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Abstract

This paper examines the contribution that three simple migration models can make towards a fuller understanding of the migration process. The models employed are a Kinematic model, a Markov Chain model and a Modified Markov model. Their capacities to reflect trends inherent in migration matrices from England and Wales, Italy, Germany, and France are examined in three ways. Firstly, the variations between the projections of each model are compared with the maximum projected changes after ten and fifty years. Secondly, the sensitivity of the models to changes in system parameters is explored in order to test the utility of the models as monitoring tools. Thirdly, the generality of the models is tested by making changes in the geographic specification of the German regional system.

An associate exercise employs a more complex model incorporating positive feedback effects in order to compare the likely redistributive effects of policy input.

1. Introduction

The purpose of this paper is to consider the likely redistributive effects on current national settlement patterns of trends inherent in present migratory movements with an assumption of stationarity in the data. The basic exercise considers how these trends will be reflected by three migration models of varying complexity. The extrapolations from these trends are not themselves regarded as predictions of future population distributions since their main aim is to provide the policy maker with information for use in planning. For this reason the models remain simple, and do not consider the effects of natural change through regional variations in fertility and mortality rates.

The intention is to achieve a realistic indication of present trends at minimum cost, and therefore the models are principally tested for compatibility and the circumstances in which compatible projections occur. In association with this main theme three other exercises were undertaken. Firstly, a comparison between variations in the projections is reported for two distinct periods of migratory movements to test the sensitivities of the models to changes in the system parameters. Secondly, the effects of different scales of regional design are reported to reflect the sensitivities of the models to geographic specification. Thirdly, the ramifications for the system population vector are explored in a situation where policy input is directed to a limited number of regions. The time horizon for the projections is basically fifty years but varies according to data availability. This is outlined in Section 2.

The exercise is part of an ongoing iterative research project carried out at IIASA and the Centre for Environmental Studies, London, which aims at a more complete understanding of the dynamics of population movements and regional economic growth. The work has been reported in Cordey-Hayes and Gleave [1], [2], and Gleave and Cordey-Hayes [4].

2. Data

The data base from which the regional population projections have been generated comprises the inter-regional migration tables of the French, German, Italian and British censuses. The inter-regional population movements in France for the periods 1954-1962 and 1962-1968 facilitate an assessment of the changes in the system parameters of the twenty-two planning regions during the two periods under consideration.

Migrations between the eleven German Länder during 1970-1971 and an aggregate of eight spatial units which fuses the city of Hamburg with the Land of Bremen, the Länder Schleswig-Holstein and Neidersachsen, and eliminates West Berlin, permit an assessment of the effects of regional design on population projections. The Italian tables refer to population shifts between twenty regions during the same time period, 1970 to 1971, whilst the British data are concerned with movements between twenty city regions during 1960-1961 (Fielding [3]). The transition matrices therefore refer to annual migrations except for the French data. In all cases a comparison of the projections of the three alternative models was possible.

3. The Models

The models used in the exercise were 1) a Kinematic model, 2) a Markov Chain model and 3) a Modified Markov model which permits feedback effects of the population attractiveness of the potential destination region. They are specified in the following way:

1) The Kinematic model is the most elementary of the three and assumes that the equilibrium state of the system is determined by two parameters for each region in the system. These parameters are the regional escape frequency, ε_i and the capture cross section μ_i which are defined as:

$$\varepsilon_i = \sum_j M_{ij}/P_i \tag{1}$$

and

$$\mu_i = \sum_j M_{ji}/\sum_i \sum_j M_{ji} \quad , \tag{2}$$

where

ε_i = escapes frequency from region i;

μ_i = capture cross section of region i;

M_{ij} = migrants from region i to region j;

P_i = population of region i.

The equilibrium population is defined as the product of the total system population (which is held constant) and the ratio of the proportion of the regions' escape section and escape frequency, divided by the system sum of this proportion. Hence

$$P_i^e = P_* \frac{M_i/\varepsilon_i}{\sum_j M_j/\varepsilon_j} \tag{3}$$

where

P_i^e = equilibrium population of region i;

P_* = system population where * means: sum the missing subscript.

This equation may be expanded in terms of basic data input in the following manner:

$$P_i^e = P_* \frac{M_{*i} P_i}{M_{**} M_{i*}} \sum_j \frac{M_{**}}{M_{*j}} \frac{M_{j*}}{P_j} \tag{4}$$

$$= P_* \frac{M_{*i}P_i}{M_{i*}} \sum_j \frac{M_{j*}}{M_{*j}P_j} \tag{5}$$

$$= A \frac{M_{*i}P_i}{M_{i*}} \tag{6}$$

where

A = system constant of proportionality ensuring $\sum_i P_i^e = P_*$.

The input for this model therefore comprises either the set of region population totals and the ratio of in-migration to out-migration or the total in-migration and per-capita out-migration. Projections can be made for an n region system with only 2n + 1 parameters. The Kinematic interpolation for the regional population projections at time t is given by:

$$P_i^t = P_i^e + (P_i^o - P_i^e)\, e^{-\varepsilon_i t} \quad . \tag{7}$$

This equation requires no further data input assuming the availability of per-capita rates of out-migration. The model therefore is easily operationalised.

2) The Markov model is somewhat more demanding in terms of data input and requires either n^2 parameters for an n region system comprising a migration transition matrix or $n^2 + 1$ parameter comprising transition probability matrix and system population total. Assuming convergence, the equilibrium population vector is independent of the initial population vector and depends only upon the transition probability matrix A where

$$a_{ij} = M_{ij}^o / P_i^o \quad . \tag{8}$$

The equilibrium population of region i is given by:

$$P_i^e = P_*(I - A)^{-1} \quad . \tag{9}$$

The population distribution of the system at any time t depends only upon the state of the system at the previous time period and the transition matrix. Hence:

$$P_i^t = \sum_j P_j^{t-1} a_{ji} \tag{10}$$

$$= \sum_j P_j^{t-1} \frac{M_{ji}^o}{P_j^o} \quad . \tag{11}$$

The transition probability elements in this approach incorporate the relative attraction of the destination region, the distance friction between the origin and destination regions and any economic and/or social factors peculiar to the determination of migration between the pair. Most interaction models, particularly the family of gravity models, postulate that the flow from origin to destination is a function of the "population" at the origin, the "population" at the destination and a function of the distance between them. That is:

$$M_{ij} = f(P_i P_j d_{ij}) \quad . \tag{12}$$

3) However, the effect of the Markov Chain model reported above is to fossilise the destination effect as it operates at time t_o since $a_{ij} = M_{ij}^o / P_i^o$, and therefore the allocation of migrants between alternative destinations remains constant. It may be reasonable, especially if the population terms are linear, to respecify the transition matrix by either taking into account the relative changes in attractiveness of alternative destinations or to respecify the transition matrix. The second approach was adopted and the concept of field strength between pairs of regions is introduced to clarify the procedure. The field strength between regions is defined as:

$$f_{ij} = \frac{M_{ij}^o}{P_i^o P_j^o} \quad . \tag{13}$$

Associated with the concept of field strength is a time constant of proportionality for each origin at each time period, a_i^t which ensures that

$$a_i^t \sum_j \frac{P_j^t M_{ij}^o}{P_i^o P_j^o} = 1 \quad . \tag{14}$$

The projection of migrants at time t therefore becomes:

$$P_i^t = \sum_j P_i^{t-1} P_i^{t-1} f_{ji} a_j^{t-1} \quad (15)$$

$$= \sum_j P_j^{t-1} P_i^{t-1} \frac{M_{ji}^o}{P_i^o P_j^o} \sum_i \frac{P_i^o P_j^o}{P^{t-1} M_{ji}^o} \quad . \quad (16)$$

The Modified Markov model projections may be estimated in terms of the same data input as the Markov model, but obviously require more computational time as the transition parameters have to be re-evaluated on each cycle.

Whilst the effects of changes in the system parameters (μ_is and ε_is) and changes in region design were evaluated in terms of their effects on the projections of the three models reported above, the ramifications of policy input were considered against a fourth migration model. The assumption adopted was that policy measures were directed at particular regions only and that the policy measures achieved the desired level of success. The purpose then was to see the overall effects on the distribution of population in the system. The model used here was the Cumulative Inertia, Differential Attractiveness (CIDA) model reported in Cordey-Hayes and Gleave [2] which predicts regional population at time t to be:

$$P_i^t = \sum_{j=1}^{n} \sum_{a=1}^{m} \sum_{d=1}^{a} P_{j\,a-d}^{t-1} \, e^{-\alpha d} \prod_{\tau=1}^{d-1} (1-e^{-\alpha\tau}) \frac{- q_i P_i^{\tau} f(c_{ji})}{\sum_{k=1}^{n} q_k P_k^{\tau} f(c_{jk})} \quad (17)$$

where

$P_{j\,a-d}^{t-d}$ = the population who moved into region j d years ago and who were then aged a-d years;

α = propensity to migrate through residence time, parameter;

q_i = "attractiveness" of region.

This model assumes that the rate of out-migration is a function of the period of residence in the region and that in-migration is a function of the "attractiveness" of the destination region modified by population and distance effects. The exercise involved varying the attractiveness parameter of selected regions.

These models are now considered in terms of the variations of their projections and capacity to represent trends inherent in the initial migration matrix.

4. Analysis of Projections

The relative performances of the three basic models may be considered for projection periods of varying length and evaluated in a number of ways. It was considered appropriate to consider the impact of inherent trends after periods of around ten and fifty years for the following reasons. The regional shifts in population over the shorter term are likely to be of interest to planners concerned with the provision of a balanced social and economic infrastructure over a period when stronger economic trends are only susceptible to limited modification. The longer-term effects are more interesting to those concerned with developing national settlement strategies and instigating a programme of long range regional economic management. However, the periods of review were selected subjectively and are therefore open to criticism.

The projections were compared by relating the variations between extreme estimates with the maximum projected growth or decline. If the models were to reflect the system changes in a similar manner, the ratio of the variations in estimate to projected population changes would tend to zero. There were no a priori grounds for anticipating that the ratio of these components would vary in a systematic way, for, whilst the Modified Markov model may be expected to render extreme projections where rapid growth or decline is a characteristic of the region, only a small variation in the projections would be needed to produce a high ratio when little regional population change was anticipated.

4.1 Ten Year Projections

The projections were contrasted by selecting from each national migration system a sample of three regions characterised by rapid growth, slow growth and decline. These are illustrated in Figure 1. These sample regions were analysed to assess whether any systematic variations existed between the differences in the model projections and the rate of population change. The per capita variation in population projections was found to have significant linear relation to the rate of population decline, as in the case of the ten year data, $r_c = 0.83$ (significant at the 5% confidence level, Figure 2). The regression co-efficient effectively measured the percentage variation between the estimates per unit population change, and for the short run sample projections was .1479. In all cases except two, the extreme projection was generated by the Modified Markov model. A second feature of interest was the relationship between the projected per

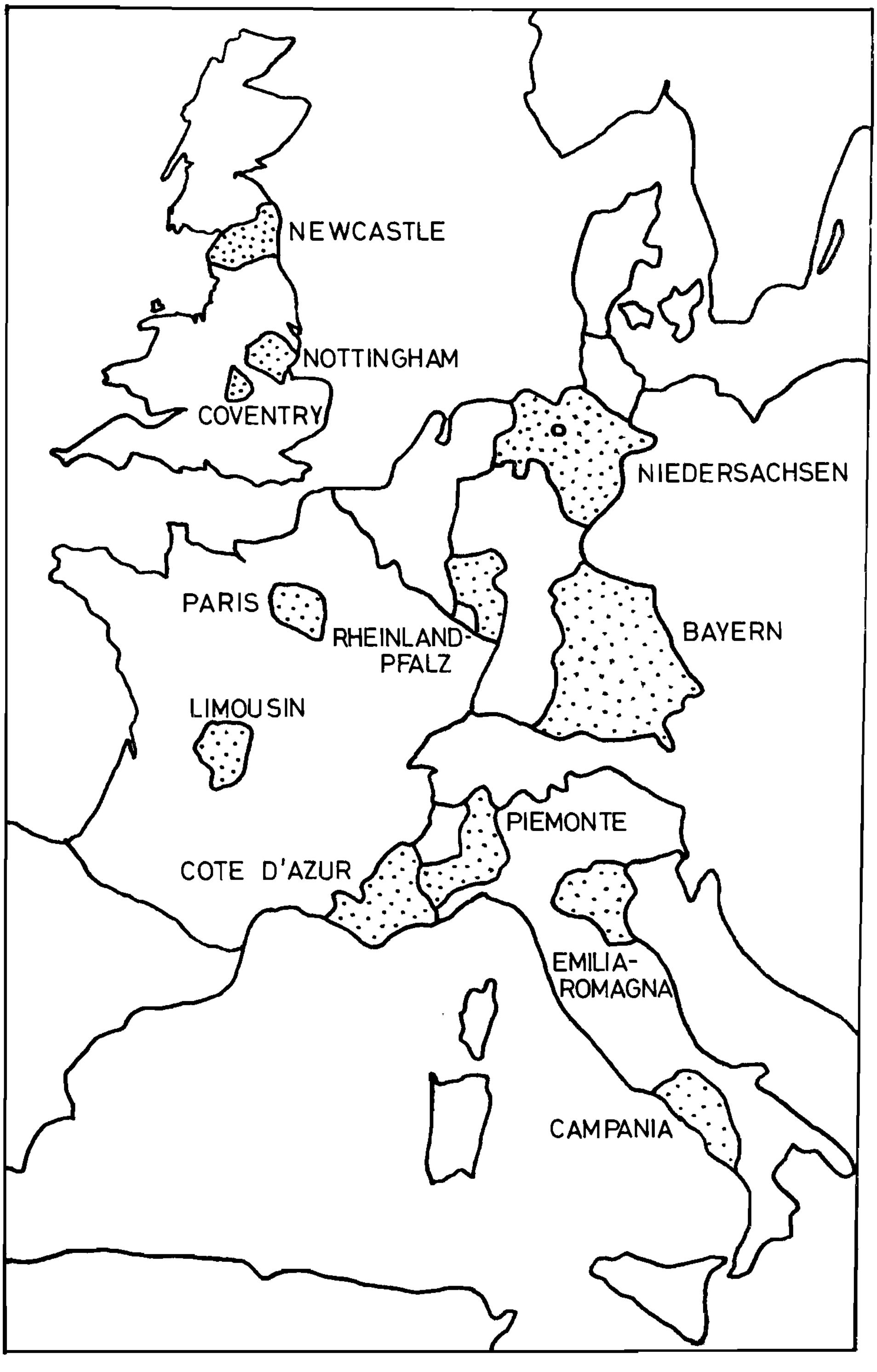

FIGURE 1. LOCATION OF REGIONS.

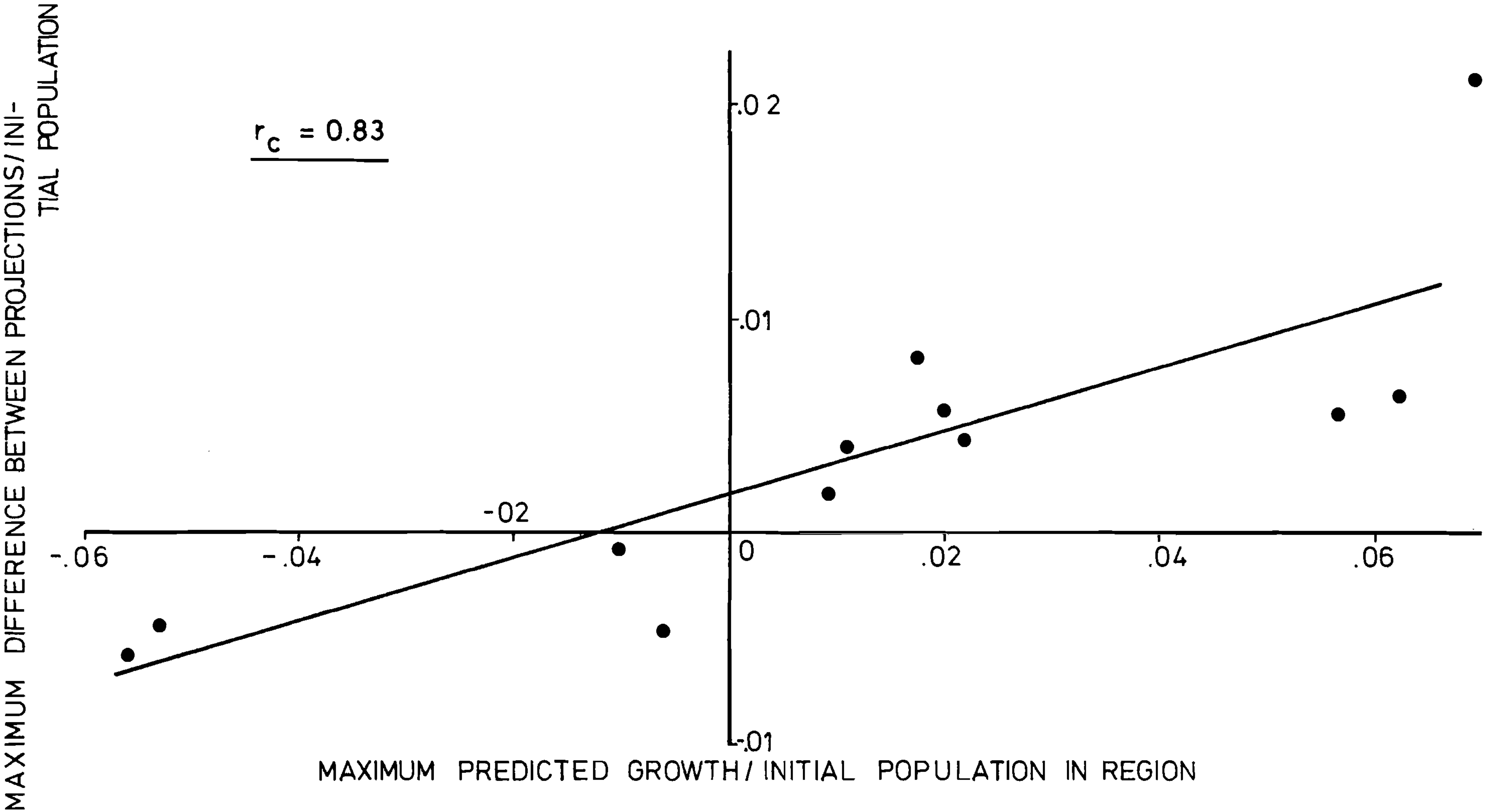

FIGURE 2. THE RELATIONSHIP BETWEEN PROJECTION VARIATION PER CAPITA AND MAXIMUM GROWTH PER CAPITA FOR TWELVE SAMPLE REGIONS AFTER TEN YEARS (TWELVE IN FRENCH CASE).

capita population change and the differences between the estimates per unit change, illustrated in Figure 3. In the case of the ten year data the relative difference between the estimates was lowest in extreme cases of growth or decline. However, it is absolute difference between projections rather than the relative difference which is of most importance, and consequently the regions of smallest population change are the easiest to plan for. The three models provide fairly compatible projections for a period of ten years or so. There was also some indication to suggest those systems closer to equilibrium were characterised by similar projections. The greatest contrasts occurred in the case of the Italian system which is at present in great disequilibrium.

4.2 Fifty Year Projections

The fifty year projections for the same twelve sample regions show some minor changes from the ten year estimates. Most expected was a divergence in the model projections, for the Modified Markov model tends to compound the attractiveness or unattractiveness of regions and hasten their rate of change whilst the Markov and Kinematic models are continuously moving towards a state of equilibrium. This is manifest in the regression co-efficient relating the differences in estimates per unit population with the maximum per capita rate of change which rose from 0.1479 to 0.3418 (Figure 4).

The relationship was more significant in the case of the fifty year data (forty-eight years in the French case), $r_c = 0.98$ (significant at the 1% confidence level). The difference between the estimates per unit maximum change showed the most increase in the case of the fast growth and rapid decline regions changing by factors of 3.9, 4.2, 2.8 and 3.4, in the cases of Côte d'Azur, Coventry, Newcastle and Campania respectively. The slower changing regions did not display this characteristic, largely because of a limited impact of the feedback effect in the Modified Markov model. For example Nottingham, Emilia-Romagna and Rheinland-Pfalz increased the variations in projection per unit change by factors of 1.02, 1.10 and 0.71.

Although the relationship between the per capita difference between projections and the per capita rate of growth shifts significantly from the ten year period to the fifty year period, the ratio of the difference between the projections and the maximum predicted regional change does not do so. This can be seen by contrasting Figures 5 and 3. Although the sample region mean increases from .255 to .372, suggesting increased variation in projections, the standard errors of the estimate are so large that there is no significant difference at the 5% confidence level between the two ratios of the sample for the two points in time (t statistic = 1.71). Consequently there is only conflicting evidence to report when

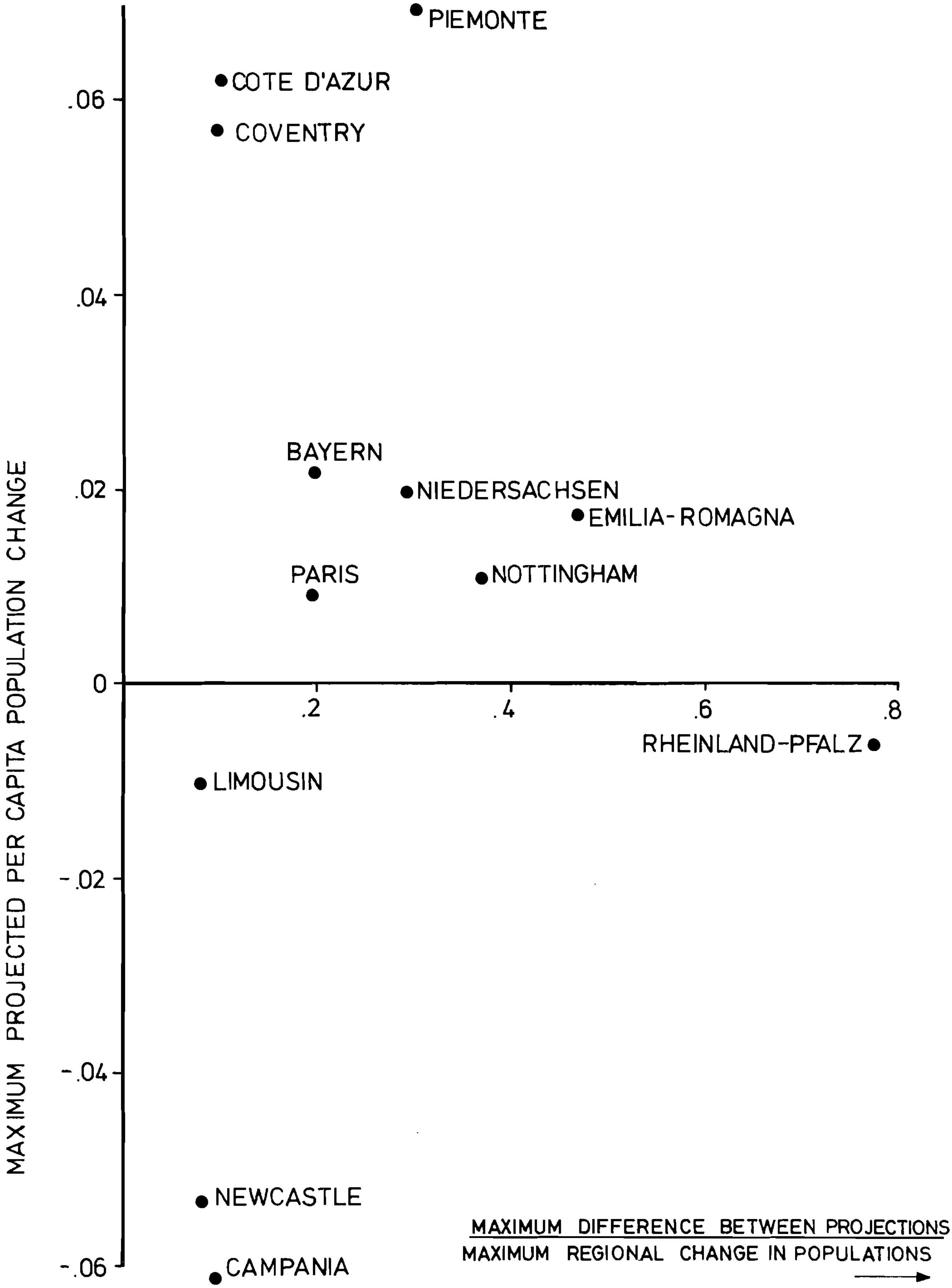

FIGURE 3. MAXIMUM PER CAPITA POPULATION CHANGE AGAINST PROJECTION VARIATIONS PER UNIT-POP. CHANGE. SAMPLE REGIONS AFTER TEN YEARS.

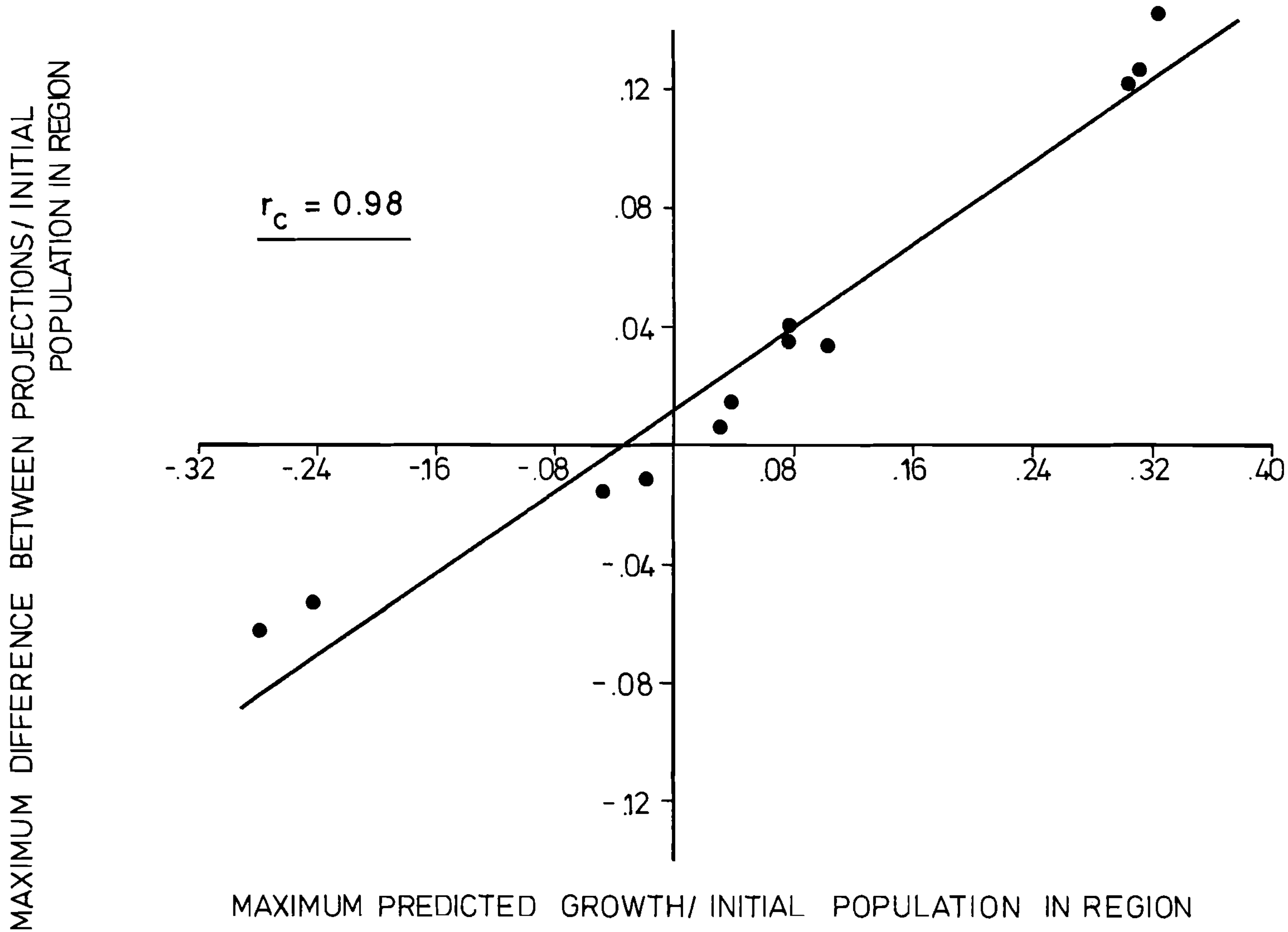

FIGURE 4. THE RELATIONSHIP BETWEEN PROJECTION VARIATION PER CAPITA AND MAXIMUM GROWTH PER CAPITA FOR TWELVE SAMPLE REGIONS AFTER FIFTY YEARS (FOURTY-EIGHT IN FRENCH CASE)

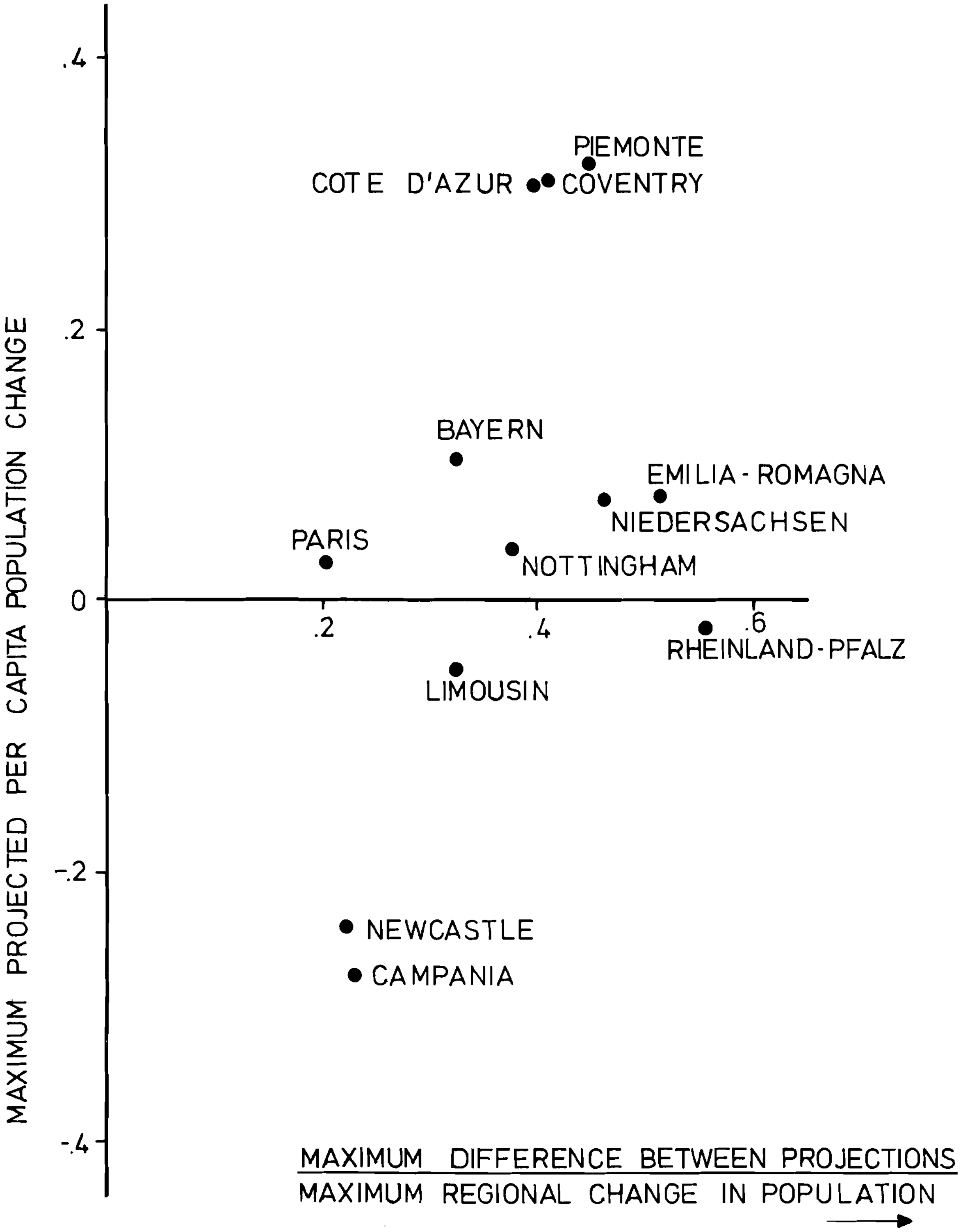

FIGURE 5. MAXIMUM PER CAPITA POPULATION CHANGE AGAINST PROJECTION VARIATIONS PER UNIT POP. CHANGE. SAMPLE REGIONS AFTER FIFTY YEARS.

seeking an answer to the question, "are the short run projections less contrasting and contradicting than the long run?" There is also no evidence to confirm that the projections are more compatible for more stable rather than less stable systems; in fact there is some suggestion to the contrary for in the long run case it was the slow growth/decline regions which manifested the greatest relative variation in their population projections. The contrasts between the projections of three models are illustrated in Figures 6 to 9 for Coventry and Newcastle, regions of rapid growth and decline, and for Paris and Emilia-Romagna, regions of compatible and contrasting projections.

4.3 Shifts in System Parameters

How efficient are the three basic models in reflecting changes in the system parameters? This aspect of the population projection problem was examined in the context of French projections based upon behaviour during the time periods 1954-1962 and 1962-1968 when a vigorous programme of decentralisation and regional assistance was in operation. Basically the growth of the Paris region was to be contained by stimulating the provincial regional economies.

How far was the success of these attempts reflected in the projections? Figures 10, 11 and 12 show the regional time paths for the Paris region, the Provence-Côte d'Azur region and Bretagne based upon the two transition matrices. The discrepancies between the six year projection from 1962 and the 1968 initial population must first be explained. Three factors account for this: a) the natural increase of population through a surplus of births over deaths, b) positive net international migration and c) the error in the projection estimate based on 1954 to 1962 migration. However, the variations in the estimates in the three cases illustrated are quite small in comparison with the difference between the 1954-1962 based projection and the actual population in 1968, particularly in the cases of Provence-Côte d'Azur and Bretagne. The changes in regional economic attractiveness of the regions indicated by the shifts in the capture cross-section parameter for Paris and for Bretagne are strongly reflected by major changes in the trajectories of the regional populations. Major changes in these parameters are strongly reflected in the projections to the extent that variations in the populations attributable to the different assumptions of the models are very small compared with variations due to parametric change. All three models appear to be quite sensitive to such changes.

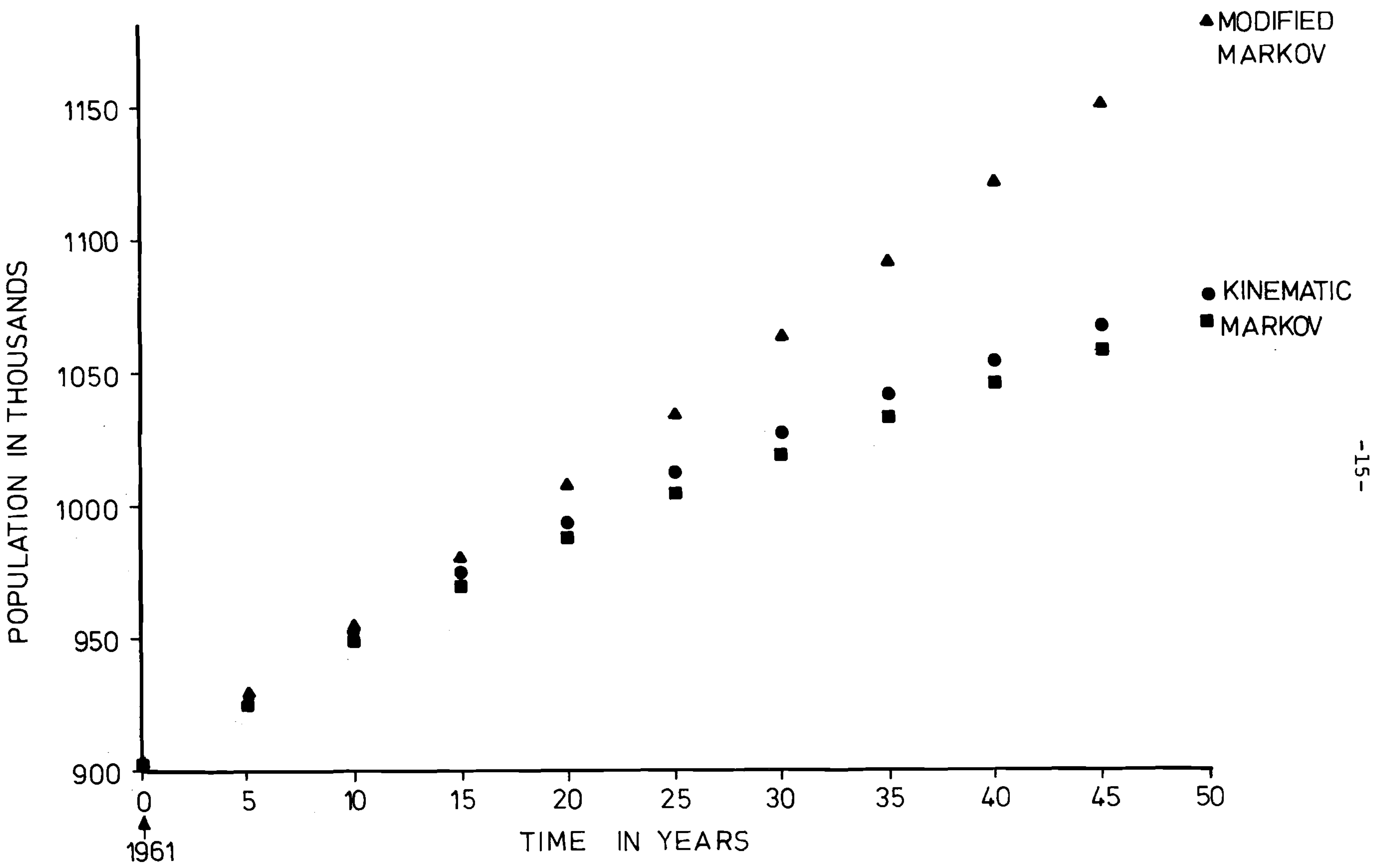

FIGURE 6. POPULATION PROJECTIONS FOR COVENTRY.

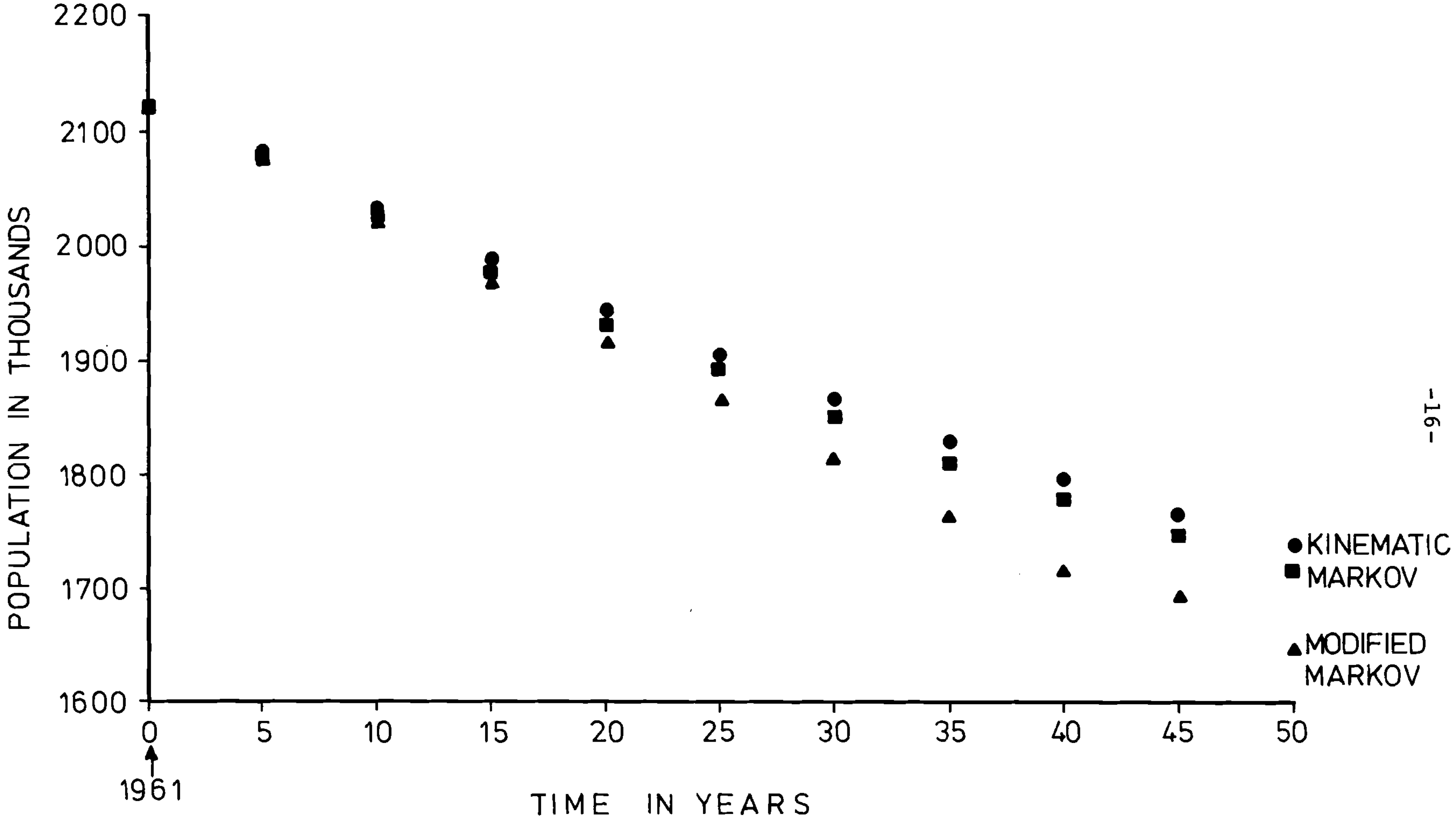

FIGURE 7. POPULATION PROJECTIONS FOR NEWCASTLE.

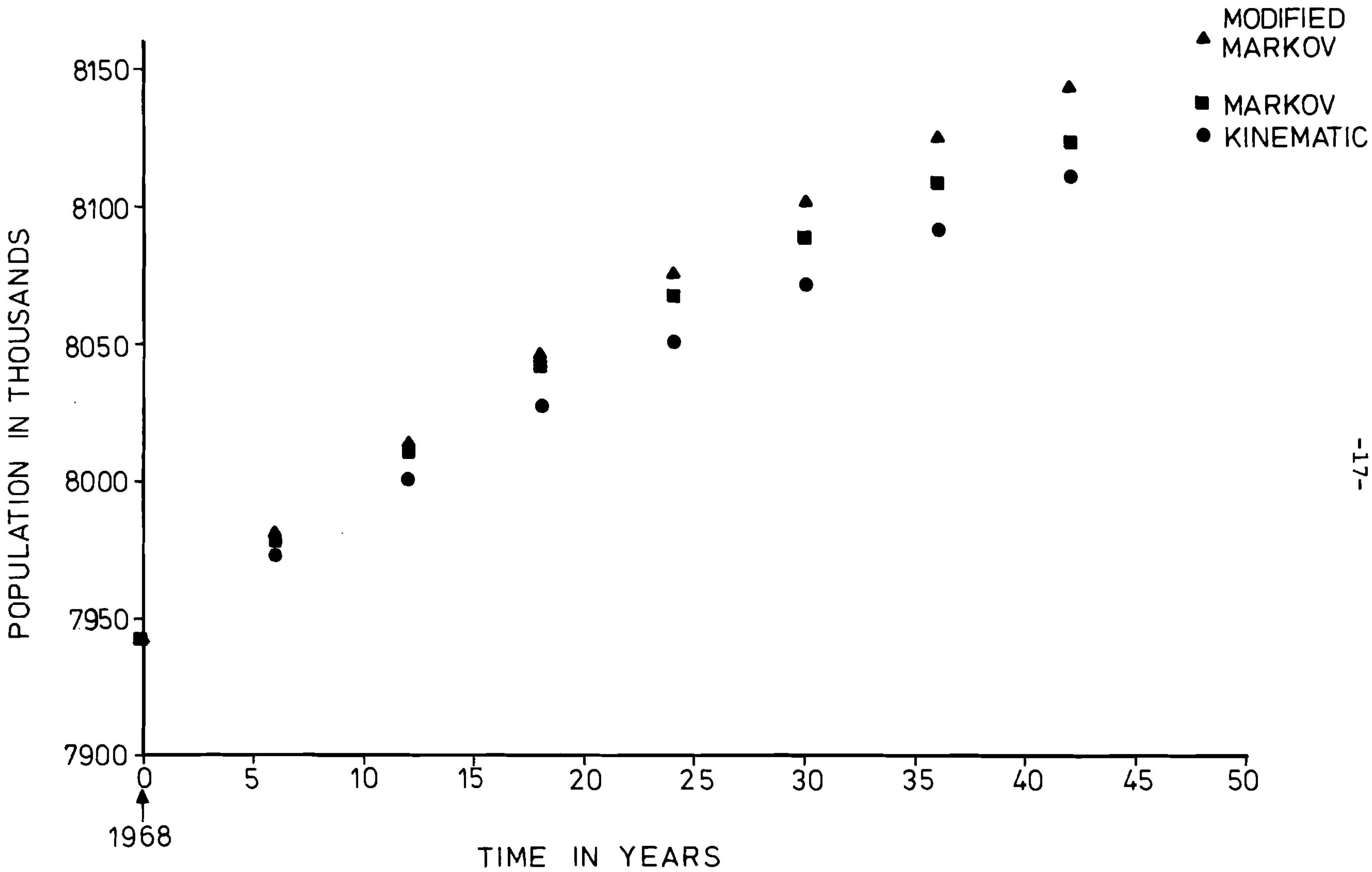

FIGURE 8. POPULATION PROJECTIONS FOR PARIS.

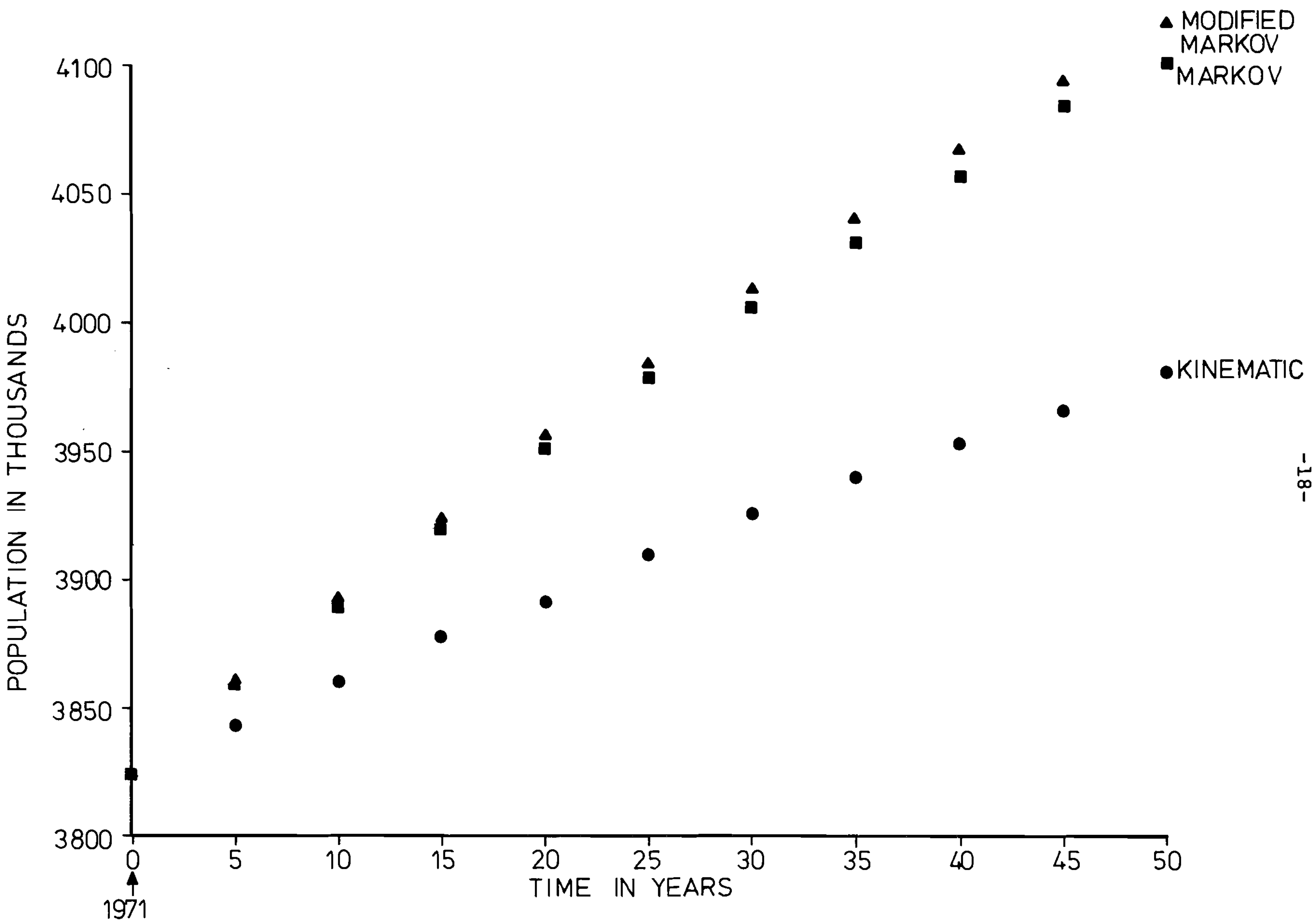

FIGURE 9. POPULATION PROJECTIONS FOR EMILIA-ROMAGNA.

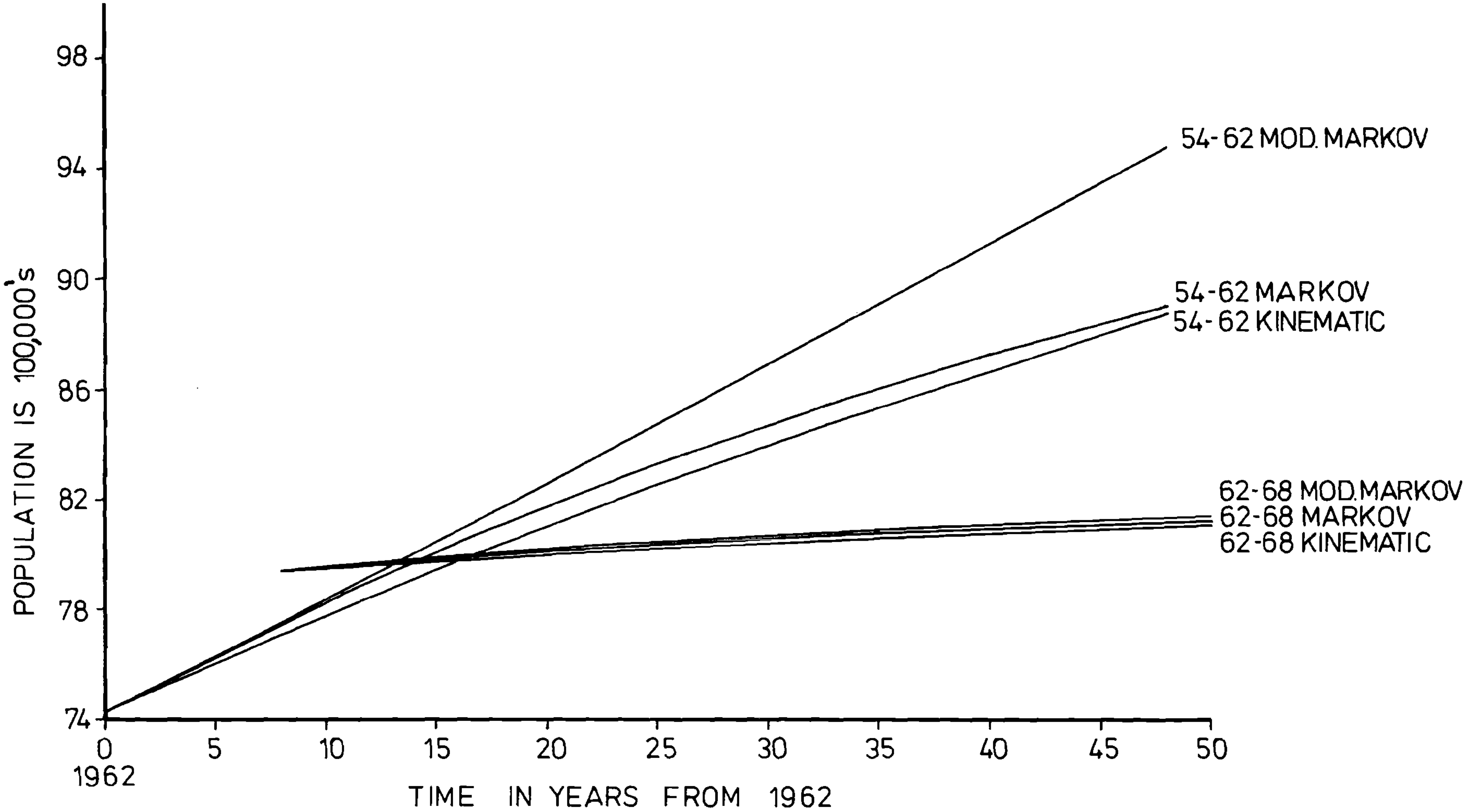

FIGURE 10. PROJECTION FOR THE PARIS REGION BASED ON 1954-62 AND 1962-68 MIGRATION MATRICES.

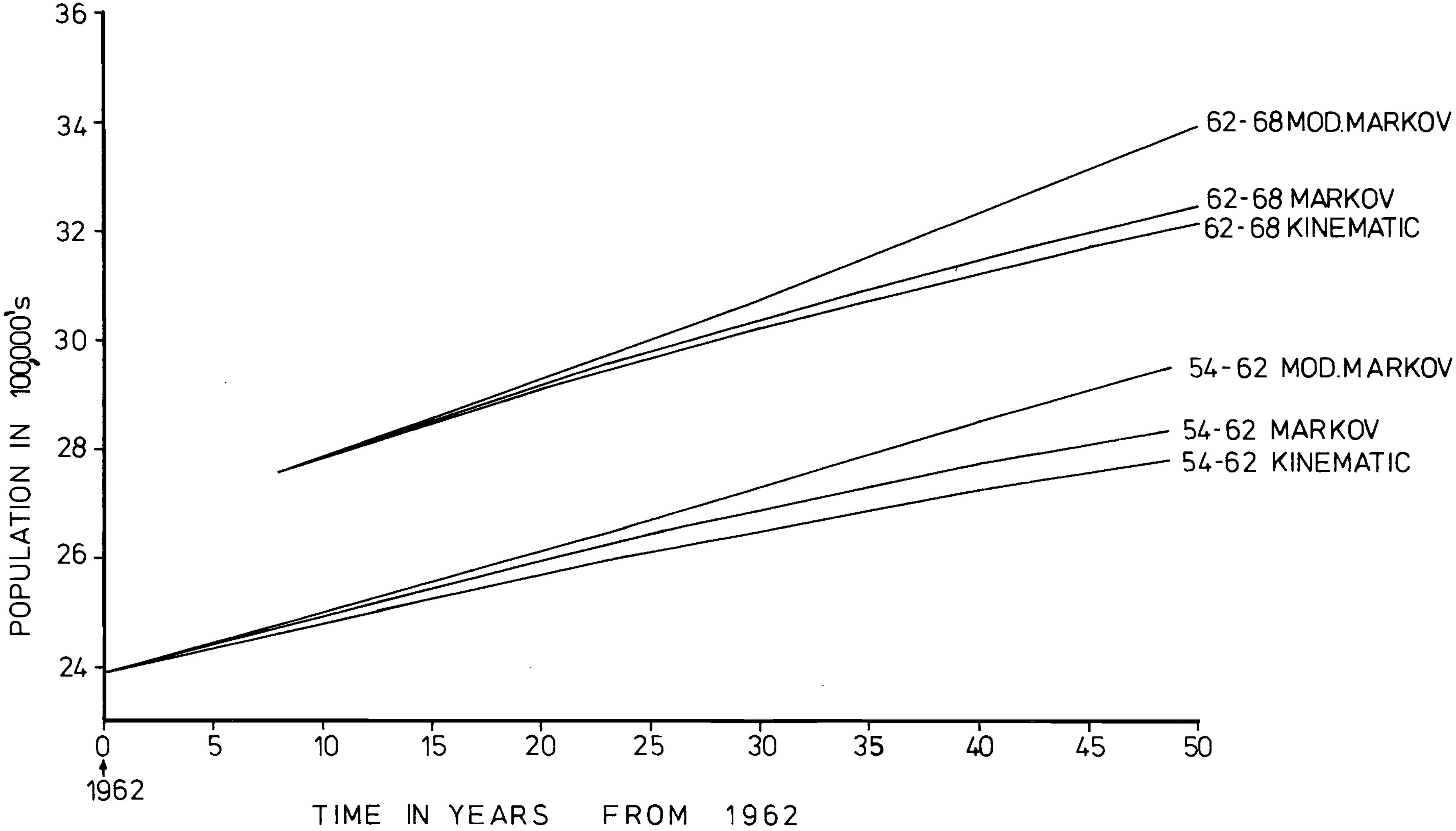

FIGURE 11. PROJECTION FOR PROVENCE - COTE D'AZUR REGION BASED ON 1954-62 AND 1962-68 MIGRATION MATRICES.

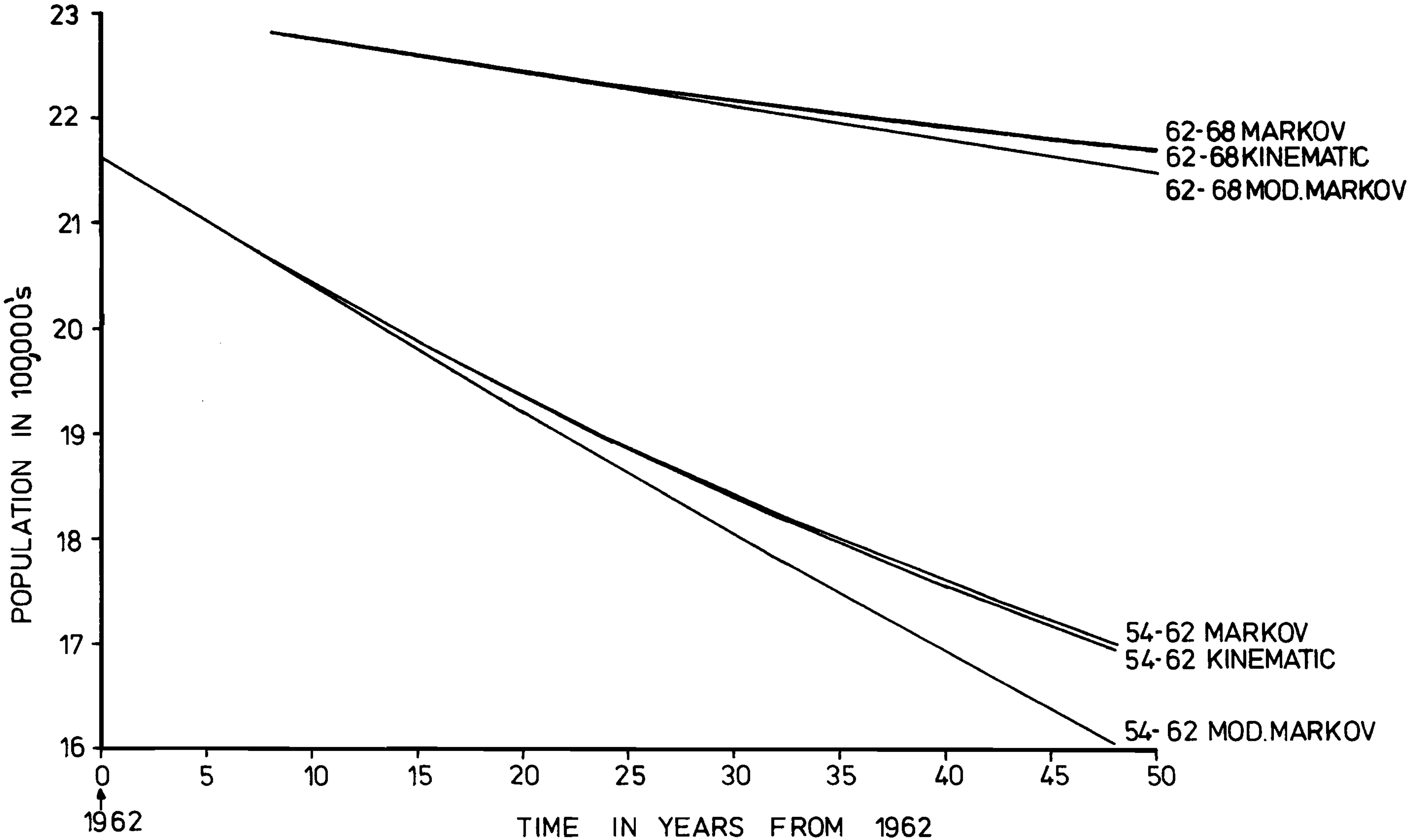

FIGURE 12. PROJECTIONS FOR THE BRETAGNE REGION BASED ON THE 1954-62 AND 1962-68 MIGRATION MATRICES.

4.4 The Effects of Region Design

The effects of respecifying the region system were explored in the case of the German data and in the manner outlined in the data section. The circumstances in which a respecification of spatial units will not effect the population distribution of the system are very limited. We need, by way of example, only consider the case where a large region is disaggregated into two subregions (or vice versa) to illustrate the point. The regional population projections of the Kinematic model will only remain the same when:

$$\frac{M_{*j}P_j}{M_{j*}} = \frac{M_{*x}P_*}{M_{x*}} + \frac{M_{*y}P_y}{M_{y*}} \tag{18}$$

where x and y are exhaustive subregions of region j such that $P_j = P_x + P_y$.

Similarly, in the case of the Markov model the regional population projections will only remain the same if the out-migration transition vectors are identical, that is:

$$a_{xi} = a_{yi} \qquad \text{for all } i, \quad i \neq x,y \tag{19}$$

and when the in-migration transition vector of one subregion is a scaler product of the second subregion and proportional to the ratio of the initial subregion populations. That is:

$$a_{xi} = k\ a_{iy} \qquad \text{for all } i, \quad i \neq x,y \tag{20}$$

where

$$k = P_x/P_y \quad .$$

More simply, the region population projections remain constant only when subregion disaggregates are homogenous or regional amalgams are unions of homogenous units. The examples cited to illustrate these points are the projections for Schleswig-Holstein and Hamburg and the projections for Bayern. Figure 13 shows the projections of the summed populations in the eleven-Länder case and the projections for the aggregate region in the 8-Länder case whilst Figure 14 shows the impact on a region whose specification does not change. The difference in the fifty year projections for Bayern is attributable not only to the heterogeneity of the two pairs of aggregate regions but also to the exclusion of Berlin.

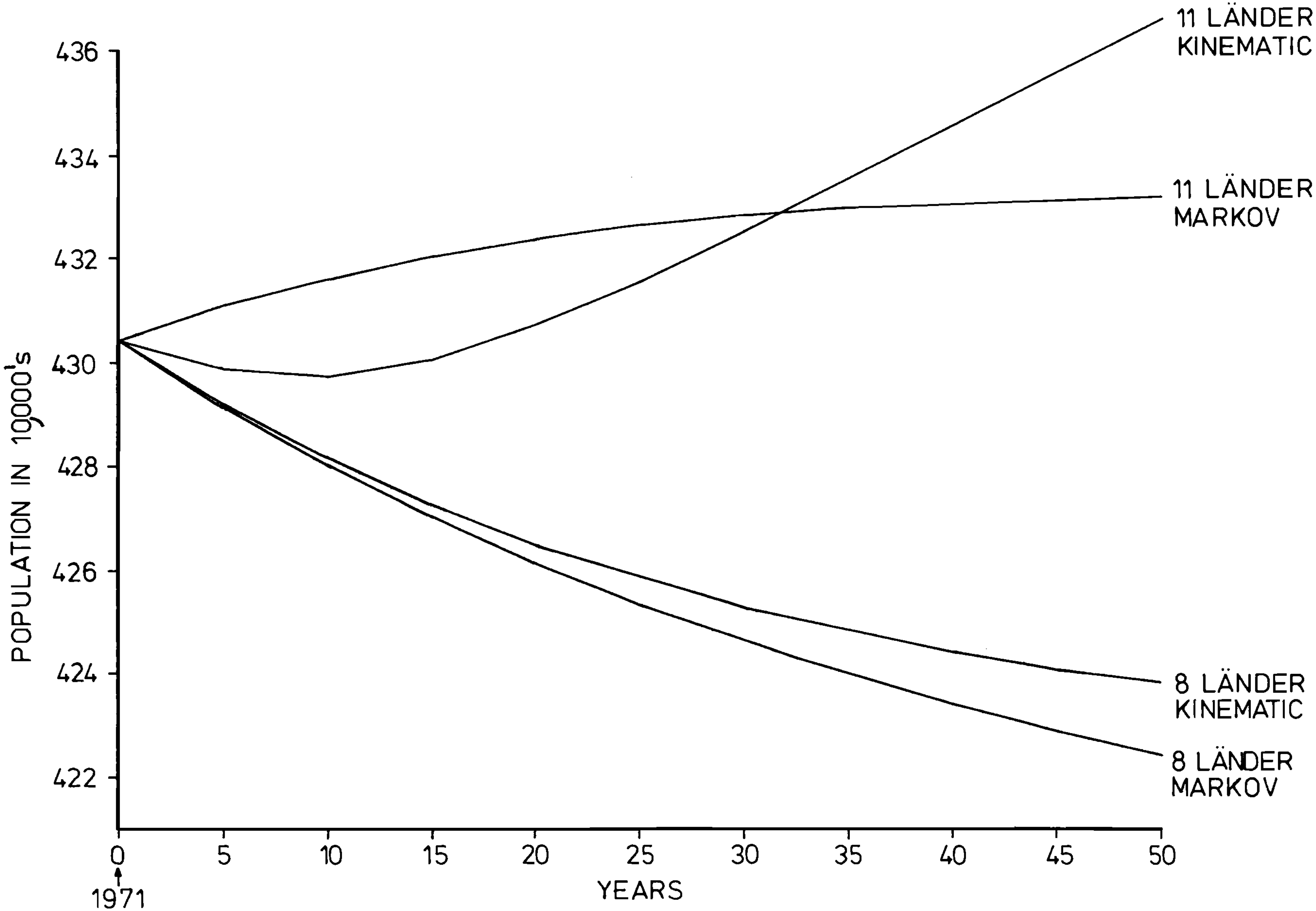

FIGURE 13. POPULATION PROJECTION OF THE SCHLESWIG-HOLSTEIN-HAMBURG AGGREGATE.

11 LÄNDER - POPULATION AGGREGATED AFTER INDEPENDENT PROJECTIONS.

8 LÄNDER - PROJECTION AFTER AGGREGATION OF INITIAL REGIONAL POPULATIONS.

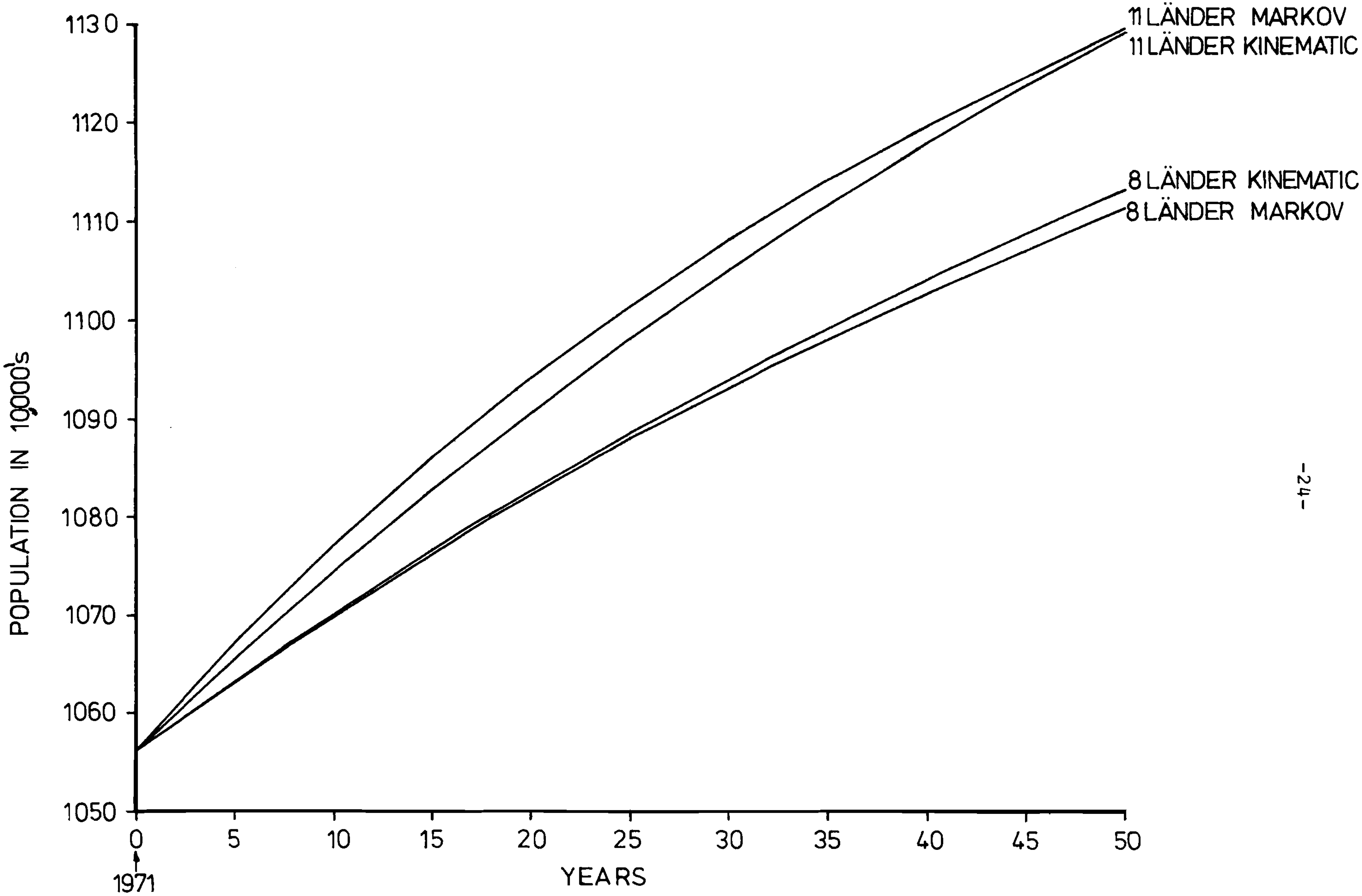

FIGURE 14. POPULATION PROJECTIONS FOR BAYERN.
11 LÄNDER - PROJECTIONS FOR BASIC REGIONS.
8 LÄNDER - PROJECTIONS FOR AGGREGATE REGIONS.

4.5 The Impact of Policy Input

The effect of policy impact was evaluated by adjusting the regional attractiveness parameters of the CIDA model in order to explore the system ramifications of 1) effective controls imposed on a fast growth region, and 2) stimulating growth in a stagnating region. Table 1 below compares the projected region population proportions after fifty years with the initial distribution. This exercise was carried out for the England and Wales planning regions.

Table 1.

	Region	Policy I Containment of Region 8	Initial Population	Policy II Stimulation of Region 1
1.	Northern England	.035	.066	.051
2.	Yorkshire-Humberside	.116	.103	.113
3.	Northwest England	.135	.143	.129
4.	East Midlands	.114	.075	.100
5.	West Midlands	.117	.111	.109
6.	East Anglia	.065	.036	.055
7.	Southeast England	.319	.357	.319
8.	Southwest England	.063	.075	.087
9.	Wales	.038	.055	.038

A rigorous constraint to growth in Southwest England was simulated by reducing the attractiveness parameter of that peripheral region by 33% and, by implication, making all other regions relatively more attractive. The economic growth of the Northern region was simulated by a threefold increase in its attractiveness parameter to bring it in line with the Southeast region.

Comparing the fifty year vectors in the case of each policy measure with the initial distribution indicates that in the case of regions unaffected by policy input only small modifications were manifest in the projections. The growing regions, particularly East Anglia and the West Midland, continued to grow at a fairly rapid rate whilst the declining regions, particularly Northwest England and Wales continued to decline. Nonetheless there were systematic variations in the trajectories of the regions unaffected by policy. The effect of limiting growth in the Southwestern region was basically to deflect growth from it to other faster growing regions in Southern England. In fact, proximity to the Southwest region itself was not the major consideration in determining the redirection of the migrant flow but rather proximity to Southeastern England which is the main reservoir for persons heading to the Southwestern region. The regions which showed the greatest readjustments were East Anglia, the East Midlands and the West Midlands. The effects on the northern regions and Wales were small. The Southwestern region itself registered a decline in population as a result of this policy measure which was as great as its anticipated growth without interference in the system.

The second policy input was more disappointing from the point of view of regional equity. The decline region, despite considerable stimulation, continued to decline, although the rate over the fifty year period was reduced from 47% to 22%. The main effect of reducing the rate of decline was to marginally reduce the growth rate in the Midlands regions and to increase the decline rate in Northwestern England. Paradoxically, the Yorkshire region adjacent to Northern England benefited from policy measure two probably by attracting a larger proportion of the increased out-migration resulting from the increased in-migration to the Northern region.

Superficially, the lesson of this particular exercise is quite clear. Any attempts to interfere with the market mechanism to bring about a planned redistribution of population must be comprehensive and basically discriminate in favour of all the declining regions to a lesser or greater extent and against the growth regions in a similar fashion. To constrain growth in one fast growth area simply serves to redirect it to other growth regions, whilst propping up individual decline regions has little or no positive impact on other regions in distress.

5. The Efficacy of Naive Projection Models

The three main models used in this exercise have now been evaluated in a largely qualitative way and it is now possible to bring together some of the general conclusions in order to make a tentative statement on their efficacy and utility.

Perhaps the most unexpected result from the analysis of the fifty year projections was the continuing compatibility of the Modified Markov model with the other two, for this former model incorporates a positive feedback effect which, although resulting in increased divergence between the estimates, still produces a projection after fifty years which is, on average, only 34% at variance with the maximum predicted regional change. More complex models of population prediction result in contrasts of this magnitude when relatively minor modifications are made to control parameters such as fertility rates. Secondly, the major contrast between the models is explained by the inherent tendencies of the Markov and Kinematic models to move towards an equilibrium state. The Modified Markov model does not have this same tendency in the periods we have been considering but the regional trajectories associated with all three models have been very similar over periods of around twenty-five years. This aspect is important because the purpose of the exercise is to discover the trends inherent in the system rather than to make accurate predictions, and when the projections are very similar over the middle run, despite the positive feedback effects of the Modified model, the message to the policy maker is quite clear.

The models are also efficient in reflecting changes in the system parameters as evidenced by the exercise on the French data. They are then useful tools for monitoring the progress of planning policy and permit a swift and easy indication of changes in migratory trends. It is important to mention that they must be applied at an appropriate scale which will be the operational policy scale; it is useless to analyse migration and population change for spatial units which have no policy context.

These favourable conclusions do not mean that more complex models of migration and population projection should be abandoned. To the contrary, the models described above are useful analytical tools for testing and varifying the theoretical content of more complex models. Until a rigorous behaviour based theoretical model(s) of migration is developed and tested, the simpler type of model will have a useful role to play in hinting at regional trends and indicating the effects of policy inputs in poorly understood migration systems.

APPENDIX

Population Projections for the Twelve Sample Regions

Ten Year Projections

Regions	a	b	c	d	e	f
BAYERN	10561100	10744953	10771844	10790518	0.43	2.17
NIEDERSACHSEN	7121800	7221089	7242842	7262006	0.57	1.97
RHEINLAND-PFALZ	3658900	3636030	3653766	3653775	0.48	-0.63
COVENTRY	902250	952222	948466	953444	0.55	5.67
NOTTINGHAM	1494540	1510787	1504792	1505103	0.40	1.09
NEWCASTLE	2139170	2034455	2029832	2025275	0.43	-5.32
COTE D'AZUR	2757550	2911090	2920697	2928420	0.63	6.20
PARIS	7942660	8001180	8013653	8015500	0.18	0.92
LIMOUSIN	662320	655576	656107	655777	0.08	-1.02
PIEMONTE	4389126	4600726	4665286	4693302	2.11	6.93
EMILIA-ROMAGNA	3825570	3860969	3891158	3892071	0.81	1.74
CAMPANIA	50-1584	4733332	4762244	4746707	0.57	-6.10

a - Initial Population,
b - Kinematic Projection,
c - Markov Projection,
d - Modified Markov Projection
e - Maximum difference between estimates / Initial Population expressed as a percentage,
f - Maximum Projected Growth / Initial Population expressed as a percentage.

Fifty Year Projections

Regions	a	b	c	d	e	f
BAYERN	10561100	11292368	11294221	11643486	3.32	10.25
NIEDERSACHSEN	7121800	7468864	7410006	7661274	3.53	7.57
RHEINLAND-PFALZ	3658900	3583883	3625221	3613309	1.13	-2.05
COVENTRY	902250	1076708	1067637	1180945	12.56	30.89
NOTTINGHAM	1494540	1551043	1529856	1533736	1.42	3.78
NEWCASTLE	2139170	1737115	1717501	1622375	5.36	-24.16
COTE D'AZUR	2757550	3264770	3346937	3598680	12.11	30.50
PARIS	7942660	8126830	8149942	8174130	0.60	2.91
LIMOUSIN	662320	641127	640331	629816	1.59	-4.91
PIEMONTE	4389126	5166185	5275179	5801297	14.47	32.17
EMILIA-ROMAGNA	3825570	3967470	4103229	4117771	3.93	7.64
CAMPANIA	5041584	3815074	3954529	3631620	6.40	-27.97

a - Initial Population,
b - Kinematic Projection,
c - Markov Projection,
d - Modified Markov Projection,
e - Maximum difference between estimates / Initial Population expressed as a percentage,
f - Maximum Projected Growth / Initial Population expressed as a percentage.

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