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An Informal Guide to Some Papers on a Theory of Money and Financial Institutions

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AN INFORMAL GUIDE TO SOME PAPERS ON A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

Martin Shubik

June 1977

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PREFACE

One of IIASA's aims is the development and application of methods of investigation and analysis for treating problems of contemporary societies. As a part of the System and Decision Sciences task in mathematical economics, Professor Martin Shubik was invited to give a series of lectures on the theory of money and financial institutions.

This paper was written to accompany the lectures. It contains a listing, together with brief summaries, of published and preliminary papers dealing with the topic of a macro-economic process approach to the theory of money and financial institutions. The approach is essentially via methods suggested by the theory of games and oligopoly theory. The purpose of this paper is to provide a brief set of guidelines to a reader interested in considering this approach to the theory.

ABSTRACT

The basic ideas behind a new approach to modeling money and financial institutions are summarized. A listing is given of a set of inter-related papers on this subject by the author, together with capsule summaries of each which can serve as a guide and introduction to the subject. Various unsolved problems and directions for further research are suggested.

This paper was written to accompany a series of lectures given by Professor Martin Shubik at IIASA in April and May 1977.

A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

1. INTRODUCTORY NOTES

On the following pages is a listing of published and preliminary papers dealing with the topic of a micro economic process approach to the theory of money and financial institutions. The approach is essentially via methods suggested by the theory of games and by oligopoly theory.

Considerable stress is laid upon information conditions and an explicit description of process involving items such as price formation and the explicit mechanisms for trade with various financial institutions.

The papers referred to here are preliminary material for a projected book. Not all of them should be regarded as a finished work. The modeling difficulties and the mathematical difficulties are considerable. With respect to the latter, I have been fortunate to have had the collaboration of several colleagues, especially Lloyd Shapley and Pradeep Dubey.

It is my belief, however, that mathematization of a subject frequently follows the formulation of the appropriate models hence it would be overly stultifying to limit the formulation of models to include only those which can be solved by immediately available mathematics.

The list of publications is divided into two parts, the first consisting of articles already published, bearing on money and financial institutions and the second containing a list of working papers.

In a previous version of these notes a list of earlier publications was included to provide a sketch of the context of the work which led me to adopt this particular approach. Needless to say my intellectual debt to the works of Cournot, Edgeworth, and von Neumann and Morgenstern is enormous. I am especially indebted to Dr. Lloyd Shapley with whom I have worked and discussed these matters for many years and to Pradeep Dubey who has worked intensively with me in the last eighteen months. I could list dozens of other individuals and references who have helped shape this approach, however, the purpose of this note is to provide a relatively brief set of guidelines to a reader who may be interested in considering this approach to a theory of money and financial institutions, but is not inclined to wade through an enormous number of articles which have not yet been integrated into a single work.

2. PUBLICATIONS AND WORKING PAPERS

Publications

1. "On Different Methods for Allocating Resources," Kyklos, XXIII, 2 (1970), pp. 332-337.

2. "Pecuniary Externalities: A Game Theoretic Analysis," American Economic Review, LXI, 4 (September 1971), pp.713-718 (CFP 355*, CFDP 288**).

3. "Fiat Money and Noncooperative Equilibrium in a Closed Economy," International Journal of Game Theory, 1, 4 (1971/72), pp 243-268. (Earlier version CFDP 330.)

4. "Fiat Money in an Economy with One Nondurable Good and No Credit (A Noncooperative Sequential Game)," with W. Whitt. TOPICS IN DIFFERENTIAL GAMES, ed. A. Blaquiere, North-Holland Publishing Co., 1973, pp. 401-448. (Also CFP 389, earlier version CFDP 355, 3/6/73.)

5. "Commodity Money, Oligopoly, Credit and Bankruptcy in a General Equilibrium Model," Western Economic Journal, X, 4 (December 1972), pp 24-38. (Also CFP 391, earlier versions CFDP 324, RAND p-4686.)

6. "Information, Duopoly and Competitive Markets: A Sensitivity Analysis," Kyklos, 26, 4 (1973), pp 736-761. (Also CFP 400, earlier versions CFDP 347 and Technische Hochschule, Vienna, RM-3, June 1972.)

7. "The General Equilibrium Model: Barter and Trust or Mass Markets with Money and Credit," Economic Record (1974) (earlier version University of Melbourne RM 4, July 1973), 50, pp 245-258.

8. "Money, Trust and Equilibrium Points in Games in Extensive Form," Zeitschrift fur Nationalokonomie, 34 (1974), pp 365-385.

9. "Competitive and Controlled Price Economies," EQUILIBRIUM AND DISEQUILIBRIUM IN ECONOMIC THEORY, ed. G. Schwodiauer, Proceedings of a conference held in Vienna, July 1974 (forthcoming, 1977), (Also CFDP 337 revised).

10. "A Dynamic Economy with Fiat Money without Banking but with Ownership Claims to Production Goods," Melanges Francois Perroux, Grenobles, (forthcoming, 1977).

11. "A Trading Model to Avoid Tatonnement Metaphysics," STUDIES IN GAME THEORY AND MATHEMATICAL ECONOMICS, Bidding and Auctioning for Procurement and Allocation, Y. Amihud (ed.) New York, NYU Press, 1976, 129-142. (Also CFDP 368 revised.)

12. "On the Eight Basic Units of a Dynamic Economy Controlled by Financial Institutions," The Review of Income and Wealth, Series 21, No. 2 (June 1975), pp 183-201.

13. "The General Equilibrium Model is Incomplete and Not Adequate for the Reconciliation of Micro and Macroeconomic Theory," Kyklos, 28, (1975), pp 545-573. (Also CFP 432.)

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* Cowles Foundation Paper.
** Cowles Foundation Discussion Paper.

14. "On the Role of Numbers and Information in Competition," Revue Economique, 26, 4 (1975), pp 605-621.

15. "Mathematical Models for a Theory of Money and Financial Institutions" ADAPTIVE ECONOMIC MODELS, ed. Richard H. Day and Theodore Groves, Academic Press, 1975, pp 513-74.

16. "A Noncooperative Model of a Closed Economy with Many Traders and Two Bankers," Zeitschrift fur Nationalokonomie, , 1976, 10-18.

17. "Competitive Equilibrium, Contingent Commodities and Information," Journal of Finance, forthcoming, 1977.

18. "Beyond General Equilibrium," Economie Applique, XXIX, 2, 1976, 319-337.

19. "Trade with Fiat Money but No Individual Trust -- A Preliminary Stage towards Banking," forthcoming 1977, in honor of Giovanni Demaria.

20. "Trade Using One Commodity as a Means of Payment," with L.S. Shapley, The Journal of Political Economy, forthcoming 1977, (also CF 403/8).

21. "Trade and Prices in a Closed Economy with Exogenous Uncertainty Different Levels of Information, Money and Compound Futures Markets," with P. Dubey, Econometrica, forthcoming, 1977.

22. "Banks, Insurance and Futures Markets, or a Trading Economy with Money, Exogenous Uncertainty and an Optimal Bankruptcy Rule," Mathematical Economics and Game Theory, ed. R. Henn and O. Moeschlin, Springer Verlag: Berlin, 1977, 668-682.

23. "A Closed Economic System with Production and Exchange Modelled as a Game of Strategy," with P. Dubey, Journal of Mathematical Economics, forthcoming, 1977.

Working Papers*

1. On Prices, Fiat Money, Credit and Transferable Utility, CFDP 166, 1/28/64. (also Measurable, Transferable and Comparable Utility and Money CFDP 200 2/8/66.)

2. A theory of Money and Banking in a General Equilibrium System, RM 48 7/70. Institute for Advanced Study, Vienna, Austria.

3. TMFI;**Part 1: The General Approach Adopted, CFDP 320, 10/12/71.

4. TMFI, Part 2: On the Paradox of the Efficient Price System in a Completely Centralized Economy and in a Capitalist Individual Ownership Economy, CFDP 322, 11/1/71. (Also RAND p-4689, August 1971.)

*5. TMFI, Part 3: The Missing Degree of Freedom: Commodity Money and Oligopoly in a General Equilibrium Model, CFDP 324, 11/10/71. (Published No. 5.)

*6. TMFI, Part 4: Fiat Money and Noncooperative Equilibrium in a Closed Economy, CFDP 330, 2/7/72. (Published No. 3.)

7. TMFI, Part 5: The Rate of Interest on Fiat Money in a Closed Economy, CFDP 338, 4/5/72. (Withdrawn for Revision; also RAND p-4723, October 1971.)

8. TMFI, Part 6: The Rate of Interest, Noncooperative Equilibrium and Bankruptcy, CFDP 334, 4/5/72. (Withdrawn for Revision.) (with L.S. Shapley)

*9. TMFI, Part 7: Money, Trust and Equilibrium Points for Games in Extensive Form, CFDP 331, 2/15/72. (Revised version, 1973) (Published No. 8.)

10. TMFI, Part 8: Transactions Costs in a Market Economy, CFDP 336, 4/5/72.

*11. TMFI, Part 9: Competitive and Controlled Price Economies; the Arrow-Debreu Model Revisited, CFDP 337, 4/5/72. (Published No. 9.)

12. TMFI, Part 10: Some Informal Notes, Preliminary Results and Models Relevant to Dynamic Monetary Economies, Preliminary CF-20509, 5/9/72.

*13. TMFI, Part 11: Trade with Fiat Money but No Individual Trust -- A Preliminary Stage towards Banking, CFDP 363, 11/6/73. (Revised to be Published No. 19.)

*14. TMFI, Part 12: A Dynamic Economy with Fiat Money without Banking and with or without Production Goods, CFDP 364, 11/13/73. (Revised and Published No. 10.)

15. TMFI, Part 13: Trade with Spot Markets, Fiat Money and Internal Banking, CFDP 366, 1/18/74.

* Part 14 is not yet completed.
** The abbreviation TMFI is used for A Theory of Money and Financial Institutions.

*16. TMFI, Part 15: A Trading Model to Avoid Tantonnement Metaphysics, CFDP 368, 2/25/74. (Revised version, May 1974, Published No. 11.)

*17. TMFI, Part 17: On the Eight Basic Units of a Dynamic Economy with Spot and Futures Markets, CFDP 367, 2/8/74. (Published No. 12.)

*18. The General Equilibrium Model is the Wrong Model and a Noncooperative Strategic Process Model is a Satisfactory Model for the Reconciliation of Micro and Macroeconomic Theory, CFDP 365, 12/6/73. (Revised October 8, 1974, Published No. 13.)

*19. On the Role of Numbers and Information in Competition, CFDP 371, 3/13/74. (Published No. 14.)

*20. TMFI, Part 18: A Noncooperative Model of a Closed Economy with Many Traders and Two Bankers, CFDP 374, 3/28/74. (Published No. 16.)

21. Some Strategic Models Related to General Equilibrium (with L.S. Shapley), mimeographed RAND, February 21, 1973.

*22. Trade Using One Commodity as a Money (with L.S. Shapley), mimeograph CF-40318, April 18, 1974. (Revised to be published No. 20.)

*23. TMFI, Part 16: Mathematical Models for a Theory of Money and Financial Institutions, CFDP 377, 10/9/74. (Published No. 15.)

*24. Competitive Equilibrium Contingent Commodities and Information, CFDP 379, 10/17/74. (Revised May 1975, see also CF-20322), (Published No. 17.)

25. TMFI, Part 19: Some Problems and Conjectures Concerning Mathematical Models of Money and Financial Institutions without Exogenous Uncertainty, CF-50401, April 1, 1975.

26. TMFI, Part 20: A Linear Exchange Model with Financial Institutions: The Rate of Interest, CF-50416, March 16, 1975.

27. TMFI, Part 21: Fiat Money, Bank Money, The Float and the Money Rate of Interest, CFDP 394, 5/8/74.

28. TMFI, Part 23: Fiat Money, Bank Money, the Force of the Rate of Interest and the Vanishing Float, CFDP 395, 5/14/74.

*29. TMFI, Part 24: Trade and Prices in a Closed Economy with Exogenous Uncertainty, Different Levels of Information, Money and Compound Futures Markets (with P. Dubey), CFDP 410R, 11/14/75. (Published No. 21.)

*30. TMFI, Part 26: On the Number of Types of Markets with Trade in Money, CFDP 416, 1/14/76. (Revised for Publication April 1977.)

*31. TMFI, Part 27: Beyond General Equilibrium, CFDP 417, 1/14/76 (Published No. 18.)

32. TMFI, Part 28: The Noncooperative Equilibria of a Closed Trading Economy with Market Supply and Bidding Strategies (with P. Dubey) CFDP 422, 2/10/76.

33. TMFI, Part 30: The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money, CFDP 424, 2/9/76. (Revised 424R with C. Wilson 6/7/76.)

*34. A Closed Economic System with Production and Exchange Modelled as a Game of Strategy (with P. Dubey) CFDP 429, 5/18/76. (Published No. 23.)

35. A Dynamic Economy with Shares Fiat, Bank and Accounting Money (with J.J.M. Evers), CFDP 431, 5/25/76.

*36. TMFI, Part 31: Banks, Insurance and Futures Markets, CF 60819, August 15, 1976. (Published No. 22.)

37. TMFI, Part 25: A Closed Economy with Exogenous Uncertainty, Different Levels of Information, Money, Futures and Spot Markets, (with P. Dubey) CFDP 414, October 11, 1976.

38. TMFI, Part 32: Bankruptcy and the Money Rate of Interest instead of Excess Supply or Demand, CF-60820, August 25, 1976.

39. TMFI, Part 33: On the Value of Market Information, CFDP 439, November 9, 1976.

40. TMFI, Part 34: A Multiperiod Trading Economy with Fiat Money, Bank Money and an Optimal Bankruptcy Rule, CFDP 441. (Withdrawn for Revision March 1977.)

41. TMFI, Part 35: Bankruptcy and Optimality in a Closed Trading Mass Economy Modelled as a Noncooperative Game, (with P. Dubey) CFDP 448, February 15, 1977.

42. TMFI, Part 36: The Money Rate of Interest, (with P. Dubey) CFDP April, 1977.

43. TMFI, Part 37: Growth in a Bounded Universe, (with P. Dubey) CFDP 1977

44. TMFI, Part 22: The Price-Quantity Bid-Offer Model, CFDP May 1977.

3. AN INFORMAL GUIDE TO THE PAPERS

3.1. General Approach

On the previous pages a listing is given of approximately twenty published papers and forty working papers on the topic of a theory of money and financial institutions. These papers represent work in progress on an attempt to construct a theory of money and financial institutions based upon reformulating the problem of exchange and production among many individuals as an n person noncooperative game.

The stress in this work is upon presenting a complete process model of production and exchange which describes economic activities in situations of equilibrium and disequilibrium. The rules required to completely specify economic trade and production as a well defined game amount to completely specifying the rudimentary institutions and processes for the carrying out of trade.

A theory which completely specifies the market mechanisms for physical goods and for financial instruments contains a rudimentary description of institutions. In other words the rules of the game are tantamount to describing the elementary institutions which carry the processes of trade, production and the exchange of financial instruments.

The construction of an adequate theory of money and financial institutions calls for the solution of a host of different but related problems. The approach adopted here is to devise a great number of models of trade with different market mechanisms and different financial institutions in order to be able to analyze and isolate related but different aspects of an economic system controlled by financial institutions using an array of financial instruments.

It has been suggested frequently that the essence of the understanding of money and financial institutions lies in models of dynamic and subjective as well as objective uncertainty. Although I believe that many of the more important macroeconomic manifestations of financial behavior are associated with subjective uncertainty and the dynamics of economies, it is my belief that the first steps which are needed to be taken in the construction of a sound theory of money and financial institutions must be taken at a much more basic level.

Specifically the question we must address is where did the Walrasian general equilibrium model go wrong when it was fully mathematized? Can we find where a money could play a natural role in a simple exchange economy with no exogenous uncertainty and with no subjective misperceptions? The approach which concentrates on these problems first, is addressed to bridging the gap between micro and macroeconomic theory.

It concentrates on specifying the basic mechanisms which not only describe states of disequilibrium but at the same time will under the appropriate circumstances be completely consistent with the general equilibrium theory. This approach is heavily oriented towards dividing difficulties. There are at least eight allied problem areas which must be investigated. These are indicated in the tables given below. The forty odd papers noted fit into

different parts of these tables. There are some areas indicated with an asterisk or a cross which indicate work in progress or work not yet attempted.

The specific way in which I have tried to split the problem is as follows: the key technical approach is to view the financial and economic system as a noncooperative closed oligopolistic game. In the work discussed here international trade and multiple currencies are ignored--not because I believe that they are not important but because I feel that it would be premature to start to model the type of international law considerations needed to understand the functioning of nation states with independent currency control.

As Hicks noted in Value and Capital an important gap in value theory was the omission of a reasonable theory which included oligopolistic elements. The work here attempts to close this gap. The spirit of the approach is to start by formulating most problems in terms of finite numbers of individuals and finite time periods. A continuum of traders and continuous time are introduced with care only where the mathematical simplification is considerable. The first set of models are for a finite length of time. If financial institutions appear they must be explicitly constructed at one point of time and dismantled at the end of the "game". In this way one must specify how financial instruments get into an economy and eventually get out of the economy. The link with general equilibrium theory can be made by replacing the finite number of traders by a countable infinity of traders or by a continuum. Either of these devices should be looked at as methods for approximating the concept of "many traders in a market".

The eight major lines of investigation followed here are as indicated:

1. Market mechanisms
2. Money and credit instruments
3. Exogenous uncertainty
   a) Insurance
   b) Nonsymmetric information
   c) Subjective probability
4. Dynamics
5. Transactions costs
6. Public goods, government and taxation
7. Labor as a special commodity
8. Utility theory requirements

The bulk of the work completed to date has been on the first four topics. The tables not only show the current state of the work but the numbers prefaced by the letter p indicate articles already or about to be published while those prefaced by the letter c indicate working papers available.

Table 1 presents the overall scheme with each subheading leading to a more detailed table. The usual programming notation is used to indicate cross linkages between topics or articles. Thus an arrow out of a box]→ⓝ with a number in a circle indicates a connection noted elsewhere as ⓝ→ [.

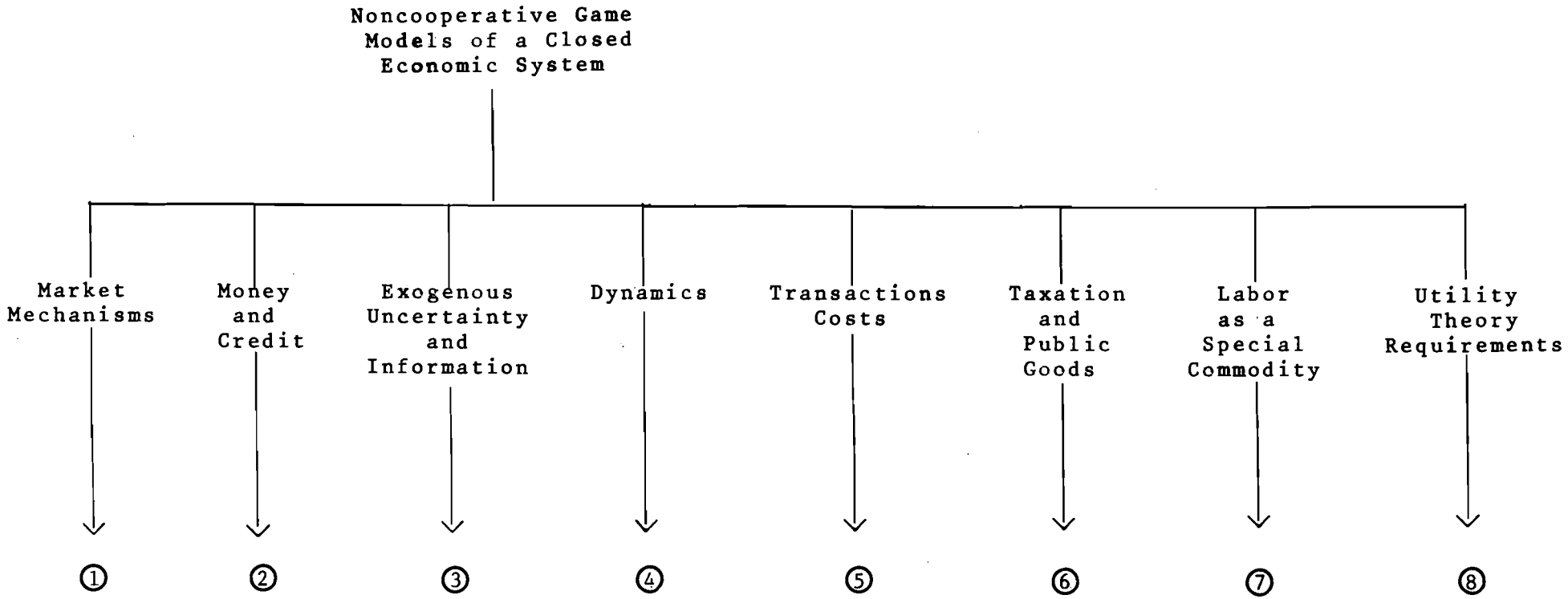

Noncooperative Game Models of a Closed Economic System
Market Mechanisms
Money and Credit
Exogenous Uncertainty and Information
Dynamics
Transactions Costs
Taxation and Public Goods
Labor as a Special Commodity
Utility Theory Requirements
①
②
③
④
⑤
⑥
⑦
⑧

Table 1

3.2. Specific Topics

(1) Market Mechanisms

The first step towards the creation of models of trade and production calls for the explicit description of the methods of trade. At first this seems to be a hopelessly institutional problem with the possibility for many thousands of different markets or other mechanisms of distribution. Fortunately when looked at a little more abstractly we may limit our investigation.

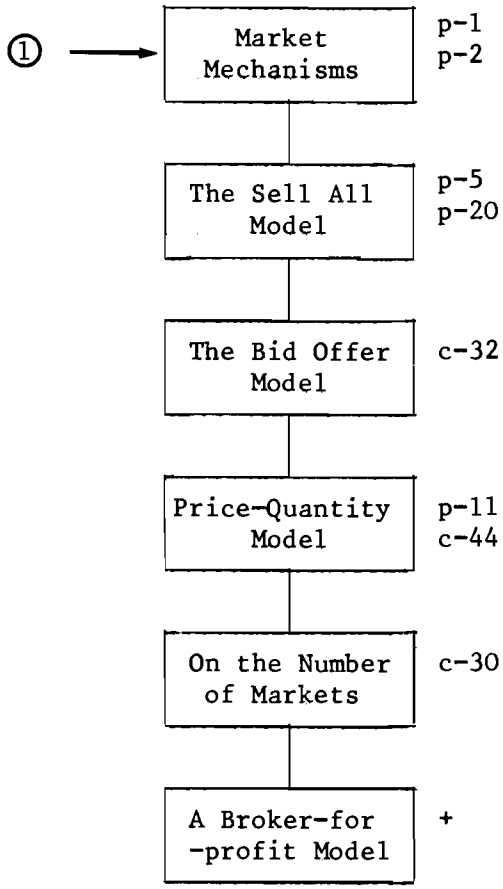


Table 1a

Prior to limiting our concern to economic markets, a reasonable question to ask is how are goods and resources distributed in society in general?

On Different Methods for Allocating Resources (1970) p-1

This paper describes ten different methods in which resources can be allocated. It serves to indicate the different types of market and non-market mechanisms as well as the interface between market and political mechanisms which must be taken into account in the development of an adequate theory of the allocation of economic goods.

Limiting ourselves to economic markets an example of where the role of money in exchange had begun to pose difficulties in microeconomic analysis is provided by Viner's discussion of pecuniary externalities.

Pecuniary Externalities: A Game Theoretic Analysis (1971) p-2

In this paper, it is shown that pecuniary externalities are in fact real, and arise from having trade take place through a specific market structure. The problem in trying to formulate the characteristic or cooperative form of a trading game is used to illustrate this point. It is easy to miss this fundamentally different aspect of partial equilibrium models of the economy in contrast with current general equilibrium models. In the former, market structure and money is implicitly assumed, in the latter, market structure and money are ruled out.

Commodity Money, Oligopoly, Credit and Bankruptcy in a General Equilibrium Model (1973) p-5

This paper presents a noncooperative model of a many-person, many-commodity trading economy. This model stands in contrast with the general equilibrium price model which is mechanistic in formulation, and the various cooperative theories of solution such as the core of a cooperative game. These latter solutions implicitly finesse the problem of describing the strategy spaces of the traders, as they go immediately to a cooperative description of trade which implicitly fails to model many of the strategic limitations on the actions of the traders forced by the process of trade. The noncooperative solution calls for the description of the game in strategic form. As such, it is necessary to specify in total detail how trade takes place. In this model, this is done and in order to have a symmetric game with only one market for each commodity, an extra commodity which serves the purposes of a medium of exchange is introduced.

It is observed that the properties of a money are manifested both in the utility structure and the strategic structure of a market. A commodity money is a substance which has utilitarian intrinsic worth to the traders beyond its value as a means of exchange. A fiat money has no intrinsic worth. It exists as a commodity with special rules concerning its role in exchange, and hence in the determination of individual strategies.

It is noted in this model that the introduction of a commodity money to play a specialized role in trade is not sufficient to guarantee that a limiting noncooperative equilibrium (limiting in the sense of replication of types of traders) will be Pareto optimal in the sense of optimality in a game without trading restraints. In order to obtain optimality, it is necessary to either have "enough of the commodity money" held by all traders or to have a credit system which enables individuals to finance their short-term purchases. However, as soon as one introduces the concept of credit it becomes necessary to take into account the possibility that the system might evolve into a state where an individual cannot pay back the sums he has borrowed. Thus in order to well define a model with credit, bankruptcy rules or rules which make bankruptcy impossible must be specified. These are described in (2) of this section under the heading: Money and Credit Instruments.

This paper does not provide a formal existence proof for the non-cooperative equilibrium and its properties under replication. This proof, due to L. Shapley, will appear in a forthcoming joint publication. A published version of a highly related proof is given by Dubey and Shubik (p-21).

Trading Using One Commodity as a Means of Payment (1974) (1977) p-20

This is a basic expository paper with Lloyd Shapley describing in detail the noncooperative solution to the model of trade in a market with money. The Edgeworth Box is utilized to illustrate the feasible set of trades and the effect on the feasible set of permitting credit. The problems of modeling bankruptcy are also discussed and a heuristic sketch of the general results is given.

The Non-cooperative Equilibria of a Closed Trading Economy with Market Supply and Bidding Strategies (1976) c-33

In the first model with trade using a commodity as a means of payment, for simplicity it was assumed that all commodities must be sold in the markets. In this joint paper with P. Dubey a strategy is a vector of 2m dimensions, where in each of m markets each individual decides both how much to offer for sale and how much he intends to bid. The intuitively appealing simplification that an individual will either offer or bid in a market, but will not do both is not made. It is shown in this paper that when the number of competitors is few it may be strategically meaningful to both bid and offer thus creating a market that is thicker than otherwise by means of "wash sales".

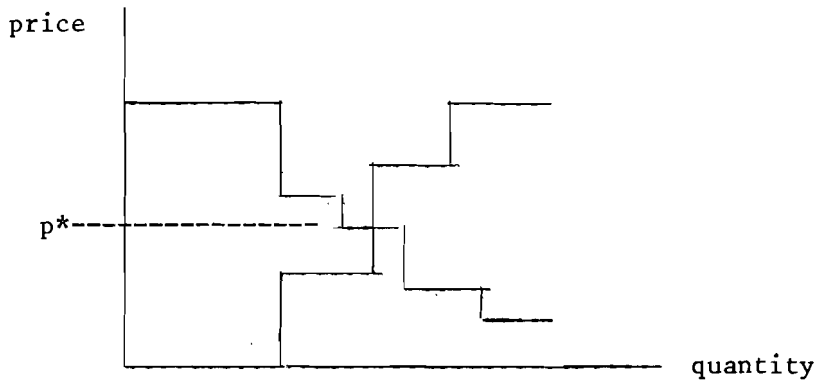
The key phenomenon of added interest in this model is the role of the volume of trade and its relationship to the thickness of markets.

A Trading Model to Avoid Tatonnement Metaphysics (1974) p-11

This is an abstract model of a two-sided auction market. The mechanism for price formation is described explictly and a proof is given that the competitive equilibrium is among the set of noncooperative equilibria for this particular model. It is also observed that this model is well suited for experimental work and the results of several preliminary experiments are reported. The effect of numbers of competitors in the market is also reported upon. In particular, it is noted that the actual process suggested here is relatively closely related to stock market price formation and to the method used for bidding for treasury paper.

The Price-Quantity, Bid-Offer Model (1977) c-44

This is an extension of the previous paper. It is shown that in a market where m commodities are being traded, a noncooperative game where each player has a strategy of 4m dimensions can be defined. Each trader names a price and quantity at which he will buy and a price and quantity at which he will sell. Price is formed at the intersection of the supply and demand histograms as illustrated.



Rationing rules must be specified to take care of excess supply or demand at the market price. It is proved that this excess is not greater than the supply or demand of a single individual. The noncooperative equilibria form a continuous set which contain the competitive equilibrium for all replications.

On the Number of Types of Markets with Trade in Money (1976,1977 revised) c-30

In this paper it is shown that when each trader's move is made simultaneously then there is only a restricted class of models involving actions naming combinations of quantities of goods, quantities of money or prices to be bid or offered. The stress here is upon how to regard markets as mechanisms which transform bids and offers into prices and distributions of goods. The previous papers p-5, p-20, c-32, p-11, c-44 covering the "sell-all"model; the "bid-offer" and the "price-quantity bid offer" variations all serve to show that the structure of trade may make an important difference when numbers are few (or when information conditions differ; see 3 of this section). However the competitive equilibria emerge as limiting noncooperative equilibria for many different market structures when there are many traders.

Some Strategic Noncooperative Models Relating to General Equilibrium (1974) c-21

These are a set of unpublished notes written jointly with Lloyd Shapley in order to formulate the basic mathematical model for the proof of the existence of a noncooperative equilibrium in a closed trading model with a commodity or fiat money.

(2) Money and Credit Instruments

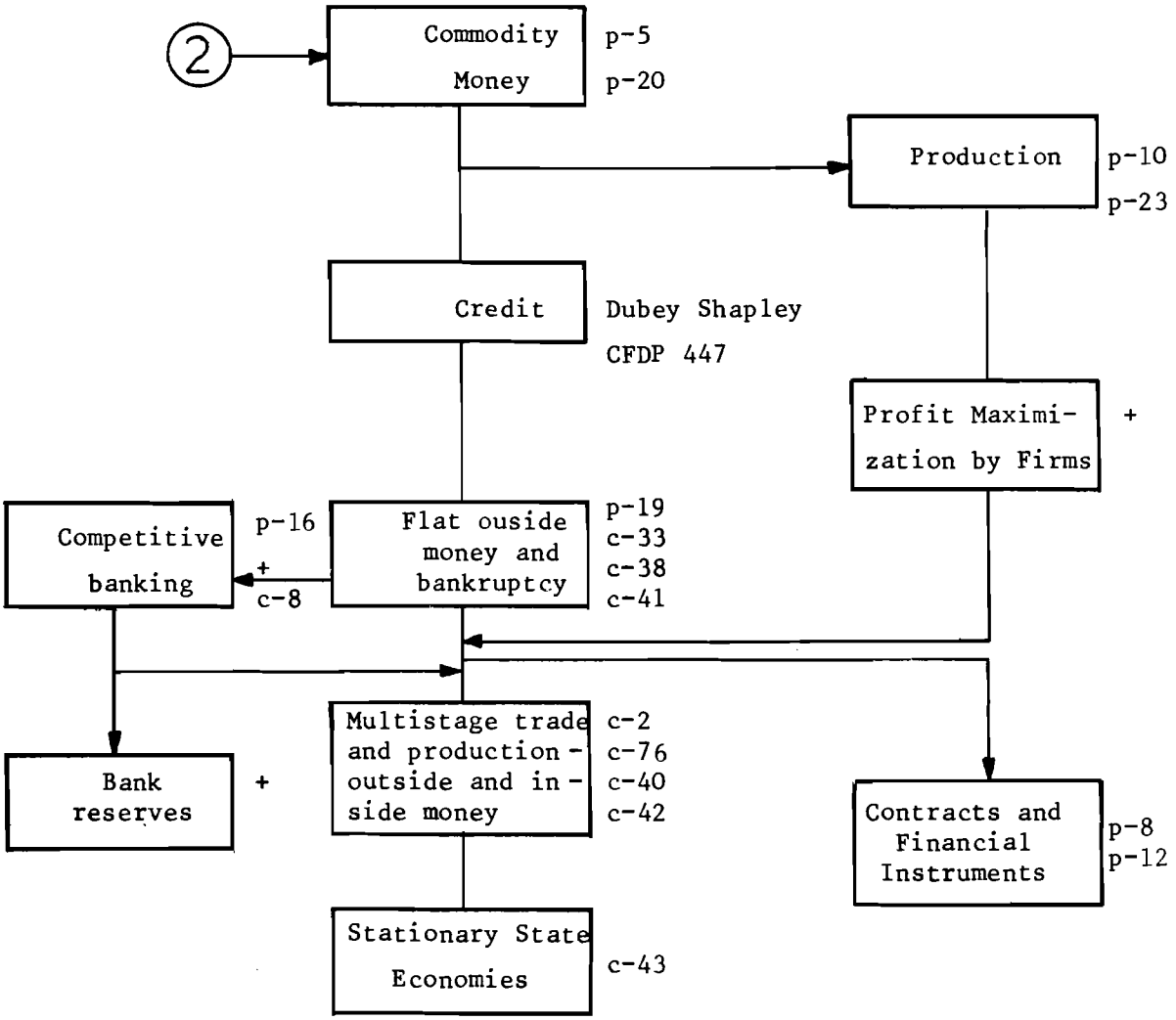


Table 1b

Given market mechanisms which generate prices and the distribution of commodities, if we postulate trade in a commodity money it is easy to show that a shortage of the money could cause trade which is less than optimal when compared with the original Pareto optimal surface unconstrained by the need to trade in a specific commodity.

In order to achieve optimality the shortage of a means of payment may be relieved by the creation of credit or of paper money.

Noncooperative Exchange with a Continuum of Traders Dubey and Shapley (1977) CFDP 447

The simplest way to introduce credit is to imagine that the traders are unconstrained in their bidding. In other words they may bid any amount of a numeraire as though each has an unlimited line of credit at a bank. After trade has taken place and all traders have received their incomes the rules require that all traders pay back whatever they have borrowed. It is possible that after trade the system is in a state such that it is not possible for every trader to pay back what he owes. A penalty must be leveled against any individual who fails to pay back his debts. Otherwise the strategies would be unbounded. If there were no penalty for default every individual could revoke his debts with impunity.

Basing their model on those of Shubik and Shapley and of Shubik, Postlethwaite and Schmeidler analyzed a game with arbitrarily high penalties levied against anyone who fails to repay. Dubey and Shapley considered similar models, but with two differences. Their approach to the measurability of sets of strategies employed by nonatomic traders differs from Postlethwaite and Schmeidler and the bankruptcy penalties rather than being arbitrarily high are introduced as parameters. It is shown that for any positive bankruptcy penalties there will exist noncooperative equilibria which yield the same allocations as the competitive equilibria, but the price level will depend upon the bankruptcy penalties.

Trade with Fiat Money but no Individual Trust--A Preliminary Stage towards Banking (1973), (1977) p-19

Several simple models are discussed and contrasted to show the possibility of using hoarding as "an Alice through the looking glass" type of banking system. The analogy is easier to see when we think of the practice of using rings* in auction bidding. This is the equivalent to having various individuals hoard their money and thereby implicitly increase the purchasing power of others at the time they are hoarding.

The Optimal Bankruptcy Rule in a Trading Economy using Fiat Money (with C. Wilson), (1976) c-33

A two stage model is constructed where in the first stage individuals bid personal I.O.U. notes in return for a share of a fixed amount of money M, supplied by a bank which may be viewed as outside of the game. An individual i will obtain $\frac{u^i}{u}$ M units of money where u^i

* Where a subgroup of individuals collude in the public auction, then by some means allocate the items purchased among themselves privately.

is the amount he bids. He is then in the position to bid on the goods for sale. He bids $(b_1^i,\ldots,b_m^i)$ where $\Sigma\, b_j^1 \leq \frac{u^i}{u} M$. Each individual attempts to maximize an objective function of the form $\phi_i(x_1^i,\ldots,x_m^i) + \mu^i \min[0,\{\frac{u^i}{u} M - \Sigma\, b_j^i + \Sigma\, p_j a_j^i - u^i\}]$ where μ^i is a bankruptcy penalty. Shubik and Wilson take a special case with log utility functions and two types of traders and calculate the noncooperative equilibrium for all parametric values of the μ^i. It is shown that when the bankruptcy penalties are set equal to the Lagrangian multipliers then under repition these are noncooperative equilibria which approcah the competitive equilibrium. If the bankruptcy penalties are set otherwise then hoarding may take place and one or both types of the traders may elect to play strategies which lead to their going bankrupt.

An optimal bankruptcy rule is one which makes strategic bankruptcy unprofitable.

Bankruptcy and Optimality in a Closed Trading Mass Economy Modelled as a Noncooperative Game (with P. Dubey), (1977) c-41

This paper contains the full mathematical generalization and proofs of the results noted in the paper above. A nonatomic game is described and the conditions for the coincidence or lack of coincidence of certain noncooperative equilibria with competitive equilibria are presented.

Bankruptcy and the Money Rate of Interest instead of Excess Supply or Demand (1976) c-38

In the two papers noted above, when there is an N.E.* with some individuals becoming bankrupt, a positive money rate of interest is present in the economy. The rate of interest is only zero when there is a coincidence in distribution between an N.E. and a C.E.** and furthermore when no individual goes bankrupt. It is suggested in this brief note that the rate of interest σ ($\frac{u}{M} = 1 + \sigma$, which can best be described as a loss reserve charge) provides a measure of the "distance" one is from a C.E. At a C.E. $\sigma = 0$. If there are k CEs then it is possible to divide the Pareto optimal surface into k zones with zero σ at the C.E. but with increasing σ in each zone.

The addition of production to a noncooperative game model of trade introduces several new phenomena. A simple example serves to indicate a special role for assets in the enforcing of certain noncooperative equilibria.

A Dynamic Economy with Fiat Money without Banking and with and without Production Goods (1973), (1977) p-10

The important feature illustrated in this model is that assets and ownership paper written against assets play a central role in the

* noncompetitive equilibrium
** competitive equilibrium

enforcement of an efficient noncooperative equilibrium. In particular, the assets can be used as hostages as in escrow arrangments or can be sold outright. Further modifications entail the issuance of ownership paper against the assets and then permitting the ownership paper to be traded as claims against the assets even though the assets themselves may not necessarily physically be moved. A simple two-sided trading example is calculated to show the value of assets as hostages.

A Closed Economic System with Production and Exchange Modelled as a Game of Strategy (with P. Dubey), (1977) p-23

A three stage model of an economy with individuals who own resources and shares in firms which are operated by profit maximizing automata, is examined. In the first stage the firms are able to obtain their inputs; in the second stage they produce and in the third stage all product is sold to the final consumers. Trade takes place using a commodity money.

As the game has several stages considerable care must be taken in specifying the information conditions in order to describe strategies. The conditions are developed for which there will exist noncooperative equilibria which coincide with competitive equilibria. In particular a new phenomenon appears in this model which can best be explained by considering trade in 2m rather than in m goods. It is possible that there may be an initial shortage of supply of a good as a factor where the good can be produced. If this happens it is possible that its price in the first stage is higher than in the third stage.

Several basic difficulties are encountered when trade and production are modelled distinguishing owner-consumers and firms run by managers. In particular:

(1) If trade in shares is permitted when does it take place? (Also when are shares ex-dividend?)

(2) Are the managers of the firms automata or strategic dummies, or do they have preferences of their own?

(3) If they have preferences of their own, when and how will their behavior differ from profit maximization?

(4) What happens to the markets if shares are voting shares? i.e. if a majority share position gives control to a group of stockholders?

(5) What happens to the size of the float in an economy of this type? It appears to depend upon the "depth" of industrialization, i.e. upon the size of intrafirm sales in relation to the size of final consumer demand (all measured in money).

A further minor difficulty with the three stages--one period model with production is that any capital goods left over would have zero worth. It is more or less clear that production is intimately connected with dynamics and a one period model clearly distincts this aspect of the process.

Under the appropriate conditions it appears to be possible to prove that managers with their own preferences may nevertheless operate a corporation in a way that maximizes profits. The specific necessary and/or sufficient conditions have not yet been developed, although a paper on this topic has been started.

In the modeling of trade with a banking system there is a problem posed in the design of a competitive mechanism which supplies the appropriate amount of money to an economy at a competitive rate of interest. When we examine a single period economy it is relatively easy to devise such a mechanism and to show that if there were many bankers, depending upon the mechanism the interest rate would equal or approach zero.

A Noncooperative Model of a Closed Trading Economy with Many Traders and Two Bankers (1974), (1976) p-16

In this model duopolistic banking is considered in two different contexts. In one context, the bankers use price, or rate-of-interest, naming strategies and in the other they use quantity or credit-rationing-strategies. An explicit model is worked out showing the difference in the two models and the change in the solution is considered as the number of traders utilizing the banking facility is increased.

Two problems which have not yet been faced are the design of a competitive banking system in a multiperiod model and the examination of bank reserve requirements. Banking for a multiperiod economy appears to pose an extra difficulty which is not present in the one period economy, that is control over deposits as well as loans, including the possibility of refinancing. It is not clear (at least to me) that it is possible to design a multiperiod model of trade and production with competitive banking which produces both a rate of interest and an appropriate amount of money.

The financing of multiperiod trade appears to be most naturally approached by devising an outside or central bank along with an inside or privately owned banking system. In doing this the central bank may be in a position to control the supply of money which can be issued by the inside banking system by having reserve requirements. We have not yet studied this problem together with the allied difficulty posed by the need to supply rules which either make bank failure impossible, or specify what happens when a bank fails.

The Rate of Interest, Noncooperative Equilibrium and Bankruptcy (with L.S. Shapley) (1972) c-8 (withdrawn for revision)

This was a somewhat premature attempt to build a model of a "money market" where the banking system is nothing more than a brokerage house for the exchange of present for one-period-in-the-future money. Several technical difficulties were not adequately covered in the original version and the paper has been withdrawn for extensive revision.

The Rate of Interest on Fiat Money in a Closed Economy (1972) c-7 (withdrawn)

This was a premature attempt to sketch a justification for a positive money rate of interest which has been completely superceded

by more recent work. It is only noted for the sake of completeness and in the belief that one should acknowledge one's false or inadequate starts.

A Theory of Money and Banking in a General Equilibrim System (1970) c-2

This paper represents an early attempt to pick up the main aspects of a theory of money and the rate of interest by means of cooperative game theory with stress upon the concept of the core of a cooperative game. Although it is possible to obtain some results in this way, this approach was abandoned as it appeared that the cooperative game formulation is not sufficiently detailed concerning the details of strategies and the mechanisms of trade to make it as good a device to study financial phenomena in mass markets as are strategic or normal form models studied for the existence of noncooperative equilibria.

A Multiperiod Trading Economy with Fiat Money, Bank Money and an Optimal Bankruptcy Rule (1977) c-40 (withdrawn for revision)

This paper contains a discursive discussion of many of the problems involved in modeling a multiperiod economy with trade and production and a banking system. The model sketched involves a game played for k periods broken into 2k + 2 stages. In the first stage of the first period a supply of money from an outside bank is auctioned off with the traders bidding using I.O.U. notes. In the second stage they use the outside money to bid for shares in a bank which may be privately held. In the third stage they borrow from or deposit in the private bank. A money rate of interest for inside money is specified exogenously; traders are not restricted in the amounts they can borrow. In the fourth stage each individual bids in the markets for goods. In all subsequent periods there are two stages, refinancing and bidding for goods.

After the k periods are over; at the start of period k + 1 there is a final settlement which involves the final settling of debts to the inside bank, the dissolving of the inside bank and the paying out of its capital, the calling of the outside money supply and finally balancing of the books and execution of the bankruptcy procedure against those who cannot pay their debts.

In the attempt to well define the multiperiod model a considerable number of modeling choices must be made, many of which raise empirical problems in finance. In particular

(1) Should the traders be allowed unrestricted refinancing, or rollover of loans? If yes, this produces a Ponzi game which enables an individual to put off bankruptcy until the end.

(2) Are the shares in the inside bank fully assessable, i.e. are losses as well as profits flowed through to the stockholders?

(3) Is accrual accounting permitted. In particular this would enable the inside bank to report profits it has not yet collected. For example, if an individual at time 1 borrows $1,000 and at time 2 he now owes $1,100 he may pay back nothing, reborrow $1,100 and the bank reports a profit (booked but not received) of $100.

(4) Who runs the inside bank and with what motivation? The simplest assumption is that it is run by a strategic dummy with no decision freedom.

(5) What are the reserve requirements and how are they set? The simplest assumption is that the ability of the bank to grant the credit is not limited.

(6) Do bank reserves earn an interest payment for the bank or are they nonearning assets?

(7) Do all individuals live for precisely the length of the multiperiod economy? Or are there generations with births and deaths and the passing on of assets?

(8) When production is introduced is the economy open or closed? i.e., does it obey laws of conservation, or are the exogenous inputs and outputs at each point in time?

The stress in this paper is on the modeling, with the analysis left to the subsequent paper.

The Money Rate of Interest (with P. Dubey), (1977) c-42

This paper contains the mathematical analysis of the model described in the previous paper noted above. There are three variants of multiperiod trade considered. In the first only outside money is issued. It is shown that noncooperative equilibria exist which coincide in the final distribution of resources with the competitive equilibria; however given only spot market trading and a fixed money supply relative prices from period to period are adjusted by hoarding.

The second variant is that of multiperiod trade without production, but with an inside bank. It is shown that there exist two solutions for which noocoperative equilibria and competitive equilibria coincide. One solution is with the inside money rate of interest $\rho = 0$ and with no one going bankrupt. It must be noted that this is almost, but not quite the same as the case with hoarding. In one instance an inside bank does not exist, in the other it does, but with $\rho = 0$ depositing in the bank cannot be operationally distinguished from hoarding.

A second solution exists with $\rho > 0$; however, although the distribution of resources at the noncooperative equilibrium is the same as at the competitive equilibrium, every trader goes bankrupt by an amount equal to the cost each incurs in financing his part of the float. The outside rate of interest is related to the relative size of the outside money supply and the total float.

It seems counterintuitive that all individuals should go bankrupt in an economy with a positive rate of interest on inside money. A third variant of the basic model can be constructed which avoids this difficulty. In this variant production is introduced. At period $k + 1$ a set of prices $\pi_1,\ldots,\pi_m$ are announced as the "salvage values" of all goods of the society left over at $k + 1$. We may imagine that after the economy closes down at period k a buyer stands ready to purchase left

over goods in order to use them as capital in starting up an economy elsewhere.

As the firms are modeled as profit maximizers the existence of salvage prices encourages them to leave over capital which is sold at the end and the profits are paid out to the shareholders of the firms. It is shown that given any set of salvage prices there will exist a nonnegative rate of interest for which noncooperative equilibria of the nonatomic game coincide in the distribution of goods and in relative prices with the competitive equilibria and no one goes bankrupt.

The relationship between an inside money rate of interest and the value of goods sold in period k + 1 is clearly a relationship between the rate of interest and capital stock although as this result is obtained from a fixed point argument which is essentially static no particular causality pattern is offered.

Quantitative change may easily imply qualitative change. In particular when the number of periods becomes extremely large it becomes somewhat difficult to accept the assumption of the existence of a single generation of long lived individuals with well defined fixed utility functions. If we accept these fairly implausible assumptions then as the time horizon grows we may expect that for bounded economies the money rate of interest approaches zero. In a further paper the possibility of many generations with both processes and intergenerational transfers is considered

Money, Trust and Equilibrim Points in Games in Extensive Form (1974) p-8

A key element that distinguishes noncooperative from competitive equilibria is that the former is much more widely defined and explicitly concerned with the nature of self policing systems where individuals may have strategic influence on the system.

When multistage processes are considered the nature of what constitutes an individual or a joint strategy becomes much more complex than it is for single stage processes. In particular it becomes reasonable to consider strategies which can be interpreted as containing enforecable or nonenforceable threats, promises or other contingencies. When we do so it appears that there is a close relationship between certain strategies, contracts and escrow arrangements.

In particular it can be illustrated that in two societies which are identical in all aspects except that one has more goods suitable for use as "hostages" than the other; then for equal levels of trust more credit arrangements can be enforced in the former than in the latter.

This paper examines the enforecment of loan agreements in terms of strategies and the types of contracts used to help to enforce the repayment of loans.

On the Eight Basic Units of a Dynamic Economy Controlled by Financial Institutions (1975) p-12

It is suggested here that a useful way of viewing the functioning of a modern economy is via eight basic classes of instruments, four of which are clearly contracts and the remaining four are essentially not contracts. The contracts are naturally derived from the other four elements.

In particular the eight elements are as follows:

1. Goods
2. Services
3. Money (outside or fiat)
4. Ownership paper (stocks, deeds, etc.)
5. Futures contracts
6. Service contracts
7. Debt
8. Warrants, calls

The first four items can each be described by a symbol of the variety a^{i}_{kt} which stands, for example, for the amount a of good k owned by individual c at time t. This indicates, a commodity with one owner at one point in time. In contrast a contract requires a more complicated representation such as $(a^{ij}_{kt}, b^{ji}_{\ell,t+1})$ which can be read as follows: an amount a of good k is given to individual i by j at time t in return for an amount b of good ℓ to be delivered to individual j by i at time t + 1.

We may consider a transition matrix which indicates what happens to all of these eight items or instruments after the passage of a unit of time. Money, ownership paper and goods are transformed into identical or similar items; pure services are used up and the contracts, if honored are converted into goods, services money or ownership paper.

It is argued that items such as insurance contracts, bonds, pension plan paper, etc....can be regarded as variants of these basic instruments. Furthermore many of the papers previously noted are specifically devoted to trying to isolate the conditions under which the invention of a new financial instrument or institution appears to be a logical necessity arising from an attempt to well define the rules designed to guide an efficient economic process.

This paper also presents a sketch of a few crude economic statistics merely to indicate where the connections must be made between macroeconomics and theoretical microeconomics.