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**IIASA Research Memorandum
June 1978**



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A STOCHASTIC MODEL OF PHOSPHORUS
LOADING FROM NON-POINT SOURCES

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June 1973

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PREFACE

Although the eutrophication--that is, the enrichment in nutrients--of natural water bodies (especially lakes and reservoirs) is a natural process, man-made interventions have accelerated it. As a result, eutrophication seems to be a world-wide environmental concern since algae and the often associated odor and taste problems prevent, or at least hinder, the planned utilization of water resources. For this reason IIASA focuses on research which surveys existing models and elaborates new ones with the ultimate goals of predicting and finding efficient ways to control eutrophication. To concretize this goal a Case Study on Lake Balaton in Hungary was recently initiated within the Resources and Environment Area of IIASA. This Research Memorandum represents a first step in modelling non-point sources phosphorus loading into a lake, which has been found to be a major component in the eutrophication process in a great number of lakes. Phosphorus data of Lake Balaton have been used, although this approach can be applied to other water bodies and other nutrients also.

Research leading to this study was finalized during a working visit of the authors to IIASA's Resources and Environment Area, and is relevant for both the subtask on the Lake Balaton Case Study, and for the task on Environmental Impact of Agriculture. The research was also supported by a U.S. National Science Foundation grant to the University of Arizona and by the Hungarian National Water Authority.

IIASA would be grateful for any comments and/or recommendations on possible improvements and developments of the present model.

ABSTRACT

A stochastic model is presented and applied for Lake Balaton, Hungary to estimate the phosphorus (P) loading from non-point sources. Rainfall events cause surface runoff events and erosion events; all three events are random. P is carried by runoff into the lake in two forms: (1) dissolved P and (2) sediment, absorbed or fixed P. P loading is thus considered as a random variable, whose probability density function (pdf) per event is to be estimated. Pdf of seasonal (e.g. annual) loading is determined as the sum of a random number of random events.

The annual mass balance of P stored in lake sediment leads to a first order difference equation, the solution of which can be used to predict the expected P available for release at future times. The model is applied for the Tetves subwatershed (70 km²) of Lake Balaton. Preliminary results show that during relatively short runoff events about as much P reaches the lake as during the rest of the year and that more sediment P is produced than dissolved P. Since a considerable variance appears in the annual amounts of P loading, the use of stochastic models to estimate the loading conditions seems to be most appropriate. The stochastic loading model should be incorporated into a broader control model. Elements of such a control model are given in the form of possible P loading reduction measures; also, economic trade-off between these measures is discussed.

1. INTRODUCTION

The purpose of this paper is to estimate the phosphorus (P) load from agricultural sources into a lake under various uncertainties, using Lake Balaton in Hungary as a case study. This estimate of P is needed to forecast and control the state of eutrophication of lakes, as P most often appears to be the limiting factor to phytoplankton growth (or primary productivity) [Wetzel, 1975]. The terms "P loading" and "state of eutrophication" are thus considered as germane concepts in the present study.

The study of eutrophication has evolved over the past seventy years from intuitive description [Naumann, 1919] to sophisticated and comprehensive computer simulation models [Imboden and Gächter, 1975]. Rodhe (1969) reviews historical developments in Northern Europe, while Edmonson (1969) presents examples of eutrophication in North America. As pointed out by Serruya and Serruya (1975) and Reckhow (1977), numerous mathematical models of phytoplankton have been published in the past few years; also, a growing number of models have been developed for the purpose of predicting the eutrophication process.

The present approach differs from the current modeling trend in one main aspect: the stochastic nature of nutrient input is recognized and modeled. On the other hand, only the limiting nutrient P of non-point origin is considered, as a random variable \tilde{P} whose probabilistic description is sought for three time scales:

1. probability density function (pdf) per event;
2. annual accumulation;
3. long-term accumulation.

Furthermore, accounting for P input is done separately for two main forms of transport of P from the watershed into the lake: dissolved, and fixed or adsorbed \tilde{P} . Finally and most importantly, the paucity of data for model calibration is recognized, so that simple sub-models based on phenomenological assumptions are used. It is hoped that such an approach will lead to system and control models which are easily transferable to lakes

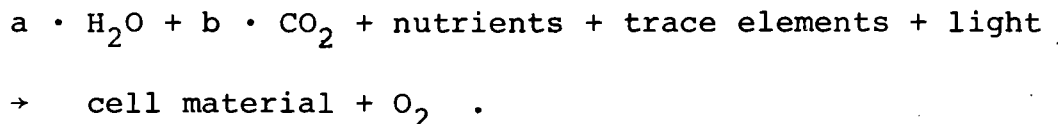
other than the one used for a case study. Note that the development of a control model is only to be done later, in an investigation using (presumably) the presently developed system model.

In view of the importance of the problem, a brief scientific background of the eutrophication problem, the P loading problem and the P cycle in the lake is given in the next section. Then, the stochastic model is described, followed by an application to a subwatershed of Lake Balaton.

2. SCIENTIFIC BACKGROUND

2.1 Lake Eutrophication

The term "eutrophication" means enrichment in nutrients; a lake poor in nutrients is said to oligotrophic. The trophic state of a lake is usually measured by primary productivity of phytoplankton, whose growth results from the following factors [Bucksteeg and Hollfelder, 1975]:



Of concern in most studies is the nutrient component of this equation; more precisely, the limiting factor to algae growth is recognized to be P [Wetzel, 1975]. Consequently, the study of P leaching into lakes and P cycle inside lakes has received considerable attention [Timmons et al., 1970, 1973; Syers et al., 1973].

According to Wetzel (1975), it is only in very exceptional cases that primary productivity may have been inhibited by carbon or trace elements. Such cases, described, for example, in Hutchinson (1973), are of no concern to the present effort, since the purpose here is to develop an approximate model valid in most cases, rather than a universal all-encompassing lake eutrophication model.

The usefulness of lake modeling is recognized by many leading limnologists [Vollenweider, 1972]; though it is difficult if not impossible to model at once all species of phytoplankton, zooplankton as well as cycles of organic and inorganic elements in a given lake. Such a model could perhaps be developed, using for example general systems theory [Wymore, 1967, 1976] in a manner akin to the general watershed model of Rogers (1971), but then the calibration problem would be insurmountable within the time horizon that decisions must be made. Thus, regardless how approximative the present P loading model may be, it hopefully will make possible to take actions, rapidly if necessary, to stop or reverse the eutrophication process, although the process may not yet be fully understood and account for various uncertainties.

2.2 Case of Lake Balaton

For Lake Balaton, the often quoted approach of Vollenweider (1968) is used to demonstrate the urgency of the P input problem. As shown in Figure 1, the trophic state of a lake may be represented as a function of phosphorus loading and volumetric exchange of water into the lake. More precisely, the abscissa is the logarithm of the exchange coefficient; i.e., the ratio of average depth to residence time, and the ordinate represents the logarithm of the loading in g/m^2 year. In this space, the lower line represents the permissible P loading level below which the lake is oligotrophic, while the upper line corresponds to the danger level, above which the state is eutrophic. The question arises as to which is the present trophic state of Lake Balaton, whose hydrologic and physical description may be found in Szesztay (1967) and Metler, et al. (1975).

According to data gathered by the Hungarian Water Research Institute (VITUKI, 1975) the average P loading of Lake Balaton is 0.3 g/m^2 year; and the average depth is 3.25 m, the average residence time is 2.15 years, which yields an exchange coefficient of $3.25/2.15 = 1.5$. It thus appears that, on the average, Lake Balaton is on the danger line at point M (Figure 1). If such is the case, the trophic state of the lake may be brought

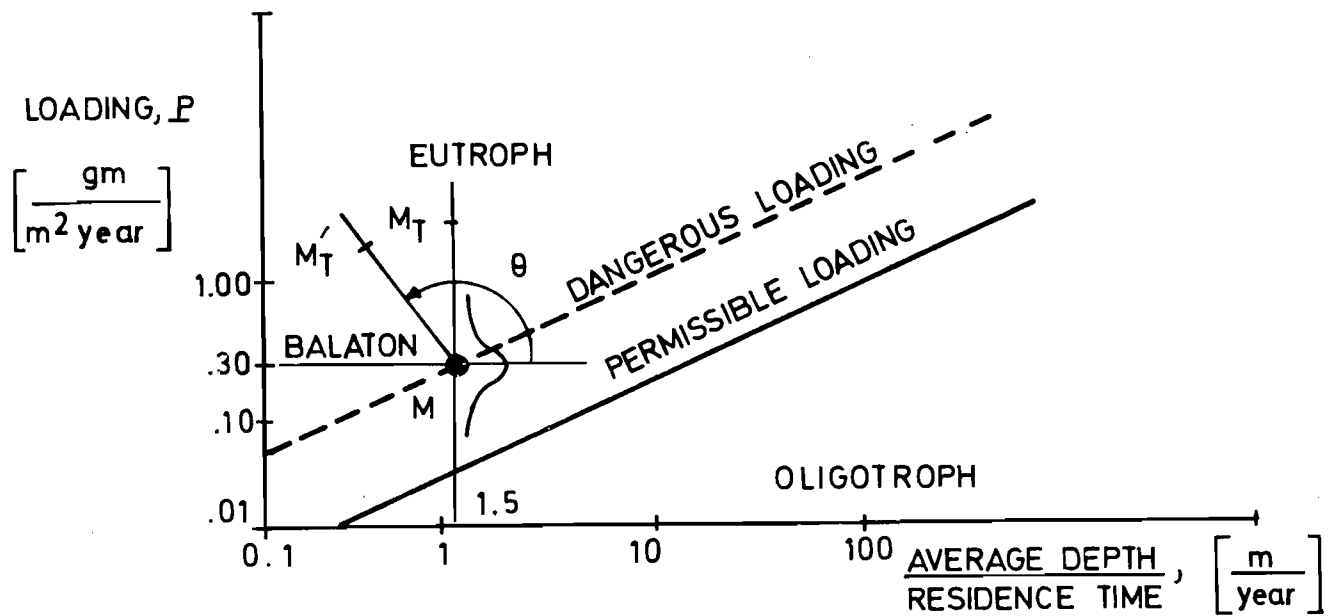


Figure 1: Trophic state of a lake as a function of exchange coefficient and loading (after Vollenweider, 1968)

back toward the permissible region in at least three ways:

1. increase the average depth;
2. decrease the residence time;
3. decrease the P loading.

For Lake Balaton, it would be conceivable to apply the second method because the Sio River, which is the release channel of the lake, has recently been enlarged for reasons other than eutrophication control. However, the third method is the most effective one.

Returning to Figure 1, many uncertainties are present in the determination of the location of point M; then, because of the spatial variability of conditions in the lake, the actual or true trophic state may be at a point M'_T . The vector \underline{MM}'_T is a random vector defined for example by magnitude and argument (MM'_T, θ) . In the present study, we fix $\theta = \frac{\pi}{2}$, so that the pdf of the random variable MM'_T , where M'_T is on the vertical of M may be sought. The ordinate of M'_T is the true value of P-loading. Note that a Bayesian viewpoint is being used [Davis, et al., 1972], since the true value of P-loading is not assumed to be known: we only have a sample in hand, and study how the true value (random as far as we are concerned) varies around our sample.

At any rate, the trophic state of Lake Balaton seems to warrant P loading control very soon. This control is to be effected dynamically, using a model that enables the decision-maker to account for present uncertainties, to improve the control performance as more experience is gained and data are gathered, and to forecast long-range effects. It is difficult to see how existing models may perform such tasks.

2.3 Sources of phosphorus

Where does the phosphorus originate in Lake Balaton (and in many other lakes around the world)? Here a distinction may be made between (a) point and (b) non-point sources.

(a) Point sources are essentially domestic or industrial waste discharge outlets, which may be on the catchment, whereby the waste is transported into the lake by rivers, or on the lakeshore; in either case, local concentration of nutrients must be taken into account to predict the trophic state of that portion of the lake [Edmonson, 1969]. To this category belong the output of sewage treatment plants and feedlots [Holt et al., 1970].

(b) Phosphorus from non-point sources, may be soluble organic (grazing animal wastes, leaching of vegetation), soluble inorganic (orthophosphate, hydrolyzable polyphosphates), suspended insoluble inorganic compounds, and sorbed or fixed phosphorus.

Because point sources are easier to monitor and control than non-point ones, the present study is concerned with the latter type of source only. Furthermore, according to numerous studies both in Hungary [Felföldy and Tóth, 1970; VITUKI, 1975] and elsewhere [Sager and Wiersma, 1975; Nakanishi, 1975; Brink, 1975; Barta, 1970], much of the non-point source P originates from commercial or natural fertilizers. Note that the utilization of manure as a fertilizer may be an important source of P. The figures below, which include all animal sources such as feedlots, have been calculated by Holt, et al., (1970) for the U.S.A.

<u>Source</u>	<u>Total P (10^9 lb/year)</u>
Human (a)	0.83
Animal (b)	3.2
Fertilizer (b)	3.7

(a) based on 4.5 lb per person for 185 million

(b) based on 1968 U.S. agricultural statistics

A similar breakdown of P source for selected lakes in Bavaria has been given in Bucksteeg and Hollfelder (1975), and the case of Japan is being investigated by Nakanishi (1976): the same proportions are found as in the U.S.A.

It is thus realized that by focusing the study on the contribution of non-point sources, only part of the problem may be modeled. For the case of Lake Balaton, fertilizers appear to be the most important controllable non-point source [Barta, 1970]; the corresponding loading mechanism is thus examined in detail in the next section.

2.4 P loading from agricultural sources

Over the recent years, P stemming from agricultural lands has been increasing for two main reasons:

1. increase of fertilizer usage, and
2. increase of the phosphorus to other elements ratio, especially nitrogeneous compounds.

This phenomenon is true worldwide.

In order to study this P loading, a distinction will be made between (a) transport mechanism from the watershed to the lake, and (b) P cycle in the lake.

(a) Transport Mechanism

Frere (1973) has given an overview of the transport mechanisms of chemicals of agricultural origin in watersheds. By definition the flow of a chemical is the product (concentration) \times (discharge), or equivalently the weight of chemical reaching the lake per event is: (concentration) \times (water volume per event).

According to numerous studies [Holt et al., 1970; Edwards and Harrold, 1970; Sievers, et al., 1970; Benoit, 1973], P is mainly transported under the form of sorbed compounds attached to sediments eroded from the watershed; a substantially smaller portion of the P reaches the lake under a dissolved form, mainly PO_4 (orthophosphate). Let C_1, C_2 be the average concentration of dissolved and adsorbed phosphorus, respectively, in a runoff event. As stated in Frere (1973), on the basis of observations by many authors which he cites, C_1 and C_2 are not really constant--both solution and adsorption phenomena are complex. However, physical models are available to estimate

C_1 and C_2 ; the alternative is to measure P concentrations in the field, following one of the procedures reviewed in Burwell, et al., (1975). It turns out in particular that P concentration measurements in three points around the peak runoff are sufficient to estimate the total volume of transported P with good reliability.

Clearly, the dissolved P can be used by phytoplankton immediately upon reaching the lake; whereas P sorbed, hence stored by sediments, must first be released. This is the rationale for a separate accounting of the loading and accumulation of P by these two different mechanisms. Note, however, that both mechanisms have the same common element: precipitation, which makes P loading a random process, as exemplified in the empirical time series of P input in Taylor, et al., (1971), and the pdf of phosphate concentration in various U.S. water bodies given in Edmonson (1969). In other words, as suggested by Frere (1973), one may consider that P loading is the result of both runoff and sediment yield. Runoff transports dissolved P, and causes erosion; adsorbed P is in turn transported with the sediments, as observed, for example, by Rogowsky and Tamura (1970) who traced sorbed chemicals down the watershed using radioactive cesium. Oddson et al., (1970) found that the P transport process reaches a rapid steady-state during a rainfall-runoff event. Thus, a stochastic event-based approach to model P loading is used in the present study; a review of this type of approach with numerous watershed management applications can be found in Fogel, et al., (1976).

It should be kept in mind that the purpose of modeling P loading is to examine the effectiveness of various control methods. These methods, to be reviewed in greater detail later in this paper, consist in changing the type, quantity or method of application of the fertilizer and in erosion control measures. Each control method may act on either mechanism or on both of them.

Since the P sources are distributed over the catchment, the hydrologic event definition should be of a space-time nature, as in the precipitation model put forth in Gupta (1973). However,

spatial models of rainfall are not yet developed well enough to be used as inputs into other hydrologic models. Also, it is not necessary or useful to provide greater resolution in the input than in other parts of the model: rainfall-runoff relationship, sediment yield, adsorption phenomenon, and nutrient cycle in the lake. Thus, the catchment is divided into subcatchments, each of which is subject to a stochastic sequence of point precipitation events, assumed to occur at a representative location in the watershed. The P loading pertaining to each subcatchment is accumulated over time (both annually and over a long time horizon); then, the total contribution of the subcatchments is estimated.

In the present investigation, P loading is modeled for only one typical subcatchment. The dependence between hydrologic events in the various subcatchments should be studied before the complete model can be developed.

The next step consists in examining what happens to the various P compounds that reach the lake, either in dissolved or in sorbed form. Note that routing models of P, as mentioned in Thomann (1972), are outside of the scope of the present study.

(b) P cycle in lakes

Only a brief overview of this complex phenomenon is given here: the reader is referred to standard texts such as Wetzel (1975) for a comprehensive description.

Essentially, dissolved P is assimilated very quickly by algae to the point where the concentration of P in the lake and the biomass of bluegreen algae can be negatively correlated [Hutchinson, 1973]. There are cases when dissolved P may be adsorbed by lake bottom sediments, although in eutrophic lakes, the release of P from sediments is more frequent than is adsorption [Holt, et al, 1970]. Such a phenomenon has also been observed in Lake Balaton [Toth, et al., 1975].

There are numerous studies of nutrient chemistry in lake sediments; P chemistry has been summarized in Syers, et al., (1973). In brief, only the upper few millimeters of sediment layer play a major role in the P exchange process. If the water contains much oxygen, no P is released; if the oxygen becomes depleted, iron and

P are released through a mechanism described from laboratory experiments in Fillos and Swanson (1975) and in Hwang, et al., (1975). In the former paper, the authors write, "...during aerobic conditions in the overlying water, phosphates emanate from the deeper anaerobic layers of the benthal deposits toward the surface. When they reach the surface, the phosphates are retained in the aerobic layer in which they are adsorbed predominantly by ferric complexes."

Then, under anaerobic conditions, the ferric complexes break down and the phosphates are released. The lake bottom sediments thus act as a P reservoir, and introduce a positive feedback effect into the eutrophication process: if primary productivity increases, because of P loading in particular, the dissolved oxygen level (DO) decreases, which causes the release of sorbed P and provokes a further increase in primary productivity.

This brief background on P cycle in the lake has been given to demonstrate the necessity of accounting for both dissolved and sorbed P in the loading from agricultural sources--but keeping a separate account of the two forms of P.

In the next section, the stochastic modeling philosophy is developed, and the model itself is presented.

3. MODEL

3.1 Model Choice

In view of the complexity of the transport and eutrophication processes, and cycle of various nutrients or trace elements in the water bodies (Everett, 1972), the model choice problem is considerably broader than in purely hydrologic problems, such as inverse problem algorithms in groundwater (Sagar, et al., 1975), watershed models (Lovell, 1975) or bivariate model of floods at the confluence of two rivers (Bogardi, et al., 1976). For purely scientific purposes, choosing no model at all (null choice) but instead observing the behavior of one species of plankton or one predator-prey relationship and describing the results scientifically is a very worthwhile pursuit. However, if a decision is to be made on the basis of the observation, then a system model is

in order, as for example in Vicens et al., (1975). Not all descriptive models are usable for decision-making: only those models capable of predicting new situations can be used. In other words, as pointed out in a system-theoretic formulation of the model choice problem in Lovell (1975) and Davis, et al., (1976), the decision to be made determines or at least guides the choice of a model. The other major consideration is the proper matching of data and models as demonstrated for hydrologic applications of multivariate models in Weber, et al., (1973,1976).

For the case of controlling the eutrophication of large lakes such as Lake Balaton, one may envision adapting existing models such as the one developed in Switzerland by Imboden and Gächter (1975).

However, as noted earlier, it would take many years to gather sufficient data to adapt and calibrate such models; then, the use of relationships such as the Michaelis-Menten equation implies steady-state. Existing models are not designed to account for long-term effects, although an additional feedback loop could be added to take care of such effects. But most of all, very few attempts to model uncertainty in eutrophication models, as in Reckhow (1977), can be found. The models proposed by Riley, et al., (1949), Steele (1965), Chen and Orlob (1968, 1972), Di Toro, et al., (1970), Park and Wilkinson (1971) are based on a set of differential equations expressing the conservation of mass in the lake body. Everett (1972) uses multiple linear regression to model the biological and chemical properties of Lake Mead on the Colorado River (Arizona). Yet there are several biological applications of stochastic models (Neyman and Scott, 1959; Bartlett, 1957) and of Monte Carlo techniques (Engstrom-Heg, 1970; Beyer, et al., 1972). In particular, the latter authors have demonstrated that their model of the wolf population in the Isle Royale biome has a good predictive value for up to ten years.

3.2 Model Specifications

The main specifications of a P loading model designed for decision-making in the lake eutrophication problem are:

1. to account for uncertainty in hydrologic events, that is, precipitation events causing transport of dissolved and adsorbed P into the lake; this uncertainty is encoded as pdf of P loading per event for each type of P;
2. to utilize existing precipitation data plus a minimum amount of data for calibration: chemical data on dissolved and adsorbed P, runoff and sediment yield data; the model thus leads to a proper allocation of data-gathering efforts;
3. to enable us to keep track of both types of P in the lake given the net release rate of P from sediments, and the residence time of water in the lake (Armstrong and Weimer, 1973);
4. to predict the effect of fertilizer control on the pdf of P loading, hence the pdf of eutrophic state of the lake.

The economic effects include cost of control methods, yielding an opportunity loss in the case of overcontrol, and the expected losses resulting from eutrophication. Alternatively, a fixed goal of P loading may be set (see Figure 1), and the minimum cost control may be sought.

The present investigation leads to a Monte Carlo simulation technique, which is applied after calibration of the stochastic model of P loading presented in the next section. Using an event-based stochastic model conjunctively with simulation, the same line of thought will be followed as in Hekman, et al., (1976) for watershed management application, in Hanes, et al., (1976) for designing storage reservoirs, and Duckstein, et al., (1977) for estimating long-range sediment yield.

3.3 Model Development

As shown in Figure 2, the system leading to eutrophication of a lake may be divided into a watershed subsystem (I), and a lake subsystem (II).

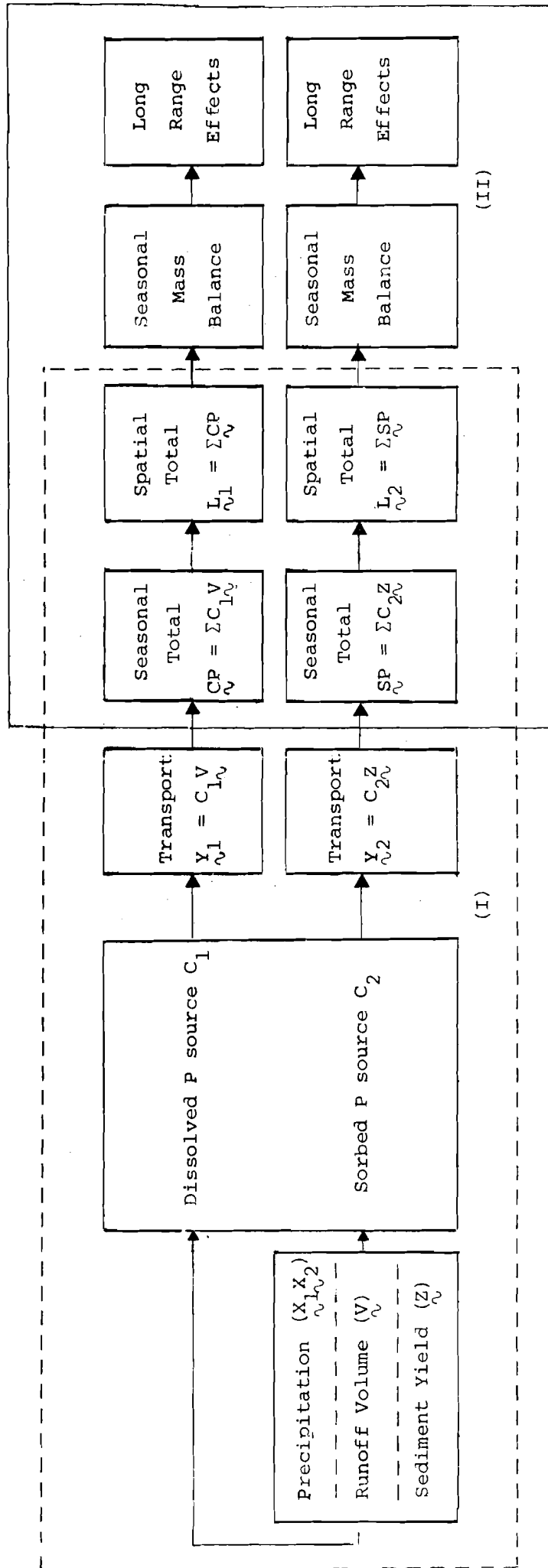


Figure 2. Elements of Stochastic P Loading System Model

Let each subsystem be composed of five vector-valued elements:

- (a) the state $S(t)$
- (b) the input $X(t)$
- (c) the state transition function F , which defines the state at time $(t+1)$ as a function of $S(t)$ and $X(t)$:

$$S(t+1) = F(S(t), X(t)) \quad (1)$$

- (d) the output $Y(t)$
- (e) the output function G , such that

$$Y(t+1) = G(S(t), X(t)) \quad (2)$$

These elements are now described for each subsystem.

3.3.1 Subsystem I: The Watershed

(A) State $S(t)$

The state description of the watershed includes physical characteristics, such as those shown in Table 1 (p.23) for the case study, as well as quantity of P available for transport into the lake, as described earlier in section 2.3.

(B) Input $X(t)$

Input consists of natural hydrologic events (rainfall-runoff-erosion) and man-made intervention in the watershed, especially the application of fertilizer.

As in Szidarovsky, et al., (1976), let a precipitation event be characterized by a random variable t describing the inter-arrival time between events and the bivariate random variable (X_1, X_2) , where

X_1 = effective rainfall depth in mm;

X_2 = rainfall duration in hours.

The generating mechanism of precipitation events is assumed to be a Poisson process. In other words, t_{\sim} is exponentially distributed [Feller, 1967]; the plausibility of this hypothesis has been demonstrated in Todorovic and Yevjevich (1969) and Fogel and Duckstein (1969) and many other subsequent studies.

The runoff volume V per precipitation event is calculated by means of the U.S. Soil Conservation Service (SCS) formula, namely

$$V_{\sim} = \frac{AX_{\sim 1}^2}{X_{\sim 1} + S} \quad (3)$$

in which

A = watershed area in ha;

V_{\sim} = runoff volume per event in mm/ha;

$X_{\sim 1}$ = effective rainfall: $X_{\sim 1} = R_{\sim} -$ (initial abstractions)

S = watershed infiltration constant, estimated from SCS tables or by a calibration procedure.

(C) State Transition Function $F(\cdot)$

This function is essentially a set of mass and chemical balance equations in the watershed. It establishes a dynamic watershed "state", i.e. amount of available dissolved P and sediment P, between hydrologic events. The amount of dissolved P available depends on agricultural practices, as described in Timmons et al., (1973). The principal component of F is the mechanism that generates sediment yield which is described next. Sediment yield Z_{\sim} per event is considered as a transformed random variable of the two dimensional variate $(X_{\sim 1}, X_{\sim 2})$ by means of combining the universal soil loss equation with SCS formulas for V_{\sim} and for peak flow; details are found in Smith, et al., (1977).

$$Z_{\sim} = w \left[\frac{a_0 A^2 X_{\sim 1}^4}{(X_{\sim 1} + S)^2 (a_1 X_{\sim 2} + a_2)} \right]^{0.56} \quad (m^3) \quad (4)$$

in which

- a_0 = conversion constant;
- a_1 = constant
- a_2 = time of concentration
- w = combined watershed factors and conversion constant.

The values of these constants may be found in Smith (1975), and can be determined, for a specific watershed, by simply using published tables as shown in Fogel, et al., (1976). Alternatively, if observations on runoff and/or sediment yield are available model calibration is the preferred method. Once the characteristics of Z_{λ} have been determined, the mass balance of sediment P on the watershed may be established.

(D) Output Y(t)

Precipitation events lead to runoff events which cause soluble P (PO_4 -compounds) to be washed off into the lake and produce erosion in which sediments with sorbed P are also carried into the lake. In this sense the per event output of the watershed is composed of mass $Y_{\lambda 1}$ of dissolved P, sediment yield Z_{λ} and mass $Y_{\lambda 2}$ of sediment P. Note that the estimation of Z_{λ} was considered as a part of the state transition function, while $Y_{\lambda 1}$ and $Y_{\lambda 2}$ are calculated by the output function.

(E) Output function G(·)

To obtain $Y_{\lambda 1}$ and $Y_{\lambda 2}$, let C_1 be the average concentration of dissolved P resulting from overland flow and let C_2 be the average concentration of P fixed to sediment which is washed off by runoff. Then the dissolved P loading $Y_{\lambda 1}$ and the sediment P loading $Y_{\lambda 2}$ are taken as, respectively,

$$Y_{\lambda 1} = C_1 Z_{\lambda} \quad \text{and} \quad Y_{\lambda 2} = C_2 Z_{\lambda} \quad . \quad (5)$$

The parameters C_1 and C_2 may not account for the detailed mechanism of P transport and sorption, as described for example in Cahill and Verhoff (1974): here, C_1 and C_2 are of a macroscopic

or lumped nature. The parameters C_1 and C_2 can be randomized in Equation 5 if experimental evidence shows high variability.

In the meantime, Equation 5 provides for a first approximation of the two forms of P loading per event from non-point agricultural sources. The output function of subsystem I also computes total seasonal loadings and adds up the contributions of the various subcatchments around the lake, as shown next.

Total Seasonal Loadings. In a season (or year), a random number J of events takes place. Then the total seasonal loading $\underset{\sim}{C}P$ of dissolved P, and $\underset{\sim}{S}P$ of sorbed P, are, respectively:

$$\begin{cases} \underset{\sim}{C}P = \sum_{\sim} Y_1(j) = C_1 \sum_{\sim} V(j) \\ \underset{\sim}{S}P = \sum_{\sim} Y_2(j) = C_2 \sum_{\sim} Z(j) \end{cases} \quad j = 1, 2, \dots, J \quad . \quad (6)$$

Total seasonal loadings are thus defined as the sum of a random number J of random loading events.

Let $f_X(x)$ be the pdf of any random variable X , $F_X(x)$ be the distribution function, DF, and $\phi_X(x)$ be the characteristic function (i.e., Fourier transform), for continuous variates, or generating function (i.e., Z-transform), for discrete variates, (Feller, 1967). Then the characteristic functions of $\underset{\sim}{C}P$ are:

$$\phi_{\underset{\sim}{C}P}(S) = \phi_{\sim J} \left[\phi_{\underset{\sim}{Y}_1}(S) \right] = \phi_{\sim J} \left[C_1 \phi_{\underset{\sim}{V}}(S) \right] \quad , \quad (7)$$

and similar formula for $\underset{\sim}{S}P$ with $\underset{\sim}{Y}_2 = \underset{\sim}{Y}_1$, $C_2 = C_1$, $\underset{\sim}{Z} = \underset{\sim}{V}$.

In other words, the pdf of $\underset{\sim}{C}P$ and $\underset{\sim}{S}P$ can be calculated by transformation from Equations 6 and 7 once the pdf of $\underset{\sim}{V}$, $\underset{\sim}{Z}$ and $\underset{\sim}{J}$ or, alternatively, of $\underset{\sim}{X}_1$, $\underset{\sim}{X}_2$, and $\underset{\sim}{J}$ are known. In general, closed form solutions are quite cumbersome, if they can be calculated at all, but numerical solutions are always obtainable, especially by simulation. For a first order analysis, in which only mean and variance of $\underset{\sim}{C}P$ and $\underset{\sim}{S}P$ are estimated, direct

formulas are available in terms of the mean and variance of \bar{J} , \bar{Y} , \bar{X} [Benjamin and Cornell, 1970]:

$$E(\bar{C}_P) = E(\bar{J})E(\bar{X}_1) = C_1 E(\bar{J})E(\bar{Y}) \quad , \quad (8)$$

$$\begin{aligned} \text{var}(\bar{C}_P) &= E(\bar{J}) \text{var}(\bar{X}_1) + [E(\bar{X}_1)]^2 \text{var}(\bar{J}) \\ &= (C_1)^2 E(\bar{J}) \text{var}(\bar{Y}) + [C_1 E(\bar{Y})]^2 \text{var}(\bar{J}) \quad . \quad (9) \end{aligned}$$

and a similar formula for $E(\bar{S}_P)$, $\text{var}(\bar{S}_P)$.

Spatial total. Let $\bar{C}_P(i)$ and $\bar{S}_P(i)$ be the seasonal contribution of the i th subwatershed. Then the P loadings for the whole lake are, respectively:

$$\bar{L}_{\bar{C}_1} = \sum \bar{C}_P(i) \quad \text{and} \quad \bar{L}_{\bar{S}_1} = \sum \bar{S}_P(i) \quad , \quad i = 1, 2, \dots, I \quad , \quad (10)$$

where I is the number of subwatersheds. The contributions of the various watersheds can be added only if local concentration effects, which may be important (Edmonson, 1969) are neglected or studied separately. For the case of Lake Balaton, good mixing can be assumed, so that it may be legitimate to neglect local effects in a first approximation. Also, it may be that the Central Limit Theorem applies to the addition of the random variables in $\bar{L}_{\bar{C}_1}$ and $\bar{L}_{\bar{S}_1}$ which would imply independence of the $\bar{C}_P(i)$ or $\bar{S}_P(i)$'s. At any rate, empirical observations may be used to estimate the mean and variance of $\bar{L}_{\bar{C}_1}$ and $\bar{L}_{\bar{S}_1}$:

$$E(\bar{L}_{\bar{C}_1}) = E[\bar{C}_P(i)] \quad ,$$

$$\text{var}(\bar{L}_{\bar{C}_1}) = \sum \text{var}[\bar{C}_P(i)] - \sum \sum \text{cov}[\bar{C}_P(i) \cdot \bar{C}_P(j)] \quad , \quad (11)$$

and similar formulas for $\bar{L}_{\bar{S}_1}$.

Depending upon the type of lake, the seasonal totals \bar{C}_P , \bar{J}_P or the spatial totals $\bar{L}_{\bar{C}_1}$, $\bar{L}_{\bar{S}_1}$, constitute output elements of subsystem I which are also input elements of subsystem II (Figure 2), to be described next.

3.3.2 Subsystem II: The Lake

(A) State S(t)

The trophic state of a lake is usually measured by primary productivity of phytoplankton, which may be represented by Secchi Disk data. The approach of Vollenweider (Figure 1) may be also used to describe the state of a lake as a function of P input.

(B) Input X(t)

In addition to the P loading represented by the bivariate vector (L_1, L_2) , the input includes the net inflow into the lake or else, the portion of lake volume Q lost per period through the outflow. The latter quantity can be used as a parameter in the state transition function, as described next.

(C) State Transition Function F(·)

This function describes the P cycle in the lake, which has already been touched upon under 2.4b.

The equation proposed by Dillon and Rigler (1974) may be used as a state transition function to predict the amount of dissolved phosphorus P^* available for plant assimilation, at a chosen point in time:

$$P^* = \frac{L(1-R)}{DQ} \quad , \quad (12)$$

in which

- L = P loading in $g/m^2/year$ in the absence of water release,
- R = retention coefficient of P,
- D = mean depth in meters,
- Q = portion of lake volume lost per period through the outflow.

L is a highly uncertain quantity estimated by means of a seasonal mass balance equation. Let L_3 be a given dissolved P loading from sources other than non-point sources. L_3 may be estimated by methods described in Vollenweider (1968) and Patalas (1972).

Then $L_{\bar{1}}$ results from the exchange between dissolved P, i.e. $(L_{\bar{1}} + L_{\bar{3}})$ and sediment P, $L_{\bar{2}}$:

$$L_{\bar{1}} = (1 - k_1) (L_{\bar{1}} + L_{\bar{3}}) + kL_{\bar{2}} \quad , \quad (13)$$

in which

k_1 = average seasonal proportion of dissolved loading $(L_{\bar{1}} + L_{\bar{3}})$ that becomes fixed to sediments or precipitated,
 k = average seasonal proportion of P released from sediment into the water.

The retention coefficient R in Equation 12 may be estimated by use of the empirical equation proposed in Dillon and Kirchner (1975):

$$R = 0.426 \exp (-0.271 q_s) + 0.574 \exp (-0.00949 q_s) \quad (14)$$

in which q_s = areal water load to the lake or lake outflow volume divided by lake surface area (m/year).

It is realized that Equation 14 may be particularly inaccurate for a shallow lake with relatively small release, such as Lake Balaton [Widger and Kimball, 1976]. Nevertheless, Equation 14 can provide an order of magnitude of R until sample measurements of P concentration in lake release are available.

(D) Output Y(t)

The output has a physical and an economic component. The physical component includes P^* from Equation 12 as well as the amount of $K_{\bar{1}}(n)$ of P stored in lake bottom sediment at the beginning of year n; $K_{\bar{1}}(n)$ depends on $K_{\bar{1}}(n-1)$ and $L_{\bar{1}}$, $L_{\bar{2}}$, and $L_{\bar{3}}$. The economic component U is a utility function associated with the costs and benefits of eutrophication control

$$U = U[Y_{\bar{1}}, Y_{\bar{2}}, P^*, K_{\bar{1}}(n)] \quad . \quad (15)$$

Note that U is defined as a function of the elements of both sub-systems I and II.

(E) Output Function

A complete output function would calculate both U and K(n). However, the costs and benefits associated with eutrophication control are next to impossible to evaluate with the state-of-the-art knowledge: before the economics of control can be studied, the system should be well defined. Even K(n) is very difficult to estimate because the initial state of lakes (including K(0)) is usually unknown [Edmonson, 1969; Hutchinson, 1973]; yet, K(n) is an indispensable quantity for studying long-range accumulation of P. The mass balance of annual sediment P is calculated as follows:

stored P at the beginning of year n:

$$(1 - k)K_{\lambda}(n) \tag{16}$$

storage of P from input of year n; i.e., the complement of release Equation 13:

$$F_{\lambda}(n) = \underbrace{(1 - k)L_{\lambda 2}(n)}_{\substack{\text{P not released} \\ \text{by sediments}}} + \underbrace{k_1 [L_{\lambda}(n) + L_3(n)]}_{\substack{\text{dissolved P becoming} \\ \text{fixed to sediments}}}$$

(17)

The P stored at the beginning year (n+1) is the sum of Equations 16 and 17:

$$K_{\lambda}(n+1) = (1 - k)K_{\lambda}(n) + F_{\lambda}(n) \tag{18}$$

Plausible solutions of the expected value of this equation are shown in section 4.32 first order analysis (p. 28). The example of the Tetves subwatershed draining directly into Lake Balaton is presented next.

4. CASE OF THE TETVES SUBWATERSHED OF LAKE BALATON

4.1 Physical properties of the subwatershed

A sketch of the entire Lake Balaton watershed is shown in Figure 3. Table 1 contains the main geographical, soil and land use data for the subwatershed considered. Though the area of the Tetves subwatershed (70 km²) is less than 2% of the total lake, watershed soil, land use and slope conditions are typical of conditions found around the lake.

The Tetves is a long watershed with a ratio of length to average width of about three. Since there is mostly agriculture and forestry over the catchment, phosphorus data observed at the downstream point refer mostly to non-point sources. Surface erosion is intensive over the catchment; a sediment retention dam built in 1970 has been filled up with 120,000 tons of sediment during four years.

Annual average rainfall is 652 mm; an average annual number of six days have more than 20 mm rainfall. The population was 3700 in 1975. There are about 2000 cattle with an average weight of 500 kg apiece. Phosphorus loading due to human and animal wastes is relatively low in the area; observed P loading data however, were adjusted to account for loading of human and animal origins.

During a 13-month observation period, (July 1, 1975, to July 31, 1976), the ratio of applied phosphorus in the form of fertilizers and observed total phosphorus at the outlet of the watershed was 6.6%.

4.2 Measurement data

Hydrological observations have been taken by the Research Institute for Water Management, VITUKI, Hungary over the watershed.

Rainfall data. Daily amounts of rainfall are available for years between 1964 and 1975 at four stations over the watershed.

Runoff data were recorded between 1964 and 1970 at the outlet of the watershed (43 events).

TABLE 1

Specification of the Tetves Watershed

Geographical data

Watershed area: 70 km²

Watershed length: 15.2 km

Watershed average width: 5.1

Average slope of the main water course: 4.1%

Soil data

Loess: 39%, Sandy loam: 30%, Gravel and sand: 27%, other: 4%

Land use data

Agricultural land: 70%, Meadow: 20%, Forest: 6%, Vineyard
and Orchard: 4%

Slope categories

0 - 5%	5 - 12%	12 - 25%	25 - 35%	Average
30%	30%	34%	6%	11%

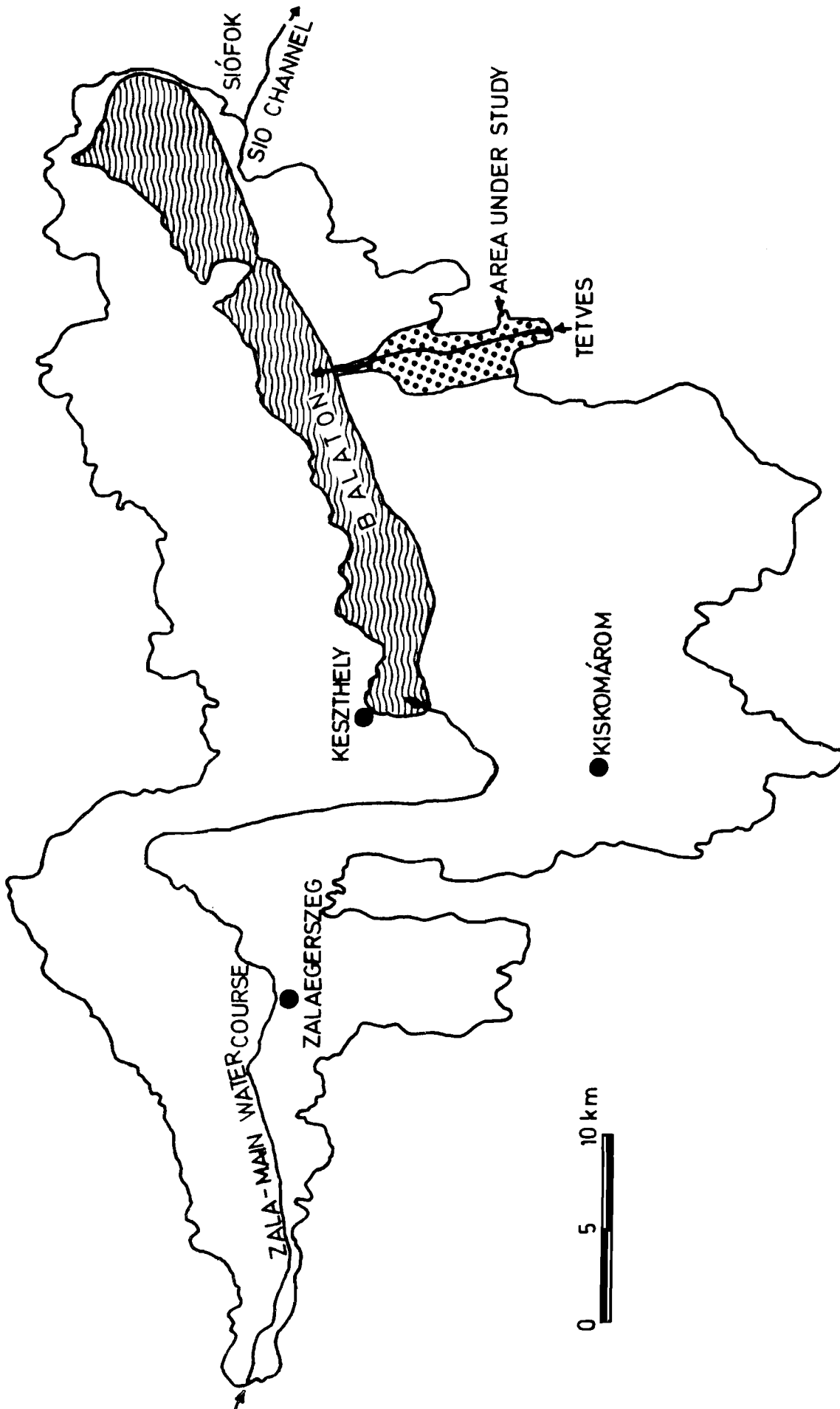


Figure 3: Watershed of Lake Balaton and Tetves subwatershed

Phosphorus loading measurements. In 1975, when the eutrophication process in the lake showed an accelerated rate, VITUKI started new observations in two forms: point measurements of discharge, then of concentration of sediment, dissolved P and fixed P. For this report, only one year of such data amounting to 49 points was available. Table 2 contains hydrologic, sediment and phosphorus data for nine events that have occurred between July 1975 and February 1976.

Next, the systematic use of these measurement data for the elements of the model (Section 3.3) will be discussed.

A. Source. At present, 49 data points mentioned are used to calibrate preliminary models for C_1 and C_2 . Ongoing measurements will be used continuously to validate and/or improve these models.

B. Precipitation events. Rainfall data are divided into two groups. First, 1964-1970 data are used to calibrate the rainfall-runoff relationship for model elements C. and D. Then rainfall data from 1971 to date provide the stochastic input for the validation of model elements C. and D.

C. Dissolved phosphorus loading. The SCS runoff model is calibrated with the help of the 43 observed rainfall-runoff events. This model is then combined with the C_1 model (point A.); this yields a calibrated dissolved P loading model which can in turn be validated with the observed runoff and loading events.

D. Sediment phosphorus loading. Since there are no observation data on event-based sediment yield, the calibrated rainfall-runoff model is used in conjunction with the universal soil loss equation. Parameters of this latter equation are taken from tables, which is the common procedure. The sediment yield model thus defined is applied in conjunction with C_2 model to yield the sediment P loading model.

This model is validated with the help of the nine observed sediment and loading events presently available.

TABLE 2
 Runoff Events in the Tetves Watershed
 Observation data of VITUKI, Budapest

Time of Event	Runoff Volume $V [10^3 m^3]$	Peak Flow $Q [m^3/s]$	Duration [hour]	Sediment			Dissolved		Phosphorus	
				Total $Z [kg]$	Average Concentration $[g/m^3]$	Total $Y_1 [kg]$	Average Concentration $C_1 [g/m^3]$	Total $Y_2 [kg]$	Average Concentration $C_2 [mg/g]$	
02.07.75	128	1.9	23	49700	388	70	0.55	627	12.6	
12.07.75	26	2.6	9	184000	7076	18	0.70	260	14.9	
19.07.75	98	2.7	22	119000	1214	70	1.05	327	4.1	
24.07.75	272	8.8	32	721000	2650	435	1.60	2830	3.9	
18.08.75	295	8.2	52	321800	1090	236	0.80	140	0.4	
03.09.75	85	5.9	18	195300	2300	13	1.50	1131	5.4	
06.09.75	235	5.5	45	159700	680	322	1.40	441	2.0	
13.09.75	114	3.3	22	132800	1160	103	0.90	182	1.4	
11.01.76	53	1.8	45	62390	1181	7	0.12	58	0.9	

With the above data in hand, numerical results may be obtained from the system model in three possible ways:

1. an analytical solution is sought, in which the random variables $(X_{\tilde{1}}, X_{\tilde{2}}, t)$ are transformed into $L_{\tilde{1}}, L_{\tilde{2}}$, hence $L_{\tilde{1}}$ or $P_{\tilde{1}}^*$;
2. a first order analysis of P loading is performed, coupled with an expected value analysis of long-term accumulation of P stored in sediments;
3. a simulation is run, starting with synthetic sequences of rainfall events, and leading to pdf of state and output variables of the system.

Examples of each of these three approaches are now given.

4.3.1 Analytical solution

A closed form solution may be obtained by effecting the transformations of random variables defined in Equations (3), (4), (5), (6), and (8) as in Smith (1975) and Smith et al., (1977). However, because the transformations are non-linear, the integrals necessary to calculate the transformed distribution functions (DF) must be evaluated numerically. Then the only possible way to obtain the DF of $L_{\tilde{1}}$ which is the sum of a random number of random P loading events $(Y_{\tilde{1}}, Y_{\tilde{2}})$ to fit a pdf to $(Y_{\tilde{1}}, Y_{\tilde{2}})$. This intermediate fit usually results in a variance estimate of $L_{\tilde{1}}$ biased downward; one possible way to avoid such a problem is to start from an empirical formula based on annual (or seasonal) quantities, such as Equation 12. Let $I_{\tilde{1}}$ and $Q_{\tilde{1}}$ be random variables with DF determined from empirical observations. Then

$$F_{P_{\tilde{1}}^*}(x) = \int_0^1 F_{L_{\tilde{1}}}\left(\frac{nq}{\alpha}\right) d F_{Q_{\tilde{1}}}(q) \quad , \quad (19)$$

with $\alpha = \frac{1-R}{D} \approx 0.2$ for Lake Balaton.

Preliminary simulation results show that L may be gamma distributed, with estimated mean and variance given in Table 2 as $m = 1.56 \times 10^3$, $s^2 = 4.7 \times 10^5$. Furthermore, a logical prior pdf for Q , defined in (0,1) is the beta distribution. Equation 19 can thus be evaluated numerically.

4.3.2 First Order Analysis

Table 3 shows the results of a first order analysis, i.e. mean and variance, of loading of dissolved P and sediment P per event, then per year. The loadings stemming from base flow included in the table and added to the loadings from runoff events assuming independence. Details of the calculations for Table 3 and possible solution of long-term accumulation equation 18 are given below.

A. Source

1. Baseflow. A constant baseflow without runoff events of $0.1 \text{ m}^3/\text{s}$ was considered throughout the year. Measurement data give the following dissolved P concentration in the baseflow:

$$E(\text{base } C_1) = 0.26 \text{ gr/m}^3 ,$$

$$\text{var}(\text{base } C_1) = 3.18 \cdot 10^{-2} [\text{gr/m}^3]^2 .$$

Thus, first order statistics for annual dissolved P in the baseflow can be calculated directly, since the annual base volume is known:

$$E(\text{base } \underset{\sim}{C}P) = 780 \text{ kg} = 0.78 \text{ ton} ,$$

$$\text{var}(\text{base } \underset{\sim}{C}P) = 2.8 \cdot 10^5 \text{ kg}^2 = 0.28(\text{ton})^2 .$$

Similarly, empirical statistics for annual amounts of fixed P in the baseflow are:

$$E(\text{base } \underset{\sim}{S}P) = 3468 \text{ kg} = 3.47 \text{ tons} ,$$

$$\text{var}(\text{base } \underset{\sim}{S}P) = 4.8664 \cdot 10^6 \text{ kg}^2 = 4.87 \text{ tons}^2 .$$

Table 3
Results of First Order Analysis

	Runoff Events				Seasonal Sum		Base Flow		Total Seasonal	
	C ₁	C ₂	X ₁	X ₂	CP ~	SP ~	CP ~	SP ~	Dissolved	Sorbed
Mean	0.95	5.15	105	700	780	5,200	780	3,470	1,560	8,670
Units (For the Mean)	g/m ³	mgP/gS	kg	kg	kg	kg	kg	kg	kg	kg
Variance	0.23	26.70	1.054x10 ⁴	7.3x10 ⁵	1.9x10 ⁵	1.02x10 ⁷	2.8x10 ⁵	4.87x10 ⁶	4.7x10 ⁵	1.507x10 ⁷

2. Runoff events. There was no seasonal pattern in observations of C_1 taken during runoff events. However, a correlation coefficient of 0.77 was found between peak flows Q and measurements of C_1 . Since runoff volume V per event and peak flows Q may also be dependent, then C_1 and V could in turn be so. Due to the paucity of data, the hypothesis $\rho(C_1, V) = 0$ could not be tested; thus, as a first approximation, independence is assumed. If further observations invalidate this hypothesis, the calculations below can be changed without problem. Empirical first order statistics of C_1 are:

$$E(C_{\nu 1}) = 0.95 \text{ g/m}^3, \quad \text{var}(C_{\nu 1}) = 0.23 [\text{g/m}^3]^2.$$

Similarly, no seasonal pattern could be detected in concentrations C_2 measured during runoff and, consequently, in sediment events. Also, sediment yield per event S_1 and C_2 seemed to be independent. Empirical first order statistics of C_2 are:

$$E(C_{\nu 2}) = 5.15 \text{ mgP/gS}, \quad \text{var}(C_{\nu 2}) = 26.70 [\text{mgP/gS}]^2.$$

B. Precipitation events

Statistical analyses of observed rainfall data strengthen prior hypotheses that the generating mechanism of precipitation events is a Poisson process and the amount of precipitation per event can be described by an exponential distribution [Fogel and Duckstein, 1969].

C. Dissolved phosphorus loading

Then the volume of runoff V_{ν} per event can be calculated from the SCS formula (Equation 3) with $X_1 = (\text{precipitation}) - 0.2S$; $S = 3.5$ in. from tables (Soil Conservation Service, 1972; Williams and Hahn, 1973). First order empirical statistics for V_{ν} from the observed 43 events are as follows:

$$E(\underset{\sim}{V}) = 10.94 \cdot 10^4 \text{ m}^3 ,$$

$$\text{var} (\underset{\sim}{V}) = 84.45 \cdot 10^8 \text{ m}^6 .$$

Now, first order statistics for events of dissolved P can be calculated as

$$E(\underset{\sim}{Y}_1 = \underset{\sim}{C}_1 \cdot \underset{\sim}{V}_1) = E(\underset{\sim}{C}_1)E(\underset{\sim}{V}) = 105 \text{ kg} = 0.105 \text{ tons} ,$$

$$\text{var} (\underset{\sim}{Y}_1) = [E(\underset{\sim}{C}_1)]^2 \text{var} \underset{\sim}{V} + [E(\underset{\sim}{V})]^2 \text{var} \underset{\sim}{C}_1 = 10535 \text{ kg}^2 .$$

D. Sediment P loading

Sediment yield $\underset{\sim}{Z}$ per event is calculated for each observed runoff event with the help of the universal soil loss equation.

$$Z = 95 \quad QV^{0.5} \quad K(LS)C \cdot P ,$$

where Z is the sediment yield per event in tons, Q is the peak flow in cfs, V is runoff volume per event in inches, K is the soil erodibility factor (0.25 in this watershed), LS is the slope length and gradient factor (0.70), C is the cropping management factor (0.10) and P is the erosion control practice factor (1.0). Values of these factors were estimated for the watershed as described in the publication: Soil Conservation Service (1972).

From these, first order statistics for $\underset{\sim}{Z}$ are as follow:

$$E(\underset{\sim}{Z}) = 136 \text{ tons} , \quad \text{var} (\underset{\sim}{Z}) = 9025 \text{ (tons)}^2 .$$

With the help of statistics for $C_{\sim 2}$, mean and variance of sediment P loadings per event are calculated as:

$$E(Y_{\sim 2} = C_{\sim 2} \cdot Z) = 700 \text{ kg} , \quad \text{var} (Y_{\sim 2}) = 7.3 \cdot 10^5 \text{ kg}^2 .$$

E. Total annual loadings

First, mean and variance of the random number of events J in a year is given, as estimated from observed runoff events:

$$E(J) = 7.43 \cdot \text{year}^{-1} , \quad \text{var} (J) = 9.92 \cdot \text{year}^{-2} .$$

Since the mean and variance are nearly equal, and interarrival times are approximately exponential, a Poisson process can be assumed.

Next, Equations 8 and 9 are used to estimate mean and variance of annual P loadings (dissolved and fixed).

$$E(CP) = E(J) E(Y_{\sim 1}) = 780 \text{ kg}$$

$$\text{var} (CP) = E(J) \text{var} (Y_{\sim 1}) + [E(Y_{\sim 1})]^2 \text{var} (J) = 1.9 \cdot 10^5 \text{ kg}^2 .$$

Similarly, the mean and variance of SP are

$$E(SP) = 52.0 \text{ kg} , \quad \text{var} (SP) = 102 \cdot 10^5 \text{ kg}^2 .$$

Finally, loadings from baseflow and runoff events can be added, leading to the following statistics:

Dissolved P:

$$E(\text{total CP}) = E(\text{base CP}) + E(CP) = 780 + 780 = 1560 \text{ kg} ,$$

$$\begin{aligned}\text{var}(\text{total } \underset{\sim}{\text{CP}}) &= \text{var}(\text{base } \underset{\sim}{\text{CP}}) + \text{var}(\underset{\sim}{\text{CP}}) \\ &= 2.8 \cdot 10^5 + 1.9 \cdot 10^5 = 4.7 \cdot 10^5 \text{kg}^2 .\end{aligned}$$

Sediment P:

$$E(\text{total } \underset{\sim}{\text{SP}}) = E(\text{base } \underset{\sim}{\text{SP}}) + E(\underset{\sim}{\text{SP}}) = 8678 \text{ kg}$$

$$\text{var}(\text{total } \underset{\sim}{\text{SP}}) = \text{var}(\text{base } \underset{\sim}{\text{SP}}) + \text{var}(\underset{\sim}{\text{SP}}) = 151 \cdot 10^5 \text{kg}^2 .$$

These preliminary results show that:

a. Approximately as much phosphorus reaches the lake during relatively short runoff events as during the rest of the year. This is especially marked for sediment P.

b. The watershed under study appears to produce more sediment P than dissolved P.

c. There is considerable variance in the annual amounts of both dissolved P and fixed P, conveyed by either baseflow or runoff events. This fact demonstrates the usefulness of stochastic models to estimate the loading conditions.

d. First order analysis is a relatively simple way to characterize natural uncertainty in P loading conditions. However, a more thorough stochastic analysis is warranted in which simulation is used with stochastic rainfall input to yield complete pdf of P loadings.

F. Average long-term accumulation

Still within the framework of first order analysis, consider the expected value of Equation 18. To study the average long-term mass balance of P loading, the generating functions (or Z transform) method is used. The expected value of Equation 18 is transformed as:

$$\frac{K(s) - K(0)}{s} = (1 - k) K(s) + F(s) ,$$

or

$$K(s) = \frac{K(0)}{1 - s(1 - k)} . \quad (20)$$

Taking the inverse transform of $K(s)$ yields

$$K(n) = K(0) (1 - k)^n + \sum_{k=0}^{n-1} f(k) (1 - k)^{n-k-1} . \quad (21)$$

Four possible cases are considered as examples:

a. $L_1 = L_2 = L_3 = 0$.

The expectation of Equation 18 then reduces to

$$K(n+1) = (1 - k_2) K(n) ,$$

whose solution is

$$K(n) = (1 - k_2)^n \cdot K(0) . \quad (22)$$

In this case the stored P decreases geometrically.

b. $F(n) = A$, a constant,
the solution is

$$K(n) = (K(0) - \frac{A}{k_2}) (1 - k_2)^n + \frac{A}{k_2} . \quad (23)$$

The stored phosphorus increases from $K(0)$ to the constant $\frac{A}{k_2}$, which is much larger than A , since the percentage, k_2 of P released from sediments is expected to be quite small. This result is (asymptotically, at least) independent of $K(0)$, but the state $K(n)$ (i.e. P stored in lake sediments) has to be known at some time n_0 if a finite horizon is being considered.

c. $F(n) = A + Bn$, a linear increase in annual loading,
the solution is

$$K(n) = (K(0) - r) (1 - k_2)^n + r + \frac{B}{k_2} n , \quad (24)$$

with the constant r defined as

$$r = \frac{1}{k_2} (A - \frac{B}{k_2}) .$$

This implies an indefinite linear growth, with a coefficient $B/k_2 \gg B$. These three cases (Equations 22, 23, and 24)

are represented in Figure 4.

d. The input is A + B in the spring and summer and A - B the rest of the year.

Hence,

$$F(n) = A + (-1)^n B \quad (25)$$

and

$$K(n) = K(0) (1 - k)^n + \left[\frac{A}{k} + \frac{(-1)^{n-1} B}{2 - k} \right] [1 - (1 - k)^n] \quad (26)$$

Let $K(0) = 30$, $k = 0.20$, $A = 8$, $B = 6$, which means that the input in the low season is 1/7 of that in the high season. Then Equation 26 becomes:

$$K(n) = 30 (0.8)^n + [40 + (-1)^{n-1} 3.33] [1 - (0.8)^n] \quad (27)$$

Asymptotically,

$$K(2n) = 36.67 \text{ and}$$

$$K(2n - 1) = 43.33$$

which means that the quantity of P in sediments in low season is 83% of the quantity in the high season, i.e. a substantial dampening of the input oscillations occurs [Duckstein and Bogardi, 1977].

Consider now how a Bayesian approach may be used in conjunction with a solution such as Equation 26. Let $B = 0$ to simplify the presentation (constant loading), hence

$$K(n) = \left[K(0) - \frac{A}{k} \right] (1 - k)^n + \frac{A}{k} \quad (28)$$

Let the parameter k be uncertain, with prior pdf uniform in (0.05, 0.35). Then the expectation of Equation 28 yields

$$\bar{K}(n) = \int_{0.05}^{0.35} \left[\left(K(0) - \frac{A}{k} \right) (1 - k)^n + \frac{A}{k} \right] dk \quad (29)$$

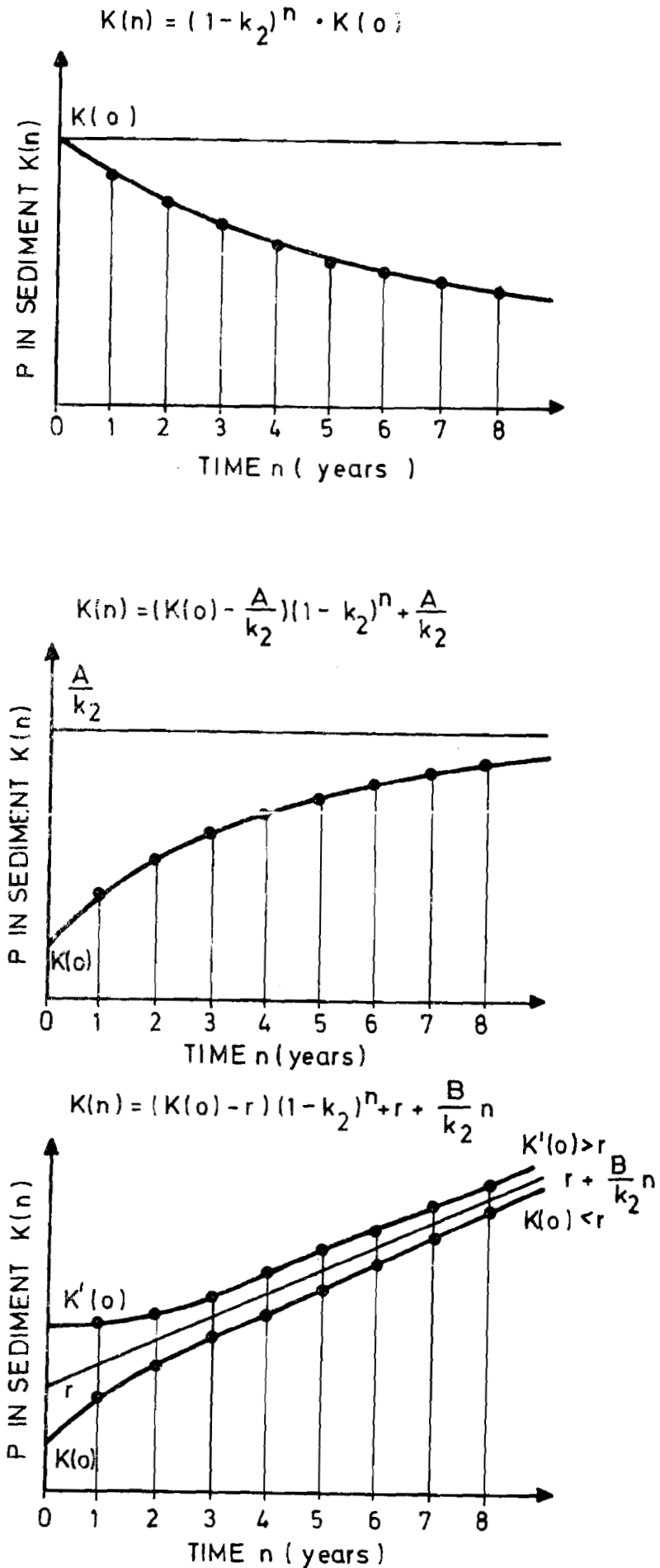


Figure 4: Long-range accumulation of P in the lake in three cases of input: null, constant, linearly increasing.

In particular, the asymptotic value of $K(n)$ is now $\frac{A}{3} \text{Ln } 7 = 6.49 A$ as compared to the (wrong) value of $5A$ obtained if the mean estimate of k had been used. If more data become available, Bayes Rule can be used to update the prior pdf and find $\bar{K}(n)$ under the posterior pdf (Davis et al., 1972).

4.3.3 Simulation

This approach is illustrated for the estimation of the DF of sediment P , $\hat{S}P$, after Szidarovsky et al., (1976). The procedure for estimating the DF of dissolved P , $\hat{L}P$, is the same, but with simpler formulas:

- a. let $T: = 0$, $ZZ: = 0$,
- b. simulate a realization t of the exponentially distributed random variable t with parameter λ and let $T = T + t$,
- c. if $T > N$ years, then go to (e). If $T \leq N$ years, then go to (d),
- d. simulate a bivariate realization of $(X_{\hat{1}}, X_{\hat{2}})$. Using Equations 4 and 5, calculate the corresponding value of Z and $Y = Y_2$, $YY = YY + Y$ and go to (b)
- e. the value of YY is a sample element of the accumulated sediment P .

When enough sample elements of YY have been simulated, an empirical DF usable in decision analysis is obtained. Numerical results of simulation are shown in Figures 5 and 6. Empirical distributions of dissolved P and fixed P per event are fitted to gamma distributions. The hypothesis of gamma distributed P - loading events cannot be rejected by the Kolmogorov-Smirnov test at the 5% probability level. Simulated seasonal (spring and summer) loadings exhibit symmetrical distribution but no formal test of fitting has been applied due to the small size of simulated data.

Simulation yields statistics that are commensurate with the results of the first order analysis (Table 3). Furthermore, the method described in Duckstein et al., (1976) may be used "Mutatis Mutandis" to minimize the expected value of a goal function $g(a, \theta)$ under both natural and parameter uncertainty. Let the decision A be the total volume of sediments retainable behind a control reservoir

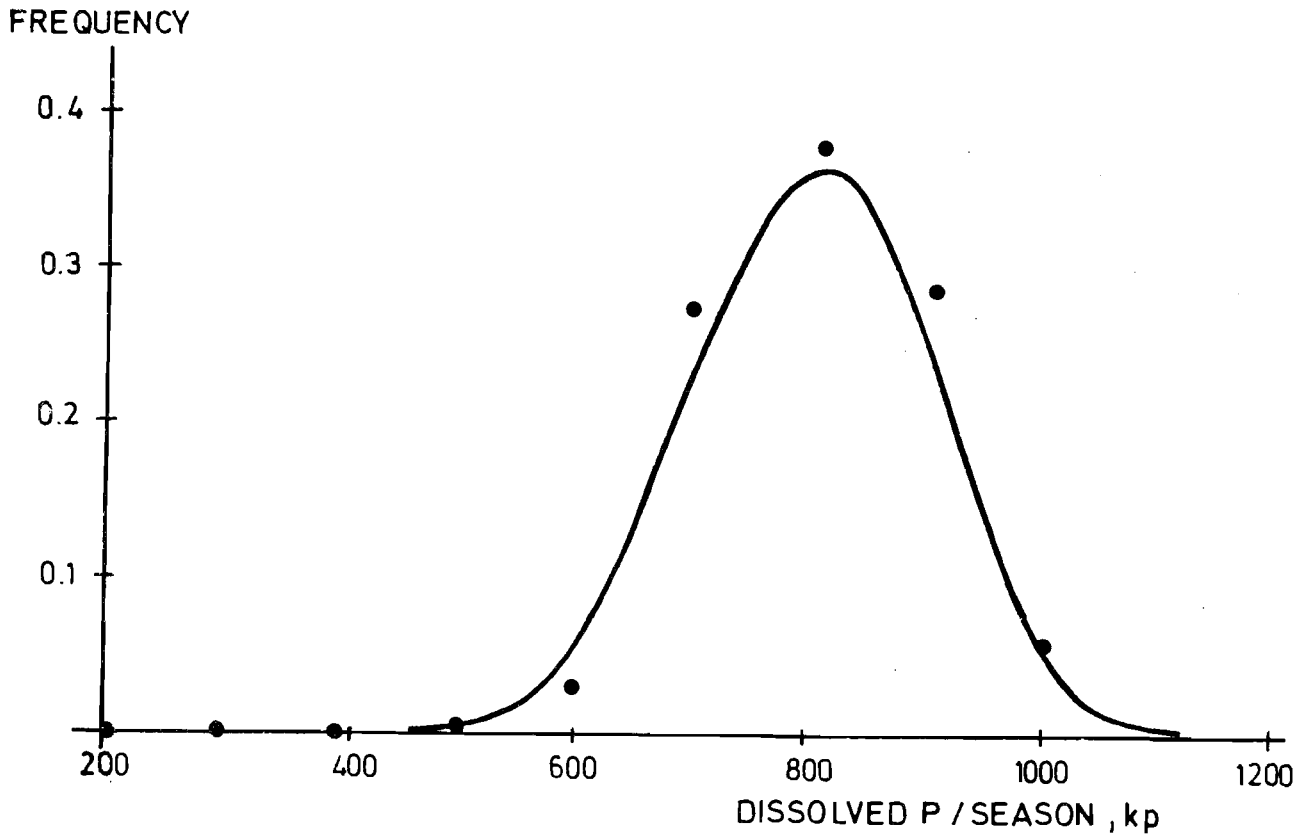
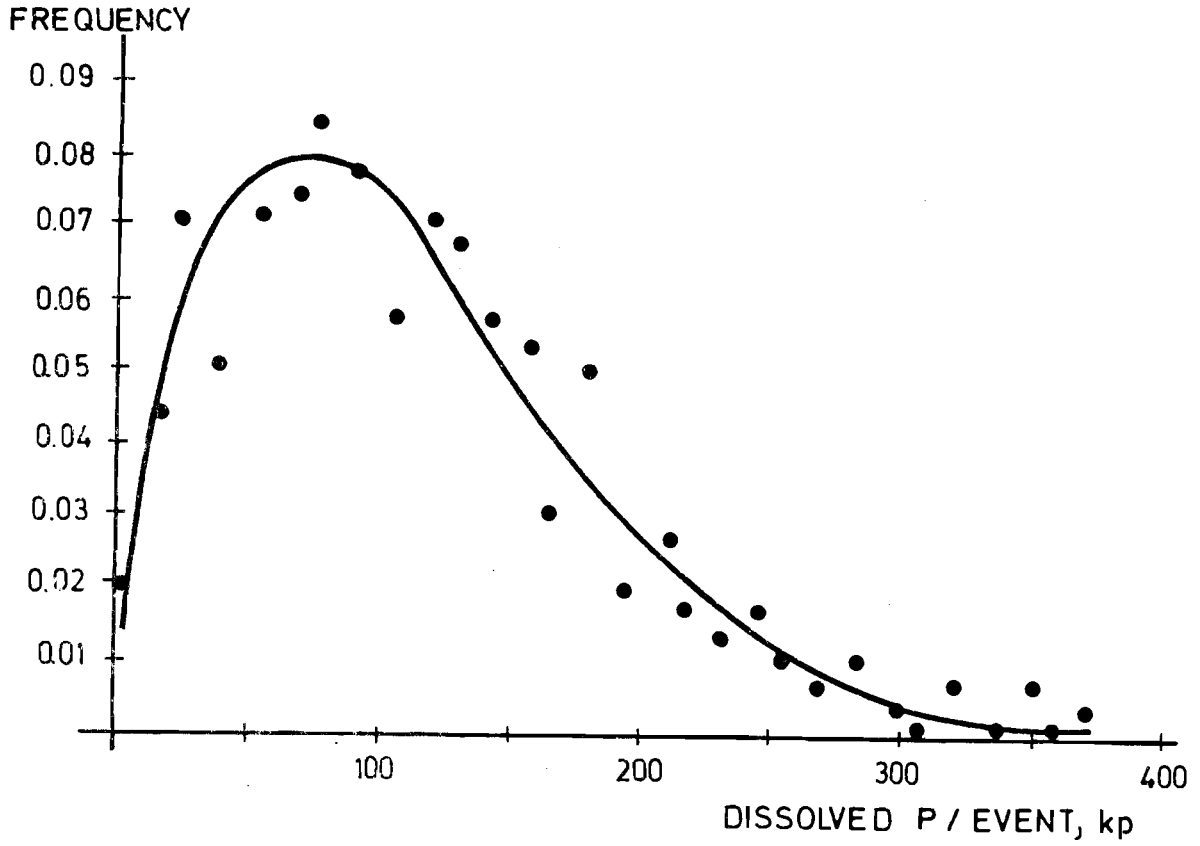


Figure 5: Simulated frequency curve and fitted pdf of dissolved P per event, y_1 , and per season, c_p .

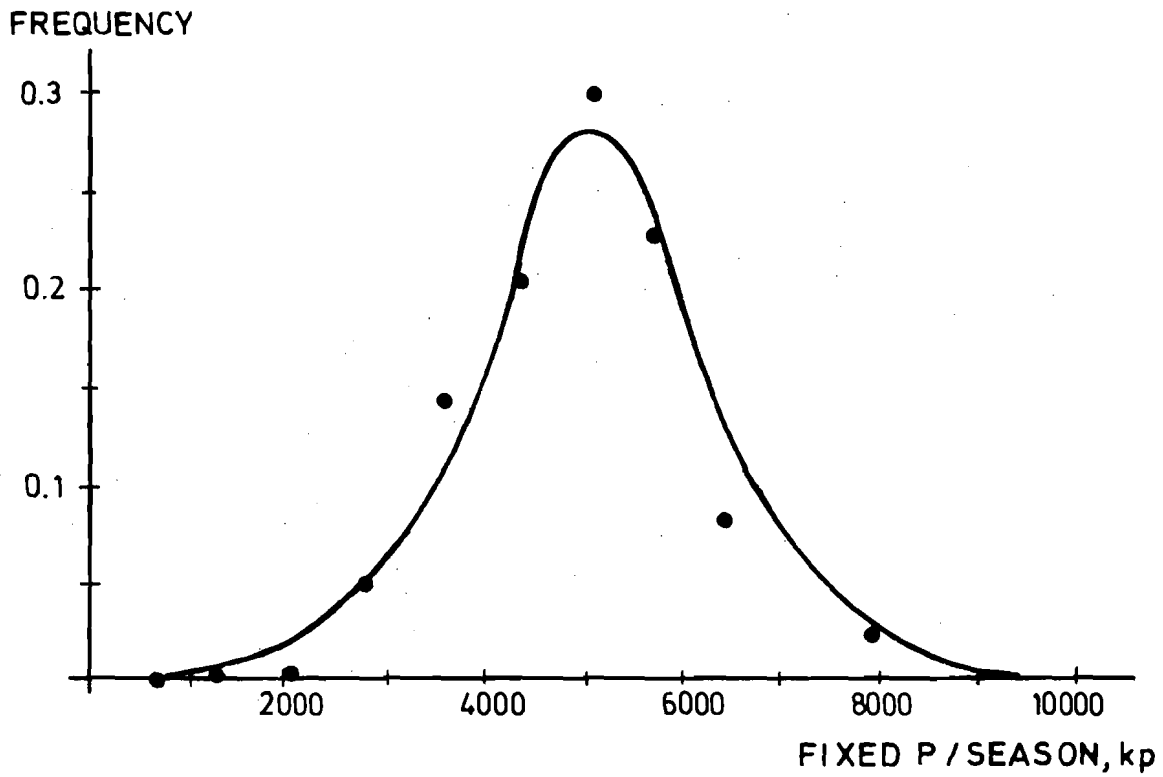
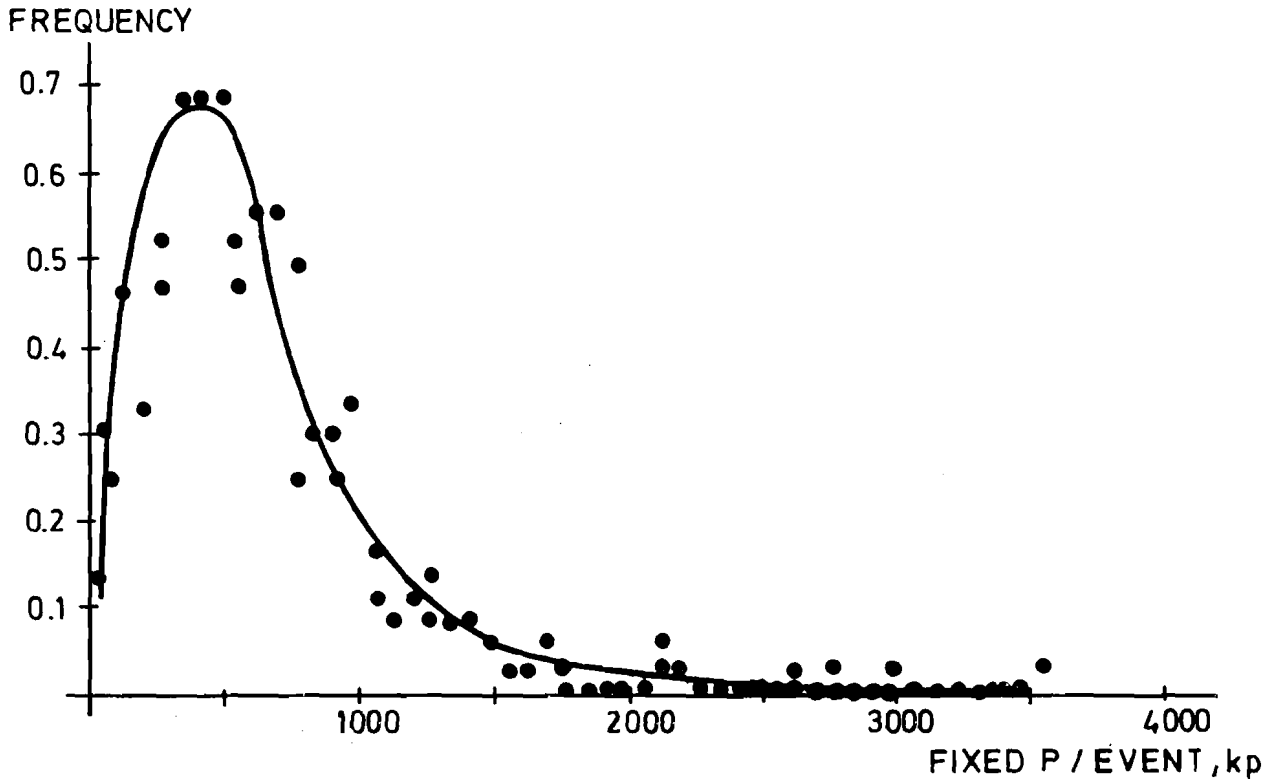


Figure 6: Simulated frequency curve and fitted pdf of P fixed to sediment per event, \tilde{Y}_2 , and per season, \tilde{S} .

of lifetime 100 years and let the loss function be piecewise linear of the opportunity loss type. Then the optimum Bayes decision a^* is shown in Figure 7, as a function of record length N .

5. DISCUSSION: CONTROL OF P LOADING

In terms of decision theory, this section exposes the decision space and reward space of the P loading problem from non-point agricultural sources. In other words, possible methods of control are discussed first; then the economics or trade-offs involved are broached.

5.1 Alternative control methods

P may be controlled at the source, in the transport phase or in the lake itself. Assume that it has been established that the larger source of P is from distributed agricultural origin.

(a) Source

The parameters C_1 and C_2 have been shown to vary as a function of the type and quantity of fertilizer (Holt, et al., 1970; Frere, 1973). Thus, one may envision changing the N:P ratio in the fertilizer utilized, or the timing of the application. On irrigated lands, timing of irrigation water and fertilizer application could be coordinated to yield maximum utilization by the crops and minimum P available for loading the lake.

An interesting case has been presented in Gburek and Heald (1974), who found that PO_4 -P loading from a Pennsylvania experimental watershed was considerably reduced if no fertilizer was applied within 50 m of streams; they suggest that this observation may hold for the Northeastern U.S.A. In this case, P loading to the stream would be reduced by avoiding crops that need high P-content fertilizer in the vicinity of the streams.

Furthermore, the mode of fertilizer spreading may have a substantial effect on P output from an agricultural watershed. A quantitative study of this factor may be found in Timmons, et al., (1973). Note that any control method has an associated cost, as discussed in Section 5.2 below. For example, a reduction in P output can be obtained if broadcast fertilizer is plowed in deeply.

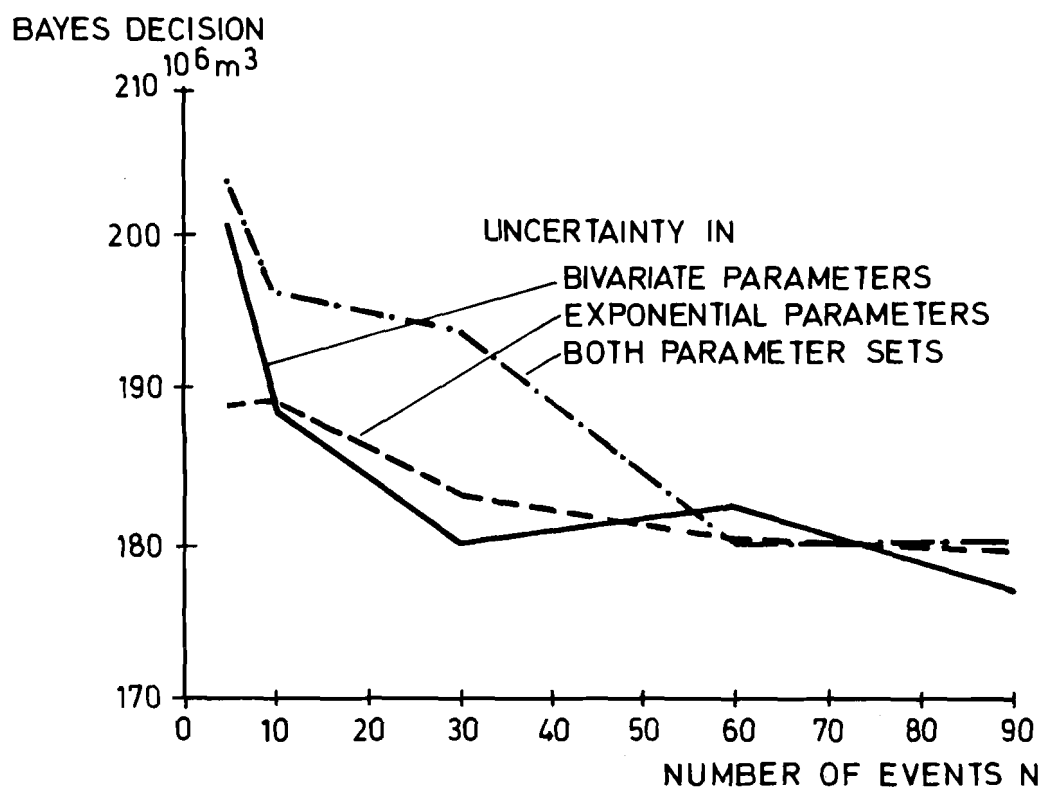


Figure 7: Optimum sediment storage volume a^* of a control reservoir versus sample record length.

(b) Transport

The quantity of dissolved P is controlled by the amount of runoff, which is generally impossible to change on agricultural lands unless retention dams are built. On the other hand, watershed management practices may have a substantial influence on runoff in other land use cases.

Sediment P depends on erosion, which can be controlled by proper soil conservation practices, such as terracing land in slopes, hedges, etc.

In general, controlling the transport of P can hardly be done only for the sake of decreasing P loading: it is incidental to other watershed control measures, such as flood or erosion control, or drainage. Further, a broader view of P loading leads to consideration of all P sources as a system; then various schemes of wastewater reuse may be studied as in Duckstein and Kisiel (1976); one of the schemes could irrigate agricultural land with secondary treated wastewater rich in nitrates and phosphates, which would otherwise have polluted the streams, groundwater, and/or other water bodies.

(c) P in the lake

The dissolved P concentration may be decreased by lowering the residence time or by increasing the average depths. These effects may be quantified by using either Figure 1 (after Vollenweider, 1968) or Equation 12 (after Dillon and Rigler, 1974). In Figure 1, the trophic state of the lake is given to the right-hand side as $\log [(\text{residence time})^{-1}(\text{average depth})]$. In Equation 12, P^* is inversely proportional to mean depth \bar{Z} and release fraction Q , which varies as the residence time. The importance of the latter quantity in determining pollutant concentration is further emphasized in Armstrong and Weimer (1973). However, as mentioned earlier, neither residence time nor average depth of a lake are easy to control. In the case of Lake Balaton, both variables are determined by the random inflow and the release policy which is based on two main considerations: minimum level control because of water quality and maximum level control because of potential wind wave damages (Metler, et al., (1975). The

amount of sediment P, once in the lake, can only be controlled by dredging, which is an expensive solution for large lakes. On the other hand, since the release of P from sediments is smaller in aerobic conditions, the control of algal growth by biological means may increase the DO level and minimize P release as suggested in Serruya and Serruya (1975). Also, the use of copper sulphate, which inhibits the growth of algae (Edmonson, 1969) may itself have a beneficial effect on keeping the phosphorus fixed to sediments. Finally, stirring up the sediment layer usually causes P release: while natural causes such as wind or currents are unavoidable, human intervention, in the form of motor boats or large numbers of swimmers can, to a certain extent, be curtailed.

5.2 Economics of control

The effect of P loading, in the cases when P is the limiting nutrient for lake eutrophication, has economic implications. However, the losses due to eutrophication are almost impossible to evaluate with any degree of accuracy. On the other hand, the costs of control can usually be estimated within several percentage points. Recognizing this fact led Miller and Byers (1973) to trade off national benefits versus sediment P in a multi-objective approach to planning a watershed project; in other words, sediment P may be used as a measure of effectiveness (MOE) in a cost-effectiveness display.

More generally, it is proposed to compare, in a later study, alternative P loading control methods using a standardized cost-effectiveness approach as presented, for example, in David and Duckstein (1976). The essence of such an approach consists of the following steps:

1. Define the goals of controlling eutrophication (in words);
2. Give engineering specifications for the control, such as minimum DO level, release constraints, number of visitors allowed;
3. Set up evaluation criteria or MOE, such as various costs or probability of algal growth;

4. Define the approach philosophy: fixed cost or fixed effectiveness; at first, a fixed effectiveness approach seems to be warranted;
5. Define the alternative control methods;
6. Investigate how effectively the alternatives operate, using the stochastic model presented herein, as well as simulation and other operation research tools.
7. Display the alternatives versus MOE (including costs) in a tableau.
8. Rank the alternative systems using a multicriterion ranking method such as ELECTRE as in David & Duckstein (1976), or a multiobjective decision-making scheme;
9. Perform sensitivity analysis on all the above steps;
10. Document the assumptions, rationale, submodels used throughout the study.

6. CONCLUSIONS

Results of this paper lead to the following concluding points:

1. The necessity of using system models to predict nutrient loadings to lakes and reservoirs has been established.
2. Available models cannot account for the inherent uncertainty in the non-point nutrient production over the watershed;
3. For Lake Balaton, Hungary, as in a great number of other lakes, phosphorus input is the controlling element of primary productivity which, in turn characterizes the eutrophication process;
4. The model recommended in this paper considers hydrologic events (rainfall-surface runoff-erosion) which trigger P loading events;
5. Amounts of dissolved P and sediment P per event are estimated from known or assumed relationships (SCS formula and the universal soil loss equation), whose parameters should be calibrated with observation data or, in the absence of these, with the help of published tables;

6. The sum of a random number of random P events leads to the pdf of seasonal (e.g. annual) amount of dissolved P loading and sediment P loading;
7. The model makes it possible to estimate long-range accumulation of P in lake sediment; such accumulation may be an essential element of a control model of lake eutrophication;
8. Elements of the model are being tested for the Tetves subwatershed of Lake Balaton. Preliminary results show that there are considerable uncertainties in P loading, thus the use of stochastic model may be quite appropriate;
9. The form of the model, as presented in this paper, may only be regarded as a first step. Estimates of model parameters, especially concentrations of dissolved P and of fixed P, should be improved.

ACKNOWLEDGEMENTS

The research leading to this paper was sponsored by funds provided by the National Water Authority in Budapest and a U.S. National Science Foundation Grant No. GF-38183, "Decision Making Under Uncertainty in Hydrologic and Other Resource Systems", and by IIASA. The administrative support of the Hungarian Institute for Cultural Relations and the University of Arizona, which made this U.S.-Hungarian cooperative research possible, is also gratefully acknowledged.

Also gratefully acknowledged is the help of Elemér Dobolyi, Martin Fogel and Géza Jolánkai whose observation data and useful advice were invaluable for preparing this paper.

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