(X_\nu) = \bar{\omega}_\nu b_\nu X_\nu^{\gamma_\nu} , \qquad \nu = 1,\ldots,m , \tag{13}$$

where $\bar{\omega}_\nu$ is the rent outside of the city.

The approximation of net output, for computational convenience, is also assumed to have the form:

$$Y(X) = pb_0 X^{\gamma_0} \qquad (\gamma_0 < 1) \tag{14}$$

It should be observed that models (13) and (14) of $Y_\nu(X_\nu)$ and $Y(X)$ constructed by ex post facto estimation of parameters $b_\nu, \gamma_\nu$ ($\nu = 0,\ldots,m$) are, generally speaking, suited for studying historical processes. One can use them a priori for short-term forecasts only. However, it is sometimes possible to construct forecasts for $X_\nu^t$, $Y_\nu^t$, $\bar{\omega}_\nu^{-t}$, $t > 0$, using more specialized submodels. For example, the demographic model may be used to forecast employment, the national economic model may be used to forecast $\bar{\omega}_\nu$ prices, and Lowry-type models (1964) may be used to forecast transport costs. When these forecasts are available, it is possible to use $Y_\nu^t$, $X_\nu^t$, $\bar{\omega}_\nu$, $t > 0$, as additional data in the regression progress. Such an approach may increase model accuracy when it is used a priori.

## 3 AN OPTIMAL STRATEGY FOR ALLOCATION OF PRODUCTION FACTORS IN A SINGLE SECTOR REGIONAL ECONOMY

Consider a single sector regional economy that is located in the CBD of an agglomeration. The production function of that economy, $X = \Phi[X_1,\ldots,X_m]$, depends on the $m$ production factors, $X_\nu$, representing labor, capital, water, land, and services (e.g., education of the labor force and health). In order to get more specific results, it is assumed that $\Phi$ is a generalized Cobb-Douglas function of the form

$$x(t) = Ae^{\mu t} \prod_{\nu=1}^{m} [x_\nu(t)]^{\beta_\nu} , \qquad \sum_{\nu=1}^{m} \beta_\nu = \beta ,$$

where instead of dealing with the aggregated factors $X_\nu$ (i.e., numbers), the intensity of change in each factor over time (i.e., $x_\nu(t)$) has been introduced. It is assumed that $A$, $\mu$, and $\beta_\nu$ are given positive parameters and that, generally speaking, economies of scale for production may be possible ($\beta \gtrless 1$).

The planner's main concern is to allocate production factors in the given planning period $[0,T]$ ($T$ in many countries is fixed; in Poland $T = 5$, 10, or 15 years) in an optimal fashion. For that purpose the following objective function is used (continuous variables are used here, instead of discrete variables, for convenience rather than general methodology):

$$Y = \int_0^T \overline{W}(t)p(x,t)x(t)\,dt = \int_0^T \overline{W}(t)y(t)\,dt\ , \tag{15}$$

where $\overline{W}(t) = e^{-\lambda t}$ and $p(x,t) =$ the price attached to $x(t)$. Following (14), we shall assume $p(x,t) = b_0 p(t)x^{\gamma_0 - 1}$; $p(t)$ will be discussed later. $b_0$ and $\gamma_0$ are given parameters. Then (15) can be written as

$$Y = \int_0^T W(t) \prod_{\nu=1}^{m} \phi_\nu(t)\,dt\ , \tag{16}$$

where

$$W(t) = p(t)ae^{\bar{\mu}t} \prod_{\nu=1}^{m} [\omega_\nu(t)]^{-\alpha_\nu}\ , \qquad a = A^{\gamma_0} b_0\ , \qquad \bar{\mu} = \mu\gamma_0 - \lambda\ ,$$

$$\alpha_\nu = \text{given positive numbers} \sum_{\nu=1}^{m} \alpha_\nu = 1\ ,$$

$$\phi_\nu(t) = [\omega_\nu(t)]^{\alpha_\nu}[x_\nu(t)]^{\bar{\beta}_\nu}\ , \qquad \bar{\beta}_\nu = \beta_\nu\gamma_0\ ,$$

$\omega_\nu(t) =$ prices attached to production factors.

The cost of production factor $x_\nu$ in $[0,T]$, according to (13), should not exceed the given number $Y_\nu$:

$$\int_0^T \omega_\nu(t)[x_\nu(t)]^{\gamma_\nu}\,dt \leqslant Y_\nu\ , \qquad \nu = 1,\ldots,m \tag{17}$$

where

$$\omega_\nu(t) = \overline{W}(t)\bar{\omega}_\nu(t)b_\nu \ ,$$

$\overline{W}(t)$ = given discount cost.

It should be observed that using the discounted values of production factors prevents the possibility of trading factors over time with profit. For example, capital cannot be traded in this way because the bank imposes an interest function of the type $e^{\lambda(T-t)}$ for each loan given at $t$ up to $t = T$.

For the output price, the efficiency conditions require the average sectoral price, $\tilde{p}(t)$, to attain a minimum, denoted $p(t)$, at the optimal allocation of the production factors, $x_\nu(t) = \hat{x}_\nu(t)$, $\nu = 1,\ldots,m$, where $\tilde{p}(t) = c(t)/x(t)$, and $c(t)$ is the input cost, $c(t) = \Sigma_{\nu=1}^{m} \omega_\nu(t)[x_\nu(t)]^{\gamma_\nu}$. From the optimality condition (to minimize the average cost):

$$\frac{\partial \tilde{p}}{\partial x} = \left[\frac{\partial c}{\partial x} x - c\right] x^{-2} = 0 \ ,$$

one gets

$$\frac{\partial c}{\partial x} = \left.\frac{c(x_1 \ldots x_m)}{x(x_1 \ldots x_m)}\right|_{x_\nu = \hat{x}_\nu} .$$

Then $\tilde{p}(t)$ for the optimal allocation of production factors attains the minimum:

$$p(t) = \frac{\sum_{\nu=1}^{m} \omega_\nu(t)[\hat{x}_\nu(t)]^{\gamma_\nu}}{ae^{\bar{\mu}t} \prod_{\nu=1}^{m} [\omega_\nu(t)]^{-\alpha_\nu} \prod_{\nu=1}^{m} [\omega_\nu(t)]^{\alpha_\nu} [\hat{x}_\nu(t)]^{\bar{\beta}_\nu}} , \qquad (18)$$

equal to the marginal cost of the inputs.

So far the optimal strategy for allocation of resources $\hat{x}_\nu(t)$, $\nu = 1,\ldots,m$, has not been explicitly determined. For the planner, the best allocation strategy should yield the maximum value of $Y$ subject to the limitations imposed on the cost of inputs (17). Among the inputs, at least one, say the employment $x_1(t)$, $t \in [0,T]$, is exogenous. Since the objective function under the condition of increasing return ($\beta > 1+\gamma_0$) is not concave, and the constraints (17) are convex, except for land, it is not yet clear whether an optimal strategy exists, and, if it does exist, whether it is unique.

The following theorem gives an answer to these questions.

THEOREM 1. *Let* $\omega_1(t)[x_1(t)]^{\gamma_1}$ *be integrable in* $[0,T]$ *and*

$$\gamma_0 \sum_{\nu=1}^{m} \beta_\nu/\gamma_\nu = 1\,, \qquad W(t) = a \text{ constant}. \tag{19}$$

*Then the nonnegative strategy that maximizes* (16) *subject to* (17),

$$\hat{x}_\nu(t) = \left\{\frac{Y_\nu \omega_1(t)}{Y_1 \omega_\nu(t)} [x_1(t)]^{\gamma_1}\right\}^{1/\gamma_\nu}, \qquad \nu = 2,\dots,m\,, \tag{20}$$

*exists and is unique. With this strategy*

$$p(t) = \frac{W}{a} \prod_{\nu=1}^{m} [\omega_\nu(t)]^{\alpha_\nu} e^{-\bar{\mu}t}\,, \qquad \alpha_\nu = \bar{\beta}_\nu/\gamma_\nu \tag{21}$$

$$W = \sum_{\nu=1}^{m} Y_\nu \prod_{\nu=1}^{m} Y_\nu^{-\alpha_\nu} \tag{22}$$

*and the output* (16) *attains the maximum value*

$$\bar{Y} = \sum_{\nu=1}^{m} Y_\nu\,. \tag{23}$$

*Proof.* Dividing the numerator and denominator of (18) by $\omega_1(t)\cdot[x_1(t)]^{\gamma_1}$ and taking into account (20), i.e.,

$$\omega_\nu(t)[\hat{x}_\nu(t)]^{\gamma_\nu}/\omega_1(t)[x_1(t)]^{\gamma_1} = Y_\nu/Y_1\,, \qquad \nu = 2,\dots,m\,,$$

one gets

$$p(t) = \frac{(1 + \sum_{\nu=2}^{m} Y_\nu/Y_1)}{ae^{\bar{\mu}t}} \prod_{\nu=1}^{m} \left[\frac{Y_1}{Y_\nu}\omega_\nu(t)\right]^{\alpha_\nu} = \frac{\sum_{\nu=1}^{m} Y_\nu \prod_{\nu=1}^{m} \left[\frac{\omega_\nu(t)}{Y_\nu}\right]^{\alpha_\nu}}{ae^{\bar{\mu}t}}\,.$$

Then the function $W(t)$ in (16) becomes

$$W(t) = \sum_{\nu=1}^{m} Y_\nu \prod_{\nu=1}^{m} Y_\nu^{-\alpha_\nu} = W = \text{a constant} ,$$

and it is possible to apply the generalized Hölder inequality to (16) which yields:

$$Y \leqslant W \prod_{\nu=1}^{m} \left\{ \int_0^T |\phi_\nu(t)| \frac{1}{\alpha_\nu} dt \right\}^{\alpha_\nu} = W \prod_{\nu=1}^{m} \left\{ \int_0^T \omega_\nu(t) [x_\nu(t)]^{\bar{\beta}_\nu/\alpha_\nu} dt \right\}^{\alpha_\nu} . \tag{24}$$

Assuming $\bar{\beta}_\nu/\alpha_\nu = \gamma_\nu$, $\nu = 1,\ldots,m$, and taking into account (17), one can write the last inequality in the form:

$$Y \leqslant W \prod_{\nu=1}^{m} Y_\nu^{\alpha_\nu} = \sum_{\nu=1}^{m} Y_\nu = \overline{Y} .$$

We will have equality in (24) if and only if

$$\phi_\nu(t) = c_\nu \phi_1(t) , \qquad \nu = 2,\ldots,m , \tag{25}$$

where the $c_\nu$ are positive constants that can be determined using the constraints of (17). Then (25) can be written in the form

$$[\hat{x}_\nu(t)]^{\gamma_\nu} = \frac{Y_\nu \omega_1(t)}{Y_1 \omega_\nu(t)} [x_1(t)]^{\gamma_1} , \qquad \nu = 2,\ldots,m , \tag{26}$$

or equivalently in the form of (20).

Obviously, the Hölder inequality can be applied when $\omega_1(t)[x_1(t)]^{\gamma_1}$ is integrable, and

$$\sum_{\nu=1}^{m} \alpha_\nu = \gamma_0 \sum_{\nu=1}^{m} \beta_\nu/\gamma_\nu = 1 .$$

It follows from (19) that the optimal strategy exists, if parameters $\beta_\nu$ and $\gamma_\nu$ satisfy the necessary condition. This condition may be interpreted as a balance requirement, i.e., increases in input costs (except land) should be compensated by increases in return from economies of scale, so that $\Sigma_{\nu=1}^{m} \bar{\beta}_\nu/\gamma_\nu$ is constant. In other words, the agglomeration that wants to grow should develop new technologies of production that can counterbalance the growing input and environmental costs due to congestion, for example.

It should be observed that the strategy derived is not complete in the sense that one does not know how to spend the output-generated $\bar{y}$ on the factor endowments $Y_\nu$, $\nu = 1,\ldots,m$. Therefore, we have to relax our assumption that $Y_\nu$, $\nu = 1,\ldots,m$, are fixed and replace it by a more flexible condition. To be more specific, let us find the strategy $Y_\nu = \hat{Y}_\nu$, $\nu = 1,\ldots,m$, that minimizes the function (21), i.e., $W(Y_1,\ldots,Y_m)$, subject to the constraint

$$\sum_{\nu=1}^{m} Y_\nu \leqslant \bar{Y}.$$

It is possible to show that $\hat{Y}_\nu = \alpha_\nu \bar{Y}$, $\nu = 1,\ldots,m$. Then $W(\hat{Y}_1,\ldots,\hat{Y}_m)$ and $p(t)$ can be derived, yielding the following:

THEOREM 2. *The unique optimal factor endowment strategy, for the model under consideration,*

$$\hat{Y}_\nu = \frac{\alpha_\nu}{\alpha_1} Y_1 = \frac{\beta_\nu \gamma_1}{\beta_1 \gamma_\nu} Y_1 , \qquad \nu = 2,\ldots,m , \tag{27}$$

*exists. For this strategy*

$$p(t) = \prod_{\nu=1}^{m} \left[ \gamma_\nu \frac{\omega_\nu(t)}{\bar{\beta}_\nu} \right]^{\bar{\beta}_\nu/\gamma_\nu} \frac{e^{-\bar{\mu}t}}{a} \tag{28}$$

*and the output becomes*

$$\hat{Y} = \prod_{\nu=1}^{m} \left( \frac{\gamma_\nu}{\bar{\beta}_\nu} Y_\nu \right)^{\bar{\beta}_\nu/\gamma_\nu} = \frac{\gamma_1}{\beta_1} Y_1 = \bar{Y}. \tag{29}$$

It should be observed that Theorems 1 and 2 can be regarded as extensions of results obtained in Kulikowski (1976) for the model with $\gamma_1 = \gamma_2 = \cdots = \gamma_m = 1$. The optimal strategy there was called *the* *principle of factor coordination.* According to that principle, factors should change in a coordinated fashion along with employment. One can consider that principle as an extension of the so-called golden rule of development, where the labor ratio is a constant. In the case of (20), the ratio $\hat{x}_\nu(t)/x_1(t)$ is changing in time along with $x_1(t)$ and $\omega_1(t)/\omega_\nu(t)$. For example, when $m = 2$, $\gamma_1 > 1$, and $\gamma_2 = 1$, one gets from (26):

$$\frac{\hat{x}_2(t)}{x_1(t)} = \frac{\beta_2\gamma_1}{\beta_1}\frac{\omega_1(t)}{\omega_2(t)}[x_1(t)]^{\gamma_1 - 1} .$$

From (27) it follows that a factor's endowment strategy is determined by $\alpha_\nu/\alpha_1$, $\nu = 2,\ldots,m$, where $\Sigma_{\nu=1}^{m} \alpha_\nu = 1$. On the other hand, assuming that the past strategy was optimal, one can estimate $\alpha_\nu$ ex post facto by regression. For this purpose, the data regarding $Y_\nu^t$, $\nu = 1,\ldots,m$, $t = -1,-2,\ldots$, should be used. When $\gamma_\nu$ and $\alpha_\nu$, $\nu = 1,\ldots,m$, are known, one can easily find the corresponding $\beta_\nu = \gamma_\nu\alpha_\nu$, $\nu = 1,\ldots,m$. Another possible way to determine $\beta_\nu$, $\nu = 1,\ldots,m$, is to use standard econometric methods.

Since in our simple model the wages $Y_1$ are spent on consumption, the consumption share in the output

$$\frac{Y_1}{\bar{Y}} = \frac{\beta_1}{\gamma_1},$$

decreases when the agglomeration congestion (i.e., $\gamma_1$) is growing faster than the "effect of labor activity" (i.e., $\beta_1$).

It should also be noted that the price $p(t)$ according to (28) increases as $\gamma_\nu/\beta_\nu$ increases. Since the price for output product, $p(t)$, should not be greater than the exogenous (outside of the agglomeration) price, $\bar{p}(t)$, one gets the condition

$$p(t) \leqslant \bar{p}(t) . \tag{30}$$

Assuming that the model of the exogenous economy is similar in form to the CBD economy (with $\bar{\gamma}_\nu/\bar{\beta}_\nu$ generally different from $\gamma_\nu/\beta_\nu$ but with the same prices for factors $\omega_\nu(t)$, $\nu = 1,\ldots,m$, and neutral technological progress $\mu$) one can assume that the product $x$ will compete with exogenous product $\bar{x}$ if, according to (28),

$$\frac{1}{a}\prod_{\nu=1}^{m} \delta_\nu^{1/\delta_\nu} \leqslant \frac{1}{\bar{a}}\prod_{\nu=1}^{m} \bar{\delta}_\nu^{1/\bar{\delta}_\nu} \tag{31}$$

where

$$\delta_\nu = \frac{\gamma_\nu}{\gamma_0 \beta_\nu}, \qquad \bar{\delta}_\nu = \frac{\bar{\gamma}_\nu}{\bar{\gamma}_0 \bar{\beta}_\nu}, \qquad \nu = 1,\ldots,m .$$

Condition (31) should also hold when products $x$ and $\bar{x}$ are perfect substitutes, and the CBD specializes in production corresponding to comparative advantages in factor use. For example, if the CBD has comparative disadvantages in labor transportation cost ($\delta_1 > \bar{\delta}_1$), it can compete successfully with the outside economy, which has comparative disadvantages in capital cost ($\bar{\delta}_2 > \delta_2$).

It should be observed that the price $p(t)$ depends, generally speaking, on the influence of consumer demand. As shown in O'Luoghaire and Himmelblau (1974), one can use the consumer demand model to find the investment strategy that yields the necessary equilibrium between supply and demand.

The investments, as well as the rest of $z_\nu(t)$, $\nu = 1,\ldots,m$, expenditures should be derived starting with the known renting costs, $\omega_\nu x_\nu(t)$. A typical problem of this type arises when one considers the investment intensity cost, $z_2(t)$, and the rent cost, $\omega_2(t)x_2(t)$.

It may be assumed that one dollar of unitary investment produces $k_2 e^{-\delta_2 t}$ units of capital, decaying with depreciation rate $\delta_2$ to zero. If that capital is rented, at rent $\omega_2$, it should produce (with discount $e^{-\lambda t}$) at least one dollar, i.e.,

$$k_2 \int_0^\infty \omega_2 e^{-\lambda t} e^{-\delta_2 t}\, dt = \frac{\omega_2 k_2}{\lambda + \delta_2} = 1 .$$

Then $k_2 = (\lambda + \delta_2)/\omega_2$ and one can write

$$\omega_2 x_2(t) = \omega_2 k_2 \int_{-\infty}^{t} e^{-\delta_2(t-\tau)} z_2(\tau - T_2)\, dt$$

$$= (\lambda + \delta_2) \int_{-\infty}^{t} e^{-\delta_2(t-\tau)} z_2(\tau - T_2)\, dt , \qquad (32)$$

where $T_2$ is the factory construction delay.

When a factor, say capital, is not rented but is created by spending $z_2(t)$ on construction expenses, it is necessary to check whether net revenue is positive, i.e.,

$$d(t) = px(t) - z_2(t - T_2) - \sum_{\substack{\nu=1 \\ \nu \neq 2}}^{m} \omega_\nu(t) x_\nu(t) \geqslant 0 , \qquad t \in [0,T] . \quad (33)$$

It is possible to show that under the optimal development strategy (20), net revenue may not be positive. We assume that $m = 2, \gamma_1 = \gamma_2 = 1$, $\beta_1 + \beta_2 = 1$, $\omega_1$ and $\omega_2$ are constant, and $T_2 = 0$. Employment $x_1(t) = 0$ for $t < 0$, and is a given function for $t \geqslant 0$. Then by using (20) and (33) we obtain

$$\omega_2 \hat{x}_2(t) = \frac{\beta_2}{\beta_1} \omega_1 x_1(t) = (\lambda + \delta_2) e^{-\delta_2 t} \int_{-\infty}^{t} e^{\delta_2 t} z_2(\tau)\, dt . \qquad (34)$$

Differentiating both sides of (34) one gets

$$z_2(t) = \frac{\beta_2 \omega_1}{\beta_1 (\lambda + \delta_2)} [x_1{}'(t) + \delta_2 x_1(t)] .$$

Under the optimal strategy the output is $y(t) = \beta_1^{-1} \omega x_1(t)$, consumption is $\omega_1 x_1(t)$, and $d(t)$ becomes

$$d(t) = \left(\frac{1}{\beta_1} - 1\right) \omega_1 x_1(t) - \frac{\beta_2 \omega_1}{\beta_1 (\lambda + \delta_2)} [x_1{}'(t) - \delta_2 x_1(t)]$$

$$= A_1 x_1(t) - A_2 x_1{}'(t) ,$$

where

$$A_1 = \frac{\omega_1}{\beta_1} \left[ 1 - \beta_1 - \frac{\beta_2 \delta_2}{\lambda + \delta_2} \right] > 0 , \qquad A_2 = \frac{\beta_2 \omega_1}{\beta_1 (\lambda + \delta_2)} > 0 ,$$

$x_1(t) = at$ , a constant ,

$$d(t) < 0 \text{ for } 0 \leqslant t < \frac{A_2}{A_1} , \qquad d(t) > 0 \text{ for } t > \frac{A_2}{A_1} .$$

Obviously, in order to realize the optimal development strategy, a loan from the bank is necessary. The loan will be repaid with interest

$\lambda_0$ in the time interval ($t > A_2/A_1$), when $d(t) > 0$. It is possible to show that for the given $\lambda_0$ there exists a minimum time interval, $T_0$, that can be derived from condition (4):

$$\int_0^{T_0} e^{\lambda_0(T_0-t)} d(t)\, dt = 0 .$$

When credits for capital investments are available, it is possible to show that regional growth is, generally speaking, faster in the case of autarky (for details see Kulikowski (1976)). In the case of planned economies, $\lambda_0$ can also be regarded as a policy instrument by which the growth of different regions can be coordinated at the national level.

As already mentioned, $x_1(t)$ denotes at least one exogenous factor. It is sometimes convenient to introduce a number of factors (i.e., labor, land, and water) as exogenous. In that case, $x_1^{\beta_1} x_3^{\beta_3} x_4^{\beta_4}$ in (16) is a given function, say $\tilde{x}_1^{\bar{\beta}_1}$ ($\bar{\beta}_1 = \beta_1 + \beta_3 + \beta_4$), and the optimal allocation of the rest of factors $\hat{x}_\nu(t)$, ($\nu \neq 1,3,4$), should follow the strategy determined by $\hat{x}_1$ (Theorems 1 and 2).

Then (29) can be written

$$\hat{Y} = \frac{1}{\tilde{\alpha}_1} \int_0^T \tilde{\omega}_1(t) \tilde{x}_1^{\tilde{\gamma}_1}(t)\, dt$$

$$= \frac{\tilde{\gamma}_1}{\beta_1} \int_0^T \tilde{\omega}_1(t) [x_1(t)]^{\tilde{\beta}_1} [x_3(t)]^{\tilde{\beta}_3} [x_4(t)]^{\tilde{\beta}_4}\, dt , \qquad (35)$$

where $\tilde{\beta}_\nu = \beta_\nu^{\tilde{\gamma}_1/\bar{\beta}_1}$, $\nu = 1,3,4$, and $\tilde{\omega}_1$ and $\tilde{\gamma}_1$ satisfy a constraint equivalent to (17); i.e.,

$$\int_0^T \tilde{\omega}_1(t) [\tilde{x}_1(t)]^{\tilde{\gamma}_1}\, dt \leqslant \tilde{Y}_1 = Y_1 + Y_3 + Y_4 .$$

## 4 OPTIMAL ALLOCATION STRATEGY IN A MULTISECTOR MODEL

As shown in section 3, in the single sector regional model studied there is a unique optimal strategy of allocation of production factors (with at least one factor exogenous). One can easily extend this model by introducing several production sectors, $S_i$, $i = 1,\ldots,n$, into the CBD. These production sectors are shown in Figure 1, where $X_i$ represents the output production of sectors $S_i$, and $X_{ji}$ the number of commodities that $S_j$ sells to $S_i$, $i,j = 1,\ldots,n$. The net outputs, $\overline{X}_i$, $i = 1,\ldots,n$, contribute to the gross regional product (GRP), which is allocated by the decision center (DC) in the form of $Z_{\nu i}$, $i = 1,\ldots,n$, $\nu = 1,\ldots,m$, to factor endowments. The input-output production function is assumed to be of the form

$$X_i = F_i^q \prod_{j=1}^{n} X_{ji}^{\alpha_{ji}} , \qquad i = 1,\ldots,n \tag{36}$$

$$q_i = 1 - \sum_{j=1}^{n} \alpha_{ji} > 0 , \qquad \alpha_{ji} \geqslant 0 , \qquad i,j = 1,\ldots,n ,$$

where $\alpha_{ji}$ are given positive numbers, and $F_i$ depend on the vector $\underline{Z}_i$ of factor costs, i.e., $F_i = F_i(\underline{Z}_i)$, $i = 1,\ldots,n$.

Assuming that a set of sector prices $p_i$, $i = 1,\ldots,n$, exists, one can design the model in monetary terms, i.e.,

$$Y_i - \sum_{j=1}^{n} Y_{ij} = \overline{Y}_i , \qquad i = 1,\ldots,n \tag{37}$$

$$Y_i = K_i \prod_{j=1}^{n} Y_{ji}^{\alpha_{ji}} \tag{38}$$

where

$$Y_{ij} = p_j X_{ji} , \qquad K_i = pF_i^q \prod_{j=1}^{n} P_j^{-\alpha_{ji}} , \qquad i,j = 1,\ldots,n .$$

We shall also assume that a decentralized structure of decisions exists. Each sector $S_i$, $i = 1,\ldots,n$, can decide how many of the inputs $Y_{ji}$, $j = 1,\ldots,n$, to buy in order to maximize profit (value added),

$$D_i = Y_i - \sum_{j=1}^{n} Y_{ji} , \qquad i = 1,\ldots,n . \tag{39}$$

The regional decision center, DC, is at the same time concerned with the

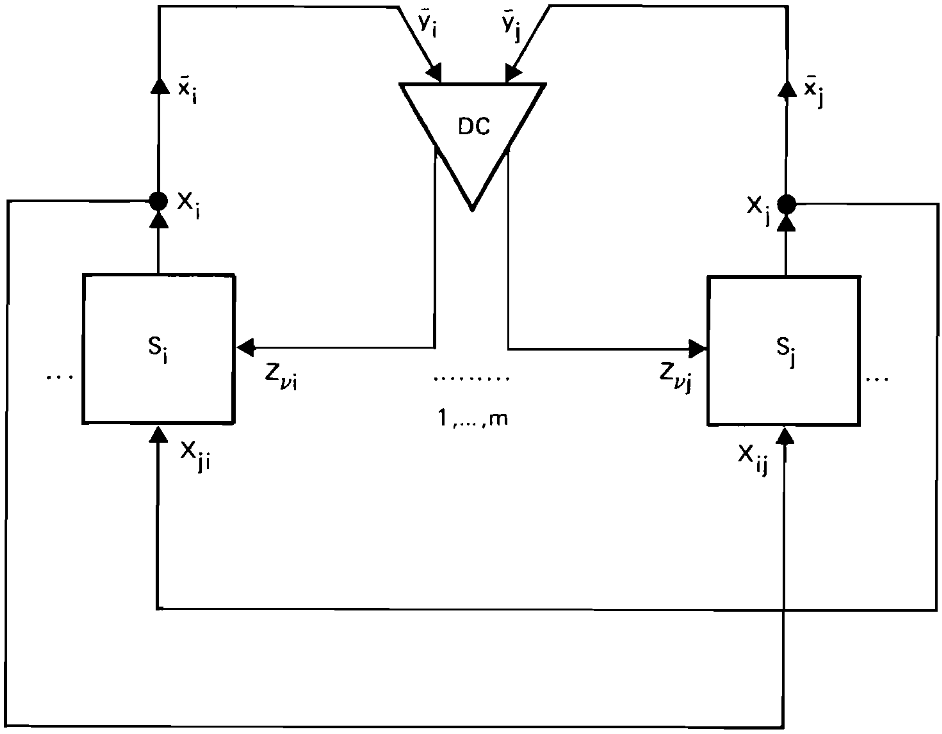


FIGURE 1 Production sectors in a regional economy.

best allocation of production factors $\underline{Z}_i$, $i = 1,\ldots,n$. Since $D_i$ is a strictly concave function, a unique set of strategies $Y_{ji} = \hat{Y}_{ij}$, $i,j = 1,\ldots,n$, exists, such that $D(\hat{Y}_{ji},\ j,i = 1,\ldots,n) = \hat{D}_i$ is a maximum. This strategy (see Kulikowski 1976) becomes

$$\hat{Y}_{ji} = \alpha_{ji}\hat{Y}_i\,, \qquad j,i = 1,\ldots,n, \tag{40}$$

where

$$\hat{Y}_i = F_i(\underline{Z}_i)\Pi_i\,, \qquad i = 1,\ldots,n\,, \tag{41}$$

$$\Pi_i = \left[\prod_{j=1}^{n}\left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}} P_i\right]^{1/q_i} .$$

When one uses this strategy, $\hat{D}_i = q_i Y_i$, and the GRP becomes

$$\hat{Y} = \sum_{i=1}^{n} \hat{D}_i = \sum_{i=1}^{n} q_i \Pi_i F_i(\underline{Z}_i) \,. \tag{42}$$

Note that output $Y$ is represented in decomposed form, i.e., it is the sum of terms $Y_i(\underline{Z}_i) = q_i \Pi_i F_i(\underline{Z}_i)$. This can be called an aggregated production function, and written in the typical (Cobb-Douglas) form (16):

$$Y_i = \int_0^T \omega_i(t) \prod_{\nu=1}^{m} \phi_{\nu i}(t)\, dt \,, \qquad i = 1,\ldots,n \,,$$

where

$$\omega_i(t) = q_i \Pi_i e^{\bar{\mu}_i t} \prod_{\nu=1}^{n} [\omega_\nu]^{-\alpha_{\nu i}} \,,$$

$$\phi_{\nu i}(t) = [\omega_\nu(t)]^{\alpha_{\nu i}} [Z_{\nu i}(t)]^{\bar{\beta}_{\nu i}} \,, \qquad \nu = 1,\ldots,m \,.$$

Using factor coordination, one can allocate factors $z_{\nu i}(t)$, $\nu = 1,\ldots,m$, $t \in [0,T]$, in an optimal manner. Kulikowski (1976) shows that it is also possible to allocate factors among sectors $i = 1,\ldots,n$, in such a way that $\Sigma_{i=1}^{n} Y_i$ is at a maximum, while prices $\bar{p}_i$ satisfy the a priori equilibrium condition between supply and demand (on the national scale), generated by the consumption submodel. Comparing the present model with the single sector model, one observes that each regional sector price $p_i(t)$ should now satisfy condition (30) (i.e., $p_i(t) \leqslant \bar{p}_i(t)$, $i = \mathrm{i},\ldots,n$, where $\bar{p}_i(t)$ is the national equilibrium price). In other words, the regional model should cooperate with the national model which uses as inputs the regional supplies and demands and which generates for regional models the average equilibrium prices $\bar{p}_i(t)$, $i = 1,\ldots,n$, $t \in [0,T]$. It should also be observed that since all sectors $S_i$ are located in the CBD, the impact of the transport costs on the sectoral inputs $Y_{ji}$ can be neglected.

In studying the multisectoral regional models, it is sometimes important to know how the existing sectors utilize the factors that are in limited quantity and whether it would pay to reallocate factors among sectors. These questions arise when one tries to formulate the optimal policy of rural–urban migration. The simple economic model that can be used to answer these questions consists of two sectors with production functions

$$Y_i = A_i K_i^q L_i^\beta \,, \qquad q = 1 - \beta \,, \qquad i = 1,2 \,, \tag{43}$$

where

$Y_i$ = output of sector $S_i$ expressed in monetary units,
$K_i$ = capital stock of sector $S_i$,
$L_i$ = employment of sector $S_i$,
$i = 1$ denotes the agricultural sector,
$i = 2$ denotes the rest of the economy.

The numerical values of $A_i$ and $\beta$ can be estimated from statistical data. It is assumed that total employment, $L$, is predetermined by demographic factors, so

$$L_1 + L_2 \leqslant L . \tag{44}$$

If one looks at a particular year, with known values of $G_i = A_i^{1/q} K_i$, $i = 1,2$, it is possible to find the employment levels $\hat{L}_i$, $i = 1,2$, that maximize the total output, i.e., the GRP,

$$Y = Y_1 + Y_2 ,$$

subject to constraint (44). One can easily show that

$$\hat{L}_i = \frac{G_i}{G_1 + G_2} L , \qquad i = 1,2 . \tag{45}$$

By examining the optimal allocation of labor $\hat{L}_i$, $i = 1,2$, one can discover that a surplus (or deficit) of labor, $L_s$, exists in agriculture with respect to the rest of the economy. Defining $L_s = xL_1$, one can use (43) and (45) to show that

$$\frac{\hat{L}_1}{L_1} : \frac{\hat{L}_2}{L_2} = \frac{1 - x}{1 + ax} = \left[\frac{y_1}{y_2}\right]^{1/q} , \tag{46}$$

where

$$a = L_1/L_2 , \qquad y_1 = Y_1/L_1 , \qquad y_2 = Y_2/L_2 .$$

Statistical data for the Polish economy (Kulikowski 1977) show that the ratio for labor productivity, $y_1/y_2$, decreased from 1970 to 1975 from 0.33 to 0.24. The surplus labor in Polish agriculture in 1970, derived from (46) for $\beta = 0.5$, was around $L_s = 4.5 \times 10^6$. Formula (46) was also used to derive the surplus labor in agriculture on a regional level (Kulikowski 1977). An extension of formula (46) for the economy with production functions

$$Y_i = G_i^q L_i^{\beta_1} \prod_{\nu=2}^{m} Z_{\nu i}^{\beta_{\nu i}} , \tag{47}$$

$$a = 1 - \beta_1 - \beta > 0 ,$$

$$\beta = \sum_{\nu=2}^{m} \beta_{\nu i} , \qquad i = 1,2$$

is possible. In the extended formula,

$$\frac{1-x}{1+ax} = \left\{ \frac{y_1 \Pi_2}{y_2 \Pi_1} \right\}^{1/q} . \tag{48}$$

The surplus, $x$, depends on the labor efficiency, $y = y_1/y_2$, and also on the ratio $\Pi = \Pi_2/\Pi_1$, which in turn depends on $Z_{\nu i}$, the expenses per head employed, i.e.,

$$\Pi_i = \prod_{\nu=2}^{m} (Z_{\nu i}/L_i)^{\beta_{\nu i}} , \qquad i = 1,2 .$$

Since $x$ is a decreasing function of $y\Pi$ (when $x = 0$, $y\Pi = 1$, and when $x = 1$, $y\Pi = 0$), the surplus of labor in agriculture increases when capital $(z_{21}/L_1)$ or education per head in agriculture increases; an increase of capital stock or education in the rest of the economy has the opposite effect.

Using the optimal allocation of labor formulae, it is also possible to find out how much the economy loses because of poor labor allocation. In the case of the simple model, (43), the following loss function can be constructed,

$$\Delta(x) = G^q L^\beta - G_1{}^q (L_1 - L_s)^\beta - G_2{}^q (L_2 + L_s)^\beta$$

$$= G^q L^\beta \left[ 1 - \left(\frac{G_1}{G}\right)^q \left(\frac{L_1}{L} - x\right)^\beta - \left(\frac{G_2}{G}\right)^q \left(\frac{L_2}{L} + x\right)^\beta \right] , \tag{49}$$

where $G^q L^\beta$ = GNP generated under the optimal labor allocation strategy. The function $\Delta(x)$ decreases monotonically to zero, at which point $x$ approaches the value determined by (46). Migration also involves housing costs, $C(x)$, which increase with $x$. Kulikowski (1977) defines the optimal labor migration per year, $\hat{x}$, as the value of $x$ that yields the minimum of the combined cost function, $\Delta(x) + C(x)$. Assuming $C(x) = r_1 x L_1$, where $r_1$ is the rent of housing and urban facilities, one finds that at optimal migration the marginal gain, $-\Delta'(x)$, is equal to $r_1 L_1$.

Migration influences the existing employment pattern and helps to create scale economies as well as diseconomies. Therefore, the surplus formulae, (46) and (48), can be improved if the results obtained in section 3 are taken into account. In particular, one can use formula (35), which, in the static case ($x_\nu(t) = X_\nu$), can be written

$$Y_i = G_i^q X_{1i}^{\gamma_{1i}} \prod_{\nu=2}^{m} X_{\nu i}^{\gamma_{\nu i}} , \qquad (50)$$

where

$$q = 1 - \gamma ,$$

$$\gamma = \sum_{\nu=1}^{m} \beta_{\nu i} ,$$

$$i = 1,2 .$$

When $\gamma_{1i} = \gamma_1$, $i = 1,2$, the optimal allocation strategy $X_{1i} = \hat{X}_{1i}$, $i = 1,2$, which maximizes $Y_1 + Y_2$ subject to the constraint $X_{11} + X_{12} \leqslant X_1$, can be derived yielding formula (48) where:

$$\Pi_i = \prod_{\nu=2}^{m} \left[\frac{X_{\nu i}}{X_{1i}}\right]^{\gamma_{\nu i}} , \qquad i = 1,2 . \qquad (51)$$

It should be observed that by the proposed methodology one can derive the optimal allocation strategies for labor as well as other factors (e.g., water, land). The factor $y_1/y_2$ in (48) is in this case the ratio of efficiencies in utilization of labor, water, land, etc.

When one finds the optimal allocation of one factor by (46), the remaining factors are kept constant. It is, however, possible to derive formulae for the allocation of all factors simultaneously. For example, assume $\gamma_{\nu i} = \gamma_\nu$, $i = 1,2$. Then the optimal strategies (those maximizing $Y_1 + Y_2$) subject to $X_{\nu 1} + X_{\nu 2} \leqslant X_\nu$, $\nu = 1,\ldots,m$, are

$$\hat{X}_{\nu 1}/\hat{X}_{\nu 2} = G_1/G_2 \;, \qquad \nu = 1,\ldots,m \;,$$

and one gets

$$\frac{\hat{X}_{11}}{\hat{X}_{12}} = \frac{X_{11}}{X_{12}} \left[ \frac{y_1}{y_2} \prod_{\nu=2}^{m} \left( \frac{u_{\nu 2}}{u_{\nu 1}} \right)^{\gamma_\nu} \right]^{1/q} , \tag{52}$$

where

$$u_{\nu i} = \hat{X}_{\nu i}/X_{1i} \;, \qquad u_{\nu 2}/u_{\nu 1} = G_2 X_{11}/G_1 X_{12} \;, \qquad \nu = 2,\ldots,m \;.$$

When the condition $\gamma_{\nu i} = \gamma_\nu$, $i = 1,2$, does not hold, one cannot use the explicit relations (48) and (52), and the optimal allocation strategy can be derived only in numerical form.

## 5 OPTIMIZATION OF REGIONAL DEVELOPMENT

Using formulae (48) and (52), one can estimate the surplus or deficits of each factor in each region. One can also regard regional surpluses estimated in the previous year as the initial supplies of factors that determine the regional comparative advantages for future development plans. The planners would like, however, to know how the supplies of factors will change in the future, i.e., in the interval $[0,T]$. To forecast with formulae (48) and (52) a priori is more difficult. One possible approach is to speculate on the future change of

$$Y = \left[ \frac{y_1}{y_2} \frac{\Pi_2}{\Pi_1} \right]^{1/q} ,$$

or

$$a = L_1/L_2 \;,$$

and their impact on

$$x = \frac{1-y}{1+ay} \,. \tag{53}$$

If, for example, one finds by regression analysis that the trends of $y(t)$ and $a(t)$ can be approximated by the functions $y(t) = 1 - e^{-\alpha t}$, $a(t) = e^{\beta t}$, the future trend of $x(t)$ can be found from (53):

$$\tilde{x}(t) = [e^{\alpha t} + e^{-\beta t}]^{-1} \; .$$

Note that $\tilde{x}(t)$ is increasing in the initial time interval. It attains a peak value, then tends to zero as $t \to \infty$. In many developed regions, migration from rural to urban areas has followed such a pattern. Knowing $\tilde{x}(t)$, one can find the expected employment in the future, i.e., for $t > 0$,

$$L_1(t) = \bar{L}_1(t)[1 - \tilde{x}(t)] \; , \qquad L_2(t) = \bar{L}_2(t)[1 + \tilde{x}(t)] \; ,$$

where $\bar{L}_1(t)$ and $\bar{L}_2(t)$ are estimated labor supplies in the rural and urban areas, respectively. Then, using the factor coordination principle, the optimal strategy of development of the rural–urban region can be easily derived.

One can determine $x(t)$ in a more accurate way by iteration, using at each step the minimum cost-benefit analysis approach. This approach minimizes the loss function $L(x) = \Delta(x) + C(x)$ (compare with (49)). Assume that we are concerned with a two-sector regional model, which uses $m$ primary resources $x_\nu$ (manpower, land, water, etc.), and under factor coordination strategy can be described by production functions with the same form as (35) [when $T$ = one year, (35) can be regarded as a static production function, i.e., the integral sign can be dropped] :

$$Y_i = G_i^q \prod_{\nu=1}^{m} X_{\nu i}^{\beta \nu} \; , \qquad \sum_{\nu=1}^{m} \beta_\nu < 1 \; , \qquad i = 1,2 \; , \tag{54}$$

where

$$X_{\nu 1} + X_{\nu 2} \leqslant X_\nu \; , \qquad \nu = 1,\ldots,m \; . \tag{55}$$

Assume that forecasts for the natural supply of each factor are given, that is, the sequences $\bar{X}_{\nu i}(t)$, $\nu = 1,\ldots,m$, $i = 1,2$, $t = 0,1,\ldots,t$, are known. When decisions $X_{\nu s}(t)$ are made regarding the transfer of factors between two sectors, the real supplies change. So we have

$$X_{\nu 1}(t+1) = \bar{X}_{\nu 1}(t+1) - \sum_{\tau=0}^{t} X_{\nu s}(\tau) \; , \tag{56}$$

$$X_{\nu 2}(t+1) = \bar{X}_{\nu 2}(t+1) + \sum_{\tau=0}^{t} X_{\nu s}(\tau)\,. \qquad (57)$$

It should be noted that a further improvement of the model can be made when one has more information concerning the transfer processes. For example, age structure and mortality may change the labor supply term, $\Sigma_{\tau=0}^{t} X_{\nu s}(\tau)$, in (56) and (57) [see Rogers 1975].

We are now in a position to evaluate the loss function $L(\underline{X}_s)$, which can be assumed to have the form:

$$L(\underline{x}) = G^q \prod_{\nu=1}^{m} X_\nu^{\beta_\nu} - G_1{}^q \prod_{\nu=1}^{m} [X_{\nu 1} - X_{\nu s}]^{\beta_\nu}$$

$$- G_2{}^q \prod_{\nu=1}^{m} [X_{\nu 2} + X_{\nu s}]^{\beta_\nu} + \sum_{\nu=1}^{m} r_\nu X_{\nu s}$$

$$= \bar{Y} \left\{ 1 - \left(\frac{G_1}{G}\right)^q \prod_{\nu=1}^{m} (x_{\nu 1} - x_\nu)^{\beta_\nu} \right.$$

$$\left. - \left(\frac{G_2}{G}\right)^q \prod_{\nu=1}^{m} (x_{\nu 2} + x_\nu)^{\beta_\nu} + \sum_{\nu=1}^{m} \frac{r_\nu}{A_\nu} x_\nu \right\}, \qquad (58)$$

where

$$\bar{Y} = G^q \prod_{\nu=1}^{m} X_\nu^{\beta_\nu}\,,$$

$$G = G_1 + G_2\,,$$

$$x_{\nu i} = X_{\nu i}/X_\nu\,,$$

$$x_\nu = X_{\nu s}/X_{\nu 1}\,,$$

$$i = 1,2\,,$$

$$A_\nu = \bar{Y}/X_{\nu 1}\,,$$

$$\nu = 1,\ldots,m\,,$$

and $r_\nu$ are the rents paid for housing services, public facilities, etc., connected with migration and transfers of all the other factors, $X_{\nu s}$.

Since (58) is a strictly convex function, a unique solution, $\hat{\underline{x}} \equiv \{\hat{x}_1,\ldots,\hat{x}_m\}$, exists that minimizes $L(\underline{x})/\overline{Y}$ subject to the constraints (55). Since $x_{\nu i}$, $\nu = 1,\ldots,m$, $i = 1,2$, are given but change each year, the optimization procedure should be repeated each year, so that a sequence of numbers $\hat{x}_\nu(t)$, $\nu = 1,\ldots,m$, $t = 0,\ldots,T$, is generated. Using these numbers one can compute $X_{\nu s}(t) = \hat{x}_\nu(t)(\overline{X}_{\nu 1}(t))$, $\nu = 1,\ldots,m$, as well as the optimal factor utilization in both sectors, (56) and (57). The complete optimal regional development for both sectors can be derived by the principle of coordination of capital and remaining factors. When the optimal allocation of production factors for the planning interval is known, it remains to check whether the assumptions regarding input and output costs hold. Since the cost functions (14) and (15) were derived from past data and subsequently used for forecasting, the allocation of, for example, labor (according to the optimal strategies) may differ from what is assumed. A possible way to avoid this drawback is to correct the input cost functions (by changing $\gamma_\nu$, $\nu = 0,\ldots,m$) in an iterative way, so the "ex post facto" and "a priori" functions coincide. A similar iterative process has been used to derive the future sectoral prices on the national level by Kulikowski (1976).

The GRP generated in the region studied can be derived by summing the sectoral outputs from (54). The transfer of factors satisfies the condition $L'(\hat{\underline{x}}) = 0$, so the gain to the regional economy resulting from the transfer of one factor unit, e.g., one worker, can be compensated for by the corresponding rent paid. Migration of factor units does not decrease the resulting GRP. On the contrary, since $L(\underline{x})$ is strictly convex, at $\underline{x} = \hat{\underline{x}}$ an increase resulting from the better allocation of factors can be recorded. The effect of rent, $r_\nu$, on migration is shown in section 6. Generally speaking, $r_\nu$ is an increasing (convex) function of $X_{\nu s}$, and the cost components $C_\nu(x_\nu) = (r_\nu/A_\nu)x_\nu$, $\nu = 1,\ldots,m$, increase along with $x_\nu$ faster than a linear function. The necessary capital investment strategy is completely determined by $r_\nu \hat{X}_{\nu s}$. In order to derive that strategy, one can use the relation (32), which explains how the investment intensities, $z_\nu(\tau)$, are related to the factor costs, $r_\nu X_\nu(t)$, representing different public service facilities (including housing, water supply system, adaption of land for urban purposes, etc.). If, for example, one dollar of unitary investment produces $k_i e^{-\delta t}$ units of $i$-th service facilities ($T_i$ years later), and these facilities are rented at rent $r$, they should produce at least one dollar in return (compare (32)), so the following relation should hold,

$$r_\nu X_{\nu s}(t) = \sum_{i=1}^{N} (\lambda + \delta) \int_{-\infty}^{t} e^{-\delta(t-\tau)} z_{\nu i}(\tau - T_i)\, d\tau \ .$$

Differentiating both sides, one gets

$$z_\nu(t) = \sum_{i=1}^{N} z_{\nu i}(t - T_i) = \frac{r_\nu}{\lambda + \delta} [X_{\nu s}(t) + \delta X'_{\nu s}(t)] \ . \qquad (59)$$

If $z_\nu(t)$ are assigned solely to the city budget, they should be compensated for by taxes imposed on inputs and outputs of the urban economy. This, in turn, increases the cost of inputs and the output price, and reduces the benefits offered by the economies of scale to the agglomeration studied. In this case, producers as well as consumers may choose to locate in different areas. For this reason, and for other reasons, urban investments in different regions in planned economies are controlled to a large extent by the central planning unit, and the city budget is supported by a central financial system.

It should also be noted that in order to derive the optimal policy for rural–urban migration, it is necessary to confront this planning model with residential utility models. These models show how the income distribution pattern, and the so-called amenity resources, influence a migrant's decision about where to live. The basic question we shall try to answer in the next section can be formulated as follows: is there an optimal migration policy that satisfies both the planner and the migrant?

## 6 OPTIMAL MIGRATION POLICY

Much has been written about the factors that influence a migrant's decision to move from a rural to an urban area; Willekens (1977) gives several references. Among these factors are wages, services (for education, recreation, public health, etc.), and the attractiveness of residential areas. Econometric models have also been constructed to find out how the migration rate $x = L_s/L_1$ depends on the factor preference ratios,

$$f_i = F_i^1 / F_i^2 \ , \qquad i = 1,\ldots,n \ ,$$

where $F_i^1$ denotes rural factor preferences and $F_i^2$ denotes urban factor preferences. For the present study, the most interesting models are those of the form

$$x = 1 - \prod_{i=1}^{n} f_i^{\delta_i}\,, \qquad \sum_{i=1}^{n} \delta_i < 1\,, \tag{60}$$

where $\delta_i$ are given positive numbers. One can regard $\delta_i$ as ratios of migration to relative factor changes, i.e.,

$$\delta_i = \frac{d[1-x]}{1-x} : \frac{df_i}{f_i}\,.$$

Obviously, when there are no factor preferences, $x = 0$, and $x$ becomes negative when rural attractiveness is higher than urban attractiveness. It should also be observed that $F_i^j$ can be measured by arbitrary units. For example, one can compare the number of rainy days and sunny days or the distance to recreation in the rural area and the urban area.

The decision maker is only interested in the $f_i$ which can be influenced by policy instruments. In this paper, we assume that there is a single service (e.g., social benefits) that has the values (rent) $\bar{r}_1$ and $\bar{r}_2$ for rural and urban people, respectively; i.e., we assume the main preference factor to be

$$f_1 = \frac{F_1^{\ 1}}{F_1^{\ 2}} = \frac{\bar{r}_1}{\bar{r}_2 + r_1}\,, \tag{61}$$

where $r_1$ = additional services in the urban area offered to migrants when they agree to live there for a long time.

In many countries, $r_1$ can be regarded as the main decision parameter. In Poland, it assumes for the most part the form of government and regional contributions to the housing expenses that face rural migrants when they live in urban areas. In this model, it should be observed that wage differences between rural and urban workers are determined by the marginal costs of the commuting labor studied in section 2 (the CBD can entice a worker away by offering him a salary that compensates him for his transportation expenses).

Substitution between migrants and commuting migrants is possible, however, when the transport costs reach the $r_1$ level. Then the simplified version of the general econometric model (60) can be assumed to have the form (suitable for regression):

$$\log(1-x) = \log a - \delta \log(b + (r_1/\bar{r}_1)) \tag{62}$$

where

$b = \bar{r}_2/\bar{r}_1$ is exogenous in our model,

$a = \prod_{i=2}^{n} f_i^{\delta_i}$ is the preference index attached to the rest of $f_i$.

Since statistical data regarding $x_t$, $r_{1t}$, $\bar{r}_{1t}$, $\bar{r}_{2t}$, $t < 0$, are usually available, it is possible to find ex post facto estimates for $a$ and $\delta$. Then one can use (62) a priori for short-term forecasts.

Two different models, describing the future migration rate $k(t)$ as a function of variable $r_1/\bar{r}_1$, can be constructed. The first model is for the planner, who is trying to minimize the loss function (58), which for $m = 1$ can be written,

$$\bar{L}(x) = \bar{\Delta}(x) + \bar{C}(x, r_1/\bar{r}_1)\ , \tag{63}$$

where (compare (49))

$$\bar{\Delta}(x) = 1 - \left(\frac{G_1}{G}\right)^q \left(\frac{L_1}{L} - x\right)^\beta - \left(\frac{G_2}{G}\right)^q \left(\frac{L_2}{L} + x\right)^\beta ,$$

$$\bar{C}(x, r_1/\bar{r}_1) = \frac{r_1 L_1}{\bar{y}} x = \frac{r_1}{A\bar{r}_1} x ,$$

and

$$A = \frac{\bar{y}}{L_1 \bar{r}_1}, \qquad (A = A_1/\bar{r}_1)\ .$$

The second model, accounting for the residential utilities, (62), as a function of $r_1/\bar{r}_1$, can be written in the form

$$\frac{r_1}{\bar{r}_1} = \left(\frac{a}{1-x}\right)^{1/\delta} - b\ . \tag{64}$$

Then it is possible to eliminate $r_1/\bar{r}_1$, i.e., replace $C(x, r_1/\bar{r}_1)$ by

$$\bar{C}(x) = \frac{x}{A}\left[\left(\frac{a}{1-x}\right)^{1/\delta} - b\right]\ . \tag{65}$$

Note that $\bar{C}(x)$ is a strictly convex function, and since $\bar{\Delta}(x)$ is convex, there exists a value, $x = \hat{x}$, that minimizes the objective function, $\bar{L}(x)$.

Figure 2 shows the graph of the function $\bar{\Delta}(x)$ at the national level, which was estimated using statistical data from the Polish economy in 1970 (see Kulikowski 1977) with $\beta = 0.5$, $L_1/L = 0.348$, $L_2/L = 0.6157$, $G_1/G = 0.0643118$, and $G_2/G = 0.935688$. The cost function, $\bar{\Delta}(x)$, decreases monotonically starting from the value $\bar{\Delta}(0) = 0.082$ and reaches zero for $x \approx 0.33$. Figure 2 also shows the graph of $\bar{C}(x)$ for the estimates $A = 25$, $a = 1.45$, $b = 1.50$, and $\delta = 0.167$. Observe that $\bar{L}(x)$ attains a minimum value for $\hat{x} \approx 1.5\%$ ($r_1/\bar{r}_1$ is then equal to 8.7). The migration rate in 1970 in Poland was $x = 1.1\%$, which is close to the value derived.

The analysis carried out in the present section shows that it is possible to choose a migration policy in terms of $r_1/\bar{r}_1$ that yields maximum regional economic growth and that satisfies migrant residential preferences. In the general case, the function $\bar{L}(x)$ can be used for forecasting. Since the labor supplies (56) and (57) change over time, the optimal migration strategies, $\hat{x}(t)$ and $\hat{r}(t)$, change as well. It should also be noted that $\hat{r}_1(t)$ determines the economically justified value of commuting migration cost, i.e., the number of commuters.

## 7 CONCLUSIONS

The present paper is concerned with the normative approach to regional planning problems. For that purpose, some extensions of the known, classical approaches are needed. First of all, it is necessary to introduce the regional scale economies and externalities, which increase the cost of input factors. Then it is necessary to find the conditions under which the scale economy comes to an equilibrium with the externalities. Theorems 1 and 2 give an answer to that question, showing at the same time the best strategy for the coordination of input factors. The next important question we deal with is the optimal allocation of resources in a multisector model.

The solution of problems, stated in sections 2, 3, and 4, were based on the assumption that the primary factors (such as labor) are exogenous and cannot be transferred (e.g., by migration) as a result of economic activity or differences in the standards of living (utilities). In sections 5 and 6, we drop that assumption and introduce factor flows (migrants), subject to a given cost function. The optimal policy of income distribution, which affects the migrant's decision to move, is also derived.

The results obtained in the present paper indicate that it is feasible to

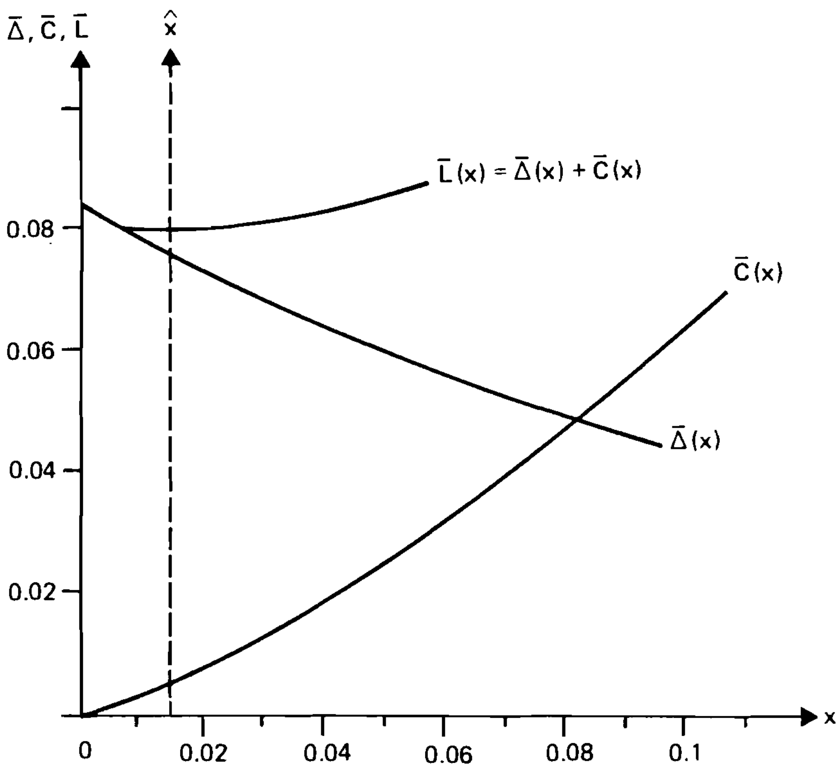


FIGURE 2 Loss function for the Polish economy in 1970.

construct a normative, dynamic model for rural–urban development. In order to explain how the computer-operated version of the model could work, let us specify the exogenous, estimated, and endogenous variables.

In the model proposed, there are the following exogenous factors and processes:

1. Forecasts of natural supplies of primary resources at urban and rural areas, $\bar{X}_{\nu i}(t)$, $\nu = 1,\ldots,m$, $i = 1,2$, $t = 0,1,\ldots,T$;
2. Rents for factors $\bar{\omega}_\nu(t)$, $\nu = 1,\ldots,m$, $t = 1,\ldots,T$ (outside the region) and prices for products $\bar{p}_i(t)$, $i = 1,\ldots,n$, $t = 1,\ldots,T$ (outside the region);
3. Rents on capital used for transfer (migration) of factors $r_\nu$, $\nu = 1,\ldots,m$;
4. Values (rents) of social benefit services $(\bar{r}_1,\bar{r}_2)$ for rural and urban people;
5. Discount rate $\lambda$, depreciation of capital $\delta$, construction delays $T_i$, and neutral technological progress rate $\mu$.

The following parameters are estimated from historical data and are used a priori:

1. Input cost parameters, $\gamma_\nu$, $\nu = 1,\ldots,m$, $b_0$;
2. Production function parameters $A$, $\beta_\nu = 1,\ldots,m$ (or alternatively $\alpha_\nu$, $\nu = 1,\ldots,m$), $G_1/G$, $G_2/G$;
3. Migrant preference function parameters $a$ and $\delta$.

The following variables are endogenous (derived by the model within the planning interval):

1. Strategies for optimal allocation of primary factors, [e.g., (56) and (57)], $X_{\nu i}(t)$, $\nu = 1,\ldots,m$, $i = 1,2$, $t = 1,2,\ldots,T$, and corresponding optimal transfer (migration) strategies, $x_\nu(t)$ [by minimizing (58)];
2. Output prices for goods produced, $p_i(t)$, $i = 1,\ldots,n$ [e.g., (28)];
3. Optimal allocation of remaining factors, $\hat{x}_\nu(t)$, $\nu = m,\ldots,\bar{m}$, $t = 1,\ldots,T$ [e.g., (20) and (27)];
4. Output production, $\bar{y}$, consumption share in the output, etc. [e.g., (29)];
5. The rent for necessary services, $r_1(t)$, $t = 1,\ldots,T$ [e.g., (64)], which should be offered to migrants when they agree to live in the urban area;
6. Necessary investments, $z_{\nu i}(t)$, $\nu = 1,\ldots,m$, $t = 0,1,\ldots,T$, that constitute part of the regional budget [e.g., (59)].

The model can be easily extended to the multisector case if necessary. As shown by Kulikowski (1977), the general methodology can be extended to economies that are described by a C.E.S., rather than a Cobb-Douglas production function.

Using the model proposed, one can also compare the impact of different decisions regarding the allocation of factors, employment, and migration on regional development and, in particular, the impact of optimal decisions. It is possible to connect the regional model to a national model, of the type described by Kulikowski (1976), which generates the exogenous variables $\omega_\nu(t)$, $p_i(t)$, $\nu = 1,\ldots,m$, $i = 1,\ldots,n$. In order to use the model in planning a computer-operated version, a special system for dialogue with the decision maker should be constructed.

A simple rural–urban version of the model, based on the methodology described, has been implemented in the computerized, interactive form. The model has been used to test the alternative water system development strategies for the Notec region in central Poland.

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