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Short-Term Planning of an Integrated Industrial Complex

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1. Introduction

The design and construction of modern integrated industrial complexes involve consideration of a great number of variables such as location, capacity, energy, economy, etc., and a number of respective calculations as well. The variables embrace the following problems:

- resource distribution tasks,

- choice of the equipment types and capacities,

- transport problems, and others.

Modern integrated industrial complexes represent a compilation of separate production divisions or integrated units which are destined to perform the successive-parallel operations.

The characteristic features of the large-scale integrated complexes include great flows of raw material, energy, final products and information. The complicated and expensive equipment included in such integrated complexes demands the perfect organization of the work, since even small inaccuracies in the design and control of the large-scale integrated complexes involve great losses of production, quality and so on.

The effectiveness of the work of these complexes is defined, to a considerable extent, not by the effectiveness of the work of the separate production divisions, but by the mutual job. This is a main characteristic feature of most of the integrated industrial complexes in the chemical, machine-building, and metallurgical industries.

As a criterion of the optimal work of the large-scale complexes, a certain summarized index, which characterizes the work of the whole complex, should be given. Sometimes such a criterion is considered to be the total idle time of all the units. In some cases this criterion proves to be insufficiently common as it does not fully characterize the mutual actions of the separate units and production divisions. A more common criterion can be the productivity of the whole industrial complex. In this case the task can be considered as minimization of the time for performing a given number of production cycles or maximization of production cycles during a given time.

Let us consider one of the possible approaches to the solution of the problems of such types.

2. Problem Statement

Although the general project scheduling problem under limited resources remains unsolved for practical-sized problems, some simplified approaches can be suggested for the rough investigation of the problem. One of these schemes is considered in this paper. It allows one to construct a rational (possibly non-optimal from a strictly mathematical standpoint) schedule through man-machine interaction.

The machine sequencing problem, which has received considerable exposure in the literature, is a special case of general project scheduling. In this case the precedence network has a special structure so that all the activities (other than what may be first and last dummy activities) have exactly one predecessor and one successor (see Figure 1). In addition it is assumed that each activity requires only one unit of one type resource and that all requirements and availabilities are constant.

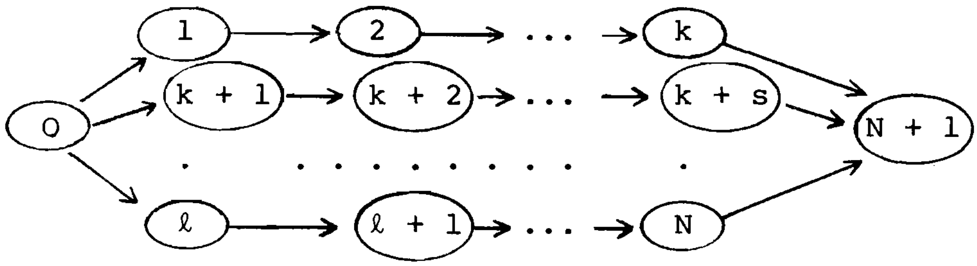

Figure 1.

Nevertheless, the assembly line balancing problem is closely related to the project scheduling problem, since it can be represented on a similar network, although the form of the resources constraints may be quite different.

In this paper we propose a method for solving the problem. It is based on the extending of the procedures originally proposed for the project scheduling problem (Zimin [2]). Note that our method is based on the control theory approach and differs very much from heuristic methods treated in Levy [1] and other approaches in which the problem is stated as a set of linear (integer) inequalities and approaches which deal with the combinatorial problem directly.

We consider the problem described by the following conditions:

A) A set of S jobs must be performed. The j-th job consists of n_j tasks numbered from 1 to n_j. The maximal time to perform each task is a known integer represented by T_{ij} for the i-th task of the j-th job.

B) A set of K different resources is available. R_k is the amount of the k-th resource available in any time. The amount of the k-th resource required by the task ij during its processing is r^k_{ij}. For example, the resources correspond to the machines of a job shop, and each task requires only a single machine during the interval of its processing.

C) No preemption of task performance is allowed. Once the task ij is started, it must be processed until completed in no longer than T_{ij} time units and no less than $\underline{T}_{ij}$ time units.

D) The start times of the tasks on a given job are constrained by a cycle-free network of the CPM-PERT type (see Figure 1).

E) We are required to find the values for t^o_{ij} (start time), $j = 1,\ldots,n_i$, $i = 1,\ldots,S$ which satisfy conditions A-E and for which the total number of jobs (or tasks) completed during a given period of planning T is maximal.

The "dual" problem of minimizing the completion time of a given set of jobs can be stated. Here we use the finite-difference equation to describe the model with:

the dynamic equation:

$$x^{ij}(t+1) = x^{ij}(t) + u^{ij}(t)\theta\,(1 - x^{i-1,j}(t)) \quad , \qquad (1)$$

where

$x^{ij}(t) \equiv$ portion of the ij-th task performed to the moment t. It could be interpreted as a portion of the total time (T_{ij}) the task required for its performance until the moment t;

$u^{ij}(t) \equiv$ performance intensity or portion of the ij-th task has been completed within $[t - 1, t]$ period;

$$\theta(y) = \begin{cases} 0, & \text{if } y < 0 \\ 1, & \text{if } y \geq 0 \end{cases} .$$

By introducing the multiplier θ to the right-hand sides of the equations, we take into account the precedence relations (see point D) with:

the initial conditions:

$$x^{ij}(0) = 0 \quad . \tag{2}$$

We assume that the ij-th task be completed if $x^{ij}(t) = 1$. Thus, we have the following constraints for every t:

$$x^{ij}(t) \leq 1 \tag{3}$$

and natural constraints for $u^{ij}(t)$

$$u^{ij}(t) \geq 0 \quad . \tag{4}$$

All relations formulated are held for:

$$i = 1,2,\ldots,n_j \quad ; \quad j = 1,2,\ldots,S \quad ; \quad t = 0,1,\ldots,T \quad . \tag{5}$$

In addition, for the final (dummy) task OS + 1 we have:

$$x^{OS+1}(t + 1) = x^{OS+1}(t) + u^{OS+1}(t) \prod_{j=1}^{S} \theta(1 - x^{n}ij(t)) \quad , \tag{6}$$

$$x^{OS+1}(0) = 0 \quad , \tag{7}$$

$$x^{OS+1}(t) \leq 1 \quad , \tag{8}$$

$$u^{OS+1}(t) \geq 0 \quad . \tag{9}$$

Resource constraints (see point B) are:

$$\sum_{i,j} r_{ij}^{k} u^{ij}(t) \leq R^{k} \quad , \quad i \neq 0 \quad , \quad j \neq S+1 \; ,$$
$$k = 1,2,\ldots,K \quad . \tag{10}$$

There are no preemption constraints (see point C):

$$u^{ij}(t) \geq \frac{1}{\underline{T}_{ij}} \theta(x^{ij}(t)) \; \theta(1 - x^{ij}(t)) \quad . \tag{11}$$

We define (see point E) t_{ij}^{o} as the first moment when the following condition is held:

$$u^{ij}(t) > 0 \quad .$$

Some limitations to the maximum performance intensity have to be present (see point C):

$$u^{ij}(t) \leq \frac{1}{\overline{T}_{ij}} \quad .$$

According to point E, the objective function could be written in the form of:

$$I(u) = \sum_{j=1}^{S} x^{n_j j}(T) \quad , \tag{12}$$

where T is a given integer.

Thus, the problem could be stated as

$$\text{Max } I(u) \quad ,$$

$$\text{s.t. } (1) - (11) \quad .$$

To solve this problem we used a modification of the method described in [2].

Note that the model could incorporate some additional factors, for example, that the job should perform without interruption. Once job j is started, all its tasks must be processed without a time lag between the finish time of the predecessor and start time of the successor. The resource consumption and supply dependence of time also could be considered, etc.

3. Complex Oxygen Processes--Continuous Casting Model (particular case)

As an example, an industrial system consisting of two complexes, oxygen-converters and continuous casting machines for steel production, is considered in this work. Such systems are considered to be effective and are being installed all over the world. According to a number of forecasts, steel production will be based mainly on these processes in the near future.

The task of the oxygen-converter complex is the production of steel of a given composition and temperature. Cyclic production is characteristic of these complexes. There are some process and production control systems being developed for implementation in this type of complex. In 1973, about eighty of these systems, using mini- and average computers, were functioning around the world.

The tasks of such systems include:

- process control,
- production control,
- data collection for statistical research, calculation, etc.

The production control systems are not included in the integrated control system of the enterprise as a rule.

The task of the continuous casting machine complex is the continuous casting of steel in slabs of given dimensions. Control systems for continuous casting mills have been developed less considerably.

When scheduling the mutual work of BOP - CC, there is the problem of choosing the rhythm for all the work in which productivity is maximal. In other words, the frequency of the heat preparation in the oxygen-converter complex should correspond to the productivity of the continuous casting machine complex.

In accordance with the given steel grade, productivity possibilities of each complex, energy expenditures and other distributions are changed. In this case the problem of operation-state scheduling arises.

In a melting and continuous casting process, every job consists of three tasks: melting, preparing for casting and casting itself. This network diagram is shown in Figure 2.

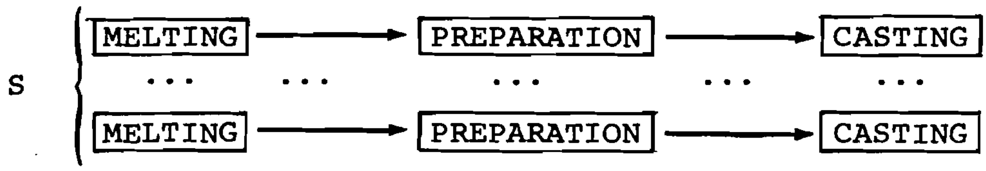

Figure 2.

Converters for the melting and casts for the casting are considered here as resources. We formulate the model by using a single index:

dynamic equations:

$$x^{3k-2}(t+1) = x^{3k-2}(t) + u^{3k-2}(t) \quad \text{(melting)} \quad ,$$

$$x^{3k-1}(t+1) = x^{3k-1}(t) + u^{3k-1}(t)\ \theta(x^{3k-2} - 1) \quad \text{(preparation)} \quad ,$$

$$x^{3k}(t+1) = x^{3k}(t) + u^{3k}(t)\ \theta(x^{3k-1} - 1) \quad \text{(casting)} \quad ,$$

$$k = 1,2,\ldots,S \quad , \quad \text{and} \qquad (13)$$

resource constraints:

$$\sum_{k=1}^{S} a_k\ u^{3k-2}(t) \le m \quad ,$$

$$\sum_{k=1}^{S} b_k\ u^{3k}(t) \le n \quad , \qquad (14)$$

where

$a_k \equiv$ time required to complete the $(3k - 2)$-th melting;

$b_k \equiv$ time required to complete the $3k$ - th casting;

$c_k \equiv$ time required to complete the $(3k - 1)$-th preparation for the $3k$-th casting;

m ≡ the number of converters;

n ≡ the number of casts.

We assumed that a preparation for castings does not require any resources.

We have no preemption constraints:

$$u^{3k-2}(t) \geq \frac{1}{a_k}\,\theta(x^{3k-2}(t))\;\theta(1 - x^{3k-2}(t)) \quad ,$$

$$u^{3k-1}(t) \geq \frac{1}{c_k}\,\theta(x^{3k-1}(t))\;\theta(1 - x^{3k-1}(t)) \quad , \qquad (15)$$

$$u^{3k}(t) \geq \frac{1}{b_k}\,\theta(x^{3k}(t))\;\theta(1 - x^{3k}(t)) \quad ,$$

$$k = 1,2,\ldots,S.$$

Constraints to the maximum performance intensity are:

$$u^{3k-2}(t) \leq \frac{1}{a_k} \quad ,$$

$$u^{3k-1}(t) \leq \frac{1}{c_k} \quad , \qquad (16)$$

$$u^{3k}(t) \leq \frac{1}{b_k} \quad ,$$

$$k = 1,2,\ldots,S.$$

Eqs. (15) and (16) indicate that for all tasks intensity could be equal to 0 or correspondingly $\frac{1}{a_i}$, $\frac{1}{c_i}$ and $\frac{1}{b_i}$.

The objective function is:

$$I(u) = T \quad ,$$

where T is the time of completion of all the jobs.

4. A Numerical Example

We have done some computational experiments with the model presented. An additional assumption is that no preemption of the job performance is allowed. The results are shown below. The network diagram is given in Figure 3.

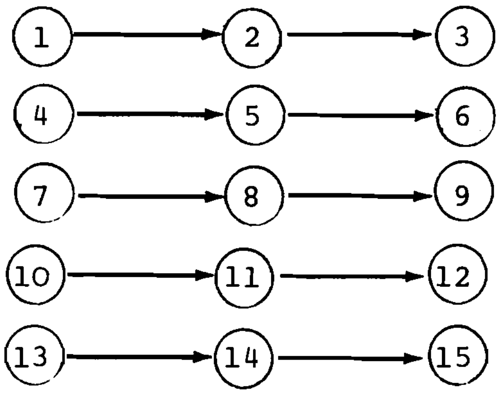

Figure 3.

Thus, S = 5, the tasks 1, 4, 7, 10, 13 correspond to the meltings; the tasks 2, 5, 8, 11, 14 are preparations for the castings; and the tasks 3, 6, 9, 15 correspond to the castings (see Figures 4 and 5). Table 1 shows the time durations for every task.

Table 1. Input data (durations of the tasks in minutes).

Job number i		Task number i		Task number i		Task number i
1	40.	(1)	25.	(2)	40.	(3)
2	35.	(4)	25.	(5)	40.	(6)
3	30.	(7)	25.	(8)	40.	(9)
4	45.	(10)	25.	(11)	40.	(12)
5	50.	(13)	25.	(14)	40.	(15)

The task numbers (in brackets) correspond to Figure 3.

VARIANT i (m = 2 , n = 3)

$T^* = 170$

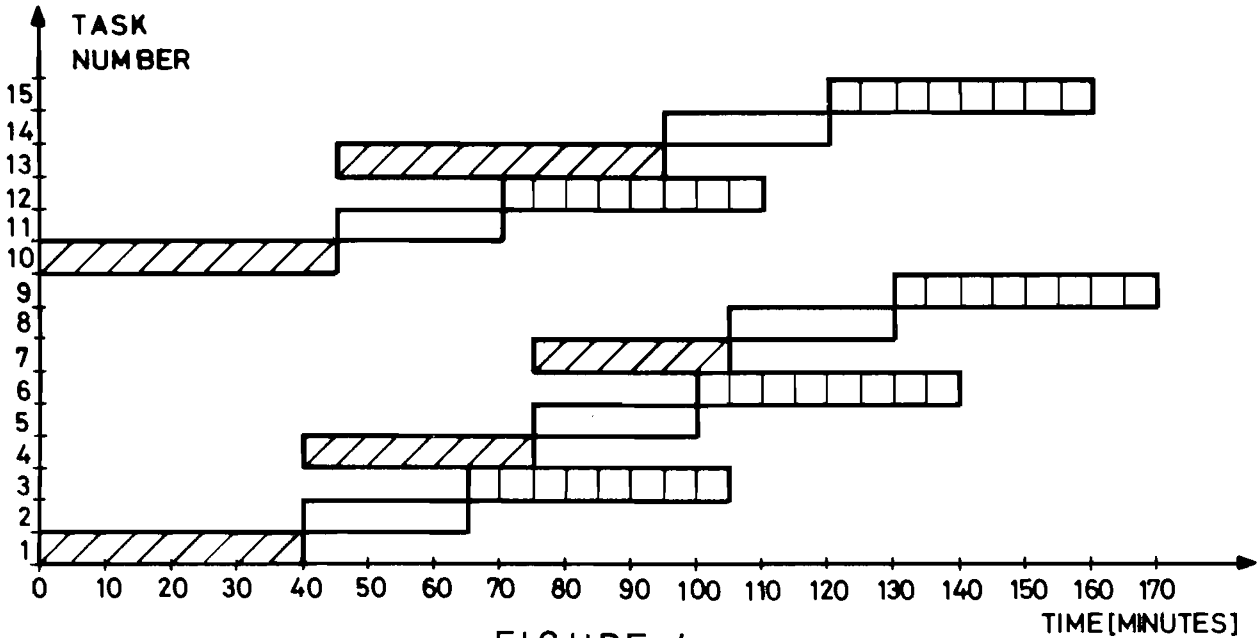

FIGURE 4

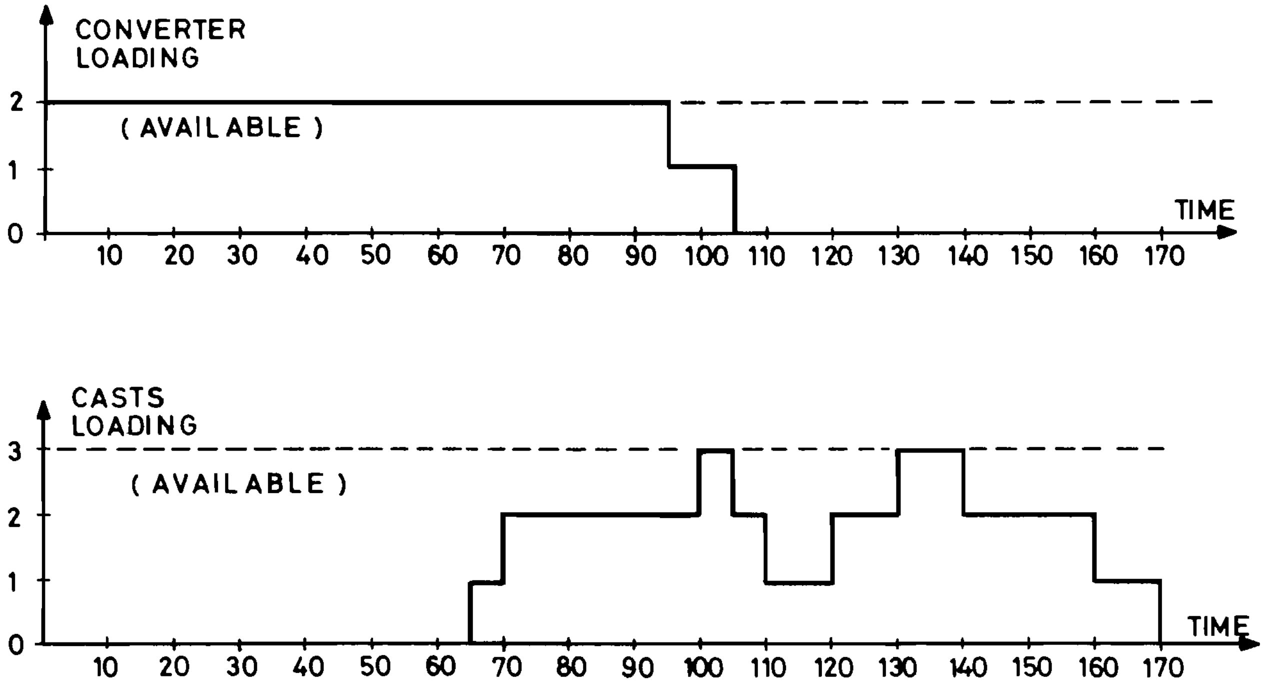

FIGURE 5

$T^* = 170$

The following variants have been computerized and examined (Table 2).

Table 2. Input data (resource supplies).

Variant number	Converters	Casts
i	2	3
ii	3	4
iii	4	4
iv	5	2

Ghant diagrams which correspond to the optimal solution of the problem and resource loadings are shown in Figures 4-13.

The construction of the surface $T^*(m, n)$ would help the management of a plant or a job shop to select the appropriate amount of facilities from the available set and to deal with the input-payoff analysis very effectively.

The surface (or table) $T^*(m, n)$ could be easily constructed on the basis of the problem solution for all the m and n. In our case the total number of these calculations is equal to twelve. Each calculation required about 0.1 sec of the CP time on the CDC-6600 computer.

Note that the solution of the problem (T') with the constraints (15) removed gives us the low boundary to the length of the optimal schedule for the initial problem (T*). That is

$$T^* \geq T' \quad .$$

In this variant the low boundary has been achieved. The solution with the constraints (15) removed is shown in Figures 6 and 7. This solution (Figure 6) is not acceptable for our initial problem due to the violation of the constraints mentioned in point C (see Figure 8 and Figure 9).

In this case we could conclude that three casts will be enough to complete the whole job within the same time: that is, the

T' = 170

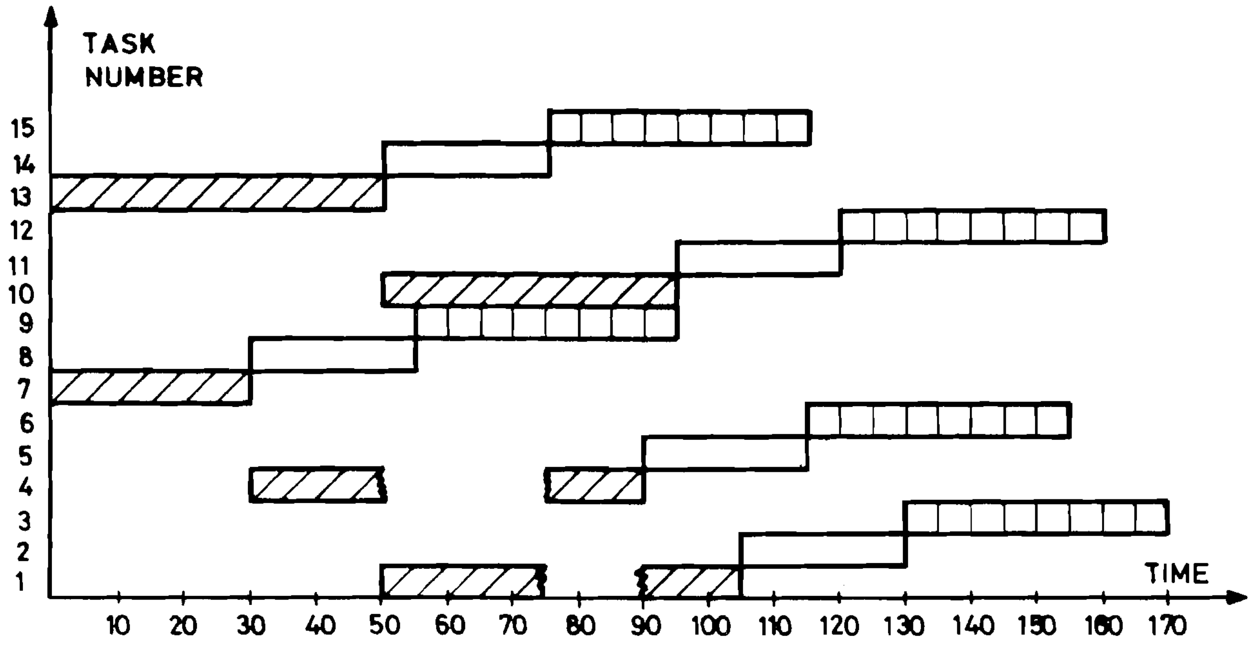

FIGURE 6

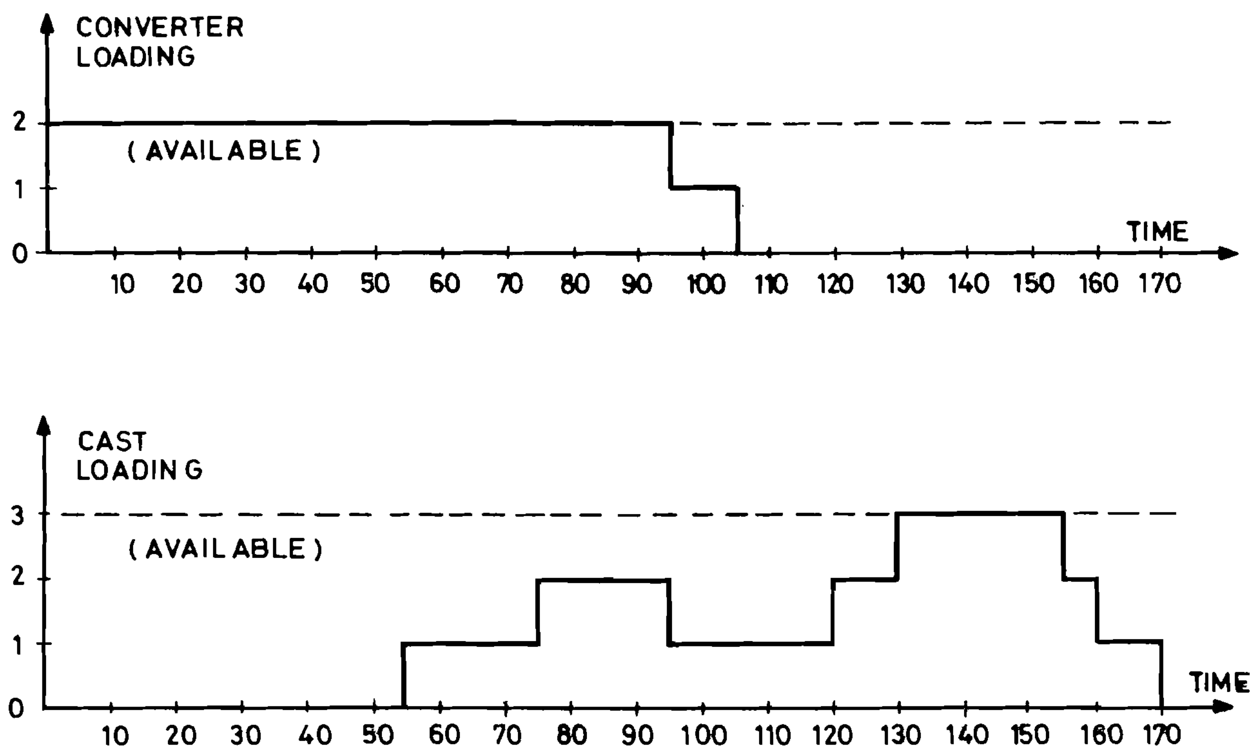

FIGURE 7

VARIANT ii (m = 3, n = 4)

$T^* = 145$

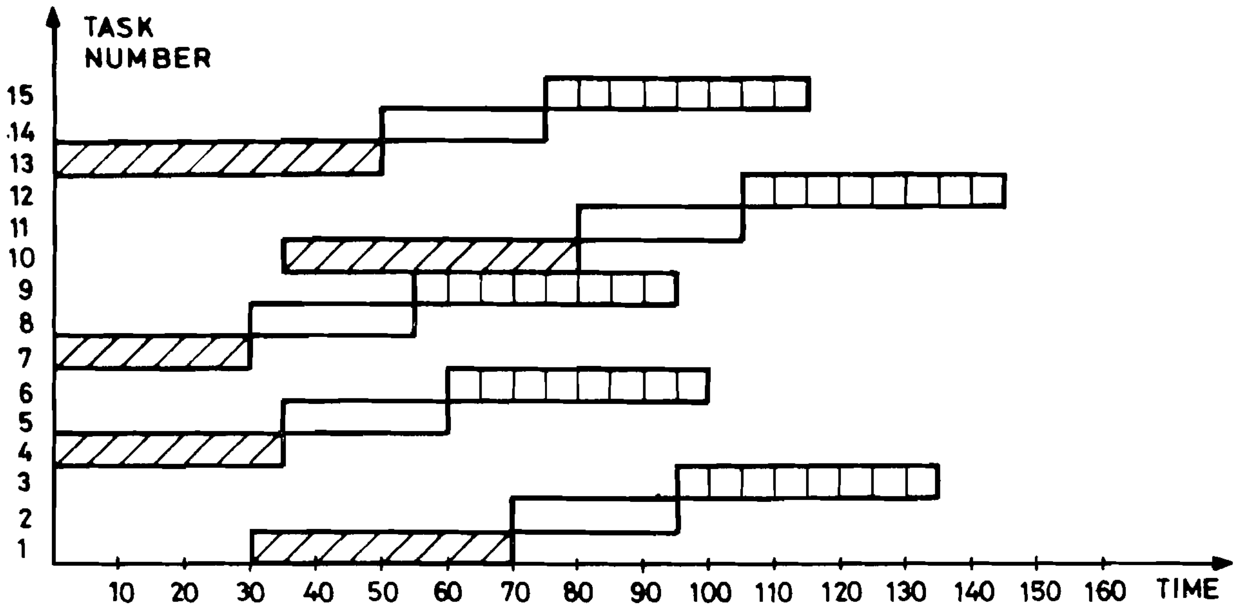

FIGURE 8

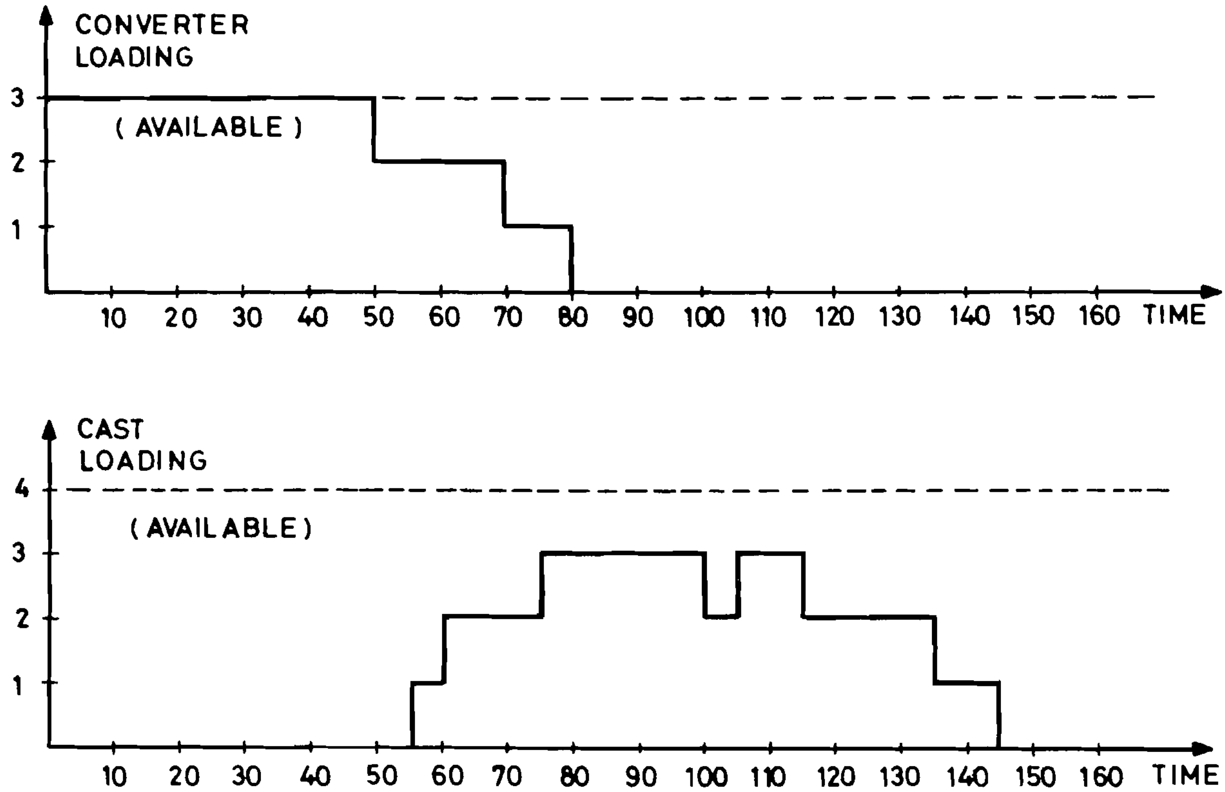

FIGURE 9

optimal number of casts for the given number of converters is equal to three (see Figure 10 and Figure 11).

In this case the increase in the number of casts will not lead to a time decrease. An increase in the number of converters (to five) will decrease the time to T* = 135. The simultaneous increase of the number of converters and casts to five will lead to the time decrease T* = 125. The latter is the minimum possible time in which the given number of jobs can be completed. It corresponds to the case in which all the resources are unlimited (see Figure 12 and Figure 13).

Conclusion

The examples of the tasks we have considered do not cover all the numerous problems which arise in operation production processes planning. We propose to continue our job of trying to find the general principles of control of the integrated production complexes of the same kind.

Acknowledgment

We are grateful to Prof. T. Koopmans and Prof. A. Cheliustkin for their discussions and useful advice.

VARIANT iii (m = 4, n = 4)

$T^* = 140$

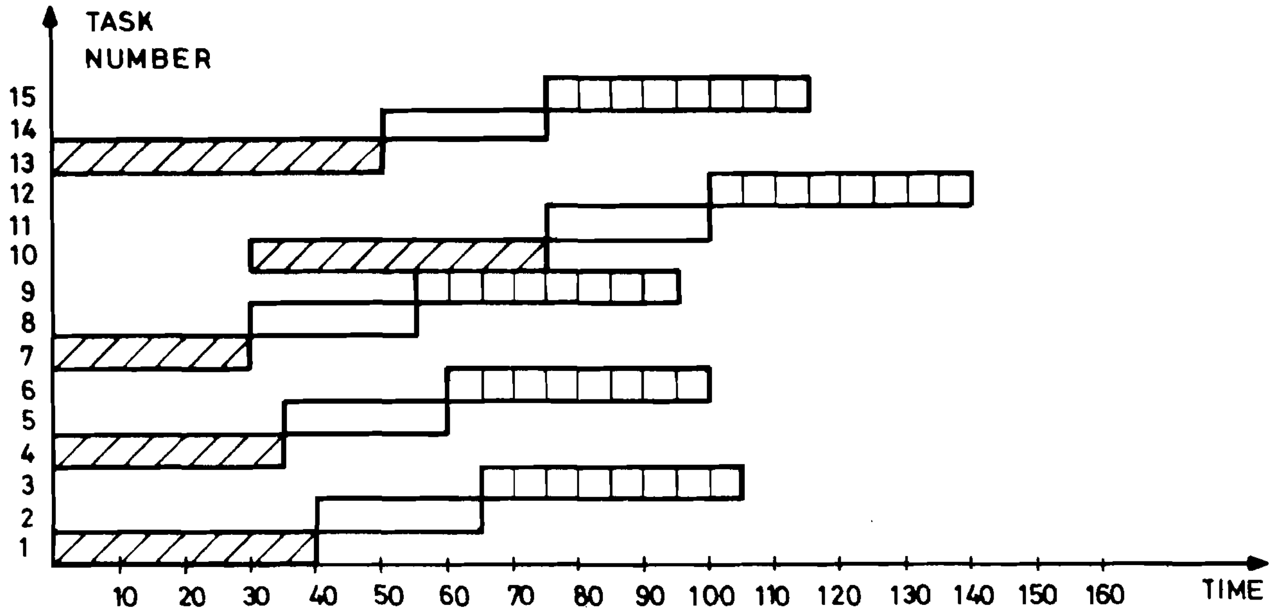

FIGURE 10

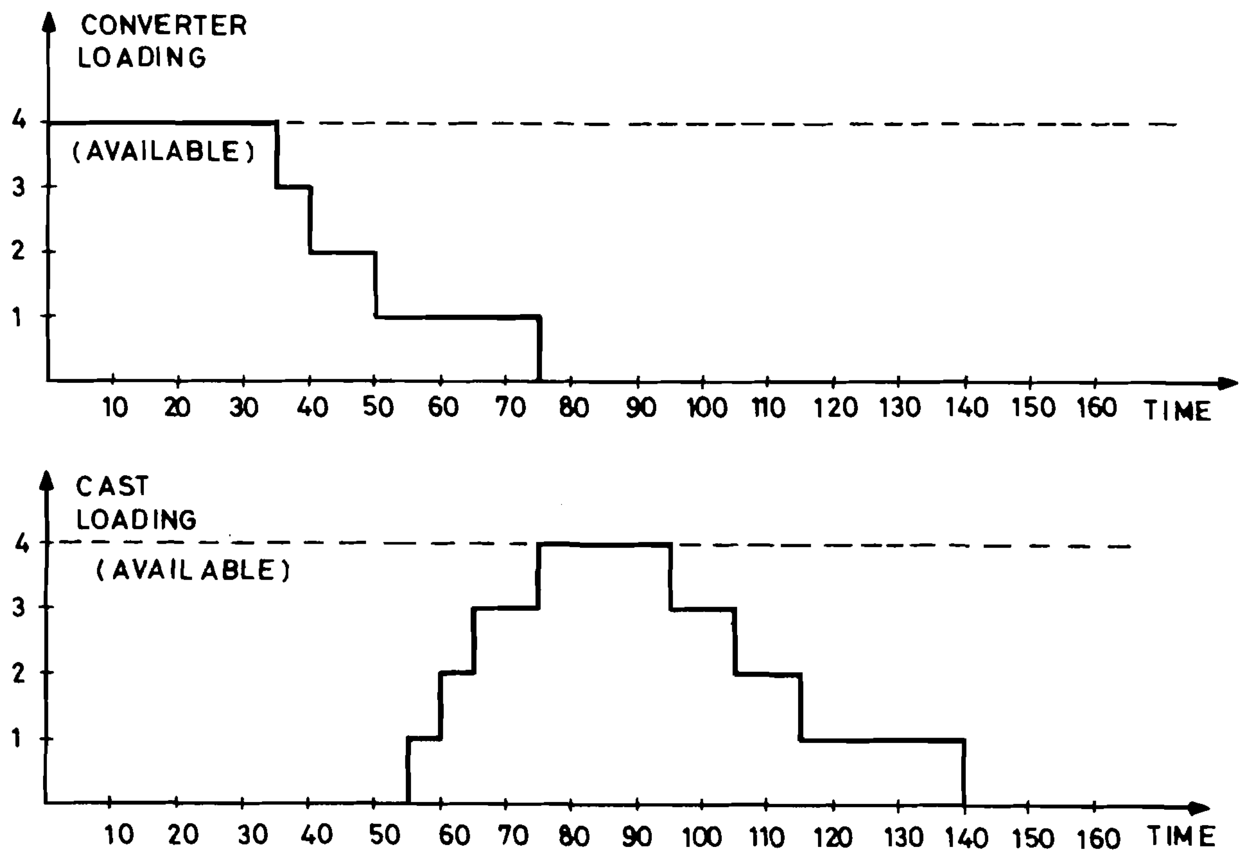

FIGURE 11

VARIANT iv (m = 5, n = 2)

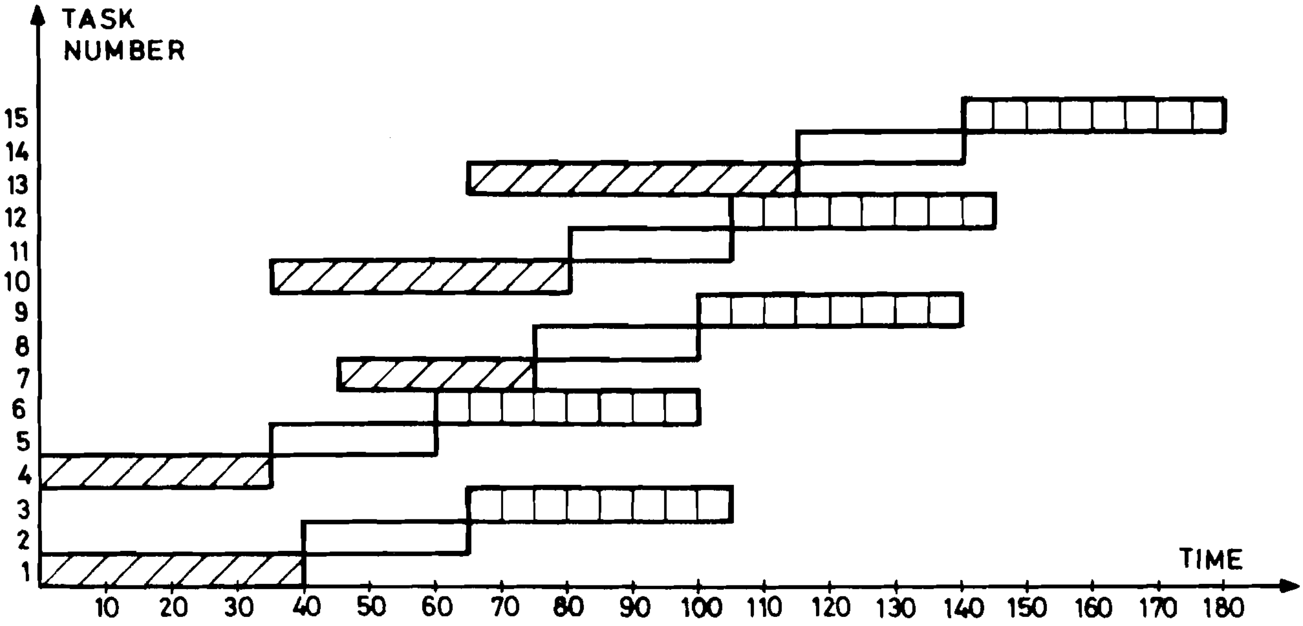

FIGURE 12

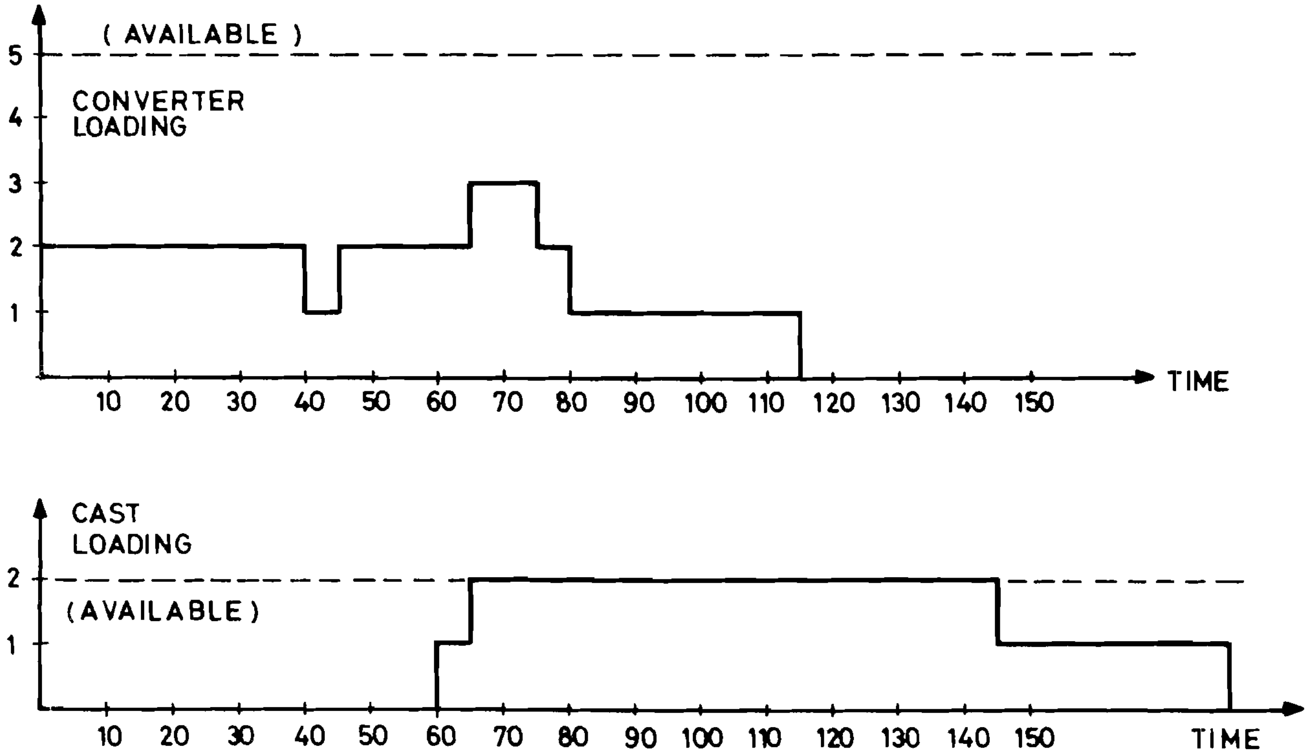

FIGURE 13

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[2] Zimin, I. "Large-Scale Scheduling Problems in Long-Range Planning." Internal paper, January 1975.