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I. Introduction

In a recent paper, G. Dantzig [2] has formulated a model for resource allocation in the so called "Divvy Institutional Economy". The author proves the existance of a general equilibrium solution to the economic problem (in terms of prices and quantities of input factors and final goods) which at the same time satisfies agreed upon shares of monetary flows allocated to input resource groups and to output consumer groups. The agreement upon the share values is carried out by a political process, while the market mechanisms adjust the prices of primary resource inputs and the relative sizes of the consumer groups until those shares are satisfied. The inputs and outputs and the production and transformation technology are presented in an Input-Output format.

The formulization of the resource allocation problem takes into account the presence of institutionalized forces together with the market mechanism. Examples can be taken from empirical observation (collective bargaining, Congressional Budget Approval, indexed prices of raw material) is per se a major innovation with respect to more classical results. In the following sections we we will try to view the Divvy results in relation to the classic economic formulation of the problem and study possible implications of it.

The general framework will be the welfare maximization problem for the economy and the general equilibrium conditions that are derived from it. The model that is going to be the background of our exposition is presented in the Appendix 1 following a classical formulation. The consumer and producer sectors of the Divvy Economy will be reformulated in the light of the behavioral models presented there.

We will try to maintain the notations in [2]. We will assume $1,\dots,r$ resource groups and $1,\dots,s$ consumer groups. There are $1,\dots,n$ economic sectors which buy resources x_i $i = 1,\dots,r$ to produce final goods y_k, $k = 1,\dots,n$. p_k, $k = 1,\dots,n$ will represent prices of final goods and and λ_i, $i = 1,\dots,r$ prices of primary resources; the relative sizes of the groups will be given by μ_j, $j = 1,\dots,s$.

2. A Welfare Maximization Problem Compatible with the Divvy Economy

Consider the following optimization problem

$$\max_{x,y\geq 0} \quad U = \prod_{j=1}^{s} \prod_{k=1}^{n} (y_{jk}\mu_j)^{\beta_{jk}} = \prod_{j=1}^{s} \left(\prod_{k=1}^{n} y_{jk}^{\beta_{jk}} \right) \mu_j^{\sum_k \beta_{jk}}$$

s.t.

$$\sum_{j=1}^{s} y_{jk}\mu_j \leq q_k \qquad \forall\ k \tag{2.1.1}$$

$$q_k = \prod_{i=1}^{r} x_{ik}^{\alpha_{ik}} \qquad \forall\ k \tag{2.1.2}$$

$$\sum_{k=1}^{n} x_{ik} \leq h_i \qquad \forall\ i \tag{2.1.3}$$

The objective function is a welfare function for the society where y_{jk} is the quantity of final good k consumed by a member of group j, and μ_j is its relative size. (2.1.1) is an availability constraint for each final good, and (2.1.2) represents the technology producing k. (2.1.3) is an availability constraint in primary inputs. Problem (2.1) does not differ significantly from the classical welfare maximization problems [3]. The only innovation is to consider the groups rather than single individuals in the index of welfare that has been chosen. If p_k is the multiplier in the combined constraints (2.1.1) and (2.1.2) and λ_i is the multiplier in (2.1.3), we can write the two behavioral models associated with (2.1).

The consumer utility maximization sub model would be

$$\max U = \prod_{j=1}^{s} \left(\prod_{k=1}^{M} y_{jk}^{\beta_{jk}} \right) \mu_j^{\sum_k \beta_{jk}} \tag{2.2}$$

s.t.

$$\sum_k \sum_j p_k y_{jk} \mu_j = 1$$

where 1 is a normalized value for the income of the society. In the Divvy Economy the y_{jk} are considered given as somehow "typical" consumption patterns for the groups, and in that case the optimization is carried over μ_j. (2.2) converts then to

$$\max_{\mu \geq 0} U = k_1 \prod_{j=1}^{s} \mu_j^{\beta_j} \tag{2.3}$$

s.t.

$$\sum_k \sum_j p_k y_{jk} \mu_j = 1 \tag{2.3.1}$$

where $\beta_j = \sum_{k=1}^{n} \beta_{kj}$ is a parameter. $\beta_j \geq 0$, $\sum_{j=1}^{s} \beta_j = 1$.

If v is the Lagrange multiplier of (2.3.1), the necessary conditions given, after solving for $v = k_1 \sum_{j=1}^{s} \mu_j^{\beta_j}$.

$$\sum_{k=1}^{M} p_k y_{jk} \mu_j = \beta_j \quad \forall j \tag{2.4}$$

together with $\mu_j \geq 0$ and (2.3.1).

Similarly, the producer efficiency maximization sub model would be,

$$\min_x \sum_{i=1}^{r} \alpha_i x_i \tag{2.5}$$

s.t.

$$y \leq \prod_{i=1}^{r} x_i^{\alpha_i} \tag{2.5.1}$$

(2.5) is actually an aggregated model of the index of output across sectors, i.e., (2.5.1) would actually be written as

$$y = \prod_{k=1}^{n} y_k^{\beta_k} = \prod_{k=1}^{n} \prod_{i=1}^{r} x_{ik}^{\alpha_{ik}} = \prod_{i=1}^{r} x_i^{\alpha_i}$$

with $\alpha_{ik} = \beta_k \alpha_i$ and $\sum_{i=1}^{n} \alpha_i = 1$, $\alpha_i \geq 0$.

Since the Divvy Economy takes λ as the variables, we want the equivalent problem to (2.5) in terms of λ. This is provided by the "dual" problem of (2.5). The concept of duality and the specific dual for problems of the form (2.5) are treated in [4].

$$\min_{\lambda} \sum_{i} \lambda_i x_i \tag{2.5)'$$

s.t.

$$k_2 \prod_{i=1}^{n} \lambda_i^{\alpha_i} \geq 1 \tag{2.5.1)'$$

where $k_2 = \prod_{i=1}^{n} \alpha_i^{-\alpha_i} y$, and $\sum_{i} \alpha_i x_i^* = k_2 \prod_{i=1}^{n} \lambda_i^{\alpha_i}$, for x_i^* solving (2.5). (2.5.1)' scalled to 1 since $\sum_{i} \lambda_i x_i = 1$.

The first order condition to (2.5)' gives

$$\frac{\lambda_i x_i}{\sum_{i=1}^{n} \lambda_i x_i} = \lambda_i x_i = \alpha_i \qquad \forall\ i \tag{2.6}$$

or in more disaggregated form,

$$\lambda_i x_i = \sum_{k=1}^{n} \lambda_i x_{ik} = \sum_{k=1}^{n} \alpha_{ik} = \alpha_i \qquad \forall\ i \tag{2.7}$$

Equations (2.4) and (2.7) together with the accounting conditions that industry makes zero profit, $\sum_{i=1}^{r} \lambda_i x_{ik} = \sum_{j=1}^{s} p_r y_{jk} \mu_j$ and aggregate value of inputs equal aggregate value of outputs, characterize the general equilibrium conditions. Since p_r, y_{jk}

and x_{ik} to achieve y_{jk} are assumed given, the conditions are stated in terms of λ and μ. Moreover, by computing the value x_{ij} across final sectors k, (2.3.1) could be written as

$$\sum_k \sum_j p_k y_{jk} \mu_j = \sum_i \sum_j \lambda_i x_{ij} \mu_i = 1$$

where x_i in (2.5) would be equal to $\sum_{j=1}^{s} x_{ij}\mu_j = x_i$.

Using these results, (2.4) and (2.7) would be now

$$\sum_{i=1}^{r} \lambda_i x_{ij} \mu_j = \beta_j \qquad \forall\, j \tag{2.8}$$

$$\sum_{j=1}^{s} \lambda_i x_{ij} \mu_j = \alpha_i \qquad \forall_i \tag{2.9}$$

(2.8) and (2.9) are equivalent to (18) in [1.p.12] for $\beta_j = \delta_j$ $\alpha_i = \lambda_i$ and $M_{ij} = x_{ij}$.

3. Critique of the Model

In the previous section we have shown that the behavioral models of consumer utility maximization and producer efficiency maximization in the way they are formulated in classical economic theory, under assumptions similar to those in Divvy Economy (fixed consumption pattern for each group), give results that are consistent with the ones implied by the Divvy Economy in terms of solutions satisfying general equilibrium conditions. However, the behavioral assumptions of both models are very differant and would call for differant understandings of the economic problem. The decision on which model truly represents the actual behavior is difficult to make because the shares of flows would have to be observed from empirical results and either model could claim that were generated under its assumptions.

But apart from this ambuiguity, the models deserve other comments on their assumptions. Divvy Economy, we think, makes a valuable and justifiable point when arguing that the shares of flows are affected by the political process, something which in the classical economic result is not very often considered.[1] Nothing is said however, about the dynamics of the political process and what social pressures actually determine those shares. Rather, by introducing a new variable, the adjustable size of the groups, the system has enough degrees of freedom to minimize the effects of political decisions in the economic sector, together with the fact that the shares are stated in monetary terms and not in real ones.

Observing the functioning of the political process, one can claim that there exist social groups bargaining for the shares of the outcome of the economy, in real terms. The groups are rather fixed and very often have strict control of membership to maintain their competitive advantage. To distinguish between resource groups and consumer groups is difficult since, except for retired people, the rest participate directly in the production process (in a broad sense), and a good part of their share is determined already by the remunerations for their contribution. The groups that do not feel satisfied by the strict economic share that marginal productivity criteriums would assign to them, make use of the political system and force redistributive actions by the Government. An example would be the "income policy" whose aim is to achieve income redistribution through taxation, an indirect consequence of the policy is its contribution to political stability by reducing social differences. Another example at the international level would be to aid programs of the developed countries in favor of the less developed ones, while a manifestation of the power of the resource groups is the ability of the OPEC countries to control the price of oil. In this last case note however, the concern of those countries in changing the prices so that their share of flows is always maintained in real terms.

The previous analysis suggests new formulations of the resource allocation problem. Although technological and economic

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[1] The introduction of budgetary constraints for the Government sector in some models [1] could be interpreted as an example of this consideration.

relationships must not be put aside, the political power of the social groups in controlling the economic resources and in the decisions about the selections of a particular technological option need to be better understood.

Appendix

Formulation of a General Equilibrium Problem Consistent with our Behavioral Models

Given a vector of final goods y produced by a technological relationship of the form $y = F(x)$, where x is a vector of inputs available in quantities h, for a utility function on y, $U(y)$, the problem is[2]

$$\max_{y} u(y)$$
$$\text{s.t.} \quad y \leq F(x) \qquad \text{A1}$$
$$x \leq h$$

The solution to A1 will satisfy the Pareto condition that $u(y^{*}) \geq U(y)$ for all feasible y.

If p and λ are the vector dual variables associated with the first and second constraint in A1, the consumer utility maximization problem subject to the budget constraint

$$\max_{y} U(y)$$
$$\text{s.t.} \quad py = M \qquad \text{A2}$$

and the producer efficiency maximization (cost minimization) problem

$$\min_{x} \; x$$
$$\text{s.t.} \quad y \leq F(x) \qquad \text{A3}$$

will provide general equilibrium conditions consistent with A1 and consequently satisfying the Paretian condition. Problems A2 and A3 are called for this reason Paretian rules [3].

[2] U and F are continuous concave twice differentiable functions.

The first order conditions are

$$U1_y - vp = 0 \qquad \text{(A.2.1)}$$

$$\lambda - pF^1x = 0 \qquad \text{(A.3.1)}$$

where v is the marginal utility of income or scaling factor that converts monetary output of the society into welfare measures in utils. F(x) is assumed homogeneous of degree one, which means that at optimal $py = \lambda x = M$. A general equilibrium solution is the vectors y, x, p, λ satisfying (A.2.1) and (A.3.1).

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