



A Maximization Problem Associated with Drew's Institutionalized Divvy Economy

Wilson, A.G.

**IIASA Research Memorandum
March 1974**



Wilson, A.G. (1974) A Maximization Problem Associated with Drew's Institutionalized Divvy Economy. IIASA Research Memorandum. Copyright © March 1974 by the author(s). <http://pure.iiasa.ac.at/199/> All rights reserved. Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage. All copies must bear this notice and the full citation on the first page. For other purposes, to republish, to post on servers or to redistribute to lists, permission must be sought by contacting repository@iiasa.ac.at

INTERNATIONAL INSTITUTE FOR **IIASA** APPLIED SYSTEMS ANALYSIS
RESEARCH MEMORANDUM

A MAXIMIZATION PROBLEM ASSOCIATED WITH
DREW'S INSTITUTIONALIZED DIVVY ECONOMY

A. G. Wilson

March 1974

SCHLOSS LAXENBURG
A-2361 AUSTRIA

Research Memoranda are informal publications relating to ongoing or projected areas of research at IIASA. The views expressed are those of the author, and do not necessarily reflect those of IIASA.



A Maximization Problem Associated with
Drew's Institutionalized Divvy Economy

A.G. Wilson

Define variables as in Dantzig [1]. Let $U_k(C_{ik})$ be the utility derived by a member of the k th group, and let

$$U = \sum_k \mu_k U_k(C_{ik}) \quad (A)$$

be an aggregate utility function. Equation numbers refer to Dantzig's paper; letters to this paper.

The economy is described by Dantzig's equations (2) - (4)--(2) being subdivided here as (2a) and (2b). The equations can be written in full, with explicit subscripts, as follows:

$$\sum_j L_{ij} - x_j = \sum_k C_{ik} \mu_k \quad (2a)$$

$$\sum_i y_i L_{ij} = \sum_\ell \lambda_\ell R_{\ell j} \quad (2b)$$

$$\sum_j \lambda_\ell R_{\ell j} x_j = \ell \gamma_\ell \quad (3)$$

$$\sum_i y_i C_{ik} \mu_k = \ell \delta_k \quad (4)$$

Equations (2a) and (2b) can be combined to determine \underline{x} and \underline{y} :

$$x_i > \sum_j \{L\}_{ij}^{-1} \sum_k C_{jk} \mu_k \quad (2a')$$

$$y_j > \sum_i \{L\}_{ij}^{-1} \sum_\ell \lambda_\ell R_{\ell i} . \quad (2b')$$

We can substitute for \underline{x} and \underline{y} in (3) and (4), but now write them as equations in λ_ℓ , μ_k and C_{ik} : equations (5) and (6) then become

$$\sum_{ik} \lambda_\ell^M M_{\ell i} \mu_k \cdot C_{ik} = \ell \gamma_\ell \quad (B)$$

$$\sum_{i\ell} \lambda_\ell^M M_{\ell i} \mu_k C_{ik} = \ell \delta_k \quad (C)$$

where

$$M_{\ell i} = \sum_j R_{\ell j} \{L\}_{ji}^{-1} . \quad (D)$$

The proposed model is

Max U in equation (A)

subject to (B) and (C).

\underline{x} and \underline{y} are given by (2a'), (2b').

Possible Uses of the Model

1) In an existing equilibrium situation, we can assume that \underline{C} , $\underline{\lambda}$ and $\underline{\mu}$ exist such that equations (C), (D), (2a'), (2b') are satisfied and U in (A) is a max.

We can then explore the use of the model to investigate the consequences of various types of change.

2) Change in tastes: $U_k \rightarrow U'_k$. Then since there are fewer constraints (B) and (C) than there are C_{ik} 's, there should exist a set of C_{ik} 's which give the new equilibrium.

3) It may be, however, that imposing conditions (B) and (C) is too strong. An alternative would be to calculate \underline{C} to maximize (A) subject to (C) (which is like a budget constraint), for initially assumed $\underline{\lambda}$; and then to compute \underline{x} from (2a'), $\underline{\lambda}$ from (B), \underline{y} from (2b') and to iterate until a new equilibrium is found.

4) Alternative schemes on the lines of (3) could be investigated for other initial changes--e.g. in $\underline{\lambda}$.

5) So far, we have assumed that the μ_k 's remain fixed. A further outer loop could be added to the iteration which solved the LP problem of max U in (A) as a function of μ_k subject to (B) and (C). Alternatively, find μ_k to equalize consumer surplus per capita.

6) A further alternative would be to produce entropy maximizing versions of these models by

$$\text{Max } S = - \sum \log C_{ik} ! \quad (E)$$

subject to, say (B) and (C) and

$$\sum u_k U_k(C_{ik}) = \bar{U} \quad (A')$$

where \bar{U} is assumed given at some suboptimal value.

References

- [1] Dantzig, G.B. "Drew's Institutionalized Divvy Economy,"
Report 73-7, revised, Dept. of Operations Research,
Stanford University, 1973.