International Institute for Applied Systems Analysis
IIASA www.iiasa.ac.at

A Decision Theoretic Model for Standard Setting and Regulation

von Winterfeldt, D.

IIASA Research Memorandum
February 1978

RM-78-7

A DECISION THEORETIC MODEL FOR STANDARD SETTING AND REGULATION

Detlof v. Winterfeldt

February 1978

PREFACE

Standard setting is one of the most commonly used regulatory tools to limit detrimental effects of technologies on human health, safety, and psychological well being. Standards also work as major constraints on technological development, particularly in the energy field. The trade-offs which have to be made between economical, engineering, environmental, and political objectives, the high uncertainty about environmental effects, and the conflicting interests of groups involved in standard setting, make the regulatory task exceedingly difficult.

Realizing this difficulty, the Volkswagen Foundation sponsored a research subtask in IIASA's Energy Program under the name "Procedures for the Establishment of Standards". The objectives of this research are to analyze existing procedures for standard setting and to develop new techniques to improve the regulatory decision making process. The research performed under this project include:

i) policy analyses of the institutional aspects of standard setting and comparisons with other regulatory tools;

ii) case studies of ongoing or past standard setting processes (e.g. oil discharge standards or noise standards);

iii) development of formal methods for standard setting based on decision and game theory;

iv) applications of these methods to real world standard setting problems.

The present research memorandum is one in a series of papers dealing with the development and application of decision theoretic methods to standard setting. It presents the formal basis of the decision theoretic model for standard setting.

ABSTRACT

This paper presents a decision theoretic model which was developed to aid regulatory agencies in standard setting and regulation tasks. The one stage three decision maker model encompasses the decision making of a regulator, a developer, and an impactee unit. Each decision unit is assumed to follow a basic decision model, which is a combination of a probability model, a difference value judgment model, and an expected utility model. The developer unit is linked to the regulator unit through possible detections of violations of a regulation and sanctions. The impactee unit is linked to the developer unit through pollution generating events stemming from the developer's actions, and the subsequent damages which may result from pollution.

This basic regulation model is then specified to safety and emission standard setting. Central in these specifications is a signal detection model which characterizes the uncertainty with which the regulator will detect or miss violations of his regulation. A multistage conditional probability model links the developer's actions, pollution generating events, amounts of pollutants, and possible effects on impactees.

A DECISION THEORETIC MODEL FOR STANDARD SETTING AND REGULATION

INTRODUCTION

Setting standards such as emission or ambient standards on chemicals polluting the air or the water is one of the most widely used regulatory tools to limit the negative effects of industrial activities on human health, safety, and psychological well being. Scientists and administrators in regulatory agencies who have to set such standards agree that the task is exceedingly difficult. There usually exists a vast amount of uncertainty about the effects of pollutants on human well being. Crucial trade-offs have to be made among multiple objectives which are often conflicting such as engineering, economic, political and environmental objectives. Conflicting interest groups are involved in standard setting each backing their case with different expert reports.

Since standard setting decisions are important, recurrent and exceedingly difficult, several attempts have been made to develop procedures along which a regulator and his experts can organize their information collection and evaluation tasks. Several authors have suggested and applied cost benefit analysis to solve the problem of standard setting and regulation (see, for example, Dorfman and Jacoby, 1970; North and Merkhofer, 1975; Karam and Morgan, 1975). But decision makers and scientists are often skeptical about the use of cost benefit analysis for such complex problems. The main reasons for this skepticism are that many values can not be expressed in Dollar terms and that the political character of the decision process is not taken into account (see, for example, Holden, 1966; Majone, 1976; Reports by the US Academy of Sciences, 1975; and by the National Research Council, 1977).

To aid regulators and scientists in standard setting tasks, therefore, new procedures and methods are called for. These procedures could include new institutional mechanisms (e.g. public participation, science courts, etc.) and new "softer" modeling approaches (e.g. decision theory and game theory). This paper concentrates on the second type of procedural innovation. It presents a formal decision theoretic model that was designed to help regulators to structure a standard setting task, to express uncertainties in a quantifiable form, and to evaluate alternative regulation and standards in the light of conflicting objectives. The paper is addressed mainly to decision theorists and operation researchers who are interested in the quantitative aspects of the model. Readers interested in the qualitative model structure are referred to von Winterfeldt (1978), and Fischer and von Winterfeldt (1978).

The paper is organized as follows. First the general decision theoretic model for a single decision maker will be developed. Readers familiar with measurement and decision theory on the level of DeGroot (1970), Fishburn (1970) and Krantz, Luce, Suppes and Tversky (1971) may wish to skip this part. The second section adapts the single decision maker model to a regulation model in which the decision making of a regulator, developer (producer) and impactee (sufferer) unit are linked. The third part of the paper details the general regulation model to the specific circumstances of standard setting.

BASIC DECISION MODEL FOR A SINGLE DECISION MAKER

The following mathematical formulation of the basic decision theoretic model is a modified version of the usual expected utility model (von Neumann and Morgenstern, 1947) which is developed, for example in Raiffa and Schlaifer (1961), DeGroot (1970) and Fishburn (1970). It differs from the basic expected utility formulation in two aspects: first, it does not assume that conditional on event and action combinations final consequences can be predicted with certainty, but it leaves the possibility that there is a residual uncertainty about final consequences; second, it does not directly construct a von Neumann and Morgenstern utility function, but rather constructs it through an additive difference value function which can be shown to be functionally related to a von Neumann and Morgenstern utility function.

Let A be the set of decision alternatives (courses of action) with typical elements $a,b \in A$. Let S be a set of mutually exclusive and exhaustive events with typical elements $s,t \in S$. Let $C = \overset{n}{\underset{i=1}{X}} C_i$ be a subset of $\mathbb{R}^n$ which characterizes the possible consequences of act-event combinations. Typical elements of C are n-dimensional vectors $\underline{c}$, $\underline{d}$. Finally, let Z be the set of possible information sources with typical elements $y,z \in Z$.

The model assumes that the decision maker and his experts can quantify their uncertainty by a judgmental probability distribution (pd) over events and a probability density function (pdf) over consequences:

1) A probability distribution p which assigns to each event $s \in S$ a probability $p(s|a,z)$ depending on information source z and act a.

2) A probability density function f which assigns to each point $\underline{c} \in C$ a probability density $f(\underline{c}|a,s,z)$, depending on an act a, event s and information source z.

The residual uncertainty expressed in f can often be represented by independent marginal pdfs f_i. Therefore the following assumptions can be made:

$$f(\underline{c}|a,s,z) = \prod_{i=1}^{n} f_i(c_i|a,s,z) \quad , \tag{1}$$

where $c_i \in C_i$.

The total uncertainty about consequences given an act a and information source z can then be expressed by the following pdf g :

$$g(\underline{c}|a,z) = \sum_{s \in S} p(s|a,z) \prod_{i=1}^{n} f_i(c_i|a,s,z) \quad . \tag{2}$$

The set of all such probability density functions will be called F with typical elements $f, g \in F$,

The model assumes further that the decision maker has preferences among probability density functions, which can be characterized by the ordered set $\langle F, \succsim \rangle$. The interpretation of "$f \succsim g$" is that "f is preferred to or indifferent to g". The assumption which will be made in the following is that <u>$\langle F, \succsim \rangle$ is an expected utility structure</u>, i.e. that $\succsim$ obeys the axioms of expected utility theory (see von Neumann and Morgenstern, 1947; Savage, 1954; Fishburn, 1970). Therefore, there exists a function u: $C \to \mathbb{R}$ such that for all $f, g \in F$

$$f \succsim g$$

if and only if

$$\int_C f(\underline{c})u(\underline{c})d\underline{c} \geq \int_C g(\underline{c})u(\underline{c})d\underline{c} \quad . \tag{3}$$

Next, the model assumes that the decision maker or his experts can express their strength of preferences of one consequence over another, which can be characterized by the ordered set $\langle C \times C, \dot{\succsim} \rangle$. The interpretation of "$(\underline{c},\underline{d}) \dot{\succsim} (\underline{c}',\underline{d}')$" is that "the degree of preference of $\underline{c}$ over $\underline{d}$ is larger or equal to the degree of preference of $\underline{c}'$ over $\underline{d}'$". In the following the assumption will be made that <u>$\langle C \times C, \dot{\succsim} \rangle$ is an algebraic difference structure</u> i.e. that $\dot{\succsim}$ obeys the axioms of algebraic difference measurement (Suppes and Winet, 1955; Krantz, Luce, Suppes and Tversky, 1971). It follows that there exists a function v: $C \to \mathbb{R}$ such that for all $\underline{c},\underline{d},\underline{c}',\underline{d}' \in C$

$$(\underline{c},\underline{d}) \dot{\succsim} (\underline{c}',\underline{d}')$$

if and only if

$$v(\underline{c})-v(\underline{d}) \geq v(\underline{c}')-v(\underline{d}') \, . \tag{4}$$

Both functions v and u express some evaluation aspect about consequences C, but there is no basis in the assumptions behind (3) and (4) that would establish a relationship between them. In particular there is no reason to assume that the expectation of v preserves the preference order of $f,g \in F$. To distinguish, u will be called a utility function, and v will be called a value function.

The remainder of the model description for the single decision maker will be concerned with establishing decomposition forms of u and v and functional relationships between u and v. In particular, independence assumptions on preferences among pdfs and on preference strength judgments will be made that lead to additive or multiplicative decompositions of u and v into single consequence functions u_i and v_i. Some further assumptions will then be used to show that the relationship between u and v must be either linear or exponential.

The reason for this type of model formulation is largely pragmatic. Assuming that v is additive, single consequence difference value functions v_i can be assessed through rather simple preference strength judgments. Given that u and v are related by a simple function, one can then construct a utility function u from v by assessing the parameters of the transformation either through sensitivity analysis or by asking a few simple insurance type questions. Thus the construction process, while ending with a von Neumann and Morgenstern utility function, circumvents the sometimes awkward lottery assessment methods which would have to be used otherwise. In addition, this type of model has the advantage of separating clearly between concepts of "marginal utility" (which has a place in the algebraic difference structure) and "risk attitude" (which has a place in the expected utility structure).

As a first step the preference relation and the preference strength relations are coupled. Preferences $\gtrsim$ are defined over F, but they can easily be applied to C by defining

$$\underline{c} \overset{'}{\gtrsim} \underline{d}$$

if and only if

$$f(\underline{c}) = g(\underline{d}) = 1 \text{ and } f \gtrsim g \ . \tag{5}$$

Another induced preference relation can be defined by

$$\underline{c} \overset{''}{\gtrsim} \underline{d}$$

if and only if

$$(\underline{c},\underline{d}) \overset{*}{\gtrsim} (\underline{d},\underline{d}) \ . \tag{6}$$

Nothing guarantees that $\overset{'}{\gtrsim} = \overset{''}{\gtrsim}$. However, from a judgmental point of view this equality seems plausible. Therefore, the following assumption will be made:

For all $\underline{c},\underline{d} \in C$

$$\underline{c} \overset{'}{\succsim} \underline{d}$$

if and only if

$$\underline{c} \overset{''}{\succsim} \underline{d} \quad . \tag{7}$$

If there are no ambiguities $\succsim$ will from now on be substituted for $\overset{'}{\succsim}$ and $\overset{''}{\succsim}$.

From definitions (5) and (6) and from assumption (7) it is obvious that there exists a functional relationship between u and v and that this relationship must be monotonically increasing:

$$\begin{array}{lll} u(\underline{c}) \geqslant u(\underline{d}) & \text{if and only if} & \\ & & \text{(by 3 and 5)} \\ \underline{c} \overset{'}{\succsim} \underline{d} & \text{if and only if} & \\ & & \text{(by 7)} \\ \underline{c} \overset{''}{\succsim} \underline{d} & \text{if and only if} & \\ & & \text{(by 6)} \\ (\underline{c},\underline{d}) \dot{\succsim} (\underline{d},\underline{d}) & \text{if and only if} & \\ & & \text{(by 4)} \\ v(\underline{c}) \geqslant v(\underline{d}), \text{ for all } \underline{c},\underline{d} \in C. & & \end{array} \tag{8}$$

Thus u and v are both order preserving functions for preference over C and by the uniqueness of such functions (see Suppes and Zinnes, 1963; Krantz, Luce, Suppes, and Tversky, 1971) two such functions must be related through a monotonically increasing transformation u=h(v).

Next, the assumption will be made that v is additive:

$$v(\underline{c}) = \sum_{i=1}^{n} v_i(c_i), \quad c_i \in C_i \quad . \tag{9}$$

In the appendix an independence condition for $\dot{\succsim}$ is defined and a proof is given that this independence condition is sufficient for (9).

With respect to the decomposition of u, two possibilities are considered in the model:

Either

$$u(\underline{c}) = \sum_{i=1}^{n} u_i(c_i) \tag{10}$$

or

$$1+ku(\underline{c}) = \prod_{i=1}^{n} [1+ku_i(c_i)] \quad . \tag{11}$$

(10) and (11) are the classic decomposition forms of u in expected utility theory, and independence assumptions and proofs for these forms can be found, for example in Fishburn, 1970; Keeney, 1974; Keeney and Raiffa,1976.

The uniqueness property of additive value functions (see Fishburn, 1970; Krantz, Luce, Suppes and Tversky, 1971) states that any two additive order preserving functions defined on C must be related by a positive linear transformation. From this uniqueness property and assuming (9) and (10) the following relationship between v and u results:

$$u = \alpha v + \beta \quad \text{for some} \quad \alpha>0,\ \beta\in\mathcal{R} \quad . \tag{12}$$

Again using the uniqueness property, but this time assuming (9) and (11) an exponential relationship results:

$$u = \frac{1}{k} e^{\alpha v+\beta} - \frac{1}{k} \quad \begin{array}{l} \text{for some } \alpha>o,\beta \text{ if } k>o \ , \\ \alpha<o,\beta \text{ if } k<o \quad . \end{array} \tag{13}$$

In (13) α is a risk attitude parameter which, unlike usual risk parameters (see Raiffa, 1968), is not confounded with marginal value considerations. Marginal value considerations are expressed solely in v. The relationship between α and k is that $\alpha>o$ if $k>o$ and $\alpha<o$ if $k<o$. If $k=o$ then (12) holds. These results are proven and discussed in the appendix.

With definitions and assumptions (1) - (13) an evaluation function U can now be defined on AxZ which is consistent with a rational decision maker's preference and probability judgments:

$$U(a,z)=\int_C [\sum_{s\in S} p(s|a,z) \prod_{i=1}^{n} f_i(c_i|a,s,z)]h[\sum_{i=1}^{n} v_i(c_i)]d\underline{c}. \tag{14}$$

where h is either linear or exponential as defined in (12) and (13).

Given the appropriate choice of the functions like p, f, h, and v the decision rule that logically follows from the assumption made is:

select $a^* \in A$ with

$$U(a^*,z) = \max_{A} U(a,z). \tag{15}$$

Clearly one could also maximize over Z, the possible information sources. This would amount to a value of information analysis. Within the proposed model this could be done provided that the cost of information is included in the consequence space C. More specific models which assume additive costs of information and decisions could also be considered. Furthermore, additional information could be considered (e.g. a research study, an independent expert estimate) and a so called preposterior analysis of the value of additional information could be carried out (see Raiffa and Schlaifer, 1961).

The model allows a simple construction of the model functions. Decision theoretic techniques are available to construct pdfs, pds, and value functions in (14), and some of them have been developed to a high degree of sophistication. To construct pds odds comparison techniques can be used (see Spetzler and von Holstein, 1975). To construct pdfs over continuous random variables one would use either of two techniques: the fractile method (see Brown, Peterson and Kahr, 1974) or direct probability estimation techniques (see Spetzler and von Holstein, 1975). To construct the value function v rating and weighting techniques can be used which approximate quite closely the theoretically feasible techniques of using indifference judgments about value differences (Edwards, 1971). Within model (14) the transformation h from v into u is already so restricted that a few questions about insurance behavior and risk preferences should be sufficient to assess the parameters of the exponential form. If the transformation h is linear, one can use the value function directly as input into (14) without choosing transformation constants α and β, since utility functions are unique up to a linear positive transformation (see Krantz et al., 1971; Keeney and Raiffa, 1976).

THE DECISION MODEL FOR ENVIRONMENTAL REGULATION

In regulatory decision making the decision theoretic model described above becomes much more complicated, since several decision makers (or better: decision making units, organizations) are involved rather than an individual. These decision units have their own objectives, alternatives, and opinions. In a previous study on regulatory decision making (Fischer and von Winterfeldt, 1978) five main decision units were identified which enter into regulatory decision making. Generically they are called here:

1. the regulator unit
2. the developer unit

3. the impactee unit
4. the expert unit
5. the exogenous unit

The regulator unit consists of the people and institutions that have the responsibility for setting regulations as, for example, the Central Unit for Environmental Pollution in the UK, the State Pollution Control Agency in Norway, or the EPA in the US. The developer unit is defined as all those people and organizations whose span of decision alternatives is restricted by the regulation. In the impactee unit all those groups are combined whose activities or perceptions are impacted upon by the development activities. The expert unit consists of researchers and other experts that can provide information bearing on the regulatory problem. The exogenous unit is defined as those national or international organizations which constrain the decision making of the regulatory unit.

A good first approximation to the regulation problem was found to be achieved by considering the first three decision units only, and by identifying the elements of the expert unit with sources of information $z \in Z$. The resulting regulator-developer-impactee model has a structure which is schematically represented in Fig.1.

Each decision unit has its own alternatives (as well as sets of relevant events, information sources, consequence spaces, pdfs, value functions, etc.). If the regulator decides on a particular regulation alternative r, the decision making of the developer will be influenced by the possibility of sanctions if violations of the regulation r are detected. Thus the developer will respond to a regulation r by an action d(r) which may not be an action he would have taken without regulation.

Through pollution and their adverse effect on health and well being the developer influences the decision making of the impactee unit. In response to a developer decision d, the impactees will choose an action a(d) which may not be the action which they would have taken without the development activity.

The idea of the model is to determine optimal decisions d(r) and a[d(r)] for the developer and the impactee as a function of r, together with the associated utilities $U_R(r), U_D[d(r)]$, $U_A\{a[d(r)]\}$. Further aggregation or Pareto optimality analysis may then be used to focus on a "good" value of r.

In the regulator-developer-impactee model each unit is represented by model (14). The notational specifications are given in Table 1.

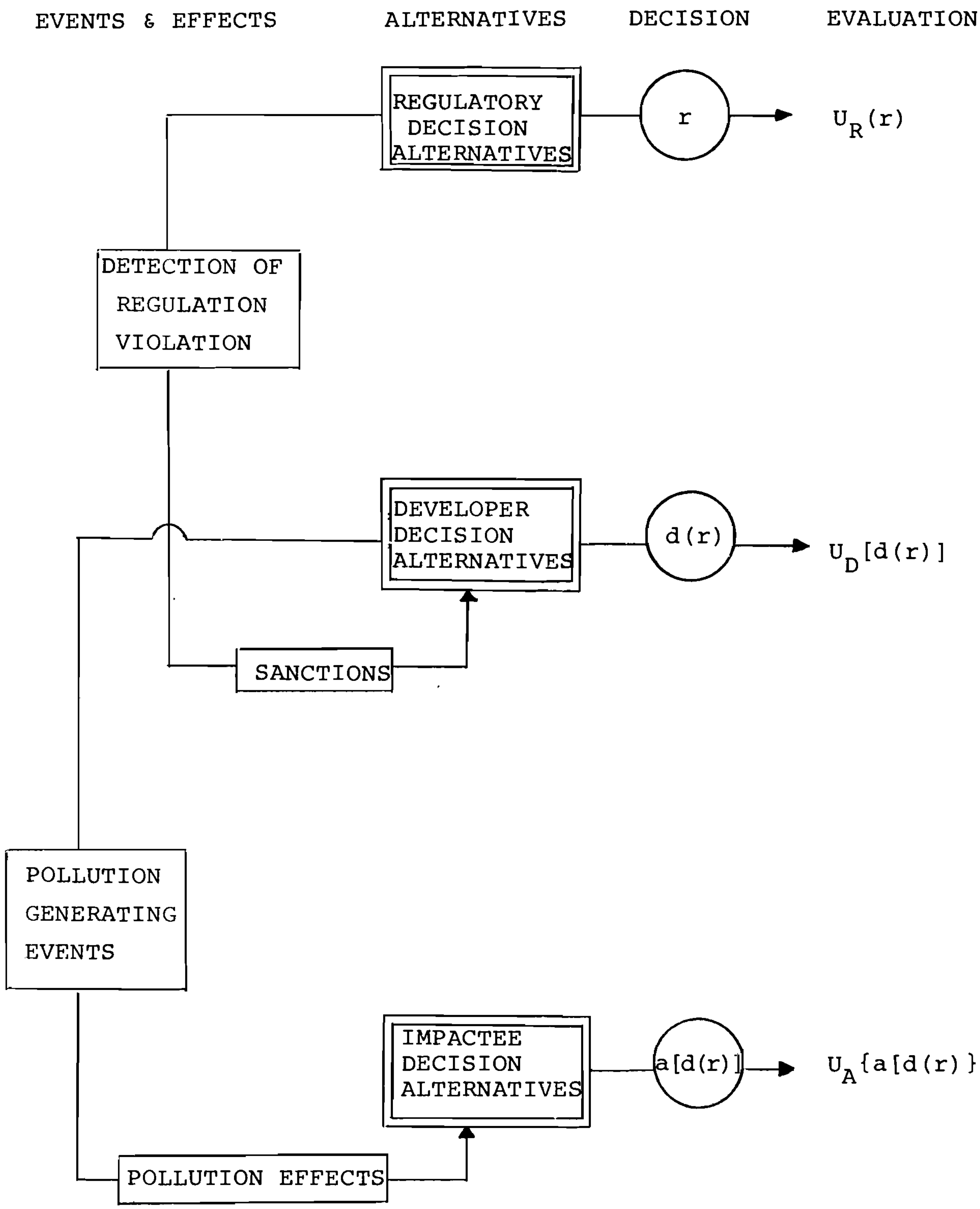

Figure 1. Schematic Representation of the Regulator-Developer-Impactee Model

Table 1. Notation for the Regulation Model

	Set	Element
Regulator's alternatives	R	r
Developer's alternatives	D	d
Impactee's alternatives	A	a
Regulator's consequences	C_R	$\underline{c}_R$
Developer's consequences	C_D	$\underline{c}_D$
Impactee's consequences	C_A	$\underline{c}_A$
Regulator's events	S_R	s_R
Developer's events	S_D	s_D
Impactee's events	S_A	s_A

The functions, pds, and pdfs $p_R, p_D, p_A, f_R, f_D, f_A, v_R, v_D, v_A, h_R, h_D, h_A$ are also assumed. For simplicity the information sources Z_R, Z_D, Z_A are not spelled out any more and it is implicitly understood that all pds and pdfs of a decision unit are conditioned on an element $z_.$ from the information source $Z_.$ belonging to his unit.

Without specifying interlinkage models "detection of violation and sanction" and "pollution generation and effects" the evaluation of acts for each decision unit according to model (14) depends on the acts of other units, since $p_.$ and $f_.$ are dependent on all acts:

$$U_.(r,d,a)=\int_{C_.}[\sum_{s_.\in S_.} p_.(s_.|r,d,a)f_.(\underline{c}_.|r,d,a,s_.)]\cdot h_.[v_.(\underline{c}_.)]d\underline{c}_. \quad (15)$$

where "." stands for R, D, and A respectively. Let Q_o and Q_1 denote the events "non-detection" and "detection" of a regulation violation, and let $s_1, s_2, \ldots, s_K$ be the set of pollution generating events (e.g. explosions, normal operation of equipment, etc.). The model defines $S_R = \emptyset$, $S_D = \{Q_o, Q_1\}$ and $S_A = \{s_1, s_2, \ldots, s_K\}$. The following crucial independence assumptions are now made to simplify (15):

$$f_R(\underline{c}_R|r,d,a) = f_R(\underline{c}_R|r) \quad , \quad (16)$$

$$p_D(Q_o|r,d,a) = p_D(Q_o|r,d) = 1 - p_D(Q_1|r,d) \quad , \qquad (17)$$

$$f_D(\underline{c}_D|r,d,a,Q_k) = f_D(\underline{c}_D|r,d,Q_k) \quad k = 1,2 \quad , \qquad (18)$$

$$p_A(s_j|r,d,a) = p_A(s_j|d) \quad j = 1,2,\ldots,K \quad , \qquad (19)$$

$$f_A(\underline{c}_A|r,d,a,s_j) = f_A(\underline{c}_A|d,a,s_j) \quad j = 1,2,\ldots,K \ . \qquad (20)$$

Verbally, these assumptions mean that:

(16) *The regulator's consequences only depend on his own action;*

(17) *Detection probabilities do not depend on the impactee's action;*

(18) *The developer's consequences do not depend on the impactee's action;*

(19) *Pollution generating event probabilities depend only on the developer's action;*

(20) *The impactee's consequences do not depend on the regulator's action.*

Therefore (15) can be written as follows:

$$U_R(r) = \int_{C_R} f_R\,(\underline{c}_R|r)\cdot h_R[v_R(\underline{c}_R)]\,d\underline{c}_R \quad , \qquad (21)$$

$$U_D(d,r) = \int_{C_D} [\sum_{k=o}^{1} p_D(Q_k|r,d) f_D(\underline{c}_D|r,d,Q_k)]\, h_D[v_D(\underline{c}_D)]\,d\underline{c}_D , \qquad (22)$$

$$U_A(a,d) = \int_{C_A} [\sum_{j=1}^{K} p_A(s_j|d) f_A(\underline{c}_A|d,a,s_j)]\, h_A[v_A(\underline{c}_A)]\,d\underline{c}_A \ . \qquad (23)$$

Independence of C_i, additivity of v, and the linear or exponential form of h lead to a further refinement of (21) - (23). Defining the optimal decisions of the developer and the impactee by

$$d(r):\ U_D[d(r),r] \geqslant U_D(d,r) \quad \text{for all } d \in D \quad , \qquad (24)$$

$$a(d):\ U_A[a(d),d] \geqslant U_A(a,d) \quad \text{for all } a \in A \quad , \qquad (25)$$

model (14) applied to all three decision units allows the determination of the optimal responses of the developer and impactee to a regulation r as well as the associated utilities $U_R(r)$, $U_D[d(r),r]$ and $U_A\{a[d(r)],d(r)\}$ for all three units as a function of r. A Pareto optimality analysis can now be performed on the $U_{\cdot}$, or, as a final step, one could postulate for some λ.

$$\bar{U}(r)=\lambda_R \cdot U_R(r) + \lambda_D U_D[d(r),r] + \lambda_A U_A\{a[d(r)],d(r)\} \quad , \quad (26)$$

with the optimal regulation defined as

$$r^* : \bar{U}(r^*) \geqslant \bar{U}(r) \quad \text{for all} \quad r \in R \quad . \quad (27)$$

THE DECISION MODEL FOR STANDARD SETTING

Model (15) - (27) will now be adapted to the specific circumstances arising in environmental standard setting as a particular type of regulation. The regulator's alternatives R will be further specified and the detection and consequence probabilities of the regulator and developer will be decomposed.

First, the regulatory alternatives R in standard setting are defined to consist of a set of standards SL, a set of monitoring and inspection procedures SM, and a set of possible sanctions for violations SS. Typical elements will be labelled *sl*, *sm*, and *ss* respectively. The developer's alternatives D are thought of as technological processes, equipments, and operations to reduce pollution risks and hazards. The impactee's alternatives are not further specified, and often treated as a dummy variable, i.e. the impactees are considered "sufferers".

Two classes of standards can be distinguished: safety standards are set by the regulator on D directly in order to reduce the probability of undesired events S_A; emission standards are set on amounts or rates of pollution in those cases in which for any choice of $d \in D$ the event "normal operation" can be taken for granted, but when such normal operations generate a constant flow of pollution.

Safety standards are considered first. In this case SL is defined as the set of all subsets of D. Consequently, if $sl \in SL$, then $sl \subseteq D$. D is considered to exist of mutually exclusive elements.

A violation of *sl* occurs if $d \notin sl$, otherwise the regulation is adhered to. The model assumes that the regulator cannot perfectly discriminate whether $d \in sl$ or $d \notin sl$, because of the limits of his monitoring and inspection procedure $sm \in SM$. Instead of d the regulator perceives $\hat{d} \in D$ with a probability distribution $p(\hat{d}|d,sm)$ which depends on d and *sm* only. A regulation violation is detected if $\hat{d} \notin sl$, not detected if $\hat{d} \in sl$. Detection probabilities (17) can now be specified:

$$p_D(Q_O|r,d) = \sum_{\hat{d} \in sl} p(\hat{d}|d,sm) \quad , \quad (28)$$

$$p_D(Q_1|r,d) = \sum_{\hat{d} \notin sl} p(\hat{d}|d,sm) \quad . \tag{29}$$

This formulation leaves the possibility open that the regulator detects a violation ($\hat{d} \notin sl$) when in fact no such violation occurred ($d \in sl$) and vice versa. In signal detection theory (Green and Swets, 1967) these cases are called "false alarm" and "miss".

The safety standards model assumes further that consequences accruing to the developer as expressed in (18) depend on regulatory action only through detections and sanctions:

$$f_D(\underline{c}_D|r,d,Q_k) = f_D(\underline{c}_D|ss,d,Q_k) \quad k = 0,1 \quad . \tag{30}$$

The total consequence distribution is then

$$f_D(\underline{c}_D|r,d) = \sum_{d \in sl} p(\hat{d}|d,sm) \cdot f_D(\underline{c}_D|ss,d,Q_o) +$$

$$+ \sum_{\hat{d} \notin sl} p(\hat{d}|d,sm) \cdot f_D(\underline{c}_D|ss,d,Q_1) \quad . \tag{31}$$

Turning now to the consequence distribution of the impactee, let $l \in L \subset \mathcal{R}$ be an amount or rate of pollution which is considered a random variable with pdf $f(l|s_j)$.

The safety standards model assumes that, given l, the consequences accruing to the impactees do not depend any more on d and s_j:

$$f_A(\underline{c}_A|d,a,s_j,l) = f_A(\underline{c}_A|a,l) \tag{32}$$

Therefore, (2) can be written as

$$f_A(\underline{c}_A|d,a,s_j) = \int_L f(l|s_j) \, f_A(\underline{c}_A|a,l)\,dl \quad . \tag{33}$$

Event probabilities s_j are assumed to be independent of impactee actions $a \in A$:

$$p_A(s_j|d,a) = p_A(s_j|d) \quad . \tag{34}$$

Therefore the total consequence distribution for the impactee unit is:

$$f_A(\underline{c}_A|d,a) = \sum_{j=1}^{K} p_A(s_j|d) \int_L f(l|s_j) f_A(\underline{c}_A|a,l)\,dl \ . \tag{35}$$

These specifications leave the regulator unit unchanged, thus (21) still is the regulator's utility function. But substituting (31) into (22) and (35) into (23) gives new utility functions for the developer and the impactee. The three full utility functions for the safety standards model are

$$U_R(r) = \int_{C_R} f_R(\underline{c}_R|r)\; h_R[v_R(\underline{c}_R)]\,d\underline{c}_R \ , \tag{36}$$

$$U_D(r,d) = \int_{C_D} \{ [\sum_{\hat{d}\in sl} p(\hat{d}|d,sm)\; f_D(\underline{c}_D|ss,d,Q_o) \; + \; + \sum_{\hat{d}\notin sl} p(\hat{d}|d,sm)\; f_D(\underline{c}_D|ss,d,Q_1)]\} \cdot \cdot\; h_D[v_D(\underline{c}_D)]\,d\underline{c}_D \tag{37}$$

$$U_A(d,a) = \int_{C_A} [\sum_{j=1}^{K} p_A(s_j|d) \int_L f(l|s_j) \cdot \cdot\; f_A(\underline{c}_A|a,l)\,dl]\; h_A[v_A(\underline{c}_A)]\,d\underline{c}_A \ . \tag{38}$$

$d(r)$, $a[d(r)]$, and $\bar{U}(r)$ are determined as in (24), (25) and (26).

The emission standard version of the model defines sl not as a subset of D, but rather as an element of L. $sl\in L$ is interpreted as a maximum admissible amount of emission. S_A is now assumed to consist only of one element, say s_1, the event of "normal operation" of $d\in D$ for all d. This leads to

$$p_A(s_j|d) = 1 \text{ for } j = 1$$
$$p_A(s_j|d) = 0 \text{ for } j \neq 1 \tag{39}$$

Let $l\in L\subset\mathcal{R}$ be the amount or rate of emissions under s_1 and d with pdf $f(l|s_1,d)$. Since s_1 is the same for all developer decisions d, it will from now on be dropped.

The regulator tries to establish whether $l>sl$ (violation) or $l\leq sl$ (no violation). However, his monitoring and inspection procedure does not provide him with perfectly reliable information. Let $\hat{l}$ be a reading of emissions that the monitoring and inspection procedure registers. Let $g(\hat{l}|l,sm)$ be the reliability pdf that characterizes the quality of sm. Let q_o be the state "violation occurs $(l>sl)$" and q_1 the state "no violation occurs ($l \leq sl$)." The probabilistic relationship between the violation and detection states Q_o, Q_1, q_o, q_1 can be expressed in a fourfold table:

	q_o	q_1	
Q_o	p_{oo}	p_{o1}	$p_D(Q_o)$
Q_1	p_{1o}	p_{11}	$p_D(Q_1)$
	$p_D(q_o)$	$p_D(q_1)$	

Where the p_{ij}'s are the joint probabilities and the marginals are defined as:

$$p_D(q_o|d,sl) = \Pr[l \leq sl|d] \tag{40a}$$

$$p_D(q_1|d,sl) = \Pr[l > sl|d] \tag{40b}$$

$$p_D(Q_o|d,sl,sm) = \Pr[\hat{l} \leq sl|d,sm] \tag{40c}$$

$$p_D(Q_1|d,sl,sm) = \Pr[\hat{l} > sl|d,sm] \tag{40d}$$

From the above definitions the marginal probabilities (40a) and (40b) can be directly inferred:

$$p_D(q_o|d,sl) = \int_{l \leq sl} f(l|d)\,dl \quad , \tag{41a}$$

$$p_D(q_1|d,sl) = \int_{l > sl} f(l|d)\,dl \quad . \tag{41b}$$

The consequences that accrue to the developer depend, however, not on the true states of violation q_k but rather on the states of detection Q_k. These probabilities can usually not be inferred directly. But with the knowledge of $g(\hat{l}|l,sm)$ the joint probabilities p_{ij} can be determined as follows:

$$p_{oo} = \int_{\hat{l} \leq sl} \int_{l \leq sl} f(l|d) \cdot g(\hat{l}|l,sm)\, dl\, d\hat{l} \quad , \tag{42a}$$

$$p_{1o} = \int_{\hat{l} > sl} \int_{l \leq sl} f(l|d) \cdot g(\hat{l}|l,sm)\, dl\, d\hat{l} \quad , \tag{42b}$$

$$p_{o1} = \int_{\hat{l} \leq sl} \int_{l > sl} f(l|d) \cdot g(\hat{l}|l,sm)\, dl\, d\hat{l} \quad , \tag{42c}$$

$$p_{11} = \int_{\hat{l} > sl} \int_{l > sl} f(l|d) \cdot g(\hat{l}|l,sm)\, dl\, d\hat{l} \quad . \tag{42d}$$

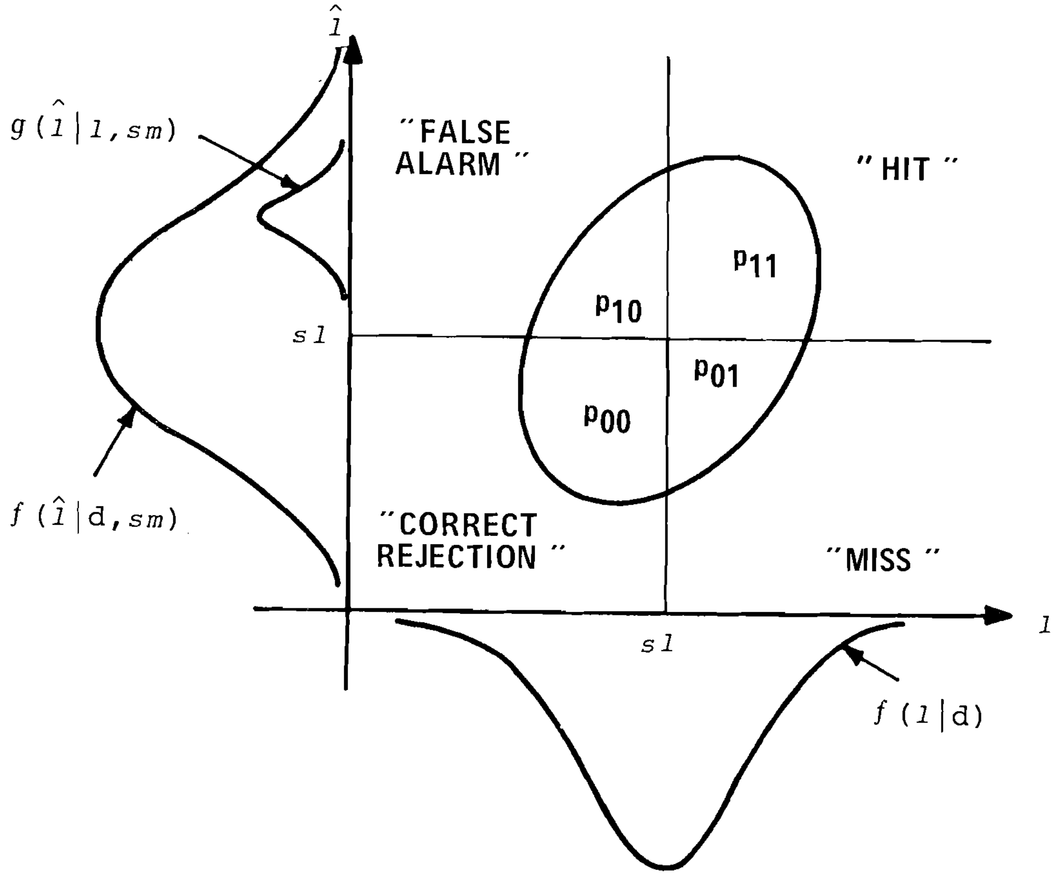

Figure 2. Graphic Representation of the Probabilistic Relationship between Violations and Detections (39-42)

The marginal probabilities of interest, $p_D(Q_o)$ and $p_D(Q_1)$, can now be determined as

$$p_D(Q_o|d,sl,sm) = \int_{\hat{l} \leq sl} \int_L f(l|d) \cdot g(\hat{l}|l,sm)\, dl d\hat{l} \quad , \quad (43a)$$

$$p_D(Q_1|d,sl,sm) = \int_{\hat{l} > sl} \int_L f(l|d) \cdot g(\hat{l}|l,sm)\, dl d\hat{l} \quad . \quad (43b)$$

These detection (non-detection) probabilities refer to a fixed time interval Δt. One could assume that this time interval is the lifetime of the plant. However, it is more realistic to assume that Δt is some "normal" time interval of inspection (e.g. one week). Then, over the lifetime of the plant T there will be $T/\Delta t$ possibilities for such detection. Let $n \cong T/\Delta t$; assuming that detection probabilities are constant over time and independent, the probability of no detection during T, labelled Q_o^n, is

$$p_D(Q_o^n|d,sl,sm) = p_D(Q_o|d,sl,sm)^n \quad . \tag{44a}$$

The probability of at least one detection during T, labelled Q_1^n, is

$$p_D(Q_1^n|d,sl,sm) = 1 - p_D(Q_o^n) \quad . \tag{44b}$$

The consequences accruing to the developer during T as expressed in f_D are now assumed to depend only on d,*ss*, and Q_k^n. (That is, as far as the developer is concerned, one detection is as bad as n):

$$f_D(\underline{c}_D|d,r) = p_D(Q_o^n|d,sl,sm) \cdot f_D(\underline{c}_D|ss,d,Q_o^n) +$$

$$+ p_D(Q_1^n|d,sl,sm) \cdot f_D(\underline{c}_D|ss,d,Q_o^n) =$$

$$= [\int_{\hat{l} \leqslant sl} \int_L f(l|d) \cdot g(\hat{l}|l,sm)\ dl d\hat{l}]^n \cdot$$

$$\cdot f_D(\underline{c}_D|ss,d,Q_o^n) +$$

$$+\{1-[\int_{\hat{l} \leqslant sl} \int_L f(l|d) \cdot g(\hat{l}|l,sm)\ dl d\hat{l}]^n\} \cdot$$

$$\cdot f_D(\underline{c}_D|ss,d,Q_1^n) \quad . \tag{45}$$

As a further decomposition of the impactee's pdf f_A is concerned, the emission standards model follows the route of the safety standards model as expressed in (35). However, in emission standards, the picture gets simplified since $p_A(s_1|d) = 1$. Therefore, (35) becomes

$$f_A(\underline{c}_A|d,a) = \int_L f(l|d) \cdot f_A(\underline{c}_A|a,l)\ dl \quad . \tag{46}$$

Equation (46) expresses the usual view of the pollution problem as mediated through the levels of discharges into the environment. For each specific level probable consequences follow for the impactees. Equation (46) is the first step in the direction of decomposing the chain of events from emissions, over ambient distributions to final value relevant consequences. One could include another probability density function over ambient levels. One could then condition the consequence distribution f_A on ambient levels, rather than on emission levels, and define the total consequence distribution as a mixture of emission, ambient and the respectively conditioned consequence distribution for given ambient levels.

The full utility equations can now be written out for the emission standards model:

$$U_R(r) = \int_{C_R} f_R(\underline{c}_R|r)\ h_r[v_R(\underline{c}_R)]\ d\underline{c}_R \qquad (47)$$

$$U_D(r,d) = \int_{C_D} \{p_D(Q_o|d,sl,sm)^n f_D(\underline{c}_D|ss,d,Q_o^n) + $$

$$+ [1-p_D(Q_o|d,sl,sm)^n] f_D(\underline{c}_D|ss,d,Q_1^n)\} \cdot$$

$$\cdot\ h_D[v_D(\underline{c}_D)]\ d\underline{c}_D \ . \qquad (48)$$

$$U_A(d,a) = \int_{C_A} [\int_L f(l|d) \cdot f_A(\underline{c}_A|a,l)\ dl] \cdot$$

$$\cdot\ h_A[v_A(\underline{c}_A)]\ d\underline{c}_A \ . \qquad (49)$$

In summary, regulation alternatives were decomposed into standards, monitoring and inspection procedures, and sanctions. Safety standards were considered to be set directly on D. The monitoring and inspection procedure was assumed to be fallible leading to the possibilities of misses or false alarms in regulation violation detection. This was the main vehicle for decomposing f_D. To decompose f_A it was assumed that pollution amounts are fully determined by s_j, the pollution generating events, and the developer decision d. Pollution amounts and impactee action alone determine the probable consequences for the impactee. The final utility equation for the safety standards model are (36-38).

The emission standard model, (47) - (49) assumed that a maximum level of emissions *sl* is set by the regulator, that pollution generating events S_A are reduced to "normal operation" for each d, that actual levels produced by d are probabilistic and can only be detected with error. This led to a rather specific definition of detection probabilities and a decomposition of f_D. f_A was decomposed as in the safety standard model but it was simplified, since pollution generating events were reduced to normal operations.

POSSIBLE USES OF THE DECISION THEORETIC MODEL

The model that has been developed here is a decision theoretic formulation of the regulatory decision problem in standard setting and it gives a stepwise solution of how a regulator might think of going through the standard setting task to solve the problem. The model prescribes an "optimal" standard setting solution that is consistent with the decision maker's preferences and opinions. However, the question arises of how the model can be applied in its full complexity to a real problem, and what its main uses and limitations are. There seem to be five ways in which a regulator could benefit from the use of the model. The model can help

- to structure the regulation problem;
- to enable regulators and their experts to express uncertainties, and intangibles in quantitative forms;
- to make trade-offs explicit;
- to identify a set of good regulatory solutions;
- to allow a study of the sensitivities of the regulatory solution to conflicting opinions, values, and information.

Structuring the Regulation Problem

The model provides a cognitive structure or roadmap along which the regulator can organize his thinking. Some of the main distinctions that the model makes are those between consequences, their values and probabilities, and between three decision-making units involved in regulation. Even if none of the further steps could be achieved (quantitative estimation of probability density functions, quantification of values) the model could already in this respect be an aid for the regulator.

Enabling Regulators and Experts to Express Uncertainties and Intangibles in a Quantitative Form.

In this presentation of the model the details of the actual quantification procedures were not given, although some methods were outlined on p.8. But through its simplifying assumptions the model is designed such that regulators and their experts can, in principle, perform the actual quantification tasks, both on the uncertainty and the value side. The model requires at no step to construct functions that would be very difficult to assess in practice (although by doing so it had to make some severe simplifying assumptions). The tools for such quantification exist and have been extensively explored in the laboratory and in real world decision problems. It remains a task of a model application to see how far one can go with the actual quantification steps in standard setting and regulations.

Making Trade-Offs Explicit

The model requires this directly by postulating quantitative trade-offs within each decision making unit (the $v_{i.}$ in the additive value function $v_{.}$), by postulating a parametrized risk transformations (the $h_{.}$), and by postulating trade-offs among decision units (the λ_{i}). Again, the question remains whether these weights and parameters that reflect the trade-offs can be realistically assessed. But the literature on multi-attribute utility theory and its applications indicate that a quantification of such trade-offs can be feasible.

Identifying a set of good standard solutions

The general regulation model (21-23), the safety standards model (36-38), and the emission standards model (47-49) all end up with the three utilities accruing to the regulator, the developer, and the impactee respectively, given that the latter two units take actions which are "optimal" in the decision theoretic sense. The search for a good standard solution can begin with an examination of the optimal developer and impactee responses d(r) and a[d(r)]. Then a Pareto optimality analysis can be applied on U_R, U_D, and U_A to eliminate obviously unsatisfactory standards. Finally a weighting scheme such as (26) can be used to further explore the changes in utility as a function of r.

Allowing a Study of the Sensitivities of the Regulatory Solution to Conflicting Values, Opinions, and Degrees of Information

In each step the values, trade-offs, and probability density functions were made explicit based on a set of information that the regulator had at hand. Each of these parameters and information variables can be pushed around to see in which areas the model is most sensitive. For example, one could run the whole model based on some information provided by developers or impactees, to see if such different information sources would lead to different standard solutions. Furthermore, one could analyze if different weights or trade-offs would change the solution, etc. Finally, as an important output of the model one can compute the value of perfect information (see DeGroot, 1970), and the value of sample information. This could then be an important input for future budgeting decisions to set up research programs to improve standards.

Decision makers and analysts will probably find many formal and substantive limitations of the model presented here, ranging from criticisms that it is an overformalization of a very complex political process to specific criticisms of independence assumptions made in the model. Probably the most persuasive way to meet these criticisms are successful applications of the model. Within IIASA's research on standard setting one such application (on chronic oil discharge standards) has been completed, another one (on noise standards) is in process. In conjunction with these applications a final evaluation of the model presented here will be possible.

APPENDIX

In this appendix the additive decomposition of an algebraic difference function and the linear or exponential relationship between a von Neumann and Morgenstern function u and the difference function v will be derived from behavioral axioms.

Let $\langle CxC, \overset{\circ}{\gtrsim} \rangle$ be an algebraic difference structure (Krantz et al., 1971). Let $\underline{c} = (c_1, c_2, \ldots, c_i, \ldots, c_n) \in C$ and let $(\underline{c},\underline{d}) \overset{\circ}{\sim} (\underline{c}',\underline{d}')$ be defined as $(\underline{c},\underline{d}) \overset{\circ}{\gtrsim} (\underline{c}',\underline{d}')$ and $(\underline{c}',\underline{d}') \overset{\circ}{\gtrsim} (\underline{c},\underline{d})$. C_i is said to be <u>difference value independent</u> of $\underset{j \neq i}{X} C_j$ if and only if for all $c_i, d_i \in C_i, c_j^o, d_j^o \in C_j$, $j \neq i$ the following condition holds:

$$[(c_1^o, c_2^o, \ldots, c_{i-1}^o, c_i, c_{i+1}^o, \ldots, c_n^o) \quad , \quad (c_1^o, c_2^o, \ldots, c_{i-1}^o, d_i, c_{i+1}^o, \ldots, c_n^o)]$$

$$\overset{\circ}{\sim}$$

$$[(d_1^o, d_2^o, \ldots, d_{i-1}^o, c_i, d_{i+1}^o, \ldots, d_n^o) \quad , \quad (d_1^o, d_2^o, \ldots, d_{i-1}^o, d_i, d_{i+1}, \ldots, d_n^o)] \; . \qquad (50)$$

<u>Decomposition of v</u>

Let $\langle CxC, \overset{\circ}{\gtrsim} \rangle$ be an algebraic difference structure with difference value function v as defined in (4). If for all i=1, 2,...,n-1, C_i is difference value independent of $\underset{j \neq i}{X} C_j$, then there exist functions $v_i : C_i \to \mathbb{R}$ such that

$$v(\underline{c}) = \sum_{i=1}^{n} v_i(c_i) \quad . \qquad (51)$$

It is clear that difference value independence is necessary for (51) to hold. To prove sufficiency an approach by Fishburn (1970) for additive utility functions is followed. Consider the following n-1 indifference equations which are a result of the difference value independence condition for arbitrary $c_i, c_i^o, \in C_i$

$$[(c_1, c_2^o, c_3^o, \ldots, c_i^o, \ldots c_n^o) ,$$
$$(c_1^o, c_2^o, c_3^o, \ldots, c_i^o, \ldots c_n^o)]$$
$$\succsim$$
$$[(c_1, c_2, c_3, \ldots, c_i, \ldots c_n) ,$$
$$(c_1^o, c_2, c_3, \ldots, c_i, \ldots c_n)] ; \qquad (52.1)$$

$$[(c_1^o, c_2, c_3^o, \ldots, c_i^o, \ldots c_n^o) ,$$
$$(c_1^o, c_2^o, c_3^o, \ldots, c_i^o, \ldots c_n^o)]$$
$$\succsim$$
$$[(c_1^o, c_2, c_3, \ldots, c_i, \ldots c_n) ,$$
$$(c_1^o, c_2^o, c_3, \ldots, c_i, \ldots c_n)] ; \qquad (52.2)$$

$$\vdots$$

$$[(c_1^o, c_2^o, \ldots, c_{i-1}^o, c_i, c_{i+1}^o, \ldots, c_n^o) ,$$
$$(c_1^o, c_2^o, \ldots, c_{i-1}^o, c_i^o, c_{i+1}^o, \ldots, c_n^o)]$$
$$\succsim$$
$$[(c_1^o, c_2^o, \ldots, c_{i-1}^o, c_i, c_{i+1}, \ldots, c_n) ,$$
$$(c_1^o, c_2^o, \ldots, c_{i-1}^o, c_i^o, c_{i+1}, \ldots, c_n)] \qquad (52.i)$$

$$\vdots$$

$$[(c_1^o, c_2^o, \ldots, c_i^o, \ldots, c_{n-2}^o, c_{n-1}, c_n^o) ,$$
$$(c_1^o, c_2^o, \ldots, c_i^o, \ldots, c_{n-2}^o, c_{n-1}^o, c_n^o)]$$
$$\succsim$$
$$[(c_1^o, c_2^o, \ldots, c_i^o, \ldots, c_{n-2}^o, c_{n-1}, c_n) ,$$
$$(c_1^o, c_2^o, \ldots, c_i^o, \ldots, c_{n-2}^o, c_{n-1}^o, c_n)] . \qquad (52.n-1)$$

Since $\langle C\times C, \overset{\circ}{\gtrsim}\rangle$ is an algebraic difference structure from (4) and (52) the following n-1 difference equations can be derived:

$$v(c_1,c_2^o,\ldots,c_i^o,\ldots,c_n^o) - v(c_1^o,c_2^o,\ldots,c_i^o,\ldots,c_n^o) =$$

$$= v(c_1,c_2,\ldots,c_i,\ldots,c_n) - v(c_1^o,c_2,\ldots,c_i,\ldots,c_n) \quad , \qquad (53.1)$$

$$v(c_1^o,c_2,c_3^o,\ldots,c_i^o,\ldots,c_n^o) - v(c_1^o,c_2^o,\ldots,c_i^o,\ldots,c_n^o) =$$

$$= v(c_1^o,c_2,c_3,\ldots,c_i,\ldots,c_n) - v(c_1^o,c_2^o,c_3,\ldots,c_i,\ldots,c_n) , \qquad (53.2)$$

$$\vdots$$

$$v(c_1^o,c_2^o,\ldots,c_{i-1}^o,c_i,c_{i+1}^o,\ldots,c_n^o) -$$

$$v(c_1^o,c_2^o,\ldots,c_{i-1}^o,c_i^o,c_{i+1}^o,\ldots,c_n^o) =$$

$$= v(c_1^o,c_2^o,\ldots,c_{i-1}^o,c_i,c_{i+1},\ldots,c_n) -$$

$$v(c_1^o,c_2^o,\ldots,c_{i-1}^o,c_i^o,c_{i+1},\ldots,c_n) \quad , \qquad (53.i)$$

$$\vdots$$

$$v(c_1^o,c_2^o,\ldots,c_i^o,\ldots,c_{n-2}^o,c_{n-1},c_n^o) -$$

$$v(c_1^o,c_2^o,\ldots,c_i^o,\ldots,c_{n-2}^o,c_{n-1}^o,c_n^o) =$$

$$= v(c_1^o,c_2^o,\ldots,c_i^o,\ldots,c_{n-2}^o,c_{n-1},c_n) -$$

$$v(c_1^o,c_2^o,\ldots,c_i^o,\ldots,c_{n-2}^o,c_{n-1}^o,c_n) \quad . \qquad (53.n-1)$$

Adding up the left and right terms and cancelling the fourth term of equation (53.i) against the third term of (53.i+1) for all i results in

$$\sum_{i=1}^{n-1} v(c_1^o,c_2^o,\ldots,c_{i-1}^o,c_i,c_{i+1}^o,\ldots,c_n^o) -$$

$$(n-1) \cdot v(c_1^o,c_2^o,\ldots,c_i^o,\ldots,c_n^o) =$$

$$= v(c_1,c_2,\ldots,c_i,\ldots,c_n) - v(c_1^o,c_2^o,\ldots,c_i^o,\ldots,c_{n-1}^o,c_n) \quad (54)$$

Defining $\underline{c}^o = (c_1^o, c_2^o, \ldots, c_i^o, \ldots, c_n^o)$ and

$$v_i(c_i) = v(c_1^o, c_2^o, \ldots, c_{i-1}^o, c_i, c_{i+1}^o, \ldots, c_n^o) - v(\underline{c}^o) \cdot \frac{(n-1)}{n}$$

gives the desired result:

$$v(c_1, c_2, \ldots, c_i, \ldots, c_n) = \sum_{i=1}^{n} v_i(c_i) \qquad (55)$$

The next decomposition is a well known result in expected utility theory. It is based on the definition of <u>utility independence</u> introduced by Keeney (see Keeney, 1974; Keeney and Raiffa, 1976). In the present terminology utility independence can be formulated as follows.

Let $\langle F, \gtrsim \rangle$ be an expected utility structure as defined in (3). Let f^i, g^i be any two probability density functions with identical degenerate marginal probability densities $f_i(c_i) = g_i(c_i) = 1$ for some $c_i \in C_i$, but otherwise unrestricted. $\underset{j \neq i}{X} C_j$ is said to be utility independent of C_i if the preference among f^i and g^i does not depend on the specific value of c_i with unity density. Weaker formulations of utility independence can be found in Keeney (1974); Fishburn and Keeney (1974); von Winterfeldt and Fischer (1975); Keeney and Raiffa (1976). The result of utility independence together with the assumption of a von Neumann and Morgenstern utility function over C is the following decomposition:

<u>Decomposition of u</u>

Let u be a von Neumann and Morgenstern utility function over C. Let $\underset{j \neq i}{X} C_j$ be utility independent of C_i for all i. Then there exist functions $u_i: C_i \rightarrow \mathcal{R}$ such that either

$$u(\underline{c}) = \sum_{i=1}^{n} u_i(c_i) \qquad (56)$$

or

$$1 + ku(\underline{c}) = \prod_{i=1}^{n} [1 + ku_i(c_i)] \; . \qquad (57)$$

The proof can be found in the papers cited above.

To relate u and v the following uniqueness theorem for additive order preserving functions is observed (see Krantz et al, 1971)*: If a function $v: C \rightarrow \mathcal{R}$ is additive and preserves

---

*A similar proof based on constant risk aversion arguments has been provided by Pratt and Meyer in Keeney and Raiffa (1976).

the order of C, and if there is another function $v':C\to\mathbb{R}$ with the same property then there exist real numbers $\alpha,\beta,\alpha>0$ such that

$$v = \alpha v' + \beta .$$

This property, now used to relate u and v with respect to the orders $\overset{'}{\gtrsim}$ and $\overset{''}{\gtrsim}$ over C, which by (7) are identical.

If (51) and (56) hold u and v are both additive and order preserving functions from $C\to\mathbb{R}$.

Therefore, there exist $\alpha>0,\beta$ such that

$$u = \alpha v + \beta \quad . \tag{58}$$

If (51) and (57) hold, then first (57) needs to be transformed into an additive order preserving function before the uniqueness theorem can be applied. Two cases have to be distinguished: if $k>0$, then $1+ku$ in (57) is an order preserving function. If $k<0$ then $1+ku$ has an inverse order.

Consider $k>0$ first:

$$\ell n[1+ku] = \ell n[\prod_{i=1}^{n} (1+ku_i)] \quad , \tag{59}$$

$$\ell n[1+ku] = \sum_{i=1}^{n} \ell n(1+ku_i) \quad . \tag{60}$$

Since $(1+ku)$ is order preserving and ℓn is strictly increasing, $\ell n(1+ku)$ is also order preserving. Therefore (60) is an additive order preserving function. By uniqueness, there exist $\alpha>0,\beta$ such that

$$\begin{aligned} \ell n[1+ku] &= \alpha v + \beta \\ 1+ku &= e^{\alpha v+\beta} \end{aligned} \tag{61}$$

$$u = \frac{1}{k}e^{\alpha v+\beta} - \frac{1}{k} \tag{62}$$

where $k>0$ and $\alpha>0$.

If $k<0$ then $(1+ku)$ is an inverse order. Since ℓn is strictly increasing $\ell n(1+ku)$ is an inverse order, and

$$- \ell n[1+ku] = - \sum_{i=1}^{n} \ell n(1+ku_i) \tag{63}$$

is again an additive order preserving function. By uniqueness, there exist $\alpha>0,\beta$ such that

$$- \ell n[1+ku] = \alpha v + \beta \tag{64}$$

$$1+ku = e^{-\alpha v-\beta}$$

$$u = \frac{1}{k}\, e^{-\alpha v-\beta} - \frac{1}{k} \tag{65}$$

where $k<0$, $\alpha>0$.

(58), (62), and (65) are the desired results.

REFERENCES

BROWN, R.V., KAHR, A.A., and PETERSON, C., Decision Analysis for the Manager. New York, Holt, Rinehart and Winston, 1974.

DEGROOT, M.H., Optimal Statistical Decisions. New York, McGraw Hill, 1970.

DORFMAN, R., and JACOBY, H.D., "A Model of Public Policy Decisions Illustrated by a Water Pollution Policy Problem." In R.H. Haveman and J. Margolis (Eds.), Public Expenditures and Policy Analysis. Chicago, Markham, 1970.

EDWARDS, W., "Social Utilities." The Engineering Economist. Summer Symposium Series VI, US Naval Academy, Annapolis, 1971.

FISHBURN, P.C., Utility Theory for Decision Making. New York, Wiley, 1970.

FISHBURN, P.C. and KEENEY, R.L., "Seven Independence Concepts and Continuous Multiattribute Utility Functions." Journal of Mathematical Psychology, 11, 1974, pp. 294-327.

FISCHER, D.W., and VON WINTERFELDT, D., "Setting Standards for Chronic Oil Discharges in the North Sea." IIASA Research Memorandum, RM-78-005, Laxenburg, 1978.

GREEN, D.M., and SWETS, J.A., Signal Detection Theory and Psychophysics. New York, Wiley, 1966.

HOLDEN, M., Pollution Control as a Bargaining Process: An Essay on Regulatory Decision Making. Cornell University, Ithaca, New York, 1966.

KARAM, R.A., and MORGAN, K.Z. (Eds.), Energy and the Environmental Cost Benefit Analysis. New York, Pergamon Press, 1975.

KEENEY, R.L., "Multiplicative Utility Functions." Operations Research, 1974, 22, pp. 22-34.

KEENEY, R.L., and RAIFFA, H., Decisions with Multiple Objectives: Preferences and Value Trade-offs. New York, Wiley, 1976.

KRANTZ, D.H., LUCE, R.D., SUPPES, P., and TVERSKY, A., Foundations of Measurement, Vol. I, New York, Academic Press, 1971.

MAJONE, G., "Choice among Policy Instruments for Pollution Control." Policy Analysis, 1976, 2, pp. 589-613.

NORTH, W.H., and MERKHOFER, M., "Analysis of Alternative Emission Control Strategies." In Air Quality and Stationary Source Emission Controls. A Report by the National Academy of Sciences to the US Senate Committee of Public Works, Washington, D.C., 1975.

National Research Council. Decision Making in the Environmental Protection Agency. A Report to the US Environmental Protection Agency from the Committee on Environmental Decision Making, National Academy of Sciences, Washington, D.C., 1977.

RAIFFA, H., Decision Analysis. Reading, Mass., Addison-Wesley, 1968.

RAIFFA, H., and SCHLAIFER, R., Applied Statistical Decision Theory. Cambridge, Mass., Harvard University Press, 1961.

SAVAGE, J.L., The Foundations of Statistics. New York, Wiley, 1954.

SPETZLER, C.S., and STAEL VON HOLSTEIN, C.A., "Probability Encoding for Decision Analysis." Management Science, 1975, 3, pp. 340-357.

SUPPES, P., and ZINNES, J.L., "Basic Measurement Theory." In R.D. Luce, R.R. Bush, and E. Galanter (Eds.), Handbook of Mathematical Psychology, Vol. I., New York, Wiley, 1963.

SUPPES, P., and WINET, M., "An Axiomatization of Utility Based on the Notion of Utility Differences." Management Science. 1955, 1, pp. 259-270.

United States Academy of Sciences. Decision Making for Regulating Chemicals in the Environment. A Report to the Environmental Protection Agency, Washington, D.C., 1975.

VON NEUMANN, J., and MORGENSTERN, O., Theory of Games and Economics Behavior. Prineton, Princeton University Press, 1947(2).

VON WINTERFELDT, D., "A Decision Aiding System for Improving the Environmental Standard Setting Process." In K. Cichocki (ed.), Systems Analysis Applications to Complex Programs. Oxford: Pergamon Press, 1978 (forthcoming).

VON WINTERFELDT, D., and FISCHER, G.W., "Multiattribute Utility Theory: Models and Assessment Procedures." In D. Wendt and C. Vlek (Eds), Subjective Probability, Utility and Decision Making. Dordrecht, Reidel, 1975.