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# The IIASA Health Care Resource Allocation Sub-Model: Mark 2 - the Allocation of Many Different Resources

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# THE IIASA HEALTH CARE RESOURCE ALLOCATION SUB-MODEL: MARK 2--THE ALLOCATION OF MANY DIFFERENT RESOURCES

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## Preface

The aim of the IIASA Modeling Health Care Systems Task is to build a National Health Care System model and apply it in collaboration with national research centers as an aid to Health Service planners. The modeling work is proceeding along the lines proposed in earlier papers. It involves the construction of linked sub-models dealing with population, disease prevalence, resource need, resource supply, and resource allocation.

In this paper, an earlier version of the resource allocation sub-model is extended to have wider application in the planning of health services, and to make direct use of historical allocation data. Both the model and parameter estimation procedures are available as computer programs, and three illustrative examples are presented.

Recent related publications of the IIASA Modeling Health Care Systems Task are listed on the back pages of this Memorandum.

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Abstract

The function of the resource allocation sub-model within the IIASA Health Care System model is to simulate how the HCS allocates limited supplies of resources between competing demands. The principal outputs of the sub-model are the numbers of patients treated, in different categories, and the modes and quotas of treatment they receive. The Mark 2 version of the sub-model described in this paper simulates the allocation of many resources within one mode of treatment. It uses the same main assumption as used in the Mark 1 version previously reported; namely that in allocating its resources the HCS attempts to optimise a utility function whose parameters can be inferred from data on past allocations. Depending upon the type of data that is available different procedures for parameter estimation are required. This paper analyses estimation procedures which use historical allocation data directly. Both these procedures and the solution algorithm have been realized in a small computer program which can be readily installed on most scientific computer installations. The use of the sub-model is illustrated by three hypothetical applications using hospital data.

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## 1. INTRODUCTION

At the International Institute for Applied Systems Analysis, a group of scientists from different countries is developing a national Health Care System (HCS) model. This model and its sub-models are designed for application with collaborating national research centres as an aid to health service planners. As described in earlier papers by Venedictov and Shigan [1] and by Gibbs [2] the research plan includes the construction of linked sub-models dealing with population, disease prevalence, resource need, resource supply and resource allocation. This paper describes the further development of the resource allocation sub-model DRAM--*disaggregated resource allocation model*. This first section reviews the role of DRAM within the IIASA National HCS model, and motivates the various developments described in the rest of this paper.

The IIASA national HCS model has at present four groups of sub-models, shown in Figure 1 and described more fully in Gibbs [2]. Within this framework the function of the resource allocation sub-model is to represent how the HCS allocates limited supplies of resources between competing demands. Accordingly it takes input data on demand and supply, uses a hypothesis about how allocation choices are made, and gives indicators of the predicted performance of the HCS.

The demand inputs are:

- the total number of individuals who need treatment, by category (from the morbidity and population sub-models),
- the policies for treatment (i.e. the feasible modes of treatment for each patient category--in-patient, out-patient, domiciliary, etc.), and

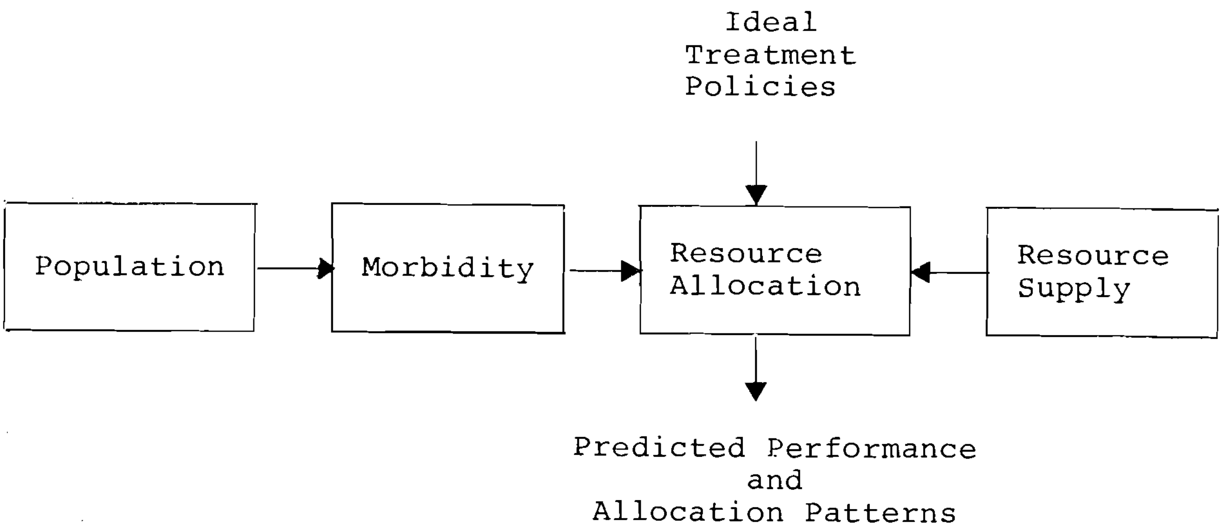


Figure 1. The four groups of sub-models in the IIASA national health care system model.

- the ideal quotas of resources needed in each patient category and mode of treatment.

The supply inputs are the amounts of resources available for use in the HCS, and their costs (from the resource supply production model).

The model's hypothesis about the behaviour of the HCS has two parts. First it assumes that there is never a sufficient supply of resources to saturate all the potential demands for them. This finding has been frequently noted in many areas of health care [7,8,9] Accordingly the sub-model represents the HCS as attempting to achieve an equilibrium between supply and demand. The second assumption is that the HCS allocates its resources so as to maximise a utility function whose parameters can be inferred from observations of past allocations. Such a model is of the behaviour simulation kind [3], and like the models of McDonald, et al. in the UK [4] and Rousseau in Canada [5], it represents the actors in the HCS striving to attain some ideal pattern of behaviour within resource constraints. If these hypotheses are sound, DRAM can not only describe past equilibria, as can classical econometric models, but it can also, unlike classical econometric models, predict how the equilibrium is likely to change in the future as a result of changes in factors such as clinical standards, disease prevalence, and the preferences and priorities operating in the HCS.

The model outputs represent the levels of satisfied demand in a HCS with limited resources. They are:

- the numbers of patients of different categories who receive treatment,
- the modes of treatment offered, and
- the quotas of resources received by each patient in each mode of treatment.

Inevitably these levels fall short of the ideal demand levels. DRAM shows how the short-falls are different for different patients in different parts of the HCS. These results can be used by health care planners to explore the consequences of alternative policies for resource production, treatment, and prevention.

DRAM Mark 1 was described in Gibbs [6]. This first version of the resource allocation sub-model demonstrates how a *single* resource is allocated between many patient categories in a single mode of treatment. The present paper describes DRAM Mark 2, in which the earlier work is developed in two respects. First, DRAM Mark 2 represents how *many* resources are allocated between many patient categories in a single mode of treatment. Thus this version approaches more closely the model of McDonald, et al. [4] in which the HCS can choose not only between resources but also between modes of treatment. Nevertheless DRAM Mark 2 retains the advantage of needing only a small computing facility. No elaborate software is required and the workings of the model can be easily explained.

A second feature of DRAM Mark 2 is the method used to estimate the parameters of the model. Information useful for this task is available from many sources, but in all cases it must reflect the way in which the HCS has solved its allocation problem up until now. Below we develop procedures for parameter estimation which use such historic data directly. The results can be usefully compared and combined with the results of other procedures which use data from special surveys and investigations.

DRAM cannot and does not represent every mechanism of the real process by which health care resources are allocated. Its purpose is rather to model a concept: namely that the HCS achieves an equilibrium by balancing the desirabilities of treating more patients of one type against treating more of other types and against treating each type of patient at a higher average standard. In the examples illustrating the use of DRAM, we examine how the HCS allocates beds and staff in the treatment of in-patients. But the underlying concept appears to be valid for many other HCS sectors (e.g. out-patient treatment) and for many resources within each sector (e.g. out-patient physicians, beds, nurses). It is therefore likely that the model could be applied quite widely.

The next section describes the *model* in mathematical terms. When the model parameters are known, the output variables can be *solved* by a simple iterative algorithm. The problem of *parameter estimation* is considered in Section 3. Section 4 gives the *results* of using DRAM on data from the United Kingdom and Czechoslovakia. We hope to extend such applications to other countries. Section 5 *concludes* and describes possible further developments of DRAM.

## 2. MODEL FORMULATION AND SOLUTION

This section decribes DRAM Mark 2 in mathematical terms, defining the variables used and making precise the underlying hypotheses. This leads to the derivation of an algorithm for finding the model outputs in terms of the model parameters.

### Model Formulation

We begin by defining some variables. DRAM is a model in which many resources are allocated between many patient categories. Define, therefore, the subscripts

$j$ = patient category (e.g. diagnosis), $j = 1,2,\dots J$

$k$ = resource type (e.g. beds, doctors), $k = 1,2,\dots K$

and the model variables

$x_j$ = numbers of individual in the $j^{th}$ patient category who receive treatment (per head of population, per year)

$y_{jk}$ = amounts or quotas of resource type k received by each treated individual in the $j^{th}$ patient category.*

It is these variables that the model seeks to predict, within certain constraints, and according to a certain criterion.

There are three constraints on the choice of x,y. They are

$$\sum_j x_j y_{jk} = R_k \qquad \forall\, k \tag{1}$$

* In the sequel, we use x,y to denote $\{x_j, j=1,2,\dots J\}$, $\{y_{jk}, j = 1, 2,\dots J, k=1,2,\dots K\}$ respectively, with a like notation for similarly subscripted variables.

$$0 < x_j < X_j \qquad \forall\, j \tag{2}$$

$$0 < y_{jk} < Y_{jk} \qquad \forall\, j,k \tag{3}$$

Equation (1) states that the total resources of the $k^{th}$ type allocated by the model are equal to

> $R_k$ = the total resources of the $k^{th}$ type available to the HCS (per head of population, per year).

In other words, all the available resources must be allocated. Equations (2) and (3) state that the demands which are input to the model

> $X_j$ = the total number of individuals in the $j^{th}$ patient category who need treatment (per head of population, per year)

> $Y_{jk}$ = the ideal standards or quotas of resource k for treating an individual in the $j^{th}$ patient category

are never exceeded by the model variables. Equations (1), (2), (3) together imply that

$$R_k < \sum_j X_j Y_{jk} \qquad \forall\, k$$

or that supply is always less than demand--the first hypothesis of the model.

The criterion used to determine x and y is the second hypothesis of the model. Specifically the model chooses x,y so as to maximize a utility function

$$U = \sum_j g_j(x_j) + \sum_j \sum_k x_j h_{jk}(y_{jk}) \tag{4}$$

in which

$$g_j(x_j) = - \frac{X_j \sum_k C_k Y_{jk}}{\alpha_j} \left(\frac{x_j}{X_j}\right)^{-\alpha_j} \tag{5}$$

$$h_{jk}(y_{jk}) = \frac{C_k Y_{jk}}{\beta_{jk}} \left\{1 - \left(\frac{y_{jk}}{Y_{jk}}\right)^{-\beta_{jk}}\right\} \tag{6}$$

and where

$\alpha_j$, $\beta_{jk}$ are strictly positive constants

$C_k$ = the marginal unit cost of resource type k when all demands are satisfied.

subject to the constraints of equations (1),(2),(3). This completes the specification of the model.

The utility function of equations (4),(5),(6) is very similar to that used in [6], and it can be derived solely from the following assumptions.

a) The utilities of treating more patients and of treating each patient with more resources, are independent, monotonically increasing, and additive across patients, patient categories, and resource types.

b) When all demands are met (x = X, y = Y), the marginal utilities of increasing the numbers treated or their resource quotas equal the corresponding marginal resource costs. In this situation, extra resources are useful only as assets and not for treating patients.

c) Percentage increases in x and y give rise to proportional percentage decreases in marginal utility at all levels of x and y. The function g and h are therefore concave, implying diminishing utility increases for large x and y. An alternative way of expressing this assumption is to suppose that marginal utility is an independent variable and to write

$$\frac{d \ln x_j}{d \ln g_j'(x_j)} = E_j < 0 \qquad \forall\ j$$

$$\frac{d \ln y_{jk}}{d \ln h_{jk}'(y_{jk})} = F_{jk} < 0 \qquad \forall\ j,k$$

This shows that the elasticities of numbers treated and resource quotas with respect to marginal utility are assumed to be constant and negative.

It is important to understand that the utility function U does *not* represent a quantity which anyone in the HCS is consciously, or even subconsciously trying to maximize. Instead it represents a hypothesis about the aggregated behaviour of the HCS, in which the parameters $\alpha, \beta$ represent the priorities implicit in the choices which are made. The utility function may appear to include both inputs (numbers of individuals) and outputs (resource quotas) of the HCS. In fact, both these variables are regarded here as outputs, with the inputs to the system being the ideal values of these variables.

## Model Solution

The remaining task for this chapter is to find expressions for the model variables x and y in terms of the model parameters $\alpha$, $\beta$, X, Y, C and R. The constrained maximisation problem in DRAM Mark 2 is similar to that which arose in DRAM Mark 1, and it can be similarly solved using the technique of Lagrange multipliers. The solution given below follows very closely that used in [6] including the use of a simple numerical technique to find the values of the multipliers.

In the normal way we adjoin the K constraint equations (1) to the utility function which is to be maximized (4) by means of K arbitrary multipliers $\lambda_k$. It is convenient for subsequent analysis to scale these multipliers by the cost of each resource type $C_k$

$$H = \sum_j g_j(x_j) + \sum_j \sum_k x_j h_{jk}(y_{jk}) + \sum_k C_k \lambda_k (R_k - \sum_j x_j y_{jk}) \quad . \quad (7)$$

In order to find the values of x and y which maximize H, we must solve the J(K + 1) + K equations

$$\frac{\partial H}{\partial x_j} = 0 \qquad \forall\ j \qquad (8)$$

$$\frac{\partial H}{\partial y_{jk}} = 0 \qquad \forall\ j,k \qquad (9)$$

$$\frac{\partial H}{\partial \lambda_k} = 0 \qquad \forall\ k \qquad (10)$$

for the J(K + 1) + K unknowns: x, y, and $\lambda$. Equation (9) gives

$$\frac{\partial H}{\partial y_{jk}} = x_j h'_{jk}(y_{jk}) - C_k \lambda_k x_j = 0$$

$$y_{jk} = h'^{-1}_{jk}(C_k \lambda_k)$$

and using the expression for $h_{jk}(u_{jk})$ given in (6), we obtain

$$y_{jk} = Y_{jk}(\lambda_k)^{\frac{-1}{\beta_{jk}+1}} \quad . \qquad (11)$$

Similarly, equation (8) gives

$$\frac{\partial H}{\partial x_j} = g'_j(x_j) + \sum_k h_{jk}(y_{jk}) - \sum_k C_k \lambda_k y_{jk} = 0$$

$$x_j = g'^{-1}_j\left(\sum_k (C_k \lambda_k y_{jk} - h_{jk}(y_{jk}))\right)$$

where $g_j'^{-1}$ is the inverse of the partial derivative with respect to $x_j$ of the function $g_j(x_j)$. Using the expression for $g_j(x_j)$ given in (5), and the solution for $y_{jk}$, we obtain

$$x_j = X_j(\mu_j)^{\frac{-1}{\alpha_j+1}} \tag{12}$$

where $\mu_j$ is a weighted sum

$$\mu_j = \sum_k C_k Y_{jk} \nu_{jk} / \sum_k C_k Y_{jk} \tag{13}$$

of the terms

$$\nu_{jk} = \left( (\beta_{jk} + 1) \lambda_k^{\frac{\beta_{jk}}{\beta_{jk}+1}} - 1 \right) / \beta_{jk} \quad . \tag{14}$$

It remains to solve equation (10) for the Lagrange multipliers $\lambda$. Substituting the results of equations (11), (12) we obtain

$$f_k(\lambda_1, \lambda_2, \ldots \lambda_K) = f_k(\lambda) = 0 \qquad \forall_k \tag{15}$$

where

$$f_k(\lambda) = -R_k + \sum_j X_j Y_{jk} (\lambda_k)^{\frac{-1}{\beta jk+1}} (\mu_j)^{\frac{-1}{\alpha_j+1}} \tag{16}$$

which must be solved by a numerical technique such as the multi-dimensional extension of the Newton-Raphson method. In this method, an approximate solution $\hat{\lambda}$ yields an improved solution $\lambda$ according to

$$\lambda = \hat{\lambda} - \left\{ \frac{\partial f_k(\lambda)}{\partial \lambda_\ell} \right\}^{-1} \left\{ f_\ell(\lambda) \right\} \tag{17}$$

where $\{a_\ell\}, \{a_{k\ell}\}$ denote the vector, matrix with typical element

$a_\ell, a_{k\ell}$. Equation (17) can be used to derive successively improved solutions until some convergence criterion is satisfied.

To show that equation (15) can be solved by the Newton-Raphson method, we note first that we are seeking solutions within the range,

$$\lambda_k > 1 \qquad \forall\ k$$

because only such solutions for $\lambda$ will give solutions for x and y satisfying

$$0 < x_j < X_j \quad , \qquad 0 < y_{jk} < Y_{jk} \qquad \forall\ j,k \quad .$$

Within this range of possible $\lambda_k$, the function $f_k(\lambda)$ is analytic and so also is its first derivative

$$\frac{\partial f_k(\lambda)}{\partial \lambda_\ell} = - \sum_j X_j Y_{jk} (\lambda_k)^{\frac{-1}{\beta_{jk}+1} - 1} (\mu_j)^{\frac{-1}{\alpha_j+1} - 1} \left\{ \frac{\mu_j}{\beta_{jk} + 1} \frac{\partial \lambda_k}{\partial \lambda_\ell} + \frac{\lambda_k}{\alpha_j + 1} \frac{\partial \mu_j}{\partial \lambda_\ell} \right\} \qquad (18)$$

where

$$\frac{\partial \lambda_k}{\partial \lambda_k} = 1 \quad , \qquad \frac{\partial \lambda_k}{\partial \lambda_\ell} = 0 \qquad \text{for } k \neq \ell$$

and

$$\frac{\partial \mu_j}{\partial \lambda_\ell} = \frac{C_\ell Y_{j\ell}}{\sum_\ell C_\ell Y_{j\ell}} (\lambda_\ell)^{\frac{-1}{\beta_{j\ell}+1}} \quad . \qquad (19)$$

Next we note that

$$\lambda_k = 1 \quad \forall\, k \implies f_k(\lambda) = -R_k + \sum_j X_j Y_{jk}$$

which is always positive for $R_k < \sum_j X_j Y_{jk}$, and that

$$\lambda_k \to \infty \quad \forall\, k \implies f_k(\lambda) \to -R_k$$

which is always negative. Finally we find that $\frac{\partial f_k(\lambda)}{\partial \lambda_\ell}$ is always negative between these points. From these facts it follows that equation (15) has only one real solution for $\lambda$ in the range $\lambda_k > 1$, $\forall$ k, and that this solution can be found by the multi-dimensional Newton-Raphson method.

This completes the solution of the model. When the $\lambda_k$ have been found by numerical solution of (15), equations (11) and (12) can be used to calculate x and y. A small computer program has been written to perform this calculation, and the Newton-Raphson procedure is found to converge rapidly. However, before this program can be used, values are needed for the model parameters $\alpha$, $\beta$, X, Y, C and R. In the next section, we consider how to estimate these parameters.

## 3. ESTIMATION OF PARAMETERS

When all the model parameters $\alpha$, $\beta$, X, Y, C and R are known, the equations given in Section 2 can be used to solve for the model variables x and y. First, however, values for these parameters must be found.

The present treatment assumes that the costs $C_k$ and availabilities $R_k$ of different resources are given exogenously. If the model is being used to simulate historic situations, values for these variables will be found in routine statistics. For runs designed to simulate future situations, values may be given by price or production models external to DRAM, or if such models are not available, values may be chosen without difficulty by the decision-maker. In the latter case, DRAM can be used to predict how resources will be used if they are available at prescribed levels and prices. The costs C must be estimated by the average or marginal costs at some arbitrary level of production. In our illustrative examples we have assumed that this is satisfactory on a national or regional planning level. Fortunately the model uses only the relative costs of different resources, and the price base of C is immaterial.

On the other hand, it is not easy for the decision-maker to choose values for the elasticities $\alpha,\beta$. Nor is this desirable since the decision-maker will be tempted to choose values which he would like to see realised. But in DRAM, the elasticities indicate, not the decision-maker's preferences, but the actual behaviour of the HCS in allocating scarce resources. We assume here, therefore, that $\alpha,\beta$ change little over some period of time or in some region, and that they can be estimated from historic data about the model variables x and y.

The same assumptions are made about the demand levels X,Y. This is in spite of the fact that the potential numbers of patients X might well be given by a morbidity model such as those of Klementiev [10] and Kaihara, et al. [11] and ideal quotas Y could be defined by professional consensus. There are three reasons for this. First, if morbidity models or professionals are not at hand, it is not immediately obvious how to choose X,Y. Secondly, it is not difficult to by-pass the estimation of X,Y if exogenous values are actually available. Thirdly, the quantities X, Y and $\alpha,\beta$ are rather closely related in DRAM and it is important that they be consistent. If exogenous estimates of X,Y are to be used which are very different from the values estimated from historic data, it may suggest that the values of $\alpha,\beta$ estimated from historic data are inappropriate, and that some different estimates should be used.

The most easily obtained data with which to calibrate the model are the model outputs: the actual numbers of patients treated x, and the quotas of resources which they receive y. Sometimes, however, other useful data is available. Feldstein used 1968 data from the 14 regional hospital areas of England to estimate how admission rates, length of stay in hospital, etc. vary with changing resource supply [9]. These *empirical elasticities* are closely related to the model elasticities $\alpha, \beta$, and were successfully used to calibrate DRAM Mark 1 [6]. Similar methods are suggested below. Often, however, empirical elasticities are not available without carrying out a major study. For this reason, we show how to calibrate DRAM using only some observed data points x,y. When in addition empirical elasticity data are available, they may be used either in calibration or for comparison with the values implied by calibration on other data.

Our task is to estimate the parameters $\alpha$, $\beta$, X, Y in order to deduce what future values of x,y will follow from alternative choices of C and R. It is convenient to solve the estimation problem in three stages; first assuming that one or the other of the pairs $\alpha, \beta$ and X,Y is known and need not be estimated, and then combining the results for the case when both pairs are unknown:

<u>Stage 1</u>: $\alpha, \beta$ are known. X,Y are to be estimated. Rearranging equations (11), (12) gives

$$X_j = x_j (\mu_j)^{\frac{1}{\alpha_j + 1}} \tag{20}$$

$$Y_{jk} = y_{jk} (\lambda_k)^{\frac{1}{\beta_{jk} + 1}} \tag{21}$$

If a single set of values for x and y are known, for example the present distribution of resources in a particular region, these equations can be used to find X and Y in terms of $\lambda$. Unfortunately, however, a single data point x,y does not give sufficient information to solve for $\lambda$. Figure 2 illustrates the problem for a single disease category and resource. The curved lines define the possible solutions for x and y, for two pairs X(i),Y(i), i = 1,2, when $\alpha$ and $\beta$ are known. By suitable choice of X(i) and Y(i), both lines may pass through the known data point. Without knowing whether

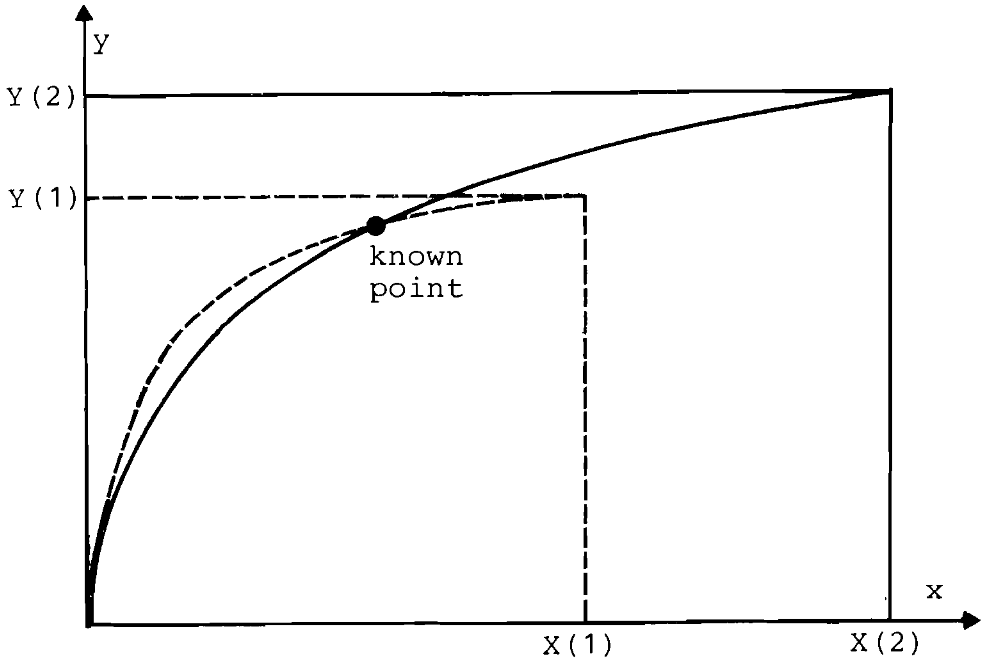


Figure 2. When $\alpha,\beta$ are known, a single data point does not uniquely identify X and Y.

the data point is near to ($\lambda$ small) or far from ($\lambda$ large) the maximum values X,Y, there is no unique solution for X,Y.

In order to constrain these K degrees of freedom in the estimation problem, we assume that we can define the resources needed to satisfy the ideal levels $X_j, Y_{jk}$ as some multiple $\Theta_k$ of the resources used at the data point

$$\sum_j X_j Y_{jk} = \Theta_k \sum_j x_j y_{jk} \qquad \forall\ k \tag{22}$$

Substituting equations (11),(12) in (22) gives

$$\tilde{f}_k(\lambda) = 0 \qquad \forall\ k \tag{23}$$

where

$$\tilde{f}_k(\lambda) = -\Theta_k \sum_j x_j y_{jk} + \sum_j x_j y_{jk} (\lambda_k)^{\frac{1}{\beta_{jk}+1}} (\mu_j)^{\frac{1}{\alpha_j+1}}$$

and where (23) must be solved for $\lambda$. The equations in $\tilde{f}$ are very similar to equations (15) in f, and provided that $\Theta_k > 1 \; \forall k$, and that all the terms except $\lambda$ are known, they may be solved in the same way to give $\lambda$. Unfortunately not all the terms are known. In particular, $\mu_j$ is a weighted average involving the terms $Y_{jk}$, which as yet are unknown. An appropriate iterative solution scheme which overcomes this problem is outlined in Stage 3.

Stage 2: X,Y are known. $\alpha,\beta$ are to be estimated. Rearranging equations (11), (12) gives

$$\alpha_j = \ln(\mu_j)/\ln\left(\frac{X_j}{x_j}\right) - 1 \tag{24}$$

$$\beta_{jk} = \ln(\lambda_k)/\ln\left(\frac{Y_{jk}}{y_{jk}}\right) - 1 \quad . \tag{25}$$

If a single set of values for x and y are known, these equations can be used to find $\alpha$ and $\beta$ in terms of $\lambda$. Again, however, $\lambda$ remains undetermined. Figure 3 illustrates that the difficulty is in knowing the shape of possible solution lines OA in the xy space. We do know, however, that $\alpha$ and $\beta$ are always positive, and equations (24), (25) then imply that

$$\lambda_k > \tilde{\lambda}_k = \max_j \left\{ \left(\frac{X_j}{x_j}\right), \left(\frac{Y_{jk}}{y_{jk}}\right) \right\} \tag{26}$$

A priori, large elasticities are unlikely, and $\lambda_k$ might be defined as some (small) multiple $\phi_k > 1$ of the minimum value $\tilde{\lambda}_k$

$$\lambda_k = \phi_k \tilde{\lambda}_k \qquad \forall \; k \quad . \tag{27}$$

Another way of estimating $\alpha,\beta$ is to use *empirical* elasticity data such as

$\gamma_{jk}$ = the elasticity of the admission rate $x_j$ to changes in the resource level $R_k$;

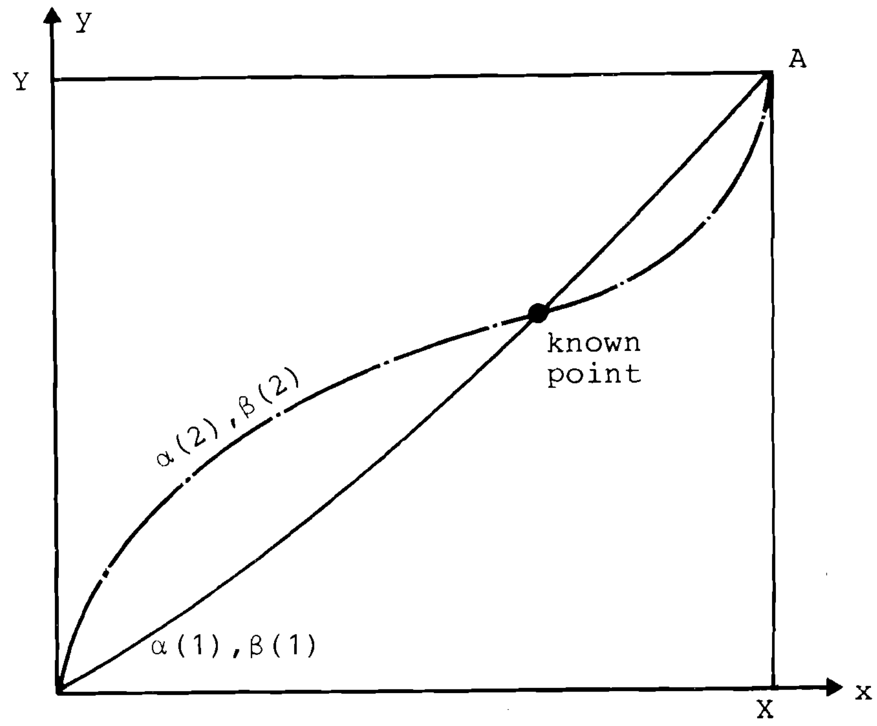


Figure 3. When X,Y are known, a single point does not uniquely identify α and β.

$\eta_{j\ell k}$ = the elasticity of the standard $y_{j\ell}$ to changes in the resource level $R_k$.

These empirical elasticities, which sometimes come from other studies, may be expressed in terms of the model elasticities α,β. For example, $\gamma_{jk}$ is

$$\gamma_{jk} = \frac{\partial \ln x_j}{\partial \ln R_k} = \sum_i \frac{\partial \ln x_j}{\partial \mu_i} \cdot \frac{\partial \mu_i}{\partial R_k} \cdot R_k$$

and using equation (11) to give an expression for $\partial \log x_j/\partial \mu_i$ gives

$$\gamma_{jk} = \frac{-R_k}{(\alpha_j + 1)\mu_j} \cdot \frac{\partial \mu_j}{\partial R_k} \quad . \tag{28}$$

Similarly

$$\eta_{j\ell k} = \frac{\partial \ln y_{j\ell}}{\partial \ln R_k} = \sum_i \frac{\partial \ln y_{j\ell}}{\partial \lambda_i} \cdot \frac{\partial \lambda_i}{\partial R_k} \cdot R_k$$

gives

$$\eta_{j\ell k} = \frac{-R_k}{(\beta_{j\ell} + 1)\lambda_\ell} \cdot \frac{\partial \lambda_\ell}{\partial R_k} \quad . \tag{29}$$

In these expressions, the partial derivatives $\partial\mu_j/\partial R_k$ may be expressed in terms of the partial derivatives $\partial\lambda_\ell/\partial R_k$ by writing

$$\frac{\partial \mu_j}{\partial R_k} = \sum_\ell \frac{\partial \mu_j}{\partial \lambda_\ell} \cdot \frac{\partial \lambda_\ell}{\partial R_k} \tag{30}$$

in which equation (19) may be used to substitute for $\partial\mu_j/\partial\lambda_\ell$. It remains to find $\partial\lambda_\ell/\partial R_k$. Writing equations (16) in the form

$$f_k(\lambda) = -R_k + \hat{f}_k(\lambda) = 0$$

we may differentiate to obtain

$$\frac{\partial R_k}{\partial \lambda_\ell} = \frac{\partial \hat{f}_k(\lambda)}{\partial \lambda_\ell}$$

at the value of $\lambda$ for which $f_k(\lambda)$ in zero. But regarding $f_k(\lambda)$ simply as a function of $\lambda$ we have

$$\frac{\partial f_k(\lambda)}{\partial \lambda_\ell} = \frac{\partial \hat{f}_k(\lambda)}{\partial \lambda_\ell} \quad .$$

It follows therefore that

$$\frac{\partial \lambda_\ell}{\partial R_k} = \left\{\frac{\partial R_k}{\partial \lambda_\ell}\right\}^{-1} = \left\{\frac{\partial f_k(\lambda)}{\partial \lambda_\ell}\right\}^{-1} .$$

These are the same derivatives that arise in the solutions of equation (15) by the Newton-Raphson technique, and they are easily calculated.

Although it is easy to express $\gamma,\eta$ in terms of $\alpha,\beta$, it is impossible to express $\alpha,\beta$ in terms of $\gamma,\eta$. This is because the various partial derivatives in these formulae depend upon $\alpha,\beta$ in such a way that they cannot be inverted. This problem arose in DRAM Mark 1 and was successfully overcome by writing equations for $\alpha_j,\beta_{jk}$ in the form

$$\beta_{jk} = \frac{-R_k}{\eta_{j\ell k}\lambda_\ell}\left(\frac{\partial f_\ell}{\partial R_k}\right)^{-1} - 1 \tag{31}$$

$$\alpha_j = \frac{-R_k}{\gamma_{jk}\mu_j}\sum_\ell\left[\frac{C_\ell Y_{j\ell}}{\sum_\ell (C_j Y_{j\ell})}(\lambda_\ell)^{\frac{-1}{\beta_{j\ell}+1}}\right]\left(\frac{\partial f_\ell}{\ell R_k}\right)^{-1} - 1 \tag{32}$$

and by using an iterative method of solution. In the present case, values of $\left(\frac{\partial f_\ell}{\partial R_k}\right)^{-1}$ may be derived from initial estimates of $\alpha$ and $\beta$. Equations (31),(32) may then be used to improve these estimates. Note, however that the estimates of $\alpha,\beta$ derived from $\gamma,\eta$ are like those derived from a single data point in that they still depend upon an unknown $\lambda$. We cannot dispense with a condition such as equation (22).

There are three technical problems associated with the use of the empirical elasticities $\gamma,\eta$ to estimate the model elasticities $\alpha,\beta$. The first problem is that equation (31) gives not just one value for $\beta_{jk}$, but K values which correspond to the K elasticities $\eta_{ij1}, \eta_{ij2}, \ldots \eta_{ijK}$. A similar problem arises in the estimation of $\alpha_j$. However, it is likely that reliable *cross-elasticity* data $\eta_{j\ell k}$, $\ell \neq k$, will be unavailable, and that $\gamma_{jk}$ will be better known for some resource type $k = \ell$ than for the others. Then $\alpha_j, \beta_{jk}$ may be estimated from $\eta_{jkk}$ and $\gamma_{j\ell}$ alone.

The second problem is that the empirical elasticities must be consistent with the ideal levels X,Y. To see why this must be so, differentiate equation (1)

$$\sum_j x_j y_{jk} = R_k \tag{1}$$

with respect to $R_k$ to give

$$\sum_j \left( x_j \frac{\partial y_{jk}}{\partial R_k} + y_{jk} \frac{\partial x_j}{\partial R_k} \right) = 1 \qquad \forall\ k$$

and use the definitions of $\gamma_{jk}$ and $\eta_{j\ell k}$ to give

$$\gamma_{jk} = \frac{R_k}{x_j} \cdot \frac{\partial x_j}{\partial R_k} , \qquad \eta_{jk\ell} = \frac{R_k}{y_{j\ell}} \cdot \frac{\partial y_{j\ell}}{\partial R_k} .$$

Combining these results gives

$$\sum_j x_j y_{j\ell} (\gamma_{jk} + \eta_{j\ell k}) = R_k \qquad \forall\ k,\ell$$

and substituting the equations for $x_j, y_{j\ell}$ (11),(12) gives

$$\sum_j X_j Y_{j\ell} (\mu_j)^{\frac{-1}{\alpha_j+1}} (\lambda_\ell)^{\frac{-1}{\beta_{j\ell}+1}} (\gamma_{jk} + \eta_{j\ell k}) = R_k \qquad \forall\ k,\ell . \tag{33}$$

If X,Y are given exogenously, this equation will generally not be satisfied during the iterative estimation of $\alpha,\beta$, and the procedure may not converge. A natural solution is to scale the elasticities at each iteration so that equation (33) is satisfied.

The third problem is that of finding suitable initial values of $\alpha,\beta$ with which to start the iterative estimation. Again following the previous approach [6] we expect $(\alpha_j + 1)$ to be of the same order of magnitude as $(\gamma_{jk})^{-1}$, and $(\beta_{jk} + 1)$ to be of the same order of magnitude as $(\eta_{j\ell k})^{-1}$. This suggests that suitable initial values will be

$$\alpha_j = \left\{\max_k \gamma_{jk}\right\}^{-1} - 1 \quad , \qquad \beta_{jk} = \left\{\max_\ell \eta_{j\ell k}\right\}^{-1} - 1 \quad .$$

<u>Stage 3</u>: $\alpha$, $\beta$, X, Y are all to be estimated. It is now clear that to estimate both pairs of model parameters $\alpha,\beta$ and X,Y, either two different data points, or one data point plus the empirical elasticities are needed. In either case the estimation problem has two degrees of freedom for each of the K resource types, which represent uncertainty about the scale and shape of solutions in the x,y space. Because empirical elasticity data is not always available, we henceforth consider only how to estimate the model parameters given two different data points. The appropriate procedure when empirical elasticity data is available is similar to that described previously [6]. We assume for simplicity that the resource costs C are the same at both data points, although this condition can easily be relaxed.

Two data points x(1),y(1) and x(2),y(2) are related by four equations, for each of all possible values of j and k

$$x_j(1) = X_j(\mu_j(1))^{\frac{-1}{\alpha_j+1}} \tag{34}$$

$$y_{jk}(1) = Y_{jk}(\lambda_k(1))^{\frac{-1}{\beta_{jk}+1}} \tag{35}$$

$$x_j(2) = X_j(\mu_j(2))^{\frac{-1}{\alpha_j+1}} \tag{36}$$

$$Y_{jk}(2) = y_{jk}(\lambda_k(2))^{\frac{-1}{\beta_{jk}+1}} \tag{37}$$

It is natural to choose the Lagrange multipliers at each point, $\lambda(1)$ and $\lambda(2)$, as the 2K degrees of freedom which we must constrain. Once these multipliers are known, equations (34)-(37) may be readily solved for the known parameters. Appropriate additional constraints may be applied as in earlier stages. Suppose that for some k, $R_k(1) < R_k(2)$. Then equation (15) ensures that $\lambda_k(1) > \lambda_k(2)$, and equations (24),(25) will give positive values for $\beta_{jk}$ only if

$$\frac{\lambda_k(1)}{\lambda_k(2)} = \phi_k \max_j \left(\frac{Y_{jk}(2)}{Y_{jk}(1)}\right) \tag{38}$$

where $\phi_k > 1$ is some small multiplier. With this result, $\beta_{jk}$ can be found directly from equations (35),(37). If also it is possible to define the resources needed to satisfy the ideal levels $X_j, Y_{jk}$ as some multiple $\Theta_k$ of the resources used at one of the data points

$$\sum_j X_j Y_{jk} = \Theta_k \sum_j x_j(1) y_{jk}(1) \qquad \forall\ k$$

then equations (23) define values for $\lambda_k(1)$, but only, as noted earlier, if all the parameters are known. This suggests the following iterative scheme for estimating $\alpha$, $\beta$, X, Y.

a) Use equation (38) to define the ratios $\lambda_k(1)/\lambda_k(2)\ \forall\ k$. Divide equation (35) by (37) and solve for $\beta$.

b) With some arbitrary value for $\lambda(1)$, use equation (35) to find Y. Equation (13) can then be used to find $\mu(1)$ and $\mu(2)$, and equations (34) and (36) then give $\alpha$ and X.

c) Use these parameters to solve equation (23) for improved values of $\lambda(1)$ and repeat from b) until convergence.

This completes the anlysis of parameter estimation in DRAM. An important feature of the analysis is that the estimates of $\alpha,\beta,X,Y$ depend strongly upon the additional constraint variables $\Theta,\phi$, both of which are somewhat arbitrary. Fortunately, this is not a problem. Although different values of $\Theta,\phi$ lead to different values for $\alpha,\beta,X,Y$, each set of parameter values will reproduce the data points used for estimation. Provided that predictive runs of the model do not involve resource levels very different from those used in estimation, the results are relatively insensitive to $\Theta,\phi$. Our illustrative examples show that the precision of model predictions is much better than the likely accuracy of the data used for parameter estimation.

A second small computer program has been written to implement the iterative estimation procedure proposed above, and when it converges, it generally does so rapidly. However, convergence cannot be guaranteed, because the structure of the model necessarily limits the set of possible data points. When the estimation procedure does not converge, it implies that the data are inconsistent with the model and that either the data or the model hypothesis is suspect. The next section gives the results of using real data in the estimation procedures described above.

## 4. ILLUSTRATIVE EXAMPLES

To illustrate how the model can be used, we shall present three hypothetical examples of HCS resource allocation problems.

### Example 1

The first example is designed to compare the parameter estimation procedures derived in Section 3, with those developed previously for DRAM Mark 1. Consider the allocation of acute hospital bed-days in the South Western Region of England between patients suffering from six diseases: varicose veins, haemorrhoids, ischaemic heart disease (excluding acute myocardial infarction), pneumonia, bronchitis and appendicitis. In this problem there is a single resource (beds), and six patient categories corresponding to the six diseases. Table 1 gives the numbers of patients admitted to hospital in 1968 with these diseases, and their average lengths of stay [12]. Gibbs used these data, together with the empirical elasticities of Feldstein [9] and exogenous estimates of the ideal levels X and U, to calibrate a predictive resource allocation model for the South Western Region [2].

Here we repeat this exercise. However, we estimate the model parameters, not using Feldstein's results, but with the other data given in Table 1: the actual admissions and lengths of stay in 1973 [13]. The assumption underlying this alternative approach to parameter estimation is that the model parameters, and especially the numbers per head of population who need treatment X, do not change with time. The admission figures in Table 1 have therefore been corrected for population age-structure changes between 1968 and 1973 which could invalidate this assumption.

Table 1 gives a set of model parameters estimated from this data. Table 2 tabulates the corresponding model outputs for the resource levels in Table 1 and for a resource level of just 800 bed-days. We find that it is impossible to calibrate a model which exactly reproduces the 1973 data. We have had to assume therefore that the increasing average length of stay for varicose veins is caused by a data anomaly. (The *median* length of

Table 1. Example 1--Input data and model parameters.

| Disease | Actual allocations of bed-days | | | | Estimated Parameters (2) | | | |
|---|---|---|---|---|---|---|---|---|
| | 1968: R = 1094.2 bed-days | | 1973: R = 613.9 bed-days | | | | | |
| | Admissions per million population | Average stay (days) | Admissions per million population | Average stay (days) | $X_j$ | $Y_j$ | $\alpha_j$ | $\beta_j$ |
| Varicose Veins | 6.3 | 11.3 | (1) 6.1 | 14.4 | 6.5 | 12.2 | 44.4 | 19.8 |
| Haemorrhoids | 4.1 | 13.1 | 4.2 | 7.7 | 4.2 | 22.4 | 43.0 | 1.9 |
| Ischaemic Heart | 4.6 | 40.2 | 5.1 | 17.4 | 4.7 | 93.3 | 38.3 | 0.8 |
| Pneumonia | 12.3 | 14.7 | 11.0 | 14.4 | 13.8 | 15.0 | 12.5 | 73.2 |
| Bronchitis | 11.8 | 27.4 | 9.7 | 16.8 | 14.8 | 44.8 | 4.7 | 2.1 |
| Appendicitis | 24.8 | 11.3 | 15.3 | 7.8 | 42.7 | 16.4 | 1.5 | 3.1 |

(1) corrected for population age structure changes between 1968 and 1973.
(2) estimated with $\Theta = \phi = 2.0$.

Table 2. Example 1--Model results.

| Disease | R = 1094.2 bed-days | | R = 613.9 bed-days | | R = 800 bed-days | |
|---|---|---|---|---|---|---|
| | Admissions per million population | Average stay (days) | Admissions per million population | Average stay (days) | Admissions per million population | Average stay (days) |
| Varicose Veins | 6.3 | 11.3 | 6.1 | 10.5 | 6.2 | 10.9 |
| Haemorrhoids | 4.1 | 13.1 | 4.0 | 7.7 | 4.0 | 9.9 |
| Ischaemic Heart | 4.6 | 40.2 | 4.5 | 17.4 | 4.5 | 26.0 |
| Pneumonia | 12.3 | 14.7 | 11.0 | 14.4 | 11.6 | 14.5 |
| Bronchitis | 11.8 | 27.4 | 9.7 | 16.8 | 10.6 | 21.3 |
| Appendicitis | 24.8 | 11.3 | 15.3 | 7.8 | 19.2 | 9.3 |

stay *decreases*.) We have also assumed that the increasing numbers of patients with heart disease reflects a true increase in morbidity which we have excluded from the model.

The allocation when just 800 bed-days are available may be usefully compared with similar predictions in [2: Table 6]. The average difference is about 17%, which is reasonable in an illustrative run. In a real application, one could use both methods of parameter estimation together with other years' data in order to calibrate a more precise model. In particular one would want to investigate the differences between the two sets of elasticities to see which are likely to be most appropriate: those estimated from historical cross-sectional surveys or those estimated from the recent dynamic behaviour of the HCS.

Example 2

The second example is designed to illustrate as simply as possible the concept modelled by Mark 2 of DRAM. Table 3 shows the numbers of patients admitted to hospitals in Czechoslovakia in 1975 in three specialties: interní (general medicine), chirurgický (general surgery), and ženský (obstetrics and gynaecology). Also shown is their average length of stay and the average number of doctor-days (all grades) per patient. The two sets of figures are for two neighbouring areas of Czechoslovakia.

We immediately observe that area A has high average lengths of stay and low doctor ratios, while area B has the opposite. It is interesting to consider for example how the HCS in area A would make use of doctors if they were available at the levels in area B. Making the assumption that elasticities and demands are the same in the neighbouring areas, we estimate the model parameters given in Table 4, which give the typical results of Table 5. For simplicity we assume that the costs of the two resources are the same.

Again, it is not possible to reproduce exactly the input data of Table 3, but the agreement is very close. The elasticities of lengths of stay to changing bed numbers are all higher

Table 3. Example 2 -- Input data.

| Speciality | Area A | R = 1677.3 bed-days 271.3 doctor-days | | Area B | R = 1233.5 bed-days 279.4 doctor-days | |
|---|---|---|---|---|---|---|
| | Admissions per million population | Average stay (days) | Average doctor rates | Admissions per million population | Average stay (days) | Average doctor rates |
| Interní | 35.8 | 16.04 | 3.00 | 21.6 | 16.02 | 4.97 |
| Chirurgický | 34.8 | 13.05 | 1.70 | 24.3 | 12.01 | 2.31 |
| Ženský (1) | 82.9 | 7.81 | 1.26 | 81.0 | 7.35 | 1.43 |

(1) Population divisors exclude males.

Table 4. Example 2 -- Model parameters.

| Speciality | $x_j$ | $\alpha_j$ | Beds | | Doctors | |
|---|---|---|---|---|---|---|
| | | | $y_{j1}$ | $\beta_{j1}$ | $y_{j2}$ | $\beta_{j2}$ |
| Interní | 93.7 | 0.03 | 16.1 | 90.8 | 5.5 | 1.1 |
| Chirurgický | 71.3 | 0.39 | 15.2 | 6.4 | 2.5 | 2.4 |
| Ženskí | 87.0 | 19.89 | 8.8 | 9.1 | 1.5 | 7.2 |

Table 5. Example 2 -- Model results.

| Specialty | R = 1677.3 bed-days 271.3 doctor days | | | R = 1233.5 bed-days 279.4 doctor days | | | R = 1677.3 bed-days 279.4 doctor-days | | |
|---|---|---|---|---|---|---|---|---|---|
| | Admissions | Stay | Doctor | Admissions | Stay | Doctor | Admission | Stay | Doctor |
| Interní | 35.1 | 15.9 | 3.0 | 21.9 | 15.8 | 4.8 | 35.2 | 15.9 | 3.1 |
| Chirurgický | 35.0 | 13.3 | 1.7 | 24.2 | 12.1 | 2.3 | 35.0 | 13.3 | 1.8 |
| Ženský | 82.8 | 7.9 | 1.3 | 80.8 | 7.4 | 1.5 | 82.8 | 7.9 | 1.3 |

than the corresponding staff parameters, and the model results are much more sensitive to the supply of beds than to doctors. Therefore, when we simulate an increase of doctors in area A, we observe relatively small changes. After a more careful estimation of the model parameters, a health planner might be able to use such a model to compare alternative policies for expanding care in area A.

Example 3

The last example also considers the allocation of beds and doctors, but using data from the South Western Region of England. Table 6 presents historic allocation data from 1968 and 1973 [14, 15] for the seven largest acute hospital specialties: general surgery, general medicine, obstetrics and gynaecology, trauma and orthopaedic surgery, ENT, paediatrics, and ophthalmology. For this example, we have tried to estimate more accurately the relative costs of beds and doctors. First, we assume that at a national or regional planning level, marginal costs will be well approximated by average costs. In other words, we assume that the aggregate of the production functions of the many different production units in the HCS will be approximately linear. Most of the average cost of a doctor is incurred by salaries and wages, which were approximately £5900 per doctor per year (all grades) in 1973/74 [15]. We associate all of the remaining current expenditure on acute care with acute beds at a rate of about £3780 per available bed per year. It is this apportionment of costs which actually defines the two resources for the model. For example, the figures given above define a "bed" as including *all* associated costs *except* doctoring, and any model results should be interpreted in this light.

Unfortunately, however, the data given in Table 6 are insufficient to derive a useful model. Although parameters can be estimated that will reproduce the input data, the $\alpha_i$ for some specialties must be negative. The implication is either that two years' data are unrepresentative, or that morbidity, ideal levels of care, or elasticities are changing with time. The structure

Table 6. Example 3--Historic resource allocations.

| Specialty | R = 940.7 bed-days, (1968) 104.1 doctor-days | | | R = 782.2 bed-days (1973) 125.9 doctor-days (3) | | |
|---|---|---|---|---|---|---|
| | Admission | Stay | Doctors | Admissions | Stay | Doctors |
| General Surgery | 19.6 | 9.5 | 1.14 | 17.3 | 8.3 | 1.27 |
| General Medicine | 12.3 | 14.2 | 1.55 | 12.4 | 11.4 | 1.79 |
| Obstet./Gynae. (1) | 33.1 | 7.5 | 0.59 | 35.0 | 6.2 | 0.67 |
| T&O Surgery | 7.1 | 17.9 | 1.28 | 7.4 | 15.0 | 1.48 |
| ENT | 5.8 | 5.2 | 0.74 | 4.1 | 4.3 | 1.22 |
| Paediatrics (2) | 15.4 | 9.7 | 1.67 | 19.0 | 7.1 | 1.92 |
| Ophthalmology | 2.4 | 10.1 | 1.68 | 1.8 | 8.6 | 3.18 |

(1) Population divisors exclude males.

(2) Population divisors exclude adults.

(3) Relative costs of doctors: beds assumed to be 1.57:1 (see text) 1973.

of the model sufficiently general that this could be tested by using other sub-regional data or other categorizations; for example, diagnostic categories or age categories. Alternatively, perhaps the in-patient treatment modelled by DRAM Mark 2 is affected by changes in out-patient treatment. This could be shown by the full version of DRAM proposed in Section 1.

## 5. CONCLUSION

The user of DRAM Mark 2 is able to explore a wider range of planning issues than with DRAM Mark 1. In particular, he may study the consequences of changing the *mix* of several different resources within a single mode of health care. The examples given in Section 4 illustrate possible applications in acute in-patient treatment, but the model should be equally applicable in other care sectors where a single patient needs many resources.

In the future we hope

- to develop more general versions of DRAM, and in particular a Mark 3 version, to include substitution between alternative treatment modes,
- to develop more general methods of parameter estimation using both cross-sectional (or sub-regional) and longitudinal (or time series) data.

Such work would give a more accurate representation of the HCS, and would be more useful to health care planners. It is also likely to involve more complicated mathematics for model solution and parameter estimation. We hope, however, to be able to retain a solution procedure which uses Lagrange multipliers rather than other optimization methods. In this way, DRAM will continue to be easily transferable and useful to scientific groups outside IIASA.

REFERENCES

[1] Venedictov, D.D., and E.N. Shigan, *The IIASA Health Care System Model*, paper presented at IIASA Conference on Modeling Health Care Systems, Laxenburg, Austria, November, 1977.

[2] Gibbs, R.J., *A Disaggregated Health Care Resource Allocation Model*, RM-78-1, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

[3] Gibbs, R.J., *Health Care Resource Allocation Models - A Critical Review*, RM-77-53, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[4] McDonald, A.G., G.C. Cuddeford, and E.M.L. Beale, Mathematical Models of the Balance of Care, *British Medical Bulletin*, 30, 3 (1974), 262-270.

[5] Rousseau, J., The Need for an Equilibrium Model for Health Care System Planning, in E.N. Shigan and R.Gibbs, eds., *Modeling Health Care Systems - Proceedings of a IIASA Workshop*, CP-77-8, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[6] Gibbs, R.J., *The IIASA Health Care Resource Allocation Sub-Model: Mark 1*, RR-78-8, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

[7] Roemer, M.I., and M. Shain, *Hospital Utilization Under Insurance*, Hospital Monograph Series, No. 6, American Hospital Association, Chicago, 1959.

[8] Harris, D.H., Effect of Population and Health Care Environment on Hospital Utilisation, *Health Services Research*, 10, 229 (1975).

[9] Feldstein, M.S., *Economic Analysis for Health Service Efficiency*, North-Holland, Amsterdam, 1967.

[10] Klementiev, A.A., *On the Estimation of Morbidity*, RM-77-43, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[11] Kaihara, S., et al., *An Approach to Building a Universal Health Care Model: Morbidity Model of Degenerative Diseases*, RM-77-06, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[12] Department of Health and Social Security, UK, *Report on Hospital In-Patient Enquiry for the Year 1968*, HMSO, 1972.

[13] Department of Health and Social Security, UK, *Report on Hospital In-Patient Enquiry for the Year 1973*, HMSO, 1977.

[14] Department of Health and Social Security, UK, *Hospital Medical Staff Tables, 1968 and 1973*, unpublished.

[15] Department of Health and Social Security, UK, *SH3 Hospital Return Summaries, 1968 and 1973*, unpublished.

[16] Department of Health and Social Security, UK, *Health and Personal Social Services Statistics for England, 1975*, HMSO, 1976.

Papers of the Modeling Health Care Systems Study

Kiselev, A., *A Systems Approach to Health Care*, RM-75-31, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975.

Venedictov, D.D., Modeling of Health Care Systems, in *IIASA Conference '76*, Vol.2, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Fleissner, P., *Comparing Health Care Systems by Socio-Economic Accounting*, RM-76-19, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Klementiev, A.A., *A Computer Method for Projecting a Population's Sex-Age Structure*, RM-76-36, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Klementiev, A.A., *Mathematical Approach to Developing a Simulation Model of a Health Care System*, RM-76-65, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Kaihara, S., et al., *An Appraoch to Building a Universal Health Care Model: Morbidity Model of Degenerative Diseases*, RM-77-06, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Shigan, E.N., *Alternative Analysis of Different Methods for Estimating Prevalence Rate*, RM-77-40, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Klementiev, A.A., *On the Estimation of Morbidity*, RM-77-43, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Fleissner, P., and A. Klementiev, *Health Care System Models: A Review*, RM-77-49, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Gibbs, R., *Health Care Resource Allocation Models - A Critical Review*, RM-77-53, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Gibbs, R., *A Disaggregated Health Care Resource Allocation Model*, RM-78-01, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

Kaihara, S., et al., *Analysis and Future Estimation of Medical Demands Using a Health Care Simulation Model: A Case Study of Japan*, RM-78-03, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

Fujimasa, I., S. Kaihara, and K. Atsumi, *A Morbidity Submodel of Infectious Diseases*, RM-78-10, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

Klementiev, A.A., and E.N. Shigan, *Aggregate Model for Estimating Health Care Systems Resource Requirements (Amer)*, RM-78-21, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

Gibbs, R.J., *The IIASA Health Care Resource Allocation Sub-Model: Mark 1*, RR-78-08, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.