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Dynamic Linear Programming Model for Agricultural Investment and Resources Utilization Policies

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DYNAMIC LINEAR PROGRAMMING MODEL FOR AGRICULTURAL INVESTMENT AND RESOURCES UTILIZATION POLICIES

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Preface

The food production is one of the most decentralised activity of man with a lot of country and region specific features. Accordingly the food and agriculture research at IIASA relies upon investigations at the national and regional level. The regional as well as micro approach enable us to study the agricultural systems in their proper local economic, social and natural environment and hopefully lead to conclusions about the universal problems of regional agricultural development.

The model presented in this paper has been developed as a part of a IIASA-Bulgarian joint research work on the methodological problems of regional agro-industrial development. This paper is closely connected with the study of Carter, H.O., Csåki, C., Propoi, A.* on dynamic linear programming models for agro-industrial development outlining a flexible procedure for modelling of agricultural investment policies and the associated resource utilization programs.

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* Carter, H.O., Csåki, C. and Propoi, A. (1977), Planning Long Range Agricultural Investment Projects: A Dynamic Linear Programming Approach. RM-77-38, IIASA, Laxenburg, Austria.

Summary

In this paper a dynamic linear programming model for large scale farm development is presented. Special emphasis is placed upon the modelling of investment policies and associated resource utilization programs.

Therefore, first a flexible modelling procedure to handle capital input constraints of farms is outlined. The method is discussed within the framework of a traditional linear programming model and a numerical example is also given showing how this procedure can be used to formulate models of farm development for decision making purposes.

In the second half of the study a dynamic linear programming model covering the whole farming system and the whole planning horizon is described. It grows out that static model which has been introduced previously optimizing both production mix and resource utilization. Beside the description of the crop and animal production as well as resource utilization subsystems the model includes a very detailed financial subsystem, too.

At the end of the paper the experiences gained in Hungary in the use of different linear and dynamic linear programming models of agricultural development are summarized.

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1. Introduction

Agricultural production is one of the most complex and many-sided activities of mankind involving coordination of biological, technical, human and economical factors. In recent years considerable efforts have been devoted to the analysis and modelling of agricultural systems. Models describing agricultural systems may be formulated emphasizing different aspects of agricultural production, using various techniques and various degrees of detail and sophistication. Using complicated stochastic-dynamic methods, an adequate description of agricultural systems can be considered. But in the present data situation and given the existing computing facilities, these models cannot serve as a basis for wider practical applications. Dynamic linear programming seems to be one of the most appropriate techniques for agricultural planning purposes.

In this paper a dynamic linear programming model for farm development is presented. Special emphasis is placed upon the modelling of investment policies and the associated resource utilization programs. The model being discussed is based on the experiences of an agricultural modelling work conducted in large scale Hungarian farms for the past few years and has been developed as a part of IIASA's research on modelling of long range agro-industrial development.* Though large scale farms and agro-industrial combines with mixed production structure are not typical for most countries, we believe that the investigation of such problems, in order to select the best investment program and the most efficient use of resources, are of interest for policy makers and farm operators everywhere in the world.

2. Modelling of Capital Inputs of Agricultural Production

Dynamic linear programming models of agricultural systems can be constructed in various ways. A general description of DLP models of agricultural development is given by H. Carter, C. Csaki, A. Propoi [5]. As it has been indicated by many authors one of the most crucial problems of these models is the handling of capital inputs and resource utilization. According to the experience acquired in several countries the practical significance of production plans derived by DLP models depends to a large degree upon the way of modelling capital inputs.

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* The author is grateful to H. Carter, A. Propoi and F. Rabar for helpful comments.

The conventional and general interpretation of dynamic linear programming is that constraints on inputs establish a relation between the derived demand of such inputs and the available supply. Yet, there exist several ways of deriving the demand for capital resources and determining the appropriate supply of them. The more articulate and flexible the formulation of this portion of the model of agricultural development, the greater the opportunity to describe the system with a satisfactory degree of realism and practicality.

Therefore, before describing the DLP model - covering the whole farming system and the whole planning horizon - a comprehensive modelling procedure to handle the capital input constraints of farms is presented. This method is discussed within the framework of a traditional linear programming model, but of course this approach is applied in the DLP model outlined under point 3. An example will also be given showing how this procedure can be used to formulate models of agricultural investments and resource utilization programs in a very flexible manner for decision-making purposes.

2.1 The Determination of the Optimal Program of Resource Utilization.

The conventional way of agricultural modelling to determine the requirements for capital inputs is preliminarily to fix the production techniques as well as technologies and the relative model coefficients. This is also the most restrictive and inflexible way. In this case, the computed program is optimal only conditionally: no optimization with respect to production technologies and techniques is considered. In general, such optimization cannot be carried out prior to solving the overall optimization problem. Clearly, an input coefficient considered optimal at the activity level of any production branch may no longer be optimal within the context of the agricultural system as a whole. Without sufficient knowledge about the entire production structure and the functional relationship of all resources involved, the decisions about the distribution of capital's fixed costs cannot be made in an optimal way. Thus, the *ex ante* specification of techniques and production coefficients can be a serious source of error.

The construction of a model which includes also the optimization of the techniques and certain elements of production technologies in relation to the availability of capital inputs is a complex undertaking, but, according to our experience a fruitful one. The procedure can be briefly outlined as follows. Special variables must be assigned to each capital resource employed in each operation (production task) in every production activity. Their values indicate the quantity of inputs needed for the production of different commodities and the various areas (activities, sectors) of resource utilization. We call these variables "resource utilization variables". Thus, the model must include as many resource utilization variables for every kind of capital input as there are production tasks that can be performed using this resource.

Additional constraints need to be introduced as well. They establish the relationship between the request for the execution of a production task and the specification of how such an operation will be best performed. A constraint is introduced for every production task associated with each production activity. It compares the required volume of tasks with the possible available techniques for performing them. Symbolically, such constraints can be specified as follows:

(a) Crop production activities:

$$(1) \quad t_{\beta(\gamma\delta\varepsilon)} x_{\gamma\delta\varepsilon} - \Sigma_\alpha g_{\alpha(\gamma\delta\varepsilon)\beta}\, v_{\alpha(\gamma\delta\varepsilon)\beta} \leq 0$$

where t_β is a coefficient which expresses the unit volume of production task β required for growing crop γ on soil δ for commodity utilization ε; $x_{\gamma\delta\varepsilon}$ is the level of production of crop γ on soil δ for commodity utilization ε; $g_{\alpha(\gamma\delta\varepsilon)\beta}$ is the coefficient which expresses the unit volume of performance of resource input α when used for task β on crop activity $(\gamma\delta\varepsilon)$; $v_{\alpha(\gamma\delta\varepsilon)\beta}$ is the volume of resource α used in performing task β on crop activity $(\gamma\delta\varepsilon)$.

(b) Animal production activities:

$$(2) \quad p_{\beta(\eta\theta)}\, y_{\eta\theta} - \Sigma_\alpha d_{\alpha(\eta\theta)\beta}\, v_{\alpha(\eta\theta)\beta} \leq 0$$

where $p_{\beta(\eta\beta)}$ is a coefficient which expresses the unit volume of production task β for raising animal type η according to method θ; $y_{\eta\theta}$ is the level of production of animal activity $\eta\theta$; $d_{\alpha(\eta\theta)\beta}$ is the coefficient which expresses the unit volume of performance of resource input α when used for task β on animal activity $\eta\theta$; $v_{\alpha(\eta\theta)\beta}$ is the level of resource α used for performing production task β associated with animal activity $(\eta\theta)$.

The number of these constraints in the model will obviously depend upon the number of production activities and tasks. If the same capital inputs are used for different activities, similar production tasks can be aggregated into one constraint. Further restrictions involving capital inputs, however, could be suggested by technological or biological reasons. Such restrictions may usually take the form of upper bounds or rates as follows:

(a) Restriction by constant:

$$(3a) \quad v_{\alpha(\gamma\delta\varepsilon)\beta} \lesseqgtr q_\beta \qquad\qquad (3b) \quad v_{\alpha(\eta\theta)\beta} \lesseqgtr r_\beta$$

where either q_β and r_p are the minimum or maximum volume of use of resource for the production task β relative to crop activity $x_{\gamma\delta\varepsilon}$ or animal activity $y_{\eta\theta}$ respectively.

(b) Restriction by rates:

(4a) $v_{\alpha(\gamma\delta\varepsilon)\beta} \lesseqgtr a_{\beta}\Sigma_{\alpha}v_{\alpha\ \gamma\delta\varepsilon\ \beta}$

(4b) $v_{\alpha(\eta\theta)\beta} \lesseqgtr c_{\beta}\Sigma_{\alpha}v_{\alpha\ \gamma\delta\varepsilon\ \beta}$

where a_β and c_β are the coefficients that express either a minimum or a maximum rate of use of resource α for the task β relative to crop activity $x_{\gamma\delta\varepsilon}$ and animal activity $y_{\eta\theta}$ respectively. According to our experience, using the latter type of constraints is more convenient. In fact, it is difficult, in general, to determine appropriately fixed levels of resource use at the time of the construction of the models.

The advantage of working with models that include the explicit optimization of the production techniques and technologies is quite obvious, they allow quite accurate and detailed determination of resource use. The size of such models, however, soon becomes a multiple of those with fixed technology. When the computing facilities are limited, the researcher may be compelled to adopt one of the following alternative procedures:

(a) preliminary optimization of the production technology;

(b) inclusion of several techniques for the production of each commodity;

(c) allowing substitution among resources;

(d) optimization of some critical aspects of technology;

(a) Preliminary optimization of production technology at the commodity level -- preliminary fixing of input requirements constitutes no problem if one can select techniques for the various sectors which are best at the level of the whole farming system. If this assumption is warranted, partial models for determining the optimal technology at the individual production sector can easily be constructed. By using the optimal techniques determined by these partial models the unconditional optimal solution of the overall farm problem can be substantially approximated.

(b) Inclusion of several techniques (complex technological systems) for the production of each commodity. -- This is one of the most common ways to tackle the problem. In general there are several ways for producing a given commodity. To account for this, the production of such commodity is represented in the model by more than one activity. Of course, one cannot guarantee the optimality of input requirements associated with the activities constituting the optimal program.

(c) Allowing substitution among resources. -- The simplest type of substitution is the one-way substitution involving only two resources α and β: resource α could be substituted for resource β.

More complex types of substitution are the bilateral and the multilateral substitution. The approach of multilateral substitution closely approximates the model for complete optimization of resource use described in this paper. The one-way substitution can be conveniently used in connection with resource bottlenecks in models based on ex ante fixed coefficients.

(d) Optimization of some critical aspects of technology. -- According to this approach, the method for optimization of the resource utilization described in this paper is applied only to production tasks judged as critical for the determination of the structure of resource use. The efficiency of this approach obviously depends upon the ability to choose a priori the critical production tasks.

2.2 The Determination of the Optimal Program of Resource Needs

The second group of crucial aspects associated with the modelling of resource utilization deals with the specification of capital inputs availability. As for the resource utilization portion of the problem, the volume of resources needed for the realization of the production plan can be assembled in various manners with different degrees of generality.

The simplest and perhaps the most common way to deal with capital input constraints is to specify a constant availability (upper bound) of the given resource throughout the production period. (e.g. ten 50 HP tractors for every month). This approach has limited validity in general. The type and volume of given resources depend closely upon the functional relation with all other elements of the production plan. Hence, large savings can be engineered through a careful specification of the input supply.

It is convenient to classify the resources of agricultural production into two groups as:

(a) Inflexible Restrictions: The available quantity of resources in this group cannot be varied during the planning horizon either because it is very expensive, or undesirable, or physically or biologically impossible, therefore they are represented as upper bounds in the model.

(b) Flexible Restrictions: The quantity of available resource belonging to this group is not known a priori, but it is determined as a decision variable by the specification of the solution of the model.

To formulate in a flexible way the portion of the model dealing with the availability of capital inputs, new variables must be introduced. They will be called resource-need variables. The value of these variables shows the (capacity) quantity of resources necessary for the realization of the optimal production plan. They may represent quantities of either owned, purchased or rented resources and directly depend on the resource-utilization variables. The particular use of resource-need variables in a dynamic linear programming model depends upon the character of the resource itself. In connection with this problem, flexible resources can be further classified into two subgroups those having a circulation fund character and those representing strictly capital inputs. No special problem arises with resources belonging to the first subgroup. Their availability to farm operators can easily be adjusted as required by the plan. It can be made up of quantities already owned or purchased by the farm.

More interesting and more complex is the specification problem dealing with resource-need variables representing capital inputs. Generally, such resource-need variables include building, machinery and equipment and, thus, either constitute the existing stocks of inputs or indicate the required new investment policies. The owned stocks of resources place an obvious limit on the resource-need variables associated with the initially existing capacity, while the variables expressing the need of new resources are restricted only by financial possibilities and other investment conditions. Within this framework, either rates or total utilization of some (or all) resources can be prescribed. A symbolic specification of the above discussion can be outlined according to whether or not the input coefficients for each production activity are selected a priori, and according to whether resource availabilities are either flexible or inflexible. Thus, in the case of pre-selected input coefficients the constraint relative tc resource α can be written as:

$$(5)\quad \Sigma_{\gamma\delta\varepsilon}\alpha_{\gamma\delta\varepsilon}x_{\gamma\delta\varepsilon} + \Sigma_{\eta\theta}\alpha_{\eta\theta}y_{\eta\theta} + \Sigma_{\omega}\alpha_{\omega}u_{\omega} - (f_{\alpha\nu}w_{\alpha\nu} + f_{\alpha\mu}w_{\alpha\mu}) \leq 0$$

where the α's indicate the various input coefficients; u_{ω} indicates the level of commercial activity ω, $w_{\alpha\nu}$ indicate the level of new investment of resource α necessary for the realization of the optimal production plan; $w_{\alpha\nu}$ is the level of the existing stock of resource α available for utilization; $f_{\alpha\nu}$ and $f_{\alpha\mu}$ are the capacity coefficients associated with the new and existing stocks of resources α. In the more general case involving the optimization of techniques and technologies, the constraints can be formulated for all types of resources as follows:

$$(6)\quad \Sigma_{\beta(\gamma\delta\varepsilon)}v_{\alpha(\gamma\delta\varepsilon)\beta} + \Sigma_{\beta(\eta\theta)}v_{\alpha(\nu\theta)\beta} - (f_{(\alpha\nu)}w_{\alpha\nu} + f_{(\alpha\mu)}w_{\alpha\mu}) \leq 0$$

In such a case, this type of constraint is utilized in conjunction with those indicated in relations (1) and (2). In order to add flexibility to the model, constraints of the type indicated above should be defined for short periods of time. Of course, this suggestion should be balanced against the rate of increase in the size of the overall model.

2.3 The Formulation of the Objective Function

Regardless of what the economic objective might be, firm managers operating in either a socialist or a capitalist economy have to find a satisfactory solution to the problem of handling "fixed" (not directly related to the level of resource utilization*) and "variable" (directly connected with the level of resource utilization) costs associated with capital inputs. By the use of the

* The "fixed" costs may vary according to the accounting system of countries.

procedure suggested above it is possible to offer an interesting answer to such a problem. In a natural way in fact, the variable costs of an input resource becomes the objective function coefficients of the resource-utilization variables, where the fixed costs are represented by the coefficients of the resource-need variables. This approach to the capital input variables allows for a more flexible and correct allocation of both variable and fixed costs, of say, machinery, equipment, and building to the individual enterprises, and, in fact, to the individual operations (tasks). In this context, the researcher can also recognize and deal efficiently with the fact that capital inputs come, in general, in nondivisible units. Thus, when funds and appropriate computing facilities are available the resource-need variables can be stated as integer variables. When specified as noninteger variables as in ordinal linear programming, the correct allocation of fixed costs is only approximated.

Symbolically the objective function may be indicated as follows:

$$(7)\quad \max(\Sigma_{\gamma\delta\varepsilon} r_{\gamma\delta\varepsilon} x_{\gamma\delta\varepsilon} + \Sigma_{\eta\theta} r_{\eta\theta} y_{\eta\theta} + \Sigma_{\omega} r_{\omega} u_{\omega} -$$

$$- \Sigma_{\alpha(\gamma\delta\varepsilon)\beta} r_{\alpha(\gamma\delta\varepsilon)\beta} v_{\alpha(\gamma\delta\varepsilon)\beta} - \Sigma_{\alpha(\eta\theta)\beta} r_{\alpha(\eta\theta)\beta} v_{\alpha(\eta\theta)\beta} -$$

$$- \Sigma_{\alpha} r_{\alpha\mu} w_{\alpha\mu} - \Sigma_{\alpha} r_{\alpha\nu} w_{\alpha\nu}).$$

Where u_{ω} is the level of commercial activity ω, $r_{\gamma\delta\varepsilon}$, $r_{\eta\theta}$ and r_{ω} are coefficients expressing the unit value of returns minus production expenses excluding those associated with capital inputs of the corresponding production and commercial variables, $r_{\alpha(\gamma\delta\varepsilon)\beta}$ and $r_{\alpha(\eta\theta)\beta}$ express the variable costs related to a specific use (for solution of production task β for growing crop γ on soil δ for commodity utilization ε, or for performing production task β associated with animal activity $(\eta\theta)$) of resource α, $r_{\alpha\mu}$ and $r_{\alpha\nu}$ are unit fixed costs on resource α.

2.4 An Hypothetical Example

To illustrate the use of the approach presented above, let us discuss the example given in Table 1. The data used are all hypothetical.

Suppose a farm is constituted by 3,000 acres, 2,500 of which are of type A soil while the remaining 500 acres are of type B soil. Wheat, corn, and alfalfa are the crop activities open to the grower. Corn and alfalfa hay can be grown only on type A soil. Alfalfa can be processed into pellets. Besides the

TABLE 1

An Example of the Use of Resource-Utilization and Resource-Needs Variablab

Constraints	Units			Crops: Wheat on type A soil	Crops: Wheat on type B soil	Crops: Corn on type A soil	Crops: Alfalfa on type A soil	Crops: Alfalfa on type B soil	Alfalfa-pellets processing	Buying and selling: Corn purchase	Buying and selling: Corn sale	Buying and selling: Pellets sale
				1	2	3	4	5	6	7	8	9
1 A-type soil	Acres	2,500	≥	1		1	1					
2 B-type soil	"	500	≥		1			1				
3 Max wheat	"	1,500	≥	1	1							
4 Max alfalfa	"	1,200	≥				1	1				
5 Period 1 - corn cultivation	"	0	≥			1						
6 Period 1 - alfalfa cultivation	"	0	≥				1	1				
7 Period 2 - wheat harvest	Cwt	0	≥	40	35							
8 Period 3 - wheat A-soil preparation	Acres	0	≥	1								
9 Period 3 - wheat B-soil praparation	"	0	≥		1							
10 Period 3 - combine corn harvest	Cwt	0	≥			20						
11 Period 3 - 75 HP - Z equipment corn harvest	Cwt	0	=									
12 50 HP tractor needs - period 1	Machines	0	≥									
13 50 HP tractor needs - period 3	"	0	≥									
14 75 HP tractor needs - period 1	"	0	≥									
15 75 HP tractor needs - period 3	"	0	≥									
16 Equipment A needs - period 1	"	0	≥									
17 Equipment A needs - period 3	"	0	≥									
18 Equipment B needs - period 1	"	0	≥									
19 Equipment B needs - period 3	"	0	≥									
20 Equipment C needs - period 1	"	0	≥									
21 Combine SZ - period 2	"	0	≥									
22 Combine SZ - period 3	"	0	≥									
23 Combine E - period 2	"	0	≥									
24 Combine E - period 3	"	0	≥									
25 Equipment E - period 3	"	0	≥									
26 Equipment Z - period 3	"	0	≥									
27 50 HP tractor-existing availability	"	10	≥									
28 Equipment A-existing availability	"	7	≥									
29 Equipment C-existing availability	"	4	≥									
30 Combine SZ-existing availability	"	3	≥									
31 Equipment Z-existing availability	"	4	≥									
32 Corn balance	Cwt	0	=			40				1	-1	
33 Alfalfa balance	Cwt	0	=				400	400	- 40			
34 Labor availability	Man/hour	40,000	≥	5	5	10	4	4	.1			
35 Capital investment	Dollar	0	≤	1,600	1,000	- 600	- 500	- 520	- 400	- 88	80	120
36 Pellets plant capacity	Cwt	3,000	≥						10			
37 Pellets balance	Cwt	0	=						10			-1
Objective function	Max			4,000	4,900	-1,500	-1,250	-1,100	-1,000	-220	200	300

TABLE 1 (Con't.)

An Example of the Use of Resource-Utilization and Resource-Needs Variables

Constraints	Units			Resource utilization variables: Period 1, Corn operations: 50 HP tractor type A equipment	50 HP tractor type B equipment	75 HP tractor type A equipment	75 HP tractor type B equipment	Alfalfa tasks: 50 HP tractor type C equipment	75 HP tractor type C equipment	Period 2, Wheat harvest: Combine SZ	Combine E	Wheat type A soil: 50 HP equipment A
				10	11	12	13	14	15	16	17	18
1 A-type soil	Acres	2,500	≥									
2 B-type soil	"	500	≥									
3 Max wheat	"	1,500	≥									
4 Max alfalfa	"	1,200	≥									
5 Period 1 - corn cultivation	"	0	≥	- 36	- 34	- 44	- 42					
6 Period 1 - alfalfa cultivation	"	0	≥					- 150	- 175			
7 Period 2 - wheat harvest	Cwt	0	≥							-9,000	-1,200	
8 Period 3 - wheat A-soil preparation	Acres	0	≥									- 50
9 Period 3 - wheat B-soil preparation	"	0	≥									
10 Period 3 - combine corn harvest	Cwt	0	≥									
11 Period 3 - 75 HP-Z equipment corn harvest	Cwt	0	=		34		42					
12 50 HP tractor needs - period 1	Machines	0	≥	1	1			1				
13 50 HP tractor needs - period 3	"	0	≥									1
14 75 HP tractor needs - period 1	"	0	≥			1	1		1			
15 75 HP tractor needs - period 3	"	0	≥									
16 Equipment A needs - period 1	"	0	≥	1		1						
17 Equipment A needs - period 3	"	0	≥									1
18 Equipment B needs - period 1	"	0	≥		1		1					
19 Equipment B needs - period 3	"	0	≥									
20 Equipment C needs - period 1	"	0	≥					1	1			
21 Combine SZ - period 2	"	0	≥							1		
22 Combine SZ - period 3	"	0	≥									
23 Combine E - period 2	"	0	≥								1	
24 Combine E - period 3	"	0	≥									
25 Equipment E - period 3	"	0	≥									
26 Equipment Z - period 3	"	0	≥									
27 50 HP tractor-existing availability	"	10	≥									
28 Equipment A-existing availability	"	7	≥									
29 Equipment C-existing availability	"	4	≥									
30 Combine SZ-existing availability	"	3	≥									
31 Equipment Z-existing availability	"	4	≥									
32 Corn balance	Cwt	0	=									
33 Alfalfa balance	Cwt	0	=									
34 Labor availability	Man/hour	40,000	≥									
35 Capital investment	Dollar	0	≤	- 840	- 880	-1,080	-1,120	- 800	-1,040	-2,700	-3,600	-1,260
36 Pellets plant capacity	Cwt	3,000	≥									
37 Pellets balance	Cwt	0	=									
Objective function	Max			-2,100	-2,200	-2,700	-2,800	-2,000	-2,600	-6,750	-9,000	-3,150

TABLE 1 (Con't.)

An Example of the Use of Resource-Utilization and Resource-Needs Variables

Constraints	Units			Resource utilization variables: Period 3: Wheat type A soil: 75 HP equip-ment A	Period 3: Soil preparation Type B soil: 50 HP equip-ment B	Period 3: Soil preparation Type B soil: 75 HP equip-ment B	Period 3: Corn harvest: Combine SZ equip-ment E	Period 3: Corn harvest: Combine E - equip-ment E	Period 3: Corn harvest: 75 HP equip-ment Z	Resource need variable: Available machinery and equipment: 50 HP tractor	Equip-ment A	Equip-ment C	Combine SZ	Equip-ment Z
				19	20	21	22	23	24	25	26	27	28	29
1 A-type soil	Acres	2,500	≥											
2 B-type soil	"	500	≥											
3 Max wheat	"	1,500	≥											
4 Max alfalfa	"	1,200	≥											
5 Period 1 - corn cultivation	"	0	≥											
6 Period 1 - alfalfa cultivation	"	0	≥											
7 Period 2 - wheat harvest	Cwt	0	≥											
8 Period 3 - wheat A-soil preparation	Acres	0	≥	- 45										
9 Period 3 - wheat B-soil preparation	"	0	≥		- 45	- 40								
10 Period 3 - combine corn harvest	Cwt	0	≥				- 180	- 225	- 180					
11 Period 3 - 75 HP - Z equipment corn harvest	Cwt	0	=						- 180					
12 50 HP tractor needs - period 1	Machines	0	≥							- 1				
13 50 HP tractor needs - period 3	"	0	≥		1					- 1				
14 75 HP tractor needs - period 1	"	0	≥											
15 75 HP tractor needs - period 3	"	0	≥	1		1			1					
16 Equipment A needs - period 1	"	0	≥								- 1			
17 Equipment A needs - period 3	"	0	≥	1							- 1			
18 Equipment B needs - period 1	"	0	≥											
19 Equipment B needs - period 3	"	0	≥		1	1								
20 Equipment C needs - period 1	"	0	≥									- 1		
21 Combine SZ - period 2	"	0	≥										- 1	
22 Combine SZ - period 3	"	0	≥				1						- 1	
23 Combine E - period 2	"	0	≥											
24 Combine E - period 3	"	0	≥					1						
25 Equipment E - period 3	"	0	≥				1	1						
26 Equipment Z - period 3	"	0	≥						1					- 1
27 50 HP tractor-existing availability	"	10	≥							1				
28 Equipment A-existing availability	"	7	≥								1			
29 Equipment C-existing availability	"	4	≥									1		
30 Combine SZ-existing availability	"	3	≥										1	
31 Equipment Z-existing availability	"	4	≥											1
32 Corn balance	Cwt	0	=											
33 Alfalfa balance	Cwt	0	=											
34 Labor availability	Man/hour	40,000	≥											
35 Capital investment	Dollar	0	≤	-1,620	-1,320	-1,680	-3,060	-3,960	-1,900	-4,000	- 800	-1,200	-12,000	-4,000
36 Pellets plant capacity	Cwt	3,000	≥											
37 Pellets balance	Cwt	0	=											
Objective function	Max			-4,050	-3,300	-4,200	-7,650	-9,500	-4,750	-10,000	-2,000	-3,000	-30,000	-10,00

TABLE 1 (Con't.)

An Example of the Use of Resource-Utilization and Resource-Needs Variables

Constraints	Units			Resource need variable: New machinery and equipment: 50 HP tractor 30	75 HP tractor 31	Equipment A 32	Equipment B 33	Equipment C 34	Combine SZ 35	Combine E 36	Equipment E 37	Equipment Z 38
1 A-type soil	Acres	2,500	≥									
2 B-type soil	"	500	≥									
3 Max wheat	"	1,500	≥									
4 Max alfalfa	"	1,200	≥									
5 Period 1 - corn cultivation	"	0	≥									
6 Period 1 - alfalfa cultivation	"	0	≥									
7 Period 2 - wheat harvest	Cwt	0	≥									
8 Period 3 - wheat A-soil preparation	Acres	0	≥									
9 Period 3 - wheat B-soil preparation	"	0	≥									
10 Period 3 - combine corn harvest	Cwt	0	≥									
11 Period 3 - 75 HP - Z equipment corn harvest	Cwt	0	=									
12 50 HP tractor needs - period 1	Machines	0	≥	- 1								
13 50 HP tractor needs - period 3	"	0	≥	- 1								
14 75 HP tractor needs - period 1	"	0	≥		- 1							
15 75 HP tractor needs - period 3	"	0	≥		- 1							
16 Equipment A needs - period 1	"	0	≥			- 1						
17 Equipment A needs - period 3	"	0	≥			- 1						
18 Equipment B needs - period 1	"	0	≥				- 1					
19 Equipment B needs - period 3	"	0	≥				- 1					
20 Equipment C needs - period 1	"	0	≥					- 1				
21 Combine SZ - period 2	"	0	≥						- 1			
22 Combine SZ - period 3	"	0	≥						- 1			
23 Combine E - period 2	"	0	≥							- 1		
24 Combine E - period 3	"	0	≥							- 1		
25 Equipment E - period 3	"	0	≥								- 1	
26 Equipment Z - period 3	"	0	≥									- 1
27 50 HP tractor-existing availability	"	10	≥									
28 Equipment A-existing availability	"	7	≥									
29 Equipment C-existing availability	"	4	≥									
30 Combine SZ-existing availability	"	3	≥									
31 Equipment Z-existing availability	"	4	≥									
32 Corn balance	Cwt	0	=									
33 Alfalfa balance	Cwt	0	=									
34 Labor availability	Man/hour	40,000	>									
35 Capital investment	Dollar	0	≤	-14,000	-21,000	-28,000	-4,200	-4,200	-4,200	-7,000	-14,000	-14,000
36 Pellets plant capacity	Cwt	3,000	≥									
37 Pellets balance	Cwt	0	=									
Objective function	Max			-10,000	-15,000	-2,000	-3,000	-3,000	-30,000	-50,000	-10,000	-10,000

production activities mentioned above the following buying and/or selling activities are contemplated: purchase* and sale of corn, and the sale of alfalfa pellets.

The constraints of the model can conveniently be divided into four groups:

(a) The first group includes constraints (Nos. 1-4) associated with soil capacity. Wheat is limited to fifty percent of the total land, while alfalfa may be grown on forty percent of all land.

(b) The second group of constraints expresses the possible use of capital inputs (machinery and equipment) in crop activities. The production period is arbitrarily divided into three subperiods of 30, 45, and 45 work-days respectively. Only few operations (production tasks) for each crop activity is considered. This group of constraints (Nos 6-11) requires the introduction of the resource-utilization variables. The coefficients of such variables show the unit task-output of various machinery-equipment combinations, during the time periods suitable for the execution of the task. The unit of measurement of the resource-utilization coefficients is the most natural to describe the performance of the various variables. Hence, the task-output of a tractor-and-implement type of machinery will be measured in acres/hour, while the performance of a wheat combine is more properly measured in, say, hundred weight/day.

(c) The third group of constraints (Nos. 12-31) expresses the relation between resource-utilization and resource-need variables. If some resource-utilization variable is called into operation to guarantee the execution of some operation, the machinery/equipment combination must be readily available. Resource-need variables may be possessed beforehand by the farm, or may be purchased in accordance with their demand. In this example, we assume that the quantities of resource-need variables associated with owned capacities are as follows: 10 50-HP tractors, 7 type-A implements, 4 hay-bailers, 3 SZ-type combines, 4 type-Z corn harvesters.

(d) Miscellaneous constraints mostly self-explanatory are described in the fourth group. A simple type of financial constraint is included: it is supposed that 40 percent of net revenue may be used for new investments.

To the constraints described in groups (b), and (c) there are two corresponding groups of activities (variables) which have been the main focus of discussion in this note: resource-utilization and resource-need activities. Their description as reported in Table 1 is self-explanatory and their interpretation is rather obvious.

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* In the example this variable has only an illustrative role.

The coefficient of the objective function are also self-evident. Perhaps a few words of comment are warranted for those coefficients associated with resource-utilization and resource-need variables. The coefficients of the former represent the variable cost of operating the specific machinery/equipment combinations. Those of the latter represent the fixed costs. The problem of defining fixed and variable costs must be considered and solved within the context of the specific economic environment to which the model refers. The number associated with these variables in the example are fictitious.

Solution of the Example

An optimal solution of the program illustrated in the above example can be seen on Tables 2 and 3. The optimal value of the objective function is: $ 6,541,920.

Table 2

Activities in the optimal basis

Activity number	Unit	Level	Activity number	Unit	Level
1	Acres	1000.00	22	Machine-Period 3	47.90
2	"	500.00	24	"	2.52
3	"	453.75	25	Machine	10.00
4	"	30.00	26	"	7.00
6	10 CWT	300.00	27	"	.20
8	CWT	18150.02	28	"	3.00
9	CWT	3000.00	29	"	2.52
11	Machine-Period 1	10.23	30	"	21.11
13	"	2.52	31	"	2.52
14	"	.20	32	"	13.00
16	Machine-Period 2	6.39	33	"	12.75
18	Machine-Period 3	20.00	35	"	44.90
20	"	11.11	37	"	47.90

Table 3

The shadow-prices corresponding to the model's constraints

Constraint number	Shadow-price	Constraint number	Shadow-price
1	0.00	20	3353.49
2	741.01	21	0.00
3	4028.39	22	42372.20
4	0.00	23	14071.90
5	38.37	24	56548.50
6	37.26	25	14124.10
7	.84	26	11178.30
8	409.40	27	2945.76
9	395.84	28	589.15
10	361.37	29	0.00
11	-158.57	30	8837.29
12	0.00	31	0.00
13	14124.10	32	-223.57
14	904.93	33	-3.58
15	20281.20	34	0.00
16	0.00	35	.29
17	2824.81	36	209.22
18	4237.22	37	-223.57
19	0.00		

The objective of this example is only to represent the basic features of the approach outlined under point 2. But even based on this hypothetical example one can notice, beside the production structure, how detailed program of resource utilization - the optimal annual stock of resources and best way of their use over time - can be computed by these models. It is not very difficult to assume that results similar to those represented on Tables 2 and 3 generally suggest strategies for saving a substantial amount of capital inputs.

3. Proposed Structure of a DLP Model for Farm Development

Now we describe a DLP model which is applicable to the elaboration of the medium and long range development of large scale farms. It grows out of that static model which has been introduced previously optimizing both product mix and resource utilization. The dynamic variant consists of several such static blocks, one for each year or desired time increment of the model of the covered planning horizon. But the DLP approach is obviously more appropriate to handle various dynamic aspects of agricultural development (e.g. investments, development of animal stock).

The variables of DLP model represents activity levels according to time increments, but some of them may refer to the whole planning horizon. (e.g. the variable expresses the growth of gross value of production during the whole covered time period.) The variables are given by the subsystems of farming as follows:

$x_i^{(t)}$ is production variable representing the scale of i th crop and animal production activity in period t. (A more detailed formulation of these variables can be seen under point 2.1);

$u_i^{(t)}$ is the level of commercial activity (buying, selling) i in period t;

$v_{\alpha i\beta}^{(t)}$ is resource utilization variable expressing the volume of resource α used in performing task β on production activity i in period t;

$w_\alpha^{(t)}$ is resource-need variable expressing the level of resource α required in period t;

$z_i^{(t)}$ is the level of financial activity i in period t.

3.1 Description of the farming system.

The production of the large scale farms with mixed production structure can be modelled according to:

- crop production and
- animal production subsystems*

3.1.1. Crop Production

Within the crop production annual and perennial crops are considered. In the case of the annual crops the only dynamic element of the system is the influence of the previous crop on the yield of the crop in the next year. The following equation expresses these connections:

$$(8)\quad \Sigma_k x_{jk}^{(t-1)} = \Sigma_a x_{aj}^{(t)}$$

where $x_{jk}^{(t-1)}$ is the scale of production crop j after crop k in period t-1 and $x_{aj}^{(t)}$ is the scale of production crop a after j in period t.

$$(9)\quad \Sigma_j x_{jk}^{(1)} = c_k^{(o)}$$

where $c_k^{(0)}$ is the initial scale of production crop k, and the available land is fixed as follows:

$$(10)\quad \Sigma_{jk}\ x_{jk}^{(t-1)} = \Sigma_{aj}\ x_{aj}^{(t)} = L_c$$

where L_c is the available land for annual crop production.

* In certain cases food processing can also be considered.

Agro-technical and diseases control considerations may limit the production of various crops. For example in most cases sugar beet can be sown on a given land only after four years. Such and similar restrictions can be formulated as:

If (11a) $x_{jk}^{(t)} \geq 0$

$j \in P$

then $x_{uj}^{(t+1)} = 0 \quad (u \in P)$

(11b) $x_{uj}^{(t+1)} \geq 0 \quad (u \notin P)$

and $x_{zu}^{(t+2)} = 0 \quad (z \in P)$

(11c) $x_{zu}^{(t+2)} \geq 0 \quad (z \notin P)$

and $x_{yz}^{(t+3)} = 0 \quad (y \in P)$

$x_{yz}^{(t+3)} \geq 0 \quad (y \notin P)$

Where P is the group of crops being restricted.

Various crops might be completely excluded as previous crops for certain others as follows:

(12a) $x_{jk}^{(t)} = 0$

if $k \in K$

(12b) $x_{jk}^{(t)} \geq 0$

if $k \notin K$

where K is the group of crops being "unacceptable" as a previous crop for crop j.

In some cases a stationary crop structure (the same for each year) is also required as:

(13) $\Sigma_k x_{jk}^{(t-1)} = \Sigma_a x_{ja}^{(t)}$

The dynamics of perennial crops can be described by the following equations:

(14a) $x_n^{(t)} = x_n^{(t-1)} + x_{nu}^{(t-k)} - x_{ns}^{(t-1)}$

$n \in F$

and (14b) $x_{ns}^{(t-1)} \geq a^{(t)} x_n^{(t-1)}$

where F is the group of perennial crops, $x_{nu}^{(t-k)}$ is the scale of new plantings become mature after the required k years and $x_{ns}^{(t-1)}$ is the level of trees cut down in period t-1, $a^{(t)}$ is a coefficient expressing the minimum requirement for trees cut down.

3.1.2. Animal Husbandry

The internal dynamic connections of animal husbandry are more complicated and may vary according to animal types. The process of breeding and feeding of animals have to be modelled. In most cases the breeding stock represents the most valuable part of animal stock, and in the other groups the animals stay generally less than a year period, therefore the dynamics between time periods can be described based on the group of animals for breeding. We formulate only the most general characteristics of animal husbandry which can be applied for most types of animals. It is obvious that the animal production can be modelled on a more detailed and less aggregated way if it is required.

The dynamics of animals for breeding considering a given replacement policy can be described as:

$$(15a) \qquad x_g^{(t-1)} + x_{gu}^{(t-1)} - x_{gc}^{(t-1)} = x_g^{(t)}$$

where $x_g^{(t)}$ is stock of animals of type g for breeding in period t, $x_{gu}^{(t-1)}$ is the increase and $x_{gc}^{(t-1)}$ is the decrease of the absolute level of stock of animals for breeding due to decision of management in period t-1, and

$$(15b) \qquad p_g^{(t)} x_g^{(t)} + x_{gu}^{(t)} - p_{(g+1)}^{(t)} x_{(g+1)}^{(t)} - x_{gc}^{(t)} + u_p^{(t)} = 0$$

where $x_{(g+1)}^{(t)}$ is the stock of young breeding animals of type g, p_g is the replacement coefficient expressing a given breeding policy, $p_{(g+1)}$ attrition rate of young breeding animals and $u_p^{(t)}$ buying $(u_p^t > 0)$ or selling $(u_p^{(t)} < 0)$ animals for breeding in period t.

In most cases a balance of the new born animal's utilization has to be formulated:

$$(16) \qquad q_g^{(t)} x_g(t) - q_{(g+1)}^{(t)} x_{(g+1)}^{(t)} - q_{(g+2)}^{(t)} x_{(g+2)}^{(t)} + u_n^{(t)} + u_h^{(t)} = 0$$

where $x_{(g+2)}^{(t)}$ is the stock of animals of type g for feeding in period t, $q_g^{(t)}$ is the birth rate, $q_{(g+1)}^{(t)}$, $q_{(g+2)}^{(t)}$, are transformation coefficients and $u_n^{(t)}$ as well as $u_h^{(t)}$ are the amount of young breeding or feeding animals bought ($u_n > 0$; $u_h > 0$) or sold ($u_n < 0$; $u_h < 0$) in period t.

The upper limit of young breeding animals has also to be restricted because of biological reasons (not all the new born animals are suitable for breeding purposes).

$$(17) \qquad b^{(t)} q_g^{(t)} - q_{(g+2)}^{(t)} x_{(g+2)}^{(t)} + u_n^{(t)} = 0$$

where $b^{(t)}$ is a coefficient expressing the ratio of new born animals suitable for breeding purposes.

3.2 Resource Utilization and Investments

A static approach for modelling of resource utilization and investment policies have been discussed under point 2. The resource utilization system of DLP models can be formulated on that basis considering (1), (2), (5) and (6).

The utilization of resources in case of "inflexible" resources can be described as:

$$(18) \qquad \Sigma_i \; \alpha_i^{(t)} \; x_i^{(t)} \leq f_\alpha^{(t)} \; R_\alpha^{(t)}$$

where $\alpha_1^{(t)}$ indicates the input coefficient $f_\alpha^{(t)}$ is the capacity coefficient associated with existing stocks of resources $R_\alpha^{(t)}$ in period t.

The "flexible" case of resource utilization:

$$(19) \qquad t_{\beta i}^{(t)} x_i^{(t)} - \Sigma_\alpha g_{\alpha i \beta}^{(t)} \; v_{\alpha i \beta}^{(t)} \leq 0$$

where $t_{\beta i}^{(t)}$ is a coefficient which expresses the unit volume of production task β required for production activity i in period t, $g_{\alpha i \beta}^{(t)}$ is the coefficient which expresses the unit volume of performance of resource input α when used for task β on production activity i, $v_{\alpha i \beta}^{(t)}$ is the volume of resource α used in performing β production task on production activity i; and the usage of available resource stocks assuming that the increase of stocks take place at the beginning of the year:

$$(20) \qquad \Sigma_{i\beta} \; v_{\alpha i \beta}^{(t)} - (f_{\alpha\nu}^{(t)} w_{\alpha\nu}^{(t)} + f_{\alpha\mu}^{(t)} w_{\alpha\mu}^{(t)}) \leq 0$$

where $w_{\alpha\nu}^{(t)}$ is the existing and $w_{\alpha\mu}^{(t)}$ the new stock of resources, $f_{\alpha\nu}^{(t)}$ and $f_{\alpha\mu}^{(t)}$ are capacity coefficients in period t.

In the DLP model further restrictions involving capital inputs similar to (3a), (3b), (4a), (4b) have also to be applied. The stocks of available "flexible" resources is determined by the following equation:

$$(21) \qquad d_{\alpha}^{(t)} w_{\alpha\nu}^{(t-1)} + w_{\alpha\mu}^{(t-1)} = w_{\alpha\nu}^{(t)}$$

where $d_{\alpha}^{(t)}$ indicates the depreciation coefficient of resource α in period t.

where d_{α} indicates the depreciation coefficient of resource α in period t.

In the model most of the resource need variables represent capital inputs (building, machinery and equipment) and the increase of the available stocks requires investments. The variables of new available resources stocks ($w_{\alpha\mu}^{(t)}$) associated with the fixed assets of farms indicate the size of the required investments. Because of the mixed production structure considered, the range of possible investments is very wide and in some cases a more sophisticated procedure has to be followed.

The increase of machinery stock does not require special treatment. This type of investments can generally be realized during a year and the above mentioned $w_{\alpha\mu}^{(t)}$ variables are quite appropriate to handle them without any addition. The modelling of investments in soil ameliorization is a similar case because the investments of various years can easily be separated.

In the case of new planting of perennial crops, the length of period of maturation has to be considered, as

$$(22) \qquad w_{\alpha\mu}^{(t)} = w_{\alpha p}^{(t-k)}$$

Where $w_{\alpha p}^{(t-k)}$ new planting of perennial crop α in period (t-k), and k is the period of maturation. (During the period of maturation, the non-mature trees have to be expressed by special variables in the model).

The most complicated task of modelling of investment can be executed during various time periods in the function of the available physical (e.g. construction capacities) and financial resources. This investment obviously has a minimum time requirement, too. To solve this problem in the DLP model one can apply variables expressing various time horizons of a specific investment as follows:

$$(23) \qquad w_{\alpha\mu}^{t} = \Sigma_a \frac{w_{\alpha\mu a}^{(t)}}{f_{\alpha a}}$$

and

$$(24) \qquad w_{\alpha\mu o}^{(t)} = \sum_t w_{\alpha a}^{(t-u)}$$

where $w_{\alpha\mu a}^{(t)}$ is the value of finished investment type a in resource α in period t, $f_{\alpha a}$ is the unit investment cost of resource α in case of investment type a and $w_{\alpha a}^{(t-u)}$ is the value of investment type a in resource α executed in (t-u) period and u is the time requirement of investment a.

3.3 Financial Subsystem

The financial subsystem has a very important role in the model. It summarizes the financial results of farming and makes the modelling of income formation and utilization possible. Because accounting and revenue systems of countries may vary to a great extent the financial subsystem is always specific to concrete conditions. The structure outlined below is one of the possible solutions expressing the existing Hungarian practice.

3.3.1 Modelling of Income Formation

First the expenses are described. The indirect expenses of farming (overhead costs) are given as a ratio of gross value of production realized:

$$z_o^{(t)} = c^{(t)} z_p^{(t)} \tag{25}$$

Where $z_o^{(t)}$ is the sum of overhead costs in period t, $z_p^{(t)}$ is the gross value of production in period t and $c^{(t)}$ is the ratio of overhead expenses to gross value of production.

Material expenses:

$$\Sigma_i m_i^{(t)} x_i^{(t)} + \Sigma_i m_i^{(t)} u_i^{(t)} + \Sigma_{\alpha i \beta} \; m_{\alpha i \beta}^{(t)} v_{\alpha i \beta}^{(t)} + \tag{26}$$

$$+ \Sigma_\alpha m_\alpha (w_{\alpha\nu}^{(t)} + w_{\alpha\mu}^{(t)} + e^{(t)} z_o^{(t)} = z_m^{(t)}$$

Where m - s are material cost input coefficients, $e^{(t)}$ is material share in overhead costs and $z_m^{(t)}$ is the material expenses in period t.

Labour expenses:

$$\Sigma_i l_i^{(t)} x_i^{(t)} + \Sigma_i l_i^{(t)} + \Sigma_{\alpha i \beta} l_{\alpha i \beta}^{(t)} v_{\alpha i \beta}^{(t)} + \tag{27}$$

$$+\Sigma_\alpha l_\alpha (w_{\alpha\nu}^{(t)} + w_{\alpha\mu}^{(t)}) + k^{(t)} z_o^{(t)} = z_l^{(t)}$$

Where $l^{(t)}$- s are labour cost input coefficients, $k^{(t)}$ is the labour cost share in overhead expenses, and $z_l^{(t)}$ is the labour expenses in period t.

Amortization costs:

$$(28) \qquad \Sigma_\alpha s_\alpha^{(t)} (w_{\alpha\mu}^{(t)} + w_{\alpha\mu}^{(t)}) = z_s^{(t)}$$

Where $s_\alpha^{(t)}$ is the amortization cost coefficients, and $z_s^{(t)}$ expresses the amortization expenses in period t.

The gross value of production:

$$(29) \qquad \Sigma_i p_i^{(t)} x_i^{(t)} + \Sigma_i p_i^{(t)} u_i^{(t)} = z_p^{(t)}$$

Where $p_i^{(t)}$ is the unit return coefficient in period (t).

The income:

Now we can describe the income of the farming in period (t)

$$(30) \qquad z_p^{(t)} - (z_m^{(t)} + z_l^{(t)} + z_s^{(t)}) = z_j^{(t)}$$

Where $z_j^{(t)}$ expresses the income of period t.

3.3.2 Income Utilization

In Hungary the utilization of realized farm income is strongly influenced by government regulations. The farms can only decide upon the usage of investment funds:

$$(31) \qquad z_j^{(t)} [1 - (e^{(t)} + t_l^{(t)} + t_g^{(t)})] = z_b^{(t)}$$

Where $e^{(t)}$ is the coefficient expressing the bonuses paid to the workers from income realized, $t_l^{(t)}$ is the local and $t_g^{(t)}$ is the government tax coefficient (e^t, $t_l^{(t)}$, $t_g^{(t)}$ are fixed by by the government) and $z_b^{(t)}$ expresses the part of the income which can be used for investments by the farm.

The usage of investment funds has one year's time lag. The investment funds realized in year (t - 1) can be used in year t.

$$(32) \qquad z_b^{(t-1)} + z_{br}^{(t-1)} + z_s^{(t)} = z_{bf}^{(t)} + z_{bc}^{(t)} + h^{(t)} z_{cs}^{(t-1)} + + z_{br}^{(t)}$$

Where $z_{br}^{(t-1)}$ expresses the unused investment funds in period (t - 1), $z_{bf}^{(t)}$ is the investments in fixed assets in period t, $z_{bc}^{(t)}$ is the amount of investments in current assets, $z_{cs}^{(t-1)}$ expresses the debts of the farm at the end of period (t-1), $h^{(t)}$ is the credit repayment coefficient.

The farm investments can be financed by the farms own resources (investment funds as explained and amortization fund), loans and government subsidy may also be available for certain investments:

$$(33a) \qquad i_{\alpha}^{(t)} w_{\alpha\mu}^{(t)} = z_{bf\alpha}^{(t)} + z_{c\alpha}^{(t)} + z_{s\alpha}^{(t)}$$

In case of investment with more than one year time horizon:

$$(33b) \qquad \Sigma_a w_{\alpha a}^{(t)} = z_{bf\alpha}^{(t)} + z_{c\alpha}^{(t)} + z_{s\alpha}^{(t)}$$

$$(33c) \qquad j_{\alpha}^{(t)} i_{\alpha}^{(t)} w_{\alpha\mu}^{(t)} = z_{s\alpha}^{(t)}$$

$$(33d) \qquad z_{bf\alpha}^{(t)} \geq o_{\alpha}^{(t)} i_{\alpha}^{(t)} w_{\alpha\mu}^{(t)}$$

Where $i_{\alpha}^{(t)}$ is the unit investment expenses, $z_{bf\alpha}^{(t)}$ expresses the farms own resources used for investments in resource α, $z_{c\alpha}^{(t)}$ is the credit to investments in resource α and $z_{s\alpha}^{(t)}$ is the state subsidy to investments in resource α.

$j_{\alpha}^{(t)}$ is a coefficient expressing the ratio of subsidies to total investment expenses given to investments in resource α, $o_{\alpha}^{(t)}$ expresses the minimum requirement for farm's own resources in investments of resource α.

In addition to the above mentioned equations we need
-the summing up of usage of the farms own investment funds:

$$(34) \qquad z_{bf}^{(t)} = \Sigma_{\alpha} z_{bf\alpha}^{(t)}$$

-the summing up of new credits and calculation of debts:

$$(35a) \qquad z_{c}^{(t)} = \Sigma_{\alpha} z_{c\alpha}^{(t)}$$

$$(35b) \qquad z_{cs}^{(t)} = z_{c}^{(t)} + z_{cs}^{(t-1)} (1 - h^{(t)})$$

Where $z_{c}^{(t)}$ is the amount of new credits in period t
-to summarize the government subsidies:

$$(36) \qquad z_{g}^{(t)} = \Sigma_{\alpha} z_{s\alpha}^{(t)}$$

Where $z_{g}^{(t)}$ expresses the total amount of government investment subsidies given to the farm in period t.

- to calculate the value of fixed assets:

$$(37) \quad z_f^{(t)} = z_f^{(t-1)} - z_s + \Sigma_\alpha i_\alpha w_{\alpha\mu}^{(t)} + \Sigma_{\alpha a} w_{\alpha\mu a}^{(t)}$$

The modelling of current assets of farms with mixed production structure cannot be done in great details within the framework of a DLP model with one year time increments. A feasible solution for this problem is the handling of current assets as a function of gross production, as the following:

$$(38a) \quad n^{(t)} z_p^{(t)} = z_v^{(t)}$$

$$(38b) \quad z_v^{(t)} = z_v^{(t-1)} + z_{pc}^{(t)}$$

Where $z_v^{(t)}$ expresses the current assets required by farming in period t and $n^{(t)}$ is a coefficient expressing the current asset requirements.

Within the financial subsector of the model other economic constraints can also be considered. For example in the Hungarian state farms the total amount of wages paid for employee is limited by the growth of gross value of production, therefore we need the following equation in the DLP model:

$$(39a) \quad z_p^{(t)} = z_p^{(o)} + z_{pg}^{(t)}$$

$$(39b) \quad z_{pg}^{(t)} = z_{pr}^{(t)} \; z_p^{(o)}$$

$$(39c) \quad z_l^{(t)} \leq \sigma z_{pr}^{(t)} z_l^{(o)}$$

Where $z_p^{(o)}$ and $z_l^{(o)}$ are the initial values of gross value of production and labour expenses, $z_{pg}^{(t)}$ expresses the growth of gross value of production from period o to period t and $z_{pr}^{(t)}$ is the growth rate of gross value of production, σ is a coefficient fixed by the government.

3.4 Objective Function

There are several options to formulate the objective function of DLP model. In any case the problem of returns in different time periods and investments in the terminal year have to be treated. One possible solution:

$$(40) \quad \Sigma_t \frac{z_j^{(t)}}{(1+\varepsilon)^{(t-1)}} + \Sigma_{\alpha t} \; r_\alpha w_{\alpha\mu}^{(t)} = \max$$

Where ε is the discount coefficient applied, and r_α is expressing the expected returns on investment completed after the covered time horizon.

4. Conclusions

The models outlined in this paper are based on research conducted in Hungary during the last several years. In the Hungarian agriculture state and cooperative farms produce and market a large share of the total agricultural output. The average size of such farms, measured in terms of marketable production is much larger than the average size of commercial farms in Western Europe: The average state farm possesses about 15,000 acres of cropland, while the average cooperative farm controls about 18,000 acres. Recently, both state and cooperative farms have undergone an intensive process of renovation which included the adoption of the newest technological advances in farm machinery and equipment as well as the application of modern managerial methods. In Hungary, agriculture constitutes a sector of the national planned economy. All farms must operate according to their explicit year and long range plans which are coordinated at both the sectorial and national level. The present system of national economic planning is inspired to a great extent by a scheme of decentralized decision making. The leaders at the farm level have great opportunity (as well as responsibility) for choosing among alternative options. This situation has stimulated great interest toward the utilization of modern methods of decision making based on mathematical programming. The adaptation of these methods to the real and concrete situation of Hungarian agriculture has required a substantial amount of research in this field. The work reported in this paper is part of that effort. The success has not been lacking. Not only have state and cooperative farms formulated their plans based on linear and dynamic linear programming, but several of them have decided to actually implement them.

The experience gained in the use of the static approach outlined under point 2 has revealed that a crucial specification is the definition of time periods within which the various tasks have to be performed. The length of time period considered is of ten days. This implies that, in general, the models are of a considerable size, but still within the capacity of the available computer facilities. For example, the model formulated for a state farm of 16,500 acres included 1149 resource-utilization variables and 89 resource-need variables. The annual plan was formulated by choosing from 17 production activities whose tasks were defined in 26 time periods (most of them are 10 day periods) for a total of 872 constraints. The result of this model suggested a strategy for saving equivalent to 20-25 percent of the cost of machinery and equipment compared to programs developed by traditional planning methods. This type of model was successfully used also for evaluating policy decisions at the national level dealing in particular with the determination of the optimal price support for machinery. In all these applications the flexibility of the model allowed a very detailed analysis of the problems and generated a considerable amount of information extremely useful for the planning agency.

In the practical application of the model the collection of required data and the construction of the relatively large-scale models caused certain difficulties and slowed down the whole procedure. To avoid these difficulties a method was developed which may serve the basis for a wider and faster practical application of the model described under point 2. [2] The kernel of the method is a special linear programming model, the so called basis model which consists of two parts: of a standard block comprising coefficients that relate to the wide scale of technological variants, and of a concrete block that takes local characteristics into consideration. The basis model contains 2799 variables and 1999 constraints. A very important element of the procedure is a special computer programme. Starting from the basis model the programme is suitable both for the generation of models (about 400 variables and constraints) providing plans for single farms, while taking into consideration concrete economic data, and later for the solution of these models.

The DLP model presented under point 3 has also been applied successfully in several cases for five year planning of farms.[7] The utilization of resources were modelled not in such details as indicated by the model under point 2. Table 4a, 4b and 5a, 5b show the structure of two models. In the first case various technological options were considered in connection with the production variables therefore only resource-need variables were applied. As we can see on Table 4a and 4b this approach led to a moderate model size of 465 variables and 538 constraints. The usage of resource utilization variables increases the model size substantially, as it is shown in table 5a and 5b (913 variables and 847 constraints). The comparison of the two models gave a good opportunity to investigate the additional planning opportunities offered by the model structure outlined under point 2. We feel that the use of resource need variables has to be considered as a minimum requirement for models of large scale farm development. The amount of information on technological options open for the farm can substantially be increased by the usage of resource utilization variables, but due to the larger model size the modelling work becomes more time and money consuming. According to our experiences at the large scale farms the range of possible technical and technological solutions is so wide, that the detailed modelling of resource utilization is also a worth while enterprise.

The advantages of DLP approach have been indicated by our practical applications. This model structure was appropriate for the handling of investment and financial problems and to consider the dynamics of agricultural production on a relatively high level of sophistication. The DLP models obviously have limitations too. The deterministic character of the model makes the handling of the stochastic elements of agricultural production very difficult. In many cases the assumption of the linearity is a very strong simplification. Difficulties connected with the terminal year of the model and the relatively large model size have also to be considered. The problems of data collection by traditional ways and the model construction in every single case gave an experimental character to these works. Therefore, we believe that large-scale practical application of these models can only be done on the basis of computerized data preparation and model construction.

Table No. 4

Structure of a DLP farm development model with preliminary fixed technological systems

a. Model variables

Type of variables	Time periods (years)					Together
	1	2	3	4	5	
Production variables	36	36	36	36	36	180
- plant production variables	15	15	15	15	15	75
- animal production variables	21	21	21	21	21	105
Commercial variables	11	11	11	11	11	55
Resource-need variables	33	26	26	26	26	137
Financial variables	11	14	14	14	14	67
Other	6	5	5	5	5	26
Together	97	92	92	92	92	465

b. Model constraints

Type of constraint	Time periods (years)					Together
	1	2	3	4	5	
Land constraints	2	2	2	2	2	10
Plant production	6	6	6	6	6	30
Animal husbandry	18	18	18	18	18	90
Feed balances	7	7	7	7	7	35
Labour balances	11	10	10	10	10	51
Technical resource balances	31	31	31	31	30	154
Resource availability	27	19	19	19	20	104
Financial subsystem	11	13	13	13	14	64
Together	113	106	106	106	107	538

Table No. 5

Structure of a DLP farm development model
with detailed submodel for resource utilization

a. Model variables:

Type of variables	Time periods (years)					together
	1	2	3	4	5	
Production variables	28	28	28	28	28	140
- plant production variables	18	18	18	18	18	90
- animal production variables	10	10	10	10	10	50
Commercial variables	8	8	8	8	8	40
Resource utilization variables	97	98	98	98	98	489
Resource-need variables	46	31	31	31	31	170
Financial variables	12	15	15	15	17	74
Together	191	180	180	180	182	913

b. Model constraints

Types of constraints	Time periods (years)					together
	1	2	3	4	5	
Land constraint	1	1	1	1	1	5
Plant production	10	10	10	10	10	50
Animal production	9	9	9	9	9	45
Feed balances	8	8	8	8	8	40
Resource utilization and need	138	120	122	122	123	625
Financial subsystem	13	16	15	15	23	82
Together	179	164	165	165	174	847

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