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Dynamic Models of the Interaction Between Migration and the Differential Growth of Cities*

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Abstract

The general area of interest of this paper is the dynamic relationship between regional imbalances, migration, and the differential growth of cities. In particular, the paper considers the feasibility of building dynamic models of the economic and demographic interactions between a set of linked cities, which could be used to explore the effects and repercussions of national settlement policies addressed at the alleviation of imbalances.

Inter-regional economic growth models are well known, and recently several papers have focused on inter-regional demographic models. There has been less research on the dynamics of the interdependent interaction between economic and demographic growth. This paper focuses on this economic-demographic adjustment for a system of linked city regions and considers research results which point to several difficulties in building theoretically well structured dynamic models of differential city growth.

Part of the paper considers the inadequacies of current theories of inter-regional population migration and a new approach based on job-search theory is outlined.

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1. The Modelling of Poorly Understood Systems

The aim of this paper is to highlight some of the difficulties in the dynamical modelling of poorly understood systems, and to call for a strategy that <u>integrates</u> fundamental research on the structure and workings of the urban system with the policy need for methodologies that analyse, through time, the effects and repercussions of alternative policies. In urban and regional analysis it is of little value to develop simulation or optimisation approaches as if dealing with well structured engineering systems. Large scale optimisation and simulation methods have been largely ineffective over two decades of development in urban planning. For poorly understood systems, it is necessary that analytical modelling be integrated into a structured learning process. Research into planning methodology requires a dialogue between hypothesis and data, and between theory and practice that can only be achieved in an iterative ongoing process.

This cyclical learning process has been well documented in the development and application of simple comparative-static land use models in the U.K. (Massey and Cordey-Hayes [19], Barras [1], Broadbent [6]). Such a view has also been put forward by Boyce [3, 4], Brewer [5], and indirectly and in much more polemical form by Lee [17]. In the current vogue for dynamical modelling (both simulation and optimisation), much of this earlier experience is being ignored. But this paper will go no further into this debate; here the aim is simply to emphasise how important it is that dynamical modelling be integrated with a serious programme of experimental research devoted to the formal understanding of urban processes.

A framework which is useful in this respect is outlined below. The approach is derived from dynamical systems theory, which itself is historically a direct outgrowth of the Lagrangian viewpoint of classical mechanics (see for example, Rosen [24]). It can be used as a dynamic optimising approach, or adapted as a heuristic or simulation method. However, in urban systems it is probably more useful simply as a general structuring framework for the experimental analysis of growth and change.

It is worth noting that this approach is derived from a state space approach to dynamical systems theory in preference to the classical transform function methods. The latter are based on a Laplace transform of a linear input-output differential equation, and thus give a high level of abstraction that is suitable for control problems in engineering, but is rather opaque to the mechanisms and behavioural changes that occur <u>within</u> the system. In urban and regional planning the explicit manner in which the system changes and the intermediate states through which it passes are of vital importance. An approach which is based on state variables and direct rates of change is preferred because it has greater transparency to the processes of change, and this makes it a much more useful conceptual framework for policy oriented dynamic analyses of urban systems.

The dynamical study of any system has two basic aspects: firstly, it must be decided what constitutes an instantaneous description of the system of interest; and secondly, the mechanisms that translate this information from one point in time to another must be understood and expressed in formal terms. An ordered n-tuple of numbers $(x_1, x_2, \ldots, x_n)$ arising from a finite set of measurements represents a possible instantaneous state of the system of interest and this notationally expresses the first step in the dynamic description. But now the manner in which this system changes over time must be specified, and this is much more difficult. Sometimes it is possible to give conditions that help in the specification of the functional dependencies that express the rates of change. For example, the rate at which a particular state variable $x_i(t)$ is changing at time t may depend only on the existing state $(x_1(t), x_2(t), \ldots, x_n(t))$, i.e.

$$\frac{dx_i}{dt} = f_i(x_1, \ldots, x_n) \qquad i = 1, \ldots, n. \qquad (1)$$

Thus in this case the dynamics of the system are determined by specifying the instantaneous description $(x_1, \ldots, x_n)$ and the functions $f_1,\ldots,f_n$. It is very rare that we are able to specify these functional dependencies adequately for urban systems, and it is considered here that the experimental deduction of these functions is the fundamental long term problem in the analysis of growth and change. The functions are essentially an expression of the exogenous "forces" that are acting upon the system and which are responsible for its dynamical behaviour. The problem of interest is how to structure experimental analysis in order to deduce these functional dependencies.

If the functions f_i were known, then in principle, it would be possible to consider how the inputs to the system could be chosen such that the trajectory to some pre-assigned state is made in a "best possible" way.[1] But we restate that for urban systems these functional dependencies are mostly unknown, and therefore an important problem is how to structure analyses in order to deduce these functions whilst concurrently addressing policy questions. The next two sections outline our attempts to do this in a dynamical study of inter-urban migration.

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[1]This leads to variational principles and "optimal controls." See, for example, McFarlane [20]; Paelinck [22] has used a similar approach in the analysis of urban systems.

2. A Framework for the Analysis of Migration

Migration is taken as the focus of the paper partly because it is the most volatile component of population change, but also because it provides the linch-pin interaction between the demographic and employment growth sectors. In many urban growth models it is assumed that semi-independent submodels of population and employment growth can be constructed with migration providing the equilibrating mechanism between these supply and demand submodels (Hamilton *et al*. [13], Forrester [10], Kadanoff [15]). Section 3 questions the validity of this traditional theory of migration and develops a more dynamic conceptual structure. This section summarises several earlier papers (Cordey-Hayes [7, 8], Cordey-Hayes and Gleave [9], Gleave and Cordey-Hayes [12]) and integrates these with more recent research.

The system considered here for migration analysis is conceptually a relatively simple one. We are interested in the rates of change of population of a set of city regions in terms of the probability of transitions (migration) between the city regions. Thus let x_i denote the population of city region i and $\frac{dx_i}{dt}$ its rate of change over time due to migration. Denote the probability per unit time of a transition from category i to j as a_{ij}. Similarly, x_j is the occupation number of j and a_{ji} is the transition coefficient which represents the probability a j to i transition in unit time. The rate of change of the occupation number of category i is thus simply related to the difference between the inward and outward flows for that category.

$$\frac{dx_i}{dt} = \sum_j (a_{ji}x_j - a_{ij}x_i) \qquad i = 1,\ldots,n. \qquad (2)$$

Rosen [24] describes how the solutions of this set of equations have the general form

$$e^{u_i t} \; e^{\sqrt{-1}v_i t}$$

where the second exponential represents an undamped oscillatory function (u_i and v_i are related to the transition parameters a_{ij}; they are the eigenvalues of the a_{ij} matrix).

The time variation of the population of a city region (x_i) can thus exhibit a variety of behaviours depending on the sign of u_i and its magnitude relative to v_i. If u_i is positive and much larger than v_i, then x_i increases almost exponentially with time; when u_i is negative and larger than v_i, the x_i decreases exponentially, and when u_i is small but v_i is large, then x_i exhibits damped oscillations. Thus a variety of trajectories are possible and the explicit properties of the dynamical system are determined by the matrix (a_{ij}) which represents the external forces acting upon the system. Given a specific functional form for (a_{ij}), then it is possible to obtain a particular solution of the basic equations (2) above, which would then, for example, analytically describe the population changes due to migration within a system of interacting city regions.

This analytical approach to urban growth and change processes suggests that a useful strategy for structuring experimental work (in this case for migration analysis) is:[2]

i) To construct an accounting matrix comprising transition probabilities from observations of past behaviour. This provides a description of the migration process through time in a useful summary form, but this must be followed by:

ii) An interpretation of these parameters in terms of hypothesised causal relationships which represent the external forces.

iii) These tested hypotheses for the particular form of a_{ij} could then give specific solutions to equations (2) which would then, in principle, give the future distributions of population over time, or at least the "behaviour modes" (approximate time path) of that system of city regions. It should also begin to provide the understanding necessary for the implementation of policies aimed at steering the system of city regions to some planned national settlement pattern.

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[2]The first two steps are similar to the strategy described by Ginsberg [11] in his work on a Semi-Markov approach to migration analysis.

Thus initially our aim is to interpret the a_{ij} for city regions in terms of the characteristics of i and j. Considerable simplification of the solution procedure of equations (2) is possible if the a_{ij} are decomposed into two components, which have been called in earlier papers an "escape probability per unit time" (ε_i), equivalently an escape frequency, and a "capture cross-section" (μ_j). This simplification separates the migration interaction into dynamic "mover pool" and "differential attraction" components. For example, from conventional spatial interaction theory it is hypothesised that the "differential attraction" depends not only upon the intrinsic attributes of city region j as perceived from i, but also upon the competing attractions from all other possible destinations j. That is, the probability of a migrant from i selecting a destination j from a competing set of city regions is

$$\mu_{ij} = \frac{q_j P_j f(c_{ij})}{\sum_k q_k P_k f(c_{ik})} , \qquad (3)$$

where q_j represents some "intrinsic attractiveness" of city region j for potential migrants; the population (P_j) and the function $f(c_{ij})$ weight this intrinsic attraction in relation to its size and distance from the origin region i. Hence, the probability of an individual in city region i migrating to city region j in unit time becomes

$$a_{ij} = \varepsilon_i \frac{q_j P_j f(c_{ij})}{\sum_k q_k P_k f(c_{ik})} \quad . \tag{4}$$

Experimental results based on an analysis of 20 city regions in the United Kingdom do suggest that there exists a concept of intrinsic attractiveness that strongly influences the differential allocation of migrants, and, furthermore, that there exists a strong feedback relationship between q_j and ε_i that dominates the properties of the mover pool. It is this latter fact that may have caused difficulty and anomalous results in many migration analyses.

The next section considers the structure of this feedback mechanism in the context of a simulation model based upon the experimental analysis mentioned above. It does so by comparing and contrasting this structure with the traditional approach to migration and reports the progress to date on a theoretical interpretation of ε_i and q_j in terms of the changing characteristics of the 20 city regions.

3. Rival Hypotheses on the Relationship between the Directional Components of Migration

The traditional economic approach to migration is summarised in the following flow diagram.

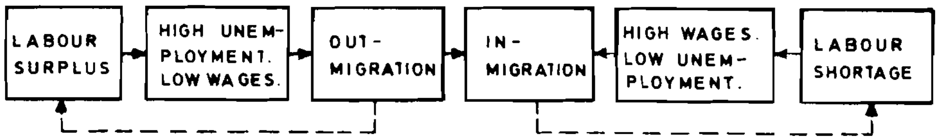

FIGURE 1

THE CHAIN OF CAUSALITY IMPLIED IN THE ECONOMIC THEORY OF MIGRATION.

The theory is essentially based on a "push-pull" phenomenon that seems intuitively sensible: migration is motivated by poor employment conditions (low wages, high unemployment) and migrants are differentially attracted to areas with high wages and low unemployment. Such a process is self-equilibrating since out-migration reduces the labour surplus and in-migration reduces the labour shortage. As mentioned above, this equilibrating mechanism between the supply and demand for labour has been used to link the employment and demographic sectors of a number of regional models (for example, Hamilton [13], Forrester [10], Kadanoff [15]). The basic hypotheses that underlie the above push-pull theory are that in-migration is directly related to the economic attractiveness of an area, and that out-migration is inversely proportional to in-migration.

A great deal of research was carried out on migration in the 1960's. The general conclusion was that in-migration could be interpreted in terms of concepts of economic attractiveness, but several researchers, in particular Lowry [18] and Lansing and Mueller [16], conclude that per capita gross out-migration is independent of the economic characteristics of the generating region. This means the per capita rates of out-migration are similar for all regions irrespective of their economic character, and therefore out-migration is dependent only on the total population within the region. Thus, there are two rival hypotheses on the relationship between the directional components of migration. These are: (i) the standard economic hypothesis that in- and out-migration are inversely related; and (ii) the empirically derived hypothesis that out-migration is independent of the economic characteristics of the area (and is therefore unrelated to in-migration which is dependent on areal characteristics). Cordey-Hayes and Gleave [9] have tested these hypotheses by reference to city-region data for England and Wales, and found that neither hypothesis was correct. In fact, a strong <u>direct</u> correlation between the per capita rates of in-migration and out-migration was observed. That is, areas with the highest rates of in-migration also had the highest per capita out-migration rates. Declining regions were observed to have the <u>lowest</u> per capita rates of out-migration and minimal in-migration. Decline should therefore be associated with a lack of compensating flow of in-migrants, rather than with high out-migration. Similar results had been obtained by Miller [21] and Stone [25].

This direct relationship between the directional components of migration was explained by a dynamic mechanism which associated regional out-migration with in-migrants from a previous time period. That is, an intrinsic attractiveness concept was used to explain the differential attraction of migrants to specific destinations, but then a positive feedback mechanism was introduced such that recent migrants to an area were more likely to move on again than were the remainder of the resident population, who had established a strong network of social and economic ties in the area. This feedback, based on a selective concentration of mobile population, satisfactorily explained the mechanism that relates the directional component of migration, but what is missing is an associated theory of labour mobility. Currently, a theory which looks at inter-urban migration as an extension of the local labour market is being developed and tested (Renshaw [23], Gleave and Cordey-Hayes [12]). This theory is outlined in the flow diagram below.

Here the probability of out-migration is hypothesised to depend on local employment conditions, the differential mobility of individuals within the region, and on their knowledge/information of the opportunities outside their own region.

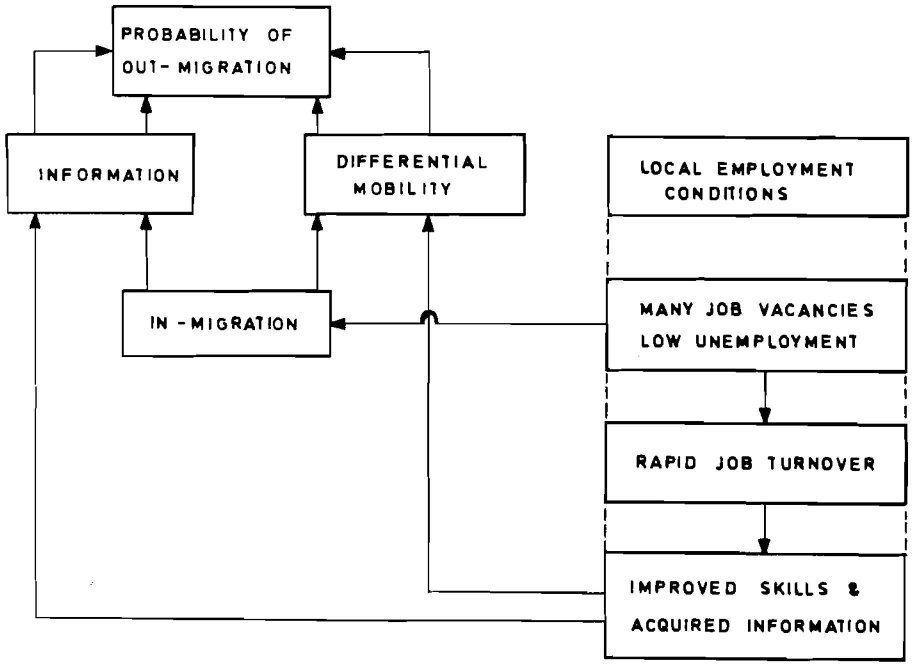

FIGURE 2
POSITIVE FEEDBACK EFFECTS RELATING OUT-MIGRATION TO IN-MIGRATION.

It is further hypothesised that a regional labour market in which there are many job vacancies and low unemployment will generally have a rapid voluntary turnover of jobs. This is because employees have a risk-free opportunity to change their jobs in an attempt to match their differential skills and tastes to a differentiated job market. This matching of abilities and tastes to jobs can be regarded as a stochastic learning process in which movers are acquiring new skills and gaining information not only of the local labour market but also of a more extensive one. That is, conditions that favour local labour market turnover are also those that induce occupational mobility and give a better knowledge of spatially more extensive labour markets. Both of these increase the likelihood of inter-urban migration, as indicated by the positive feedback arrows in the flow diagram. High numbers of job vacancies also attract migrants from other regions and therefore growth regions will be characterised by high in-migration and high out-migration. Conversely, low vacancies and high unemployment lead to a static local labour market with few people changing jobs. The tightness of the labour market produces caution, over-specialisation, and possibly low productivity. Low turnover results in individuals not having the opportunity to enter the learning process that was outlined above, and hence there is little occupational mobility and low out-migration.

These arguments are presented here[3] simply to recast inter-urban migration in a way that has implications for both research and policy. Firstly, consider briefly the implications for research.

i) The results suggest that the tradional demographic-employment linkage based on migration as the equilibrating mechanism between the supply and demand for labour is incorrect. Therefore, the results of regional and inter-regional models based on this conceptual linkage should be treated with caution (Hamilton [13], Forrester [10], Kadanoff [15]).

ii) It is of interest to compare the flow diagrams for the "standard economic theory"(p. 11) and the "mobility theory" sketched above. The former are based on a deterministic chain of mono-causality and are static. The latter is stochastic, with many feedback loops giving multi-causality and is dynamic. Whereas the concepts of the traditional economic approach are homogeneous labour, complete information and perfect mobility, the approach argued here is based upon heterogeneous labour, partial information and strong differential mobility. It considers that the labour market is a complex stochastic process that involves interactions amongst many participants--and therefore should be modelled as such. One of the key variables in a stochastic linkage between the demographic and employment sectors is the dynamic concept of

[3]A more formal presentation of the above migration theory will be published in the forthcoming project report by the present authors.

job vacancy. In a separate context (that of manpower planning), several labour market models based on a dynamic treatment of job vacancies have been developed (Holt and David [14], Harrison White [26], Bartholomew [2]), and these are currently being reviewed in an attempt to link migration to a stochastic treatment of the components of employment change (Gleave and Cordey-Hayes [12]).

iii) More generally, the above research on migration suggests that it would be very useful to develop a methodology capable of treating occupational mobility and geographical mobility within the same analytical framework. Potentially, the framework would be able to integrate spatial settlement systems with sociological concepts of relative opportunity, occupational mobility and equity.

On the policy side, the differences between the three rival hypotheses have important implications. For example, the second implies that policies aimed at halting out-migration are likely to be ineffective and the best that could be achieved is to steer out-migrants away from congested cities to selected growth areas. The results suggest that:

i) A large part of inter-urban migration in developed countries is a spontaneous movement of individuals moving from economic "strength to strength," rather than being "pushed" from economically weak regions. This is of interest because it suggests that there may be a more subtle restructuring occuring than simply the growth and decline of some regions. Also, the result implies that a national settlement policy does not need to stimulate migration but simply to channel it to selected growth areas. But these growth areas will be more successful if they have a diverse range of job opportunities, providing a setting for voluntary job turnover and occupational mobility, rather than a highly specialised employment structure based on one or two large plants.

ii) The results also suggest that it is inevitable that each region lose a substantial proportion of its young, dynamic and most mobile population each year, and therefore national settlement policy should recognise that new or expanding towns will have to be continually attracting population to compensate for the large numbers who will leave irrespective of how successful the town may be.

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