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Long-Term Normative Model of Development: Methodological Aspects

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LONG TERM, NORMATIVE MODEL OF DEVELOPMENT: METHODOLOGICAL ASPECTS

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INTRODUCTION

Few academic discussions in recent years have become as controversial as those regarding the application of computer-operated mathematical models (see Reference [7, 10]) which forecast the future of mankind.

It has been argued, for example, that the modelling methodology usually takes a descriptive, i.e. passive, attitude with regard to global development processes. As a result, even slight tendencies towards a crisis may appear as if they had been determined, in which case the whole future development appears predestined and "doomsday" seems inevitable. It has also been argued that descriptive models do not take into account the changes in the system of socio-economic values and development goals, which have a direct effect on the consumption structure, allocation of resources, prices, fertility, and the growth of population, etc. Since, in descriptive models, decisions cannot be introduced explicitly, one cannot find out what can be done when the crisis is in sight and what chances one has in trying to avoid or reduce the effects of the crisis. It was proposed that, in order to have a realistic global development model, the normative rather than descriptive approach should be used.

First of all, that approach requires the existing system of national development goals and the decision system, which is capable of implementing these goals, should be investigated. It is also necessary to investigate to what extent the system of development goals can be realized when one takes into account the contraints imposed by shortages of resources, protection of the environment, etc.

Since most of the decisions regarding the allocation of resources take place at the national or regional level, it is convenient to start global modelling efforts with relatively simple national models. The next feasible step is to see whether the national development policies could be coordinated in order to make the best contribution to the welfare of global development. It is possible to observe that, using the above approach, one arrives at the global model by a "bottom-up" process of construction and linkage of national submodels. Another possible approach employs the "top-down", or decomposition technique. The advantage of the "bottom-up" technique is that one can use the original data base, whereas the "top-down" technique uses mostly aggregated data (from international organizations) which must again be decomposed when the model is extended.

An additional advantage of the "bottom-up" approach is that it enables many questions to be answered regarding the perspectives of national development. In particular, it is possible to learn how global development affects the national plans of development; what the possible national specialization in production, trade, research and development, etc., could be; what the best policy is in population growth, specialization in education, science, etc. Such an approach is also helpful when trying to answer the basic question: How much can a country benefit from international cooperation? Should it follow the strategy of complex isolation and autarky or engage in any of the various forms of international cooperation?

Using the optimization theory, we shall demonstrate that the strategy of broad international cooperation yields a faster socio-economic growth. The strategy of international cooperation (e.g. The Council for Mutual Economic Assistance (CMEA) created by the socialist countries) has already proven to be beneficial to each member country.

The approach used in this paper can be called "optimistic" (compared to the approaches used in the so called "Doomsday Models") in the sense that it relys on the realistic assumption that mankind will choose the policy of international cooperation and will optimize the allocation of scarce resources rather than follow the passive attitude and apathy when confronted with a crisis or catastrophy. The increase in international cooperation, especially after the Helsinki Peace Conference, indicates that such a strategy is feasible.

Following the "bottom-up" approach, outlined above, research on the construction of a long-term, normative model of national development was begun in 1972 at the Polish Academy of Sciences. As a result of that research, a number of models (labelled MRI, I = 0,1,2,3,...) were constructed. The purpose of the present paper is to describe the general methodology which was used for the construction of MRI models.

The most difficult problem the modellers faced was the necessity to formalize the development goals which were formulated in a descriptive form. The next problem was to describe the system under consideration in mathematical language; i.e. find the appropriate models of production, consumption and environmental subsystems, as well as the set of development constraints. In order to do that we have been trying to use, as much as possible, the existing methodology in macro-economics, environmental studies, system analysis and computer sciences. In this respect, papers written by scientists from socialist countries [1, 3, 9, 13, 17, 38, 42, 43, 44] were of particular help. However, many sacrifices were inevitable, especially in the cases where existing theories did not fit well together. Some extensions of the concepts commonly used were also necessary.

As far as the development goal was concerned, the classical utility theory seemed appropriate but an extension enabling us to deal with dynamic processes was necessary. The Polish national development goals and policy objectives are clearly outlined in the Constitution of the Polish People's Republic. According to the Constitution, "national policy should contribute to the country's full political, socio-economical and cultural development. It should contribute to the national strength and independence, realization of socialist ideology, strengthing of friendship and cooperation with allied countries and all peaceful nations in order to consolidate and secure global peace". The Constitution also states that the main socio-economic goal of Polish internal policy is "systematic improvement of the material, social and cultural living conditions and the constant development of production factors". Realizing these goals, the Polish government submits the budgets to the Polish parliament at the end of each year for approval. In other words, the nation decides each year how much of the GNP generated should be spent on individual consumption, productive investments and government expenditure, i.e. the aggregated consumption including education, medical care, research and development, etc.

According to our methodology, the 'ex post' data for the allocation of GNP can be used for the construction of a national utility functional. That functional can be used 'ex ante' to derive the future strategies of resources allocation which will also be called here, "the development factors".

We have assumed that all the development factors contribute to the development, i.e. the GNP growth and a generalized Cobb-Douglas function has been used to describe the corresponding development functional. The functional takes into account all the inertial effects, caused by the delays in investment, education, research and development, etc. processes. Then the development problem can be formulated in terms of the optimum allocation of development factors. One should obviously follow such an allocation strategy that yields the maximum value of the GNP, generated within the planning interval, and is subject to the constraints imposed by the quality of environment, shortages of natural resources, foreign trade balances, etc.

Since the utility and development functional parameters are closely related, the maximization of the integrated GNP is equivalent to the maximization of the utility functional. As a result, the development goal takes into account all the socio-economic sub-goals which are represented in the utility functional by the corresponding factor endowments.

It should be observed that the optimization of development plays an important role in the policy of socialist countries, which use an effective system of allocation of development factors. The importance of mathematical methods in the planning practice of these countries has been fully recognized [1, 9, 13, 43].

From a mathematical point of view, our optimization problem boils down to the maximimization of an integral, nonlinear functional, subject to a number of integral and/or amplitude type constraints. The solution to multi-variable problems of that type is, generally speaking, not easy. As shown in References [19 - 31], the method based on the generalized Hölder and Minkowski inequalities is, in that respect, very effective. A solution to the optimization problem exists and it is unique. It can be derived in an explicit form, independent of the number of variables involved. Another advantage of the approach used is that under optimum strategy, the sectorial development functionals can be aggregated to yield a simple resulting development function. In other words, the optimization process can be regarded as a useful aggregation device. Using this device, it is possible to allocate the development factors in the optimum manner along the hierarchical, multilevel structure, which corresponds to the decentralized decision system of Polish economy. According to that system, productive investments, labour and other development factors can be allocated by the decision centre (which is the Planning Commission) among the production sectors and services. The sectors, in turn, allocate the factors among the corresponding subsystems or individual factories. The factors are also allocated among different regions of the country. In addition, the sectors decide on how much of the commodities (produced by the other sectors) to purchase in order to have the next sector profit maximized. The model incudes a system of prices, controlled by special government agencies in such a way as to achieve the equilibrium between the supply and demand. The prices are also supposed to realize a number of further requirements which will be discussed in Section 2.3.

From the point of view of control theory, the model discussed can be regarded as a complex, nonlinear system with inertial feedbacks controlled by the hierarchical decision system. The system parameters (i.e. the technological coefficients) change slowly in time. In order to control such a system the methods known as "adaptive control" can be applied. As will be shown later, an "adaptive control" model of the technological change can be constructed. Using this approach, the model parameters are "adapted" to the changing reality, and a high degree of accuracy can be achieved. An interesting feature of the model discussed is the slow change of the goal (i.e. utility) coefficients. This kind of system is unknown in classical control theory. In order to solve the problem effectively, a moving time horizon technique has been used. This technique is based on the consecutive solutions of the optimization problems in the planning intervals [k, T + k], k = 0,1,2... and adjustments of the resulting strategies. It should be noted that the methodology used for the construction of MRI models, can also be applied to modelling the development of other countries, primarily all the CMEA countries. The exogenous variables, such as foreign trade price indices, then become endogenous; i.e. they can be derived from the aggregated multinational model.

Bearing in mind the general philosophy outlined above, the present paper has been divided into three parts. The first part, consisting of five sections, deals with the single-sector model. In the first section the development factors and development functions are introduced. In the simple case of two development factors (capital and labour), the development function coincides with the known production function used in macroeconomics. The next section deals with optimum development problems. Here, the important "principle of factor coordination" has been formulated, and optimization strategies derived in the explicit form. The third section, where the optimization of population policy has been formulated, deals with long-term development planning problems. Then the pollution impact on envir onment and development was investigated. It was shown that an increase in the pollution abatement level decreases the growth of GNP. The last section deals with development objectives and utility functionals where an extension of the "golden rule" of development has been formulated.

The second part of the paper -- the multi-sector case -- deals with an extension of the results obtained in Part I. In the first section it is shown that with optimum strategy the sector contributions to GNP do not depend on the intersector flows so that decomposition of optimization strategy is possible. In the second section, the optimum strategies have been derived by the explicit form and the sector coordination principle has been formulated. The third section deals with the model of sector prices and the last section deals with the "adaptive" model of technological and structural change. The third part of the paper deals with the regional and international cooperation models. Further details concerning the methodology used and models constructed can be found in References [19 - 37].

The author appreciates receiving the invitations to visit the International Institute for Applied Systems Analysis, where he has been able to study the global modelling problems. Discussions with Prof. T. Koopmans, Prof. F. Rabar and other scientists have been very helpful.

Special thanks should also be given to the Soviet scientists from the Central Economic and Mathematical Institute and the Institute of Management Problems in Moscow, for the valuable comments on the modelling problems which have been presented in the paper.

Long Term, Normative Model of Development: Methodological Aspects

R. Kulikowski

I. Single-Sector Model

1.1. Development factors and functions

By the development process here we understand a complex dynamic process described by the vector $\underline{X} \equiv \{X_1(t), X_2(t), \ldots, X_n(t)\}$ which depends on the vector $\underline{Y} \equiv \{Y_1(t), \ldots, Y_m(t)\}$ of the development factors $Y_i(t)$, $i = 1, \ldots, m$.

A typical example of such a process is a n-sector economic system with the outputs $\underline{X}$ depending on the production factors: labour and capital stock.

In a more general case (illustrated by Figure 1), one can assume that $\underline{X}$ depends as well on social and environmental factors such as health, education, research and development level, housing, transport, quality of air, water etc. The factors Y_i, $i = 1, \ldots, m$, are regarded as external (exogenous) while X_i, $i = 1, \ldots, n$, are dependent (endogenous) variables.

We shall assume that the $\underline{X}(\underline{Y})$ relation can be described by n development functions

$$F_i[\underline{X}, \underline{Y}, t] = 0 \quad , \qquad i = 1, \ldots, n \quad , \tag{1}$$

Since the direct solution of (1) is usually not easy, a linearization technique may be applied. Assume for that purpose that around the point $(\underline{X}_0, \underline{Y}_0, t_0)$ the functions (1) are continuously differentiable so that (1) can be replaced by

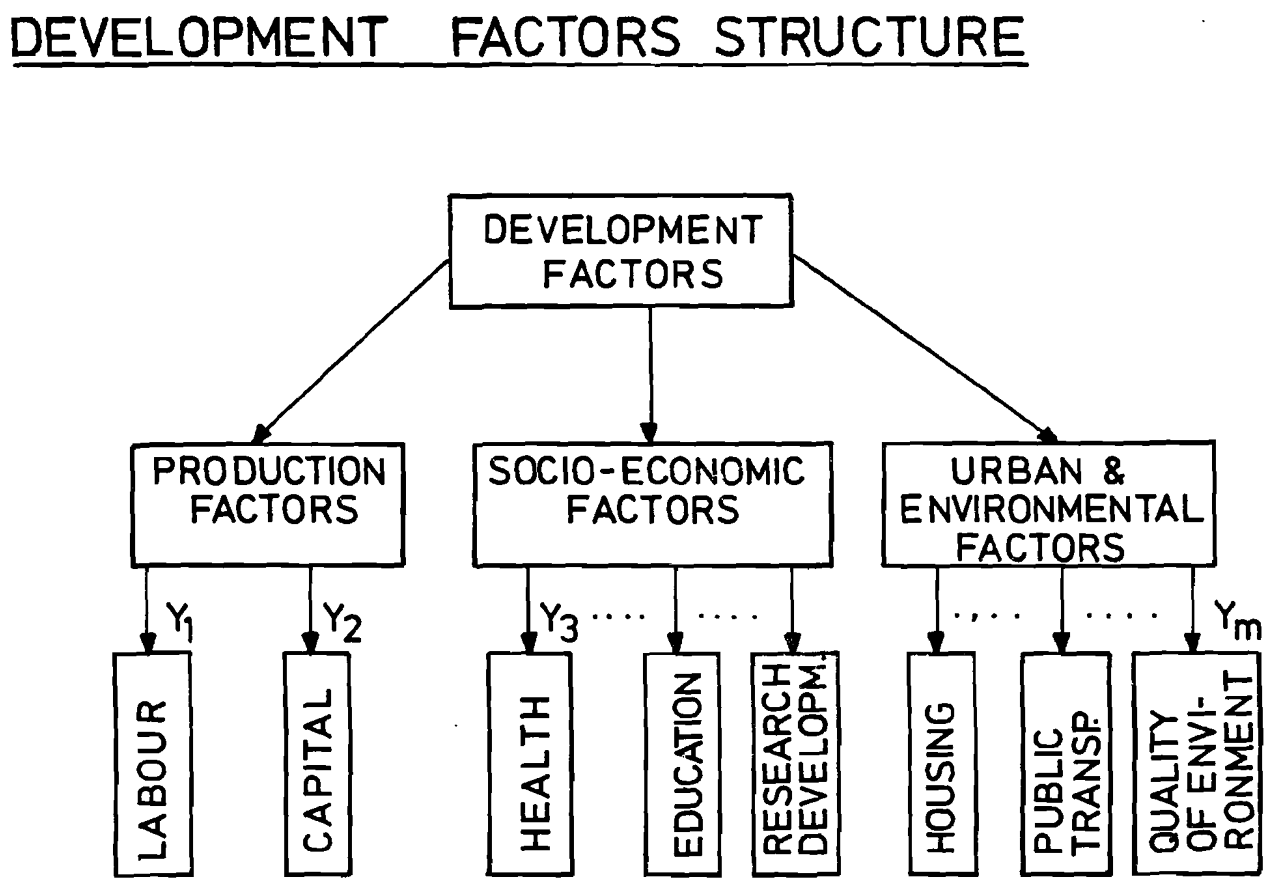
DEVELOPMENT FACTORS STRUCTURE
DEVELOPMENT FACTORS
PRODUCTION FACTORS
SOCIO-ECONOMIC FACTORS
URBAN & ENVIRONMENTAL FACTORS
Y1
Y2
Y3
Ym
LABOUR
CAPITAL
HEALTH
EDUCATION
RESEARCH DEVELOPM.
HOUSING
PUBLIC TRANSP.
QUALITY OF ENVI-RONMENT

FIGURE 1

$$\sum_{j=i}^{n} F'_{ix_j} \dot{X}_j + \sum_{\nu=1}^{m} F'_{iy_\nu} \dot{Y}_\nu + F'_{it} = 0 \tag{2}$$
$$i = 1,\ldots,n \quad ,$$

where

F'_{ix_j}, F'_{iy_ν} , F'_{it} are partial derivatives of F_i.

The next assumption is that around $(\underline{X}_0,\underline{Y}_0,t_0)$

$$F'_{ix}X_j \triangleq a_{ji} \quad , \qquad F'_{iy_\nu} Y_\nu \triangleq b_{\nu i} \quad ,$$
$$i,j = 1,\ldots,n \quad , \qquad \nu = 1,\ldots,m \quad ,$$

do not change much so that (1) can be approximated by the linear equations:

$$\sum_{j=1}^{n} a_{ji}\rho_{xj} + \sum_{\nu=1}^{m} b_{\nu i}\rho_y + F'_{it} = 0 \quad , \tag{3}$$
$$i = 1,\ldots,n \quad ,$$

where

$\rho_{xj} = \frac{\dot{X}_j}{X_j}$, $\rho_{y\nu} = \frac{\dot{Y}_\nu}{Y_\nu}$ are called the growth coefficients.

When the determinant of $[\alpha_{ji}]$ is not zero, the system (3) can be solved with respect to ρ_{xi}, $i = 1,\ldots,n$, i.e.

$$\rho_{xi} = \frac{\dot{X}_i}{X_i} = \mu_i(t) + \sum_{\nu=1}^{m} \beta_{\nu i} \frac{\dot{Y}_\nu}{Y_\nu} \quad , \qquad i = 1,\ldots,n \quad , \tag{4}$$

where

$\beta_{\nu i}$ = given numbers;

$\mu_i(t)$ = given functions.

Assuming $x_i(0)$ to be given, one can integrate the system (4) and obtain (see [36]):

$$X_i(t) = K_i e^{\int_0^t \mu_i(\tau)d\tau} \prod_{\nu=1}^{m} [Y_\nu(t)]^{\beta_{\nu i}} dt \quad , \qquad (5)$$

$$K_i = X_i(0) \prod_{\nu=1}^{m} [Y_\nu(0)]^{-\beta_{\nu i}} \quad , \qquad i = 1,\ldots,n \quad .$$

The functions (5) should be regarded as an approximation of the more general relations (1). They express however the "input-output" relation in the explicit form. The functions (5) shall be called the (explicit) development functions. In the simple case of a single sector and m = 2, and when $\mu_i(t) = \mu = \text{const.}$ and Y_1, Y_2 represent the labour and capital respectively, the function

$$X = A\, e^{\mu t}\, Y_1^{\beta_1}\, Y_2^{\beta_2} \quad , \qquad \beta_1 + \beta_2 \le 1 \quad , \qquad (6)$$

is the classical Cobb-Douglas production function with the neutral technical progress represented by μ.

The development factors obviously depend on the expenditure intensity $z_\nu(t)$, $\nu = 1,\ldots,m$. For example, investments are dependent upon the capital stock; the education level, R & D, medical care level, etc., depend on government expenditures. These relations can generally be described by the strictly concave, integral operators [10]:

$$Y_\nu(z_\nu) = Y_\nu(t) = \int_{-\infty}^{t} K_\nu(t - \tau)\; [z_\nu(\tau)]^{\alpha} d\tau \qquad (7)$$

where

$K_\nu(t)$ is a non-negative function, which can be approximated by:

$$K_\nu(t) = \begin{cases} K_\nu e^{-\delta_\nu (t-T_\nu)} \quad , & t > T_\nu \\ 0 \quad , & t < T_\nu \end{cases} \qquad (8)$$

K_ν, δ_ν, T_ν = given positive constants; and $0 < \alpha < 1$.

There is a simple interpretation of $Y_\nu(z_\nu)$ in the case of $\nu = 2$, when Y_2 represents the capital stock and z_2 is the investment intensity. Here the capital represents the accumulated investments $z_2(\tau)$ for $\tau \leq t$; δ_2 represents the depreciation (aging, wear and tear) of capital stock in time while T_2 is the construction delay, i.e. the time required for investment funds to materialise in the form of new production capacity. The inertial effect of investments on the plant capacity $Y_2(z_2)$ is illustrated by Figure 2, for the case $z_2(\tau)$ = const. $0 \leq \tau \leq T_2$. It is possible to observe that the plant capacity decreases for $t > 2T_2$ if no investments are being made after $t > T_2$. The construction delay T_ν for different branches of economy, ranges from one to four years, while δ_ν is usually assumed 0.1 - 0.05.

A similar interpretation (except labour) can be given for the rest of $Y_\nu(z_\nu)$, $\nu = 1,\ldots,m$, factors. In the case of the health service ($\nu = 3$ according to Figure 1) one can assume, as a first approximation, that most government expenditure is used for new facilities (hospitals, medical equipment, etc.) so that $Y_3(z_3)$ behaves in a similar way to $Y_2(z_2)$. The same assumption can be made with respect to the rest of the development factors. However, that approach neglects the effects of current expenditures and qualitative factors, such as knowledge and qualified services. For example, the health-level depends greatly on the training of medical and scientific staff, i.e. on the "investment" in employed specialists. The depreciation of Y_ν ($\nu \geq 3$) in time, results not only from aging of equipment, but also from the depreciation of

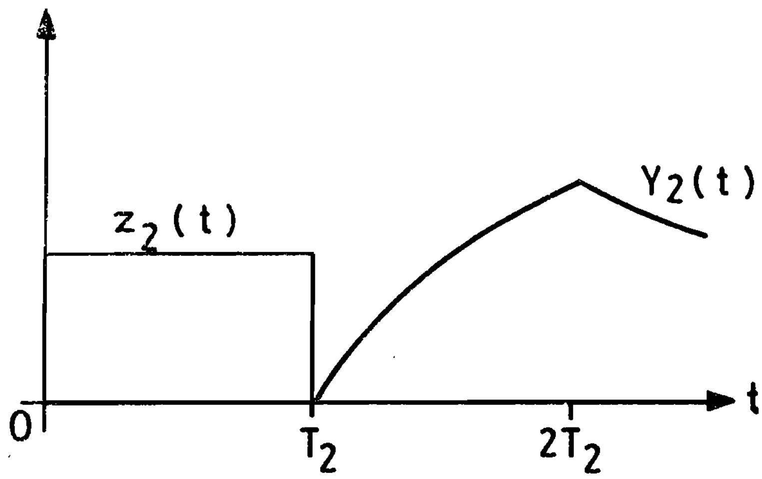

FIGURE 2

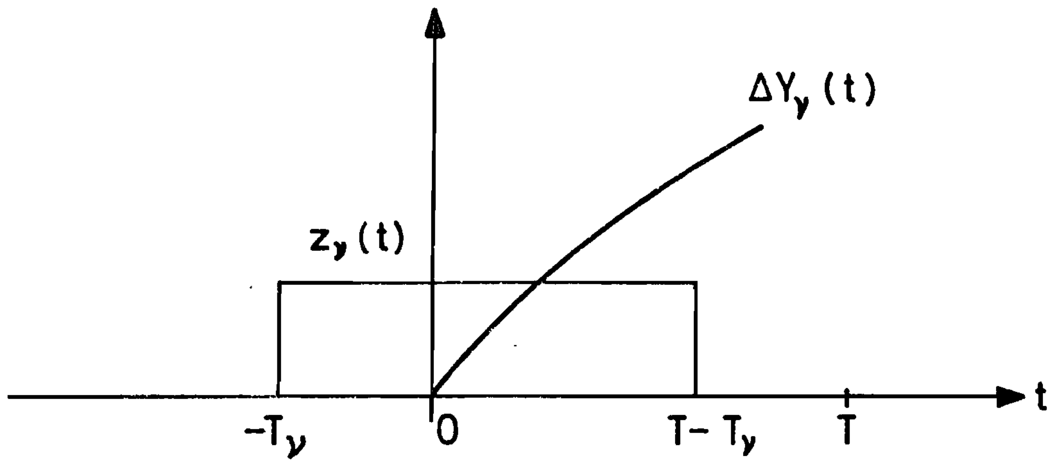

FIGURE 3

knowledge acquired in the past. The "production" delay T_ν in the case of education is the basic tuition time period, which, at present in Poland, is 10 years.

The observation made above indicates that the possible development-factor-parameters may vary in a wide range of values.

The general expression (7) simplifies in the case of labour force ($\nu = 1$). It can be assumed that labour does not depend on past salaries, so

$$Y_1(z_1) = K_1[z_1(t)]^\alpha \quad . \tag{9}$$

The same effect can be obtained assuming in (7):

$$\tilde{K}_1(t) = K_1\delta(t) \quad ,$$

where

$\delta(t)$ = Dirac's pulse.

In the production model under consideration it is important to deal with strictly concave operators so the numbers $\beta_{\nu i}$, α, satisfy the conditions:

$$\sum_{\nu=1}^{m} \beta_{\nu i} = 1 \quad , \qquad i = 1,\dots,n \quad , \qquad 0 < \alpha < 1 \quad ,$$

which are called the "decreasing return to scale".

It should be noted that (7) can be written alternatively as

$$Y_\nu(t) = \bar{Y}_\nu(t) + \int_0^t K_\nu(t - \tau)[z_\nu(\tau)]^\alpha d\tau \quad , \tag{10}$$

where

$\bar{Y}_\nu(t) = \int_{-\infty}^{0} K_\nu(t - \tau)[z_\nu(\tau)]^\alpha d\tau$ represents the effect of past expenditures (i.e. $z_\nu(\tau)$, $t < 0$).

In the case of (8) one can also write

$$Y_\nu(t) = e^{-\delta_\nu t}\left\{Y_\nu(0) + \int_0^t K_\nu e^{\delta_\nu \tau}[z_\nu(\tau - T_\nu)]^\alpha d\tau\right\} \quad (11)$$

which can be regarded as the solution to the differential equation

$$\dot{Y}_\nu(t) + \delta_\nu Y_\nu(t) = K_\nu[z_\nu(t - T_\nu)]^\alpha \quad ,$$

with the given initial condition $Y_\nu(0)$.

It should be observed that the function

$$\begin{aligned}\Delta Y_\nu(t) &= Y_\nu(t) - e^{-\delta_\nu t}Y_\nu(0) \\ &= \int_0^t K_\nu e^{-\delta_\nu(t-\tau)}[z_\nu(\tau - T_\nu)]^\alpha d\tau\end{aligned}$$

in (11), can be regarded as a present contribution to the development resulting from previous expenditures (i.e. the expenditures shifted by T_ν as shown in Figure 3).

Since the statistical data regarding the production effects and expenditures are usually collected periodically (e.g. once a year). It is convenient to deal with the discrete (instead of continuous) development functions (7) - (11). In particular (11) can be written

$$Y_\nu(t) = e^{-\delta_\nu t}\left\{Y_\nu(0) + \sum_{\tau=0}^{t} K_\nu e^{\delta_\nu \tau}[z_\nu(\tau - T_\nu]^\alpha\right\} \quad .$$
$$t = 0,1,2,\ldots$$

Using the discrete form of development factors, the output is represented by development capacities belonging to different generations, For example, capital stock can be regarded as being composed of equipment acquired in different years. In the subsequent computations we shall use the continuous form of $Y_\nu(t)$, which is a matter of convenience rather than general methodology.

It should also be observed that in dealing with the development function of the form

$$X = A\, e^{\mu_o t} \prod_{\nu=1}^{m} Y_\nu^{\beta_\nu} \quad , \qquad \sum_{\nu=1}^{m} \beta_\nu = 1 \tag{12}$$

instead of (6), it is possible to express the contribution to development which results from government expenditure in different areas. In order to do that it is convenient to introduce the notion of the growth coefficient $\rho_x = \frac{\dot{x}}{x}$, of the differentiable function x(t). It is easy to show that the growth coefficients derived for the functions (6), (12) become

$$\rho_x = \mu + \beta_1 \rho_{y_1} + \beta_2 \rho_{y_2} \quad ,$$

$$\rho_x = \mu_o + \sum_{\nu=1}^{m} \beta_\nu \rho_{y_\nu} \quad ,$$

respectively. In the case where $\beta_1 + \beta_2 = \beta < 1$, the terms $\sum_{\nu=3}^{m} \beta_\nu \rho_{Y_\nu}$ can be regarded as a contribution of Y_ν - factors, $(\nu > 2)$, to the development. Such an approach has already been suggested by many authors (e.g. Denison). The difficulty is, that the data regarding ρ_{Y_ν} are usually not available. One usually has, however, data for the expenditures growth, i.e. $\rho_{z_\nu} = \frac{\dot{z}_\nu}{z_\nu}$, $\nu = 1,\ldots,m$, which are related to Y_ν by the integral equations (7).

As will be shown in the following section, it is possible to express the contribution to growth which results from expenditures $z_\nu(t)$, $\nu = 1,\ldots,m$, but the contribution is expressed in the integrated form (25) rather than the discrete one (in terms of ρ_{z_ν}).

It should also be noted that, when $\mu_0 = 0$, the value of $\sum_{\nu=3}^{m} \beta_\nu \rho_{Y_\nu}$ can be interpreted as the neutral technical progress in Hicks' sense. It incorporates the parts which magnify the labour as well as the capital stock level. The relation to government expenditure in this case, however, is explicit.

1.2. Development feed-backs and optimum development problems

Development in the model analysed is a result of positive feedbacks which exist between the output $Y = pX$ (where p = price attached to the production X) and the expenditures z_ν, $\nu = 1,\ldots,m$, which constitute the given parts of Y -- the GNP generated by the economy (Figure 4). The decision center (D.C.) allocates the GNP between different development factors, including labour (i.e. private consumption) and government expenditures (health, education, R & D, etc.) in such a way that the given development goal attains its maximum.

The aggregated production function can be written, according to (5) - (11) as

$$Y(t) = K e^{\mu t} \prod_{\nu=1}^{m} [Y_\nu(t)]^{\beta_\nu} , \qquad (13)$$

$$Y_\nu(t) = \int_{-\infty}^{t} e^{-\delta_\nu (t-\tau)} [z_\nu(\tau - t_\nu)]^\alpha d\tau , \qquad (14)$$

$$\sum_{\nu=1}^{m} \beta_\nu = 1 , \quad 0 < \alpha < 1 , \quad \mu, \delta_\nu > 0 ,$$

where $z_\nu(t)$ should satisfy, generally speaking, one of the following two sets of constraints:

a) the amplitude-type of constraints:

$$\sum_{\nu=1}^{m} z_\nu(t - T_\nu) \leq Z(t) , \qquad (15)$$

$$z_\nu(t - T_\nu) \geq 0 , \quad t \in [0,T] , \quad \nu = 1,\ldots,m , \qquad (16)$$

b) the integral type of constraints:

$$\int_0^T W_\nu(\tau) z_\nu(\tau - T_\nu) d\tau \leq Z_\nu . \quad \nu = 1,\ldots,m , \qquad (17)$$

$$\sum_{\nu=1}^{m} Z_\nu \leq Z , \quad Z_\nu \geq 0 , \quad \nu = 1,\ldots,m , \qquad (18)$$

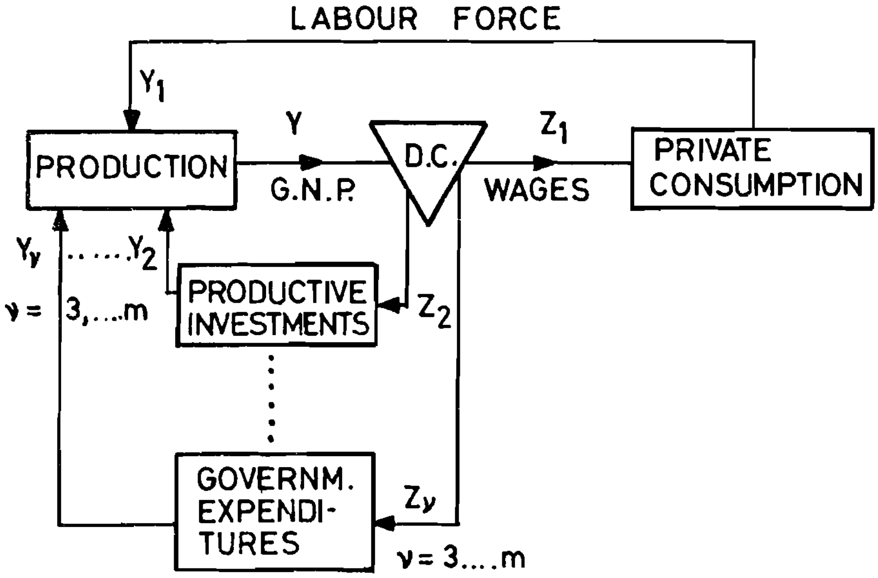

FIGURE 4

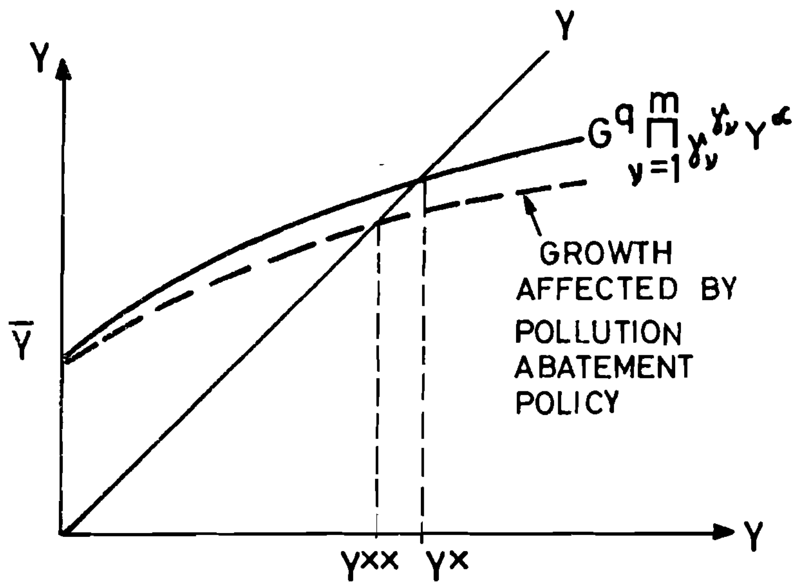

FIGURE 5

where

T is the given planning horizon and $W_\nu(t)$, $\nu = 1,\ldots,m$, the given weight functions.

In the most simple case $T_\nu = 0$, $\nu = 1,\ldots,m$ it is possible to replace $Z(t)$ by $Y(t-1)$. In that case the constraint (15) has the following meaning. The GNP generated at the end of the year $t-1$ is allocated at the year t[†] among m development factors, i.e.

$$z_\nu(t) = \gamma_\nu(t)Y(t-1) \quad , \qquad \nu = 1,\ldots,m \quad ,$$

where

$$\sum_{\nu=1}^{m} \gamma_\nu(t) = 1 \quad , \qquad t = 1,2,\ldots,$$

$\gamma_\nu(t)$ are decision variables.

In the case when some of the government expenditures, say $z_0 = \gamma_0 Y(t-1)$, have no productive effects, one should write

$$Z(t) = (1-\gamma_0)Y(t-1) \quad .$$

In the case when $T_\nu = T_0$, $\nu = 1,\ldots,m$, one obtains

$$z_\nu(t-T) = \gamma_\nu(t-T_0)Y(t-T_0-1) \quad , \qquad \nu = 1,\ldots,m \quad .$$

When $T_\nu > T$ the corresponding $z_\nu(t-T_\nu)$, $t \in [0,T]$ strategy shifts outside the planning interval (see Figure 3). In that case $z_\nu(t-T_\nu)$ should be regarded as an exogenous (given) function.

[†] In economic literature the delay taking place between the output y and the imput z_ν, is called the Robertson's postponement [2].

In the case when T_ν is different but $T_\nu < T$, one obtains

$$Z(t) = \sum_{\nu=1}^{m} \gamma_\nu(t-T_\nu)Y(t-T_\nu-1) \quad .$$

Assuming that $Y(t-T_\nu-1) = Y(t)e^{-\rho(T_\nu+1)}$, where ρ is the average annual growth of GNP in $[t-T_\nu,t]$, the above relation can be also witten as

$$Z(t) = \sum_{\nu=1}^{m} Z_\nu(t) \quad , \quad Z_\nu(t) = \tilde{\gamma}_\nu(t)Y(t) \quad , \qquad (19)$$

where

$$\tilde{\gamma}_\nu(t) = \gamma_\nu(t-T_\nu)e^{-\rho(T_\nu+1)} \quad .$$

It should be observed that,

$$\sum_{\nu=1}^{m} \tilde{\gamma}_\nu(t) < 1.$$

The number Z in (17) can be assumed equal to

$$Z = \int_0^T w(t)\ Z(t)\ dt \quad , \quad w(t) = \text{given discount function} \qquad (20)$$

The amplitude constraints are characteristic of the development in a closed economy (autarky) when expenditure used for factor endowments (i.e. investments, labour, education, R & D, etc.) are strictly limited by the GNP currently achieved.

In the case of integral constraints, it is possible to make use of international cooperation by taking foreign credits, exchange in skilled labour, expertise, etc. The only requirement is that foreign credits should be paid back together with the interest, described by the functions $W_\nu(t)$, of the type

$$W_\nu(t) = (1 + \varepsilon_\nu)^{T-t} \quad , \quad \nu = 1,\ldots,m \quad .$$

As will be shown later in the case of cooperative development (i.e. development subject to (17) and (18) only) growth is faster than in the case of complete autarky (i.e. development which is subject to (15) and (16) only).

Before we formulate the corresponding optimization problems, the optimization goals should be introduced.

In the present section, the discounted and integrated GNP, (within the planning interval)

$$Y = \int_0^T w(t)\ Y(t)\ dt, \tag{21}$$

where

$$w(t) = (1 + \varepsilon)^{-t} = e^{-\lambda t} \quad , \qquad (\lambda = \ln(1 + \varepsilon)),$$

ε = the given discount rate ;

will be used.

<u>The problem of optimization of development can be formulated as follows. Find the strategies $z_\nu(t) = \hat{z}_\nu(t)$, $\nu = 1,\ldots,m$, which maximize (21) subject to the integral (17), (18) and/or amplitude (15), (16) constraints.</u>

From the mathematical point of view, the optimization problem formulated boils down to the maximization of the strictly concave functional subject to a number of functional (17), (18) and operator type (15), (16) of constraint. There are many well known techniques suitable for attacking that problem. We shall use here, and in the next sections, a relatively simple technique which makes use of some well known integral inequalities (Hölder and Minkowski inequalities, mainly). As shown in Reference [20], that technique enables the expression of optimum strategies in a simple and explicit form.

The functional (21), where Y(t) is described by (13) and (14), can be written in the following form

$$Y(\underline{z}) = K\int_0^T \prod_{\nu=1}^{m} f_\nu(t)\, dt \quad , \tag{22}$$

$$f_\nu(t) = \left\{ e^{(\mu-\lambda)\frac{t}{\beta_\nu}} Y_\nu(z_\nu) \right\}^{\beta_\nu} \quad . \tag{23}$$

Using the generalized Hölder inequality, it is possible to derive the upper bound for $Y(\underline{z})$:

$$Y(\underline{z}) \le K \prod_{\nu=1}^{m} \left\{ \int_0^T |f_\nu^{\frac{1}{\beta_\nu}}(t)|\, dt \right\}^{\beta_\nu} \quad .$$

The upper bound is attained if the following conditions:
$c_\nu f_\nu^{1/\beta_\nu}(t) = c_1 f_1^{1/\beta_1}(t)$, c_ν = const, $\nu = 2,\ldots,m$, or

$$c_\nu e^{\vartheta_\nu t} Y_\nu(z_\nu) = c_1 Y_1(z_1) \quad , \quad \vartheta_\nu = (\mu-\lambda)\ (\beta_\nu^{-1} - \beta_1^{-1}) \quad , \tag{24}$$

$\nu = 2,\ldots,m$; $t \in [0,T]$,† hold.

The basic idea behind the optimization of development is to satisfy the "factor coordination principle" (24) and choose the parameters c_ν, $\nu = 2,\ldots,m$ in such a way that the constraints (15) - (18) hold.

<u>The factor coordination principle can be regarded as the necessary conditions of optimality.</u> <u>According to that principle, in order to get the maximum of Y, it is necessary to spend the z_ν expenditures in such a way that the development factors $Y_\nu(z_\nu)$ rise in fixed proportions</u>. In other words, it does not pay to increase the capital stock in the production sector if

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† The functions $Y_\nu(t)$ are assumed to be integrable and the conditions (24) should hold almost everywhere.

there is no skilled labour available or if the education level is not adequate.

When the coordinated growth strategy is used it is possible to separate the contribution of $\underline{z}$ to growth, i.e.

$$Y = K \prod_{\nu=2}^{m} c_\nu^{-\beta_\nu} \int_0^T dt \; e^{\vartheta_1 t} \int_{-\infty}^{t} e^{-\delta_1 (t-\tau)} \; [z_1(\tau - T_1)]^\alpha \, d\tau$$

$$= \bar{Y} + \Delta Y \quad , \tag{25}$$

where $\bar{Y}$ is the contribution to GNP resulting from past decisions: $z_\nu(t)$, $t < -T_\nu$, $\nu = 1,\ldots,m$, and

$$\Delta Y = K \prod_{\nu=2}^{m} c_\nu^{-\beta_\nu} \int_0^T dt \; e^{\vartheta_1 t} \int_0^t e^{-\delta_1 (t-\tau)} \; [z_1(\tau - T_1)]^\alpha \, d\tau \tag{26}$$

represents the contribution to the GNP in the planning interval $[0,T]$, resulting from the expenditures $z_\nu(t)$ $t \in [-T_\nu, T - T_\nu]$, $\nu = 1,\ldots,m$, as shown in Figure 3.

First of all we shall solve the optimization problem subject to the integral constraints (17) only. Changing the integration order in (26) one can express (26) in the form:

$$\Delta Y = \prod_{\nu=2}^{m} c_\nu^{-\beta_\nu} \int_0^T [w_1(\tau) z_1(\tau - T_1)]^\alpha \; [w_1(\tau)\varphi_1(t)]^q \, d\tau \quad ,$$

$$q = 1 - \alpha \; , \tag{27}$$

where

$$\varphi_1(\tau) = \left\{ K W_1(\tau)^{-q} \int_\tau^T e^{\vartheta_1 t - \delta_1 (t-\tau)} \, dt \right\}^{\frac{1}{q}}$$
$$= \left\{ K \frac{w_1(\tau)^{-q}}{\delta_1 - \vartheta_1} \; e^{\delta_1 \tau} \left[e^{(\vartheta_1 - \delta_1)\tau} - e^{(\vartheta_1 - \delta_1) T} \right] \right\}^{\frac{1}{q}} \; .$$

Applying the Hölder inequality to (27) one gets

$$\Delta Y \leq \prod_{\nu=2}^{m} c_\nu^{-\beta_\nu} \left\{ \int_0^T |w_1(\tau) z_1(\tau - T_1)| \, d\tau \right\}^\alpha \left\{ \int_0^T |w_1(\tau)\varphi_1(\tau)| \, d\tau \right\}^q ,$$

where the equality sign holds when

$$\hat{z}_1(\tau - T_1) = c_1 \varphi_1(t) \quad , \qquad c_1 = \text{const}, \quad \tau \in [0,T] \quad .$$

The constant c_1 can be derived by (17) yielding

$$\hat{z}_1(\tau - T_1) = \frac{\varphi_1(t)}{\int_0^T w_1(\tau)\varphi_1(\tau)\, d\tau} Z_1 \quad . \tag{28}$$

In order to find the remaining strategies $\hat{z}_\nu(t)$, $\nu = 2,\dots,m$, it is necessary to solve the set of integral equations (24):

$$c_\nu e^{\vartheta_\nu t} \int_0^t e^{-\delta_\nu(t-\tau)} [z_\nu(\tau - T_\nu)]^\alpha d\tau$$

$$= \int_0^t e^{-\delta_1(t-\tau)} [z_1 (\tau - T_1)]^\alpha d\tau \quad , \qquad \nu = 2,\dots,m \quad . \tag{29}$$

For that purpose it is convenient to use the Laplace transforms of (29) [24]. For example, in the case when $\vartheta_\nu = 0$, $\nu = 2,\dots,m$, one gets

$$\hat{z}_\nu (\tau - T_\nu) = c_\nu \varphi_\nu(\tau) \quad , \qquad \nu = 2,\dots,m$$

where

$$\varphi_\nu(\tau) = [\hat{z}_1^\alpha(\tau - T_1) + (\delta_\nu - \delta_1) \int_0^\tau e^{-\delta_1(\tau-t)} \hat{z}_1^\alpha(t - T_1)\, dt]^{\frac{1}{\alpha}} ,$$

and the constants c_ν can be derived by (17) yielding

$$\hat{z}_\nu(\tau - T_\nu) = \frac{\varphi_\nu(\tau)\; Z_\nu}{\int_0^T w_\nu(t)\, \varphi_\nu(\tau)\, d\tau} \quad , \quad \nu = 1,\dots,m \quad . \tag{30}$$

As follows from (30) in order to have the strategies $\hat{z}_\nu$, $\nu = 1,\dots,m$, which satisfy the conditions (16) it is necessary to enumerate the development factors in such a way that $\delta_\nu \geq \delta_1$ $\nu = 2,\dots,m$, i.e. $\nu = 1$, should be assigned to that factor which has the smallest depreciation in time.

Since $\dot{z}_\nu(t)$, depend in linear manner on Z_ν it is possible to see that the GNP value (21) under optimum strategy can be written in the form

$$Y(\underline{z}) = \bar{Y} + \Delta Y(\underline{\hat{z}}) \quad ,$$

where

$$\Delta Y(\underline{\hat{z}}) = G^q \prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu} \quad , \qquad \gamma_\nu = \alpha\beta_\nu \quad , \tag{31}$$

G = a number depending on K, T, T_ν, δ_ν, w_ν parameters. Now it is possible to derive the optimum values of expenditures $Z_\nu = \hat{Z}_\nu$, $\nu = 1,\ldots,m$, which maximize (31) subject to the constraint (18). Since (31) is strictly concave in the compact set (18), a unique optimum solution exists and can be derived by the following formula:

$$\hat{Z}_\nu = \frac{\gamma_\nu}{\sum_{\nu=1}^{m} \gamma_\nu} Z = \beta_\nu Z \quad , \qquad \nu = 1,\ldots,m \tag{32}$$

When the optimum strategy (32) is set in (31) one obtains

$$\Delta Y = \Delta\bar{Y} = G^q \prod_{\nu=1}^{m} \beta_\nu^{\gamma_\nu} \quad Z^\alpha \quad ,$$

which represents the maximum possible development increase under integral constraints. The value of Z here, represents the total resources which have been produced and used in the planning interval [0,T].

As follows from relations (19), (20) one can assume γ_ν equal to the average of $\tilde{\gamma}_\nu(t)$, $\nu = 1,\ldots,m$, so that $\sum_{\nu=1}^{m} \gamma_\nu = \alpha$, and $Z = \alpha Y$. Then

$$Y = \bar{Y} + G^q \prod_{\nu=1}^{m} \gamma_\nu^{\gamma_\nu} \quad Y^\alpha \quad . \tag{33}$$

As shown in Figure 5, a unique solution $Y = Y^*$ of the equation (33) exists, which determines the GNP generated within $[0,T]$ under optimum strategy.

Since $\alpha < 1$, the contraction property of the right side of (33) takes place for any given T or G(T). It should also be noted that when α approaches unity $q \to 0$ and the function $\varphi_1(\tau)$, which determines $\hat{z}_1(t)$, goes at $t = 0$ to infinity, i.e. it approximates the Dirac's $\delta(t)$ function.

It is also possible to show that the optimum strategy, derived for the amplitude constraints (15), (16) degenerates (for $\alpha \to 1$) in the so called "bang-bang" strategy, which requires maximum expenditures at the starting subinterval of $[0,T]$ and expenditures equal zero for the rest of the planning interval. Such strategies cannot be implemented in the real systems. That was one of the reasons why the complex nonlinear production functions (14) have been used here instead of relatively simple linear ($\alpha = 1$) relations.

It should also be observed that when one sets $Z = \gamma Y$ the open loop solutions (30) and (32) become the "closed loop" solutions. Such a procedure is in agreement with the planning practice, which makes projections of GNP rise in order to determine the amount of resources which can be spent on investments and other government expenditures.

Using the general solution method described above, it is possible to investigate some important, special cases. For example, consider the problem of optimization of functional:

$$\Delta Y = \int_0^T \left\{ \int_0^t e^{-\delta(t-\tau)} \; [z_2(\tau)]^\alpha \, d\tau \right\}^\beta \; [z_1(t)]^{\alpha(1-\beta)} \, dt \; , \qquad (34)$$

subject to

$$\int_0^T z_1(\tau)\, d\tau \le Z_1 \quad , \qquad z_1(\tau) \ge 0 \quad , \tag{35}$$

$$\int_0^T z_2(\tau)\, d\tau \le Z_2 \quad , \qquad z_2(\tau) \ge 0 \quad , \qquad t \in [0,T] \tag{36}$$

The variable $z_2(\tau)$ can be interpreted as investment intensity, while $z_1(\tau)$ is the labour employed. The total amount of capital and labour in the planning interval [0,T] is limited according to (36) and (35). The problem consists of finding the optimum strategy of investments $\hat{z}_2(t)$ and employment $\hat{z}_1(t)$.

The strategy $\hat{z}_2(t)$ becomes, by (28):

$$\hat{z}_2 = \frac{\varphi_2(\tau)}{\int_0^T \varphi_2(\tau)\, d\tau} Z_2 \quad ,$$

where

$$\varphi_2(\tau) = \left\{\int_\tau^T e^{-\delta(t-\tau)}\, dt\right\}^{\frac{1}{q}} = \left\{\frac{1}{\delta}\,[1 - e^{-\delta(T-\tau)}]\right\}^{\frac{1}{q}} .$$

The equation (29) becomes

$$c_1 \; z_1(t)^{\alpha} = \int_0^t e^{-\delta(t-\tau)}\, [\hat{z}_2(\tau)]^{\alpha}\, d\tau \quad . \tag{29'}$$

Then

$$\hat{z}_1(t) = \frac{\varphi_1(t)}{\int_0^T \varphi_1(t)\, d\tau} Z_1 \quad ,$$

where

$$\varphi_1(t) = \left\{\int_0^t e^{-\delta(t-\tau)}\, [\hat{z}_2(\tau)]\, d\tau\right\}^{\frac{1}{\alpha}} .$$

The functions $\hat{z}_1(t)$, $\hat{z}_2(t)$, $\varphi_1(t)$, for $\alpha = \frac{1}{2}$, $\delta T = 4$, and with accuracy to the degree of constant multipliers, have been shown in Figure 6.

The optimum strategy of investments $\hat{z}_2(t)$ decreases to zero when $t \to T$, while the labour employed $\hat{z}_1(t)$ increases along with t, according to the factor coordination principle, in such a way

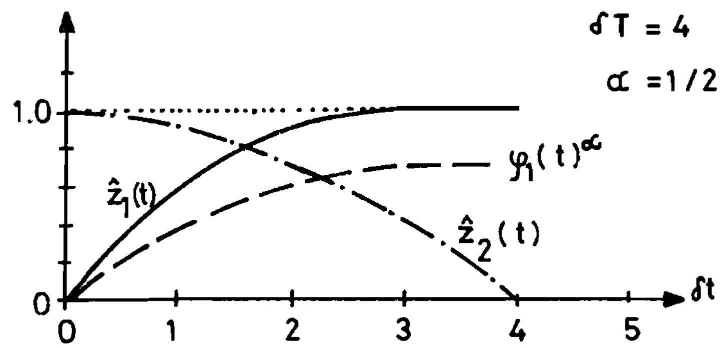


FIGURE 6

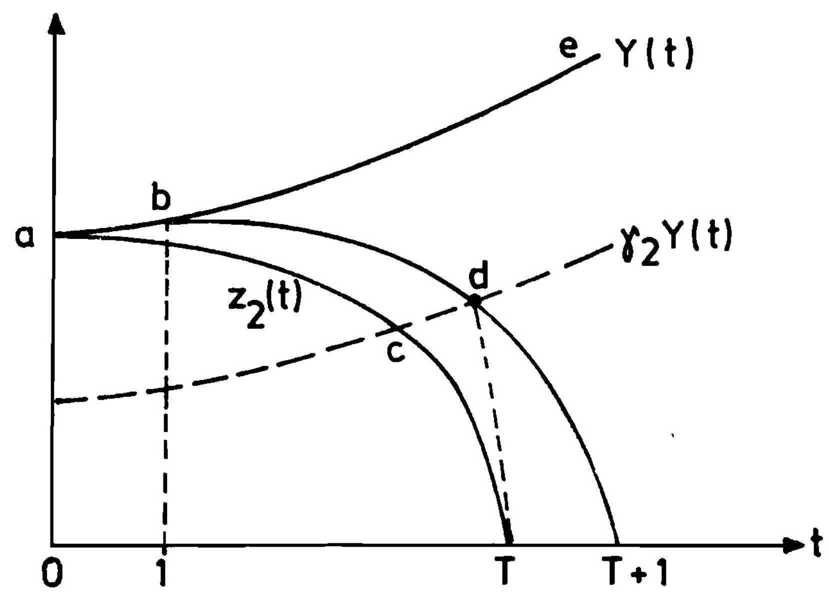


FIGURE 7

that it is proportional to the capital stock $\varphi_1(t)$. It is also possible to solve the problem when $z_1(t)$ is given exogenously, while $\hat{z}_2(t)$ is chosen in such a way as to satisfy (29').

Since the optimum investment strategy $z_2(t)$, for integral constraints may violate the admissible values $\gamma_2 Y(t)$, γ_2 = given number, at the beginning of the planning interval, as shown in Figure 7, it is also necessary to find the optimum strategy for the amplitude constraints (15). Since no harm can be done when one assumes the equality sign in (15), the unknown $\hat{z}_1(t)$ can be derived from the equation

$$z(t) = \sum_{\nu=1}^{m} \hat{z}_\nu(t - T_\nu) = \hat{z}_1(t - T_1)$$

$$+ \sum_{\nu=2}^{n} c_\nu \left[\hat{z}_1^{\alpha}(t - T_1) + (\delta_\nu - \delta_1) \int_0^t e^{-\delta_1(t-\tau)} \hat{z}_1^{\alpha}(\tau - T_1)\, d\tau \right]^{\frac{1}{\alpha}} .$$

In the case when $\alpha \approx 1$, one can solve that equation by means of Laplace Transforms:

$$\hat{z}_1(t - T_1) \cong \mathscr{L}^{-1} \left\{ \frac{\bar{Z}(p)}{\sum_{\nu=2}^{m} c_\nu \frac{p + \delta_1}{p + \delta_\nu} + 1} \right\} , \qquad (37)$$

where

$$\bar{Z}(p) = \mathscr{L}\{Z(t)\} .$$

The numbers c_ν, $\nu = 2,\dots,m$ in (37) can be chosen in such a way that:

$$\frac{\int_0^T z_\nu(\tau - T_\nu)\, d\tau}{\sum_{\nu=1}^{m} \int_0^T z_\nu(\tau - T_\nu)\, d\tau} = \gamma_\nu , \qquad \nu = 1,\dots,m ,$$

where

$$\gamma_\nu \text{ - given numbers} , \qquad \sum_{\nu=1}^{m} \gamma_\nu = 1 .$$

It should also be noted that the optimization method described in the present section can be used to deal with the situation when the development factors and the expenditures z_ν , $\nu = 1,\dots,m$, are subdivided into different, hierarchically ordered categories, as shown in Figure 8. For example, the productive investments can be split among m subcategories (e.g., the investment goods supplied from the different sectors, imported investment goods, foreign credits, etc.). Education can be subdivided into elementary, secondary and higher education, etc.

For example, consider z_1 in (26) as being composed of $z_\nu^{\gamma_\nu}$, $\nu = 1,\dots,m$ factors, so that the production function can be written:

$$\Delta Y = K \int_0^T dt\; e^{\vartheta t} \int_0^t e^{-\delta(t-\tau)} \prod_{\nu=1}^{m} [z_\nu(\tau - T_\nu)]^{\gamma_\nu}\, d\tau \quad , \quad (38)$$

where

$$\int_0^T w_\nu(\tau)\; z_\nu\, (\tau - T_\nu)\; d\tau \le Z_\nu \quad , \qquad \nu = 1,\dots,m \quad , \qquad (39)$$

$$\sum_{\nu=1}^{m} Z_\nu \leqq Z \quad , \qquad (40)$$

$$z_\nu(t - T_\nu) \ge 0 \;, \quad Z_\nu \ge 0 \;, \quad \nu = 1,\dots,m \;, \quad t \in [0,T] \;, \quad (41)$$

$$\sum_{\nu=1}^{m} \gamma_\nu = \alpha < 1 \quad .$$

In order to find the strategies $z_\nu = \hat{z}_\nu$, $\nu = 1,\dots,m$, which maximize (38), subject to the constraints (39) - (41), change the integration order in (38) and use the generalized Hölder inequality:

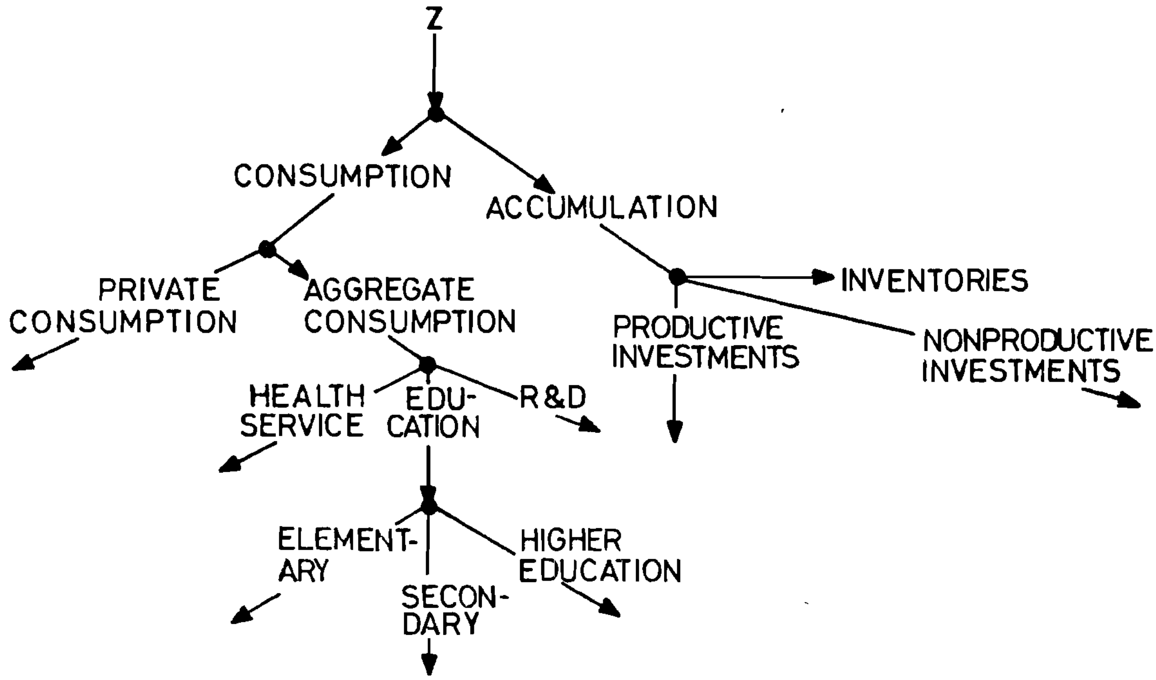


FIGURE 8

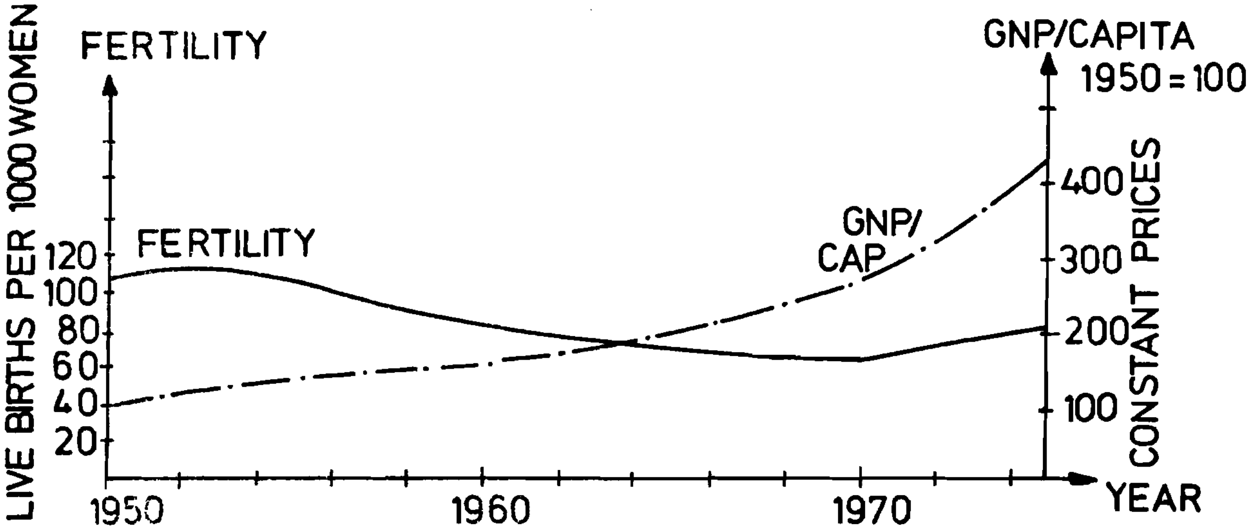


FIGURE 9

$$\Delta Y = \int_0^T \prod_{\nu=1}^m [z_\nu(\tau - T_\nu)\ w_\nu(\tau)]^{\gamma_\nu}\ [\varphi(\tau)]^q\, d\tau \quad , \quad q = 1 - \alpha$$

$$\varphi(\tau) = \left\{K \prod_{\nu=1}^m w_\nu(\tau)^{-\gamma_\nu} \int_\tau^T e^{\vartheta t - \delta(t-\tau)}\, dt\right\}^{\frac{1}{q}} \quad ;$$

$$\Delta Y \leq \prod_{\nu=1}^m \left\{\int_0^T |z_\nu(\tau - T_\nu)\ w_\nu(\tau)|\ d\tau\right\}^{\gamma_\nu} \left\{\int_0^T |\varphi(\tau)|\ d\tau\right\}^q . \tag{42}$$

The equality sign in (42) appears when

$$\hat{z}_\nu(\tau - T_\nu)\ w_\nu(\tau) = c_\nu\, \varphi(\tau) \quad , \quad \nu = 1,\ldots,m \ , \quad \tau \in [0,T] \ .$$

The constants c_ν can be derived by (39), yielding

$$\hat{z}_\nu(\tau - T_\nu) = \frac{\varphi(\tau)\ Z_\nu}{w_\nu(\tau)\ G} \quad , \quad G = \int_0^T \varphi(\tau)\, d\tau \quad , \quad \nu = 1,\ldots,m \tag{43}$$

One gets, therefore

$$\Delta Y\ (\underline{\hat{z}}) = G^q \prod_{\nu=1}^m Z_\nu^{\gamma_\nu} \ . \tag{44}$$

Now it is possible to derive $Z_\nu = \hat{Z}_\nu$, $\nu = 1,\ldots,m$, which maximize (44) subject to (40):

$$\hat{Z}_\nu = \frac{\gamma_\nu}{\sum_\nu \gamma_\nu} Z = \beta_\nu Z \quad , \quad \nu = 1,\ldots,m \tag{45}$$

It should also be observed that the government expenditures can generally be subdivided into "inertial and noninertial" categories. For example, the part of these expenditures which goes into capital investments in R & D, education, health, etc., behaves in the same way as the "productive" investments in the general expression (13), (14). In a similar way, the labour employed in services (i.e. education, health, R & D, etc.) has the same (noninertial) effect on the growth of GNP as the productive labour. The corresponding elasticities (β_ν in (14)) are, of course, different.

1.3. The long term development planning problems

Planning in the socialist economy is mostly concerned with an optimum allocation of resources. In the present section we shall investigate that problem from the point of view of allocation of GNP among the development factors. Since planning involves information about the future state of the system concerned and the system parameters change slowly, and usually in an unpredictable fashion, the standard procedure is to use the "moving planning horizon approach". According to that approach, the planning process is repeated each year with new statistical information regarding the changes of system parameters, constraints and development objectives. The strategy derived at each new planning interval should replace the previous strategies in such a way that the continuation of a general development strategy is possible. For example, expenditures in investments should cover the expenditures connected with the continuation of construction processes of factories originated in the past as well as they should cover the expenditures connected with the present and future constructions. The optimum investment strategy for the integral constraints decreases when time approaches the end of the planning horizon T, as shown in Figure 7 by acT-curve. The optimum strategy derived for the planning interval [1, T + 1] shown in Figure 7 by bdT + 1 curve also decreases in time when $t \to T + 1$. However, the effectively spent investments at each consecutive year represent the abe-curve which is rising in given proportion to the GNP growth. In other words, the optimum investment strategy does not terminate in time. The same statement is true for the strategy with amplitude constraints, as shown in Figure 7. The optimum investment strategy moves along with the curve fcd, which is described by $\gamma_2 Y(t)$, where γ_2 is a given coefficient.

As stated in Section 1.2., the fastest growth, in term of GNP, can be obtained when the expenditure strategy follows the "coordinated growth principle" (24). In order to implement that strategy in the case of development factors with long delays (T_ν) a prediction of future development strategy is needed. Consider, for example, the education system in Poland which has $T_\nu \cong 10$ years. In order to get the maximum benefits out of education, it is necessary to teach the presently young generation the skills required, not by the present but by the expected socio-economic needs in the year 1986. It is therefore important to predict the future professional qualification structure, specialization, etc. The same arguments can be used to prove that the prediction of future development strategy is required when planning the other inertial factors of development -- first of all, R & D, environment protection, demographic policy. In the short range planning problems (e.g. T = 5 years) many of the factor endowments have a negligible effect on the development, as shown in Figure 3 for $T_\nu > T$. Since one cannot affect the development by changing these factors, it has become a standard practice to replace them by the exogenous term $e^{\mu t}$, as shown in Section 1.1., and deal with the single production function of the form (6). That approach constitutes the main concern of macroeconomic planning. The extension of the planning interval necessitates, however, the inclusion of all the other important socio-economic and environmental factors in the planning process.

One of the most important factors is the labour force. In short range planning the labour is usually regarded as an exogenous factor. That can be explained by taking into account that it takes almost eighteen years before there is any effect which results from a change in the demographic policy. When, for example, the system of social benefits for families with many children is changed, it may affect the female fertility factor instantly, but the labour force will change only when the new born children reach the age of adolescence. In other words, in order to have an effective planning system for demographic policy, it is necessary to deal with the planning horizon T longer than the demographic delay $T_d \approx 18$ years.

It is well known that in many developed countries the female fertility F decreases in time. That can be explained by assuming that F depends on such factors as the value of GNP per capita, health service level, family planning level, social care program, traditions, religions, etc. In particular, it is believed that a strong correlation of F to the GNP per capita exsists and as a result F decreases along with the rise of GNP/capita. Figure 9 shows the change in recent years of female fertility in Poland and the GNP/capita. Fertility decreases up to 1970. A slow down and a slow rise of fertility (starting from 1970) can probably be attributed to a constant increase in the social benefits for families with many children. In 1960, about 1.85% of GNP was being spent on the additional monthly allowances which rises in proportion to the number of children. The present system of social benefits in Poland (supplemented with Acts of Law 14, 1972 and December 17, 1974) includes in addition:

- 16-18 weeks paid maternity leave;
- 60 days of paid leave per annum for taking care of a sick child;
- protection against dismissal from the job during maternity;
- leave with the option of transfer to suitable employment during pregnancy;
- health insurance benefits and many others.

It is expected that the above program will be further developed and extended.

The second group of social policy measures is aimed at reducing the cost of child upbringing. It is believed that the successful implementation of the above programmes, serving the purpose of social justice and welfare, will help in preparing the ground of a policy stimulating population growth, should the need arise [48].

From the point of view of system analysis, it is important to know the elasticity α_d of fertility with respect to the social care expenditures z_d:

$$\alpha_d = \frac{dF}{F} : \frac{dz_d}{z_d}$$

Then one can try to construct a model of the general form:

$$F = F\ [\underline{x}, z_d] = \bar{F}\ [\underline{x}]\, z_d^{\alpha_d} \quad , \tag{46}$$

where $\underline{x}$ = the vector of exogenous variables including such factors as GNP/capita, health-service level, etc.

The next step is to find the relation between the fertility and total labour force L, which enters as a development factor in (13). In order to do that, it is necessary to employ a model of population growth. Keeping in mind the general form of development factor levels (7), it is convenient here to use a continuous (Lotka) version of population processes. Following Ref. [15], assume that the births of the community concerned have gone through a certain trajectory, described by B(t) -- the density of births -- and a fixed life table gives the number of surviving to age a on radix unity, $p(a)^{\dagger}$. Then the number of persons at each age a of time t is determinate and equal to $B(t - a)\, p(a)$ and by integration, the total population at time t must be

$$N(t) = \int_0^{\infty} B(t - a)\, p(a)\, da \quad , \tag{47}$$

where $p(a) = 0$ for $a > w$ = last age of life table.

In order to get the amount of people in the productive age, one has to set

$$\bar{p}(a) = p(a) \quad , \quad a \geq T_d \quad ,$$
$$= 0 \quad , \quad a < T_d \quad ,$$

T_d = the age of adolescence.

Assuming that a part $\xi (0<\xi<1)$ of the total population in the productive age group can be employed and introducing the new variable $\tau = t - a + T_d$ in the integral (47) one gets the labour employed:

$$L(t) = \xi \int_{-\infty}^{t} \bar{p}(t - \tau + T_d)\, B\,(\tau - T_d)\, d\tau \quad . \tag{48}$$

Since $B(\tau - T_d)$ can be regarded as the product of total (female) population in reproductive age $\bar{N}(t)$, which is a part of 0 N(t), and fertility $F(\tau - T_d)$ one can write (48) in the form

---

†p(a) can also be interpreted as a chance of living a years by a person born at t = 0, i.e.

$$p(a) = \exp[-\int_0^a \mu(t)\, dt] \quad ,$$

where $\mu(t)$ is the instantaneous death rate.

$$L(t) = \int_{-\infty}^{t} k_d(t, \tau)\, z_d^{\alpha_d} (\tau - T_d)\, d\tau \quad , \tag{49}$$

where

$$k_d(t, \tau) = \xi\, \bar{p}(t - \tau + T_d)\, \bar{F}\, [\underline{x}]\, \dot{N}(\tau - T_d) \tag{50}$$

It can be observed that (49) does not differ much when compared to (7), (14). The only difference is that (50) is generally not stationary in time, i.e. $k_d(t, \tau) \neq k_d(t - \tau)$. However, as shown in Ref. [22], the general optimization technique, which has been used so far, can be easily extended to the non-stationary systems described by (49). The same remark concerns α_d which may be, generally speaking, different from α. Apart from that, the optimization of development methods, previously described, may be easily extended to the case which includes the expenditures spent on social benefits connected with labour and population policy. The impact of the expenditures z_d change on the labour level change is (in the present model) similar to the impact of investment change on the capital stock level. One can "invest" here in the population sector out of the present resources (i.e. GNP) in order to increase the labour force, which is the main production factor, for future development.

It is possible to observe, also, that L(t) depends directly on the ξ parameter, which represents the employment share in the total productive population. In the case where one changes the duration of working time, employment of women, etc., the value of $\xi(t)$ can be regarded as the decision parameters. The shortening of working time can be regarded as contribution to the social benefits programme. The leisure time increased in that way can be used effectively for relaxation, better education, wider participation of the whole population in cultural and social life, etc. On the other hand, the factor $\bar{F}[\underline{x}]$ depends on the exogenous parameters, or on the parameters which are endogenous, but shifted in time by T_d so they can be regarded as given (i.e. determined at the previous optimization intervals).

It should be noted that the problem of optimization of population policy involves many different factors and aspects. The model analyzed here takes into account the "productive aspects" of population change mainly in development.

In order to use that model for optimization of long term development, it is also necessary to investigate the influence of all the factors affecting fertility and to construct the corresponding explicit relationship $\bar{F}[\underline{x}]$.

1.4. Pollution impact on environment and development

The productive activity is usually accompanied by side production of waste materials which are generally harmful to the human environment. In the present state of science and technology, most of the waste materials can be purified, utilized or recycled. However, the cost of purifying waste materials increases rapidly when a high degree of purity is required. Since the environment has an ability of clearing itself with the waste decay ratio, (depending on the waste ingredients) the following approach to the pollution problem has been proposed: Minimize the cost of waste and pollution treatment subject to the conditions that the degree of environment pollution is less than a given value. Following that approach, consider the pollution control model shown in Figure 10.

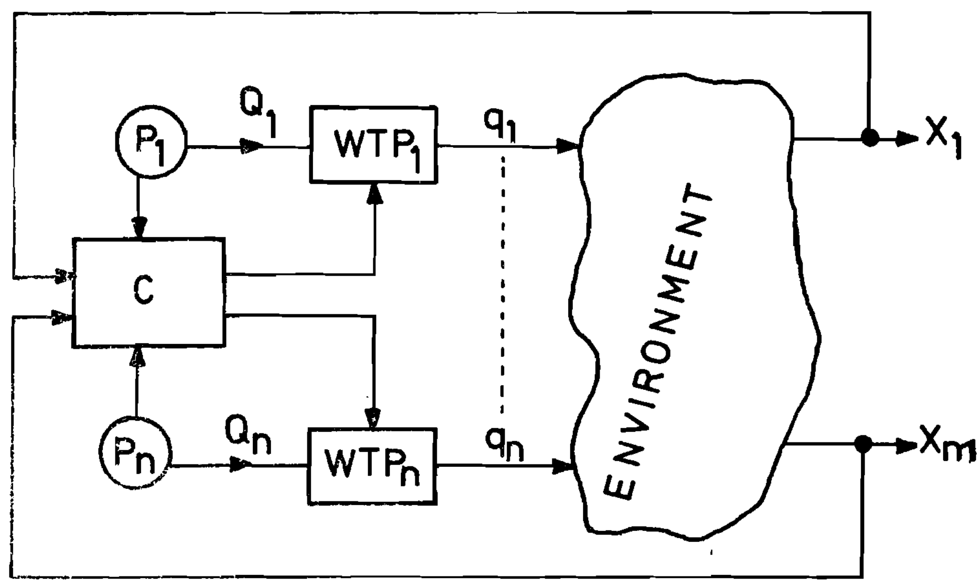


FIGURE 10

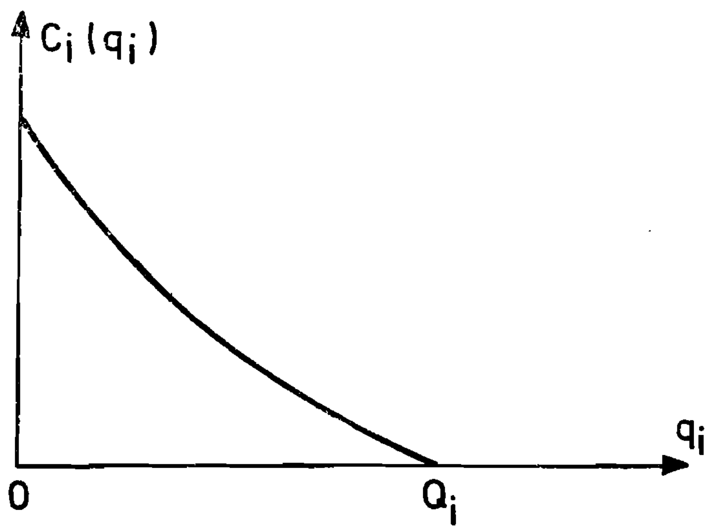


FIGURE 11

Assume that the waste with intensity $Q_i(t)$, $i = 1,\ldots,n$, generated by n given polluters P_i (such as factories, power plants, urban centers, etc.) is being treated by the waste treatment plants WTP_i and with the intensity q_i is discharged into the environment (i.e. into air, water or soil). Q_i in turn may depend on the sectors production. The degree of environment contamination (expressed by such factors as pollutant fall out, dissolvent oxygen (D.O.) concentration, or the biological oxygen demand (B.O.D.)) $x_i(t)$ can be observed by the pollution sensitive devices in the m given points or areas. The information obtained in that way together with the information regarding the weather forecast etc. is being used by the controller C to optimize the decision variables q_i, $i = 1,\ldots,n$.

The performance of pollution control can be measured by the functional

$$\Phi = \sum_{i=1}^{n} \int_0^T w_i(t) x_i(t) dt \quad , \tag{51}$$

where

$w_i(t)$ = given non-negative continuous weight functions,

T = optimization horizon.

The input-output dynamical properties of the environment according to the theoretical and experimental data can be approximated by the Volterra operator [27].

$$x_j(t) = \sum_{i=1}^{n} \int_0^T K_{ij}(t,\tau) q_i(\tau) d\tau \quad , \qquad j = 1,\ldots,m \quad , \tag{52}$$

where

$K_{ij}(t,\tau)$ = given non-negative continuous functions which satisfy the causality condition $K_{ij}(t,\tau) = 0$ for $t < \tau$.

A typical example of the cost function of the waste treatment plant has been shown in Figure 11. It can be approximated by the function

$$C_i(q_i) = k_i^{1-\alpha}(Q_i - q_i)^{\alpha} \quad , \qquad \alpha > 1 \quad , \qquad k_i > 0 \quad . \tag{53}$$

It is also assumed that the total waste treatment cost is limited, i.e.

$$\sum_{i=1}^{n} \int_0^T C_i(q_i)dt \leq C \quad , \tag{54}$$

where

C = given positive number.

<u>The pollution treatment optimization problem can be formulated as follows. Find the non-negative strategy $c_i = \hat{c}_i$, $i = 1,\ldots,n$, such that</u>

$$\Phi(c) = \sum_{j=1}^{m} \int_0^T w_j(t) \sum_{i=1}^{n} \int_0^T K_{ij}(t,\tau) \ [Q_i(\tau)+k_i^{1-\beta}c_i^{\beta}(\tau)]d\tau\, dt \quad ,$$

<u>where</u>

<u>$\beta = 1/\alpha$ attains for $c = \hat{c}$ the minimum value subject to the constraints (54), and</u>

$$Q_i(\tau) - k_i^{1-\beta} c_i^{\beta}(\tau) \geq 0 \quad , \quad t \in [0,T] \quad , \tag{55}$$

$$c_i(\tau) \geq 0 \quad . \tag{56}$$

Since the term

$$\Phi = \sum_{j=1}^{m} \sum_{i=1}^{n} \int_0^T w_j(t) \int_0^t K_{ij}(t,\tau) Q_i(\tau) d\tau \; dt$$

is a constant, the problem boils down to the maximization of

$$F(c) = \sum_{i=1}^{n} \int_0^T f_i^q(\tau) c_i^{\beta}(\tau) d\tau \quad ,$$

where

$$f_i(\tau) = \left[\int_{\tau}^{T} k_i^{1-\beta} \sum_{j=1}^{m} w_j(t) K_{ij}(t,\tau) dt \right]^{1/q} \quad , \qquad q = 1 - \beta$$

subject to the constraints (54) - (56).

It is obvious that when the constraint (55) is not active, the optimum control strategy can be derived by using the method of Section 1.3. When (55) is active the optimum strategy can be derived from the equation:

$$Q_i(t) - k_i^{1-\beta} \hat{c}_i^{\beta}(t) = 0 \quad .$$

Then the following theorem can be proved (for details see Ref. [27]).

<u>An optimum pollution strategy</u>

$$c_i(t) = \hat{c}_i(t) = \tilde{f}_i(t) \frac{C}{F} \quad , \qquad i = 1,\ldots,n, \qquad t \in [0,T] \quad , \tag{57}$$

where

$$\tilde{f}_i(t) = \begin{cases} f_i(t) & \text{for } t \notin S_i \\ \frac{F}{C} k_i^{1-1/\beta} Q_i^{1/\beta}(t) & \text{for } t \in S_i \end{cases} ,$$

$$S_i = \left\{ t : f_i(t) > \frac{F}{C} k_i^{1-1/\beta} Q_i^{1/\beta}(t) \quad , \qquad t \in [0,T], \right.$$

$$\left. i=1,\ldots,n \right\} \quad ,$$

$$F = \sum_{i=1}^{n} \int_0^T \tilde{f}_i(t)\, dt$$

exists, such that

$$\Phi(\hat{C}) = \min_{C=\Omega} [\bar{\Phi} - F(C)] = \bar{\Phi} - F^{1-\beta}C^{\beta} \tag{58}$$

and Ω is the admissible control set defined by (54) - (56).

The optimum waste discharge strategies become

$$\hat{q}_i(t) = k_i^{1-\beta}\hat{c}_i^{\beta}(t) \quad , \qquad i = 1,\ldots,n \quad .$$

Using that theorem, it is possible for a given admissible pollution level ε to find the corresponding minimum waste treatment cost $\hat{C}$ (by solving the equation $\bar{\Phi} - \varepsilon = F^{1-\beta}C^{\beta}$)

$$\tilde{C} = \left[\frac{\bar{\Phi} - \varepsilon}{F^{1-\beta}} \right]^{1/\beta} \tag{59}$$

When the function $\bar{\Phi}(Y)$, which expresses the pollution level in terms of production Y, increases rapidly it may happen that the corresponding waste treatment cost becomes greater than the production income. In that case a new technology of production or new waste treatment plant should be developed.

That requires capital investment which can be optimized by the methods described in Section 1.2 and 2.2.

Using the aggregation formula (58) it is also possible to optimize a complex hierarchical system of environment pollution control (for details see Ref. [27]).

On the macro-level, the pollution cost C impact on the development can be analysed starting with formula (33). Obviously the pollution cost should be subtracted from the GNP generated in the planning interval Y so that Z = Y - C(Y) and (33) can be written

$$Y = \bar{Y} + G^q \prod_{y=1}^{m} Y_\nu^{\prime\nu} \{Y - [\frac{\bar{\Phi}(Y) - \varepsilon}{F^q}]^{\frac{1}{\beta}}\}^\alpha \tag{60}$$

Since $\bar{\Phi}$ depends on Q_i in a linear fashion, it is natural to assume that $\bar{\Phi}(Y) = aY$, where a is a constant. Then it is possible to see that the right side of (60) decreases along with the admissible pollution level ε. As a result the solution Y^{**} of equation (60) decreases along with ε as shown in Figure 5. In other words the higher the standards are set, with respect to the pollution abatement policy, the lower the integrated GNP can be obtained. The admissible value of ε depends of course on many factors and the feedbacks exist between ε and government expenditures in health, housing and urban quality areas. In the present model ε will be regarded as a given (exogeneous) parameter representing the existing or planned standards in pollution abatement policy.

1.5 Development objectives, utility functionals

When one solves a complex optimum development problem the main concern is to choose the appropriate development goal. So far the integrated GNP has been used for that purpose. Doubts have frequently been expressed that it can hardly be used as the universal goal for development. It has been argued that the maximization of consumpation per capita might be an alternative. Assuming the latter as a goal, one begins to wonder whether he really needs to spend much on development factors. However, rigorous analysis shows that it is impossible to consume, without financing, the development factors. We shall show that in the simplified situation, when the development is described by the function

$$Y(t) = K \prod_{\nu=1}^{m} [Y_\nu(t)]^{\beta_\nu} , \qquad \sum_{\nu=1}^{m} \beta_\nu = 1 , \tag{61}$$

$$Y_\nu(t) = \int_{-\infty}^{t} z_\nu(\tau) dt ,$$

i.e. $\delta_\nu = T_\nu = 0$, $\alpha = K_\nu = 1$, $\nu = 1,\dots,m$.

We shall also assume that labour is exogenously introduced and is growing in an exponential fashion:

$$L(t) = \bar{L} e^{\lambda t} , \qquad \lambda > 0 .$$

The GNP generated per capita can be written in the form

$$Y/L = K \prod_{\nu=1}^{m} W_\nu^{\beta_\nu} ,$$

where

$W_\nu = Y_\nu/L$ = development factor level/capita

The GNP is allocated in such a way that

$$Y_1(t) = Y(t) - \sum_{\nu=2}^{m} z_\nu(t) = Y(t) - \sum_{\nu=2}^{m} \dot{Y}_\nu(t)$$

represents the consumption.

Consider the steady-state of development

$$\dot{W}_\nu(t) = \frac{\dot{Y}_\nu}{L} - \frac{Y_\nu}{L^2}\dot{L} = \frac{\dot{Y}_\nu}{L} - \lambda W_\nu = 0 \quad , \qquad \nu = 2,\ldots,m,$$

$t = 1,\ldots$.

The consumption per capita becomes

$$C = \frac{Y_1}{L} = \frac{Y}{L} - \sum_{\nu=2}^{m} W_\nu \lambda = K \prod_{\nu=2}^{m} W_\nu^{\beta_\nu} - \sum_{\nu=2}^{m} W_\nu \lambda \quad . \tag{62}$$

<u>The following optimization problem can be formulated: find the values $W_\nu = \hat{W}_\nu$, $\nu = 2,\ldots,m$ which maximize the consumption per capita C.</u> Since (62) is concave the unique optimum strategy exists and it can be derived as the solution of the equations

$$\beta_\nu W_\nu^{\beta_\nu - 1} \prod_{\substack{i=2 \\ i\neq\nu}}^{m} W_i^{\beta_i} = \lambda/K \quad , \qquad \nu = 2,\ldots,m \quad ,$$

or

$$\begin{aligned} (\beta_2-1)\ln W_2 + \cdots + \beta_m \ln W_m &= \ln \frac{\lambda}{K\beta_2} \quad , \\ \vdots \qquad\qquad \cdots \qquad\qquad & \qquad \cdots \\ \beta_2 \ln W_2 + \cdots + (\beta_m-1)\ln W_m &= \ln \frac{\lambda}{K\beta_m} \quad . \end{aligned} \tag{63}$$

When

$$D = \begin{vmatrix} \beta_2-1 \; , & \beta_3 \cdots \beta_m \\ & \\ \beta_2 \; , & \beta_3 \cdots \beta_m-1 \end{vmatrix} \neq 0 \quad ,$$

there exists a unique positive solution of (63) $W_\nu = \hat{W}_\nu$, $\nu = 2,\dots,m$.

The result obtained can be formulated as follows. Among the systems having different development factor levels; W_ν, $\nu = 2,\dots,m$, and $D \neq 0$, the highest consumption per capita can be achieved for $W_\nu = \hat{W}_\nu$, $\nu = 2,\dots,m$. One can regard that result as an extension of the "classical golden rule of development" formulated originally for the simple-two factor production function (6).

What the present version of the golden rule says is, that in the steady-state the maximum consumption per capita can be enjoyed if the development factors have a predetermined level. When one is determined to reach that level, the expenditures in development factors are inevitable. One can also consider the development factors Y_ν, $\nu = 1,\dots,m$, as valuable assets and regard the function (61) as a utility function. The last interpretation suggests that the capital stock, health, R & D, education facilities, etc. are public property. That assumption is fully justified in the socialist countries.

A known approach in the utility theory consists in finding the decision maker's strategy $Y_\nu = \hat{Y}_\nu$, $\nu = 1,\dots,m$, which maximizes the utility

$$U = K \prod_{\nu=1}^{m} Y_\nu^{\beta_\nu} , \tag{64}$$

subject to the budget constraints

$$\sum_{\nu=1}^{m} \omega_\nu Y_\nu \leq Z \tag{65}$$

where

ω_ν - prices attached to Y_ν , $\nu = 1,\dots,m$.

The optimum solution (in steady state) becomes

$$\hat{Y} = \frac{\beta_\nu}{\omega_\nu} Z \quad , \qquad \nu = 1,\dots,m \quad . \tag{66}$$

Since the prices ω_ν (excluding the average wage) are generally unknown and $Y_\nu(t)$ depend in an inertial way on $z_\nu(\tau)$ the strategy (66) cannot be derived in the effective way.

One can, however, formulate a problem of maximization of the functional (21), which represents the integrated GNP, in terms of utility function approach. Since the factor levels Y_ν in (13) depend in the inertial fashion on the expenditures z_ν, and the $Y_\nu(t)$ change in time we have here a problem of preferences in time. Then it is natural to use the functional (21) with the budgetary constraints (15) - (18), as the expected utility in the planning interval [0,T]. Assuming that the central decision maker allocates the resources in the optimum manner, we have seen that the utility satisfies the additive property (25), i.e. one can deal with the increase of utility functional $\Delta U = \Delta Y(\hat{\underline{Z}})$ resulting out of the "present" time optimum expenditures strategy. Under that strategy one arrives at the "static" function (31):

$$\Delta U = G^q \prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu} \quad , \qquad \gamma_\nu = \alpha\beta_\nu \quad , \tag{67}$$

with the budget constraint

$$\sum_{\nu=1}^{m} Z_\nu \leq Z \quad . \tag{68}$$

The values of Z_ν represent here, however, the total (i.e. integrated within [0,T]) expenditures i.e. the values of factors or services acquired in [0,T]. When the prices ω_ν do not change in [0,T] it is also possible to write (67) and (68) in the form similar to (64) and (65):

$$\Delta U = \bar{G}^q \prod_{\nu=1}^{m} \bar{Y}_\nu^{\gamma_\nu} , \tag{69}$$

$$\sum_{\nu=1}^{m} \omega_\nu \bar{Y}_\nu \leq Z , \tag{70}$$

where

$$\bar{Y}_\nu = Z_\nu/\omega_\nu \quad , \qquad \bar{G}^q = G^q \prod_\nu \omega_\nu^{-\gamma_\nu} .$$

One of the <u>important problems in modelling practice is the experimental determination of the utility function parameters</u>. One possible approach is <u>to learn about the utility from the decisions already taken in the past</u>. For example, the optimum decisions which maximize (67) subject to (68), according to (32), become

$$\hat{Z}_\nu = \beta_\nu Z \quad , \qquad \nu = 1,\ldots,m . \tag{71}$$

Assume that the "moving horizon" technique has been used for the past sequence of planning intervals $[-\tau, T-\tau]$, $\tau = 1,2,\ldots$ and the corresponding statistical data: $Z(\tau)$, $\hat{Z}_\nu(\tau)$, $\tau = 1,2,\ldots$ $\nu = 1,\ldots,m$, are available. Then it is possible to estimate the values of β_ν, $\nu = 1,\ldots,m$, from (71) by known statistical methods. The estimated values $\hat{\beta}_\nu$ of β_ν can be used 'ex ante' for the planning interval [0,T]. When $\hat{\beta}_\nu$ change slowly in time, a prediction technique (as will be shown later) can be also applied.

It should be observed that the proposed estimation technique is based on the assumption that the decision regarding the allocation of Z_ν expenditures are optimum with respect to the goal function (67). The β_ν parameters, which were regarded so far as inherent to the development mechanisms (including the technology of production), are generally unknown 'ex ante' to the decision maker but one is inclined to treat them as "stable" or slightly changing in time. On the other hand the observed, from the statistical data values $\tilde{\beta}_\nu(\tau) = \frac{Z_\nu(\tau)}{Z(\tau)}$, $\nu = 1,\ldots,m$, $\tau = 0,-1,\ldots$ change usually randomly in time, which can hardly be attributed to the change of development mechanisms. Rather, these changes should be attributed to the slight variations in the allocation strategy: $\tilde{Z}_\nu = \tilde{\beta}_\nu Z$, $\nu = 1,\ldots,m$, as a result of which the utility decreases by the factor δ, i.e. $\tilde{\Delta U} = \delta \Delta U$, where

$$\delta = \prod_{\nu=1}^{m} \left(\frac{\tilde{\beta}_\nu}{\beta_\nu}\right)^{\gamma_\nu} .$$

One can assume, however, that the values estimated by known statistical methods (e.g. the mean squares) $\hat{\beta}_\nu$, $\nu = 1,\ldots,m$, are close to β_ν , $\nu = 1,\ldots,m$. Otherwise the utility $\tilde{\Delta U}$ would be much less than the potentially possible value ΔU.

Much that has been said about the allocation of government expenditures concerns the private (personal) consumption. In particular, the dynamic model of consumption expenditures can be used here. As shown in Ref. [23] the consumers change their preferences with age; they spend, for instance, less out of the income on durable goods at the end of their life. Since the consumer's preference structure also depends on the price structure, while the prices are determined by the supply-demand relations with respect to the commodities generated by the productive sectors, we shall study that model later.

When the prices are known it is also possible to express the integrated GNP (21) and the utility (67) in constant prices per capita. The maximization of that functional will be regarded as the main optimization goal which, as is believed, is close enough to the notion of maximum personal and social satisfaction and well being. It enables, as it will be seen later, the effective planning of long term development subject to different development opportunities and constraints.

II. Multi-sector Model

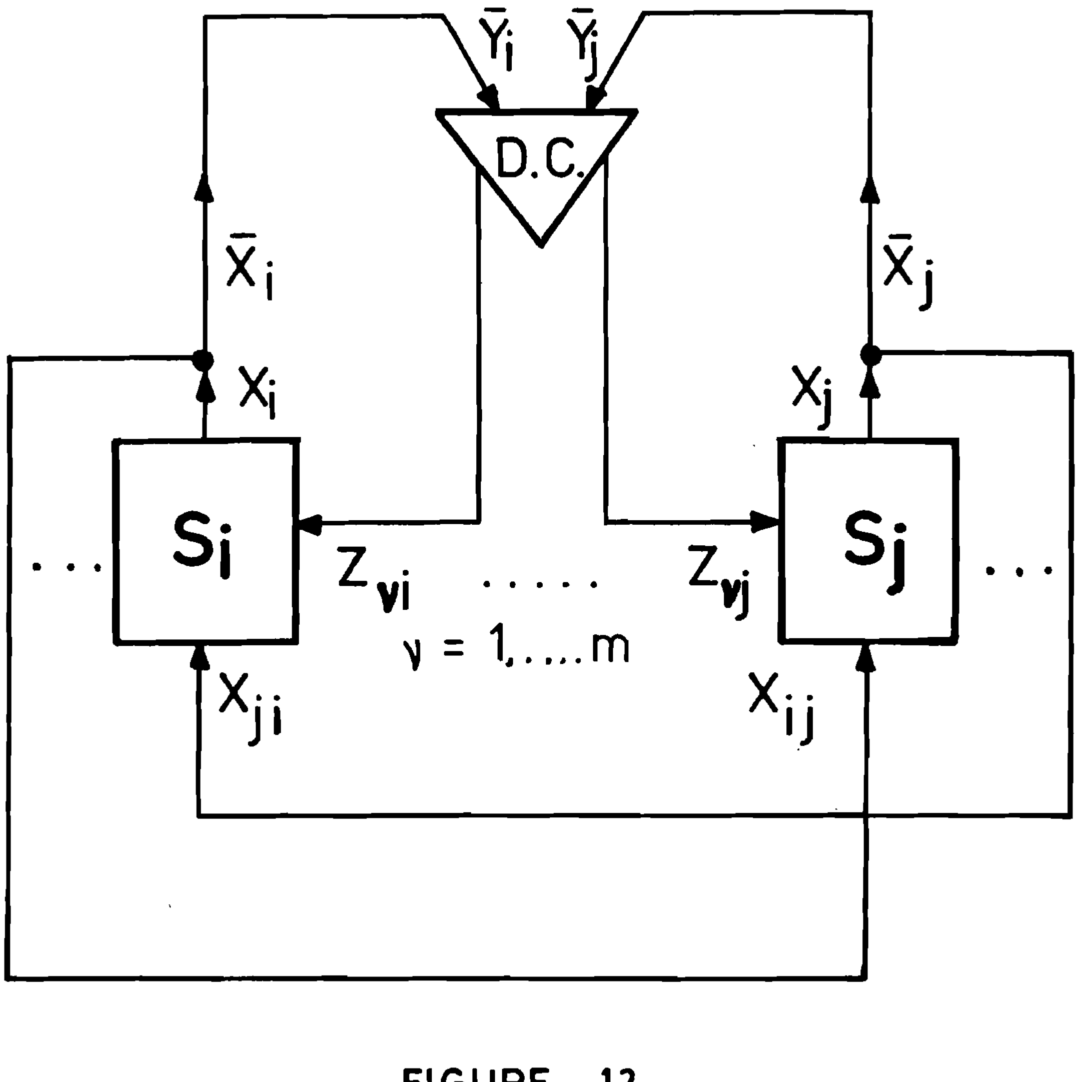
2.1 Optimization of sectorial strategies

In order to describe in a more general way the production processes it is necessary to investigate the n-sector system shown in Fig. 12, where X_i represents the output production of sector S_i and X_{ji} - the amount of commodities which S_j is selling to S_i, $i,j = 1,\dots,n$. The net outputs $\bar{X}_i$, $i = 1,\dots,n$ contribute to the GNP, which is allocated by the decision center (DC) among the S_i sectors. Adopting the structure of Fig.12 the next step is to describe the input-output relation for each sector (i.e. the sector production functions). At that step one is tempted to use the linear model (e.g. the Leontief input-output model), which is attractive from the point of view of analytic simplicity and an easy way of estimating the technological coefficients. However, the linear models are not so attractive from the point of view of macro-economic growth theory and can hardly be used effectively to describe the long-range development. In order to describe the technological change and substitution among the production factors, one would rather adopt the nonlinear production function of the Cobb-Douglas or C.E.S. type.

In the present section we shall show that a sort of reconciliation between these two approaches is also possible.

We shall start with the simple Cobb-Douglas production functions:

$$X_i = F_i^{q_i} \prod_{j=1}^{n} X_{ji}^{\alpha_{ji}} \quad , \qquad i = 1,\dots,n \quad , \tag{1}$$



$\bar{Y}_i$
$\bar{Y}_j$
D.C.
$\bar{X}_i$
$\bar{X}_j$
X_i
X_j
S_i
S_j
$Z_{\nu i}$
$Z_{\nu j}$
$\nu = 1, \ldots, m$
X_{ji}
X_{ij}

FIGURE 12

$$q_i = 1 - \sum_{j=1}^{n} \alpha_{ji} > 0 \quad , \qquad \alpha_{ji} \geq 0 \quad , \qquad i,j = 1,\ldots,n \tag{2}$$

where α_{ji}, $i,j = 1,\ldots,n$, given positive numbers while F_i depend on the vector $\underline{Z}_i$, of development factors allocated by DC, i.e. $F_i = F_i(\underline{Z}_i)$, $i = 1,\ldots,n$.

We shall also assume that a set of sector prices p_i, $i = 1,\ldots,n$, is given so the model of Fig.12 can be described in monetary terms by the Eqs.

$$Y_i - \sum_{j=1}^{n} Y_{ij} = \bar{Y}_i \quad , \qquad i = 1,\ldots,n \quad , \tag{3}$$

$$Y_i = K_i \prod_{j=1}^{n} Y_{ji}^{\alpha_{ji}} \quad , \tag{4}$$

where $Y_{ji} = p_j X_{ji}$, $K_i = p_i F_i^{q_i} \prod_{j=1}^{n} p_j^{-\alpha_{ji}}$, $i,j = 1,\ldots,n$.

Now it is possible to introduce the decision structure. <u>We shall assume that each sector S_i, $i = 1,\ldots,n$, can decide how much of input Y_{ji}, $i = 1,\ldots,n$ to buy in order to maximate the net profit (value added)</u>

$$D_i = Y_i - \sum_{j=1}^{n} Y_{ji} \quad , \qquad i = 1,\ldots,n \quad . \tag{5}$$

It should be noted that the profit maximizing strategy is a micro-economic concept. Since the sector consists of a given number of factories and firms, it is natural to extend that concept to the macro-model.

Since, by virtue of (2), D_i is strictly a concave function, a unique set of strategies $Y_{ji} = \hat{Y}_{ji}$, $i,j = 1,\ldots,n$ exists such that $D_i(\hat{Y}_{ji}, j,i = 1,\ldots,n) = \hat{D}_i$ is maximum.

That strategy, as shown in Ref.[30], becomes

$$\hat{Y}_{ji} = \alpha_{ji}\hat{Y}_i \quad , \qquad j,i = 1,\ldots,n \tag{6}$$

where

$$\hat{Y}_i = F_i \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}/q_i} p_i^{1/q_i} \quad , \qquad i = 1,\ldots,n \tag{7}$$

When one uses that strategy

$$\hat{D}_i = q_i\hat{Y}_i \quad , \qquad i = 1,\ldots,n \tag{8}$$

and the gross product becomes

$$Y = \sum_{i=1}^{n} \bar{Y}_i = \sum_{i=1}^{n} D_i \quad . \tag{9}$$

The two important results follow from the relations (6)÷(9):

I. As follows from (6) the normative n-sector nonlinear model (3)÷(5) behaves under optimum strategy in a similar way to the linear Leontief model with the technological coefficients α_{ji}, $i,j = 1,\ldots,n$. However, the outputs $\hat{Y}_i$, $i = 1,\ldots,n$ in the nonlinear model are specified in an unique manner by prices p_j, $j = 1,\ldots,n$ and F_i coefficients.

II. When prices are fixed the sector net productions $\bar{Y}_i$ $i = 1,\ldots,n$, do not depend on sector interactions (in terms of Y_{ji}, $j,i = 1,\ldots,n$), and the gross product is a linear function of F_i, $i = 1,\ldots,n$.

Result I can be used for a simple estimation procedure of α_{ji}, $j,i = 1,\ldots,n$, coefficients. Assume for that purpose that the input-output tables for an n-sector economy are given.

Assume for that purpose that the input-output tables for an n-sector economy be given. Assume also that each sector optimizes the net profit (5) so that relation (6) is valid. Then it is possible to derive a sequence of numbers

$$\tilde{\alpha}_{ji}(t) = \frac{Y_{ji}(t)}{Y_i(t)} = \frac{\hat{Y}_{ji}(t)}{\hat{Y}_i(t)} \quad , \qquad j,i = 1,\dots,n \qquad (10)$$
$$t = -1,-2,\dots$$

As follows from real statistical data, for each fixed j,i, the sequence $\tilde{\alpha}_{ji}(t)$, $t = -1,-2,\dots$ of past data is generally a random sequence.

Since

$$\tilde{\alpha}_{ji} = \frac{p_j X_{ji}}{p_i X_i} \qquad (11)$$

the variation of $\tilde{\alpha}_{ji}$ can be attributed to the technological change, i.e. the change of $\frac{X_{ji}}{X_i}$, or the variation of sectorial prices p_j/p_i .

As far as the interpretation of α_{ji} is concerned, it is also possible to regard α_{ji} as a characteristic of a technological relation independent of market phenomena like prices. L.R. Klein in the paper [16] has shown for example that the relation (6) can also be obtained by an assumption connecting the pricing system to the technology. He assumes for the competitive market

$$\frac{\partial F_i}{\partial X_i} = -\lambda_i p_i \quad , \qquad \frac{\partial F_i}{\partial X_{ji}} = \lambda_i p_i \quad , \qquad i,j = 1,\dots,n \quad ,$$

where $F_i(X_i, X_{ji}, j=1,\dots,n) = 0$, is the production function and for the ratio in (11) obtains the system of partial differential eqs:

$$-\frac{\frac{\partial F_i}{\partial X_{ji}} X_{ji}}{\frac{\partial F_i}{\partial X_i} X_i} = \alpha_{ji} \quad , \qquad j = 1,\ldots, i-1, \quad i+1,\ldots,n \quad ,$$

which as a solution have F_i functions. For the case of Cobb-Douglas functions, one gets from here the formulae equivalent to the (obtained in a different way) relations (6).

Since the statistical data regarding X_i, X_{ji}, $i,j = 1,\ldots,n$ are usually not available the only way to learn about α_{ji} is from $\tilde{\alpha}_{ji}(t)$ series. In the model constructed by Saito [41], the input-output for a chosen year has been used for that purpose. In the MRI models, constructed in Poland, the statistical estimates $\bar{\alpha}_{ji}$ of α_{ji} derived by using the ex post data $\tilde{\alpha}_{ji}(t)$, $t = -1,-2$, have mostly been used. That approach enables us to neglect the price-variation effect on possible suboptimal strategies in a sectorial policy.

Then the estimates $\bar{\alpha}_{ji}$ can be used ex ante for modeling the future development. In the long-term planning, the model of technological changes (for $\tilde{\alpha}_{ji}$ coefficients) can be used (as shown in Sec. 2.4).

It should also be noted that the decomposition method, which yields the important results (6)-(9), can also be used effectively in the model described by C.E.S. production function (instead of (1)). [20].

Another possible extension concerns the labor force. In the input-output tables, it is customary, for example, to regard labor as additional inputs to the productive sectors. As shown in Ref. [30], it is also possible to regard labor as the output of an additional sector S_o , which cooperates with the production system. The production function of that sector

$$Y_o = K_o \prod_{j=1}^{n} Y_{jo}^{\alpha_{jo}} \quad , \quad q_o = 1 - \sum_{j=1}^{n} \alpha_{jo} > 0$$

where

$$K_o = p_o F_o^{q_o} \prod_{j=1}^{n} p_j^{-\alpha_{jo}}$$

Y_{jo} - cost of goods produced by sectors S_j and consumed by S_o ,

$Y_o = p_o X_o + \bar{Y}_o$ - total value of employment in monetary units, p_o - average net wage,

X_o - number of employees, $\bar{Y}_o$ - other means of income.

The sector S_o can be treated as a "productive" sector with the value added (savings)

$$D_o = Y_o - \sum_{j=1}^{n} Y_{jo} \ .$$

One can assume that the S_o objective is to maximize D_o (i.e. to maximize wages subject to the given consumption; the savings earned in this way can be used for purchases of durable goods). Then according to (6)-(8) one gets

$$\hat{Y}_o = F_o \prod_{j=1}^{n} \left(\frac{\alpha_{jo}}{p_j}\right)^{\alpha_{jo}/q_o} p_o^{\frac{1}{q_o}} \quad ,$$

$$\hat{Y}_{jo} = \alpha_{jo} \hat{Y}_o \quad ,$$

$$\hat{D}_o = q_o \hat{Y}_o \quad .$$

In that model each sector employs $\hat{Y}_{oi}$ value of labor, $i = 1,\ldots,n$, while the rest goes to nonproductive sectors.

2.2 Decomposition and optimization of decision strategies

Using the result II of section 2.1, it is possible to decompose the decision structure. The higher level decision center (D.C.) is concerned mainly with an optimum allocation of GNP generated (at the year t-1) among the sectors S_i, $i = 1,\ldots,n$. The allocation strategy $z_{\nu i}(t)$ in the general case may concern all the development factors ($\nu = 1,\ldots,m$) and sectors ($i = 1,\ldots,n$), or may be limited to a given number of factors (e.g. the investments or investment and labor only).

The production factors endowments received by sectors S_i, according to the decision of D.C. are regarded as exogenous, by the sectorial decision units, which are concerned with optimization of input mix Y_{ji}, $i,j = 1,\ldots,n$, mainly. On the other hand, the decision center, according to the result II, does not need to worry about choosing the best intersector flows $\hat{Y}_{ji}$, $j,i = 1,\ldots,n$, which are subject to sectorial policy.

In other words, a decentralization of management structure, which corresponds to the existing structure of management of the planned economies, is possible.

When prices and technological coefficients are fixed the sector contribution to GNP: $D_i = q_i\hat{Y}_i$, where $\hat{Y}_i$ is defined by (7), becomes proportional to $F_i(\underline{z}_i)$. Taking into account the development models which were developed in Sec. 1.1, 1.2, it is natural to assume that sectorial production functions are of the form (12)(14) or (38) while the optimization goal is to maximize

$$Y = \sum_{i=1}^{n} \int_{0}^{T} e^{-\lambda t} Y_i(t)\,dt \triangleq \bar{Y} + \Delta Y \quad .$$

We shall start with the model which is an n-sector extension of (38)÷(41). The problem consists of finding the strategy $z_{\nu i}(\tau) = \hat{z}_{\nu i}(\tau)$, $\nu = 1,\ldots,m$, $i = 1,\ldots,n$, $\tau\varepsilon[0,T)$, which maximizes:

$$\Delta Y = \sum_{i=1}^{n} \Delta Y_i = \sum_{i=1}^{n} \int_{0}^{T} dt\, e^{\vartheta_i t} \int_{0}^{t} K_i e^{-\delta_i (t-\tau)}$$

$$\prod_{\nu=1}^{m} [z_{\nu i}(\tau - T_{\nu i})]^{\gamma_\nu}\, d\tau \quad , \tag{12}$$

where K_i, ϑ_i, δ_i, γ_ν, $T_{\nu i}$ = given constants. $(\vartheta_i = \mu_i - \lambda)$, subject to the constraints:

$$\sum_{i=1}^{n} \int_{0}^{T} w(\tau) z_{\nu i}(\tau - T_{\nu i})\,d\tau \leq Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \tag{13}$$

$$\sum_{\nu=1}^{n} Z_\nu \leq Z \quad , \tag{14}$$

$$z_{\nu i}(\tau - T_\nu) \geq 0 \quad , \qquad Z_\nu \geq 0 \quad , \qquad \nu = 1,\ldots,m, \quad i = 1,\ldots,n \quad , \tag{15}$$

$t\varepsilon[0,T]$, $\quad \sum_{\nu=1}^{m} \gamma_\nu = \alpha < 1$, $\quad w(t)$ = given weight function.

The expression (12) can be written in the form (27) of Sec. 1.2:

$$\Delta Y = \sum_{i=1}^{n} \int_{0}^{T} \prod_{\nu=1}^{m} [w(\tau) z_{\nu i}(\tau - T_{\nu i})]^{\gamma_\nu} [w(\tau)\varphi_i(\tau)]^q \, d\tau \quad , \quad q = 1 - \alpha \,, \tag{16}$$

where

$$\varphi_i(\tau) = \left\{ K_i w(\tau)^{-q} \int_{\tau}^{T} e^{\vartheta_i t - \delta_i (t-\tau)} dt \right\}^{1/q} .$$

Then the problem of maximizing (16) subject to (13) can be solved. Using the Holder and Minkovski inequalities [20], one obtains:

$$\hat{z}_{\nu i}(\tau - T_{\nu i}) = \frac{\varphi_i(\tau)}{G} Z_\nu \quad , \qquad i = 1,\dots,n \quad , \quad \nu = 1,\dots,m \tag{17}$$

where

$$G = \int_{0}^{T} \sum_{i=1}^{n} \varphi_i(\tau) w(\tau) d\tau \quad .$$

The value of $\Delta Y(\hat{\underline{z}})$ under optimum decisions becomes

$$\Delta Y(\hat{\underline{z}}) = G^q \prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu} \quad . \tag{18}$$

The optimum strategies $Z_\nu = \hat{Z}_\nu$, $\nu = 1,\dots,m$, which maximize (18) subject to (14) can be derived by (32) of Sec.1.2. Then by virtue of (33) one can write

$$Y = \bar{Y} + G^q \prod_{\nu=1}^{m} \gamma_\nu^{\gamma_\nu} Y^\alpha \quad , \qquad (19)$$

where

$$\gamma_\nu = \beta_\nu \alpha \quad .$$

Obviously, for $0 < \alpha < 1$ a positive solution $Y = Y^*$ of (19) exists and the numbers $Z = Y^*$, $\hat{Z}_\nu = \beta_\nu Z$, $\nu = 1,\ldots,m$ and strategies (17), are completely determined in the "closed loop" conditions.

The relation (18) can be regarded as the aggregated development function of the n-sector model (12).

In the case when the integral constraint is replaced by the amplitude constraints:

$$\sum_{i=1}^{n} z_{\nu i}(t-T_{\nu i}) \leq Z_\nu(t) \quad , \qquad t \varepsilon [0,T] \quad , \qquad (20)$$

$$Z_\nu(t) = \text{given functions} \quad , \qquad \nu = 1,\ldots,m \quad ;$$

the corresponding solution becomes [20]:

$$\hat{z}_{\nu i}(\tau-T_{\nu i}) = \frac{\varphi_i(\tau)}{\varphi(\tau)} Z_\nu(\tau) \quad , \qquad i = 1,\ldots,n \quad , \qquad (21)$$

$$\nu = 1,\ldots,m \quad ,$$

where

$$\varphi(\tau) = \sum_{i=1}^{n} \varphi_i(\tau) \quad ,$$

and

$$\Delta Y(\hat{\underline{z}}) = \int_{0}^{T} \varphi^{q}(\tau) \prod_{\nu=1}^{m} [Z_{\nu}(\tau)]^{\gamma_{\nu}} d\tau \quad . \qquad (22)$$

Since $\sum_{\nu=1}^{m} Z_{\nu}(t) = Z(t)$, $\tau\varepsilon[0,T]$, and $Z(t)$ by virtue of the relation (19) of Sec. 1.2 depends on the past value of GNP, the solution (21) should be regarded as the closed loop solution.

One should observe that according to (17),(21), <u>in order to get maximum growth, the sectorial strategies $z_{\nu i}(\tau)$ should be chosen in such a way that they are proportional to $\varphi_i(\tau)$ functions</u>. We can call that strategy the"<u>sectors coordination principle</u>".
It should be noted that using the sectors and factors coordination principles [(24) of Sec. 1.2] one can derive a unique optimum allocation strategy $\hat{z}_{\nu i}(t)$, $\nu = 1,\ldots,m$, $i = 1,\ldots n$, $\tau\varepsilon[0,T]$, for the n-sector system, described by the equations:

$$\Delta Y = \sum_{i=1}^{n} \int_{0}^{T} \prod_{\nu=1}^{m} f_{\nu i}(t)dt \quad , \qquad (23)$$

$$f_{\nu i}(t) = \left\{ e^{\vartheta_i \frac{t}{\beta_{\nu i}}} \int_{0}^{t} K_i e^{-\delta_{\nu i}(t-\tau)} [z_{\nu i}(\tau - T_{\nu i})]^{\alpha} d\tau \right\}^{\beta_{\nu i}}$$

$$\sum_{\nu=1}^{m} \beta_{\nu i} = 1 \quad , \qquad i = 1,\ldots,n \quad ,$$

which can be regarded as an extension of (21)-(23) of the Sec. 1.2 formula. Indeed, using the factor coordination principle for each sector S_i one finds $\hat{z}_{\nu i}$, $\nu \geq 2$ and can represent (23) as the of ΔY_i components $(\sum_{i=1}^{n} \Delta Y_i = \Delta Y)$, where according to (26) of Sec.2.1:

$$\Delta Y_i = \prod_{\nu=2}^{m} C_{\nu i}^{-\beta_{\nu i}} \int_0^T dt\, e^{\vartheta_i t} \int_0^t K_i e^{-\delta_{\nu i}(t-\tau)} [z_{1i}(\tau - T_{1i})]^{\alpha} d\tau \quad ,$$

$$i = 1,\ldots,n \quad , \tag{24}$$

and $C_{\nu i}$ can be determined from the corresponding constraints.

Since the problem of maximizing $\sum_{i=1}^{n} \Delta Y_i (Z_{1i})$ subject to the integral:

$$\sum_{i=1}^{n} \int_0^T w(\tau) z_{1i}(\tau - T_{1i}) d\tau \leq Z_1 \quad ,$$

or amplitude

$$\sum_{i=1}^{n} z_{1i}(t - T_{1i}) \leq Z_1(t) \quad , \quad \tau \varepsilon [0,T] \quad ,$$

constraint is a special case of the problem (12)-(15) it is obvious that <u>the optimum strategy $\hat{z}_{\nu i}(\tau)$, $\nu = 1,\ldots,m$, $i = 1,\ldots,m$, which maximizes the integrated output (23) subject to the amplitude or integral constraints exists and can be derived in an explicit manner</u>. When one does not count the different possible formulations of constraints (i.e. amplitude and/or integral constraints) the strategy derived is unique.

One should observe that the solution of the general optimization problem, with n-sector nonlinear development functions (23) degenerates when $\alpha \to 1$ (i.e. $q \to 0$) and the optimum strategies described by (17),(21) degenerate into Dirac's $\delta(t)$ functions or "bang-bang" solutions. These strategies are not unique and the analytic derivation of explicit solutions in the general case of the n-sector system is not easy. In other words, the linearization of our problem does not simplify it.

It should be also noted that in the model (23) the sectorial development factors $f_{\nu i}$, $\nu = 1,\ldots,m$, depend on the "directed" expenditures $z_{\nu i}$. In the real systems part of the general Z_ν expenditures goes to the sectors for financing the specialized education, health service, research and development, etc. The rest (z_ν) has a "universal" character, e.g. the expenditures on general education, health care, science, etc. In the more general model, one can represent $f_{\nu i}$ in the form

$$f_{\nu i}(t) = \left\{ e^{\vartheta_i \frac{t}{\beta_{\nu i}}} \int_0^t K_i e^{-\delta_{\nu i}(t-\tau)} [z_{\nu i}(\tau - T_{\nu i})]^{\alpha_1} [z_\nu(\tau - T_\nu)]^{\alpha_2} d\tau \right\}^{\beta_{\nu i}} \tag{25}$$

where $\alpha_1 + \alpha_2 = \alpha$, and replace (13)(20) by

$$\sum_{i=1}^{n} \int_0^T w(\tau)[z_{\nu i}(\tau - T_{\nu i}) + Z_\nu(\tau - T\ ]d\tau \le Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \tag{26}$$

$$\sum_{i=1}^{n} z_{\nu i}(\tau - T_{\nu i}) + z_\nu(\tau - T\ ) \le Z_\nu(t) \quad , \qquad \tau \varepsilon [0,T] \quad , \quad \nu = 1,\ldots,m$$

respectively. It is possible to show that the present problem can also be solved effectively by the methodology used so far. The main advantage of that methodology is that it constitutes a useful aggregation device at the same time. For example, the complicated n-sector production functions in (12) aggregate under optimum strategy to the simple formula (18). At the next aggregation step (18) aggregates to (19), etc.

On the other hand it is possible to decompose (18) into the given number of specialized productive or regional subsystems. To show that possibility, consider N subsystems, each described by the development function:

$$\Delta Y_j = F_j^q \prod_{\nu=1}^{m} Z_{\nu j}^{\gamma_\nu} \quad , \qquad j = 1,\ldots,N \quad , \tag{27}$$

The problem consists of finding the nonnegative strategy $Z_{\nu j} = \hat{Z}_{\nu j}$, $\nu = 1,\ldots,m$, $j = 1,\ldots,N$, such that $\Delta Y = \sum_{j=1}^{N} \Delta Y_j$, attains maximum subject to the constraints

$$\sum_{j=1}^{N} Z_{\nu j} \leq Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad . \tag{28}$$

It is easy to show that the optimum strategy becomes

$$\hat{Z}_{\nu j} = \frac{F_j}{F} Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \quad j = 1,\ldots,N \quad , \tag{29}$$

where

$$F = \sum_{i=1}^{N} F_i \quad ,$$

and the value of ΔY under optimum strategy is

$$\Delta Y(\hat{\underline{Z}}) = F^q \prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu} \quad . \tag{30}$$

The results obtained so far indicate that it is possible to use the proposed methodology for optimization of the decentralized multi-level decision structure shown in Fig.13.

At the second-level the expenditures Z_ν , $\nu = 1,\ldots,m$, are allocated among sectors according to (17),(21).

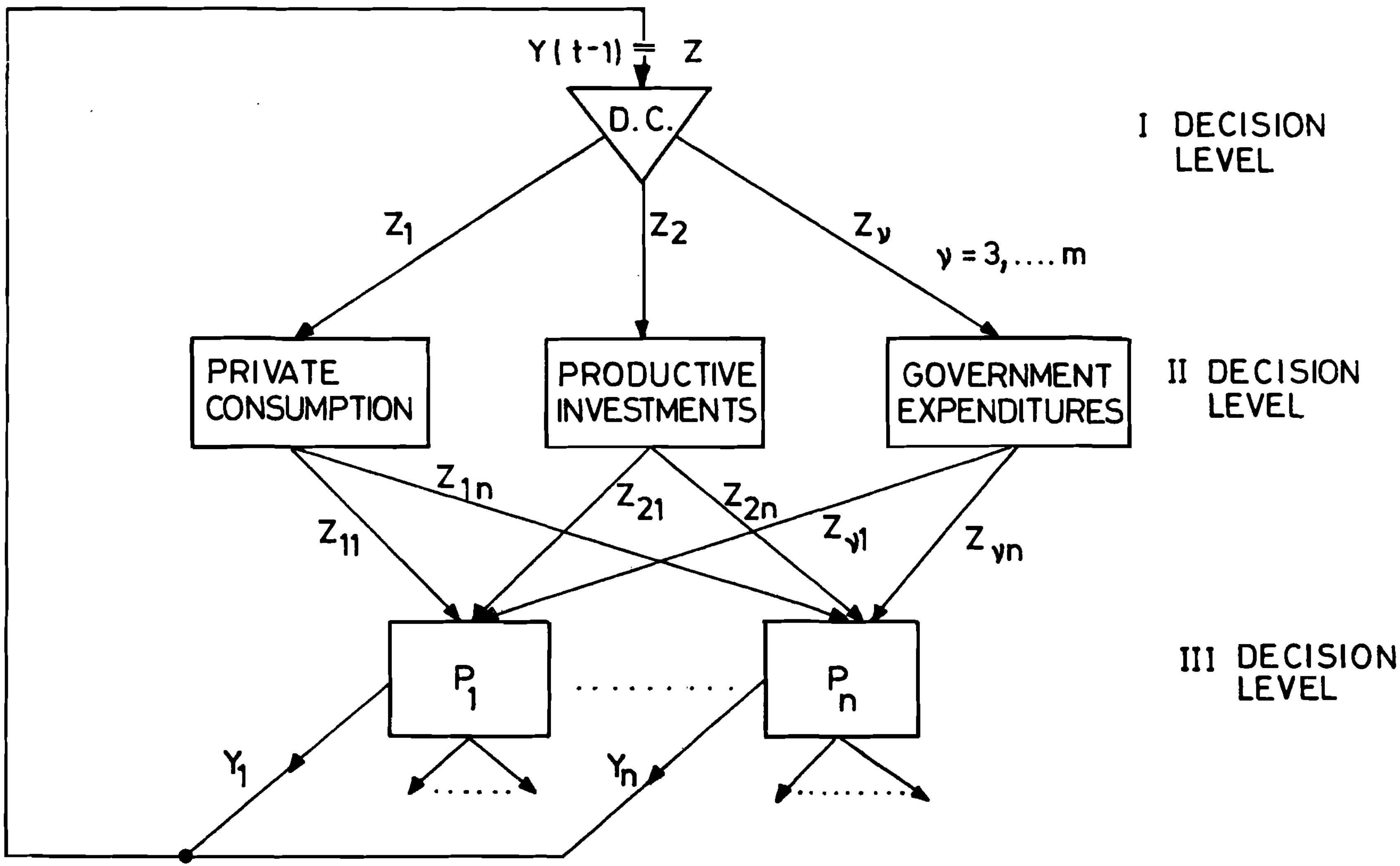

Y(t-1) = Z
D.C.
I DECISION LEVEL
Z1
Z2
Zν
ν = 3, ... m
PRIVATE CONSUMPTION
PRODUCTIVE INVESTMENTS
GOVERNMENT EXPENDITURES
II DECISION LEVEL
Z1n
Z11
Z21
Z2n
Zν1
Zνn
P1
Pn
III DECISION LEVEL
Y1
Yn

FIGURE 13

At the third (and possible lower-levels) the expenditures can be allocated in more specialized or regional ways.

The outputs generated by Sectors $S_1,\ldots,S_n$ constitute the GNP

$$Y(t) = \sum_{i=1}^{n} \bar{Y}_i(t) \quad ,$$

allocated at the next year.

In Refs. [19-37] more detailed aspects of optimum allocation of resources in production, consumption, environment and regional subsystems have been studied, using the present methodology. It should also be noted that the methodology presented can be extended in different directions. One can, for example, solve the problem when the integral and amplitude constraints are acting at the same time, or when $w(\tau)$ in (13) is different for different ν-factors. Another possible extension concerns the prices. As follows from (7) of Sec.1.2, the prices' effect on the output production can be represented by the factors

$$\Pi_i = \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}/q_i} p_i^{1/q_i} \quad , \qquad i = 1,\ldots,n \quad ,$$

which change in time, along with $p_j(t)$, $j = 1,\ldots,n$.

One does not know the future $\Pi_i(t)$ change, so it was tacitly assumed that the factors $K_i e^{\mu_i t}$ in (23) will take care of that change. Since the parameters K_i, μ_i in the model under consideration are estimated ex post from statistical data, regarding $\hat{Y}_i(t)$ and $\hat{z}_{\nu i}(t)$, $\nu = 1,\ldots,m$, $i = 1,\ldots,n$, $t = -1,-2,\ldots$, one cannot be certain that they are accurately describing $\Pi_i(t)$ "ex ante". The strategies $\hat{z}_{\nu i}(t)$, derived at the present section should be therefore regarded as the first approximation $\hat{z}_{\nu i}^{(o)}$

of the optimum strategy. However, it is possible to construct an iterational process $\hat{z}_{\nu i}^{(k)}$, $k = 0,1,\ldots$ of approximations converging for $k \to \infty$ to the optimum strategy.

The main idea behind that process consists in computation of the prices resulting from the $\hat{z}_{\nu i}^{(k)}$ strategy and finding the corresponding $\Pi_i^{(k)}$, which can be used for computation of the next iteration $\hat{z}_{\nu i}^{(k+1)}$ etc.

2.3 The model of prices

The prices play an important role in the optimally planned, socialist economy and they are supposed to fulfil many functions. In particular, it is important that prices reflect the social expenses and results of production efficiency. They should stimulate the incentives of producers and influence the consumption structure in such a way that a maximum social utility is obtained. The prices should also guarantee the market equilibrium. In addition, by a system of indirect taxation of output and government dotation, it is possible to change the prices in such a way that the widely used consumption goods are cheap, compared to some luxury goods. In that way, a just social policy, which serves the purpose of fair income distribution among different wage classes, can be implemented.

From those introductory remarks, one can see that the construction of a good mathematical price-model for a socialist economy is not easy and a number of simplifying assumptions is inevitable. First of all it is necessary to observe that we are interested in the highly aggregated sector prices. These prices, say p_i, are usually defined as an arithmetic weighted mean of a number M of commodities X_{ji} with prices p_{ji}, i.e.

$$p_i \triangleq \sum_{j=1}^{M} \frac{X_{ji}p_{ji}}{X_i} \quad , \qquad X_i = \sum_{j=1}^{M} X_{ji} \quad . \tag{31}$$

Since p_i change in time in the statistical tables, the relative prices, or price indices

$$p_i^t = {p_i(t)}/{p_i(t-1)} \quad , \qquad t = 0,1,\ldots$$

are usually given.

The price indices can be used to express the value of product $Y_i(t)$, at the basic year (t=0) prices. Obviously, for that purpose it is necessary to divide $Y_i(t)$ by $\prod_{\tau=1}^{t} p_i^\tau$. Since in the model discussed it is important to express the GNP in constant (i.e. basic year) prices, our first objective is to derive the aggregated sector price indices p_i^t , $t = 1,2,\ldots$

So far (in Sec.2.2) the supply part of the model has been studied.

On the demand side of the model, one faces in each year t the expenditures

$$Z_\nu(t) = \gamma_\nu Z(t) \quad , \quad \sum_{\nu=1}^{m} \gamma_\nu = 1 \quad , \quad t = 0,1,\ldots \tag{32}$$

which are spent on the purchase of goods $\bar{Y}_i$, $i = 1,\ldots,n$, produced by the S_i-sectors, i.e.

$$\bar{Y}_i(t) = \sum_{\nu=1}^{m} \lambda_{\nu i} Z_\nu(t) = Z(t) \sum_{\nu=1}^{m} \lambda_{\nu i}\gamma_\nu \quad , \tag{33}$$

where $\lambda_{\nu i}$ = given non-negative coefficients determining the ν-th expenditure contribution to the demand confronting the i-th production sector, and

$$\sum_{\nu=1}^{m} \lambda_{\nu i} = 1 \quad , \qquad \nu = 1,\ldots,m \quad . \tag{34}$$

The coefficients $\lambda_{\nu i}$, γ_ν , $\nu = 1,\ldots,m$, $i = 1,\ldots,n$ depend on prices $\underset{\sim}{p}_i$, $i = 1,\ldots,n$, GNP percapita in constant prices $(^{\tilde{Y}}/N)$ etc. In rather simple models one can assume that

$$\gamma_\nu = a_\nu (^{\tilde{Y}}/N)^{\varepsilon_\nu} \quad , \tag{35}$$

$$\lambda_{\nu i} = b_{\nu i}(p_i)^{-E_{\nu i}} \quad , \qquad \nu = 1,\ldots,m \quad , \quad i = 1,\ldots,n$$

where the parameters a_ν, ε_ν, $b_{\nu i}$, $E_{\nu i}$ can be estimated ex post by known statistical methods. Care should be taken that γ_ν , $\lambda_{\nu i}$ satisfy the (32),(34) conditions. Since $\underline{\bar{Y}} = [1-\underline{A}]\underline{\hat{Y}}$, $\underline{A} \triangleq [\alpha_{ij}]$, the equilibrium conditions (33) can be written in the form

$$\underline{\hat{Y}} = [1 - \underline{A}]^{-1}[\underline{\lambda}]\underline{\gamma} Z \triangleq \underline{\ell} Z \quad . \tag{36}$$

Since $Z(t)$ is determined by the GNP generated at $t - 1$, the last relation can also be written as

$$\hat{Y}_i(t) = \ell_i[Y(t-1),\underline{p}(t-1)]Y(t-1) \quad , \qquad i = 1,\ldots,n \tag{37}$$

In order to satisfy (37) it is necessary to choose the sector prices in the proper way. For that purpose, consider the i-th sector output, which according to (1) can be written

$$X_i = \left[F_i(^{Y_{1i}}/_{\omega_1}, ^{Y_{2i}}/_{\omega_2}, \ldots, ^{Y_{mi}}/_{\omega_m})\right]^{q_i} \prod_{j=1}^{n} X_{ji}^{\alpha_{ji}} \quad , \quad i = 1,\ldots,n$$

where $Y_{\nu i}/\omega_\nu$ = development factors used in production by the i-th sector. The production cost C_i consists, generally speaking, of two parts; the first connected with the materials cost $(\sum_{j=1}^{n} Y_{ji})$, and the second with the cost of production factors $(\sum_{\nu=1}^{m} Y_{\nu i})$† so that $C_i = \sum_{j=1}^{n} Y_{ji} + \sum_{\nu=1}^{m} Y_{\nu i}$. At the sectorial level it is natural to assume that the GNP, which consists of the sum of values added by sectors, is spent on the production factors endowments.

The average sectorial price

$$\tilde{p}_i = \frac{C_i}{X_i} \quad ,$$

depends on the allocation of Y_{ji}, $Y_{\nu i}$, $j,i = 1,\ldots,n$, $\nu = 1,\ldots,m$. As already noted, the planned socialist economy is very effective in the efficient allocation of resources, i.e. the development factors and materials used in production. The efficiency conditions obviously require that $Y_{ji}, Y_{\nu i}$, be chosen in such a way that $\tilde{p}_i$ attains a minimum to equal p_i , for $\hat{Y}_{ji} = Y_{ji}$, $Y_{\nu i} = \hat{Y}_{\nu i}$, $j,i = 1,\ldots,n$, $\nu = 1,\ldots,m$. In that case, the consumer's utility attains maximum, and from the condition

$$\frac{\partial \tilde{p}_i}{\partial X_i} = \left[\frac{\partial C_i}{\partial X_i} X_i - C_i\right] X_i^{-2} = 0 \quad ,$$

---

†After the reforms which took place in socialist countries in 1961-1971, and in Poland in 1971, it bacame a common practice to take into account the cost of production factors, and first of all the cost of labor and capital, when calculating prices at the micro-level.

one gets

$$\frac{\partial C_i}{\partial X_i} = \frac{C_i[\hat{Y}_{ji}, \hat{Y}_{\nu i}, j,i = 1,\ldots,n, \nu = 1,\ldots,m]}{X_i[\hat{Y}_{ji}, \hat{Y}_{\nu i}, j,i = 1,\ldots,n, \nu = 1,\ldots,m]} = p_i \quad ,$$

$$i = 1,\ldots,n \quad ,$$

i.e. the equilibrium price p_i is equal to the marginal production costs $\partial C_i/\partial X_i$.

Since

$$\hat{X}_i = \left[F_i(\hat{Y}_{1j},\ldots,\hat{Y}_{mi})\right]^{q_i} \prod_{\nu=1}^{m} \omega_\nu^{-\beta_{\nu i} q_i} \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_i}\right)^{\alpha_{ji}} \hat{Y}_i^{1-q_i}$$

and

$$\hat{C}_i = \sum_{j=1}^{n} \alpha_{ji} \hat{Y}_i + q_i \prod_{\nu=1}^{m} \beta_{\nu i} \hat{Y}_i = \hat{Y}_i \quad , \qquad i = 1,\ldots,n \quad ,$$

one gets

$$p_i = \frac{\hat{C}_i}{\hat{X}_i} = \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{-\alpha_{ji}} \prod_{\nu=1}^{m} \omega_\nu^{\beta_{\nu i} q_i} \hat{Y}_i^{q_i} F_i^{-q_i} \quad , \quad i = 1,\ldots,n \quad .$$

Introducing variables $p_i^t = p_i(t)/p_i(t-1)$, $i = 1,\ldots,n,$

$\omega_\nu^t = \omega_\nu(t)/\omega_\nu(t-1)$, $\nu = 1,\ldots,m$, $\hat{Y}_i^t = \hat{Y}_i(t)/\hat{Y}_i(t-1)$,

$F_i^t = F_i(t)/F_i(t-1)$, and taking logarithms from both sides of the last equation, one gets

$$\ell n p_i^t - \sum_{j=1}^{n} \alpha_{ji} \ell n p_j^t = q_i \left[\ell n \frac{\hat{Y}_i^t}{F_i^t} + \sum_{\nu=1}^{m} \beta_{\nu i} \ell n\, \omega_\nu^t\right] , \quad i = 1,\ldots,n \quad , \tag{38}$$

where according to (37)

$$\hat{Y}_i^t = \ell_i^{t-1} \, Y^{t-1} = \ell_i^{t-1} \sum_{j=1}^{n} \hat{Y}_j^{t-1} \quad ,$$

and

$$\ell_i^{t-1} = \frac{\ell_i[Y(t-1),\underline{P}(t-1)]}{\ell_i[Y(t-2),\underline{P}(t-2)]} \quad .$$

Since for the real economy the determinant

$$D = \begin{vmatrix} 1, & -\alpha_{11}, & \dots & -\alpha_{n1} \\ \vdots & & & \\ -\alpha_{1n}, & -\alpha_{2n} & \dots & 1 \end{vmatrix} \neq 0$$

there exists a unique solution to the set of eqs. (38). It yields a set of positive price indices p_i^t , $i = 1,\dots,n$.

It will be shown now that the formulae (38) can be used to find p_i^t , $i = 1,\dots,n$, $t = 1,2,\dots,T$, in an iterative way, starting with the known base year $t = 0$ prices.

The term

$$\frac{\hat{Y}_i^t}{F_i^t} = \frac{\ell_i^{t-1} Y^{t-1}}{\hat{Y}_i^t} \; \Pi_i^t \quad ,$$

in (38), where $\Pi_i^t = \Pi_i(t)/\Pi_i(t-1)$, and

$$\Pi_i(t) = \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}/q_i} \; p_i^{1/q_i} \prod_{\nu=1}^{m} \omega_\nu^{-\beta_{\nu i}} \quad ,$$

determines the demand to supply ratio.

For a given strategy $\underline{z}^k \triangleq \left\{\hat{z}_{\nu i}^{(k)}(t),\ \nu = 1,\dots,m\ ,\right.$ $\left. i = 1,\dots,n\ ,\ \tau\varepsilon[0,T]\right\}$, $k = 0,1,\dots$, the supply $\hat{Y}_i^t(\underline{z}^k) \triangleq S_i^k$ is a known function of time, while the term Π_i^t has been derived by using price indices $\underline{p}^{k-1} \triangleq \left\{p_i^t\ ,\ i = 1,\dots,n,\ k-1\right\}$ (from the previous step of computations) so $\Pi_i^t(\underline{p}^{k-1}) \triangleq \Pi_i^{k-1}$.

The demand term $\ell_i^{t-1}\ Y^{t-1}$ depends in turn on $\underline{z}^k$ and $\underline{p}^{t-1}$, so one can write

$$\ell_i^{t-1}\ Y^{t-1} \triangleq D_i^k \quad ,$$

and

$$\frac{\hat{Y}_i^t}{F_i^t} = \frac{D_i^k}{S_i^k}\,\Pi_i^{k-1} \quad , \qquad i = 1,\dots,n \quad . \tag{39}$$

The process of computations can be briefly described as follows. One starts at the initial step k = 1, with the $\Pi_i^o = A_i e^{\mu_i t}$ and derives by formulae of Sec.2.2 the $\underline{z}^1$ strategy which yields the supplies S_i^1 , $i = 1,\dots,n$. At the same time, the corresponding prices $\underline{p}^1$ and demands D_i^1, $i = 1,\dots,n$ by formulae (38) are being derived. When $\frac{D_i^1}{S_i^1} = 1$, $i = 1,\dots,n$, it means that our assumption regarding Π_i^o , $i = 1,\dots,n$, was correct; so the process terminates.

When $\frac{D_i^1}{S_i^1} \neq 1$, at least for one index i, the prices computed by (38): $\underline{p}^1$ should be used for computation of Π_i^1 and $\underline{z}^2$ can be computed in order to derive $\frac{D_i^2}{S_i^2}$ etc. It should be observed that in the case when Π_i^o for a particular "i" has been chosen too large, the optimum strategy $\hat{z}_{i\nu}^1$, $\nu = 1,\ldots,m$, increases the S_i term and as a result the equilibrium price p_i^1 decreases. That in turn decreases Π_i^1 , i.e. $\Pi_i^1 < \Pi_i^o$. In other words, the iterational process has a tendency to correct the unjustified projections of Π_i^o . When, after a number of iterations one arrives at the situation where $D_i^k \approx S_i^k$, $i = 1,\ldots,n$ the values of $\underline{p}^k$ can be regarded as the prices corresponding to the optimum planning strategy in $[O,T]$, i.e. $\underline{z}^k = \hat{\underline{z}}$.

The proposed methodology can also be used in the case when one wants to maximize the GNP in constant (e.g. the base year) prices. In that case, the optimization goal is to maximize the functional:

$$\bar{Y} = \sum_{i=1}^{n} \int_{o}^{T} e^{-\lambda t} [p_i(o)/p_i(t)] Y_i(t) dt \quad ,$$

and the problem boils down to using the functions

$$\bar{\Pi}_i^t = \Pi_i^t / p_i^t \quad , \qquad i = 1,\ldots,n \quad ,$$

instead of Π_i^t , in the computation process already described.

The next problem which should be clarified is the computation of $\sum_{\nu=1}^{m} \beta_{\nu i} \ell n\, \omega_\nu^t$ in (38). The prices ω_ν , of the development factors (beside labor) are in general not included in the statistical data available.

The average salary $\omega_1 = \frac{Z_1}{L} = \gamma_1 \frac{Z}{L}$ depends on the GNP ($Z(t) = Y(t-1)$) and the employment L which is exogenous, or derived from the population-employment submodel. In the simple model therefore, it is possible to regard $\bar{\mu}_i = \sum_{\nu=2}^{m} \beta_{\nu i} \ln \omega_\nu^t$, as exogenous parameters which can be estimated by fitting $\bar{\mu}_i$, ex post to the eqs. (38). These parameters should also be regarded as part of Π_i^t , responsible for the technical progress resulting from the government expenditure, $Z_{\nu i}$, $\nu \geq 3$.

In a more detailed employment model, it is possible to introduce the sectorial employments L_i and sectorial wages $\omega_{1i} = \frac{Z_{1i}}{L_i}$, $i = 1,\ldots,n$, where $\sum_{i=1}^{n} L_i = L$, $\sum_{i=1}^{n} Z_{1i} = Z$. The average sectorial wage ω_{1i}^t may change as a result of labor efficiency change in the productive sectors.

In the price model discussed so far, we did not take into account the taxation of the outputs of some of the sectors and corresponding government subsidies to the other sectors. The general requirement is that the taxes ($t_i > 0$) and subsidies ($t_i < 0$) should satisfy the balance equation:

$$\sum_{i=1}^{n} p_i t_i X_i = o \quad , \; i = 1,\ldots,n \tag{40}$$

where X_i - sector production.

The result of taxation is the change in values added (5):

$$\tilde{D}_i = Y_i(1-t_i) - \sum_{j=1}^{m} Y_{ji} \quad , \qquad i = 1,\ldots,n \; . \tag{41}$$

It is possible to see that (due to (40) the GNP

$$Y = \sum_{i=1}^{n} \tilde{D}_i = \sum_{i=1}^{n} D_i \quad ,$$

so the taxation-dotation process does not change the GNP.

Since according to (8) and (41)

$$\tilde{D}_i = \hat{Y}_i(1-t_i) - \sum_{j=1}^{n} \alpha_{ji}\hat{Y}_i = (1-t-\sum_{j=1}^{n} \alpha_{ji})\hat{Y}_i$$

the result of taxation is equivalent to change of α_{ii} , i.e. $\tilde{D}_i$ can be written

$$\tilde{D}_i = \tilde{q}_i\tilde{Y}_i \quad , \quad \tilde{q}_i = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} \alpha_{ji} - \tilde{\alpha}_{ii} \quad , \qquad (42)$$

where $\tilde{\alpha}_{ii} = \alpha_{ii} + t_i$.

As a result of taxation the price indices $\tilde{p}_i^t$ derived by (38) (with α_{ii} replaced by $\tilde{\alpha}_{ii}$) change. By using (38), it is also possible to derive the tax-dotation system, which keeps the given sectorial price $\tilde{p}_j^t$ constant. For that purpose it is necessary to regard t_j as the unknown parameter. It should also be observed that the introduction of taxation-dotation changes the consumption strategy, expressed in terms of the utility function parameters $\lambda_{\nu i}\gamma_\nu$. It is possible to show that the taxation-dotation process decreases the utility. As already mentioned, that process is motivated by the social policy aspects, which have not been introduced explicitly in our simple utility function (67) of Sec. 1.5. Therefore, we have to regard t_i parameter as exogenous variables rather than decision variables. These parameters for Polish economy do not change much. The subsidies for the market commodities are around 1.8 percent of the GNP (30 percent of that sum goes to the grain products, and 12 percent to the milk products); around 8 percent of the GNP goes for subsidizing the agriculture (fertilizers, pesticides, machinery, fodders, etc.). As a result of that policy, the basic food products did not change much. However, in recent years, a growing concern with regard to food-prices stability is frequently expressed. The problem obviously boils down to choosing the proper values for t_i. By using different values of t_i in relation (42) and by using the price model proposed, it is possible to investigate the taxation policy impacts on the development goal values.

2.4 Adaptive model of technological and structural change

By the technological change, we shall understand here the change of production technology, motivated mainly by possible substitutions of expensive imports by their cheaper equivalents. As mentioned previously in Sec.2.1, the technological change is the main factor influencing the $\tilde{\alpha}_{ji}(t)$ coefficients derived ex post from the statistical data by formula (10). As follows from the analysis carried out for the Polish economy, the $\tilde{\alpha}_{ji}(t)$ functions have a trend (mainly decreasing) in time. It is usually assumed that there are three main factors responsible for that trend: new capital investments and modernization, technical progress, and change of aggregated sector prices.

Consider first of all the change of technological coefficients due to new investments. Assume that in the year $t = t_1$ there exists a production sector S^e with the output $Y^e(t_1)$, which continues the production at $t = t_2$, so that

$$Y^e(t_2) = Y^e(t_1) \quad .$$

The sector input Y_i^e (supplied by another sector) becomes

$$Y_i^e(t_2) = Y_i^e(t_1) = \alpha^e(t_1)Y^e(t_1)^{\dagger} \quad .$$

Assume also that at $t = t_2$ a new factory (which is classified as belonging to the same sector S^e) with the output $Y^n(t_2)$ starts to operate so that the resulting production increases from $Y(t_1) = Y^e(t_1)$ to

$$Y(t_2) = Y^e(t_2) + Y^n(t_2) \quad .$$

As a result, the sector technological coefficient changes from the value

$$\alpha(t_1) = \frac{Y_i^e(t_1)}{Y^e(t_1)} \quad ,$$

to the value of

$$\alpha(t_2) = \frac{Y_i^e(t_2)+Y_i^n(t_2)}{Y(t_2)} = \alpha^e(t_1)\,\frac{Y^e(t_2)}{Y(t_2)} + \alpha^n(t_2)\,\frac{Y^n(t_2)}{Y(t_2)}$$

$$= \alpha^e(t_1)\,\frac{Y(t_1)}{Y(t_2)} + \alpha^n(t_2)\,\frac{Y(t_2)-Y(t_1)}{Y(t_2)} \quad . \qquad (43)$$

Assume the value of $\alpha^n(t_2)$ to differ from $\alpha^e(t_1)$ by a

---

† The indices j,i at α_{ji} have been dropped for the sake of simplicity in notation.

constant multiplier:

$$\alpha^{n}(t_2) = b\ \alpha^{e}(t_1) \quad , \qquad (44)$$

where b is a positive number.

The assumption (44) expresses the fact that the level of new technology is proportional to the technical level achieved in the old (existing) technology. Using (44) and assuming that $t_1 \to t_2$ we get by (43)

$$\frac{d\alpha}{\alpha} = (b - 1)\ \frac{dY}{Y} \quad . \qquad (45)$$

Integrating (45) one gets

$$\alpha(t) = \alpha_o[Y(t)]^{b-1} \quad , \qquad \alpha_o = \alpha^{e}(t_1)[Y(t_1)]^{1-b} \quad . \qquad (46)$$

When $b < 1$, the $\alpha(t)$ decreases along with an increase of output production $Y(t)$.

As follows from the $Y(t)$ and $\alpha(t)$ trends analysis, these functions can be approximated by the exponential functions:

$$\hat{\alpha}(t) = A\ e^{-\bar{a}t} \quad , \qquad \hat{Y}(t) = B\ e^{\rho t} \quad ,$$

where A, B, $\bar{a}$, ρ - given numbers.

Taking into account (45) one obtains

$$b = 1 - \frac{\bar{a}}{\rho} \quad . \qquad (47)$$

Then it is possible to express the estimated value of $\hat{\alpha}(t_2)$ by the estimated value $\hat{\alpha}(t_1)$ and $\tilde{\alpha}(t_1)$, which is taken from the last (ex post) observation:

$$\alpha(t_2) = \hat{\alpha}(t_1)\, e^{-\rho(t_2-t_1)} + b\tilde{\alpha}(t_1)\left[1 - e^{-\rho(t_2-t_1)}\right] .$$

Assuming $t_2 - t_1 = 1$, it is also possible to construct an iterative process

$$\hat{\alpha}(t_{i+1}) = \hat{\alpha}(t_i)e^{-\rho} + b\tilde{\alpha}(t_i)\left[1 - e^{-\rho}\right] , \quad i = 1,2,... \tag{48}$$

The estimation $\hat{\alpha}(t_1)$ of $\alpha(t_1)$ can be obtained by taking the mean value of N observations $\tilde{\alpha}(t)$, in the identification interval $[T_1,T_0]$ (Fig.14), i.e.

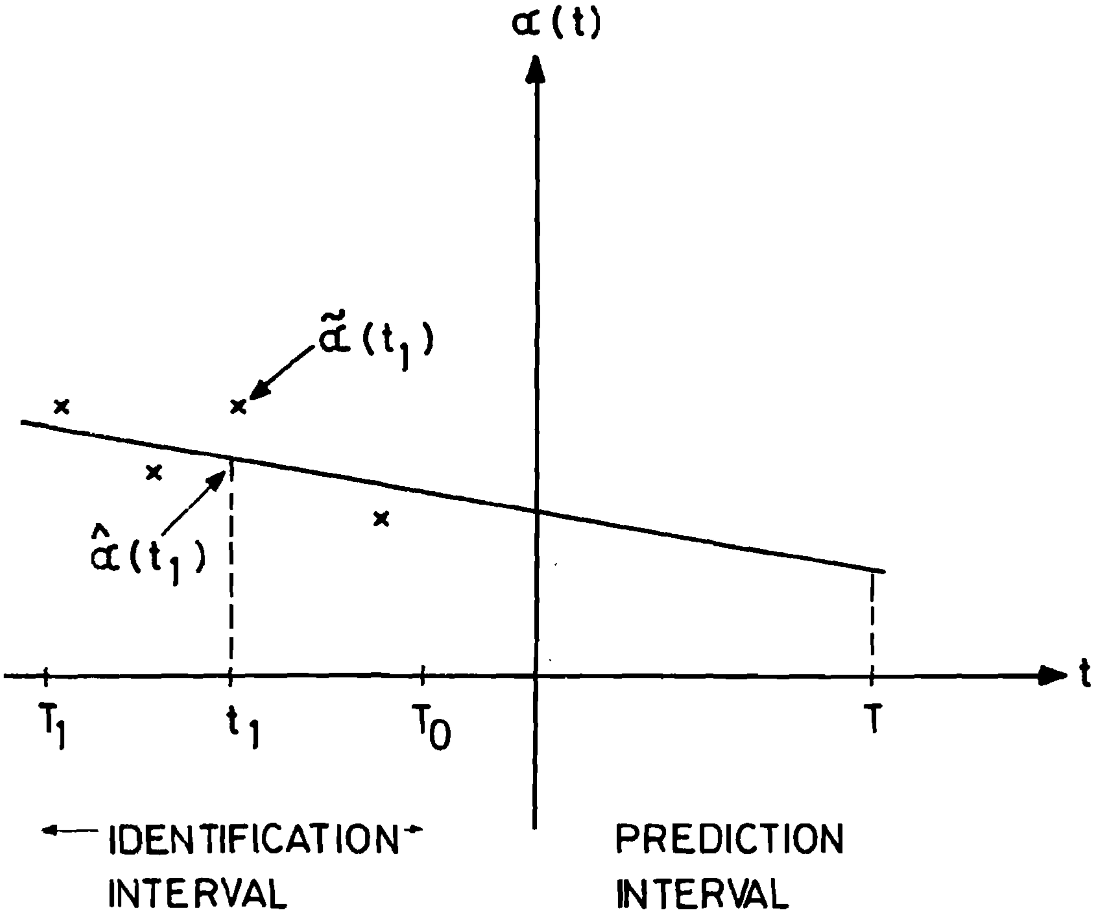
$$\hat{\alpha}(t_1) = \frac{1}{N} \sum_{t=T_1}^{t=T_0} \tilde{\alpha}(t) \quad , \tag{49}$$

Since $\bar{a}$ is a small number, in the case of short $[T_1,T_0]$ interval, $\hat{\alpha}(t)$ is almost linear and one can assume

$$t_1 \approx \frac{T_0 + T_1}{2} .$$

The formula (48) is similar to the exponential smoothing form of adaptive forecasting methods (see Ref.[6]) with the weights $e^{-\rho(t_2-t_1)}$, $b\left[1-e^{-\rho(t_2-t_1)}\right]$, attached to the estimate $\hat{\alpha}(t_1)$ and the observed value $\tilde{\alpha}(t_1)$.

It should be observed that in standard adaptive algorithms the weights attached to $\hat{\alpha}(t_1)$ and $\tilde{\alpha}(t_2)$ are unknown and are chosen experimentally to yield the best results in historical runs. Since in the practical situations one is dealing with the short



$\alpha(t)$
$\tilde{\alpha}(t_1)$
$\hat{\alpha}(t_1)$
t
T_1
t_1
T_0
T
IDENTIFICATION
INTERVAL
PREDICTION
INTERVAL

FIGURE 14

time sequences of $\alpha(t)$ the expressions with different weights are not effective and we prefer to choose the weights by using a macroeconomic model of technological change.

The forecasting of $\alpha(t)$ trend outside $[T_1,T_0]$ can be obtained by a linear or exponential extrapolation of trend function. In the last case the ratio

$$r = \hat{\alpha}(T_0-1)/\hat{\alpha}(T_0)$$

can be used for evaluation of the exponent a in the prediction formula

$$\hat{\alpha}(t) = \hat{\alpha}(T_0)e^{-a(t-T_0)} \quad , \; t \geq T_0 \quad , \tag{50}$$

i.e.

$$a = \ln r \quad .$$

It should also be observed that the model (43)-(46) does not take into account the influence of the neutral technical progress. A possible way to avoid that drawback is to replace (45) by

$$\frac{d\alpha}{\alpha} = (b - 1)\, \frac{dY}{Y} + \mu \quad , \tag{51}$$

and obtain instead of (46)

$$\alpha(t) = \alpha_0 \; e^{\mu t}[Y(t)]^{b-1} \tag{52}$$

In the present model the change of α also depends on the exogenous variable μ representing the new achievements in science and technology. It is possible to show that when the data regarding the time series $\ln \alpha(t)/\alpha(t-1)$ and $\ln Y(t)/Y(t-1)$, $t = 1,2,...$

are available, the unknown parameters μ, b, can easily be estimated by means of the least squares method.

Now we should investigate the effect of price changes on the estimation of technological coefficients. Since the sector generally consists of factories with different technological coefficients, the observed sector technological coefficients $\tilde{\alpha}_{ji}(t)$, may differ from a particular factory (which started to produce at $t = t_0$) coefficients $\bar{\alpha}_{ji}$. Assume that $\bar{\alpha}_{ji}$ have been chosen (by proper design and construction) in such a way that $\bar{\alpha}_{ji}$ are "the best" values at the existing prices $p_i(t_0) = \bar{p}_i$, $i = 1,\ldots,n$, so that

$$\bar{\alpha}_{ji} = \hat{Y}_{ji}(t_0)/\hat{Y}_i(t_0) = \frac{X_{ji}(t_0)\bar{p}_j}{X_i(t_0)\bar{p}_i} , \qquad j,i, = 1,\ldots,n \quad .$$

It is also assumed that the factory under consideration is "small enough" so its production has negligible effect on the prices and they can be regarded as exogenous variables. We shall also assume that "the input compositions" of goods sold by sector S_j to S_i do not change in time and as a result, we can deal with p_j instead of p_{ji} .

The output and net profit of the factory becomes

$$\hat{Y}_i = F_i \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{\bar{p}_j}\right)^{\bar{\alpha}_{ji}/\bar{q}_i} \bar{p}_i^{1/\bar{q}_i} , \quad \bar{D}_i = \bar{q}_i\hat{Y}_i \quad . \qquad (53)$$

Suppose now that at t the prices have changed but the technology of our factory requires that $\frac{X_{ji}(t)}{X_i(t)} = \frac{X_{ji}(t_0)}{X_i(t_0)}$. Then technological coefficients α_{ji} have to change and become

$$\tilde{\alpha}_{ji}(t) = \frac{p_j(t)X_{ji}(t)}{p_i(t)X_i(t)} = \frac{p_j^t}{p_i^t}\bar{\alpha}_{ji} \quad . \tag{54}$$

where $p_i^t = p_i(t)/p_i(t_0)$, $i = 1,\ldots,n$.

As a result, the value of net profit of the factory becomes

$$\tilde{D}_i = R_i\bar{D}_i \quad , \tag{55}$$

where R_i is the "profitability" coefficient

$$R_i = \frac{1-\sum_{j=1}^{n}\tilde{\alpha}_{ji}}{1-\sum_{j=1}^{n}\bar{\alpha}_{ji}} \prod_{j=1}^{n}\left(\frac{\tilde{\alpha}_{ji}}{\bar{\alpha}_{ji}}\right)^{\bar{\alpha}_{ji}/\bar{q}_i} \quad . \tag{56}$$

Since $\tilde{\alpha}_{ji}(t)$ are changing in time, it is interesting to find that value of $\tilde{\alpha}_{ji}$ which produces the highest profit of a particular factory. Since the value added by the factory is usually calculated by taking into account the value of productive factors (i.e. the exogenous factors in our analysis) we shall assume that

$$\sum_{j=1}^{n}\tilde{\alpha}_{ji} = \sum_{j=1}^{n}\bar{\alpha}_{ji} = \alpha_i \tag{57}$$

where α_i is given.

Then the problem consists of finding

$$\max_{\tilde{\alpha}_{ji}} R_i(\tilde{\alpha}_{ji})$$

subject to (57) . It can easily be shown that the maximum value $R_i = 1$ is obtained if

$$\tilde{\alpha}_{ji} = \frac{\bar{\alpha}_{ji}}{\sum_{j=1}^{n} \bar{\alpha}_{ji}} \alpha_i = \bar{\alpha}_{ji} \quad , \qquad j,i = 1,\ldots,n \quad . \tag{58}$$

When a factory employs an obsolete technology and as a result $\bar{\alpha}_{ji} \neq \tilde{\alpha}_{ji}$, $j,i = 1,\ldots,n$, it suffers losses. Most of the factories will try to change $\bar{\alpha}_{ji}$ by modernization of technology in order to achieve at least the average sectorial level of technology, characterized by $\bar{\alpha}_{ji} = \tilde{\alpha}_{ji}$, $j,i = 1,\ldots,n$.

As follows from statistical data, the value added α_i is usually decreasing in time. In that case, one should replace the coefficient α_i in (58) by $\tilde{\alpha}_i < \alpha_i$, and as a result

$$\tilde{\alpha}_{ji} = \delta_i \bar{\alpha}_{ji} \quad ,$$

where

$$\delta_i = \tilde{\alpha}_i \Big/ \sum_{j=1}^{n} \bar{\alpha}_{ji} < 1 \quad ,$$

In that case the R_i coefficient in (56) should be multiplied by

$$r_i = \frac{1/\bar{\alpha}_i - \delta_i}{1/\bar{\alpha}_i - 1} \div \delta_i \bar{\alpha}_i / 1 - \bar{\alpha}_i < 1 \quad .$$

In order to maximize profit, the factory should in addition increase δ_i , i.e. increase the value added.

In the hypothetical situation when no investments and moderization takes place, so that $\alpha_{ji}(t_0) = a_{ji}(t) = \bar{\alpha}_{ji}$, the observed $\tilde{\alpha}_{ji}$ change should be assigned solely to p_j^t/p_i^t , which can be estimated from the obvious relation

$$p_j^t/p_i^t = \tilde{\alpha}_{ji}(t)/\tilde{\alpha}_{ji}(t_0) \quad . \tag{59}$$

On the other hand, when the prices do not change in $[t_0,t]$ (e.g. the prices for coal, steel, electric energy in Poland are quite stable), $\tilde{\alpha}_{ji}$ change can be assigned solely to the technological change.

When there are data on sector price indices p_j^t , $j = 1,\ldots,n$, available, it is possible to compare them with (59) in order to determine the "pure" technological change.

It can also be demonstrated that the technological change resulting from modernization is generally lagging behind the price change.

Since data regarding α_{ji} at the sectorial level are usually not available, there is a tendency to use $\tilde{\alpha}_{ji}(t)$ for that purpose (as an approximation) estimated from input-output tables of national economy.

As follows from (55), the change of α_{ji} in the production function results in decreasing of the net output by a factor $R_i \leq 1$. Since in the model under consideration the K_i parameters in (12) are being identified by least squares, the accurate identification of α_{ji} is not so important and one can use the estimated by (50) $\hat{\alpha}_{ji}(t)$ as proxy for α_{ji} .

Now let us turn our attention to the construction of adaptive models for the consumption-structure change. The γ_ν coefficients which express the social preferences, regarding the allocation of expenditures among the different spheres of activity (production and development factors), can be estimated ex post by the following relations:

$$\tilde{\gamma}_\nu(t) = \tilde{Z}_\nu(t)/Z(t) \quad , \quad [Z(t) = Y(t-1)] \quad , \quad \nu = 1,\ldots,m \; ,$$

where $\tilde{Z}_\nu(t)$ - the government expenditures in fiscal t year known from statistical data.

The γ_ν coefficients should satisfy the following relations:

$$\sum_{\nu=0}^{m} \gamma_\nu(t) = 1 \quad . \tag{60}$$

The change of $\gamma_\nu(t)$ in time can be regarded as due to an additional social goal $\gamma_\nu^n(t_2) = g_\nu \gamma_\nu(t_1)$ and exogenous influence μ_ν . Then by the similar arguments and relations (43)-(51), one arrives at the formulae equivalent to (51),(48):

$$\gamma_\nu(t) = \gamma_{\nu 0} \, e^{\mu_\nu t} [Z_\nu(t)]^{g_\nu - 1} \quad , \tag{61}$$

$$\hat{\gamma}_\nu(t_{i+1}) = \hat{\gamma}_\nu(t_i) e^{-\rho_\nu} + g_\nu \tilde{\gamma}_\nu(t_1)[1 - e^{-\rho_\nu}] \quad . \tag{62}$$

However, care should be taken while g_ν are being estimated, in order to satisfy condition (60).

It should be noted that the adaptive approach to the modeling of technological and structural change has several advantages. First of all, it should be observed that only the short time sequences of $\tilde{a}_{ji}(t)$, $j,i = 1,\ldots,n$, $t = -1,-2,\ldots,T_1$ are available. For example, the reliable input-output tables for Polish economy are available only for the last eight years. The a priori knowledge, regarding the statistical properties of coefficients estimated, and the random factors involved, is limited. In that situation the application of standard econometric methods (such as least squares) would hardly give the positive results. On the other hand, the application of the adaptive forecasting models , in such situations, have been shown to be effective [6].

The adaptive algorithms (48) are simple enough so the estimation of the α_{ji} coefficients for each planning interval is easy.

When the "moving-horizant" planning technique is used, it is necessary to derive the projected values of $\alpha_{ji}(t_k)$ for the $[k,k+T]$, $(k=0,1,2,...)$ subinterval. Using the methodology described, one should derive the projection time intervals by

$$t_k = k + \frac{T}{2} \quad , \qquad k = 0,1,...$$

and then use the prediction formula (50):

$$\hat{\alpha}_{ji}(k+\frac{T}{2}) = \hat{\alpha}_{ji}(k)e^{-a_{ji}(k)\frac{T}{2}} \quad , \qquad k = 0,1,... \qquad (63)$$

In each planning interval, the α_{ji} are kept constant and equal $\hat{\alpha}_{ji}(t_k)$. These values are changed when one passes to the next planning interval. At the same time, the new estimates of $\hat{\alpha}_{ji}(k+1)$ are derived by (48), so that

$$a_{ji}(k+1) = \ln \frac{\hat{\alpha}_{ji}(k)}{\hat{\alpha}_{ji}(k+1)} \quad , \qquad k = 0,1,... \qquad (64)$$

As a result, at each planning interval [k,k+T], one deals with a new steady-state set of technological coefficients, which are approximating the expected change of technology, as shown in Fig.14. In other words, we approximate the continuously changing technology by a set of stationary (within each planning interval) models, or - the model is adapting to the changing technology. It should also be noted that an adaptive approach enables us to deal with the optimization and price submodels, which require that α_{ji} do not change within the optimization interval.

The adaptive approach in estimation of consumption structure change has many advantages as well. Besides what has already been said with regard to the technological change, it should be noticed that the utility function parameters γ_ν can hardly be regarded as determined completely by GNP and prices only (as the model (35) may suggest). Instead of taking into account a number of factors, social values and goals which can hardly be explicitly expressed, it is better to learn the effects resulting from these goals in terms of expenditures policy, i.e. $\gamma_\nu(t)$, $\nu = 1,\dots,m$. The adaptive forecasting algorithms for $\gamma_\nu(t)$ change seem to be best suited for that purpose.

III. Interregional and International Cooperation Models

3.1 Optimization of regional development

The national model of development can be regarded as the aggregation of submodels, which have been constructed for all the regions of the country. It is important that the methodology used for construction of regional submodels enables the aggregation, which yields the consistent relations between the regional and national (core) model variables and model equations.

So far, we have been dealing with the methodology, which is useful for modeling and planning of long-term development in n-sector production systems. The purpose of the present section is to show how that methodology could be used for modeling of regional development and regional planning.

First of all, it can be observed that the national model can be regarded as the sum of regional submodels if all the statistical data are available. One can consider also a particular regional model S_r cooperating with the rest of the country S_c (Fig.15). All the submodels' technological (and other) coefficients should be estimated or chosen in such a way that the aggregated submodels give the same set of basic relations as the core national model. Then the sector's strategies, regarding the allocation of production factors among the set of regions, can be analyzed. If we consider, e.g. a particular production sector S_i and N regional production functions of the general type (13) of Sec. 1.2, the contribution of j-th region to the regional production $y_{ij}(t)$ can be written as

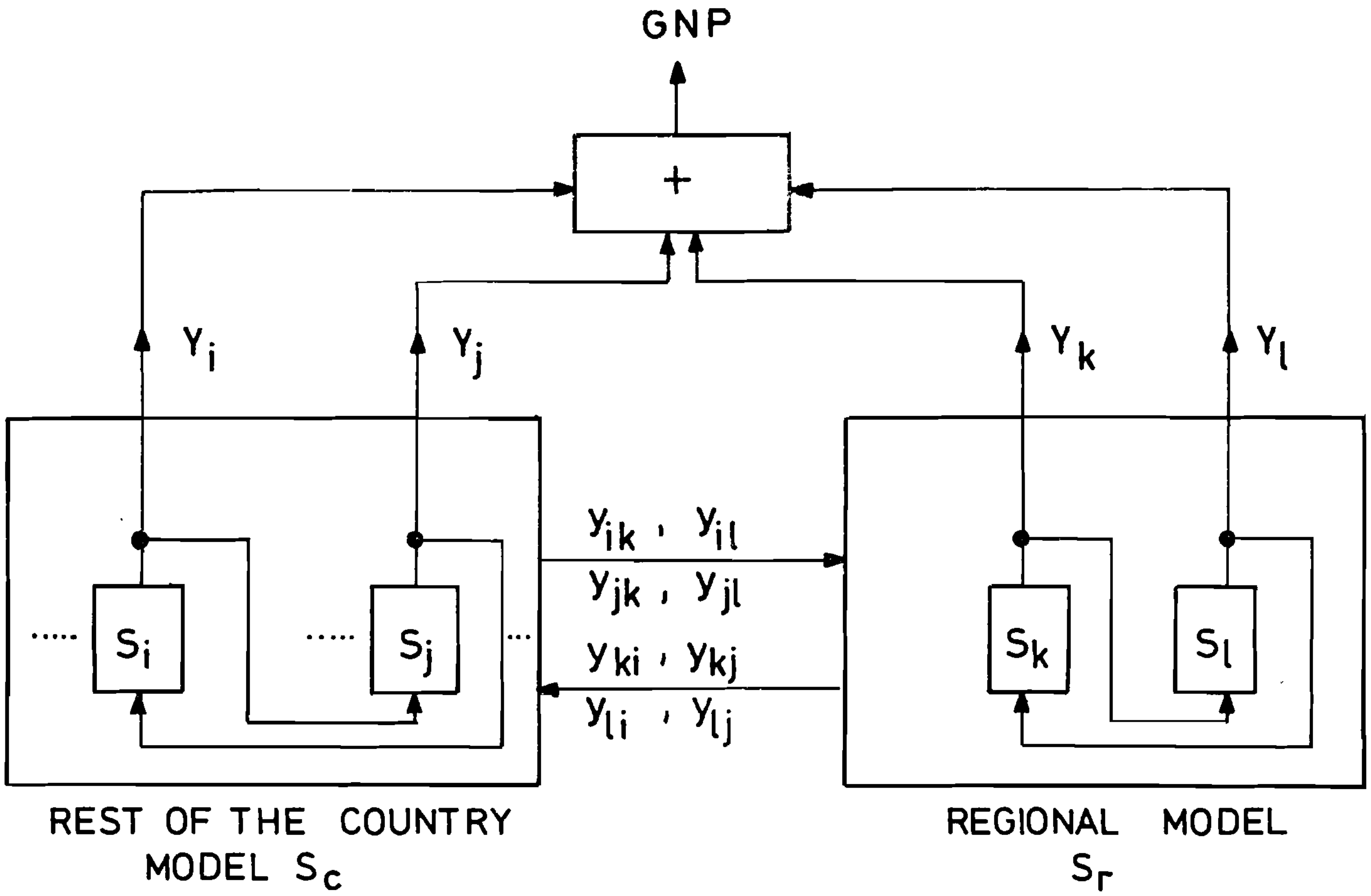

GNP
+
Y_i
Y_j
Y_k
Y_l
y_{ik} , y_{il}
y_{jk} , y_{jl}
y_{ki} , y_{kj}
y_{li} , y_{lj}
S_i
S_j
S_k
S_l
REST OF THE COUNTRY MODEL S_c
REGIONAL MODEL S_r

FIGURE 15

$$y_{ij}(t) = \prod_{\nu=1}^{m} \left\{ f_{\nu ij}(t) \right\}^{\beta_\nu} \tag{1}$$

where

$$f_{\nu ij}(t) = \int_{-\infty}^{t} k_{\nu ij}(t - \tau)[z_{\nu ij}(\tau)]^{\alpha} \, dt \quad ,$$

$$k_{\nu ij}(t) = \begin{cases} K_{\nu ij} e^{-\delta_{\nu i}(t-T_{\nu i})} & , \quad t > T_{\nu i} \\ 0 & , \quad t < T_{\nu i} \end{cases} .$$

We shall also assume, for the moment, that the total regional resources $Z_{\nu j}$, $\nu = 1,\ldots,m$, $j = 1,\ldots,N$ be given. Then it is possible to find the regional optimum development strategy $z_{\nu ij}(t) = \hat{z}_{\nu ij}(t)$, $\nu = 1,\ldots,m$, $i = 1,\ldots,n$, $j = 1,\ldots,N$, $\tau \in [0,T]$, such that

$$Y_j = \int_0^T (1 + \varepsilon)^{-t} \sum_{i=1}^{n} y_{ij}(t) \, dt \tag{2}$$

is maximum subject to

$$\sum_{i=1}^{n} \int_0^T z_{\nu ij}(t) \, dt \leq Z_{\nu j} \quad , \quad \nu = 1,\ldots,m \quad , \quad j = 1,\ldots,N \tag{3}$$

$$z_{\nu ij}(t) \geq 0 \quad , \quad i = 1,\ldots,n \quad , \quad \tau \in [0,T] \quad .$$

The optimum strategies can be used to derive the value of $\Delta Y_j = \hat{\Delta Y}_j$, which takes the form (31) of Sec. 1.2.

$$\hat{\Delta Y}_j = G_j^q \prod_{\nu=1}^{m} Z_{\nu j}^{\delta_\nu} , \quad \gamma_\nu = \alpha\beta_\nu , \quad j = 1,\dots,N . \tag{4}$$

The problem which presently faces us is to derive the optimum values of $Z_{\nu j} = \hat{Z}_{\nu j}$, $\nu = 1,\dots,m$, $j = 1,\dots,N$, which would maximize

$$\Delta Y = \sum_{j=1}^{N} G_j^q \prod_{\nu=1}^{m} Z_{\nu j}^{\gamma_\nu} , \tag{5}$$

subject to

$$\sum_{j=1}^{N} Z_{\nu j} \leq Z_\nu , \quad \nu = 1,\dots,m , \tag{6}$$

$$Z_{\nu j} \geq 0 , \quad \nu = 1,\dots,m , \quad j = 1,\dots,N . \tag{7}$$

It can easily be verified that a unique optimum strategy exists and it can be derived by the formulae:

$$\hat{Z}_{\nu j} = \frac{G_j}{G} Z_\nu , \quad j = 1,\dots,N , \quad \nu = 1,\dots,m , \tag{8}$$

where

$$G = \sum_{j=1}^{N} G_j .$$

Using the present method, we can derive the optimum allocation of production factors and government expenditures among different regions and production sectors within the planning interval. There is, however, an obvious drawback to the present approach: it is very much production oriented, i.e. it takes into consideration, first of all, the efficient allocation of resources. The government expenditures in education and health services are treated here as complementary (i.e. supporting) production factors. A possible way to avoid that drawback is to assume that a part of the government budget is used for an increased financing of these regions which are behind the average country's figures. In that case, we can use the production function (25) of Sec. 2.2, where $z_{\nu j}(\tau)$ represent that part of government expenditures which has a universal (with respect to a particular technology) production effect. In order to allocate that part of government expenditure in an explicit form, a method described in [26] can be applied. According to that method, a regional dissatisfaction function can be constructed of the general form:

$$D_j(Z) = d_j \prod_{\nu} |\bar{Z}_{\nu j} - Z''_{\nu j}|^{\beta_\nu} , \qquad j = 1,\ldots,N$$

where d_j , β_ν - given positive numbers,

$\bar{Z}_{\nu j}$ - given country's average (per capita) of government expenditure level.

The problem consists in finding $Z_{\nu j} = \hat{Z}''_{\nu j}$, $j = 1,\ldots,N$, $\nu = 1,\ldots,m$, such that

$$D = \sum_j D_j$$

is minimum, subject to

$$\sum_j Z''_{\nu j} \leq a''_\nu Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \qquad 0 < a''_\nu < 1 \quad ,$$

$$Z''_{\nu j} \geq 0 \quad , \qquad j = 1,\ldots,N \quad .$$

The numerical values of a''_ν , $\nu = 1,\ldots,m$, which represent the share of universal expenditures in Z_ν , can be estimated from past (historical) data, or considered as decision variables.

Using that approach, the regional benefit (utility) function (4) can be written as

$$\Delta Y_j = G_j^q \prod_{\nu=1}^{m} (Z'_{\nu j})^{\gamma'_\nu} (Z''_{\nu j})^{\gamma''_\nu} \quad , \qquad \gamma'_\nu + \gamma''_\nu = \gamma_\nu \quad ,$$

$$j = 1,\ldots,N \quad , \qquad (9)$$

which shows the contribution of all government expenditures to the regional welfare. That contribution can be regarded in two possible ways. The direct way in the form of salaries (Z'_{1j}) , education, medical and social care organized by production sectors $(Z'_{\nu j})$ and the indirect way (expressed by $Z''_{\nu j}$) in the form of public education, social and medical care, and environment protection organized by regional and government institutions. The main factor, determining the regional growth in terms of Y_j is, of course, G_j , which depends on the $K_{\nu ij}$, $i = 1,\ldots,n$, $\nu = 1,\ldots,m$ factors. Since the numerical values of $K_{\nu ij}$ in the model under consideration are being determined ex post from statistical data, the model has a tendency to maintain the existing development trends. However, it is a rather common situation that the regional growth depends as well on new geological discoveries, for example, which change the existing regional production structure. For that reason, a more detailed location analysis and optimization is needed. In particular, it is necessary to analyze the change of model technological coefficients, resulting from the change of location of production sectors.

Consider a simple model, shown in Figure 16, where the national core model cooperates with a new production sector S_r being planned at the given region r. It is assumed that the core model projections of the total investment intensity ($Z_2(t)$), labor cost ($Z_1(t)$) and other government expenditures ($Z_\nu(t)$, $\nu = 3,\ldots,m$) in the planning interval $[0,T]$ are given. The expenditure intensities connected with the regional project $C_i(t)$, $i = 1,\ldots,m$ are assumed to be known. It is assumed that the central planning unit considers a number (M) of different regional projects characterized by given cost functions $C_i^j(t)$, $i = 1,\ldots,m$, $j = 1,\ldots,M$, where generally

$$C_i^j(T) \leq Z_i(t) \quad , \qquad i = 1,\ldots,m \quad , \qquad j = 1,\ldots,M \quad ,$$

$$\tau \in [0,T] \quad ,$$

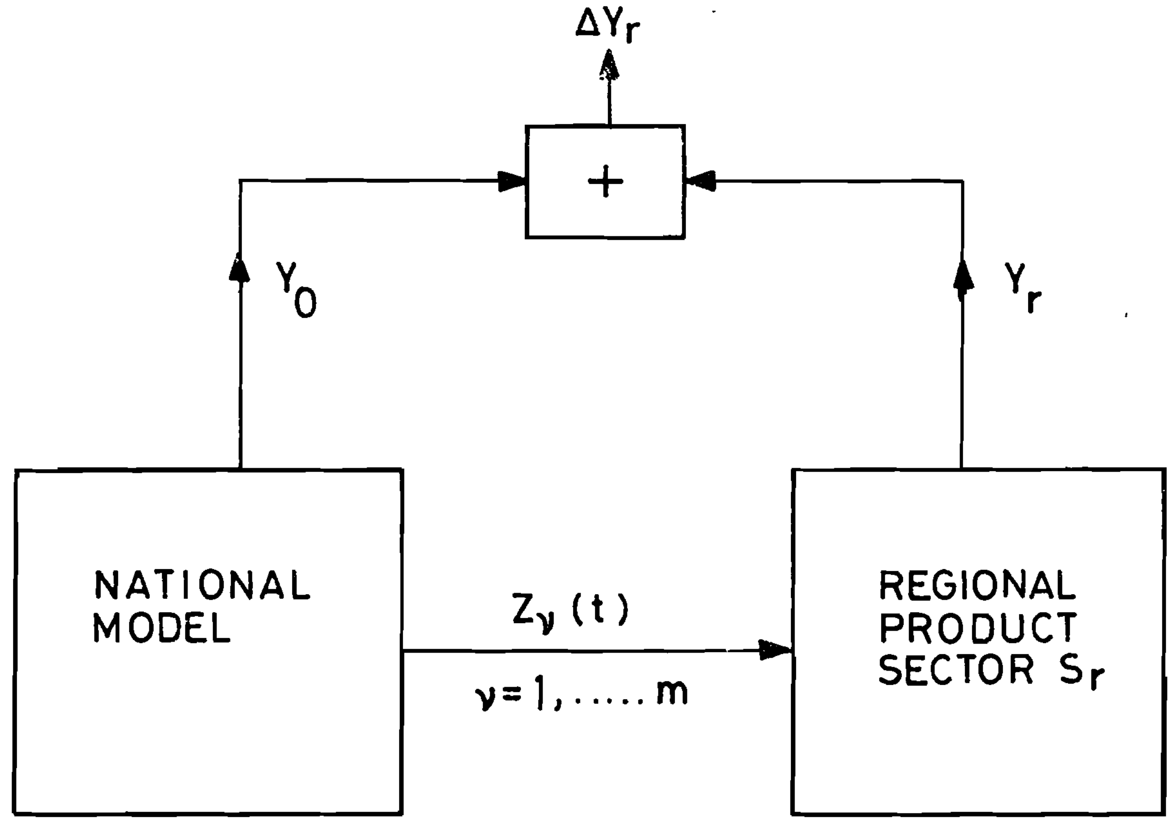


FIGURE 16

but

$$\sum_{j=1}^{M} C_i^j(t) > Z_i(t) \quad ,$$

at least for some $i \in [1,\ldots,m]$, $t \in [0,T]$. Then it is necessary to choose a subset $M' \in M$ of these projects which are most effective for national and regional development. Generally speaking, the projects can be realized at N different regions yielding different values of expected GNP increases:

$$\Delta Y_j = \hat{Y}_j - Y_0 \quad , \qquad j \in N \quad ,$$

where Y_0 = the GNP generated within the planning interval $[0,T]$ by the core model when all the resources are allocated in optimal manner, but no specific regional project is indicated,

Y_j = the GNP generated within the planning interval, by the core model and regional project, when the cost of regional project resources is shifted from core to regional project.

Since, generally speaking, the change of project location will induce the corresponding change of transport costs and prices for S_r output and other sectors' outputs, it is necessary to derive ΔY_j , $j = 1,\ldots,N$, in constant prices. In that way, one takes into account the direct economic effects of regional location as well as the indirect effects resulting from price changes within the whole socio-economic systems. Some of these changes can be regarded as beneficial (for example, an increase of regional production may decrease the product price and increase the consumption), while at the same time the industrial growth may induce more pollution, decrease the agriculture productivity, etc. Another reason is that dealing with output expressed in constant prices, it is possible to neglect the inflationary effects on the economic growth.

Suppose that at the first stage of regional planning each project has been checked for an optimum location. To do that, it is necessary to find $j = r$, such that $\Delta Y_r = \max\{\Delta Y_j\}$ $j \in N$. When the project inputs and outputs are traded with the core mainly (at least during the planning interval), that process gives us the optimum location of individual projects among the possible regions.

The next step is to choose the best portfolio of projects satisfying the constraints on the available resources generated by the core model. In order to solve that problem, one can use the well-known integer programming method. In order to do that, introduce the discrete variables $X_j \in [0,1]$, $j = 1,\ldots,M$. The problem consists in finding the strategy $X_j = \hat{X}_j$, $k = 1,\ldots,M$, such that

$$\Delta Y = \sum_{j=1}^{M} X_j \Delta Y_j \quad , \tag{10}$$

attains maximum subject to the constraints

$$\sum_{j=1}^{M} C_i^j(t)\, X_j \leq Z_i(t) \quad , \quad i = 1,\ldots,m \quad , \quad t = 0,\ldots,T \quad . \tag{11}$$

The present method can easily be extended to the case when the regional project involves a complex of n' sectors S_{ri} , $i = 1,\ldots,n' \leq n$, which exchange the products with core as well as among themselves. A typical example is an energy complex which involves the coal mine, electric power station, which consumes coal and generates electricity, utilized together with coal to produce chemicals, etc. In the last case, it is necessary to coordinate the core expenditures assigned to different production sectors.

In order to use the proposed methodology for optimization of regional allocation of resources, it is necessary to introduce the regional aspects in the regional (S_r) production function. The main factor which should be taken into account is the change of technological coefficients and prices resulting from the transport cost changes. Consider as an example the core sector production function (4) of Sec. 2.1, which corresponds to a fixed location. For the optimum sector production strategy one gets

$$\alpha_{ji} = \frac{\hat{Y}_{ji}}{\hat{Y}_i} = \frac{p_j\hat{X}_{ji}}{p_i\hat{X}_i} \quad , \qquad j,i = 1,\ldots,n \quad . \tag{12}$$

Suppose that the project under consideration has been located at the same place as the core production sector and the same technology (requiring the given ratios of $\hat{X}_{ji}/\hat{X}_i$, $j,i = 1,\ldots,n$) has been adopted. In that case, the project technological coefficients are determined by (12).

Suppose now that the location of the project S_r has been changed (with respect to core sector location) and the cost $\hat{Y}_{jr}$ of the inputs $\hat{X}_{jr}$ has changed to become

$$\tilde{Y}_{jr} = \hat{Y}_{jr}(1 + t_{jr}) \tag{13}$$

where t_{jr} - an increasing function of distance between the old and new location. The effect on the economy is the same as if the α_{jr} of S_r had changed to become:

$$\tilde{\alpha}_{jr} = \alpha_{jr}(1 + t_{jr}) \quad . \tag{14}$$

Besides the transport costs which depend on S_r location a new production project may also use more advanced technology, which changes $\hat{X}_{jr}/\hat{X}_r$, $j = 1,\ldots,n$. That process is, however, neutral with respect to location of the project. In a similar way, the change of S_r location affects the α_{ri} and ℓ_γ coefficients in equations (36)-(38) of Sec. 2.3. The final result of these changes is a change of price indices p_i^t , $i = 1,\ldots,n$ and the corresponding change of ΔY_j (in constant prices).

In order to derive the effect of $\tilde{\alpha}_{jr}$, $\tilde{\alpha}_{ri}$ on the resulting national model output, one can also consider S_r as an independent sector with the given $\tilde{\alpha}_{jr}$, $i = 1,\ldots,n$ technological coefficient and the price index p_r^t , which can be derived from the extended set of equations (38) of Sec. 2.3:

$$\ell n p_i^t - \sum_{j=1}^{n} \alpha_{ji} \ell n p_j^t - \tilde{\alpha}_{ri} \ell n p_r^t = q_i \left[\ell n \frac{\ell_i^t Y^{t-1}}{F_i^t} + \sum_{\nu=1}^{m} \beta_\nu \ell n \omega_\nu^t \right], \quad i = 1, \ldots, n$$

$$\ell n p_r^t - \sum_{j=1}^{n} \tilde{\alpha}_{jr} \ell n p_j^t = q_r \left[\ell n \frac{\ell_r^t Y^{t-1}}{F_r^t} + \sum_{\nu=1}^{m} \beta_\nu \ell n \omega_\nu^t \right] . \qquad (15)$$

The next step is an aggregation of sector S_r with the corresponding sector in the core model. As shown in [33], such an aggregation results in a new set of aggregated technological coefficients and a new sector price index. It can be observed that a regional location process has an important effect on the technological change and development on the regional, as well as the national level.

It should be observed that in the regional models discussed, the labor force, as well as all the other development factors, is regarded as mobile so that it can be transferred from the higher to the lower populated areas if necessary. That, however, involves additional costs (mainly in housing), which are usually assigned to the new investment costs. As a result, the effectiveness of regional investment may decrease when there is a shortage of labor present.

Another important factor which decreases the expected (within the planning interval) gross regional product (GRP) is the cost of protection of the environment. As already mentioned, the harm done to the environment depends much on the regional location of new production and consumption centers. For example, in a heavily industrialized or populated area, the construction of a new factory is generally more harmful than in the desert. The pollution control cost necessary to satisfy the regional standards depends, therefore, on the location of each new factory. As a result, the GRP generated in the planning interval according to (60) of Sec. 1.4 depends on the location and the technology used.

In order to achieve the fastest GRP growth, it is necessary to choose not only the location, but the area of productive specialization as well. It is also important that the chosen location ensures the availability of natural resources. It happens, for example, that industry competes with agriculture for supplies of fresh water. As a result, the price of water depends on the location, and changes with time.

3.2 International cooperation models

As already mentioned, the main idea behind the normative modeling of development is to find the best allocation of development factors strategy (which yield the maximum integrated output) subject to the constraints imposed on the flow of resources. At the regional level, as already mentioned, the availability of the labor force, natural resources (e.g. water, energy, etc.) and environment protection, represent the typical constraints of development, while the amount of financial resources depends on the higher level (i.e. government) decisions. At the national level, the constraints take the additional form of monetary constraints. For example, the GNP generated by the economy should be in balance with the government expenditures, as shown in Sec. 1.2. Since in an open economy the foreign trade influences the GNP level, the foreign trade-balance should also be observed. In addition, since the international cooperation also involves crediting of investments, exchange of "know-how", patents, skilled labor, scientific and technical cooperation, etc., the general international cooperation model takes a complex form as shown in Fig.17. The general idea behind the model is that all the factors contributing to the development can be subject to international cooperation. That usually involves international agreements on a government level. Government decisions are based on the system of values and development goals which again are subject to international exchange in values and ideology. In the case of Poland, for example, close cooperation with the Soviet Union and socialist countries is very important in that respect. It stimulates international cooperation at government and even lower levels. The creation of international organizations such as the Council for Mutual Economic Assistance (CMEA), involving nine socialist countries, has had a great impact on the rapid development of all of the member countries. The creation of the International Bank of Economic Assistance (in 1963) and the International Investment Bank of CMEA (in 1970) stimulated foreign trade, mutual assistance in the development of new branches of industry, and utilization of scarce resources among the socialist countries. The close cooperation among the CMEA countries also

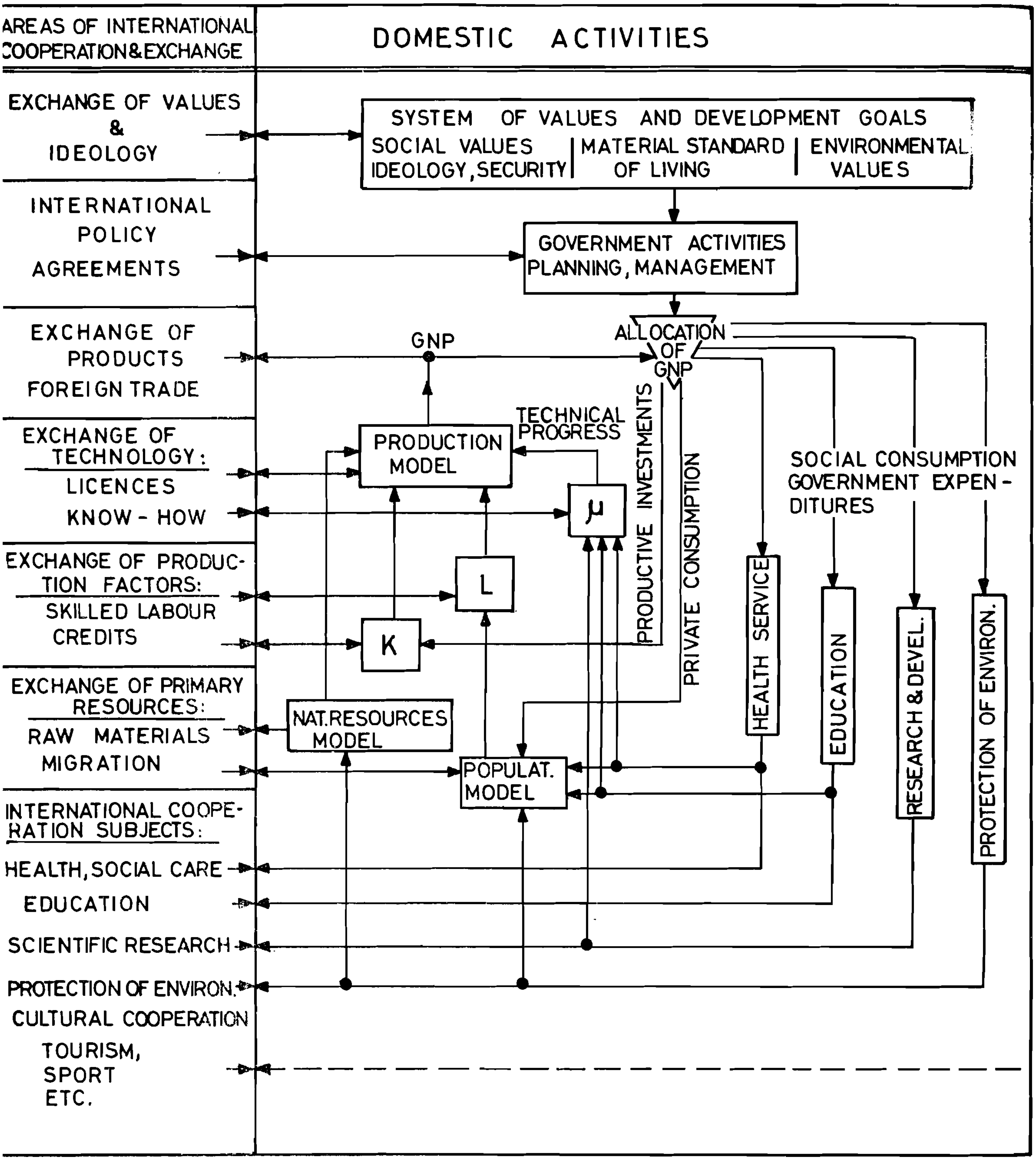


FIGURE 17. MODEL OF INTERNATIONAL COOPERATION

exists in science and technology, education, and cultural life.

The cooperation of CMEA countries with the rest of the world, especially in trade, science and technology, is rapidly developing as well.

The benefits which result from foreign trade development for each participating country are well-known and are analyzed in the "pure theory of foreign trade" [14]. Little comprehensive theory exists, however, on the benefits resulting from general international cooperation, according to the activities shown in Fig.17. Our first objective in the present section is, therefore, to find the impact of trade as well as of mutual assistance in crediting the national investments, exchange in science and technology, education and other national activities.

It is assumed that international organizations and banks have certain stocks of initial resources (created by contributions of participants according to the mutual agreements), which are used as aid in financing the development plans proposed by individual countries. The loans are given, generally speaking, with interest rates, which help the less-developed countries to grow at a faster rate. In that way, the socio-economic development of different countries may rise to predetermined levels should the need arise.

Since the loans obtained should usually be paid (including interest) back, from the national and the long-term development points of view, it is important to know how many benefits one can expect, using foreign credits.

From the systems analysis point of view, that problem boils down (as mentioned in Sec. 1.2) to the comparison of relative advantages of optimal development strategies, which are subject to the amplitude (i.e. full autarky) or integral (i.e. international credits available) constraints. In order to solve that problem, consider first of all the model of international trade shown in Fig.18. The model consists of national production-sectors S_i , $i = 1,\dots,n$, which export part Y_{iz} of their products to the foreign market S_z . On the other hand, S_z is exporting $\bar{Y}_z$ $(1-q_2)$ part of the total export $\bar{Y}_z$, to the national system, consisting of production and consumption subsystems. The "production function" of S_z (in market S_z currency) can be written in similar form to the S_i - sectors functions, i.e.

$$\bar{Y}_z = F_z^{q_z} \prod_{i=1}^{n} \bar{Y}_{iz}^{\alpha_{iz}} \bar{p}_z \prod_{i=1}^{n} (\bar{p}_{iz})^{-\alpha_{iz}} , \qquad (16)$$

where

$$q_z = 1 - \sum_{i=1}^{n} \alpha_{iz} > 0 ,$$

$\bar{p}_{iz}$ = prices of exported sectorial commodities in S_z currency.

The production functions of S_i , $i = 1,\dots,n$, sectors are supplemented with $Y_{zi}^{\alpha_{zi}}$ terms, i.e.

$$Y_i = F_i^{q_i} \prod_{j=1}^{n} Y_{ji}^{\alpha_{ji}} Y_{zi}^{\alpha_{zi}} \cdot P_i \prod_{j=1}^{n} P_j^{-\alpha_{ji}} p_{zi}^{-\alpha_{zi}} ,$$

$$q_i = 1 - \sum_j \alpha_{ji} - \alpha_{zi} \qquad (17)$$

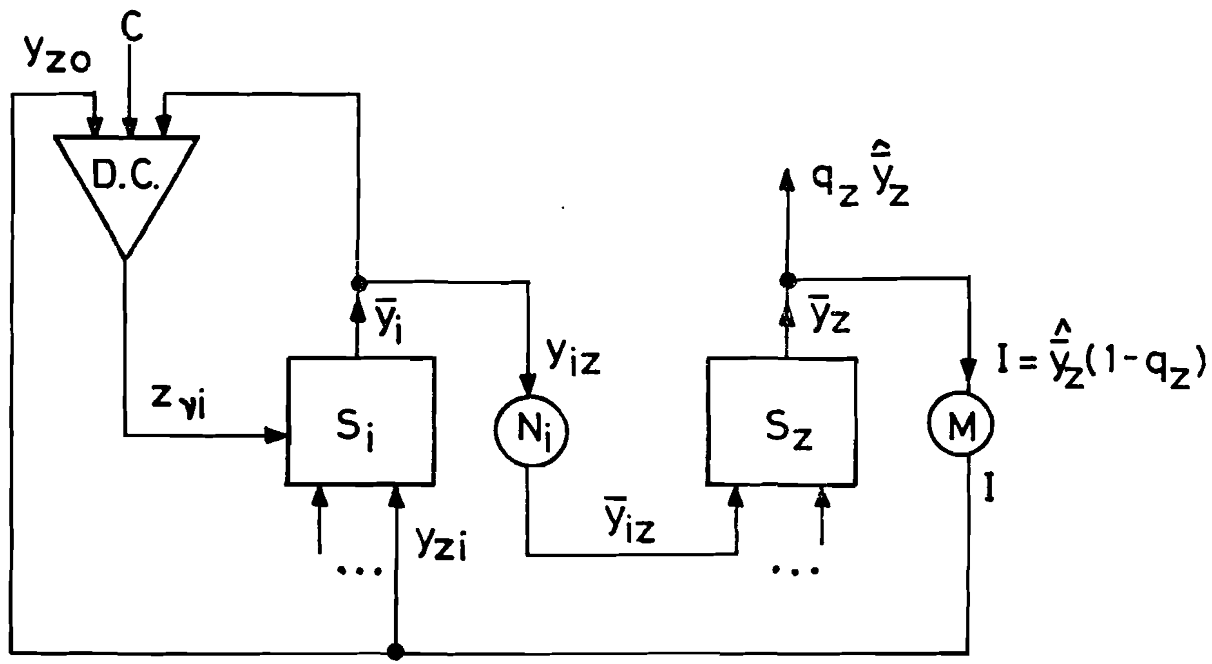


FIGURE 18

where

p_{zi} = the price of imported commodities in the domestic currency.

Since Y_{iz} , $i = 1,\ldots,n$, are measured in domestic currency it is necessary to introduce the following exchange rates:

a) the export exchange rate

$$N_i = \bar{Y}_{iz}/Y_{iz} = \bar{p}_{iz}/p_i \quad , \qquad i = 1,\ldots,n \quad , \tag{18}$$

where

p_i - sectorial prices in domestic currency.

b) the import exchange rate

$$M = I/\bar{I} \quad . \tag{19}$$

It is assumed that sector S_z maximizes the net output

$$\bar{D}_z = \bar{Y}_z - \sum_{i=1}^{n} \bar{Y}_{zi} \quad ,$$

in the same way as the domestic sectors S_i , $i = 1,\ldots,n$, do .

As a result of that strategy the corresponding trade flows (according to (6)(7) of Sec. 2.1) become

$$\hat{Y}_{zi} = \alpha_{zi}\hat{Y}_i \quad , \qquad i = 1,\ldots,n \quad , \tag{20}$$

$$\hat{\bar{Y}}_{iz} = \alpha_{iz}\,\hat{\bar{Y}}_z \quad , \qquad i = 1,\ldots,n \quad , \tag{21}$$

where

$$\hat{Y}_i = F_i \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}/q_i} p_i^{1/q_i} \cdot \left(\frac{\alpha_{zi}}{p_{zi}}\right)^{\alpha_{zi}/q_i} ,$$

$$q_i = 1 - \sum_{i=1}^{n} \alpha_{ji} - \alpha_{zi} > 0 ,$$

p_{zi} = the price of imported commodities in domestic currency, $p_{zi} = \bar{p}_{zi}/N_i$

$$\hat{\bar{Y}}_z = \bar{F}_z \prod_{i=1}^{n} \left(\frac{\alpha_{iz}}{\bar{p}_{iz}}\right)^{\alpha_{iz}/q_z} \bar{p}_z^{1/q_z} .$$

Since

$$I = \sum_{i=1}^{n} \hat{Y}_{zi} + Y_{zo} = \sum_{i=1}^{n} \alpha_{zi}\hat{Y}_i + Y_{zo}$$

where

Y_{zo} = the part of import used for factor endowments (mainly the productive investments, some government expenditures, and part of private consumption) and, on the other hand

$$\bar{I} = \hat{\bar{Y}}_z(1 - q_z) , \qquad (22)$$

one gets

$$M = \frac{I}{\bar{I}} = \frac{\sum_{i=1}^{n} \alpha_{zi}\hat{Y}_i + Y_{zo}}{\hat{\bar{Y}}_z(1-q_z)} \quad . \tag{23}$$

According to (23) the import exchange rate M increases along with the increase of demands for imports of production inputs $\alpha_{zi}\hat{Y}_i$ (claimed by sectors) and demands for development factors Y_{zo}. M decreases when the supply of import $\acute{\bar{Y}}_z(1-q_z)$ increases. From the domestic point of view, the lower M is, the greater the benefits.

The value of export under optimum strategy becomes

$$E = \sum_{i=1}^{n} \hat{Y}_{iz} = \sum_{i=1}^{n} \hat{\bar{Y}}_{iz/N_i} = \hat{\bar{Y}}_z \sum_{i=1}^{n} \frac{\alpha_{iz}}{N_i} \quad . \tag{24}$$

When the balance of payment: I - E = 0 should be observed one gets by (22)(24)

$$M = \frac{E}{\bar{I}} = \frac{1}{1-q_z} \sum_{i=1}^{n} \frac{\alpha_{iz}}{N_i} = \sum_{i=1}^{n} \bar{\alpha}_{iz/N_i} \quad , \tag{25}$$

where

$$\bar{\alpha}_{iz} = \frac{\hat{Y}_{iz}}{\sum_{i=1}^{n} Y_{iz}}$$

is the production of the S_i share in the total export E.

As follows from (25) the resulting import exchange rate M, under the balance of payment condition I - E = 0 , is determined by shares $\bar{\alpha}_{iz}$ and export exchange rates N_i . In order to decrease M, the domestic system should increase N_i by decreasing p_i prices with respect to $\bar{p}_{iz}$. That can be done by investments and technical and organizational progress in these sectors S_i which yield the greatest values of N_i . In the case when $N_i = N =$ const. for all i = 1,...,n , one gets by (25) $M = {}^1/_N$. The last condition should be regarded as an approximation. In the foreign trade practice, the commodities offered for export are usually classified according to the values of $\bar{p}_{iz}/p_i$. There is a tendency to export first of all these commodities which have the greatest values of $\bar{p}_{iz}/p_i$. That tendency can be interpreted as a willingness to keep $N_i \approx \frac{1}{M}$, i = 1,...,n . Assuming that the condition $N_i = \frac{1}{M}$, i = 1,...,n holds, it is possible to express $p_{zi} = M\bar{p}_{zi}$ in terms of export (fob) to import (cif) price ratios $T_{zi}(t) = \bar{p}_{iz}(t)/\bar{p}_{zi}(t)$, i.e.

$$p_{zi}(t) = p_i(t)\,\frac{\bar{p}_{zi}(t)}{\bar{p}_{iz}(t)} = p_i(t)/T_{zi}(t) \quad , \quad i = 1,\dots,n \;.$$

Now we can investigate the impact of $T_{zi}(t)$ change on the domestic price system. For that purpose, we can use the eq.(38) of Sec. 2.3. Since the domestic production function (17) includes in addition the $Y_{zi}^{\alpha_{zi}}\, p_{zi}^{-\alpha_{zi}}$ factor, the term

$$\alpha_{zi} \ln p_{zi}^t(t) = \alpha_{zi} \ln p_i^t(t)/T_{zi}^t(t) \quad ,$$

where

$$T_{zi}^{t}(t) = T_{zi}(t)/T_{zi}(t-1) \quad ;$$

should be added to the left side of eq.(38), Sec.2.3.

In the general case, the foreign trade system consists of many markets S_z , $z = 1,\ldots,s$, and it is necessary to have the balance of payment with each of the markets (though some transfers of payments between markets are sometimes possible). Then supplementing (38)(Sec. 2.3)with the additional factor

$$\prod_{z=1}^{s} Y_{zi}^{\alpha_{zi}} p_{zi}^{-\alpha_{zi}}$$

one obtains

$$(1 - \sum_{z=1}^{s} \alpha_{zi}) \ln p_i^t - \sum_{j=1}^{n} \alpha_{ji} \ln p_j^t = q_i \left[\frac{\hat{Y}_i^t l_i^t}{F_i^t} + \sum_{\nu=1}^{m} \beta_{\nu i} \ln \omega_\nu^t \right]$$

$$- \sum_{z=1}^{s} \alpha_{zi} \ln T_{zi}^t \quad ,$$

$$i = 1,\ldots,n \qquad (26)$$

where

$$q_i = 1 - \sum_j \alpha_{ji} - \sum_z \alpha_{zi} > 0 \quad .$$

It should be observed that the increase of T_{zi}^t decreases the domestic prices p_i^t , $i = 1,\ldots,n$. Since we have no foreign trade price model, the T_{zi}^t should be regarded as exogenous variables. For forecasting purposes, one can extrapolate the $\bar{p}_{zi}$, $\bar{p}_{iz}$ prices by exponential functions:

$$\bar{p}_{zi}(t) = a_i \, e^{\alpha_i t} \quad , \qquad \bar{p}_{iz}(t) = b_i \, e^{\beta_i t} \quad ,$$

and find

$$\ln T_{zi}^t = \beta_i - \alpha_i \quad , \qquad i = 1,\ldots,n \quad .$$

It should be observed that the assumption of exponential price trends for the depletable natural resources (i.e. coal and oil) can be justified by theoretical arguments as shown in Ref. [47].

The derivation of T_{zi}^t is especially simple when the sector trade is homogenous. For example, the sector of energy production of the Polish economy exports coal and imports oil, so $\ln T_{zi}^t$ is the difference of annual rates of price-change for these two commodities. Since oil price rises at a faster rate than coal price, $\ln T_{zi}^t$ for energy is negative, and as a result, the domestic prices p_i^t , $i = 1,\ldots,n$ increase. The impact of T_{zi}^t change on domestic prices depends also on the α_{zi} , i.e. the share of imported commodities in the total sector input, characterized by $1 - q_i$. Since the optimization goals , which is the integrated GNP in constant prices, depends on the sectorial prices, the optimum allocation of $z_{\nu i}$ strategy depends on T_{zi}^t change.

It is possible to show that when $\ln T_{zi}^{t}$ change, (e.g. from positive to negative values) for a number of sectors a reorientation of existing development strategy may follow which gives, in the planning interval [0,T], preferences to the development of these sectors of national economy which contribute most to the general development goal.

It should be observed that the proposed methodology of planning the optimum development depends a great deal on the accuracy of world prices forecasts. Obviously, the best results would be obtained if one had a world trade model including all the markets under consideration and trading the commodities which correspond to the sectorial aggregates.

In order to show how such a model can be constructed by using the present methodology, consider a simple model consisting of two submodels (representing two countries), each consisting of a number of production sectors S_i , $i = 1,\ldots,n$, S_z , $z = 1,\ldots,N$. Suppose two sectors S_i , S_z have engaged in trade and it is necessary to find out what terms of trade can be obtained, when the countries have agreed to keep the constant balance of payment (e.g. equal zero).

The production function of S_i can be written as

$$Y_i = F_i^{q_i} \prod_{j=1}^{n} Y_{ji}^{\alpha_{ji}} Y_{zi}^{\alpha_{zi}} = F_i^{\tilde{q}_i} Y_{zi}^{\alpha_{zi}} \quad ,$$

$$\tilde{q}_i = 1 - \alpha_{zi} = q_i + \sum_j \alpha_{ji} \quad .$$

In the same way for the S_z sector one gets

$$\bar{Y}_z = \tilde{F}_z^{\tilde{q}_z} \bar{Y}_{iz}^{\alpha_{iz}} \quad ,$$

where $\bar{Y}_{iz} = M^{-1} Y_{iz}$, Y_{iz} - export from S_i to S_z ,

M = exchange rate of S_z currency for S_i currency.

The optimum cooperation requires [see (6) of Sec. 2.1]:

$$\hat{\bar{Y}}_{iz} = \alpha_{iz} \hat{\bar{Y}}_z \quad , \qquad \hat{Y}_{zi} = \alpha_{zi} \hat{Y}_i \quad ,$$

where

$$\hat{\bar{Y}}_z = \tilde{F}_z \left(\frac{\bar{p}_{iz}}{\alpha_{iz}} \right)^{\alpha_{iz}/\tilde{q}_z} \bar{p}_z^{\frac{1}{\bar{q}_z}} \quad ,$$

$$\hat{Y}_i = \tilde{F}_i \left(\frac{p_{zi}}{\alpha_{zi}} \right)^{\alpha_{zi}/\tilde{q}_i} (p_i)^{\frac{1}{\bar{q}_i}} \quad .$$

and

$$\bar{p}_{iz} = M^{-1} p_i \quad , \qquad p_{zi} = M \bar{p}_z \quad .$$

Introducing the term of trade $T_{zi} = \bar{p}_{iz} / \bar{p}_z$ one gets

$$M = \frac{p_i}{\bar{p}_z T_{zi}} \quad .$$

From the balance of payment condition $M\hat{\hat{Y}}_{iz} = \hat{Y}_{zi}$ one obtains

$$\frac{p_i}{\bar{p}_z T_{zi}} \alpha_{iz} \hat{\hat{Y}}_z = \alpha_{zi} \hat{Y}_i \quad ,$$

which can be written in the form similar to (26):

$$(\alpha_{iz/\tilde{q}_z} + \alpha_{zi/\tilde{q}_i} - 1)\ln T_{zi}^t - 2\alpha_{zi/\tilde{q}_i} \ln p_i^t + 2\alpha_{iz/\tilde{q}_z} \ln \bar{p}_z^t$$

$$= \ln\left[\alpha_{zi}^{1-\alpha_{zi/\tilde{q}_i}} \alpha_{iz}^{\alpha_{zi/\tilde{q}_z}-1} \tilde{F}_{i/\tilde{F}_z}\right] \quad . \qquad (27)$$

The equation (27) combines the two systems of linear equations of the type (26), which refer to the two countries and the respective systems of prices p_i^t , $i = 1,\dots,n$, $\bar{p}_z^t$, $z = 1,\dots,N$. In order to find the development strategies which are beneficial for both countries, it is necessary that the investment-price adjustment technique, described in Sec. 2.3, is extended and coordinated by taking into account the interactions specified by (27). The resulting terms of trade T_{zi} , $T_{iz} = T_{zi}^{-1}$ are then derived endogenously. The coordination of the development process can be extended to the general case of all the countries wishing to cooperate in the given areas (sectors) of foreign trade. Using the present approach, it is possible to find these areas of international specialization which give the highest possible comparative advantages.

It should be observed that in the literature on the comparative advantages resulting from different growth policies and corresponding terms of trade, much confusion exists. The problem has been especially controversial since a paper was written by J.R. Hicks (1953), in which he had shown that export-biased growth would turn the terms of trade against a country, and that import-biased growth would improve a country's terms of trade. The so-called British school held that the terms of trade would go against the developed countries, while another school (represented by Singer and Prebish) maintained that the terms of trade would more likely have gone against the less-developed countries.

The resulting confusion is presumably due to the complex nature of international growth equations. In the situation when "everything depends on something else", it is possible to obscure and hopelessly entangle the whole issue when one starts to speculate on the possible solutions of nonlinear, many-variables equations in qualitative terms. It is probably better to leave the whole thing to the normative mathematical models and computers, which can tell us in monetary terms who will benefit and how much they will benefit, given the concrete values and constraints.

It can be shown that using a model of trading countries with the production function (which use as inputs the exports from the remaining countries) and observing the balance of payment conditions, it is possible to find a unique system of exchange ratios. That means that M in (23) should be regarded as given exogenously (from the world monetary model) and as a result, the value Y_{zo} is bounded from above. In other words, it is not economically justifiable to decrease M (e.g. by enforcing export) in order to get additional Y_{zo} for factor endowments. Then the only possible way to increase the factor endowments (and the rate of growth) is to take foreign credits. Of course the credits should be paid back (including interest) in the planning interval [0,T].

In order to derive the comparative advantages when using credits, one can apply the aggregated model of Sec.1.2, described by equations (13)-(20), or the simple version (38)-(41). In the last case, the computations are much simpler. Since the credits are usually and most often used for financing the productive investments, we shall assume also m = 2. The optimum strategy of investments, according to (43) of Sec.1.2†

$$\hat{z}(\tau-T) = \frac{\varphi(\tau)}{w(\tau)} \cdot \frac{Y^*}{G} \quad ,$$

where

$$\varphi(\tau) = \left\{ K\omega(\tau)^{-\gamma} \int_{\tau}^{T} e^{\vartheta t-\delta(t-\tau)} \, dt \right\}^{\frac{1}{q}} \quad ,$$

$$G = \int_{0}^{T} \varphi(\tau) \, d\tau \quad , \qquad q = 1 - \alpha \quad ,$$

and Y^* is the solution of the equation:

$$Y = \bar{Y} + G^q \left[\beta^{\beta} (1-\beta)^{1-\beta} \alpha \right]^{\alpha} Y^{\alpha} \quad .$$

† The index $\nu = 2$ can be dropped here.

In Fig. 19 the typical forms of the functions $\hat{z}(t)$, $Y(\hat{z}) = \hat{Y}(t)$ have been shown. It is possible to show that $\hat{Y}(t)$ increases at a faster rate than $Y(\bar{z})$, which corresponds to the strategy $\bar{z}(t) = \gamma Y(t-1)$, i.e. the amplitude constraints or-autarky. That phenomenon occurs even in the case when the total amounts of capital investments (i.e. the areas under $\hat{z}(t)$ and $\bar{z}(t)$) are equal, and it can be explained by the fact that in the inertial system it pays to allocate most of the resources at the beginning of the planning interval [0,T]. That can be done, of course, with the mobile part of capital investments only when foreign credit is used for purchasing investment goods abroad. The credit, together with the interest, should be repaid at $t = T$. That requires that foreign and domestic mobile investment goods can be traded without any restrictions. However, when the GNP increases according to $\hat{Y}(t)$, the part $\tilde{z}(t) = \gamma\hat{Y}(t-1)$ can be used for investments and only the shaded area between the curves $\hat{z}(t) - \tilde{z}(t)$, $t \in [0,T]$, as shown in Fig.19, represents the necessary total credits required. The corresponding area for $t \in [T_1 T]$ represents the domestic "savings" of mobile capital investments which can be traded or used for further investment purposes. Then the net credit value C, which is necessary for optimum development within [0,T], can be written as

$$C = \int_0^{T_1} [\hat{z}(t)-\tilde{z}(t)]w\ (t)dt - \int_{T_1}^{T} [\tilde{z}(t)-\hat{z}(t)]w\ (t)dt \quad . \tag{28}$$

Since the function w (t) increases along with interest rate ε , as shown in Fig.20, the first term in (28) increases at a

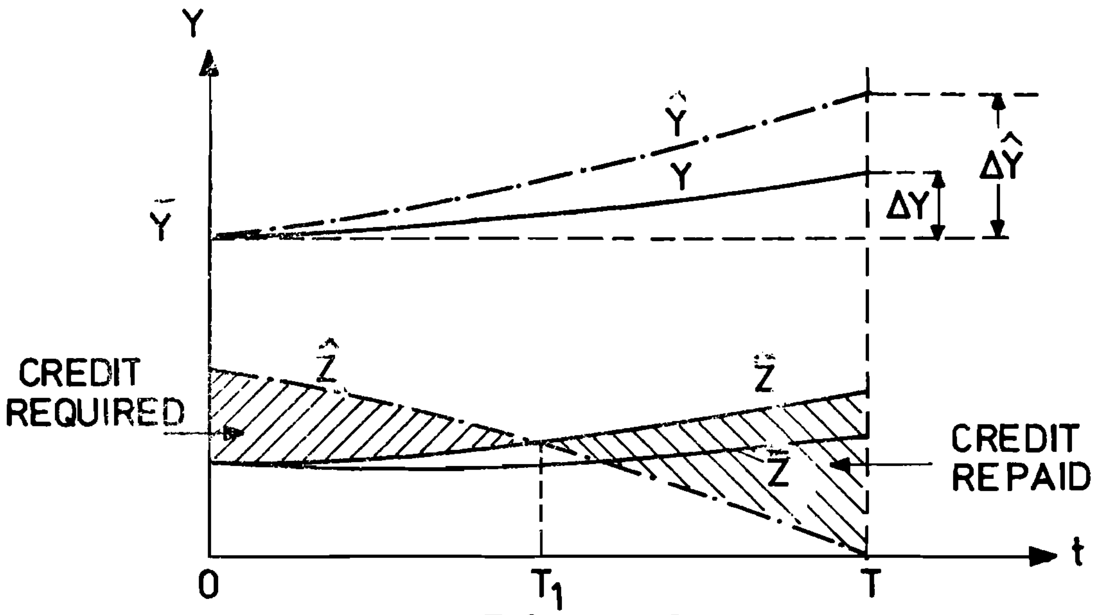


FIGURE 19

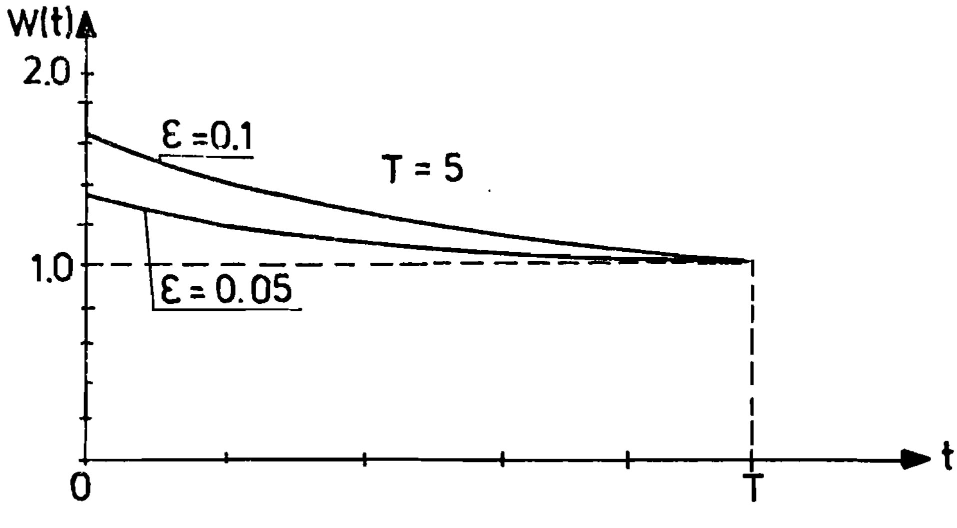


FIGURE 20

faster rate than the second term. As a result, C is an increasing function of ε and the generated within [0,T] increase of GNP (ΔY_C), which takes into account the repayments of credits, becomes

$$\Delta Y_C = \Delta\hat{Y} - C \quad . \tag{29}$$

That value (which is a decreasing function of ε) should be compared with ΔY, corresponding to the development in autarky. For a given planning interval T, there exists a value of credit rate $\bar{\varepsilon}$, such that $\Delta Y_C \leq \Delta Y$ for $\varepsilon \geq \bar{\varepsilon}$ and it does not pay to take credits with $\varepsilon \geq \bar{\varepsilon}$. The value of $\bar{\varepsilon}$ depends on the effectiveness of investments, which in turn depends on the model K, δ, β parameters and prices.

In the planning practice, the following optimization problem is also formulated. What is the minimum value of time $T = T_m$ and credits $C = C_m$, which will guarantee the given development rate $r = \Delta Y/_Y$?

That problem can be solved by finding the required GNP (Y^*) and T_m from the equations:

$$Y = \frac{G^q(T)}{r^{1/q}} \left[\beta^{\beta} \; (1-\beta^{\;\;1-\beta} \;\; \alpha \right]^{\alpha} \tag{30}$$

$$r = \frac{Y - \bar{Y}(T)}{Y} \quad , \tag{31}$$

Then from equation (28) one can find the necessary minimum value of $C = C_m$, and the optimum crediting strategy can be derived

$$C(t) = \hat{Z}(t) - \tilde{Z}(t) \quad , \quad t \in [0,T] \quad .$$

When instead of r the required development rate is defined as $\zeta = \Delta Y/\bar{Y}$ one easily finds $r = \zeta/(1 + \zeta)$.

It is necessary to observe that in the present model, r cannot be an arbitrarily great number. The increase of r increases $\hat{Z}(t) - \tilde{z}(t)$, which represents the mobile part of the investments. In the real economy, the mobile part cannot exceed a certain fraction of the whole investment volume, and as a result, r is bounded from above. The mobile part can be financed out of the credits available at different ε rates at corresponding markets. Since the change of investment strategy also affects the sector price indices, it is necessary to express ΔY, ΔY_C in constant prices. Then the crediting strategy depends on the expected terms of trade T_{zi}^{t}, $t \in [0,T]$.

The present model can easily be extended to the general model of international cooperation of Fig.17, which involves all the development factors. It can easily be shown that the rate of development can be increased when one follows a broad international cooperation program in the factor endowments policy. To what extent that program can be used depends, of course, on the international agreements, which depend in turn on the admissable ratios of mobile to immobile parts of development factors available. Since the general trend in development is a national specialization in production areas and the development plans do not coincide in time, it is often possible to coordinate the plans of exchange of scarce development factors. For example, when a particular country is planning to develop a new branch of industry it can "import" the technology, specialists, etc. at the initial part $[0,T_1]$ of the planning interval in order to "export" them in the interval $[T_1T]$, when the development reaches a sufficiently high level.

IV. Conclusions

As already stated, the main motivation behind the present research was to develop a consistent methodology enabling the construction of long-term, normative development models. That methodology enables in particular the construction of national models, starting with a data base regarding the sectorial or regional activity, and the construction of global models, starting with a national data base.

The normative approach also enables the derivation of national development strategies, which maximize the national goals and satisfy the national aspirations, subject to the constraints which result from the shortages of natural resources and an equilibrium in international relations.

The model then enables the derivation of the system of price indices, which revaluate the resources available, and help to relocate the development factors in an optimum manner. The technological and structural changes take place, and as a result, the utility functional of the optimally planned country increases along with the time horizon. In that sense, no crisis is in sight in the model investigated.

As shown in the present paper, the construction of national and then global models based on the methodology proposed is feasible.

As shown in Ref.[37], a number of MRI models, constructed at the Polish Academy of Sciences, have proved to be exact enough (in the historical runs) to be used for projections of development in Poland.

The methodology used can also be applied for the construction of a development model for C.M.E.A. countries which have similar development objectives and management structure, and which use a similar long-term planning method for allocation of development factors.

The methodology proposed should, however, be extended in the case of countries which use a higher degree of decentralization in their production factor endowments. As shown in Ref. [18], in cases where the sectors (rather than the government) decide how much capital to invest, one can easily extend the proposed methodology. In the case of underdeveloped countries, it is necessary to take into account the additional constraints resulting from shortages in food supply, population explosions, etc.

Since the proposed methodology uses a "bottom-up" technique, the problem of who should construct the national submodels and how the national submodels could be linked together arises. Before answering those questions, it is necessary to observe that in most of the countries, short-term, descriptive econometric models already exist. International cooperation in the area of econometric models resulted in the creation of the project LINK (Ref.[4]), which enables the linkage of national econometric models.

The present problem, then, involves extending the modeling efforts, in order to create a long-term, normative and complex global development model.

It seems that what should be done first is to develop a unified long-term, normative linkage methodology which would be accepted by national model builders. Then the linkage of national models in order to yield a global development model would be comparatively easy.

The role of international institutions such as IIASA is especially important in that respect.

It should also be noted that the aggregation methodology proposed in the present paper enables the decomposition and extension of national models to the sectorial and regional submodels. That approach has already been used in the system of models constructed by the Polish Academy of Sciences, where the aggregated national MRI models are regarded as the "core" models. The core cooperates with the global model (by means of foreign trade, crediting, etc.), and with a number of regional and sectorial submodels, as shown in Fig.21. The structure of sectorial and regional submodels is more complex. For example, in MR1 the agriculture is represented by one sector only, while the extended food and agriculture submodel consists of many sectors which are responsible for the production of grain, meat, etc., and for the food industry. The extended food model cooperates with the core by receiving information regarding the GNP and investment endowments, as well as the intersector flow of fertilizers and other chemicals used in food production, agricultural machinery, fodders, etc. It sends information to the core

THE SYSTEM OF MODELS

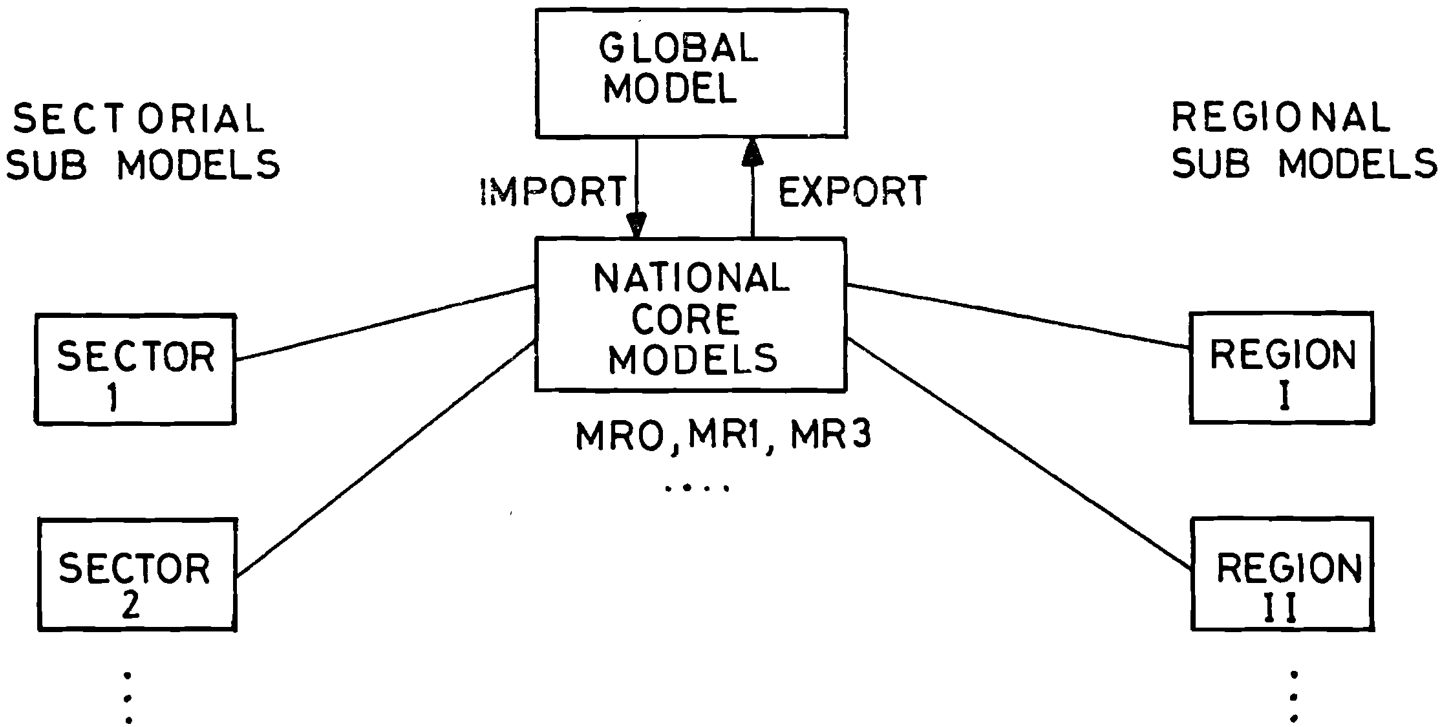


FIGURE 21

regarding the food supply structure, export and import demands, etc.

By using regional and sectorial models, and cooperating with the core model, it is possible to extend the main planning and decision area to the needs of low-level or local decision units. However, the central decision system obtains more information regarding local problems and aspirations.

As a closing observation, it is necessary to notice that the modeling of complex national and global development is a long process of successive approximations of the constantly changing reality. The approximations take place in time and space. Scientists all over the world are trying to contribute to the final result, which is a clear path of development leading to global welfare.

The present paper should be regarded, therefore, as a small step in the long approximation process. There is still much to be done before we have a good model of national and global development.

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