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Stehfest, H.

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H. Stehfest*

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* Part of this work was done at the Nuclear Research Center, Karlsruhe, Federal Republic of Germany.

International Institute for Applied Systems Analysis
A-2361 Laxenburg, Austria

Preface

A problem widely discussed in the field of water resources research is the degree of complexity a water quality model should have if it is used for water quality management. In the present work this problem is investigated by means of an example: An optimal control problem for river quality is solved for both a simple Streeter-Phelps model and a more complex ecological model. Through decision theoretical arguments it is shown how the two optimal solutions can be compared in a rational way. As in previous IIASA reports on the application of systems theory and operation research to river quality management, the methodologies described are applied to a section of the Rhine River in the Federal Republic of Germany.

On The Monetary Value of an Ecological River Quality Model

INTRODUCTION

When river pollution abatement measures are planned, often mathematical river quality models are used which relate wastewater discharges (model input) to certain water quality characteristics of the receiving river (model output). If the river quality problems are caused by organic pollutants the essential quality characteristics are dissolved oxygen (DO) concentration and concentration of organic matter, measured as biochemical oxygen demand (BOD) or as chemical oxygen demand (COD). In this case the most widely used river quality model is the Streeter-Phelps model (or modifications thereof). It describes the self-purification processes in the river as a first order chemical reaction between pollutants and dissolved oxygen. However, in reality, self-purification consists of a great variety of nutritional conversions within a complex food web, each of which requires oxygen. A few models have been built which reflect explicitly these ecological interactions, see, for example, Kelly (1976), Stehfest (1973), and Boes (1975). Although these more complex, ecological models may describe self-purification more accurately than Streeter-Phelps type models, the question arises whether it is worthwhile using them, particularly in view of the higher measurement effort needed to determine the values of their parameters. In the following this question is investigated in detail by means of an example. Both a Streeter-Phelps type model and an ecological model are used to devise an optimal river sanitation program for a section of the Rhine in the Federal Republic of Germany. For the ecological model this is quite an intricate problem because of the high dimension of the state vector. Therefore, the solution to the optimal control problem is described in detail. Finally, the two optimal strategies are compared in a quantitative way in order to find the one with the lower expected cost.

DESCRIPTION OF THE TWO MODELS

The two models and their properties have been described in detail in the literature (Stehfest, 1973, Rinaldi et al., 1978). Therefore, only a brief outline is given, which should be sufficient to understand the discussion of the optimal control problem.

The first model is the well known Streeter-Phelps model augmented by an equation for the accumulation of the nondegradable pollutants. Hence the model equations are:

$$\dot{b} = -k_1 b + L_b(t) \tag{1a}$$

$$\dot{r} = L_r(t) \tag{1b}$$

$$\dot{c} = k_2(c_s - c) - k_1 b \tag{1c}$$

where

- b: BOD;
- r: concentration of nondegradable pollutants;
- c: oxygen concentration;
- L_b: BOD input;
- L_r: input of nondegradable pollutants;
- c_s: oxygen saturation concentration;
- k_1, k_2: parameters.

The independent variable of the Streeter-Phelps model is flow time. Obviously, dispersion phenomena are neglected. For the sake of simplicity, changes of flow rate Q along the river are left out in equations (1a-c), though they are taken into account in the model used.

The ecological model has the following state variables: easily degradable, slowly degradable, and nondegradable pollutants, bacteria, protozoa, oxygen; in any case the measure is mass concentration. The model equations are:

$$\dot{b}_1 = -a_{11} \frac{a_{41} b_1}{a_{42} + b_1} B + L_1(t) \tag{2a}$$

$$\dot{b}_2 = -a_{21} \frac{a_{43} b_2}{a_{44} + b_2 + a_{45} b_1} B + L_2(t) \tag{2b}$$

$$\dot{r} = L_r(t) \tag{2c}$$

$$\dot{B} = \frac{a_{41}b_1}{a_{42} + b_1} B + \frac{a_{43}b_2}{a_{44} + b_2 + a_{45}b_1} B - a_{46} \frac{a_{51}B}{a_{52} + B} P - a_{47}B \tag{2d}$$

$$\dot{P} = \frac{a_{51}B}{a_{52} + B} P - a_{53}P \tag{2e}$$

$$\dot{c} = a_{61}(c_s - c) - a_{62} \frac{a_{41}b_1}{a_{42} + b_1} B - a_{63} \frac{a_{43}b_2}{a_{44} + b_2 + a_{45}b_1} B - a_{64}a_{47}B - a_{65} \frac{a_{51}B}{a_{52} + B} P - a_{66}a_{53}P + q \tag{2f}$$

where

- b_1: concentration of easily degradable pollutants (measured as e.g. BOD);
- b_2: concentration of slowly degradable pollutants (measured as e.g. BOD);
- r: concentration of nondegradable pollutants (measured as e.g. COD);
- B: bacterial mass concentration;
- P: mass concentration of protozoa feeding on bacteria;
- c: oxygen concentration;
- L_1, L_2, L_r: pollutant inputs;
- a_{ik}: parameters;
- q: biogenic oxygen input.

Equation (2a) describes the degradation of the easily degradable pollutants by bacteria as a Michaelis-Menten type process. The degradation of the slowly degradable pollutants, which is described by equation (2b), has the same kinetics if no easily degradable pollutants are present. The latter are assumed to inhibit the degradation of the slowly degradable pollutants according to the kinetics of competitive enzymatic inhibition. The nondegradable pollutants accumulate along the river, as expressed by equation (2c). The first two terms on the right hand side of equation (2d) give the increase of bacterial mass due to degradation activity, the last two terms account for the losses of bacterial mass due to protozoan grazing and endogenous bacterial respiration, respectively. Equation (2e) expresses the changes of the protozoan mass as the difference between the increase due to feeding on bacteria and the loss due to endogenous respiration. The latter term may also be thought of as taking approximately into account the grazing activity of higher organisms on protozoa. In equation (2f) the oxygen consumptions of all processes described by the previous equations are subtracted from the terms for physical and biogenic aeration to give the oxygen balance of the river.

The parameters of both models have been estimated such that measurements on the Rhine are fitted well in the least square sense. Besides the measurements, the estimate is also based on a priori knowledge about the magnitude of some parameter values, the estimation technique used is the quasilinearization technique (Stehfest, 1973). Only measurements taken at a particular flow rate and temperature during the period 1969-1971 are used for the estimation. Figures 1 and 2 show how the models fit those measurements. One can see that the measurements are very sparse, in particular between Mainz and Cologne. Moreover, the measurement uncertainties are high, in many cases the values given had to be derived from measurements of other quantities (Stehfest, 1973).

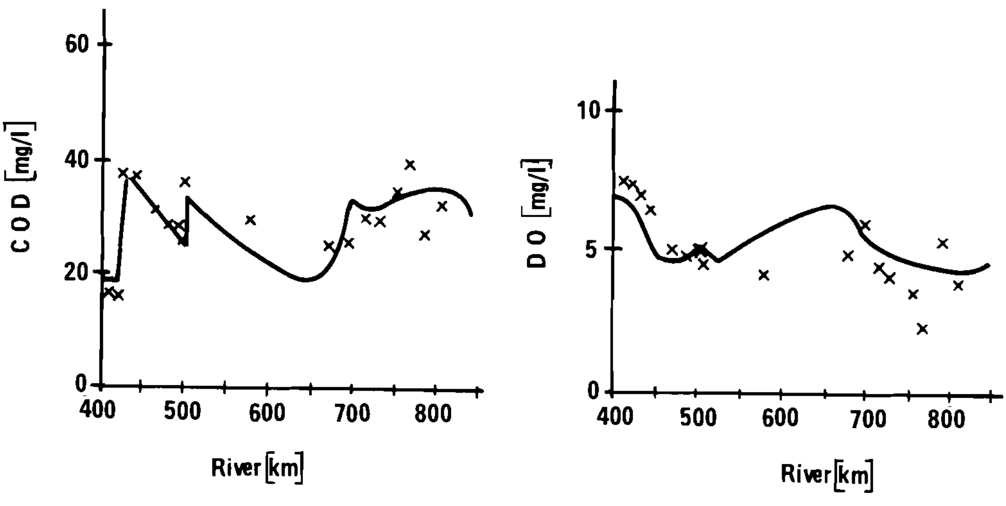

Figure 1. Fitting of the Streeter-Phelps model to measurements on the Rhine.

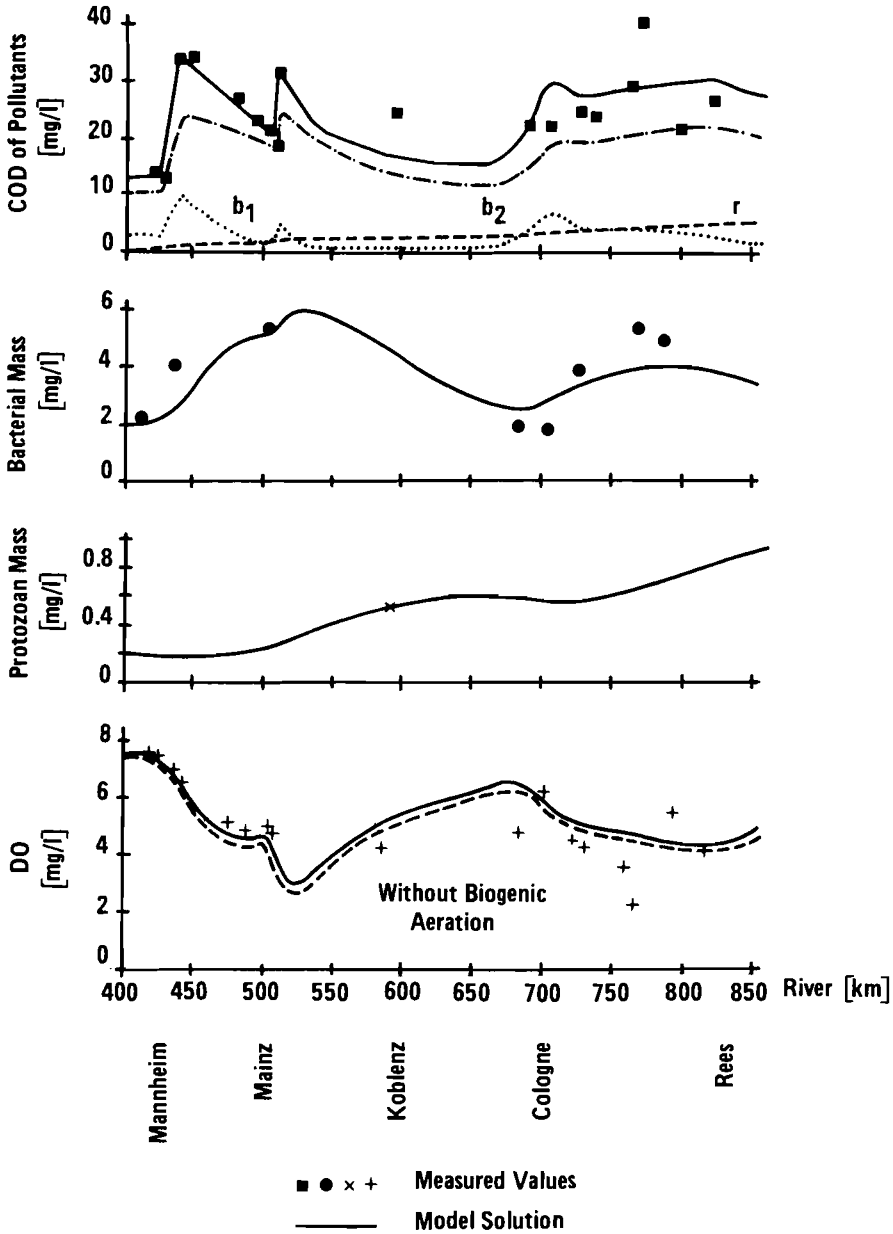

Figure 2. Fitting of the ecological river quality model to measurements on the Rhine.

Because of the defectiveness and uncertainty of the measurements the accuracy of the parameter estimates is not too high. For other flow rates and temperatures fewer measurements are available. They were used to validate the models after parameter estimation (Stehfest 1973, 1978).

Comparing the two figures, one recognizes that for the variables that are common to both models the ecological model fits the observations somewhat better than the Streeter-Phelps model, but the differences are not dramatic. However, if the models are used to design a sanitation program they may lead to different recommendations, because in this case they have to be applied to a wastewater discharge pattern for which no observations are available. Indications for this are given in Stehfest (1973) (see in particular Figure 4.17). Therefore, it is not the best strategy just to choose the model that fits the available observation better. Confidence in the model, if used under changed circumstances, plays a role as well. To determine the differences in river sanitation programs when different quality models are applied, both the Streeter-Phelps and the ecological models have been used in an optimal control study on the same section of the Rhine River for which the models were built.

USE OF THE MODELS IN AN OPTIMAL CONTROL STUDY

The optimal control problem to be studied is the least costly distribution of wastewater treatment effort along the Rhine River section such that certain quality standards are met for the wastewater production pattern expected for 1985. As for model building, the river section considered was divided into reaches within which the wastewater discharge was assumed to be uniformly distributed. The values of the wastewater production in the reaches were derived from the values used for model building (given in Stehfest 1973) by applying factors that account for the growth from 1970 to 1985; the growth factors do not differ from reach to reach, but are different for the three categories of pollutants. The relative growth of the three sewage components is shown in Figure 3 which corresponds to present trends in wastewater production. Two types of treatment were considered: a) mechanical and biological treatment, and b) precipitation with lime and adsorption with activated carbon. Of course, type b is considered only as an addition to type a. The costs differ from reach to reach because the plant sizes are assumed to differ. The underlying plant sizes were estimated mainly on the basis of industrial and population density within the reaches. These densities determine how the sewage transportation costs increase when the plant size increases and hence they determine the optimal plant size. The exclusion of the possibility of modifying the waste generating processes gives rise to overestimates of the cost of abatement. Nevertheless, we are forced to restrict our analysis to wastewater treatment alone, since reliable data on costs of changing the production are not available.

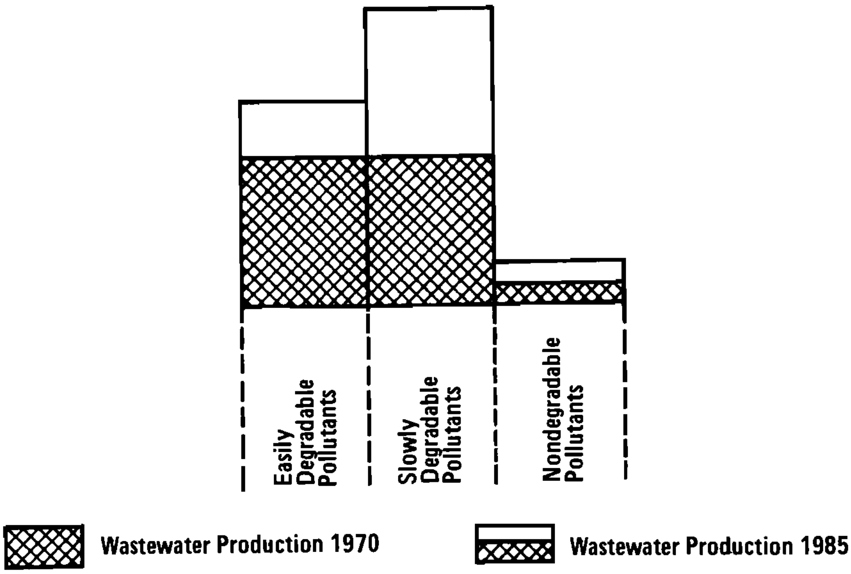

Figure 3. Assumed increase of wastewater production from 1970 to 1985 (COD production in arbitrary units).

Standards were imposed on both oxygen concentration and nondegradable pollutant concentration. Oxygen concentration standards seek to guarantee acceptable conditions for the aquatic life, while nondegradable pollutant concentration standards limit the expenditure necessary for drinking water production. The river conditions under which the standards have to be met are $T = 20°C$ and $Q = 0.56$ MQ; the latter value corresponds approximately to the flow rate which is exceeded with 80 percent probability at Cologne. These conditions have been chosen because it is generally believed that the joint occurrence of high temperature and low flow is the most unfavorable event with regard to water quality.

If we look at the problem described above from a flow time point of view, we see that it is a multistage decision problem: each time a new section is passed, a decision has to be taken as to the kind and extent of wastewater treatment in this reach. Since, in addition, the objective function (namely the total treatment cost) is a sum of terms each of which depends on the decision in one reach only, the problem has exactly the structure of a dynamic programming problem. Therefore, the dynamic programming technique can be used to solve the problem, as it has been done many times for similar problems (see, for example, Liebman and Lynn, 1966; Dysart and Hines, 1970; Shih, 1970; Converse, 1972). The main advantages of this approach are its simplicity and flexibility: it can easily take into account any kind of constraints and can deal with stochastic phenomena; one need not worry about being caught by a suboptimum. On the other hand, this approach has serious drawbacks: if the number n of state variables becomes too high ($n > 5$) the requirements for computer storage and computing time become, in general, prohibitive.

The scheme for the dynamic programming solution to the problem is shown in Figure 4. Only one state variable is shown, on which a standard (in the form of a lower bound) is imposed, which varies in space. The section $\ell_i - \ell_{i-1}$ on the abscissa axis correspond to the river reaches. The optimal control is calculated in the following way: starting from the last reach, for all reaches i, $i = N, N-1,\ldots,1$, the functions $H_i(x_i)$ are calculated, which give the minimum of treatment cost for reaches i, $i+1,\ldots,N$ if one starts from state x_i at the upstream end of reach i. Each $H_i(x_i)$ is calculated on the basis of $H_{i+1}(x_i)$ by solving the following minimization problem:

$$H_i(x_i) = \min_{u_i} \left[C_i(u_i) + H_{i+1}(f_i(x_i,u_i))\right] , \qquad (3)$$

where $C_i(u_i)$ gives the treatment cost in reach i, and $f_i(x_i,u_i)$ is the state resulting at the end of reach i if the control action

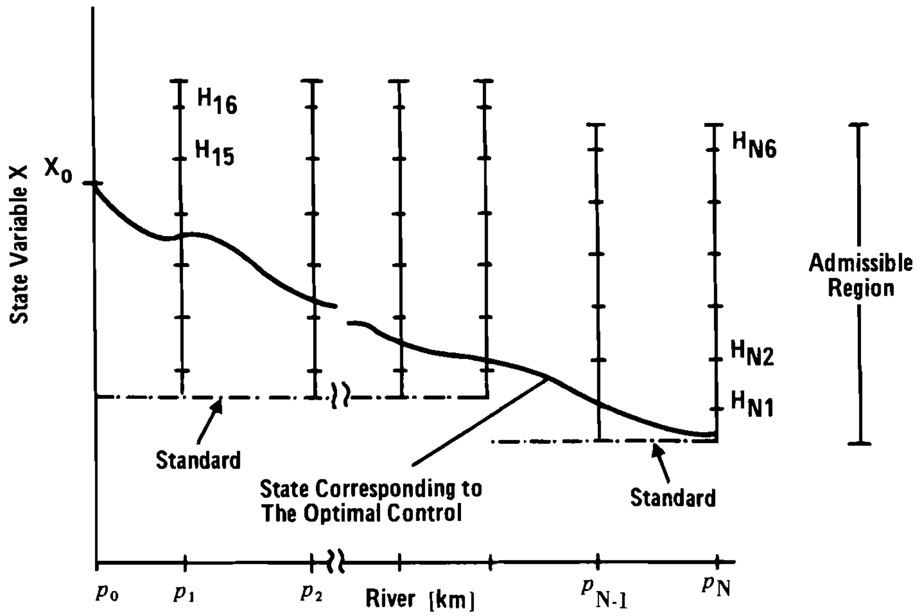

Figure 4. Structure of the dynamic programming approach as applied to river quality control.

u_i is taken. Only those control actions and state values are admitted for which $f_i(x_i,u_i)$ satisfies the standards. After having calculated in the backward direction all functions $H_i(x_i)$, the optimal control strategy is constructed by starting from the prescribed initial state x_o at the upstream end of the river section considered, and calculating for each reach the control action which minimizes the sum of the treatment costs in this and all downstream reaches, whereby the functions H_i are used. During the backward calculation one could also record the optimal control actions belonging to each state. In this case the forward calculation would not require new minimizations. This scheme has certain computational drawbacks, however, and will therefore not be applied here.

In general, the functions $H_i(x_i)$ cannot be given in closed form, but can be evaluated only numerically in certain grid points x_{ik} of the feasibility set x_i (the value of $H_i(x_i)$ in grid point x_{ik} has been denoted by H_{ik} in Figure 4). This implies that for the calculation of $H_{i-1}(x_{i-1})$ and for the final forward calculation one has, in general, to interpolate between grid points. (In the most practical cases the functions $H_i(x_i)$ are continuous, so that interpolation poses no particular problems. The optimal decision attached to each state during the backward calculation, however, is frequently not a continuous function of x_i. This is one of the computational drawbacks mentioned above for the scheme in which the optimal decisions during backward calculation are recorded.)

A simplification of the computational scheme described consists in using the value of $H_i(x_i)$ at the closest grid point instead of interpolating. Following this principle, and assuming that the control variables u_i are discretized, too, the backward calculation becomes superfluous. Starting from the prescribed initial value, one may directly follow all possible policies (i.e., sets of control actions) in the forward direction, and store for each grid point reached the accumulated costs and the last decision. If two policies lead to one and the same grid point, the more costly policy is discarded, which is the application of the principle of optimality. Finally, the optimal policy is constructed in the backward direction starting from that state in the last reach which has the lowest accumulated cost attached to it. This simple scheme has been used sometimes for water quality control studies (e.g., Liebman and Lynn, 1966; Newsome, 1972), but no clear advantage can be seen in comparison with the scheme using interpolation, because the avoidance of interpolation has to be paid for with a finer grid, which requires more computations and storage. For example, for the ecological model (n = 6) the storage requirements would be prohibitive.

A crucial point for the numerical calculation is the selection of the admissible region for each reach, i.e., the subset of the feasibility region covered by the grid. The admissible region should be as large as possible so as not to exclude any possible control policy. On the other hand, the number of grid points is limited by the need to obtain a reasonable computing time and by the amount of storage available. Hence, if the admissible regions are large, the distance between the grid points is large, too, and consequently, interpolation may yield inaccurate results. For high dimensional problems it may be impossible to resolve this trade-off in a satisfactory way. Then one has to resort to iterative techniques which may lead, however, to suboptimal solutions. One can first solve the optimal control problem with sufficiently large admissible regions; because of great interpolation errors one obtains only a crude approximation to the optimal solution. Then a better approximation is calculated by solving the problem again with smaller admissible regions which are centered around the previously obtained solution *(the coarse-fine grid method)*. This procedure may be repeated.

Another way of approaching the solution to the dynamic programming problem iteratively starts with admissible regions which are sufficiently small for accurate interpolation. The solution is enforced to remain within these regions by imposing appropriate constraints. (Compliance with these constraints during interpolation is guaranteed by attaching a very high value of $H_i(x_i)$, $i = 1,\ldots,N$ to all gird points for which no admissible control u_i exists such that $f_i(x_i,u_i)$ is within the admissible region of reach $i + 1$.) If those constraints turn out to be active, a second run is made with the admissible regions centered around the first solution. One continues in this manner until the solution is no longer deformed by those boundaries of the admissible regions which do not coincide with a water quality standard *(the relaxation method)*.

For the application in which the ecological model is used, a combination of both refining techniques has been used, while the corresponding application based on the Streeter-Phelps model could be solved in one shot. Each variable of the ecological model was discretized into 6 values, and the section of the Rhine considered was divided into 16 reaches. Hence, for the problem with the ecological model the values of $H_i(x_i)$, $i = 1,\ldots,N$ in $16 \times 6^6 \simeq 7.5 \times 10^5$ grid points had to be evaluated, whereby each value required the solution to a minimization problem of type (3). Minimization was carried out by discretizing the decision variables (i.e., the total capacity of the treatment plants in each reach) and by comparing the costs given by the bracket expression in Equation (3) for all admissible control actions. The state transition function $f_i(x,u_i)$ for the ecological model was evaluated by solving system (2) between the boundaries of reach i by means of a Runge-Kutta integration scheme.

For the Streeter-Phelps model the analytical solution to equation (1) could have been used. In order to compare the two solutions consistently, however, the same integration procedure (with the same step width) was used. The function $f_i(x,u_i)$ for the ecological model need not be evaluated in this way for each grid point because of the special structure of the model: since the equation for oxygen concentration c is linear in c, and because the other equations in system (2) are independent of c, the evolution of c can be split up into a so-called "free motion", which is the solution to the homogeneous part of equation (2f), and a "forced motion", which does not depend on the initial value of c. Consequently, the whole system (2) has to be integrated only once for all grid points which differ only with respect to the value of the state coordinate c. The resulting value of $f_i(x,u_i)$ is then corrected for the various initial oxygen values, using the simple analytic solution of the homogeneous part of the oxygen equation. The same argument applies to equation (2c) for the nondegradable pollutants, so that for each control option u_i of the full system (2) has to be solved only 6^4 times instead of 6^6 times as one could expect at first glance. For interpolation a simple linear scheme was used.

In Figure 5 the optimization results for the two models are shown under the constraints (standards) $r \leq 20$ mg/ℓ and $c \geq 4$ mg/ℓ. For the sake of simplicity, an example has been chosen where the treatment effort on the tributaries was held fixed (which is not shown in Figure 5). The standard on r affects the solution only slightly, and the solution does not contain advanced treatment plants. The latter holds for any reasonable oxygen standard if the standard on r is sufficiently high. Both solutions show clearly that the treatment effort is mainly allocated to the river reaches with high wastewater production; in these reaches the optimal plant size is large, i.e., the specific treatment cost is low. The actual abatement measures during the last few years show the same tendency. If we compare the two solutions (cf. Figure 5c), the most obvious feature is that with the ecological model more treatment effort is located in the upstream part of the river section. The differences in the ninth and tenth reaches are not as relevant as they might seem at first glance. Since both reaches are short the deviation from the optimum will be very small if some treatment effort is shifted from one of the reaches to the other. Hence, for evaluating visually the difference between the two solutions one should look at the difference between the differences of reaches nine and ten. The total annual costs for the two solutions are as follows:

$C^1 = 1.08 \times 10^9$ DM*/a for the Streeter-Phelps model,

$C^2 = 1.16 \times 10^9$ DM*/a for the ecological model.

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*German marks.

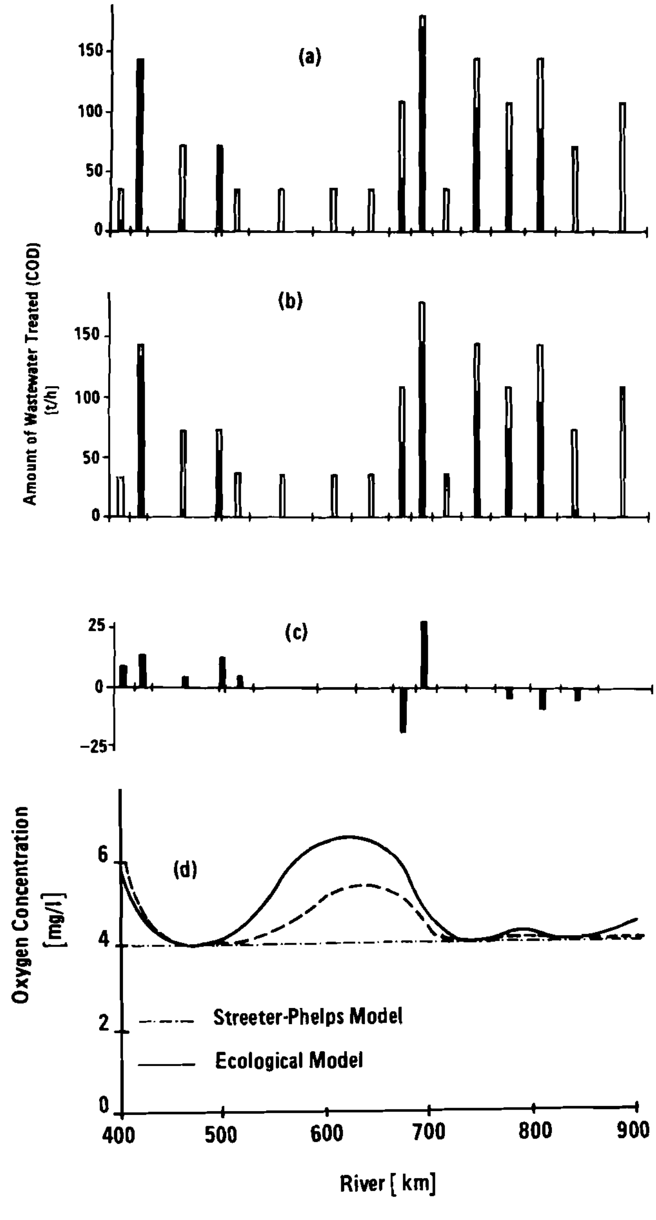

Figure 5. Optimal allocation of wastewater treatment effort along the Rhine River ($Q = 0.56MQ$, $T = 20°C$, $r \leqslant 20$ mg/ℓ, $c \geqslant 4$ mg/ℓ). The black part of the columns indicates the amount of wastewater treated, while the total columns give the amount of wastewater produced in each reach. (a) treatment effort determined with the ecological model, (b) treatment effort determined with the Streeter-Phelps model; (c) difference between a) and b); and (d) oxygen concentration for optimally allocated treatment effort.

DISCRIMINATION BETWEEN THE TWO MODELS

Even though the difference between the two solutions is not dramatic the question may arise which solution should be implemented. For example, a decisionmaker may be faced with the problem of having received two proposals for river sanitation from two consultants. It is not sufficient for him to check how well the underlying models fit actual measurements. If a model has a sufficient number of parameters the fit can always be made perfect, but the model may give very inaccurate results when applied to circumstances which differ considerably from the ones for which the parameters have been estimated and for which the model has been validated. Hence, the decision maker's confidence in the concept of the model will play a role in his choice. This confidence is certainly subjective, and may be influenced, for instance, by the reputation of the model builders. Another aspect to be included is the costs associated with each solution and the costs to be incurred if the solution selected turns out to be wrong. Although the choice involves subjective judgments, the integration of all the aspects of the choice may be formalized, and hencewith made lucid to a certain extent by means of decision theoretical arguments.

The problem to be solved is a secondary optimization problem, i.e., the optimal choice has to be made between optimal solutions, which is typical for decisionmaking. As for the primary optimizations, minimization of cost seems to be a reasonable objective, but because of the uncertainty as to which model is better we can only speak of expected cost. For simplicity in notation, we look upon the two models as one aggregate model, which contains a parameter z that can assume two values z^1 and z^2. This aggregate model may be imagined as the union of the two models plus a linear output transformation containing the parameter z. Depending on the value of z, the output variables of the aggregate model are equal to those variables of one model or the other on which standards are imposed. (Of course, a variable on which a standard is imposed must be contained in either model.) The secondary optimization problem appears now as the problem of estimating the parameter z. Obviously, this reasoning may be generalized to more than two models, and there is no restriction as to the type of the models.

If we assume a priori that one of the two models correctly describes the real behavior of the system, the expectation C of the total cost is given by the following expression:

$$C(z') = C(z',z^1)p(z^1|Y) + C(z',z^2)p(z^2|Y) \qquad (4)$$

where $C(z',z^i)$ denotes the cost to be incurred if z^i is the "right" value and the model characterized by z' is used, and $p(z^i|Y)$ is the judgmental probability for z^i being the "right" value given set Y of observations. Expression (4) has to be minimized over $z' \varepsilon \{z^1, z^2\}$. The set Y comprises all information on which the decisionmaker decides on the two models. The probability $p(z^i|Y)$ could be resolved according to Bayes' formula into

$$p(z^i|Y) = \frac{p(z^i|Y \cap Y_o)\ p(Y_o|z^i)}{p(Y_o)} ,$$

where Y_o is the set of observations of the variables on which standards are imposed. The a priori probability $p(z^i|Y \cap Y_o)$ would have to be quantified judgmentally by the decisionmaker (see for example Raiffa, 1968), while the other probabilities in principle could be calculated from the models. In view of the other uncertainties, however, it seems more appropriate to have estimated $p(z^i|Y)$ directly by the decisionmaker; for the sake of notational simplicity, $p(z^i|Y)$ will be denoted by p_i in the following. The cost $C(z',z^i)$ to be incurred if $z' = z^i$ is obviously the cost C' of the optimal control S^i belonging to the model characterized by z'. For $z' \neq z^i$ it is reasonable to assume that the costs for realizing the solution belonging to z' have to be incurred, too, because planning and construction of the treatment plants have to begin long before it becomes manifest that the wrong model was relied upon. Then these costs have to be corrected for the fact that either the standard is not met or too many treatment plants have been built. A reasonable correction would be to add the costs necessary to make the solution S' meet the standards given the z^i is the "right" value. This would be consistent with the maxim behind standard setting, which says that no damage occurs if the standard is met, while the damage is very high if it is violated. The corrections can be determined by the same optimization procedure used to find the optimal solutions among which one has to choose. If in the case of $z' \neq z^i$ the solution S' meets the standards even with z^i being the "right" value, the correction might be negative if shutting down of treatment plants is considered (which saves operational costs). If we denote by Δ_{ik} the cost of modifying the optimal control S^i such that it meets the standards if z^k is "right", minimization of expression (4) means comparing the two expressions

$$C^1 + \Delta_{12}(1-p_1) \qquad (5a)$$

and

$$C^2 + \Delta_{21}p_1 \quad . \qquad (5b)$$

If 5a is smaller than 5b the decisionmaker should choose solution S^1 in order to minimize the expected cost; otherwise S^2 should be chosen. The two expressions (5a, 5b) as functions of p_1 are shown in Figure 6. Obviously, the decisionmaker need not assign a specific value to p_1; he only has to specify whether p_1 is smaller or greater than the abscissa of the intersection point of the two lines representing the expressions (5a, 5b). For our optimal control problem on the Rhine the costs are as follows:

$$C^1 = 1.080 \times 10^9 \text{ DM/a} \qquad \Delta_{12} = 0.102 \times 10^9 \text{ DM/a} \quad ,$$

and

$$C^2 = 1.162 \times 10^9 \text{ DM/a} \qquad \Delta_{21} = 0.017 \times 10^9 \text{ DM/a} \quad .$$

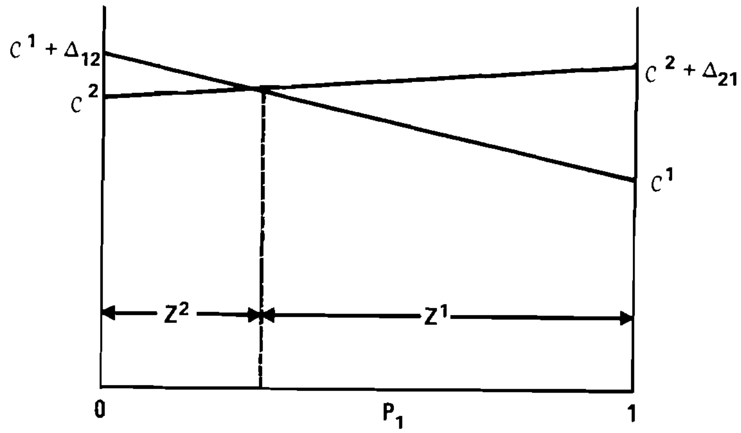

Figure 6. Choice between two optimal control strategies based on two different river quality models: expected cost versus probability p_1 given that the model is correct.

Index 1 refers to the Streeter-Phelps model and index 2 to the ecological model. Hence, the control S^2 does not satisfy the standards if the Streeter-Phelps model is the "right" one, even though the treatment effort for S^2 is considerably higher than for S^1. The abscissa of the intersection point is in this case about 0.17.

The scheme described may be generalized to cover other options that may differ even qualitatively. It could be, for example, that the consultants who determined the two different solutions to the optimal control problem propose (as usual) to take more measurements on the river in order to get a better validated model. Hence, besides realization of S^1 or S^2, the decisionmaker has the third option of first taking more measurements at cost C_m. He can assume that the model emerging from the new measurements will give the control that is optimal in reality, but he does not yet know anything about the costs. In this case, the following judgmental probabilities of the decisionmaker have to be assessed:

- p_1: probability for z^1 giving an output of the aggregate model that agrees with reality within the desired accuracy;
- p_2: probability for z^2 giving an output of the aggregate model that agrees with reality within the desired accuracy;
- p_3: probability that the model describing reality adequately differs from the two models offered to the decisionmaker by more than the prescribed accuracy.

The three probabilities add up to one if the two given models differ by more than the prescribed accuracy. For the costs C^3 of realizing the optimal control for the third yet unknown model, the only reasonable assumption that can be made is

$$C^3 = \frac{p_1 C^1 + p_2 C^2}{p_1 + p_2}$$

i.e., C^3 is the mean of C^1 and C^2 weighted with the probabilities for the corresponding models. Similarly, for the cost Δ_{13} to be incurred if optimal control S^1 is realized and neither z^1 nor z^2 gives the "right" model, the only reasonable assumption is $\Delta_{13} = \Delta_{12}$. Analogously, $\Delta_{23} = \Delta_{21}$ must be assumed. Hence, minimization of the expected cost means searching for the smallest one among the following three expressions:

$$C^1 + \Delta_{12}(1-p_1) \tag{6a}$$

$$C^2 + \Delta_{21}(1-p_2) \tag{6b}$$

$$C_m = \frac{p_1 C^1 + p_2 C^2}{p_1 + p_2} \tag{6c}$$

Figure 7 shows, in analogy to Figure 6, expressions (6a-6c) for a fixed value of $p_1 + p_2 = 1 - p_3$ as a function of $p_1/(p_1 + p_2)$.

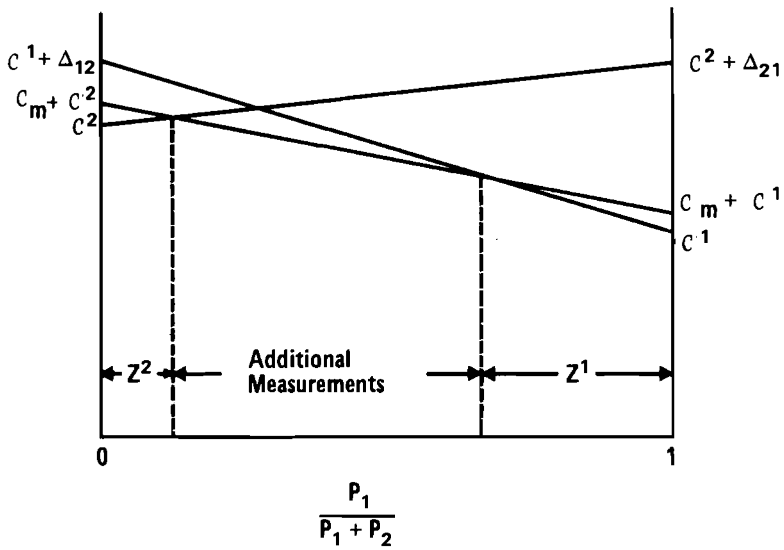

Figure 7. Choice among three options for realizing a river sanitation program: expected cost versus probability for model 1 being correct, given that either model 1 or 2 is correct.

CONCLUSIONS

Two river quality models have been applied to a steady state optimal control problem on a section of the Rhine River. One model is the well known Streeter-Phelps model, the other an ecological model with six state variables. The optimal control problem consisted in finding the least costly distribution of wastewater treatment effort on the river such that standards on oxygen and nondegradable pollutants concentration are met. The optimal control problem could be solved via dynamic programming for both models, despite the high dimension of the ecological model. If only an oxygen standard is imposed for any reasonable standard, no advanced treatment plant appears in the optimal solution. The differences between the two optimal solutions (corresponding to the two models) are remarkable, but not as large as one might expect from the differences in the model structure. For the choice between the two optimal solutions, in which subjective aspects are encountered, a formal procedure has been proposed which finds the solution with the lowest expected cost.

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