



Graph Theory Methods in Analysis of Model Structures

Aven, P.O., Borodkin, L. and Muchnik, I.

IIASA Working Paper

WP-78-011

1978



Aven, P.O., Borodkin, L. and Muchnik, I. (1978) Graph Theory Methods in Analysis of Model Structures. IIASA Working Paper. WP-78-011 Copyright © 1978 by the author(s). <http://pure.iiasa.ac.at/892/>

Working Papers on work of the International Institute for Applied Systems Analysis receive only limited review. Views or opinions expressed herein do not necessarily represent those of the Institute, its National Member Organizations, or other organizations supporting the work. All rights reserved. Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage. All copies must bear this notice and the full citation on the first page. For other purposes, to republish, to post on servers or to redistribute to lists, permission must be sought by contacting repository@iiasa.ac.at

GRAPH THEORY METHODS IN
ANALYSIS OF MODEL STRUCTURE

P.O. Aven
L.I. Borodkin
I.B. Muchnik

April 1978

WP-78-11

Working Papers are internal publications intended for circulation within the Institute only. Opinions or views contained herein are solely those of the author.

2361
Laxenburg
Austria

International Institute for Applied Systems Analysis

Preface

Further development of modelling of socio-economic systems demands a certain methodological base. One of the main problems raised in that field is the problem of the choice of indicators used in models and the connection between these indicators.

The starting point in the Food and Agriculture research of IIASA is modelling of national food and agricultural systems. Further junction of national models as a whole will be done. Such generalization of national models calls for certain homogeneity of the sets of models' indicators, particularly for homogeneity of commodity lists of the models. The importance of the problem demands not only conceptual analysis but also formal procedures for analyzing and evaluating the given set of indicators. Some of these procedures are the subject of the present Memorandum.

Abstract

The present paper consists of an introduction, in two parts and an appendix.

The Introduction is devoted to the definition of the problem and describes, as a whole, the methods used. The close connection between choosing the set of indicators and constructing the structural scheme of the model is discussed here. The possibility for using graph theory methods for analyzing and evaluating the given set of indicators is shown.

Part I deals with the formal definition of the problem and describes graph theory algorithms employed in model structure analysis.

In Part II, brief characteristics of the models under investigation and analysis of their graph models are given. The methods suggested in the present work appeared in the analysis of three global models: World 3, Mesarovic and Pestel model and MOIRA.

In the Appendix, the sets of indicators of the graph models and the graph models themselves are given.

C O N T E N T S

	<u>Page</u>
Preface	
Abstract	
<u>Introduction:</u> Definition of the Problem	1
<u>Part I:</u> Formal Definition of the Problem-- Algorithms for Solution	6
1.1 Constructing the Acyclic Graph	6
1.2 Constructing the Weights of Arcs in the Acyclic Graph	9
1.3 Isolation of the Set of Vertices Strongly Connected with the Basic Vertex	12
<u>Part II:</u> Formation of the Graph Model	18
2.1 Brief Characteristic of the Models	18
2.2 Construction of the Set of Vertices of the Graph Model	20
2.3 Construction of the Set of Arcs of Graph Model	23
<u>Appendix:</u> Lists of Indicators for Graph Models	26
<u>References</u>	40

Introduction: DEFINITION OF THE PROBLEM

The construction of global models attracts great attention today. By global models we mean models of world evolution as a whole, usually with emphasis on questions of population growth and its supply of all the necessary resources.

A few global models [4], [5], [6], have already been constructed and the exploration of new models is being continued. Further development of global modelling demands improvement of the methodological base. One of the problems raised in scientific research is the choice of indicators, used in global models, and the connection between these indicators. The sets of indicators differentiate given models both with respect to the structure and behaviour.

Due to the fact that the construction of the systems of socio-economic indicators is considered to be so important, many scientific studies are devoted to investigations in that field today. Special committees for the construction of the systems of socio-economic indicators exist in UNO and UNESCO. A permanent committee for problems of social indicators is found at the International Sociological Association as well as in a section of the Soviet Sociological Association.

In 1976, the International Symposium on the Exploration of the Systems of Social Indicators and Global Modelling was held in Moscow. Yet the situation leaves much to be desired. Methods for choosing a set of indicators for constructing a model are not yet elaborated, and each group of investigators offers its own set without explaining why certain indicators and not others are included.

The problem of devising the structural scheme of the model, i.e. of the connection among the indicators used in it, is even less elaborated. Note that this problem--the design of the structural scheme of the model--is closely connected with that of choosing the set of indicators mentioned above. It is possible to understand the role of some indicator in the model and very often even its qualitative sense, merely by knowing its connection with the rest of the indicators within the set. This can be accounted for also by the difference in meaning of qualitatively identical indicators in different models and also by the different interpretation of the accepted economic term in various countries.

This state of affairs demands not only conceptual analysis of all the problems but also objective criteria for evaluating the given set of indicators as well as a structural scheme in the model. The importance of these criteria becomes evident with the growing number of modifications of existing models and the creation of new ones. At the same time, the question of acceptability of the set of indicators or the structure of its functional links can be viewed as merely conceptual. Thus, we are

faced with a certain paradox: the conceptual analysis calls for the establishment of objective criteria which include subjective elements.

To escape this paradox, we tried to find a compromise solution. Instead of formal criteria for evaluating the given set of indicators we propose constructing formal procedures for precise description of the main structural peculiarities of the global models under investigation. The approach is based on the analysis of the graph models of these models. The vertices of graph models correspond to the chosen indicators; the arcs reflect the links between them. This graph model is in fact the structural scheme of the model. The method of graph approximation provides the possibility of describing the most important points in the structure of the graph model in an aggregated form.

A number of recent investigations employed formal methods of analyzing global models. Some were devoted to the study of the dynamics of the main indicators in the model with variation of constant parameters in equations or change in the character of functional links, but retention of the variable parameters in each model equation. In (1), the interesting case is examined when the model is set in conditions of control in accordance with certain criteria. In all these studies, it is the system of model equations that becomes the object under investigation. Even if some of these studies deal with the research of model structure--for example, with determining the set of indicators whose dynamics do not influence the behaviour of the remaining indicators [8]--the analysis is based either on the investigation of equations or on experiments with a model already constructed. Only after this work was finished did we read the paper by McLean and Shepherd [7]. The authors treat analysis of model structure with the help of graph models, but in fact their analysis, at its most interesting point, can be realized only after the model has been built. In order to represent the global model by the graph in [7], a weight is considered to correspond to each arc in the graph, the weight is equal to the partial derivative of the function which connects the arc's variables. It is clear that partial derivatives can be found only after specification of the model equations. However, it is necessary to be able to analyze the model structure at the stage of its inception before constructing the equations themselves.

The methods we suggest for solving this problem are directed to the study of the structural scheme of a global model (in other works, its graph model) in which the basic indicator is defined. This role may be played by the output parameters of the main model sectors--population, capital, food production, etc. In this case, we propose organizing the model analysis in three stages:

1. First, we pick out, from the graph model, the acyclic sub-graph-network, with a reference point to which the basic indicator corresponds. The network is constructed as follows: we begin by involving vertices from which there are arcs to the basic vertex, and after that we select from the remaining vertices those that have the arcs entering the vertices which were selected at the first step, and so on. Each time we add new vertices together with these vertices, the arcs that go out from these vertices into the ones already involved in the network, are engaged. Then a simple procedure follows which is arrived at by increasing the total number of arcs (without destroying the acyclic character of the graph) in order to achieve a better reflection of the initial graph by the network.

The presence of the acyclic graph, the structure of which closely resembles the structure of the initial graph of a global model, allows us to analyze the set of indicators from a specific point of view, classifying the indicators by the degree of their influence on the basic indicator. It is not difficult to show that all the vertices of the graph (and only these) from which the basic vertex is derived-- i.e., those that correspond to the global model indicators, which have an influence on the basic indicator, are involved in the network.

2. For each arc of the constructed network, the coefficient of its value (i.e. weight) is computed. It is defined as a number of paths penetrating through this arc in the acyclic graph. This coefficient is further employed for evaluating the weight of the vertices connected with this arc. It is essential to point out that the value of the network's vertices is analyzed only from the point of view of its influence on the basic vertex in the graph model. Such weighting of the arcs of the acyclic graph naturally provides new information for classification of model indicators on the basis of their role in the definition of the basic indicator's behaviour.
 3. We now find the subgraph of the constructed acyclic graph which, together with the basic vertex, contains several other vertices forming the so-called "kernel" of the basic one. In formulating the concept of the "kernel" we proceeded as follows. The easiest way to define the "kernel" is by fixing a value threshold and dividing the arcs according to their weight into essential and non-essential, thereby dividing the vertices of the network. At the same time, it is hardly possible to evaluate such a threshold by informal considerations. Thus, we wanted to find a definition of the "kernel" such, that the threshold could be implicitly defined.
-

We managed to do this because, within the method of graph approximation, the task of selecting from the graph some extreme subset vertices which, to a certain extent, are strongly connected with the vertex fixed from the very beginning, was naturally formulated. The subset defined by this task was called the "kernel". As a result of the analysis, the whole set of vertices of the graph model can be divided into three parts: the "kernel" of the vertices of the basic vertex; the subset of vertices which, although they have a path to the basic vertex, do not belong to the "kernel"; and, lastly, the remaining vertices placed beyond the network constructed for the basic vertex.

Such classification of the set of indicators from the point of view of the single indicator allows us to evaluate, quickly and as a whole, what had been missing in the characteristics of the indicators' position in the system. Moreover, it helps to improve the structural scheme of the model by modifying it until the classification of the model's indicators corresponds to the investigator's perception of reality. It is obvious that analysis by the method described can be carried out several times with varying fixation of the basic vertex.

Moreover, each vertex of the graph can be examined as a basic vertex; and some vertices which never appeared in the "kernel" may be found among the graph vertices. It is clear that, in some sense, these vertices do not influence the behaviour of the model, indicating that they might not play an important role in it.

After describing our method as a whole, it is pertinent to remark once again on the work by McLean and Shepherd. It is not difficult to understand that the second part of our techniques is, in its purpose, very similar to the search for critical components as in [7]. The authors defined for each pair of indicators the number of paths connecting them in the graph. Such characteristics are also computed in our program; but as we assumed the number of paths passing through the arc to be more important, the first characteristic was calculated only for the purpose of calculating the others. In any case, the method considered by McLean and Shepherd is difficult to be realized in practice. In [7] they raised the adjacency graph matrix to some power which is increased by one at each subsequent step until it becomes equal to the longest chain of connection within the model. A "total connection" matrix is then constructed by summing all matrices constructed at each step. It is not clear how this procedure can be realized for cyclic graphs. The matrix constructed as a result, in the case of the cyclic graph, will contain information that does not deal at all with the number of paths connecting the vertices. Nevertheless, this procedure requires a great deal of time. In [7], an example corresponding to the described method can be found. The graph analyzed contains 28 vertices and the maximum length of path within the graph is equal to 8. In reality, however, the graph of global models can contain hundreds of vertices and can be quite cyclic, and in the

longest chain hundreds of arcs can be included. According to the last part of [7], it seems to be more interesting not just to divide the model's indicators into several blocks where each indicator can only be placed in one block, but to define the set of indicators strongly interacting with some previously marked indicator. Here, each indicator can appear in any number of "kernels", which provide more information about the model structure.

Our method is the very first step to solving all the problem stated above. At the next stage some new algorithms can be employed for structural analysis of global models, (for instance, algorithm dealing directly with the cyclic graph of the model). On the other hand it seems to be necessary to develop some new algorithms for direct definition and eliminating all the drawbacks found (and so the problem raises the formal definition of what the drawbacks are). Finally, in order to analyze the global model itself (but not only the set of its indicators) it seems to be useful to include in the graph some information about model's functions (by weighting the arcs in the graph).

The present work consists of two parts and the Appendix. In the first part the algorithms of graph model analysis, on which the method is built, are described. In the second part, characteristics of the global models under investigation and analysis of their graph models are given. We should like to draw attention to the informal character of the graph model's construction and the formal character of the analysis. In the Appendix, the set of indicators of the graph models and the graph models themselves are given.

The work has a methodological character. The results of the experiment with the suggested methods would become the subject of future research. The description of the computer program corresponding to the algorithms constructed will be published separately.

Part I: FORMAL DEFINITION OF THE PROBLEM--ALGORITHMS FOR SOLUTION

1.1 Constructing the Acyclic Graph

Let Γ be an oriented graph with its adjacency matrix $||d_{ij}||_q^q$. Graph Γ is spanned by elements of some ordered set $D = \{d_1, \dots, d_q\}$, where each element d_i , vertex of the graph, is connected with some indicator of the global model. Let us describe formally an algorithm for constructing on the basis of graph Γ an acyclic directed graph Γ^0 - a network with reference point d_k (d_k is the basic, already defined vertex). Let $||d_{ij}^0||_n^n$ be an adjacency matrix of graph Γ^0 ($n \leq q$).

Algorithm I.

Step 0. We take the vertex d_k . Let all elements of the matrix $||d_{ij}^0||_n^n$ be equal to zero.

Step 1. We choose subset $D_1 \subset D$, so that vertex $d_i \in D_1$, only when $d_{ik} = 1$. Set $d_{ik}^0 = 1$, if $d_i \in D_1$.

Step i. We choose subset $D_i \subset D$, so that $d_e \in D_i$, only when both the following conditions are satisfied:

$$(a) \quad d_e \notin \bigcup_{j=1}^{i-1} D_j,$$

$$(b) \quad \exists d_p \in D_{i-1}, \text{ such that } d_{ep}^0 = 1,$$

For each pair (e,p) , which satisfies conditions (a)-(b) set $d_{ep}^0 = 1$.

The algorithm terminates and graph Γ^0 is completed when at the end of step S , the constructed set is empty: $D_S = \emptyset$.

It is obvious that constructed by algorithm 1 graph Γ^0 is acyclic, and set of its vertices \hat{D} (generally speaking not equal to set D ; $\hat{D} \subset D$) is a set of such vertices from which the basic vertex is attainable. Graph Γ^0 is a network, the vertices of which are distributed among "levels" corresponding to subsets D_i .

Clearly the result of constructing the acyclic part of basic graph Γ can be recognized as "good" if the number of removed

arcs of graph Γ is not too large. The following procedures raise the effectiveness of the described algorithm by increasing the number of arcs in acyclic graph Γ^0 .

Procedure 1.1

We examine in turn all levels of graph Γ^0 ; if, on some level, exogenous vertices are found (i.e. vertices in which no arc enters, they are raised to the last "upper" level (or S-level). For each such vertex d_m all levels with numbers less than S, are examined. If $d_{m_p} = 1$, we construct an arc from vertex d_m to vertex d_p .

Procedure 1.2

We examine in turn all levels of the acyclic graph constructed after the completion of procedure 1.1. In this examination each level D_i ($i-1, S$) is divided into n_s sublevels, according to the number of vertices at that level. The rules used for dividing are the following:

- for each vertex d_1 at level D_i calculate the number of arcs leading from d_1 to the other vertices of the same level;
- on the upper sublevel (n_s), place vertex d_{1_1} , for which that number Z_{1_1} is maximum. (If there are a few such vertices their sequence is constructed according to the number of arcs which lead from one such vertex to all the rest);
- place on the next sublevel (n_{s-1}) the vertex d_{1_2} with the maximum number Z_{1_2} among all the vertices of that level except d_{1_1} ;
- the procedure is finished when, for each level D_i , all the sublevels n_s are filled.

The graph constructed after the completion of that procedure will certainly be acyclic. Moreover, all the graph vertices will be distributed among the levels, the number of which is equal to the number of vertices (only one vertex is placed in each level).

Thus, each vertex of the graph is now characterized by two numbers: the first is the basic number of the vertex and the second is the number of the level on which this vertex is placed in the graph.

Procedure 1.3

This procedure consists of a sequence of cycles. In each cycle we examine in turn each pair of vertices, based on neighboring levels: we try to shift the levels for the vertices of the pair, and if such a change increases the quantity of arcs in the graph, it is accepted; otherwise, the order of the vertices does not change. It is obvious that transposing two vertices--one from level i , and another from level $(i+1)$ --will increase the total number of arcs if the basic graph Γ does not contain an arc leading from the vertex of level $(i+1)$ to the vertex of level i , and at the same time an arc exists that leads from the vertex of level (i) to the vertex of level $(i+1)$. That arc will now be constructed in graph Γ^0 and all arcs already constructed will remain in the graph.

The procedure will stop if, during a cycle, the distribution of the vertices does not change. As a result, the sequence constructed (i.e. the acyclic graph $\hat{\Gamma}$) is locally extremum: it is impossible to increase the number of arcs in the graph when the neighboring vertices of that sequence are being transposed.

It is easy to show that, as graph $\hat{\Gamma}$ is acyclic, the elements of set \hat{D} may be re-ordered so that if $i > j$, $\hat{d}_{ij} = 0^*$. This reenumeration of the graph vertices is, in fact, connected with distribution of the vertices among the levels in graph $\hat{\Gamma}$. Such enumeration of the vertices is inverse to the enumeration of the levels: the vertex of the last level (having the number n) will now take the number 1; the basic vertex, placed on the first "lower" level, will receive the number n .

For our purposes, the presence of linear distribution of the vertices among the graph levels (only one vertex placed on each level) is not important. Let us assume in this case, that elements of set \hat{D} are ordered not only in such a manner that arcs in graph $\hat{\Gamma}$ lead from vertices with higher numbers to vertices with lower ones,

* $|| \hat{d}_{ij} ||_n^n$ is the adjacency matrix of graph $\hat{\Gamma}$ after re-ordering the elements of the set \hat{D} .

but that the hierarchy of the levels is also established; i.e. the set \hat{D} is divided into S subsets (levels), such that $D_1 \cap D_k = \emptyset$ ($1, k = \overline{1, S}; 1 \neq k$); $\bigcup_{j=1}^S D_j = \hat{D}$, and if $\hat{d}_i \in D_1$, $\hat{d}_j \in D_p$, $p > 1$, then $\hat{d}_{j_i} = 0$ and $j > i$. If $\hat{d}_i \in D_1$, $\hat{d}_j \in D_1$, then $\hat{d}_{i_j} = 0$ ($i, j = \overline{1, n}; 1 = \overline{1, S}$).

1.2 Constructing the Weights of Arcs in the Acyclic Graph

Let us call a path in graph $\hat{\Gamma}$ an ordered set of arcs $L = \{l_1, \dots, l_f\}$, where $1 \leq f \leq S - 1$; so that arc l_1 has its beginning in some vertex of the first level; arc l_f has its end in some vertex of the last level*, and the beginning of arc l_k ($1 < k \leq f$) coincides with the end of arc l_{k-1} . We now have the problem of computing the number of paths passing through each arc in the graph**.

In solving the problem, we use the following considerations. Let the arc l_{i_j} lead from the vertex \hat{d}_i to the vertex \hat{d}_j . Then the number of paths, passing through this arc is equal to the product of two numbers: the first is the number of paths entering vertex \hat{d}_i , the second is the number of paths going out from vertex \hat{d}_j . (By the path entering the vertex, we mean the path which begins in the first level and ends in that vertex; by analogy, the path leaving the vertex--the path which begins in that vertex and ends in the last level). Thus, the problem arrives at the construction of two files of one dimension where information about the number of entering and leaving paths for each vertex is placed.

Let G be a file containing the information about entering paths, and file U for the ones leaving. The dimension of G is equal to $n-m$, where m is the number of vertices in the first level (due to the distribution of vertices among levels, the vertices of the first level have first m numbers). It is clear that information about the number of paths entering the vertices of the first level

* In our graph this is the basic vertex.

** This task was originally formulated in [2].

is not useful because no arc enters such a vertex. By analogy, the dimension of file u is $n-v$, where v is the number of vertices in the last level.* If $G(i) = Q$, it means that Q paths enter the vertex \hat{d}_{i-m} of graph $\hat{\Gamma}$. If $U(i) = p$ it means that p paths leave the vertex \hat{d}_i .

For constructing the two files G and U , we travel from the matrix $\|\hat{d}_{ij}\|_n^n$ to the matrices $\|g_{ij}\|_{n-v}^t$ and $\|u_{ij}\|_{n-m}^w$, where t and w are the maximum numbers of arcs, respectively going out from or entering one vertex in graph $\hat{\Gamma}$. Matrix $\|g_{ij}\|$ is the matrix of arcs going out, if $\hat{d}_{kp} = 1$, i.e. in the graph an arc exists which leads from \hat{d}_k to \hat{d}_p ($p > k$), then in row k of matrix $\|g_{ij}\|$ the number v can definitely be found. By analogy, the arc $1k_p$ corresponds with the existence of number k in the row $(p-m)$ in the matrix $\|u_{ij}\|$, i.e. $\|u_{ij}\|$ contains the information about the arcs, which protrude from any vertex. (Displacement on m is connected with the absence of entered arcs for the first m vertices). The absence of zeros between the elements of matrices $\|g_{ij}\|$ and $\|u_{ij}\|$ is also required.

Thus any row of matrix $\|g_{ij}\|$ (for example, with number p) is constructed by the following algorithm. From the first column, elements of the row are the numbers of those vertices of the graph $\hat{\Gamma}$, for which arcs exist, leading from \hat{d}_p to them. Let t_p be a number of such vertices. If $t_p < t$, then a part of the row p is not filled. It is filled by zeros. By analogy, the rows of the matrix $\|u_{ij}\|$ is constructed in the following way: the elements of the row are the numbers of such graph vertices that the arcs leading from them, end in \hat{d}_{1+m} . The rest of the row is filled by zeros.

Graph $\hat{\Gamma}$ can be set first by one or both of the matrices

* We make an abstraction of the fact that in our graph $\hat{\Gamma}$ only one vertex is placed in the last level, i.e. $v = 1$

$\| g_{ij} \|$, $\| u_{ij} \|$ or by two files H and K, whose dimensions are equal to the number of arcs in graph $\hat{\Gamma}$. The file H represents numbers of vertices, at the beginning of the arc. The file U contains numbers of vertices, at the end of the arc. So, if $H(i) = p$ and $K(i) = q$, it means that $\hat{d}_{pq} = 1 (p, q = \overline{1, n}; p < q)$. Then, by examining the files H and k it is easy to construct both matrices $\| g_{ij} \|$ and $\| u_{ij} \|$.

In constructing the file G on the base of matrix $\| g_{ij} \|$ we consider that the number of paths entering any vertex in the graph is equal to the sum of numbers of the paths entering such vertices in the graph, which have arcs entering the given vertex. So, $G(i) = \sum_j G(j)$, where j is such, that $\hat{d}_{ji} = 1$.

If the arc l_{ji} begins in the first level, then the number of paths, added by it, is naturally equal to one.

By analogy, for computing the number of paths, which go out from some vertex, we summarize the number of paths, which protrude from all the vertices in the graph in which the arc from the given vertex comes.

$$U(i) = \sum_j U(j), \text{ where } \hat{d}_{ij} = 1$$

If the arc l_{ij} has it's end in a vertex of the last level, then the number of paths, which it adds in the sum, is equal to one.

Thus, for constructing file G, we examine in turn all rows of the matrix $\| g_{ij} \|$, beginning from the first one. If in some row with the number p the number q is found then there are two possibilities: 1. $p \leq m$. Then $G(q-m)$ is increased by one.

2. $p \geq m$. Then $G(q-m)$ is increased by $G(p-m)$.

By analogy the file U is constructed. We examine all rows of the matrix $\| u_{ij} \|$, beginning from the last one. If, in row p of the matrix $\| u_{ij} \|$ the number q appeared (it means that the arc $l_{q, p-m}$ exists in the graph) there are again two possibilities:

1. $p \geq n-v-m$. Then $U(q)$ is increased by one
2. $p < n-v-m$. Then $U(q)$ is increased by $U(p+m)$

Here the first case corresponds with the presence of the vertex \hat{d}_{p-m} is the last level of the graph.

After constructing the files G and U , for computing the quantity of paths lying across an arc, we only have to multiply appropriate elements of those files. As a result, the matrix $\|a_{ij}\|$ (matrix of the weighted graph $\hat{\Gamma}^1$ corresponding with the acyclic graph Γ), will be constructed as follows:

$$a_{ij} = \begin{cases} 0, & \text{if } \hat{d}_{ij} = 0 \\ G(i-m) \cdot U(j), & \text{if } i > m; j < n-v; \hat{d}_{ij} = 1 \\ U(j) & \text{if } i \leq m; j < n-v; \hat{d}_{ij} = 1 \\ G(i-m), & \text{if } i > m; j \geq n-v; \hat{d}_{ij} = 1 \\ 1, & \text{if } i \leq m; j \geq n-v; \hat{d}_{ij} = 1 \end{cases}$$

For each arc of the graph $\hat{\Gamma}$, and thus for each pair of vertices (\hat{d}_i, \hat{d}_j) , the number \hat{d}_{ij} serves as a measure of their influence on the basic vertex in the graph. The presence of such a measure in the graph allows us to pick out the set of vertices, "strongly connected" with the basic vertex.

1.3 Isolating the Set of Vertices, Strongly Connected with the Basic Vertex

The problem of isolating the set of vertices, strongly connected with the basic vertex \hat{d}_n can be formalized by employing the method of the graph's approximation [3]. Conforming to the given task, this method calls for the pursuit of a subset $R \subset \hat{D}$, such that links of the vertices from R with the basic vertex \hat{d}_n are similar, and in some sense essential, i.e. do not differ strongly from some value λ . Thus the problem can be formulated as a problem of a minimization functional (1). In the following we will identify the indices of the elements of the set as well as the elements themselves.

$$(1) \quad J_1 = \sum_{i,j=1}^n (a_{ij} - r_{ij}\lambda)^2$$

where

$$r_{ij} = \begin{cases} 1, & \text{if } i \in R, j = n \\ 0, & \text{otherwise} \end{cases}$$

From (1) it is clear that link of any vertex $i \in R$ with the basic vertex (n) is approximated by λ , and all the other links in the graph $\hat{\Gamma}^1$ are approximated by zero. Thus from (1), our purpose is to solve the following problem:

$$\min_{R, \lambda} J_1 (R, \lambda)$$

By fixing a subset $R^* \subset \hat{D}$, it is not difficult to define the value λ , which minimizes the functional J_1 .

$$\frac{\partial J_1}{\partial \lambda} = -2 \sum_{i,j}^n r_{ij} (a_{ij} - \lambda r_{ij}) = -2 \sum_{i \in R^*}^n (a_{in} - \lambda) = 0$$

It gives:

$$\sum_{i \in R^*} a_{in} = m_{R^*} \lambda,$$

where m_{R^*} is the number of elements in subset R^* , and

$$(2) \quad \lambda = \frac{\sum_{i \in R^*} a_{in}}{m_{R^*}}$$

From (2) it is clear, that λ is the average value of weights of the arcs, which link the vertices from R^* with the basic vertex n .

Taking (2) into consideration, we can obtain more simple (and time saving) expression for the function J_1 :

$$\begin{aligned}
 J_1 &= \sum_{i,j=1}^n (a_{ij} - r_{ij}\lambda)^2 = \sum_{i,j=1}^n (a_{ij}^2 - 2r_{ij}\lambda a_{ij} + \lambda^2 r_{ij}^2) = \\
 &= \sum_{i,j=1}^n a_{ij}^2 - 2\lambda \sum_{i \in R} a_{in} + \sum_{i \in R} \lambda^2 = \sum_{i,j=1}^n a_{ij}^2 - 2\lambda \cdot m_R \lambda + \\
 &+ m_R \lambda^2 = C - m_R \lambda^2
 \end{aligned}$$

Here $c = \sum_{i,j=1}^n a_{ij}^2$ - a constant (for given matrix A)

so the problem of J_1 minimization is equivalent to the problem of maximization of the functional $J_2 = m_R \lambda^2$ (where λ is defined according to (2)):

$$\min_{R, \lambda} J_1(R, \lambda) \Leftrightarrow \max_R J_2(R)$$

We solve the problem of J_2 maximization by using the algorithm of local optimization [3].

The algorithm begins with fixing some subset $R^* \subset \hat{D}$, randomly defined (but such, that $n \notin R^*$). The algorithm consists of a sequence of cycles, each cycle includes $(n-1)$ steps. On step $i (i = \overline{1, n-1})$ we try to include element i to R (if that element did not previously belong to R), or to exclude element i from R (if that element belonged to R). If such a step leads to the positive increase of the functional, such a modification is accepted; otherwise the subset R does not change. The algorithm is terminated when during the course of a cycle, the subset R did not change at all. The subset R , constructed as a result, corresponds with the local extremum of the functional J_2 . Convergence of the algorithm for the finite number of steps is guaranteed by the finite number of elements in the set \hat{D} .

Note that according to the local character of the modification of the subset R on each step, the increase in the value of the functional may be calculated economically, with the use of recursive dependencies. Let i be a step of the algorithm, then the increase in the value of the functional J_2 on the step i can be computed as:

$$\Delta J_2 = J_2(i) - J_2(i - 1) = (m_R \pm 1) \lambda(i) - m_R^2 \lambda(i - 1)$$

Here the sign "+" (or "-") corresponds to the case, when the number of elements of set R is raised (or reduced) on step i by 1.

It is easy to see, that the value of $\lambda(i)$ can be computed with the help of the following recurring formula:

$$(3) \quad \lambda(i) = \alpha(i) \lambda(i - 1) + \beta(i),$$

where

$$\alpha(i) = \frac{m_R \lambda(i - 1)}{m_R \pm 1}; \quad \beta(i) = \frac{\pm a_{in}}{m_R \pm 1}$$

Taking into consideration the economy of computation provided by formula (3), realization of the algorithm requires of the order $O(n)$ operations:

The method described above leads to the fixing of the kernel of the vertices \hat{R} , in which only vertices directly connected with the basic vertex, are included (but not all such vertices, only those, which are "strongly connected" with the basic vertex). For the purpose of the qualitative analysis of concrete global models, it is essential to define other vertices not directly connected with the basic vertex, but indirectly strongly influencing it (i.e. through vertices from \hat{R}). Moreover, it is interesting to define the indirect links of "second order", "third order" etc., (that is to define the number of unintersected subsets of vertices, setting the hierarchy of indirect links of influence on the basic vertex).

The formalization of such a problem can be made on the basis of a process, where the procedure of kernel \hat{R} construction, is modified in each stage. In such a modification, the first stage of the process is the same as the procedure, used to construct set \hat{R} (let us call it \hat{R}_1 in the following).

Let stage t be completed, i.e. let us assume, that the set defining the indirect links of "t-order", is constructed. Then the aim of stage $(t + 1)$ is the construction of the set $\hat{R}_{(t + 1)}$ which defines the indirect links of "order $t + 1$ ", i.e. the construction of a set of vertices which are strongly connected with the vertices from \hat{R}_t . According to the method of graph approximation, this task can be formulated as the problem of minimization of the functional;

$$(1') \quad J_1' = \sum_{i,j=1}^n (a_{ij} - r_{ij} \lambda)^2,$$

where

$$r_{ij} = \begin{cases} 1, & \text{if } j \in \hat{R}_t, i \in R_{t+1} \\ 0, & \text{otherwise} \end{cases}$$

It is easy to show, that for R_t and R_{t+1} fixed, the optimal value of λ is defined as an average weight of links, leading from the vertices of set R_{t+1} to the vertices of set R_t .

$$(2') \quad \lambda = \frac{\sum_{i \in R_{t+1}} \sum_{j \in R_t} a_{ij}}{m_t m_{t+1}}$$

where m_t and m_{t+1} are numbers of elements of the sets R_t and R_{t+1} , respectively.

From (2') it is not difficult to show, that the problem of minimization of functional J_1' is equivalent to the problem of maximization of the functional J_2' :

$$J_2' = \lambda^2 m_t m_{t+1};$$

$$\min_{R, \lambda} J_1'(R, \lambda) \Leftrightarrow \max_R J_2'(R)$$

For the solution of that problem, the algorithm of local optimization, analogous to the one stated above, is used. Set \hat{R}_t , constructed on the preceding step, is fixed (as the basic vertex n was fixed previously), and a search for set \hat{R}_{t+1} , which maximizes the functional J_2' is made. The recurring formulas (3) for λ definition are modified as:

$$\alpha(i) = \frac{m_{t+1} * \lambda (i - 1)}{m_{t+1} \pm 1}; \quad \beta(i) = \frac{\pm \sum_{i \in \hat{R}_t} a_{ij}}{m_t (m_{t+1} \pm 1)}$$

The process of constructing the sequence $\{\hat{R}_t\}$ will stop if, after the realization of step s , the constructed set \hat{R}_s is empty*:

$$\hat{R}_s = \emptyset$$

The vertex \hat{d}_i is included in the kernel of the basic vertex only when, the set \hat{R}_j such, that $\hat{d}_i \in \hat{R}_j$ ($1 \leq j \leq s - 1$) exists. A characteristic of the vertex is not only its presence in the kernel, but also the number of the set from the sequence $\{\hat{R}_t\}$, to which that vertex belongs.

* This means, that the set \hat{R}_{s-1} consists of exogenous vertices

Part II: THE FORMATION OF GRAPH MODELS

The methods suggested in the present work appeared in the analysis of three global models. These are: (a) Model World 3 (the Meadows model) constructed at M.I.T. in 1972; (b) the model constructed by the team of investigators under the supervision of M. Mesarovic and E. Pestel in 1974 (the Mesarovic model), from which we took only the agricultural submodel; (c) MOIRA (the Linnemann model), which is being constructed today in The Netherlands.

In the process of our work with each model, publications [4], [5] and [6] were used.

2.1 Brief Characteristics of the Models

The three models are similar as well as different. Thus, from the very title of the Linnemann model (Model of International Relations in Agriculture), its agricultural emphasis becomes evident whereas the Meadows and Mesarovic models do not show such a clear course. The Meadows model consists of five interwoven submodels (agriculture, population, capital, persistent pollution, natural resources), while in the Mesarovic model the submodels (economics, population, food, energy, environment) are not at all connected and division into interacting sections exists within each one. The Mesarovic and Linnemann models are regionalized whereas the Meadows model presupposes the absence of division of land into regions. The Meadows model is closed, i.e. there are no exogenous inputs, while in the Linnemann and Mesarovic models these exogenous inputs, allowing play over the different variants of development at each stage of the modelling work exist.

In the Meadows model the unit of dimension for food production is vegetable-equivalent kilogram while in the Mesarovic and Linnemann models the food production is measured by kilograms of consumable protein.

The similarity and the difference in models could be clearly followed in the construction of the graph models. The construc-

tion of graph models is a non-formal process, besides the degree of accounting for qualitative factors differs for the different models. Thus, for the Meadows and Mesarovic models the equations are solved according to the left side, i.e. for each variable the set of other variables influencing it can be found simultaneously and uniquely. In the Linnemann model such a solution is absent. Here variables are connected either with the set of equations ([6] p. 240), unsolved according to their left side or, the value of variables can be found by solving the optimization problem ([6], p. 161), where it is necessary to maximize a certain function (the income) with particular restrictions.

This absence of equation solution in the Linnemann model demanded that the authors of the present work prepare the preliminary qualitative analysis of the system which came before the graph model construction (it is obvious for instance that for the system of equations 5.18-5.20 ([6] p. 240) where the equations, in accordance to the left side are not solved, it is possible to construct several structural graphs).

The necessity for qualitative analysis in constructing the graph model also became clear from the question of including certain indicators in a graph model (which will be discussed later). There was also an opportunity of constructing new indicators on the basis of the global models. In the Meadows and Mesarovic models there is an indicator characterizing the food production per capita. We took this indicator as the basic and main indicator which, at the same time, is lacking from the Linnemann model (in the chapters we analysed). We constructed this indicator on the basis of the model (TYPK, Appendix I, List of the Linnemann Graph Model Indicators No. 35), which provided us with the possibility of carrying out similar experiments with each model according to the methods described above. In general, the process of graph model construction can be divided into two parts: (a) constructing the set of vertices of each graph (the list of indicators of the graph model); (b) constructing the set of arcs (the structural scheme of the model).

2.2 Construction of the Set of Vertices of the Graph Model

In constructing the set of vertices of the graph model, it seemed to be necessary to find out which global model indicators should be included in the graph model. It is essential to point out that the authors of the models make very much the same distinction between the indicators. Thus, in the Mesarovic model all the indicators are divided into three groups: the first consists of 'variables'; the second, 'parameters'; and the third, the smallest of the 'scenario variables'. In this case, group 2 'parameters' includes indicators which do not change in value during the process of the model's work (in spite of indicators' economic content). These are: depreciation rates, basic year's prices on the same products, coefficients of protein content in these products, and the rapid coefficients of some equations. Group 1 'variables', includes all the indicators whose value changes in the process of the work. Altogether with the indicators which change are of interest, here are included the indicators constructed only for the convenience of recording the model's equations (to avoid the overloaded recordings) and which play an auxiliary role. (For example, [5] p. B-576-- multiplier connecting the growth of mortality with protein deficiency (PROPCN)). The third group, 'scenario variables' comprises indicators for which the values are changed by the investigator at each stage of the work. (These are the controlling influences or exogenous variables, allowing play over the different variants of world evolution. According to the words of the model's authors, the placing of some indicators in the group 'parameters' instead of the group 'scenario variables' is connected only with imperfection of the model with its incapability to take into account certain factors ([5] p. B-575, the indicator MORT -- the coefficient of mortality). It is clear that in the Mesarovic model the division of indicators into three groups is mainly underlined not by qualitative economic considerations but by 'model' considerations connected first of all with the convenience of reading the work and making experiments with the model and secondly by a certain lack of knowledge of the real world.

Such division of the model components can be easily made in Meadows model. The Meadows model is closed, i.e. it has no exogenous inputs. At the same time, there are many parameters in the model that can be divided into three parts: the first is constituted by the coefficients of the model's functions; the second includes the values of all the variables in the basic year; and the third is the evaluation of the earth's resources (for instance, potentially arable land). The rest of the model indicators are just variables for which the values can be computed endogenously.

In constructing the graph model of the Linnemann model we used publication [6] which is the preliminary edition of the work. This version does not contain certain chapters (notably the two with some model equations) and therefore the graph model we constructed cannot be considered as complete. At the same time, the indicators in the Linnemann model (as well as in that of Mesarovic) were divided into 'parameters' and 'variables' by the authors themselves. (The sense of this concept has been seen in the Meadows and Mesarovic models.) We should like to point out that in the Linnemann model (as well as in Meadow), in comparison with Mesarovic, there are fewer variables constructed only according to some inner 'model' considerations, without clear economic content (though there are some of that kind in [6] p. 219, indicator GPO--annual rate of increase in the level of food processing).

It is clear that including the whole set of global model indicators into the system of graph indicators is senseless, because there would be too many elements in the graph which could not be economically interpreted. According to the methods described above, recognizing the role of these economically uninterpretable indicators in their influence over the basic indicator may be of some interest, but yet it seems to be more important to indentify some economically meaningful elements of the model, most strongly connected with the indicator marked beforehand. Here qualitative

analysis is required: when recognizing that a certain indicator is interesting for the investigator, then that indicator has to be included in the list of graph indicators (one more vertex appears in the graph); if not, it does not. In our work, we included in the set of graph indicators all the "variable" indicators only. Certainly among the graph model indicators, some can be found whose use is questionable in common economic practice. At the same time, indicators which can be easily interpreted (as some evaluations of natural resources, for example) do not belong to the system of graph indicators. Nevertheless, we suppose that if the set of graph model indicators is constructed with the use of some qualitative analysis, the set would not differ greatly from the one which we constructed.

Thus, the set of graph model indicators is equivalent to the set of variable indicators of the global models (both endogenous and exogenous. Some exceptions to that rule occur in the Linnemann model and are indicated below.

Lists of indicators of the graph models are given in the Appendix.

The graph model of the Meadow model consists of 104 elements. Indicators from 1 to 39 refer to the agricultural submodel of World 3 (in which we were mainly interested). Here, only the first 35 indicators belong to the agricultural submodel itself, indicators from 36 to 39 are the exogenous inputs to the submodel from other submodels in the system (36-POP-population; 37-IOPC-industrial output per capita; 38-IO-industrial output; 39-PPOLX-index of persistent pollution). Indicators 40-61 refer to the submodel "capital"; indicators 62-69 refer to the submodel "persistent pollution" and lastly, indicators 100-104 belong to the resources submodel of the Meadows model. The enumeration of the elements in each submodel corresponds with the order to their appearance in [4]. (The only exceptions are the exogenous inputs to the agricultural submodel-indicators 36-39, which are included in the agricultural part of the system, because this area holds our main interest).

The set of indicators of the Linnemann model, included in the graph model which we constructed, consists of 35 elements. Their enumeration also corresponds, as a rule, with the order of the indicators appearing in [6]. Also, the first indicator in our set--MPDMI--maximum production in dry-matter units (with the help of artificial irrigation) is included in the list despite the general practice of including only variable indicators of global models. The same can be said about indicators EF, UCCL and some others (Appendix, graph model of Linnemann

10 and 14). The decision to include certain indicators in the set was based on various qualitative considerations and on comprehension of the fact that the model has not yet been completed. We constructed the indicator TYPK (Nr.35 in the Appendix), which was later chosen as the basic indicator to provide the feasibility of conducting experiments with the Linnemann model, similar to those done with the Meadows and Mesarovic models.

The set of graph model indicators in the Mesarovic model consists of 136 elements. We note that some equations and the names of some indicators have been omitted in [5]. (For example, p. B-596--the list of variables is absent; p. B-589--the equation for variable CLWGR is missing.) Therefore, we gave names to some indicators of the model and these names may differ from those given by the authors of the global model. On the whole, the enumeration of the indicators in our list corresponds completely with the order of indicators appearing in [5].

2.3 Construction of the Set of Arcs of the Graph Model

After constructing the set of indicators for each graph model (vertices of the graph) a question arose concerning the direct construction of the graph models themselves (i.e. the question of definition for each vertex of the graph and the set of vertices connected with it). This work could be easily prepared for the Meadows and Mesarovic models (except for difficulties where there were omissions in [5]). For each vertex of the graph, the corresponding equation was found and then those elements of the set of indicators (vertices of the graph), to be

placed on the right side of that equation, were defined. From such vertices we constructed arcs to the vertex given, corresponding to the indicator placed on the left side of the equations.

It was impossible to construct the graph model for the Linnemann model as easily as for the other two due to the absence, for some equations, of the solution to the left side of the equation (this problem was discussed earlier). The preliminary character of publication [6] sometimes led to several different recordings of one equation. Our work is based mainly on qualitative considerations. Particularly in analyzing the optimization model 4.17-4.23 ([6], p. 161), the price indicators (P, CMON, FMON) were assumed to beogenous and to influence the indicator CE (capital use) only, which defines the value of the maximized function. The indicators Y and F (numbers 2 and 3 in our list) were assumed to be dependent on indicator CE (taking into account the problem constraints). It is clear that the preliminary nature of the particular Linnemann model we used provides possibilities for another construction of the model's structural scheme.

It is essential to point out the similarity of the Meadows and Mesarovic models and their difference from the Linnemann model. The graph model of the Meadows model consists of 175 arcs (i.e. approximately 1.75 arcs per vertex). The greatest density is in the agricultural sector of the model and in the sector "capital". In analyzing the adjacency matrix of the Meadows model's graph, the vast number of units, situated near the main diagonal, is prominent. Such a matrix structure is connected with the consistent enumeration of the model's indicators according to their appearance in [4] and with the fact that the majority of indicators are employed only in one of the system's equations (there are 69 such indicators from a total of 104 in the Meadows model). The greatest number of links have those indicators which connect different submodels of the system and the indicator AL (No. 2 in our list of indicators). The greatest number of arcs going out from one vertex is thirteen, this vertex corresponds with the indicator POP (population--No. 36 in our list). Nine arcs going out from the vertex corresponding to the indicator IOPC (No. 37 in the list).

A similar structure of the adjacency matrix of the graph takes place in the graph model of the Mesarovic model. There are 218 arcs in the graph; its density is almost 1.6. Similar to the Meadows model, the greatest number of arcs go out from the vertex corresponding to the indicator POP (No. 1 in the list of indicators for the Mesarovic model). At the same time, in the Mesarovic graph model there are many vertices which do not have arcs going out. These variables are only computed to provide some additional information and do not play any role in defining the behaviour of other indicators (for example, indicators No. 45 or No. 132 in our list).

The graph model of Linnemann model differs from the Meadows and Mesarovic graph models because of its greater density. There are 68 arcs in the graph, i.e. approximately two arcs for each vertex. In the graphs of the Meadows and Mesarovic models there is no vertex which is entered by more than four arcs. In the Linnemann model, the maximum number of arcs entering one vertex is equal to eight. It is interesting to see the similarity to the two other models: in the Linnemann model the greatest number of arcs go out from the vertex, corresponding with the indicator characterizing population (although this is only agricultural population). This indicator is L--No. 15 in the list of the Linnemann model indicators.

The graph models of all three models can be easily reconstructed on the basis of the Appendix. For each model the list of indicators is constructed in the following way. All the indicators are ordered according to their numbers. For each indicator, its name and list of indicators which influence it in the model are given in the Appendix. Thus, the list of indicators consists of four columns. In the first column is the number of the indicator in the graph model; in the second is the list of influencing indicators; in the third, the abbreviated name of that indicator; and in the fourth the full name of the indicator in the global model. For those indicators whose dimension could be found in the global model, that dimension is quoted.

APPENDIX

Lists of Graph Model Indicators

I. <u>The Meadows Model</u>			
1.	2	LFC	Land fraction cultivated (dimensionless)
2.	2,4,5,6	AL	Arable land (hectares)
3.	3,4	PAL	Potentially arable land (hectares)
4.	10,12,13	LDR	Land development rate (hectares/year)
5.	5,25	LER	Land erosion rate (hectares/year)
6.	28,29	LRUI	Land removal for urban-industrial use (hectares/year)
7.	2,19	F	Food (vegetable-equivalent kilograms/year)
8.	7,36	FPC	Food per capita (vegetable-equivalent kilograms/person-year)
9.	37	IFPC	Indicated food per capita (vegetable-equivalent kilograms/person-year)
10.	11,38	TAI	Total agricultural investment (dollars/year)
11.	8,9	FIOAA	Fraction of industrial output allocated to agriculture (dimensionless)
12.	3	DCPH	Development cost per hectare (dollars/hectare)
13.	22,23	FIALD	Fraction of inputs allocated to land development (dimensionless)
14.	10,13	CAI	Current agricultural inputs (dollars/year)
15.	14	AI	Agricultural inputs (dollar/year)
16.	2,15,17,	AIPH	Agricultural inputs per hectare (dollar/hectare-year)
17.	34	FALM	Fraction of investment allocated to land maintenance (dimensionless)

18.	16	LYMC	Land yield multiplier from capital (dimensionless)
19.	18,20,21	LY	Land yield (vegetable-equivalent kilograms/hectare-year)
20.	20,30,31	LFERT	Land fertility (vegetable-equivalent kilograms/hectare-year)
21.	38	LYMAP	Land yield multiplier from air pollution (dimensionless)
22.	12,19	MPLD	Marginal productivity of land development (vegetable-equivalent kilograms/dollar)
23.	18,19,24	MPAI	Marginal productivity of agricultural inputs (vegetable-equivalent kilograms/dollar)
24.	16	MLYMC	Marginal land yield multiplier from capital (hectares/dollar)
25.	26	ALL	Average life of land (years)
26.	19	LLMY	Land life multiplier from yield (dimensionless)
27.	37	UILPC	Urban-industrial land per capita (hectares/person)
28.	27,36	UILR	Urban-industrial land required (hectares)
29.	6,29	UIL	Urban-industrial land (hectares)
30.	20,33	LFR	Land fertility regeneration (vegetable equivalent kilograms/hectare-year-year)
31.	20,32	LFD	Land fertility degradation (vegetable-equivalent kilograms/hectare-year-year)
32.	39	LFDR	Land fertility degradation rate (1/year)
33.	17	LFRT	Land fertility regeneration time (years)
34.	35	FPR	Perceived food ratio (dimensionless)
35.	38	FR	Food ratio (dimensionless)
36.	36,62,63	POP	Population (persons)

37.	36,38	IOPC	Industrial output per capita (dollars/person-year)
38.	44,49,103	IO	Industrial output (dollars/year)
39.	94	PPOLX	Index of persistent pollution (dimensionless)
40.	16	JPH	Jobs per hectare (persons/hectare)
41.	2,40	PJAS	Potential jobs in service sector (persons)
42.	44	ICDR	Industrial capital depreciation rate (dollars/year)
43.	44,45	PJIS	Potential jobs in industrial sector (persons)
44.	42,44,46	IC	Industrial capital (dollars)
45.	37	JPICU	Jobs per industrial capital unit (persons/dollar)
46.	38,47	ICIR	Industrial capital investment rates (dollars/years)
47.	11,54	FIOAI	Fraction of industrial output allocated to industry (dimensionless)
48.	41,43,60	J	Jobs (persons)
49.	51	CUF	Capital utilization fraction (dimensionless)
50.	37	ISOPC	Indicated service output per capita (dollars/person-year)
51.	56	LUFD	Labor utilization fraction delayed (dimensionless)
52.	36,53	SOPC	Service output per capita (dollars/ person-year)
53.	49,58	SO	Service output (dollars/year)
54.	50,52	FIOAS	Fraction of industrial output allocated to services (dimensionless)
55.	38,54	SCIR	Service capital investment rate (dollar/year)
56.	48,59	LUF	Labor utilization fraction (dimensionless)
57.	52	JPSCU	Jobs per service capital unit (persons/dollar)

58.	50,58,61	SC	Service capital (dollars)
59.	36	LF	Labor force (persons)
60.	57,58	PJSS	Potential jobs in service sector (persons)
61.	58	SCDR	Service capital depreciation rate (dollars/year)
62.	36,80	B	Births per year (persons/year)
63.	36,64	D	Deaths per year (persons/year)
64.	66,67,68, 69	LE	Life expectancy (years)
65.	36,63	CDR	Crude death rate (deaths/1000 persons-years)
66.	39	LMP	Lifetime multiplier from persistent pollution (dimensionless)
67.	8	LMF	Lifetime multiplier from food (dimensionless)
68.	70,71	LMC	Lifetime multiplier from crowding (dimensionless)
69.	72	LMHS	Lifetime multiplier from health services (dimensionless)
70.	36	FPU	Fraction of population urban (dimensionless)
71.	37	CMI	Crowding multiplier from industrialization (dimensionless)
72.	73	EHSPC	Effective health services per capita (dollars/person-year)
73.	52	HSAPC	Health services allocations per capita (dollars/person-year)
74.	36,62	CBR	Crude birth rate (births/1000 person-years)
75.	64	FM	Fecundity multiplier (dimensionless)
76.	64	PLE	Perceived life expectancy (dimensionless)
77.	76	CMPLE	Compensory multiplier from perceived life expectancy (dimensionless)
78.	75	MTF	Maximum total fertility (dimensionless)

79.	78,85	NFC	Need for fertility control (dimensionless)
80.	78,81,85	TF	Total fertility (dimensionless)
81.	82	FCE	Fertility control effectiveness (dimensionless)
82.	83	FCFPC	Fertility control facilities per capita (dollars/person-year)
83.	52,84	FCAPC	Fertility control allocations per capita (dollars/person-year)
84.	79	FSAFC	Fraction of services allocated to fertility control (dimensionless)
85.	77,86	DTF	Desired total fertility (dimension- less)
86.	87,88	DCF	Desired completed family size (dimensionless)
87.	91	SFSN	Social family size norm (dimension- less)
88.	89	FRSN	Family response to social norm (dimensionless)
89.	37,90	FIE	Family income expectation (dimensionless)
90.	37	AIOPC	Average industrial output per capita (dollars/person-year)
91.	37	DIOPC	Delayed industrial output per capita (dollars/person-year)
92.	93,94	PPASR	Persistent pollution assimilation rate (pollution units/year)
93.	95	AHL	Assimilation half-life (years)
94.	92,94,96	PPOL	Persistent pollution (pollution years)
95.	39	AHLM	Assimilation half-life multiplier (dimensionless)
96.	97	PPAPR	Persistent pollution appearance rate (pollution units/year)
97.	98,99	PPGR	Persistent pollution generation rate (pollution units/year)

98.	36,100	PPGIO	Persistent pollution generated by industrial output (pollution units/year)
99.	2,16	PPGAO	Persistent pollution generated by agricultural output (pollution units/year)
100.	37	PCRUM	Per capita resource usage multiplier (resource units/person-year)
101.	36,100	NRUR	Non-renewable resources usage rate (resource units/year)
102.	101,102,	NR	Non-renewable resources (resource units)
103.	104	FCAOR	Fraction of capital allocated to obtaining resources (dimensionless)
104.	102	NRFR	Non-renewable resource fraction remaining (dimensionless)

II The Mesarovic Model

1.	2,6	POP	Total population
2.	2,4,	AP(I)	Population, by age category
3.	2	BABIES	Number of live Births
4.	2,11	DN(I)	Number of deaths by age category
5.	3,12	DN(O)	Infant mortality
6.	3,5	AP(O)	People, aged 0-0,5 years, after infant mortality has been taken into account
7.	Exog.	PRODST*	Protein distribution factor
8.	7,9	PROPCI	Daily per capita protein consumption
9.	105	PTPCR	Annual protein per capita produced regionally
10.	Ex	E	Multiplier denoting sensitivity, by age group, to protein deficiency
11.	8,10	AMPF(I)	Effects of protein starvation on mortality
12.	8	AMPF(O)	Effects of protein starvation on mortality in the age group 0-0,5

13.	4	DCHLD	Total child deaths, ages 0-15
14.	1,3	CBR	Crude birth rate
15.	1	CDR	Crude death rate
16.	1,3	POPGR	Population growth rate
17.	17,18	KA	Capital stock, agricultural sector
18.	35,38,32	IA	Investment, agricultural sector
19.	17	YAX	Agricultural output, computed within the economic stratum
20.	20,22	KNA	Capital stock, non-agricultural sector
21.	17	KDA	Amount of depreciation, agricultural sector
22.	18,35	INA	Investment, non-agricultural sector
23.	20	KDNA	Amount of depreciation, non-agricultural sector
24.	36,125	Y	Gross regional product
25.	36,125	Z (1)	Gross output, agricultural sector
26.	36,215	Z (2)	Gross output, non-agricultural sector
27.	25,26	U (1)	Intermediate demand, agricultural sector
28.	26	U (2)	Intermediate demand, non-agricultural sector
29.	26	UA	Intermediate demand from agriculture
30.	29	UAF	Total expenditures on fertilizer and related productive factors
31.	Ex	IAKS*	Shift of investment of agricultural sector
32.	Ex.	K1*	Fraction of investment to agricultural capital stock
33.	Ex	IAPK*	Investment in agricultural production coefficient
34.	Ex	IALVK*	Investment in livestock, coefficient
35.	36,37	I	Total investment
36.	20	YNA	Regional product, non-agricultural sector

37.	24,36	SYSYNA	Ratio of GRP to regional product, non-agricultural sector
38.	35,31	IAS	Amount of investment, shifted from agricultural sector to non-agricultural sector
39.	35,40	IMN	Imports of investment capital needed
40.	35	IR	Regional investment
41.	18,33	IAP	Investment in non-land agricultural capital stock
42.	18,34	IALV	Investment in livestock development
43.	18,41,42	IALD	Investment in land development
44.	24	C	Consumption
45.	24	G	Governmental expenditures
46.	24	M	Imports
47.	46	MA	Imports, agriculture
48.	46,47,49	MC	Imports for consumption
49.	39,46,47	MI	Imports, investment
50.	50,51,52	CLGR	Cultivated land, grain
51.	50,54,66	CLWGR	Cultivated land withdrawal, grain
52.	50,54,63	CLWGR	Cultivated land withdrawal, grain
53.	50	CLNG	Cultivated land, non-grain
54.	50,53	CL	Cultivated land
55.	56	GL	Grazing land
56.	60,64	GLW	Grazing land withdrawal for urbanization and economic development
57.	55,58	TLLS	Total land for livestock support
58.	54,64	CLR	Cultivable land remaining
59.	60	TLWR	Ratio of land withdrawn to maximum total land
60.	60,62	TLW	Total land withdrawn for urbanization and economic development

61.	59	TLWM	Land withdrawal multiplier
62.	3,5,61	TLAW	Annual withdrawn of land for urbanization and economic development
63.	62	CLAW	Annual withdrawal of cultivated land
64.	60	CLW	Total cultivable land withdrawn
65.	54,64	FCLR	Fraction of cultivated land remaining
66.	43,67	CLD	Cultivated land developed
67.	65	KCLDH	Capital cost of land development per hectare
68.	53,54,66	CLDNG	Cultivated land developed, non-grain
69.	70	PMCI	Productivity coefficient from infrastructure
70.	1,36	YNAPC	Regional product, non-agricultural, per capita
71.	17,54	KAPH	Agricultural capital, per hectare
72.	69,73	FA	Saturation level for grain production
73.	71	PTFC	Productivity coefficient from capital investment
74.	50,75,82	ZPHG	Use, per hectare, of fertilizer and related productive factors
75.	30,76	TPF	Total use of fertilizer and related productive factors
76.	76	PXPF	Price of fertilizer and related productive factors
77.	72	TEMP	Intermediate value used in computation of productivity
78.	72,74,77	GRPH	Grain production per hectare
79.	50,78	GRGP	Gross production, grain crops
80.	79	NGGP	Gross production, non-grain crops
81.	57	SLVMA	Total livestock supportable on available grazing land

82.	87	SLVP (J)	Meat production from livestock, by type
83.	89	SLVA	Total livestock in animal units
84.	81,83	SLVAR	Total livestock, animal use ratio
85.	84	LVPLM	Livestock, price land multiplier
86.	42,89,83	UALV (J)	Investment in additional livestock, by type
87.	86,88	ALVI (J)	Additional livestock, by type
88.	85	LVPL (J)	Development capital cost per livestock
89.	89,87	SLV (J)	Total livestock, by type
90.	95	AWFM	Additional marine fish production
91.	96	AUFWP	Additional land in pond culture
92.	96	FWCP	Pond fish production
93.	92,95	FWCT	Total catch of fish
94.	93	FWT	Total fish meat production
95.	95,90	FWCM	Catch of marine fish
96.	96,91	UFWP	Land in pond culture
97.	98	SFT (J)	Seed food total, by category
98.	94,82,79 80.	FGP (J)	Gross regional food production, by category
99.	98	FTS (J)	Gross human food supply, by type
100.	99	LSFT (J)	Livestock food total, by type
101.	99	FTG (J)	Gross human food supply, by type
102.	101	FTN (J)	Net human food supply, by type
103.	1,102	FSRPC (J)	Regionally produced food, by category
104.	103	VCLPPCR (J)	Calories per capita, regional by category
105.	103	VPTPCR (J)	Protein per capita, regional, by category

106.	104	CLPCR	Calories per capita, regional
107.	105	PTAPCR	Annual protein per capita, regional
108.	110	PTNM	Protein needs multiplier
109.	Ex	PTPCB	Protein per capita base
110.	1,24	YPC	Gross regional product, per capita
111.	108,109	PTPCN	Protein per capita needed
112.	9,111	SPTPC	Per capita protein surplus
113.	9,111	DPTPC	Per capita protein deficit
114.	9,111	PTPCSN	Protein per capita, ratio of supply to needs
115.	1,111	PTN	Total protein needs
116.	1,9	PTR	Regional protein
117.	1,113	DPT	Regional protein deficit
118.	9	PTPCDR	Regional daily protein per capita
119.	Ex	PXLVP	Adjusted price of meat
120.	79	GRV	Dollar value, grain production
121.	80	NGV	Dollar value, non-grain production
122.	82	SLVV	Total livestock meat production
123.	122	LVV	Dollar value, livestock production
124.	94	FSV	Dollar value, fish production
125.	120,121, 123,124	YA	Gross regional product, agriculture
126.	1,125	YAPC	Gross regional product, per capita
127.	127	PXPTM	Price of protein imports
128.	8,125,135	FDXAR	Ratio of value of exports to gross regional product in agriculture
129.	128	ENZ	Energy required for plant food production
130.	1,106,129	ENFZR	Ratio of energy in food produced to that required for plant food production

131.	117,127	FDMV	Dollar value of food imports
132.	125,131	FDMAR	Ratio of food imports to agricultural production
133.	24,131	FDMYR	Ratio of food imports to total GRP
134.	46,131	FDMMR	Ratio of food imports to total imports
135.	116,125, 136	FDXV	Value of exports
136.	1,112	SPT	Surplus protein

III The Linnemann Model

1.	Exog	MPDMI	Maximum of production in dry matters units (with the help of artificial irrigation)
2.	6,7	Y	Yield per hectare of A (Kg. of consumable protein/ha.)
3.	2	FA	Fertilizer application per hectare
4.	2,5	TY	Total yield (kg. of consumable protein)
5.	Exog	A	Total agricultural land (potentially arable land) (millions of hectares)
6.	1	YASY	Maximum yield per hectare of A (kg. of consumable protein/ha.)
7.	5,8,9,10	Z	Auxiliary variable (merely as a label: "input mix per hectare")
8.	8,9,14,15	LE	Labour (persons)
9.	2,9,3,8,14,CE 11,12,13		Capital which increases labour productivity (number of tractors)
10.	Exog	EF	Structural characteristic, reflecting differences in land use functions

* These indicators are "scenario variables" of the model.

11.	Exog	CMON	Current price of capital's unit
12.	Exog	FMON	Current price of unit of fertilizer
13.	22,23,25	P	Price of food for producer
14.	Exog	UCCL	The upper limit of the ratio of capital to labour
15.	15,16	L	Agricultural population (persons)
16.	15,17,18	LO	Annual rate of labour outflow from agriculture
17.	15,18,19	LI	Annual rate of labour inflow in non-agriculture
18.	17,18	NPOP	Non-agricultural population (persons)
19.	20,21	TENS	Ratio of non-agricultural real income per capita and agricultural income per capita
20.	Exog	NRVLU	Non-agricultural real income
21.	Exog	RURU	Agricultural income per capita
22.	22,26	PO	Basic price of agricultural production, reflecting the level of food processing
23.	Exog	NPI	Price index for non-food items in the country
24.	15	NLO (T)	Labour outflow from agriculture during T years
25.	Exog	DFPE	Difference between the current price of unprocessed consumable protein and its price in the base year, within the country
26.	27,29	GPO	The annual rate of increase in the level of food processing
27.	15,18,28, 31	CONS	Food consumption per capita, (kg. of consumable protein)
28.	23,31,32, 33	R	Real income per capita
29.	15,18,31, 28	NCONC	Food consumption of the non-agricultural sector, per capita (kg. of consumable protein)

30.	18,25,29	TR	Tariff receipts/expenditures
31.	Exog	FP	Basic price of food per unit of consumable protein
32.	18,30	VALU	Current income per capita (dollars/person)
33.	23,25,31	TPF	Current price of consumable protein
34.	27,28,31	W	Real expenditure per capita of non-food items
35.	4,15,18	TYPK	Total yield per capita

REFERENCES

- [1] Gelovani, V.A., et al., *K probleme upravlenia v globalnoi modeli World 3 (To the Control Problem in World 3 Global Model)*. Institute of Control Problems, Moscow, 1975. (In Russian)
- [2] Mullat, I., *Extreme Subsystems of Monotone Systems*. Automatica i Telemekhanica, 1976, N5. (In Russian)
- [3] Muchnik, I., *Structural Analysis of Experimental Graphs*. Automatica i Telemekhanica, 1974, N9. (In Russian)
- [4] Meadows, D.K., et al., *Dynamics of Growth in a Finite World*. Cambridge, Mass., U.S.A., Wright-Allen Press, Inc., 1974.
- [5] Mesarovic, M. and E. Pestel (editors). *Multilevel Computer Model of a World Development System*. Extract from the Symposium Proceedings held at IIASA, Laxenburg, April 29 - May 3, 1974.
- [6] Linnemann, Hans, (editor), *A Model of International Relations in Agriculture*. A Russian translation of material from IIASA's Third Global Modelling Conference, September, 1975 .
- [7] McLean, J.M., and Paul Shepherd, *The Importance of Model Structure "Futures"*. Vol. 8, No. 1, February, 1976.
- [8] Thissen, W. and de Mol, *On Analysis of the World 3 Agricultural Submodel*, Project Group "Global Dynamics", Eindhoven, prepared for presentation at IIASA's Third Global Modelling Conference, September, 1975 .