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A Dynamic Model for Setting Railway Noise Standards

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A DYNAMIC MODEL FOR SETTING RAILWAY NOISE STANDARDS

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Preface

Standard setting is one of the most commonly used regulatory tools to limit detrimental effects of technologies on human health, safety, and psychological well-being. Standards also work as a major constraint on technological development, particularly in the energy field. The trade-offs which have to be made between economical, engineering, environmental, and political objectives, the high uncertainty about environmental effects, and the conflicting interests of groups involved in standard setting, make the regulatory task exceedingly difficult.

Realizing this difficulty, the Volkswagen Foundation sponsored a research subtask in IIASA's Energy Program under the name *Procedures for the Establishment of Standards*. The objective of this research are to analyze existing procedures for standard setting and to develop new techniques to improve the regulatory decision making process. The research performed under this project include:

i) policy analyses of the institutional aspects of standard setting and comparisons with other regulatory tools;

ii) case studies of ongoing or past standard setting processes (e.g., oil discharge standards or noise standards);

iii) development of formal methods for standard setting based on game and decision theory;

iv) applications of these methods to real world standard setting problems.

The present research memorandum is one in a series of papers dealing with the development and application of game-theoretic models for standard setting. It presents an illustrative application of a model developed at IIASA to the problem of setting railway noise standards.

Wolf Häfele

Abstract

This paper describes the application of a multistage game theoretical model to setting noise standards which is illustrated by the case of trains. The problem was structured to match the decision problem which the Environment Agency faced when setting standards for Shinkansen trains. The model considers three players: the regulator (environment agency), the producer (railway corporation), and the impactees (residents along the railway line who suffer from noise). The game has seven stages characterized by the actions of the impactees ranging from petitions to legal litigation. The final stages are the outcomes of a possible lawsuit. The case is either won by the producer or the impactees, or a compromise is reached. Transition probabilities between stages are considered parameters of the game. They depend mainly on the noise level the impactees consider acceptable, the standard set by the regulator, and the actual level of noise emitted. Only the regulator and the producer are active players in the sense that they have a set of choices characterized as standard levels (regulator) and noise protection measures (producer). The impactees are modeled as a response function. Several solutions according to a hierarchical solution concept of the game are derived. In particular, conditions are given under which the regulator or the producer would prefer a compromise solution to awaiting the outcome of the court case. These conditions can be expressed directly as functions of noise levels and transition probabilities, given some simple assumptions about the shape of the utility functions of the regulator and the producer.

A DYNAMIC MODEL FOR SETTING RAILWAY NOISE STANDARDS

1. INTRODUCTION

Since the superrapid "bullet train", the Shinkansen, began operations in Japan in 1964, complaints about train noise have never ceased. Peak noise levels can reach over 100 dB leading to substantial disturbances of residential living. Since the responses of the government and the railway corporation to these complaints have been slow, citizens began to go through various forms of protest, including petitions, organizations, and legal litigation. In 1972 the government asked the railway corporation to take urgent steps against Shinkansen noise. But it was not until 1975 that noise standards (70-75 dB) were issued to force the railway corporation to respond to the citizens' need for quietness. Residents, however, were not content with these standards and the railway corporation's subsequent attempts at improving sound protection measures. A legal battle between residents and the railway corporation is still going on in which residents ask to reduce Shinkansen noise to a "nondisturbing" level.

In a recent paper (see [1]) the decision process of the Environment Agency and the railway corporation was described and analyzed. In this analysis the need was recognized for more formal methodologies to study decision making involving the conflict between environmental and developmental interests. The present paper is an attempt at developing such a methodology based on dynamic game theoretic models. The purpose of such models is to explore alternative strategies of the conflicting actors in environmental standard setting decisions, and to derive "optimal" strategies depending on the parameters of the game and alternative solution concepts.

Essentially three groups are involved in typical environment-development conflicts: the regulator, the producer (developer), and the impactee (sufferer of pollution). In the case of train noise these groups are an environmental agency (regulator), a railway corporation (producer), and the residents along the line (impactees). Neglecting institutional arrangements, the regulator and the producer are considered single rational players for the purposes of the model. The decisions of the residents are considered (possibly probabilistic) reactions to the decision of the regulator and the producer. Thus the impactee is not modeled as a rational player but rather as a response function. The conflict situation between regulator, producer, and residents is formalized as a multistage two-person game, where a stage is characterized by the action of the residents or a judgment by a court.

2. THE MODEL

Two-person dynamic or multistage games in extensive form (see [2] or [3]) are regarded that are similar to stochastic games. At each stage a component game of perfect information is played that is completely specified by a state. The players' choices do not control only the payoffs but also the transition probabilities governing the component game to be played at the next stage. It is assumed that the regulator and the producer have the same estimates of the transition probabilities.

The states of the game are a subset of

$$\{(i,L)\,|\,i = 1,\dots,7; \quad \underline{n} \leq L \leq \overline{n}\} \quad ,$$

where i indicates the last action or measure of the residents or the court. L denotes an upper bound for the admitted noise level, $\overline{n}$ the maximum value of noise produced by the train without special sound protection measures, $\underline{n} > o$ the minimum value of noise under which the train can be run under economic considerations, and (1,L) is the first state after construction of the railway line. Hence $(1,L) = (1,\overline{n})$. State (2,L) indicates that a petition has taken place. (3,L) states that the population affected by noise has built up an organization for negotiations with government in order to arrive at a low noise standard. If the negotiations fail the residents can start a lawsuit. This is indicated by (4,L). (4,L) can be followed by states of type (5,L), (6,L), or (7,L). (5,L) stands for a permanent compromise between all parties with upper bound L for noise. (6,L) indicates that the lawsuit was decided in a neutral or positive way for the railway corporation and the government, and (7,L) that the lawsuit was decided in favor of the residents. (5,L), (6,L), and (7,L) are final or absorbing states. See also Figure 1. For each class of states the component game and the transition probability are specified separately.

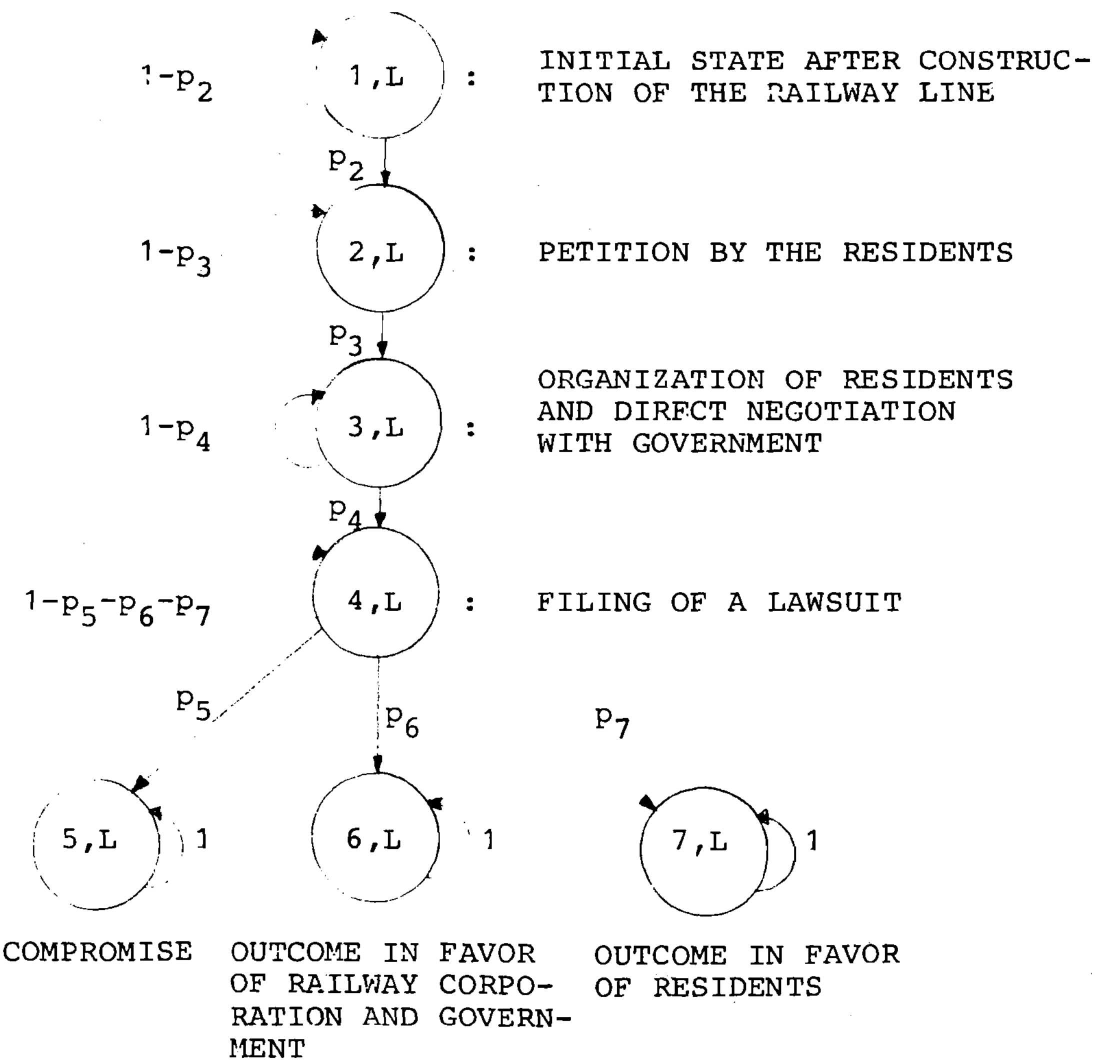

Figure 1. States of the game and transition probabilities (p_i).

The model assumes that the costs and benefits of restricting or increasing noise levels from the train can be expressed as utility functions on noise levels. The utility function of the railway corporation is given as

$$u_P : [\underline{n},\bar{n}] \to \mathbb{R} \quad ,$$

as long as there is no effective action by the residents. In general, this function will be strictly increasing. In fact, there exists evidence that within reasonable values of $\underline{n}$ and $\bar{n}$ (e.g. 60 and 100 dB, respectively) this function may be linear (see [1]). Thus in some cases it may be possible to express u_P as

$$u_P(n) = n + e \quad ,$$

neglecting a scaling factor.

The utility function of the regulator is also assumed to be defined directly on noise levels:

$$u_R : [\underline{n},\bar{n}] \to \mathbb{R} \ .$$

u_R is to reflect a compromise between the economic importance of the train and the noise pollution effects on residents along the line. In the model u_R is assumed to be unimodal with a peak at $\underline{n} \leq L^+ \leq \bar{n}$. The following argument supports the assumption that u_R is unimodal. Assuming that u_R balances environmental and developmental interests, a crude approximation of u_R could be given by

$$u_R = Wu_P + u_I \quad ,$$

where $W > o$ is an importance weight factor which indicates the relative weight of economic considerations, and u_I is the impactee's utility function. From survey data [4,5] one can infer that the strength of complaints to noise (an indicator of u_I) is approximately quadratically related to noise level. Thus neglecting scaling factors

$$u_I = -(n - \underline{n})^2 + f \quad .$$

Substituting u_I and u_P in u_R gives

$$u_R = W(n + e) - (n - \underline{n})^2 + f \quad ,$$

which is unimodal with a maximum at $L^+ = \frac{W}{2} + \underline{n}$.

In case of the first state $(1,L) = (1,\overline{n})$ the component game is specified as follows. First the regulator chooses his measure $m_R \varepsilon M_R(1,\overline{n})$, where $M_R(1,\overline{n})$ denotes the set of measures available to him. Knowing m_R the producer chooses $m_P \varepsilon M_P(1,\overline{n},m_R)$ where $M_P(1,\overline{n},m_R)$ is the set of measures available to him. M_R and M_P are specified by

$$M_R(1,\overline{n}) := \{l \mid \underline{n} \leq l \leq \overline{n}\} \quad ,$$

$$M_P(1,\overline{n},l) := \{\underline{n} \leq n \leq l\} \quad ,$$

where l denotes the highest level of noise the regulator allows, and n the value of noise generated by operating the railway. The residents' choices are not specified because they are formalized by a response function resulting in special transition probabilities.

A substantial property of the model is the assumption of a threshold $n_I \varepsilon [\underline{n},\overline{n}]$, so that a noise level below n_I is not considered a relevant disturbance of the residents.

Given state $(1,\overline{n})$ only states $(1,\overline{n})$ and $(2,\overline{n})$ can succeed. Regulator and producer believe the transition probabilities to be

$$P((1,\overline{n}) \mid 1,\overline{n},l,n) = \begin{cases} 1 & \text{if } n \leq n_I \\ 1-p_2 & \text{if } n > n_I \end{cases} \quad ,$$

and

$$P((2,\bar{n})|1,\bar{n},l,n) = 1 - P((1,\bar{n})|1,\bar{n},l,n)$$

where $p_2 > 0$ represents the experts' subjective probability that the residents will choose a petition if $n > n_I$. The utilities are given by

$$u_j(1,\bar{n},l,n) = u_j(n) \quad (j = R,P; \quad \underline{n} \leq n \leq l) \quad .$$

The state $(2,\bar{n})$ can either remain or be replaced by $(3,\bar{n})$ denoting the formation of an organization. We assume the following transition probabilities:

$$P((2,\bar{n})|2,\bar{n},l,n) = \begin{cases} 1 & \text{if } n \leq n_I \\ 1-p_3 & \text{if } n > n_I \end{cases} ,$$

and

$$P((3,\bar{n})|2,\bar{n},l,n) = 1 - P((2,\bar{n})|2,\bar{n},l,n) \quad ,$$

where $p_3 > 0$. The idea is that $n \leq n_I$ is generated either by the regulator $(l \leq n_I)$ or by the producer $(n \leq n_I < l)$ giving in to the residents' demands. The payoffs are specified by

$$U_j(2,\bar{n},l,n) := u_j(n) \quad (j = R,P; \quad \underline{n} \leq n \leq l) \quad .$$

In case of a formation of an organization $(3,\bar{n})$ residents will begin negotiations aimed at forcing the regulator to give in and set an acceptable standard. Let the measure sets of the regulator and the producer be given by

$$M_R(3,\bar{n}) := \{l | \underline{n} \leq l \leq \bar{n}\} \quad ,$$

$$M_P(3,\bar{n},l) := \{n | \underline{n} \leq n \leq l\} \quad .$$

Then

$$P((3,\bar{n})|3,\bar{n},1,n) = \begin{cases} 1 & \text{if } 1 \leq n_I \\ 1-p_4 & \text{if } 1 > n_I \end{cases} ,$$

$$P((4,\bar{n})|3,\bar{n},1,n) = 1 - P((3,\bar{n})|3,\bar{n},1,n) \quad ,$$

where $p_4 > 0$ and $(4,\bar{n})$ denotes the start of a lawsuit. Let

$$U_j(3,\bar{n},1,n) = u_j(n)(j = R,P; \quad \underline{n} \leq n \leq 1) \quad .$$

Three outcomes of a lawsuit are considered. There is a compromise (5,L) suspending the lawsuit, or a judgment in favor of regulator and producer (6,L), or a judgment in favor of the residents (7,L). Let

$$M_R(4,\bar{n}) = \{1|\underline{n} \leq 1 \leq \bar{n}\} \cup \{(1,\Lambda)|\underline{n} \leq 1 \leq \Lambda < \bar{n}\} \quad ,$$

$$M_P(4,\bar{n},1) = M_P(4,\bar{n},1,\Lambda) = \{n|\underline{n} \leq n \leq 1\} \cup \{(n,N)|\underline{n} \leq n \leq$$

$$\leq N < \bar{n}, \; n \leq 1\} \quad .$$

$(1,\Lambda)$ indicates that the regulator fixes a bound 1 for the noise at the current stage and at the same time makes a permanent commitment for a fixed bound Λ in later stages. Λ could be interpreted as a quality standard to be effective permanently after a fixed period of time has passed. For simplicity we assume that Λ becomes effective immediately. Analogously n in (n,N) denotes the actual noise level at the current stage, while N denotes a commitment made by the producer to regard this limit from now on. Let

$$M_C = \{(1,\Lambda,m_P)|(1,\Lambda)\varepsilon M_R(4,\bar{n}), \quad \Lambda \leq n_I, m_P \varepsilon M_P(4,\bar{n},1)\}$$

$$\cup \{(m_R; \quad n,N)|m_R \varepsilon M_R(4,\bar{n}), \quad (n,N)\varepsilon M_P(\bar{n},4,m_R), N \leq n_I\}$$

be called the set of compromise pairs of choices. M_C contains just the pairs (m_R, m_P) of measures guaranteeing to the residents that from now on no noise level greater than n_I will occur. Then we assume

$$P((5,L)|4,\bar{n},m_R,m_P) = \begin{cases} 1 & \text{if } (m_R,m_P) \in M_C \text{ and } L = \min(\Lambda, N) \\ & \text{where } \Lambda := +\infty \text{ or } N := +\infty \text{ in} \\ & \text{case it is not defined} \\ 0 & \text{else} \end{cases},$$

$$P((6,L)|4,\bar{n},m_R,m_P) = \begin{cases} p_6 & \text{if } L = n_R \text{ and } (m_R,m_P) \notin M_C \\ 0 & \text{else} \end{cases},$$

$$P((7,L)|4,\bar{n},m_R,m_P) = \begin{cases} p_7 & \text{if } L = n_I \text{ and } (m_R,m_P) \notin M_C \\ 0 & \text{else} \end{cases},$$

where $\underline{n} \leq n_I \leq n_R \leq \bar{n}$ holds for the maximal noise level n_R decreed by a court judgment in favor of the producer, and $p_6 + p_7$ need not equal 1. Hence

$$P((4,\bar{n})|4,\bar{n},m_R,m_P) = \begin{cases} 0 & \text{if } (m_R,m_P) \in M_C \\ 1-p_6-p_7 & \text{if } (m_R,m_P) \notin M_C \end{cases},$$

The payoffs are specified by

$$U_j(4,\bar{n},m_R,n) = u_j(4,\bar{n},m_R,n,N) = u_j(n)\ (j = R,P;\ \ \underline{n} \leq n \leq l) \quad .$$

State (5,L) means that either the regulator has agreed to take $L \leq n_I$ as the maximal level of noise, or that the producer has bound himself to noise levels not larger than $L \leq n_I$. Let the sets of measures be given by

$$M_R(5,L) := \{l | \underline{n} \leq l \leq L\} \quad ,$$

$$M_P(5,L,l) := \{n | \underline{n} \leq n \leq l\} \quad .$$

Then

$$P((5,L)|5,L,1,n) = 1 \quad .$$

The payoffs are specified by

$$U_j(5,L,1,n) = u_j(n) \; (j = R,P; \;\; \underline{n} \leq n \leq 1) \quad .$$

State $(6,n_R)$ indicates a judgment unfavorable to the residents. Let

$$M_R(6,n_R) = \{1|\underline{n} \leq 1 \leq n_R\} \quad ,$$

$$M_P(6,n_R,1) = \{n|\underline{n} \leq n \leq 1\} \quad .$$

Then

$$P((6,n_R)|6,n_R,1,n) = 1 \text{ and } u_j(6,n_R,1,n) = u_j(n) \; (j = R,P) \quad .$$

State $(7,n_I)$ denotes a judgment unfavorable to regulator and producer. Let

$$M_R(7,n_I) = \{1|\underline{n} \leq 1 \leq n_I\} \quad ,$$

$$M_P(7,n_I,1) = \{n|\underline{n} \leq n \leq 1\} \quad .$$

Then

$$P((7,n_I)|7,n_I,1,n) = 1 \quad .$$

In the case of a lost lawsuit the producer's and the regulator's utilities change. This is because such a judgment would have much wider reaching consequences than a voluntary agreement to a standard. First of all, implementation time, rules of operation, etc. prescribed in a judgment would mean substantial restriction of freedom to the railway corporation. Secondly, the sentence would most likely be applied throughout the railway network.

Thus the model assumes that

$$U_P(7,n_I,1,n) = u_P(n) + c_P \quad ,$$

where $c_P < 0$ is a fixed penalty as a result of the sentence. Also, the regulator stands to lose both in prestige and in lost flexibility if the court should decide in favor of the impactees. Again this loss is expressed in his utility function.

$$U_R(7,n_I,1,n) = u_R(n) + c_R \quad .$$

In the case of $L^+ > n_I$ it appears not unreasonable to assume that $c_j (j = P,R)$ is a negative multiple m_j of $u_j(L^+) - u_j(n_I)$, i.e.

$$c_j = - m_j[u_j(L^+) - u_j(n_I)] \quad , \quad (j = P,R) \quad .$$

A play π of the game is given by an infinite sequence $(s^1,m_R^1, m_P^1; \ s^2,m_R^2,m_P^2; \ ...)$ of states and measures. We define the utility of a play π by the discounted infinite sum of the stage utilities

$$\underline{U}_j(\pi) := \sum_{i=1}^{\infty} \rho^{i-1} U_j(s^i,m_R^i,m_P^i) \quad (j = R,P) \quad ,$$

where $0 < \rho < 1$ is a discount factor.

The game is now completely described except for the definition of strategies and the solution concept. For simplification we admit only stationary strategies where the choices depend only on the last state and the last measures of the other players.

Definition: A strategy σ_R of the regulator is a map

$$\sigma_R : S \to [\underline{n},\overline{n}] \quad ,$$

such that

$$\sigma_R(s)\varepsilon M_R(s) \quad (s\varepsilon S) \quad ,$$

where S denotes the set of states.

A strategy σ_P of the producer is a map

$$\sigma_P : \{(s,l)|s\varepsilon S, l\varepsilon M_R(s)\} \rightarrow [\underline{n},\overline{n}] \quad ,$$

such that

$$\sigma_P(s,l)\varepsilon M_P(s,l) \quad .$$

The sets of strategies are denoted by Σ_R and Σ_P.

For each strategy pair (σ_R,σ_P) a play $\pi = (s^1,l^1,n^1; \; s^2,l^2, n^2; \; ...)$ is realized. Since the strategies are stationary, two components (s^i,l^i,n^i) and (s^r,l^r,n^r) are equal as soon as $s^i = s^r$. By the definition of the transition probabilities at most seven states can occur with probability greater than zero and only one will be repeated infinitely often. From this it follows that the set $\Pi(\sigma_R,\sigma_P)$ of possibly realized plays π is finite or denumerable. The probability $P(\pi|\sigma_R,\sigma_P)$ for $\pi\varepsilon\Pi(\sigma_R,\sigma_P)$ is given as an infinite product of the terms $P(s^{i+1}|s^i,l^i,n^i)$ defined above. The payoff of player $j\varepsilon\{R,P\}$ is supposed to be his expected utility of the plays:

$$V_j(\sigma_R,\sigma_P) = \sum_{\pi\varepsilon\Pi(\sigma_R,\sigma_P)} \underline{U}_j(\pi)P(\pi|\sigma_R,\sigma_P) \quad .$$

The strategies are to be determined according to the following solution concept.

<u>Definition</u>: A hierarchical solution is a pair (τ_R,τ_P) of a strategy $\tau_R\varepsilon\Sigma_R$ and a map $\tau_P : \Sigma_R \rightarrow \Sigma_P$ such that

$$V_P(\sigma_R,\tau_P(\sigma_R)) = \max_{\sigma_P \varepsilon \Sigma_P} V_P(\sigma_R,\sigma_P) \quad ,$$

$$V_R(\tau_R,\tau_P(\tau_R)) = \max_{\sigma_R \varepsilon \Sigma_R} V_R(\sigma_R,\tau_P(\sigma_R)) \quad ,$$

3. THE GAME-THEORETIC SOLUTION

In order to keep the analytical part as small as possible we shall only discuss heuristic equations which, however, can be justified as soon as one establishes the analytical framework in full detail. At least part of it can be found in [7]. Because of the definiton of the component game payoffs and the transition probabilitics, the measures $(1,\Lambda)$ and $(1,n_I)$, or (n,N) and (n,n_I), respectively, have the same effect in the case of $\Lambda < n_I$, respectively. This also holds in the case of 1 and $(1,\bar{n})$ or n and $(n,\bar{n})$. Hence, without loss of generality, we can reduce the measure sets $M_R(4,\bar{n},m_R)$ and $M_R\ (4,\bar{n})$ to

$$M_R(4) : = \{(1,\Lambda)\,|\,n_I \leq \Lambda \leq \bar{n}, \underline{n} \leq 1 \leq \Lambda\} \quad ,$$

$$M_P(4,1,\Lambda) : = \{(n,N)\,|\,n_I \leq N \leq \bar{n}, n \leq N, n \leq 1\} \quad .$$

Since then only the states $(1,\bar{n})$, $(2,\bar{n})$, $(3,\bar{n})$, $(4,\bar{n})$, $(5,n_I)$, $(6,n_R)$, $(7,n_I)$ can occur, the states are completely fixed by their first component. We therefore drop the second component in all the terms.

For the rest of the paper let $\Gamma_b (b = 1,\ldots,7)$ denote subgames of the original game such that b ist the first state. Hence Γ_1 is the original game. $\Gamma_b (b = 5,6,7)$ has only the state b. $\Gamma_b (b = 1,2,3,4)$ covers states b, $b + 1,\ldots,7$. Though in principle one has to distinguish the strategies for different Γ_b we denote by abuse of notation the reduction of $\sigma_j \varepsilon \Sigma_j$ to Γ_b by σ_j. Let $V_{j,b}$ denote the payoff function for player j in game

Γ_b. Without the simple proof we state that

$$V_{j,7}(\sigma_R,\sigma_P) = \frac{1}{1-\rho}\,(u_j(\sigma_P(7,\sigma_R(7)) + c_j) \quad .$$

$$V_{j,b}(\sigma_R,\sigma_P) = \frac{1}{1-\rho}\,u_j(\sigma_P(b,\sigma_R(b))) \qquad (b = 5,6) \quad ,$$

for $j = R,P$. Now let $(\sigma_R,\sigma_P)(i) := (\sigma_R(i),\sigma_P(i,\sigma_R(i))(i = 1,\ldots,7)$. Then

$$\begin{aligned} V_{j,4}(\sigma_R,\sigma_P) &= u_j(\sigma_P'(4,\sigma_R(4))) \\ &+ \rho\ P(4|4,(\sigma_R,\sigma_P)(4))V_{j,4}(\sigma_R,\sigma_P) \\ &+ \rho\ P(5|4,(\sigma_R,\sigma_P)(4))V_{j,5}(\sigma_R,\sigma_P) \\ &+ \rho\ P(6|4,(\sigma_R,\sigma_P)(4))V_{j,6}(\sigma_R,\sigma_P) \\ &+ \rho\ P(7|4,(\sigma_R,\sigma_P)(4))V_{j,7}(\sigma_R,\sigma_P) \quad , \end{aligned}$$

where σ_P' $(4,1,\Lambda)$ denotes the first component of $\sigma_P(4,1,\Lambda)$. By backward iteration

$$\begin{aligned} V_{j,b}(\sigma_R,\sigma_P) &= u_j(\sigma_P(b,\sigma_R(b))) \\ &+ \rho\ P(b|b,(\sigma_R,\sigma_P)(b))V_{j,b}(\sigma_R,\sigma_P) \\ &+ \rho\ P(b + 1|b,(\sigma_R,\sigma_P)(b))V_{j,b+1}(\sigma_R,\sigma_P) \\ &\qquad (b = 1,2,3; \quad j = R,P) \quad , \end{aligned}$$

Three situations are conceivable:

(1) The regulator can enforce his maximum utility;

(2) If regulator and producer have won the lawsuit, the regulator has to offer $l > L^+$ in order to keep the producer from compromising;

(3) If regulator and producer have won the lawsuit, not even the offer $l = n_R$ can keep the producer from compromising.

Though the calculation of the hierarchical solution for three situations is not difficult for any given set of values of the parameters, the derivation of the hierarchical solution as a function of the parameters would require a lot of space. Therefore we consider only the first and the third situations. The two classes of parameters given, however, do not in general exhaust the set of all the parameter values possible.

At first we establish a pair of strategies yielding the maximum utility to the regulator.

<u>Definition</u>: Let $L^+ > n_I$. The vector of real numbers $L^+, n_I, n_R, \rho, p_6, p_7)$ satisfies the compromise condition of player j (C,j) if

$$u_j(n_I) > \frac{1}{1-\rho(1-p_6-p_7)} \{(1-\rho)u_j(L^+) + p_6\rho\, u_j(\min(L^+, n_R)) + p_7\rho[u_j(n_I) + c_j]\}$$

holds.

As can be seen by the formulae above, (C,j) indicates that a compromise is more advantageous to player j.

<u>Theorem</u>: Let $\phi \varepsilon \Sigma_R$ and $\Psi \varepsilon \Sigma_P$ be defined by

$$\phi(i) := L^+ (i = 1,2,3), \quad \phi(5) := \phi(7) := n_I,$$

$$\phi(6) := \min(L^+, n_R)$$

$$\phi(4) := \begin{cases} (n_I, n_I) & \text{if } L^+ \geq n_I \text{ and (C,R) holds} \\ (L^+, L^+) & \text{if } L^+ \leq n_I \text{ or (C,R) is violated} \end{cases}$$

$$\Psi(i,1) := 1 (i = 1,2,3,5,6,7) \quad ,$$

$$\Psi(4,1,\Lambda) := (1,\Lambda) \quad .$$

Then (ϕ,Ψ) yields the maximal utility to the regulator:

$$V_R(\phi,\Psi) = \sup_{\Sigma_R\times\Sigma_P} V_R(\sigma_R,\sigma_P) \quad .$$

In order to avoid a lengthy and not instructive proof we only give the idea of the proof. First let $L^+ \leq n_I$. Because of the definition of $U_R(i,m_R,m_P)$ the inequality $U_R(i,m_R,m_P) \leq u_R(L^+)$ holds for all possible states i and measures m_R and m_P. Hence $V_R(\sigma_R,\sigma_P) \leq \frac{1}{1-\rho} u_R(L^+)$ for each $(\sigma_R,\sigma_P)\varepsilon\Sigma_R \times \Sigma_P$. But $V_R(\phi,4) = \frac{1}{1-\rho} u_R(L^+)$ because of $\phi(1) = L^+$ and $P(1|1,(\phi,\Psi)(1)) = 1$. Now let $L^+ > n_I$. Obviously $V_{R,j}(\sigma_R,\sigma_P) \leq V_{R,j}(\phi,\Psi)(j = 5,6,7)$. Then (σ_R,σ_P) with $(\sigma_R,\sigma_P)(i) = (\phi,\Psi)(i)(i = 5,6,7)$ maximizes $V_{R,4}(\sigma_R,\sigma_P)$ if $(\sigma_R,\sigma_P)(4) = (\phi,\Psi)(4)$ under consideration of the compromise condition (C,R). Hence $V_{R,4}(\sigma_R,\sigma_P) \leq V_{R,4}(\phi,\Psi)$. The final step of the backward iteration yields $V_R(\sigma_R,\sigma_P) \leq V_R(\phi,\Psi)$ for each pair $(\sigma_R,\sigma_P)\varepsilon\Sigma_R \times \Sigma_P$.

If Ψ is an optimal response to ϕ , i.e. $V_P(\phi,4) = \sup_{\Sigma_P} V_P(\phi,\sigma_P)$, it is not important to derive a hierarchical solution since the regulator can enforce his maximum payoff.

<u>Definition</u>: The payoff vector $(V_R(\sigma_R,\sigma_P), V_P(\sigma_R,\sigma_P))$ is Pareto-optimal if there is no other strategy pair $(\sigma_R',\sigma_P'\varepsilon\Sigma_R\times\Sigma_P)$ such that $V_j(\sigma_R,\sigma_P) \leq V_j(\sigma_R',\sigma_P')(j = R,P)$ and that at least one inequality is strict.

<u>Theorem</u>: Let $(\phi,\Psi)\varepsilon\Sigma_R\times\Sigma_P$ be defined as in the preceding theorem. Then Ψ is an optimal response to ϕ, i.e. $V_P(\phi,\Psi) \geq$ $\geq V_P(\phi,\sigma_P)(\sigma_P\varepsilon\Sigma_P)$, and $V_R(\phi,\Psi) > V_R(\sigma_R,\sigma_P)(\sigma_R\varepsilon\Sigma_R, \sigma_P\varepsilon\Sigma_P)$ if one of the following conditions holds

(i) $L^+ \leq n_I$;
(ii) $L^+ > n_I$ and (C,R);
(iii) $L^+ > n_I$ and not only (C,R) but also (C,P) is violated.

In these cases $(V_R(\phi,\Psi),V_P(\phi,\Psi))$ is a Pareto-optimal payoff vector.

<u>Sketched proof</u>: Case (i): Because of $\phi(1) = L^+ \leq n_I$, $P(1|1,(\phi,\sigma_P)(1)) = 1$. Hence $V_P(\phi,\sigma_P) = \frac{1}{1-\rho} u_P(\sigma_P(1,L^+))$ where $\sigma_P(1,L^+) \leq L^+$ is maximized by Ψ. In order to obtain a greater payoff $V_P(\sigma_R,\sigma_P)$ for one stage at least L^+ has to be replaced by $n > L^+$. But then the regulator's payoff is smaller because of $u_R(n) < u_R(L^+)$.

Case (ii): $P(5|4, (\phi,\sigma_P)(4)) = 1$ because of $\phi(4) = (n_I,n_I)$. By backward iteration evaluating $V_{P,i}(\phi,\sigma_P)$ $(i = 5,4,3,2,1)$ one immediately sees that Ψ maximizes $V_P(\phi,.)$. The proof of the Pareto-optimality relies on the fact that only strategies σ_R with $\sigma_R(i) = \phi(i)$ $(i = 1,\ldots,5)$ give maximal payoff to the regulator. The verification of this fact requires a lengthy and uninstructive discussion which we therefore omit.

Case (iii): Given ϕ the assessment $\Psi(i,1) := 1$ $(i = 5,6,7)$ belongs to an optimal response for all values of the parameters. Because of $\phi(4) = (L^+,L^+)$ and $L^+ > n_I$ a strategy σ_P maximizing $V_{P,4}(\phi,.)$ takes either the value $\sigma_P(4,L^+,L^+) = (n_I,n_I)$ or the value (L^+,L^+). Since (C,P) is violated the second assessment yields a larger utility. Hence Ψ maximizes $V_{P,4}(\phi,.)$. Then obviously Ψ maximizes $V_{P,i}(\phi,.)$ $(i = 3,2,1)$. The Pareto-optimality of $(V_R(\phi,\Psi), V_P(\phi,\Psi))$ can again be verified by changing some values of $(\phi,\Psi)(i)$ proving that they reduce the regulator's payoff.

If (C,P) holds and (C,R) is violated the strategy Ψ is generally not an optimal response of ϕ. The situation can arise where the regulator by reduction of his own payoff can force the maximizing producer to a no-compromise strategy. In order to keep the analytical part small we only treat a special case where this situation cannot arise.

<u>Definition</u>: The vector $(\underline{n},n_I,n_R,\bar{n},\rho,p_6,p_7)$ satisfies the strict compromise condition (SC) if

$$u_P(\underline{n}) > \frac{1}{1-\rho(1-p_6-p_7)}\{(1-\rho)u_P(\bar{n}) + p_6\rho u_P(n_R) + p_7\rho[u_P(n_I) + c_P]\}$$

holds.

(SC) can be interpreted by the way that the utmost offer and threat of the regulator cannot match the value of a compromise for the producer.

<u>Theorem</u>: Let (SC) hold. A hierarchical solution (τ_R, τ_P) is given by $\tau_R = \phi$ and $\tau_P(\sigma_R) = \gamma \varepsilon \Sigma_P$ for each $\sigma_R \varepsilon \Sigma_R$ where

$$\gamma(i,1) := 1 \quad (i = 1,2,3,5,6,7) \quad ,$$

$$\gamma(4,1,\Lambda) := (\min(1,n_I),n_I) \quad .$$

<u>Sketched proof</u>: Because of (SC) the second component of $\sigma_P(4,\sigma_R(4))$ equals n_I for any optimal response σ_P of any $\sigma_R \varepsilon \Sigma_R$. By backward iteration one immediately sees that γ is an optimal response of each $\sigma_R \varepsilon \Sigma_R$, i.e. $V_P(\sigma_R \cdot \sigma_P) \leq V_P(\sigma_R, \gamma)$. $V_{R,5}(\cdot,\gamma)$ is maximized by ϕ and, more generally, $V_{R,i}(\cdot,\gamma)\,(i = 4,3,2,1)$ as one can see by backward iteration.

<u>Remark</u>: In case of $L^+ > n_I$ and (SC) but violated (C,R) the regulator generally does not obtain the possible maximum payoff

$$\dot{V}_R(\phi,\Psi) \gneqq V_R(\phi,\gamma) \quad .$$

Part of the results can be given in a more illustrative way. In the case of $n_I < L \leq n_R$ let

$$c_j = - m_j(u_j(L^+) - u_j(n_I)) \quad (j = R,P) \quad .$$

m_j is assumed to be a constant positive factor. It specifies the weight of the severe consequences of a judgment for noise reduction which has to be considered for all other later noise-producing activities. A short calculation yields that (C,j) is equivalent to

$$m_j p_7 \rho > 1 - \rho(1 - p_6) \quad (j = R,P) \quad .$$

The second theorem implies that in the case of $n_I < L^+ \leq n_R$ and $m_R p_7 \rho > 1 - \rho(1 - p_6)$ the regulator prefers the compromise: $\phi(4) = (n_I, n_I)$. In case of $n_I < L^+ \leq n_R$ and $m_j p_7 \rho \leq 1 - \rho(1 - p_6)$ ($j = R,P$), however, the lawsuit will result in a judgment: $(\phi,\Psi)(4) = (L^+, L^+; \; L^+, L^+)$.

An elementray calculation shows that the expected duration d of the lawsuit is $d = \frac{1}{p_6 + p_7}$. Given d, condition (C,j) is equivalent to

$$\frac{1}{d} \geq p_7 > \frac{1 - \rho + \frac{\rho}{d}}{\rho(m_j + 1)} \qquad (j = R,P) \quad .$$

The following example illustrates the relevance of the results. Let $d = 4$ years, $\rho = 0.9$ and $m_R = m_P = 10$. Then (C,j) ($j = R,P$) is approximately given by $p_7 > 0.03$. Hence a lawsuit should only be filed and pursued to final judgment if the probability for a judgment in favor of the residents in one year is not greater than three percent. If $p_7 = 0.03$ then the probability of such a judgment being pronounced at all is $dp_7 = 0.12$.

4. CONCLUSIONS

A main element of the model is the consideration of the impactees' reactions in standard setting. Under certain assumptions the model could identify the important areas in the decision process of the regulator and the producer. In particular the decision about offering and accepting or rejecting a compromise turned out to be of crucial importance. This decision could be determined as a function of the model parameters in which the subjective probabilities of the outcome of the court proceedings can play a major role.

Model limitations include the "short-sightedness" of the impactees' response which only covers present standards and noise levels. Consequently the strategies of the regulator and the producer do not include commitments for later time periods, e.g. in the form of quality standards. The model results indicate, however, that such extensions are feasible, although at a substantially greater effort. For example, strategies could be in the form of long-term noise reduction plans instead of short-term standards, and impactees' responses would take into account the nature of these plans.

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