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Abstract

The forests of Eastern Canada are subject to periodic outbreaks of a pest which devastates the trees causing major disruption to the logging industry. This paper gives details of a study to find a criterion by which management policy alternatives could be evaluated in conjunction with a simulation model of the forest. It describes the manner in which the important decision factors, or attributes, were determined and how a value function and a utility function were assessed over these attributes, taking into account the long time horizon involved of 50-100 years.

The report which follows describes an attempt to determine and quantify preferences for a forest region in New Brunswick, Canada. The forest is subject to outbreaks of a pest called the Spruce Budworm which does great damage to the trees and thus to the logging industry, a major part of the economy of the province. DDT has been sprayed extensively for the last twenty years so that now if the spraying were to stop a widespread outbreak would occur. The Ecology Project at the International Institute for Applied Systems Analysis (IIASA) were using a detailed simulation model of the forest to examine possible strategies for handling the pest, see Holling et al. [5], the Methodology Project contributing to the study by creating a Dynamic Programming Optimization Algorithm, Winkler [11]; and the study outlined here started when I attended a meeting of the Ecology and Methodology Projects together with some experts from the Canadian Forestry Commission. They were trying to establish an objective function for the optimization model by fitting values c_i to the linear formula

$$c_1(\text{Egg Density}) + c_2(\text{Stress})^1 + c_3(\text{Proportion of Old Trees}) + c_4(\text{Proportion of New Trees}) \quad . \tag{0.1}$$

I was disturbed by this process for two reasons. Firstly, they did not appear to have a very accurate way of arriving at the parameters, and secondly the only concern of the experts seemed to be the monetary gains and losses to the logging industry whereas I had always supposed that our Ecology and Environment Project would also be concerned with the protection of wildlife and scenery. So I began this study with two aims:

i) to derive the parameters c_i for the optimization model by different means as a comparison,

ii) to discover the true preferences of the members of the Ecology Project regarding trade-offs between profits, wildlife and the environment.

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[1] Stress is a measure of the health of the trees measured by the amount of defoliation in current and previous years caused by the budworm.

This paper tells of my progress, spread over the next eighteen months, towards achieving these aims. In performing the analysis, inevitably many mistakes were made and if I were to repeat this on a similar study, I would do a great many things differently, however I have chosen to describe here what actually happened rather than to serve up a neat exposition of decision analysis at its best. It should be borne in mind that this study was not planned in detail ahead, rather it developed more on a week by week basis and was subject to constant interruptions including two six-month separations of analyst from decision maker.

The benefits of presenting it like this, I hope, are that on the one hand a number of theoretical issues are raised to which some attention should be paid and on the other it might encourage potential analysts who may feel daunted by the imposing literature on decision analysis to give it a try themselves.

The first section of this report deals with the initial investigation I made to check whether the coefficients of the linear objective function were accurate and recounts the way in which we attempted to resolve apparent discrepancies in the preferences of the different Ecology group members by finding alternative sets of forest statistics which better enabled the ecologists to agree.

The second section describes the way in which I attempted to assess a value function for the preferences of one of the ecologists over attributes which were important to him. The difficulties associated with collapsing indicators over time is raised and discussed.

Section three represents stage two of the whole analysis. In this a utility function is assessed for the same "decision maker" which incorporates many of the complicating factors which hindered the assessment of the value function such as interdependencies of preferences for outcomes in different periods.

Section four summarizes the preference assumptions which were used in the assessment of the utility function. Section five presents a review of the whole procedure, discussing some of the issues raised and the pitfalls encountered.

In order to keep this paper to a reasonable length many of the concepts used from decision analysis such as value function, utility function and various independence assumptions are described rather cursorily, the reader who is not well acquainted with these definitions should consult Raiffa [10] or Keeney and Raiffa [7].

1. Preliminary Analysis

I began by asking five of the conference participants to rank a list of states of the forest, exhibited in Figure 1, by preference and after they had done this, asked them to give a value 0-100 to each state indicating its "worth." They were to rank the list by taking any pair of forest states (summarized by the five data points) and decide which state they would prefer the forest to be in, assuming that from then on nature and man would be required to deal normally with it. The value they gave to each state could be derived by any reasoning they wished save that the ordering of preferences and of values should be the same.

I then used a statistical software package to obtain regression coefficients, see for example [3], for the linear formula (0.1) by using Egg Density, Stress, Proportion of Old and Young Trees as independent variables and the value as the dependent variable, deriving one formula for each of the five participants.

The formulas I derived from the rankings of the two Forestry Commission members were very close to the parameters c_i actually obtained at the meeting (despite my misgivings) but those of the three Ecology Project members were quite different from the other two and from each other.

I discussed with the Ecologists the reasons for their differences. The feeling emerged that the states in Figure 1 were meaningless because the whole forest could not be composed uniformally.[2] Indeed, if it were, all the twenty-seven states would be equally terrible. So I asked them whether they could describe a new state vector which would be meaningful.

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[2] The forest covers about 15,000 square miles.

	Prop. of Young Trees	Medium Age Trees	Old Trees	Stress	Egg Density
1	.10	.3	.6	0	0.3
2	.15	.35	.5	0	0.6
3	.10	.4	.5	0	0.5
4	.20	.5	.3	20	1.0
5	.10	.3	.6	10	0.1
6	.10	.3	.6	40	0.1
7	.10	.4	.5	0	0.5
8	.15	.35	.5	0	0.6
9	.5	.1	.4	0	2.0
10	.2	.5	.3	20	1.0
11	.2	.2	.6	20	10
12	.1	.3	.6	50	10
13	.2	.3	.5	20	10
14	.2	.3	.5	50	10
15	.2	.4	.4	20	10
16	.2	.4	.4	50	10
17	.3	.4	.3	20	10
18	.2	.5	.3	50	10
19	.3	.4	.3	30	80
20	.3	.3	.4	0	50
21	.2	.2	.6	0	150
22	.1	.6	.3	10	200
23	.2	.2	.6	0	500
24	.1	.3	.6	40	500
25	.3	.3	.4	40	500
26	.3	.4	.3	0	500
27	.3	.4	.3	40	500

Figure 1. Forest States.

1.1 Defining a Meaningful State Description

Professor Holling then devised a list of seven typical endemic conditions of a sub-forest (Figure 2) together with their appropriate vector state classification as in Figure 1. Then a new list was drawn up (Figure 3) where the states of the forest were described by seven parameters (summing to 1) giving the proportion or mix of the total forest in each condition category.

All four members of the Ecology group were then asked for their preference rankings of these twenty states.* In addition I calculated the ranking implied by the objective function from the stand model used in the Dynamic Programming formulation which used the maximization of forest profits as the objective. This is labeled "Forest Industry" in Figure 4 which gives the correlation between the five rankings. The marked difference between the ecologists and the "Forest Industry" partly reflects the fact that the experts were asked to think only in terms of the immediate future whereas the members of the Ecology group were thinking of the long term implications of the various states.

However, there were still differences in preferences within the group. Those of Holling and Clark were essentially the same, though they arrived at their orderings in completely different ways. Holling first created seven functions $v_1(p_1)$, $v_2(p_2), \ldots, v_7(p_7)$ which gave his subjective "value" to having a proportion p_i of the forest in condition i.[3] Hence he gave a value of

$$v_1(.0023) + v_2(.0061) + \cdots + v_7(0)$$

to forest state 2 in Figure 3, and then used these values to obtain his ranking. Clark fixed his sights on having about 5-10% of forest in condition 4 (outbreak) and on keeping the predictability of the forest high (by having the proportions in conditions 3 and 7 low). He was aiming for a manageable forest.

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*The Canadian Forestry Commission experts had returned to Canada.

[3]Note that he has thus made some assumption of independence between the parameters. For a discussion on this topic see Section 3.5 in [7].

Condition of Sub Region	State No.	Proportion 0-9 Yrs.	10-30 Yrs.	30-70+ Yrs.	Stress	Eggs
Post Outbreak Endemic	1	.5	.3	.2	40	.03
Mid-Endemic	2	.4	.4	.2	0	.03
Potential Outbreak	3	.15	.35	.5	0	.03
Triggered Outbreak	4	.15	.35	.5	0	2
Mid-Outbreak	5	.2	.4	.4	40	500
Disaster	6	.3	.4	.3	.6	100
Budworm Extinct	7	.15	.35	.5	0	0

Figure 2. Classification of Possible Stand Conditions.

Forest Mix No.	Proportion of Land In Condition Category 1	2	3	4	5	6	7
1	0	0	1.0	0	0	0	0
2	.0023	.0061	.975	.0016	.0083	.0017	0
3	.0047	.0122	.96	.0033	.0165	.0033	0
4	.0122	.0122	.95	.0033	.0165	.0033	.0025
5	.04	.04	.85	.01	.05	.01	0
6	.045	.05	.80	.02	.06	.02	.005
7	.08	.08	.70	.02	.10	.02	0
8	.026	.226	.70	.007	.033	.007	.001
9	.06	.04	.66	.08	.10	.02	.02
10	.08	.04	.66	.08	.10	.02	0
11	.03	.27	.53	.06	.15	.03	0
12	.12	.10	.53	.06	.15	.03	0
13	.0244	.48	.48	.0033	.0165	.0033	.0025
14	.04	.44	.45	.01	.05	.01	0
15	.045	.42	.43	.02	.06	.02	.005
16	.052	.41	.41	.041	.058	.012	.001
17	.16	.16	.4	.04	.2	.04	0
18	.35	.08	.35	.08	.10	.02	0
19	.08	.35	.35	.08	.10	.02	0
20	0	0	0	0	0	0	1

Figure 3. Types Of Forest Mixes.

	Rashid	Clark	Holling	Jones	Forest Industry
Rashid	1.00	.69	.40	.21	-0.52
Clark		1.00	.80	.39	-0.80
Holling			1.00	.63	-0.46
Jones				1.00	-0.34
Forest Industry					1.00

Figure 4. Correlation Matrix.

This led to a general discussion of what was desirable. Predictability seemed to be one preference.[4] Another was a desire to take the observed historical budworm outbreaks over time (a cycle of the forest moving through conditions 1-6 sequentially) into the same pattern over space that is, have the same proportion of the forest in each condition at any given time: "Controlled Outbreaks."

It was decided that the seven statistics used were not sufficient to describe the state of the forest and Holling set to work to come up with a more comprehensive list of indicators. The aim was to devise a system whereby we could place a decision maker in a chair where he could wave a magic wand and place the forest in condition A or condition B, where A and B were described by a set of summary statistics. Which statistics would he like to see to enable him to make a decision?

If he were a logger he would want to know the amount of wood in good condition for logging and the forest's potential for the next few years indicated by the level of budworm and so on.

For any given decision maker we would like to build up a set of statistics (indicators) which tells him all (or virtually all) that he wants to know in order to choose between A and B from his point of view.

To put this into practice one member of the group, Bill Clark, who is well acquainted with the problems of the area was appointed as a decision maker. After Holling had drawn up a long list of possible indicators we three had a meeting to discuss this list with Clark. Which ones was he interested in?

We then ran into a problem. When a decision maker evaluates the state that the forest is in now, he has to look to the future. He has to predict how the forest will behave, keeping in mind the present number of budworm, for example. Hence when he evaluates the forest condition he amalgamates in his mind how the forest will develop in the future. Now the way in which the forest develops depends on the method of treatment, that is, on the policies being used for logging, spraying and the like.

[4] I received a new perspective to the problem when I asked Holling why he ranked Forest Mix Number 20 in Figure 3 last. "Worst thing that could possibly happen," he said.

Now recall that we are looking for an objective function which we can optimize to find a best policy for treating the forest. But if the decision maker had known of this "best policy" he might have evaluated the forests differently, which changes the best policy. Right? As an example suppose that a simple device is discovered which removes all possibility of a budworm outbreak. The forest preferences of the decision maker will be altered. Although the result of the optimization procedure may not be as good as this "device" it nevertheless may change his preferences. What is needed is a set of statistics such that preferences for their values are independent of the policy being used.

This was achieved by letting the decision maker view a stream of statistics about the conditions of the forest over a sufficiently long time horizon. Hence the decision maker need not predict anything. He is to evaluate the stream of statistics as one single finished product and is not to worry about how likely they are or to wonder what policy achieved them. Then it is the job of the simulator to adjust its internal policies to maximize the value assigned by the decision maker.

Note then that now the type of statistics required has changed. It is not necessary to know the density of budworm at any given time; that was only needed to get an idea about the future state of the trees. Since we can also see the quantity of lumber obtained for the next 100 years and the amount spent on spraying, it is irrelevant to know how much budworm is present. (Indeed, it is probably irrelevant to know how much was spent on spraying--a simple net profit or loss may be sufficient.)

1.2 Finding the Attributes Relevant to our Decision Maker

Clark went through Holling's list of indicators deleting, adding and modifying. Some were discarded for being too minor, that is, not likely to influence his decisions, others because their implications were too difficult to understand (particularly standard deviations of data over space). The following list emerged of statistics for each year which Clark felt would affect his decisions.

Financial

X_1 = Profit of logging industry

X_2 = Cost of logging

X_3 = Cost of spraying

Logging Potential of Forest

X_4 = Amount of harvestable wood

X_5 = Percent of X_4 actually harvested in the given year

Forest Composition

X_6 = Diversity, a measure of the mixture of differing classes, age type of trees for recreational purposes. The higher the diversity the better

X_7 = Percentage of old trees

Observable Damage

X_8 = Percentage of defoliated trees

X_9 = Percentage of dead trees

X_{10} = Percentage of logged areas (no trees, stumps, etc.)

Social

X_{11} = Unemployment (measured by taking a certain logging level as full mill capacity)

Insecticide

X_{12} = Average dosage per sprayed plot.

In addition to the list above, a variance for these statistics taken over the 265 states was also included in some cases.

Ignoring the variances for a moment this still leaves $12 \times T$ statistics for a history of T periods. Indeed, eight of these statistics were originally intended for each site which would have given $(4 + 265 \times 8)T$ statistics.

Two fifty-year histories were generated by the simulation model with an initial set of internal policies and these statistics generated. Clark studied these listings and, following his earlier procedure for ordering the listing on Figure 3, essentially picked a few key statistics which he desired to maintain at a certain level and then checked to see that the others were not seriously out of line.

The idea at this stage was to give him a sequence of twelve or so such fifty-year listings of statistics and ask him to order them. Then he would be given the complete simulation outputs and asked to rank those; then the two lists would be compared. In this way the list of statistics would be modified and he would learn better what were their implications, so that eventually he would be able to arrive at the same orderings for the complete listings and the reduced set of statistic listings.

Owing to the mechanical difficulty of keeping IIASA's computer in operation and lack of time this was not done. For the sake of outlining the full procedure, let us assume that this was done.

We then set about the remaining list of statistics (X_1 to X_{12}) to reduce it to a manageable size of at most five or six per year.

I successfully argued that since the potentially harvestable wood, potentially harvestable wood harvested, cost of spraying and insecticide (X_4, X_5, X_3, X_{12}) were given over all periods, if these four attributes were going seriously wrong it would show up eventually somewhere else. The cost of logging could be deduced approximately from the profit figure and the unemployment level (which is proportional to wood harvested).

This left Profit, Diversity, Old Trees, Defoliation, Dead Trees, Logging Effects and Unemployment. It seems clear that all but the first and last are related to recreational, visual and environmental considerations. Could not these five statistics be amalgamated into a single statistic of recreation? Then we would have:

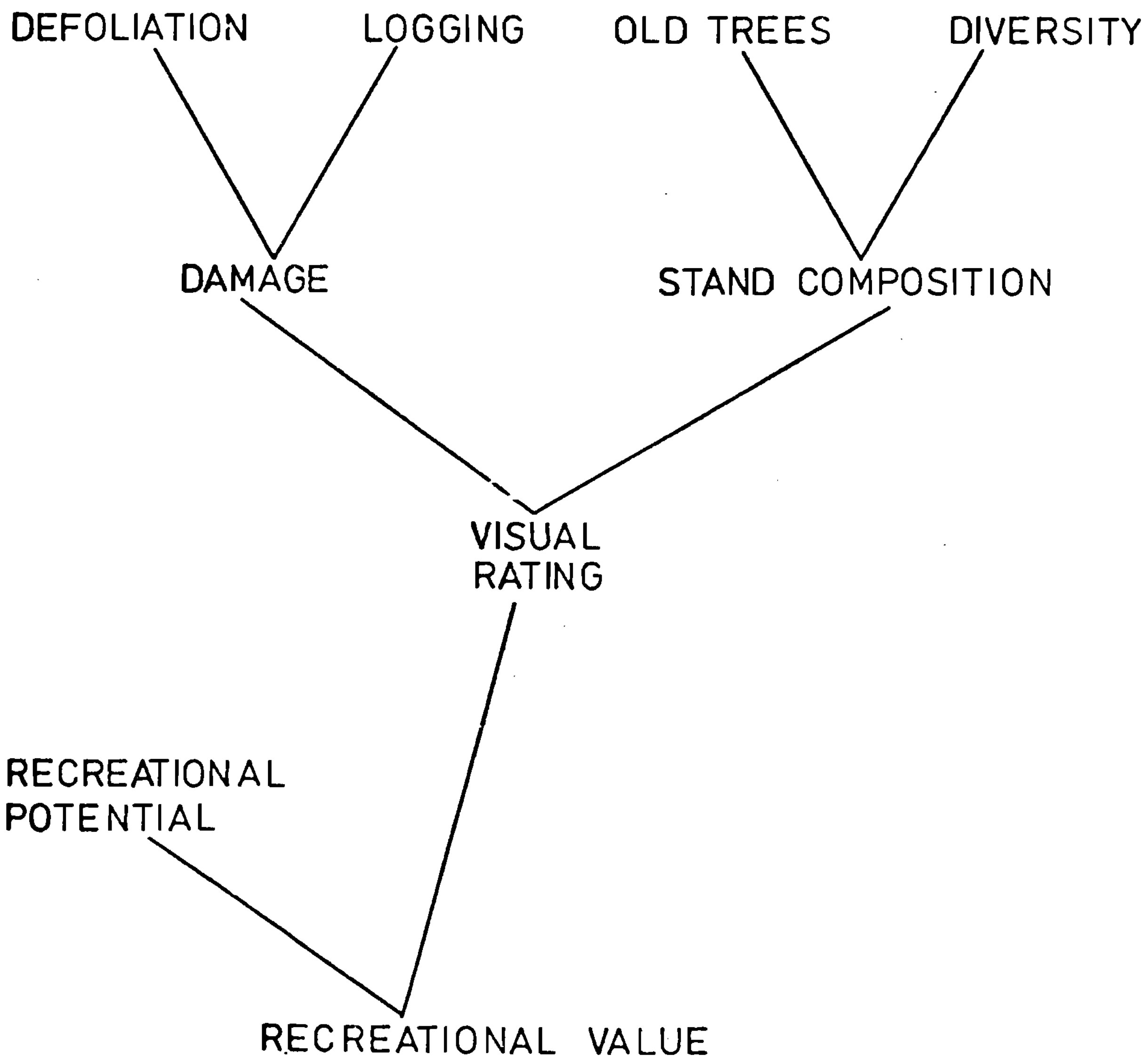
DEFOLIATION
LOGGING
OLD TREES
DIVERSITY
DAMAGE
STAND COMPOSITION
VISUAL RATING
RECREATIONAL POTENTIAL
RECREATIONAL VALUE

Figure 5.

P = Profit
U = Unemployment
R = Recreational Value of Forest

as attributes for each time period.

The general plan used by Clark for producing a recreational index is shown in Figure 5.

The recreational potential is a value assigned by the Canadian Forestry Commission to each region of the forest, indicating its accessibility to tourists and quality of surroundings (streams, lakes, gorges). Each region has a value 0, 30, 70, or 100.

For all the attributes in Figure 5 Clark divided the possible range into three classifications, for example, for defoliation a stand with 0-15% defoliation was good, 15-45% medium, 45-100% bad. Then where two attributes were combined in Figure 5 he used the rule displayed in Figure 6.

2 \ 1	GOOD	MEDIUM	BAD
GOOD	GOOD	MEDIUM	BAD
MEDIUM	MEDIUM	MEDIUM	BAD
BAD	BAD	BAD	BAD

Figure 6.

Hence a stand would be given a visual rating equal to the worst rating of its components. The final composition of recreational potential and visual rating was achieved by Figure 7.

	0	30	70	100
GOOD	BAD	MEDIUM	GOOD	GOOD
MEDIUM	BAD	MEDIUM	MEDIUM	MEDIUM
BAD	BAD	BAD	BAD	BAD

Figure 7.

Because some of the regions of the forest are not suitable for re-creation even under the best of conditions, the following are the number of regions possible in each recreation category.

$$0 \le \text{GOOD} \le 38$$

$$0 \le \text{MEDIUM} \le 262$$

$$3 \le \text{BAD} \le 265 \quad .$$

Since the total number of regions is fixed (265) it is only necessary to specify two of the above classifications; hence the final list of statistics to be tabulated for each period is:

P = Profit
U = Unemployment
G = Number of Good Recreational Regions
B = Number of Bad Recreational Regions.

2. Assessing a Value Function

The aim now is to derive a formula which takes the statistics (P_t, U_t, G_t, B_t) $t = 0, 1, 2 \ldots,$ and produces a value V such that if forest history α is preferred to forest history β then

$$V(\alpha) > V(\beta) \quad .$$

Over recent years a great deal of research has gone into devising good techniques for the assessment of value functions [7, 10]. These techniques were not tried on this problem. At the time of the study the methodology group at IIASA was experimenting with linear programming (L. P.) software and was eager for examples with which to work. I combined our two aims and used the following linear programming approach to find value functions.

Consider a value function V having two variables x, y. Suppose the decision maker has said that in the following pairs the first one in each is preferred by him to the second:

$$(2,\ 5)\ ,\ (\ 3,\ 0)$$
$$(3,-7)\ ,\ (\ 1,\ 1)$$
$$(0,\ 2)\ ,\ (-1,\ 2)\quad .$$

Thus

$$V(2,\ 5) - V(\ 3,\ 0) > 0$$
$$V(3,\ -7) - V(\ 1,\ 1) > 0 \qquad (2.1)$$

and

$$V(0,\ 2) - V(-1,\ 2) > 0\ .$$

Suppose we approximate V with a quadratic polynomial

$$V(a, y) = ax + by + cxy + dx^2 + ey^2 \quad ;$$

then we have that

$$\begin{aligned} -a + 5b + 10c - 5d + 25e &> 0 \\ 2a - 8b - 22c + 8d + 48e &> 0 \\ a \quad + 2c - d \quad &> 0 \end{aligned} \qquad (2.2)$$

are necessary requirements for V to be a valid function. Examples of polynomial expressions whose coefficients satisfy (2.2) are:

$$\begin{aligned} V_1(x, y) &= xy + y^2 \\ V_2(x, y) &= x + y^2 \\ V_3(x, y) &= -x^2 + y^2\ . \end{aligned} \qquad (2.3)$$

By obtaining more pairs of preference orderings, the set of possible coefficient values (a, b, c, d, e) may be reduced, for example, if we now find that in addition

$$(3,\ 2) > (0,\ 3)$$

then only the first of the three examples above is still valid.

If there are many alternative value functions for a given data set an L. P. algorithm will arbitrarily choose one of them unless it is given some selection criterion. Supplying an objective function for the L. P. problem gives the advantage that with the same data set the L. P. will always choose the same value function; hence as the data set alters slightly (because of new orderings) it is easier to see its effect on the resulting value function.

Note that if (a, b, c, d, e) is a solution of (2. 2) then so is any positive multiple of it; hence the arbitrary constraint

$$|a| + |b| + |c| + |d| + |e| = 100$$

was added to bound the problem.[5]

The objective criterion used was to maximize the minimum gap between preference rankings. In the example used above the gaps between the left hand side of (2. 1) and the right hand side (zero) using V_1 are 35, 26, 2; for V_2 are 24, 51, 1; and for V_3 are 30, 40, 1. Hence the minimum gap in each is 2, 1, 1, and so the maximum minimum gap is 2 and V_1 would be the preferred polynomial from that list.

In general, for a list of preferences

$$x_i^1 > x_i^2 , \qquad i = 1, 2, 3, \ldots, k$$

(> reads "is preferred to")
the full linear program would be

$$\begin{aligned} & s^* = \text{Max } s \\ \text{s.t.} \quad & V(x_i^1) - V(x_i^2) \geq s \qquad i = 1, \ldots, k \qquad (2.4) \\ & |a| + |b| + |c| + |d| + \cdots = 100 . \end{aligned}$$

Note that a valid function exists if and only if $s^* > 0$. If $s^* \leq 0$ the decision maker would be questioned more closely on doubtful orderings, or if he is resolute, a higher order approximation should be taken.

Returning to our study, with four attributes (P, U, G, B) per time period two qualitative assumptions were made by Clark (with my prompting) that were felt to be reasonable (in the first case) or necessary (in the second).

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[5] $|a|$ means $+a$ if $a > 0$, $-a$ if $a < 0$.

a) Preferences for profit and unemployment were "independent" from those of recreation. That is, the relative orderings of (P, U) pairs were independent of the level of the recreation so long as it was the same in each case.[6] The reverse was also felt to be true, that preferences for recreational alternatives were independent of profit/unemployment levels so long as these remained constant.

b) Clark's preferences for profit and unemployment levels in a year depended on what those levels were last year and would be next year. For example, a drop in profits to gain fuller employment is not too serious if compensatingly larger profits are made in the surrounding years. Also, an unemployment level of 10% is worse if it follows a year of full employment than if it follows a year of 10% unemployment; that is, he prefers a steady level to one which oscillates.

Clark felt that if we replaced P_t as a statistic by

$$Q_t = \frac{P_{t-1} + P_t + P_{t+1}}{3} ,$$

we might better justify a separable value function such as

$$V = \Sigma \, V_t(Q_t, U_t, G_t, B_t)$$

where V_t is a value function based on the figures for year t alone.

These assumptions enabled us to work with a value function

$$V_t(Q_t, U_t, G_t, B_t) = V_t(X(Q_t, U_t), Y(G_t, B_t)) ,$$

allowing us to calculate a value function for recreation independently of that for profit and unemployment.

Figure 8 shows the rankings given by Clark for the two value functions X, Y for any time period. Note that for (Q, U) it is an

[6]Preferential Independence, see Chapter 6 of [7].

ordered list and the rankings for recreation include some equalities. The last three in the recreation list were added when I discovered that the first polynomial expression was not suitably monotonic for extreme values.

These rankings produce the following value functions, a quadratic and a cubic polynomial approximation being used respectively.

$$X = 84.16Q + 2.26QU - 3.11Q^2 - 10.45\,U^2$$

and

$$Y = (71.8 - 1.88G)G^2 - B^2(5.88 + .00134B) + GB(19.63 - 0.597G + 0.185B)$$

(Q, U)		(G, B)		
(10, 0)	>	(15, 50)	>	(14, 0)
(0, 0)	>	(25, 50)	>	(24, 0)
(7, 8)	>	(34, 0)	>	(35, 50)
(20, 10)	>	(26, 0)	>	(38, 100)
(0, 5)	>	(28, 100)	=	(22, 0)
(4, 8)	>	(28, 130)	=	(16, 0)
(7, 10)	>	(38, 227)	=	(22, 50)
(-5, 0)	>	(26, 200)	=	(20, 150)
(30, 15)	>	(4, 0)	>	(1, 0)
(-5, 10)	>	(0, 50)	>	(0, 100)
(25, 25).		(30, 100)	>	(25, 100)

Figure 8

Then Clark gave the following orderings for sets of all four attributes (Figure 9). The groups are lists with each member of a group being preferred to the one below it.

10,	0,	16,	30	10,	10,	16,	30
25,	0,	16,	100	25,	10,	16,	100
0,	0,	16,	0	0,	10,	16,	0
-5,	0,	16,	50	-5,	10,	16,	50
-5,	0,	0,	50				
				5,	0,	10,	50
5,	4,	16,	50	10,	0,	10,	100
5,	7,	16,	30	0,	0,	10,	30
5,	0,	16,	100	-5,	0,	10,	0
5,	10,	16,	0				
0,	10,	16,	0				
				5,	0,	10,	40
				10,	0,	2,	40
				0,	0,	16,	40
				-5,	0,	25,	40

Figure 9.

With the aid of the functions X, Y these lists may be reduced to lists of two attributes; for example the first list becomes:

X(10, 0) , Y(16, 30)
X(25, 0) , Y(16, 100)
:
X(-5, 0) : Y(0, 50) .

The cubic approximation technique was used again to find a combined value function of

$$15.5y^2 + 357.3xy + 48.8x^3 + 1.8x^2y$$
$$-9{,}053x^2 - 3{,}039{,}500x - 195{,}197y$$

2.1 The Time Problem

So far the analysis has reduced the simulated history of the forest into a time stream of values, one per year. For two simulated histories with output values

$$(V_1^1, V_2^1, V_3^1, V_4^1, \ldots)$$

and

$$(V_1^2, V_2^2, V_3^2, V_4^2, \ldots) \quad ,$$

it is reasonable to suppose that the decision maker prefers the first history to the second if $V_k^1 \geq V_k^2$ for all k and if this inequality is strict for some k.[7]

But it is not possible at this stage for the analyst to say whether Clark would prefer a five year history

$$(2, 3, -1, 999, 7)$$

to one of

$$(2, 3, -1, 4, 8)$$

because we have no rules for intertemporal trade-offs. The only manageable model for such trade-offs is a linear assumption that

$$V = \Sigma \ a_t \ V(Q_t, U_t, G_t, B_t)$$

for some coefficients a_t, where presumably $a_t \geq a_{t+1} \geq 0$ for all t.

---

[7] Even this dominance argument is only valid because we are assuming that there are no interperiod dependencies of preferences. For example we could imagine that the 5 year stream

$$(1, 2, 3, 4, 5)$$

would be preferred to

$$(10, 9, 8, 7, 6)$$

if the decision maker abhorred a drop from one period to the next.

Had time permitted[8] we could have found viable values for the coefficients a_t by using the same technique which led to the coefficients in the second value function

$$V(X(Q, U), Y(G, B)) \quad .$$

However, at that stage we agreed that the simulation model should generate different histories using a variety of policies and calculate the value

$$V = \Sigma a^t V(Q_t, U_t, G_t, B_t)$$

for a range of constants a, $0 < a < 1$.

3. The Assessment of a Utility Function

Even if we ignore the crude manner in which the time streams of the attributes were evaluated there remains another important element in the effective evaluation of policies by use of an objective function. The particular history generated by the simulator depends upon the initial condition of the forest, the many complex equations governing the growth of budworm, trees, the effects of predators and other factors, but all of these are deterministic only if the weather pattern is known. Different weather patterns will produce different histories and hence a policy cannot be judged purely on the results of one run, its effects must be considered under all types of weather futures. Fortunately, this problem may be overcome if a utility function is used rather than a value function. A utility function not only has the properties of a value function, but in situations in which outcomes are uncertain, its expected value provides a valid quantity for making rankings.

That is, if $u(P, W)$ represents the utility (or value) of the forest history which results from using policy P when weather history W occurs and $f(W)$ is the probability that weather pattern W does occur then

$$\Sigma \; u(P, W) \, f(W) = V(P)$$

where the sum is taken over all possible weather patterns, is a legitimate value function over policies P.

[8] Bill Clark returned to Canada in July 1974.

Assessment procedures for utility functions are similar to those for value functions except that in addition, the decision maker's attitude towards risk taking must be incorporated. As with value functions it is useful to recognize assumptions that will break down the assessment of one function with many attributes into one of assessing several utility functions each having at most one or two attributes.

One such assumption is utility independence. For a utility function $u(x, y)$, where x and y might be vectors of attributes, if the decision maker's attitude towards risk taking in situations where only the outcome of x is uncertain but y is fixed and known, is independent of what that fixed value of y is, then attributes X are said to be utility independent of Y. It is important to realize that X may be utility independent of Y even if X and Y involve factors which in other respects are closely related. For more information and examples see Chapter 5 of Keeney and Raiffa [7]. The functional statement of this property is that for any two values of Y, y^1 and y^2 say,

$$u(x, y^1) = a + b\, u(x, y^2) ,$$

for some constants a and b, where b must be positive.

In our problem which has four attributes per year, with a horizon of T periods (T will be in the range 50-200) we require a utility function of $4T$ attributes so that some extensive assumptions will be required. Meyer [9] for example has shown that for a utility function $u(x_1, x_2, \ldots, x_T)$ if each subset of attributes $\{X_1, \ldots, X_t\}$ is considered to be utility independent of $\{X_{t+1}, \ldots, X_T\}$ and vice versa, then the utility function has either an additive form

$$\sum_{t=1}^{T} a_t\, u_t\, (x_t) \qquad (3.1)$$

for some positive constants a_t or a multiplicative form

$$\prod_{t=1}^{T} (b_t + c_t\, u_t(x_t)) \qquad (3.2)$$

for some constants b_t and c_t, where in each case $u_t(x_t)$ is a utility function over X_t alone.

These forms were inappropriate for our case principally because Clark's attitude towards risk taking for levels of unemployment in one period depended on the levels of unemployment in the year before, and the year after, and hence Meyer's assumptions of utility independence did not apply.

Not only that but Clark wished to make an assumption of <u>stationarity</u> (see Koopmans [8]) that is, he wished to treat all years equally, both with regard to value orderings and in risk taking. This meant that the coefficients a_t, b_t, c_t and the functions u_t would all be independent of their suffix t implying that all time streams which were merely permutations of one another would be assigned equal utility, which was not the case. For example, dealing only with levels of employment he preferred the stream (100, 100, 90, 90, 100) to (100, 90, 100, 90, 100) because of the reduced variance between years.[9]

Fishburn [4] used assumptions called Markovian dependence to produce a form

$$u(x_1, \ldots, x_T) = \sum_{t=1}^{T} u_t(x_t, x_{t+1}) - \sum_{t=2}^{T-1} u_t(x_t, x^0) \tag{3.3}$$

where $u_t(x_t, x_{t+1})$ is a utility function over the two attributes x_t, x_{t+1}. Whilst this does allow for some interdependency between attributes in neighbouring periods Clark was quite firm in preferring the lottery

$$L_1: \begin{cases} \xrightarrow{1/4} (100,\ 100,\ 100,\ 100,\ 100) \\ \xrightarrow{1/4} (100,\ 50,\ 100,\ 50,\ 100) \\ \xrightarrow{1/4} (\ 50,\ 100,\ 50,\ 100,\ 50) \\ \xrightarrow{1/4} (\ 50,\ 50,\ 50,\ 50,\ 50) \end{cases}$$

to that of

$$L_2: \begin{cases} \xrightarrow{1/4} (100,\ 100,\ 50,\ 50,\ 100) \\ \xrightarrow{1/4} (\ 50,\ 50,\ 100,\ 100,\ 50) \\ \xrightarrow{1/4} (\ 50,\ 100,\ 100,\ 50,\ 50) \\ \xrightarrow{1/4} (100,\ 50,\ 50,\ 100,\ 100) \end{cases}$$

---

[9] We switched from talking in terms of unemployment to employment so that the symbol u would not be used simultaneously for utility and a level of unemployment. $E_t = 100 - U_t$ is the new attribute.

where the figures are percentage of employment in 5 successive years. For (3. 3) to be valid for Clark's preferences he should have been indifferent between the two lotteries.

3. 1 Finding Appropriate Assumptions

To find a functional form that would be acceptable to him I considered assumptions involving conditional utility independence. This condition says, in essence, that if the set of attributes is divided into three parts X, Y and Z then X is conditionally utility independent of Y if whenever Z is fixed at some level and we regard the problem as now only having two attributes X and Y then X is utility independent of Y and that this is true for all fixed values of Z. For more detailed expositions of this concept see Chapter 6, Keeney and Raiffa [7] or Bell [2].

The idea was to assume that each subset $\{X_1, \ldots, X_{t-1}\}$ was conditionally utility independent of $\{X_{t+1}, \ldots, X_T\}$ and vice versa. This is quite similar to the assumptions used by Meyer to obtain (3. 1) and (3. 2) but does not make any assumption of independence of preferences for X_t on either X_{t-1} or X_{t+1}.

These assumptions led (for $T \geq 4$) to the result that either

$$u(x_1, x_2, \ldots, x_T) = \sum_{t=1}^{T-1} u_t(x_t, x_{t+1}) - \sum_{t=2}^{T-1} u_t(x_t, x_{t+1}^0) \tag{3.4}$$

or

$$u(x_1, x_2, \ldots, x_T) = \left[\prod_{t=2}^{T-1} (\lambda + u_t(x_t, x_{t+1}^0))\right]^{-1} \left[\prod_{t=1}^{T-1} (\lambda + u_t(x_t, x_{t+1}))\right] - \lambda \tag{3.5}$$

where λ is a constant and

$$u_t(x_t, x_{t+1}) \equiv u(x_1^0, x_2^0, \ldots, x_{t-1}^0, x_t, x_{t+1}, x_{t+2}^0, \ldots, x_T^0)$$

where x_i^0 is any fixed value of X_i, so that for example $u_t(x_t^0, x_{t+1}) = u_{t+1}(x_{t+1}, x_{t+2}^0)$, and where u was scaled so that $u(x_1^0, \ldots, x_T^0) = 0$. For a proof of this result see Bell [2].

Note that (3.4) is exactly (3.3) but that (3.5) not only allows interperiod dependencies but also is able to differentiate between the lotteries L_1 and L_2.

Bill Clark returned to IIASA for the summer of 1975 and I quizzed him on the appropriateness of the assumptions which led to (3.5). He agreed that they seemed appropriate and so we proceeded to assess his utility function over the attributes $\{P_t, E_t, G_t, B_t\}$, $t = 1, \ldots, T$.

Questioning soon established that his preferences for the recreation time streams $\{G_1, B_1, G_2, B_2, \ldots, G_T, B_T\}$ were mutually utility independent with those of profit and employment $\{P_1, E_1, P_2, E_2, \ldots, P_T, E_T\}$, enabling us to use the formula (see Keeney [6])

$$u(\underline{p}, \underline{e}, \underline{g}, \underline{b}) = u_R(\underline{g}, \underline{b}) + k_1\, u_S(\underline{p}, \underline{e}) + k_2\, u_R(\underline{g}, \underline{b})\, u_S(\underline{p}, \underline{e}) \qquad (3.6)$$

where u_R is a utility function for recreation and u_S a social utility function, k_1 and k_2 being constants, k_2 later being identified as zero. I should emphasize that Clark was not one to make assumptions out of expediency, whenever he agreed that an assumption was valid, we had discussed the implications at length and verified that his preferences reflected the required pattern or were sufficiently close.

3.2 The Utility Function for Recreation

For the recreation streams, he felt that the assumptions of Meyer were appropriate and in addition that in any given time period G_T and B_T were mutually utility independent. To determine whether the additive form (3.1) or multiplicative form (3.2) was the appropriate one to use I asked him if he had any preference between the following two lotteries

$$L_3: \begin{cases} 1/2 & G_1 = 30,\ G_2 = 30 \\ 1/2 & G_1 = 0,\ G_2 = 0 \end{cases} \qquad\qquad L_4: \begin{cases} 1/2 & G_1 = 30,\ G_2 = 0 \\ 1/2 & G_1 = 0,\ G_2 = 30 \end{cases}$$

where G_1 and G_2 are the number of good recreational areas in two successive years and B_1 and B_2 are in all outcomes assumed to be fixed.[10] If the additive form (3.1) was to be appropriate he should have been indifferent between the two but in fact he preferred the second lottery on the grounds that he was very averse to having two very bad years together. This meant that the form of the recreational utility function was

$$u_R(\underline{g}, \underline{b}) = \prod_{t=1}^{T} (\alpha + \beta(k_1 + k_2\, u_G(g_t) + k_3 u_B(b_t) + k_4 u_G(g_t) u_B(b_t))) \qquad (3.7)$$

where the various constants are independent of the time subscript because of the assumption of stationarity.

The marginal utility functions u_G and u_B for the number of good and bad areas were assessed in the usual manner (see for example Raiffa [10]) by asking questions of the form "what value $G = g^*$ for certain do you feel is equally preferable to a 50-50 gamble between $G = 20$ and $G = 5$?"

Thus $u_G(g)$ was assessed as in Figure 10 which was fitted quite closely by the exponential curve $u_G(g) = 1 - \exp(-0.08g)$: The function $u_B(b)$ was slightly more complicated (Figure 11) being fitted in two pieces by

$$u_B(b) = 0.7 + 0.35(1.0176 - 0.0176 \exp(.0225b)) \qquad b < 180$$

$$u_B(b) = -0.3 + 0.35(+1.463 + 28.222 \exp(-0.0164b)) \qquad b \geq 180$$

The constants k_1, k_2, k_3, k_4 were calculated by fixing

$$\begin{aligned} &k_1 + k_2 u_G(40) + k_3 u_B(0) + k_4 u_G(40) u_B(0) = 1 \\ &\text{and} \\ &k_1 + k_2 u_G(0) + k_3 u_B(265) + k_4 u_G(0) u_B(265) = 0 \end{aligned} \qquad (3.8)$$

and then using indifferent pairs given by Clark

$$\begin{aligned} (20, 150) &\sim (9, 0) \\ (15, 100) &\sim (25, 150) \\ (7, 0) &\sim (15, 150) \end{aligned} \qquad (3.9)$$

[10] Recall that because G_t and B_t are mutually utility independent it is not necessary to specify at what level B_1 and B_2 are fixed.

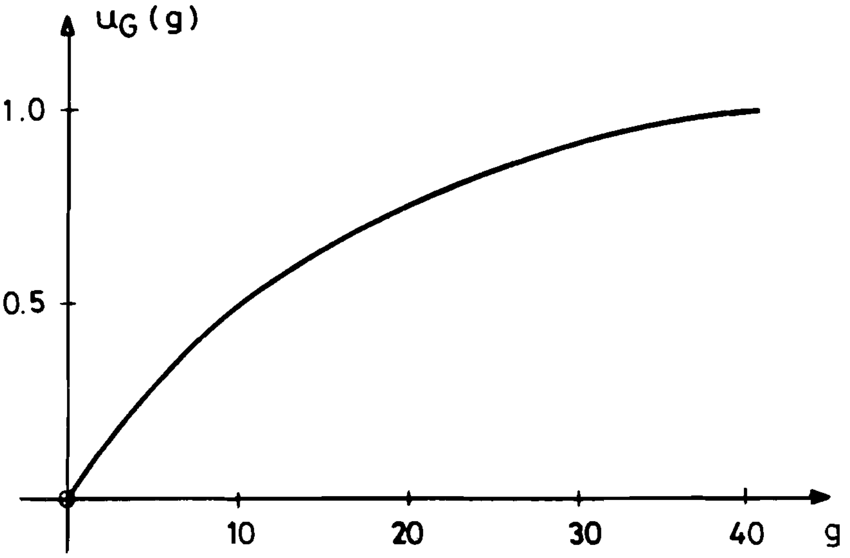

Figure 10. Utility Function for the Number of Good Recreational Areas.

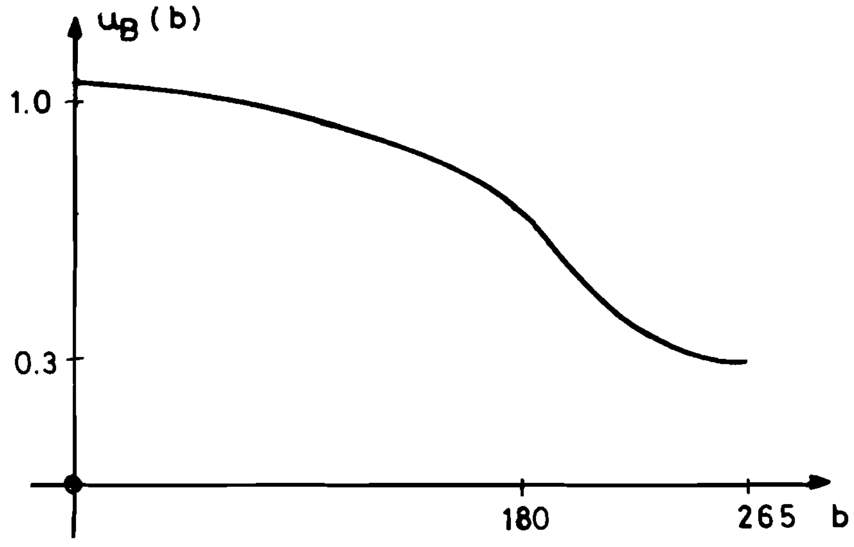

Figure 11. Utility Function for the Number of Bad Recreational Areas.

to form three more equations in the k_i's. Taking all the combinations of two pairs from (3. 9) together with (3. 8) provided three solutions for the k's which are exhibited in Figure 12.

Pair	k_1	k_2	k_3	k_4
1+2	-.48	-.60	.15	1.42
2+3	-.55	-.57	.17	1.37
1+3	-.71	-.45	.22	1.23

Figure 12

The pair 1+3 seemed to be the least reliable of the three since it involves two pairs that are quite similar. Also since Clark always prefers to increase the number of good areas if possible, the constraint

$$k_2 + k_4\, u_B(b) > 0 \tag{3.10}$$

should be true for all b, and for similar reasons also

$$k_3 + k_4\, u_G(g) > 0 \tag{3.11}$$

although since k_3 and k_4 are positive this is not important. The smallest value of $u_B(b)$ is 0. 32 and hence the coefficients should be chosen so that

$$k_2 > -0.32\, k_4\ ,$$

which none of the solutions in Figure 12 satisfy. However extrapolating the first two sets of coefficients until (3. 10) was satisfied gave coefficients of

$$k_1 = -1.201 \qquad k_2 = -.291 \qquad k_3 = .356 \qquad k_4 = .905$$

and the implied utility function using these coefficients made all of the equivalences in (3. 9) hold almost exactly!

To finalize the recreational utility function now only required the knowledge of α and β in (3. 7).

For this I asked him to consider a time stream in which all values after year 2 are assumed fixed and that the number of bad areas is fixed at 100 for years 1 and 2. So, considering only vectors of the type (number of good areas in year 1, number of good areas in year 2) he was to give values g_1, g_2, g_3, g_4, g_5 such that

$$\begin{aligned} (g_1, g_1) &\sim (15,\ 5) \\ (g_2, g_2) &\sim (20,\ 5) \\ (g_3, g_3) &\sim (25,\ 5) \\ (g_4, g_4) &\sim (30,\ 5) \\ (g_5, g_5) &\sim (35,\ 5) \end{aligned} \tag{3. 12}$$

His answers were 9, 10, 12, 14 and 15 respectively. In attempting to solve (3. 7) with this information it became clear that in fact the additive form (3. 1) rather than the multiplicative fits (3. 12). Referring this apparent inconsistency back to Clark we established that his preference between the lotteries L_3 and L_4 was caused by the rather extreme nature of the consequence in L_3 of two successive years with zero good recreational areas. When I replaced the zeros in L_3 and L_4 with something positive he became indifferent. Perhaps this should indicate a singularity in the function u_G at $G = 0$ but I chose to ignore this.

Thus the recreational utility function was established as

$$u_R(\underline{g}, \underline{b}) = \sum_{t=1}^{T} \{.356\, u_B(b_t) + [.905\, u_B(b_t) - 0.291]\, u_G(g_t)\} \tag{3. 13}$$

3. 3 <u>The Social Utility Function</u>

Clark having accepted the conditional utility independence assumptions necessary to validify the use of equation (3. 5) we chose fixed levels of $p_t^0 = 0$ million dollars per year and $e_t^0 = 100$ percent employment. The main task was thus to assess, for each $t = 1, 2, \ldots, T-1$, the function

$$u_S(p_1^0, e_1^0, \ldots, p_{t-1}^0, e_{t-1}^0, p_t, e_t, p_{t+1}, e_{t+1}, p_{t+2}^0, e_{t+2}^0, \ldots, p_T^0, e_T^0)$$

or in a shorthand notation where we omit explicit reference of attributes at their fixed values, $u_S(p_t, e_t, p_{t+1}, e_{t+1})$. Whilst previous assumptions about independence between attributes had either appeared from questioning or had been prompted by me, on this occasion Clark volunteered the information that when considering his preferences for employment in a given year, he was only concerned with the levels of profit in the same year and the levels of employment in the previous and later year, and that his preferences for profit in a given year depended only upon the level of employment in that year. This implied that for the attributes $P_t, E_t, P_{t+1}, E_{t+1}$ we could assert that P_t was mutually conditionally utility independent with P_{t+1} and E_{t+1} and similarly that P_{t+1} was mutually conditionally utility independent (m. c. u. i.) with P_t and E_t. This set of additional assumptions proved to be most useful. Consider the assumptions leading to (3.4) and (3.5) for $T = 4$. In full they are

$$X_1 \quad \text{m. c. u. i.} \quad \{X_3, X_4\}$$

and

$$\{X_1, X_2\} \quad \text{m. c. u. i.} \quad X_4$$

Those that Clark had proposed were

$$P_t \quad \text{m. c. u. i.} \quad \{E_{t+1}, P_{t+1}\}$$

and

$$\{P_t, E_t\} \quad \text{m. c. u. i.} \quad P_{t+1} ,$$

showing that (3.4) or (3.5) was appropriate for the restricted function $u_S(p_t, e_t, p_{t+1}, e_{t+1})$.

It is easy to show (set all attributes at their fixed level except for P_2) that the assumption of stationarity forces $u_S(p_t, e_t, p_{t+1}, e_{t+1})$ and the full function $u_S(\underline{p}, \underline{e})$ either both to be additive or both to be multiplicative and if multiplicative to have the same parameter λ. The non indifference between lotteries L_1 and L_2 showed the multiplicative form to be the appropriate one. Hence using all the declared independence assumptions, the social utility function could be expressed as

$$u_S(\underline{p}, \underline{e}) = \frac{\prod_{t=1}^{T} [\lambda + u_A(p_t, e_t)] \prod_{t=1}^{T-1} [\lambda + u_B(e_t, e_{t+1})]}{\prod_{t=1}^{T} [\lambda + u_B(e_t, e_{t+1}^0)] \prod_{t=2}^{T-1} [\lambda + u_B(e_t, e_{t+1}^0)]} - \lambda \qquad (3.14)$$

for some constant λ, where u_A and u_B are each two attribute utility functions for which $u_A(p_t^0, e_t) = u_B(e_t, e_{t+1}^0) = u_B(e_{t-1}^0, e_t)$. Thus the assessment problem rested on finding u_A, u_B and λ .

3.3.1 The Interperiod Employment Function

We began with u_B. Recall that $u_B(e_t, e_{t+1})$ is, in effect, $u_S(\underline{p}^0, e_1^0, \ldots, e_{t-1}^0, e_t, e_{t+1}, e_{t+2}^0, \ldots, e_T^0)$ so that when questioned about his preferences he was to compare employment streams of the form $(100, 100, \ldots, 100, e_t, e_{t+1}, 100, \ldots, 100)$. I proceeded by fixing the level of E_t at some value $\bar{e}_t$ and then assessing the one attribute function $u_1(e_{t+1} | E_t = \bar{e}_t)$. It appeared that for lotteries involving levels of E_{t+1} that were higher than $\bar{e}_t$ he was risk averse but was risk prone for levels of E_{t+1} lower than $\bar{e}_t$. The reason was that the previous year's employment level represented a goal or aspiration level for the present year, part of his desire for stability in employment levels. The only departure from this was that if I fixed $\bar{e}_t$ at anything higher than 80% he was never risk prone for values of $e_{t+1} \geq 80$, because any year in which employment was at least 80% was "satisfactory". Hence his "goal" was $\min\{\bar{e}_t, 80\}$. A typical graph of $u_1(e_{t+1} | E_t = \bar{e}_t)$ is shown in Figure 13.

The two piece function was fitted again quite closely by an exponential curve of the form

$$-\exp\{-0.03\, e_{t+1}\} \qquad e_{t+1} \geq \min\{80, \bar{e}_t\} \qquad (3.15a)$$

and

$$-\exp\{+0.03\, e_{t+1}\} \qquad e_{t+1} \leq \min\{80, \bar{e}_t\} \qquad (3.15b)$$

In a similar way $u_2(e_t | E_{t+1} = \bar{e}_{t+1})$ was assessed, exhibiting much the same features. Since E_{t-1} was fixed at 100 there was a desire to achieve this goal with e_t but this was tempered by the opposite desire not to exceed $\bar{e}_{t+1}$. The result seemed to be that Clark preferred the pattern (100, 80, 90, 100) to (100, 90, 80, 100); that although a drop from 100 to 80 was serious it was better to suffer that and follow it with two years of improvement than be faced with two years of falling employment, even though this was ultimately followed by an increase from 80 to 100. The function $u_2(e_t | E_{t+1} = \bar{e}_{t+1})$ was of the form

$$-\exp\{-.03\, e_t\} \qquad e_t \geq \min\{80, \bar{e}_{t+1} - 5\} \tag{3.16a}$$

and

$$-\exp\{+.03\, e_t\} \qquad e_t \leq \min\{80, \bar{e}_{t+1} - 5\} \tag{3.16b}$$

That the exponential coefficients in (3.15) and (3.16) are all shown equal to 0.03 was because they were all fairly close and the implications seemed insensitive to this parameter.

To obtain the combined function $u_B(e_t, e_{t+1})$ I used the fact that

$$u_B(e_t, e_{t+1}) = f(e_{t+1}) + g(e_{t+1})\, u_2(e_t | E_{t+1} = e_{t+1}) \tag{3.17}$$

and

$$u_B(e_t, e_{t+1}) = h(e_t) + k(e_t)\, u_1(e_{t+1} | E_t = e_t) \tag{3.18}$$

for some functions f, g, h, k, and solving these gives

$$[u_2(e_t^0 | e_{t+1}) - u_2(e_t^* | e_{t+1})]u_B(e_t, e_{t+1}) = (a_1 + a_2 u_1(e_{t+1} | e_t^*))[u_2(e_t^0 | e_{t+1}) - u_2(e_t | e_{t+1})]$$
$$- (a_3 + a_4 u_1(e_{t+1} | e_t^0))[u_2(e_t^* | e_{t+1}) - u_2(e_t | e_{t+1})] \tag{3.19}$$

where a_1, a_2, a_3, a_4 were constants calculated in much the same manner as the constants k_1, k_2, k_3, k_4 were for the recreational function, and $e_t^* \neq e_t^0$ was any constant, chosen to be 50.

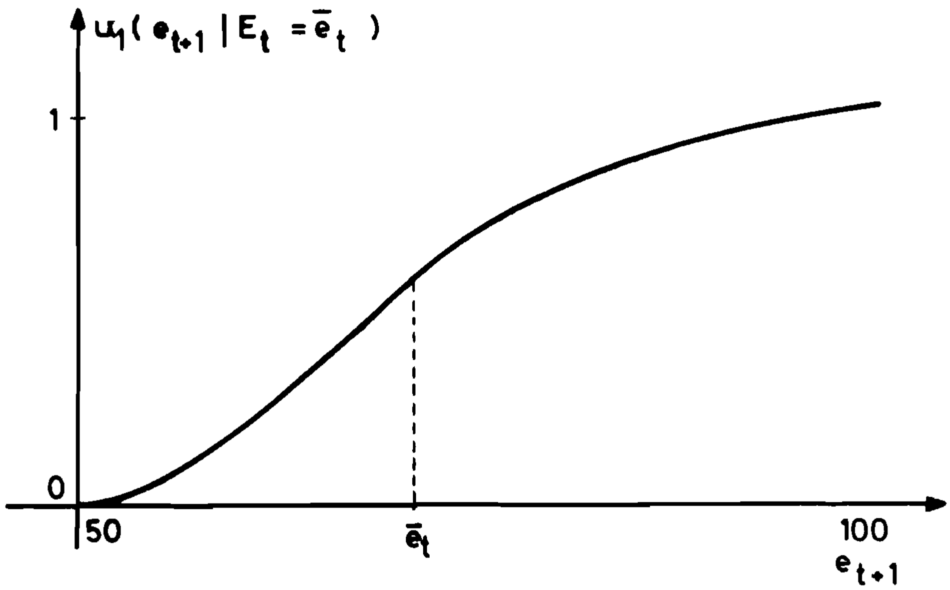

Figure 13. Utility Function for Employment, Conditional on the Level of Employment in the Previous Year.

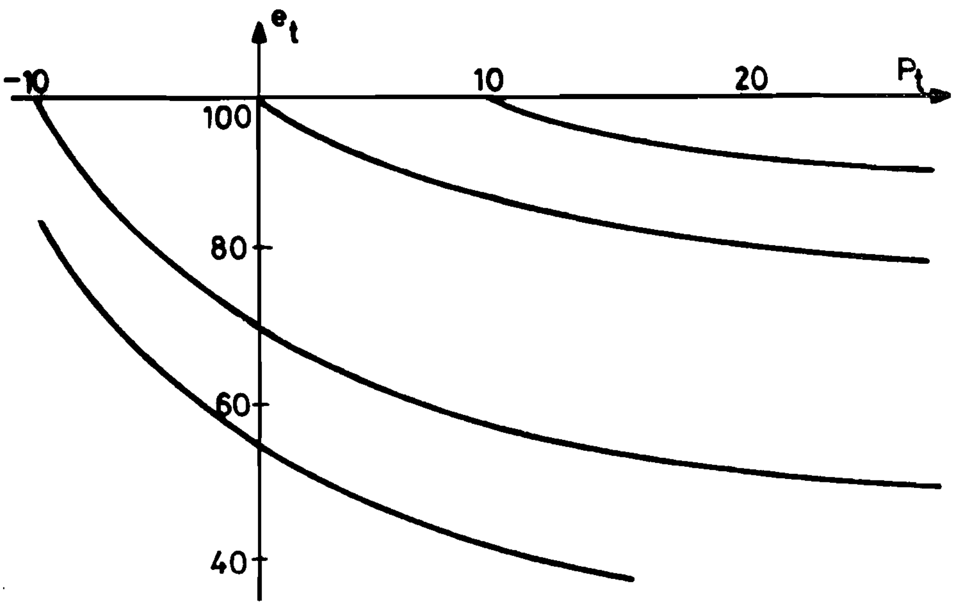

Figure 14. Profit/Employment Indifference Curves for a Single Year.

3.3.2 The Profit-Employment Tradeoffs

The next step was to calculate $u_A(p_t, e_t)$. This could have been done in the same way as for u_B but Clark found it easier to think in terms of indifference curves between p_t, e_t pairs. Hence on graph paper with axes of e_t from 50 to 100 and of p_t from -10 to + 30 we located on it pairs (p_t^1, e_t^1), (p_t^2, e_t^2) between which Clark was indifferent, again bearing in mind that all other attributes were at their fixed levels, and then fairing in sample indifference curves. The result is exhibited in Figure 14.

What was delightful to me as the analyst was that if we describe the above indifference curves by the functional relationship

$$\phi(p_t, e_t) = \text{constant}$$

for varying constants it was empirically observable that

$$\varphi(p_t^1, e_t^1) = \phi(p_t^2, e_t^2) \Longleftrightarrow \varphi(p_t^1, e_t^1 + \varepsilon) = \phi(p_t^2, e_t^2 + \varepsilon) \qquad (3.20)$$

for all values of ε. This meant that quantification of ϕ was easy. I used a polynomial curve fitting program on one of the indifference curves, finding that a quadratic was sufficiently accurate and by substituting in the other curves confirmed directly the visual observation that property (3.20) held.[11]

The indifference curves were

$$\phi(p_t, e_t) = e_t + 1.9\ p_t - 0.04\ p_t^2 = \text{constant} .$$

The next assessment task was now to assess a utility function over ϕ, the value function. Using the indifference curves, any pair (p_t, e_t) could be replaced by an equivalent pair $(p_t^*, 100)$ where $\phi(p_t^*, 100) = \phi(p_t, e_t)$ or

$$p_t^* = 23.75 - \frac{1}{2}\sqrt{2,256.3 + 100(100-e_t) - 190\ p_t + 4\ p_t^2} .$$

[11] For an example of this property in connection with time streams see Bell [1].

The one dimensional utility function $u_A(p_t, 100)$ had earlier been assessed in the usual manner in the range -8 to + 26, the result depicted in Figure 15.

Recall that if Clark has been consistent we should be able to observe that $u_A(0, e_t) = u_B(e_t, 100)$. As a check I calculated the <u>implied</u> function $u_A(p_t, 100)$ using $u_B(e_t, 100)$ and $\phi(p_t, e_t)$. Actually, comparison was only possible between $-8 \le p_t \le 0$ but here the agreement was close. The full implied function $u_A(p_t, 100)$ is shown in Figure 16 for $p_t \le 0$.

Note that because $\phi(-9.15, 100) = \phi(0, 80)$ the implied function $u_A(p_t, 100)$ becomes risk prone for $p_t \le -9.15$. From a consistency check point of view we were perhaps fortunate that the direct assessment of $u_A(p_t, 100)$ did not involve a range that low!

For later calculations the value of $u_A(p_t, e_t)$ was taken to be

$$1 - \exp[-.055\, p_t^*] \qquad \text{for} \qquad \phi(p_t, e_t) \ge 100$$

where

$$\phi(p_t^*, 100) = \phi(p_t, e_t)$$

and

$$u_B(e_t^*, e_{t+1}^0) \qquad \text{for} \qquad \phi(p_t, e_t) < 100$$

where

$$\phi(0, e_t^*) = \phi(p_t, e_t) \quad .$$

The functions u_A and u_B were scaled so that $u_A(0, 100) = u_B(100, 100) = 0$ and $u_A(0, 50) = u_B(50, 100) = u_B(100, 50) = -1$.

3.3.3 <u>Evaluating the Constant λ</u>

To complete the assessment of $u_S(\underline{p}, \underline{e})$ it remained to calculate λ, the constant in equation (3.14). What this constant controls is the degree to which the decision maker prefers a mixture of good years and bad years to appear in bunches or interspersed. So I began by asking Clark if he had to arrange 50 good years and 50 bad years in a sequence of 100, how would he do it ? Recall that if we were not using

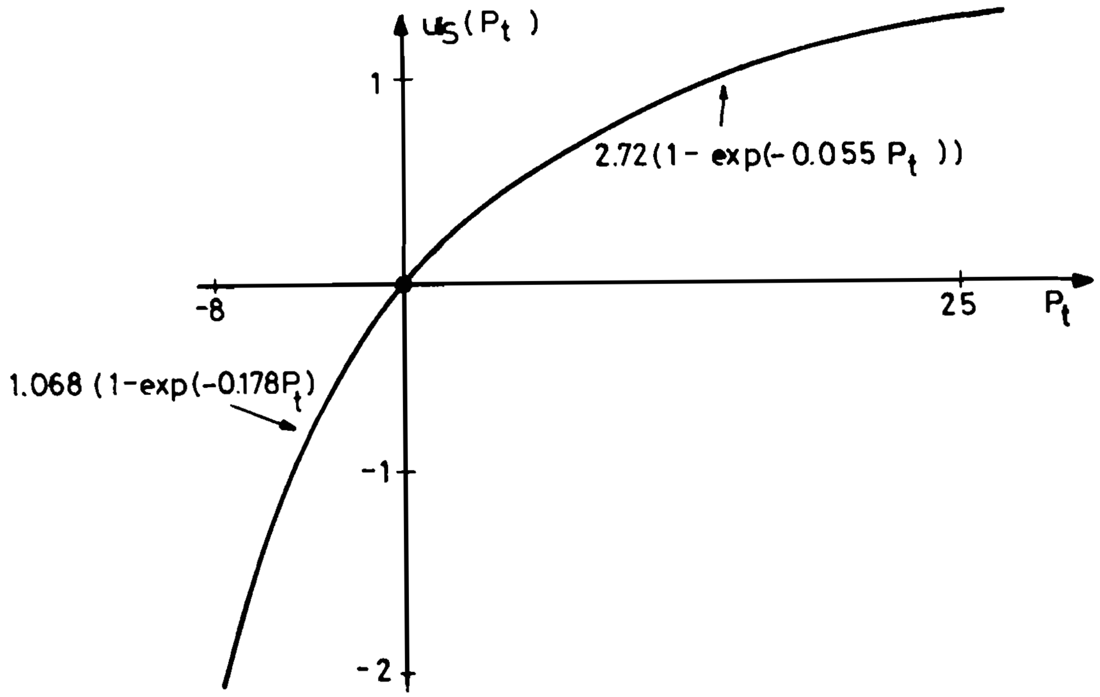

Figure 15. Marginal Utility Function for a Single Year's Profits.

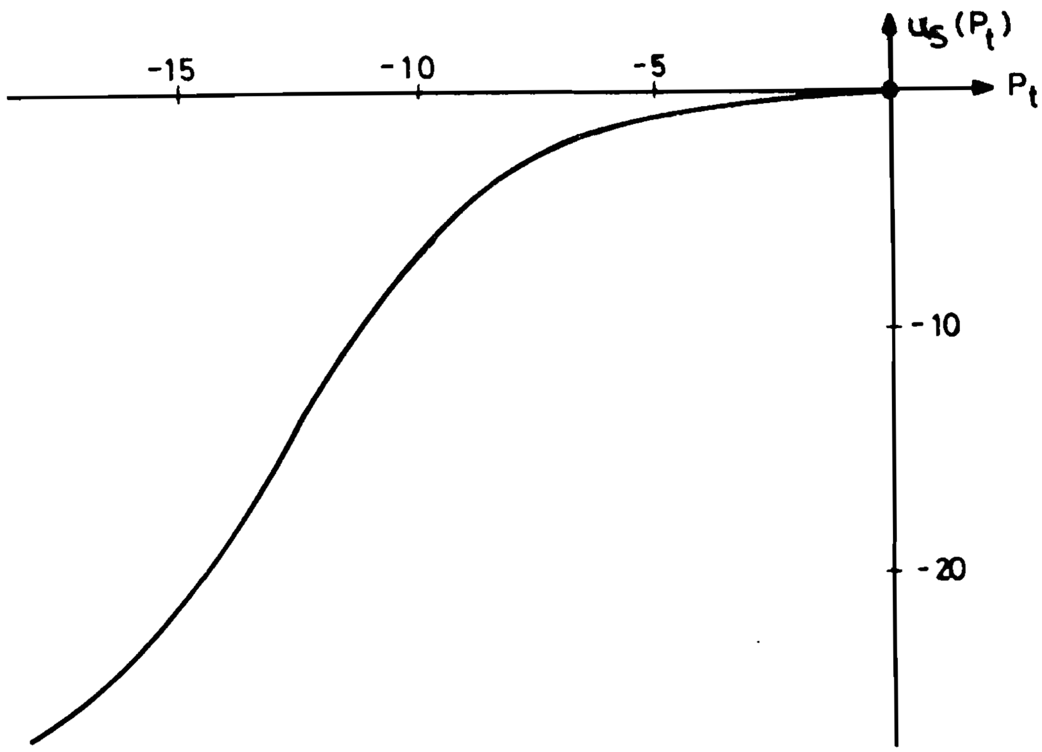

Figure 16. The Marginal Utility Function for Profit Implied by Figures 13 and 14.

functions with interperiod dependencies such a question would not arise since all permutations would be equally preferred since Clark is adopting a "no discounting" policy. He certainly disliked both the options in which good and bad alternated and in which all 50 good years came together. As $1/\lambda$ becomes larger the tendency is for the utility function to prefer smaller blocks and as it becomes smaller (and negative) to prefer the large bunching.

I asked Clark to consider the following four streams of seven year employment figures

(i)	100, 100, 70, 70, 70, 100, 100
(ii)	100, 70, 70, 70, 70, 100, 100
(iii)	100, 70, 70, 70, 70, 70, 100
(iv)	100, 70, 70, 100, 70, 70, 100

and tell me what statements he could make regarding his preferences between them. He established that (i) was the best, (iii) the worst and felt that (iv) was preferable to (ii) "if anything."

I drew the following graph (Figure 17) which shows the utility of (i) fixed at 1, the utility of (iii) fixed at zero and the corresponding utilities of (ii) and (iv) as functions of $1/\lambda$ using (3. 14). Note that $1/\lambda = 0$ corresponds to the additive case.

The near indifference of (ii) and (iv) suggested that $1/\lambda$ should be chosen to be about 1 but there were other considerations. In order to avoid discontinuities in $u_S(\underline{p}, \underline{e})$ it must be the case that $\lambda + u_B(e_t, e_{t+1})$ and $\lambda + u_A(p_t, e_t)$ are either always both negative or always both positive. If both positive then

$$\lambda \geq \max\{-u_B(50, 50), \ -u_A(-20, 50)\}$$

is a constant and if both negative then

$$\lambda \leq \min\{-u_B(100, 100), -u_A(20, 100) \ .$$

Clearly it is the former case which is appropriate here and so $\lambda \geq 1.487$ or $1/\lambda \leq .6735$. From Figure 17 this means that to be consistent, Clark

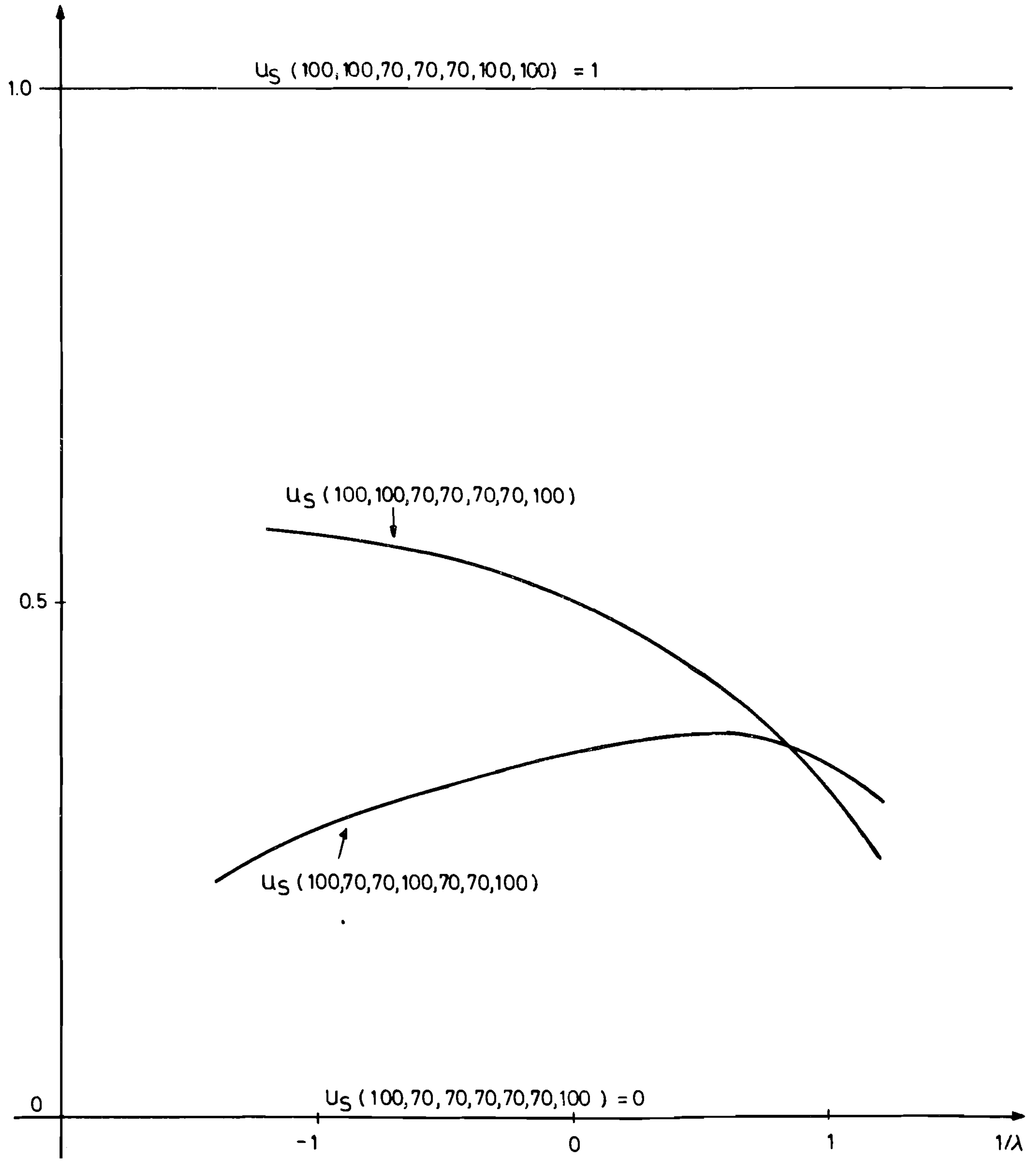

Figure 17. The Relative Utilities of Four Time Streams as λ Varies.

should prefer (ii) to (iv). He also felt that he would prefer (iv) to a 50-50 gamble between (i) and (iii) which with the existing function u_B is never possible. This discrepancy was not resolved. Clark later decided that his answer to the question of sequencing 50 good and 50 bad years was to do it in alternating four year blocks. This answer contradicts his preference of (iv) over (ii). As the mini-computer I was using only allowed up to 10 year time streams I could not experiment with graphs such as Figure 17 for longer blocks but this should be possible later. A value of $\lambda = 1.61$ is currently being used.

3.4 The Balance Between Recreation and Social Benefits

Everything was now reduced to finding the constants k_1 and k_2 of formula (3.6):

$$u(\underline{p}, \underline{e}, \underline{g}, \underline{b}) = u_R(\underline{g}, \underline{b}) + k_1\, u_S(\underline{p}, \underline{e}) + k_2\, u_R(\underline{g}, \underline{b})\, u_S(\underline{p}, \underline{e}) \qquad (3.21)$$

The constant k_2 was quickly established to be zero because Clark felt that lotteries of the following type

L_5: with probability $\frac{1}{2}$: u_R high, u_S high; with probability $\frac{1}{2}$: u_R low, u_S low

L_6: with probability $\frac{1}{2}$: u_R high, u_S low; with probability $\frac{1}{2}$: u_R low, u_S high

were indifferent. Actually over the period of the analysis he alternated between the following two arguments:

1) When unemployment is high, the people should at least be able to spend their enforced free time enjoying the forest and when business is booming, bad recreational facilities can be overlooked. At least <u>something</u> should be good.

and

2) It is probably not the unemployed who do take advantage of weekends in the forest and in any case the forest as a recreational

area serves a far greater number of people than are associated with the logging industry. Hence there is likely to be an outcry if people notice high profits in the logging business and poor recreation.

Argument 1) favours L_6 and argument 2) favours L_5. He finally converged on indifference. Thus $k_2 = 0$.

Since, in some sense, k_1 now embodies the tradeoff to be made between "profits" and "environment" some care was necessary in its calculation. First I asked for a profit level x such that for a single year

$$(x, 100, 38, 100) \sim \begin{cases} \frac{1}{2} & (10, 100, 38, 100) \\ \frac{1}{2} & (10, 100, 0, 100) \end{cases}$$

where the vectors are (P_t, E_t, G_t, B_t) all other periods being assumed to be at fixed values, and to find a number of good areas y such that

$$(10, 100, y, 100) \sim \begin{cases} \frac{1}{2} & (10, 100, 15, 100) \\ \frac{1}{2} & (-5, 100, 15, 100) \end{cases}$$

His answers of $x = -3$ and $y = 6$ yielded after substitution in (3.21) the values $k_1 = 7.9$ and $k_1 = 5.0$. To provide further evidence I asked for a number of bad areas z such that (0, 100, 15, z) was indifferent to (0, 95, 15, 100) and a level of employment w such that

$$(0, w, 15, 100) \sim \begin{cases} \frac{1}{2} & (0, 100, 30, 100) \\ \frac{1}{2} & (0, 100, 5, 200) \end{cases}$$

His answers here were $z = 155$ and $w = 90$ giving $k_1 = 9.4$ and 5.3 respectively.

I felt his answer x was the most reliable and that of w the least reliable (because of the difficult tradeoffs involved) and so $k_1 = 7.5$ seemed an appropriate compromise.

4. A Summary of the Assumptions

Let us review all of the assumptions which were used concerning the decision maker's preferences over the attributes $\{P_1, E_1, G_1, B_1, \ldots, P_T, E_T, G_T, B_T\}$. Recall that the mathematical result of asserting that attribute(s) X is conditionally utility independent of Y when Z is fixed is that for any values x, y, z of X, Y, Z and some fixed value y^0 of Y

$$u(x, y, z) = f(y, z) + g(y, z)\, u(x, y^0, z)$$

for some functions f, g. Utility independence is the special case when the set Z is empty.

The assumptions made were:

(i) The attributes $\{P_1, E_1, P_2, E_2, \ldots, P_T, E_T\}$ are mutually utility independent with $\{G_1, B_1, G_2, B_2, \ldots, G_T, B_T\}$.

(ii) For each $t = 1, \ldots, T-2$, the set $\{P_1, E_1, P_2, E_2, \ldots, P_t, E_t\}$ is mutually conditionally utility independent with $\{P_{t+2}, E_{t+2}, \ldots, P_T, E_T\}$.

(iii) For each $t = 1, \ldots, T-1$, P_t is mutually conditionally utility independent with $\{P_{t+1}, E_{t+1}\}$ and P_{t+1} is mutually conditionally utility independent with $\{P_t, E_t\}$.

(iv) For all $t = 1, \ldots, T-1$, $\{G_1, B_1, G_2, B_2, \ldots, G_t, B_t\}$ is mutually conditionally utility independent with $\{G_{t+1}, B_{t+1}, \ldots, G_T, B_T\}$

(v) For all $t = 1, \ldots, T$ G_t is mutually conditionally utility independent with B_t.

(vi) Preferences over time are stationary, that is, ignoring end effects, if the time index in any situation were altered by an equal constant amount for all outcomes, relative preferences would be unaffected.

These assumptions need not be this strong to imply the results used but since (3.14) and (3.21) together imply all of the above it seems worthwhile to state them in full.

5. Thoughts on the Whole Procedure

The motivation of this study gradually shifted in emphasis from a casual curiousness by me into the profit/environmental tradeoffs of the IIASA Ecology group, to an eagerness by that group to obtain an objective function with which to evaluate policies and finally to a searching examination by Clark of the ability of "decision analysis" to handle complex problems.

It could be that little more will be gained in terms of establishing better management policies using the complex objective function assessed here then if the original linear function were maintained. But if the policy evaluations are different then this study will have achieved a great deal for then attention can be focussed on the reasons for the differences and the implications hopefully resolved.

The Ecology project members have benefited from this study by having to discuss in concrete terms (seemingly for the first time) about their precise objectives of "what they want out of their forest." It is quite remarkable how a group who on the face of it agree "in principle" can differ diametrically when it comes to quantification.

I began this study as an advocate of decision analysis as a means of raising important issues in a decision context, but as a skeptic when it came to its ability to deal with anything more complicated than the handling of minor monetary decisions with uncertain payoffs, and my own interest was to see what I could do with the theory in a "real" situation. I am encouraged. With more practice a lot of the errors and lack of sophistication can be eliminated in future studies. For example I would concentrate much more on extracting information from the decision maker about which he was sure at an early stage. After a long period of questioning, decision making seems to get harder for the decision maker rather than easier. I would be inclined to keep questions which involve uncertainty down to a minimum as a good feel for probability is rather rare--my decision maker flatly refused to discuss any lottery which wasn't a case of an equal probability for each consequence. In theory it is possible to first evaluate a value function over all the attributes and then assess a single one attribute utility over that value function, but that is a little extreme as it loses a lot of the structure offered by the utility independence concepts.

I was initially skeptical also about the extent to which simplifying assumptions were "natural" as opposed to being forced on an unwilling decision maker as a matter of expediency. It certainly appears that often such assumptions are empirically observable or are sufficiently closely approximated that little accuracy is lost. After all, much of the weighting between different outcomes stems from the major constants such as k_1 and k_2 in (3.21) and λ in (3.14) rather than, say, the particular choice of coefficient for the exponential curve fitted to $u_G(g)$.

On a practical matter, as this study was completed on a part time on-off fashion over 18 months it was inevitable that the decision maker's preferences gradually altered over this time; adding to this my own frequent numerical and programming errors, meant that I was often forced to start from the beginning and rework most of the calculations. It was only towards the very end of this study that I learned to save the Fortran programs which performed many of these calculations. As a result, a lot of the early assessments were not reworked or subjected to a sensitivity analysis. A golden rule for those undertaking any major assessment which is likely to involve complex tradeoffs is to program everything!

6. Postscript

A monograph summarizing the "Budworm Project," of which this paper describes a part, should be available within a few months with contributions from those of us who have worked on it at the Institute of Resource Analysis of the University of British Columbia, the International Institute for Applied Systems Analysis in Laxenburg, Austria and the Environmental Systems Program at Harvard University. That should include comparisons of policy options using the linear objective function, the value function and the utility function mentioned here, as well as a range of other strategic objectives. As with any multi-person project,

the work described in this paper owes a lot to others, in particular to Bill Clark with whom I spent many hours, not only on matters of specific assessment but also in discussing ways to make decision analysis more applicable to ecological problems and in philosophizing on the ''time problem,'' the question of how to assess preferences over time. Professor Holling, the leader of the Ecology Project, spent days clarifying questions on the model and making (endless) lists of possible indicators for our, then as yet unnamed, decision maker to study. I am grateful to George Dantzig for his encouragement and meticulous reading of an earlier draft of this paper and to Ralph Keeney both for introducing me to the subject of decision analysis and for the many stimulating discussions that we have had on this project and other topics since.

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