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IIASA Working Paper

WP-73-011

November 1973

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Alan S. Manne

1. Introduction

These calculations are intended as a highly aggregated description of the energy sector--with a time horizon extending well into the 21st century. The geographical scope is that of a self-contained "model society" with a technology, income and population similar to that of the U.S. or Western Europe. The focus is upon the transition to zero population growth and zero per capita increase in the use of energy.

In this eventually stationary economy, there can no longer be any reliance upon fossil fuels. Energy is to be derived from resources that are in virtually infinite supply, e.g. through solar, geothermal, fusion and breeder fission. In their current work, Haefele and Schikorr have focussed upon fission technologies. Asymptotically, 50% of the energy is to be produced in the form of electricity by fast breeder reactors (FBR's), and the other half will be produced in the form of hydrogen by high temperature gas reactors (THTR's). (See Figure 1.) It is proposed that hydrogen, a non-polluting fuel, will replace petroleum in such uses as automotive and air transport.

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*See W. Haefele and W. Schikorr [1] and also C. Marchetti [2]. The numerical values here refer to "case 1" only. Certain modifications have been made to the original model. For computer programming of the current version, the author is indebted to Leo Schrattenholzer.

There is a synergistic relation between the various reactor types. During the transition phase (Figure 2), the light water reactors (LWR's) produce plutonium for starting up the FBR's. The FBR's produce just enough plutonium to sustain their own fission reaction. Their breeding gain is directed toward converting thorium into U-233, the fuel for the THTR. Since the LWR's require uranium 235--and since the low-cost supplies of this isotope are quite limited--this reactor type does not appear in the asymptotic technology (Figure 1). LWR's are therefore phased out just as rapidly as they can be replaced by breeders.

Upper bounds are specified for the rate of introduction of these reactors, e.g., there is an upper bound of 100 GW thermal upon the combined total capacity of LWR's and FBR's that can be introduced in any one year. Moreover, breeders cannot be introduced in large quantities until after 1988.

Demands are specified in the form of logistic curves--starting from an initial configuration in which there is a population of 250 10^6, a per capita power consumption of 10 KW, and 25% of the energy requirements are in the form of electricity. The transition is to a population of 362 10^6 persons, 20 KW per capita, and 50% electric energy. (See Figure 3.) Non-electric energy is termed "process fuel." This is a shorthand expression for petroleum and natural gas plus hydrogen.

In the transient phase, there is insufficient nuclear capacity to satisfy the various demands, and fossil fuels are employed to whatever extent is needed to avoid shortages. Thus, electricity demands are satisfied by energy sources in the following order of preference: (1) FBR's; (2) LWR's; and (3) fossil electric. Similarly, process fuel demands are satisfied first by THTR's and then by fossil fuels. For each phase of the system's evolution, there is a triangular system of linear difference equations. There is no explicit objective function other than these rankings of reactor types. The present paper is intended as a step toward the construction of an explicit optimizing model, one in which there are built-in possibilities for demand substitution between electric energy and process fuel.

2. Definition of Unknowns and Constraints

Let the unknown P_i^t denote the thermal TW equivalent of energy source i in available in year t. The subscript i identifies the energy sources as follows:

L : LWR

B : FBR

H : THTR

FE : fossil electric (typically coal in U.S.; petroleum in Western Europe)

FP : fossil process (petroleum and natural gas)

The base date (t = 0) is 1970. Calculations are made for each three-year interval thereafter. With 20 time periods, the planning horizon extends to 2030.

Let $\bar{P}_{EL}^t$ and $\bar{P}_P^t$ denote the exogenously specified demands at time t for electricity and process fuel respectively. (See the logistic curves, Figure 3.[1])

These demands are to be covered by one or another of the energy sources. That is:

$$P_{FE}^t + P_L^t + P_B^t = \bar{P}_{EL}^t \qquad (1)$$

$$P_{FP}^t + P_H^t = \bar{P}_P^t \quad . \qquad (2)$$

Let $\dot{P}_i^t$ denote the annual rate of installation of capacity type i during the representative year t. It is understood that $P_i^t \geq 0$, but that $\dot{P}_i^t$ is unrestricted in sign. With three years per period and no replacement requirements, we then have the following dynamic equations:

$$P_i^t = P_i^{t-3} + 3\dot{P}_i^t \quad , \qquad (i=L,B,H,FE,FP) \quad . \qquad (3)$$

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[1]Let $\bar{P}_j^t$ denote the demand at time t for energy type j (j = EL,P). For short, we shall omit the subscript j hereafter and will also omit the bar over P. The logistic curve is a three-parameter curve, and is defined as:

$$P^t = \frac{P^\infty}{1 + \alpha e^{-\beta t}} \quad .$$

The asymptotic value P^∞ is a datum, and the parameters α and β are calculated as follows from the values at time 0 and τ :

$$\alpha = (P^\infty - P^0)/P^0$$

$$\beta = -\frac{1}{\tau} \ln \left[\frac{P^0(P^\infty - P^\tau)}{P^\tau(P^\infty - P^0)} \right] \quad .$$

The initial conditions are as follows:

$$P_{FE}^{0} = .625\ TW_{th} = 625\ GW_{th}$$

$$P_{FP}^{0} = 1.875\ TW_{th} = 1,875\ GW_{th}$$

$$P_{i}^{0} = 0 \quad , \quad (i=L,B,H) \quad .$$

If the entire breeding gain is employed to produce U-233 for the THTR--and if this technology is introduced simultaneously with the FBR--we have:

$$P_{H}^{t} = b\ P_{B}^{t} \qquad (4)$$

where b denotes the U-233 fuel coupling factor, here taken to be 1.0.

Because of capacity limits on the nuclear construction industry, we have the following constraints:

$$\dot{P}_{L}^{t} + \dot{P}_{B}^{t} \leq A^{t} = \min\left[.1, \frac{(t-1970)}{100}\right] TW_{th} \quad . \qquad (5)$$

Note that there are ten unknowns P_{i}^{t} and $\dot{P}_{i}^{t}$ per time period, but that there are only 8 equations in (1) - (4).

The system is closed by supposing that there are four phases of the system's evolution as shown in Table 1. In phase I, there are no breeders or THTR's, and LWR's are built at the maximum annual rate, A. (Recall constraint (5).) Similarly, in phase II, both FBR's and THTR's are built at the maximum rates specified by (4) and (5). Fossil electric plants are gradually replaced by FBR's. This process comes

Table 1. Four phases of evolution.

Phase	I	II	III	IV
Representative years t	'70...'88	'91...S	S+3...T	T+3...'30
$\dot{P}_L$	A	0	*	*
$\dot{P}_B$	*	A	A	*
$\dot{P}_H$	*	*	*	*
$\dot{P}_{FE}$	*	*	*	*
$\dot{P}_{FP}$	*	*	*	*
P_L	*	*	$\overline{P}_{EL}-P_B$	0
P_B	0	*	*	$\overline{P}_{EL}$
P_H	0	bP_B	bP_B	bP_B
P_{FE}	$\overline{P}_{EL}-P_L-P_B$	$\overline{P}_{EL}-P_L-P_B$	0	0
P_{FP}	$\overline{P}_P-P_H$	$\overline{P}_P-P_H$	$\overline{P}_P-P_H$	$\overline{P}_P-P_H$

N.B. It is understood that each of these unknowns refers to period t. Accordingly, the superscript t is omitted. The asterisked quantities are calculated by direct substitution into the dynamic equations (3).

to an end in year S, the last year in which $P_{FE} > 0$. During phase III, $P_{FE} = \dot{P}_{FE} = 0$, and LWR's are in their turn superseded by FBR's. Phase III terminates in year T, the last year in which $P_L > 0$. Finally, in phase IV, all electricity demands are met by breeders. This equipment is increased by just the amount required to meet the demands, $\overline{P}_{EL}$. (See Figures 4 and 5.) In this numerical example, it turns out that the phase termination dates are as follows:

$$S = 2003$$
$$T = 2021 \quad .$$

3. Implications for the Fuel Supplying Industries

Given the time paths P_i^t and $\dot{P}_i^t$, we may calculate the cumulative requirements for fossil and nuclear fuels as follows:

cumulative fossil fuel requirements for electricity generation, as of end of period t $= CFE^t$

(unit: 10^{18}BTU = 1 Q) $= CFE^{t-3} + (3/33.4)P_{FE}^t$

cumulative fossil fuel requirements for process uses, as of end of period t $= CFP^t$

(unit: 10^{18}BTU = 1 Q) $= CFP^{t-3} + (3/33.4)P_{FP}^t$

cumulative requirements for natural uranium ore, as of end of period t $= CNU^t$

(unit: 10^6 metric tons) $= CNU^{t-3} + .500\ \dot{P}_L^t + .180\ P_L^t + .540\ \dot{P}_H^t$.

As of the end of period t, the cumulative plutonium stockpile will be:

$$CPU^t = CPU^{t-3} + .17\ P_L^t - 3.0\ \dot{P}_B^t \ .$$

(unit: 10^3 metric tons)

The annual requirements for separative work during period t will be:

$$SWU^t = \frac{.230}{3}\dot{P}_L^t + \frac{.110}{3}P_L^t + \frac{.438}{3}\dot{P}_H^t \ .$$

(unit: 10^6 metric tons per year)

For numerical results, see Figures 6 - 8. Note that the plutonium stockpile remains positive until 2015. We have not attempted to ensure the nonnegativity of this stockpile by reducing the rate of decline of the LWR's during phase III. This difficulty did not arise in the original Haefele-Schikorr paper because they assumed a constant value of the nuclear construction capacity limit, A^t. Moreover, this difficulty will not arise with a linear programming model in which it is stipulated that $CPU^t \geq 0$.

4. Sensitivity Analysis for Nuclear Construction Capacity, A^t

Instead of supposing that nuclear construction capacity will be expanded as rapidly as given in (5), we shall assume a slower rate of LWR buildup--one in which it takes until 1985 rather than 1980 to reach the annual output level of .1 TW_{th} = 33.3 GW_{el}. The qualitative results remain quite similar. The principal quantitative differences are given in Table 2.

Table 2. Effects of slower rate of LWR construction.

	Basic Case	Slower Rate of LWR Construction
A^t	$\min\left[.1, \frac{(t-1970)}{100}\right]$	$\min\left[.1, \frac{(t-1970)}{150}\right]$
S, end of phase II	2003	2006
T, end of phase III	2021	2021
CFE^{2030}, cumulative fossil fuel requirements for electricity generation (unit: 10^{18}BTU = 1 Q)	.517	.729
CFP^{2030}, cumulative fossil fuel requirements for process uses (unit: 10^{18}BTU = 1 Q)	3.028	3.028
CNU^{2030}, cumulative requirements for natural uranium ore (unit: 10^6 metric tons)	3.614	3.190
CPU^{2030}, cumulative plutonium stockpile (unit: 10^3 metric tons)	-.799	-1.199
SWU^{1991}, maximum requirements for separative work (unit: 10^6 metric tons per year)	.067	.059

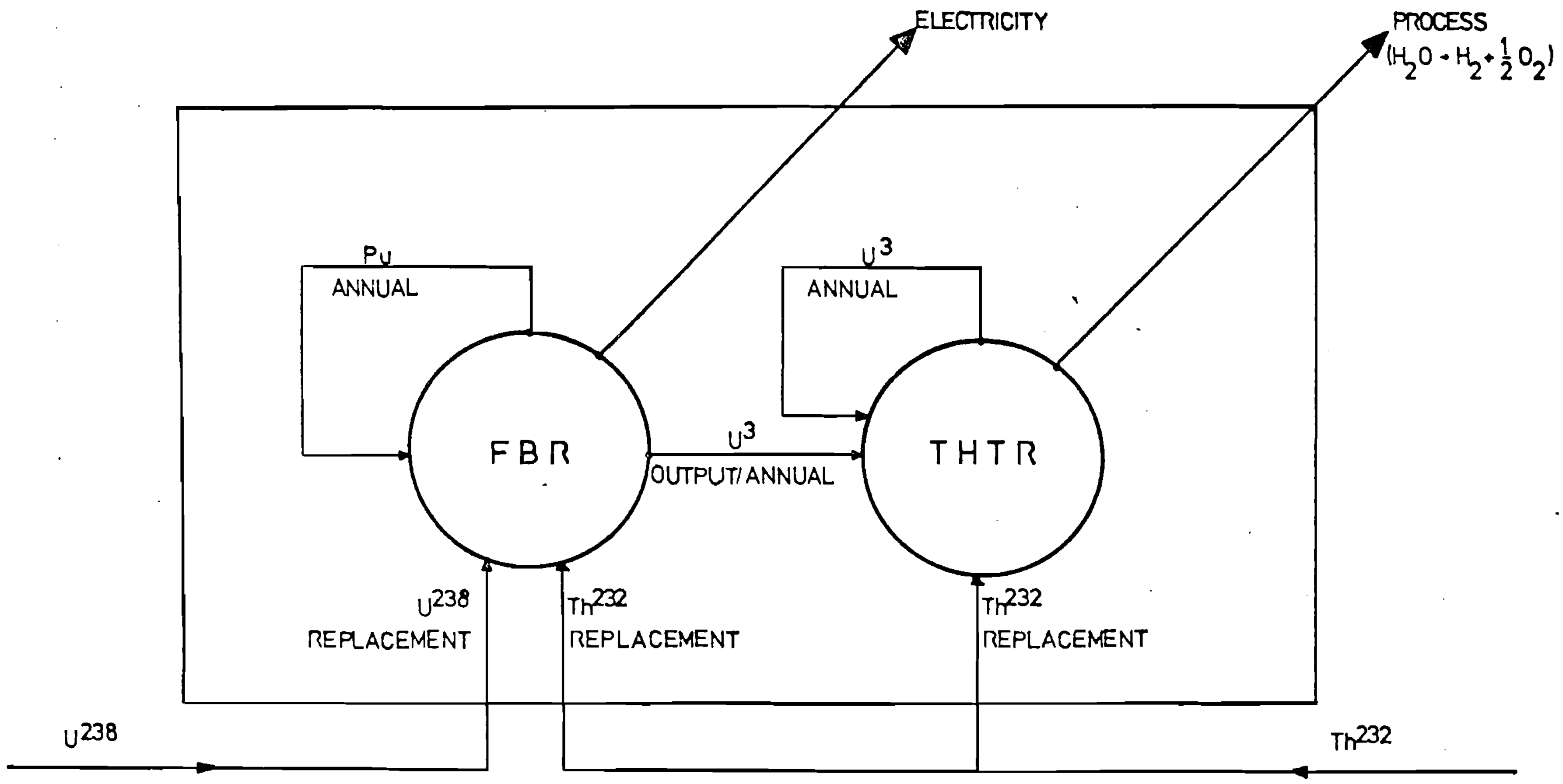


FIG.1 ASYMPTOTIC INTEGRATED REACTOR SYSTEM

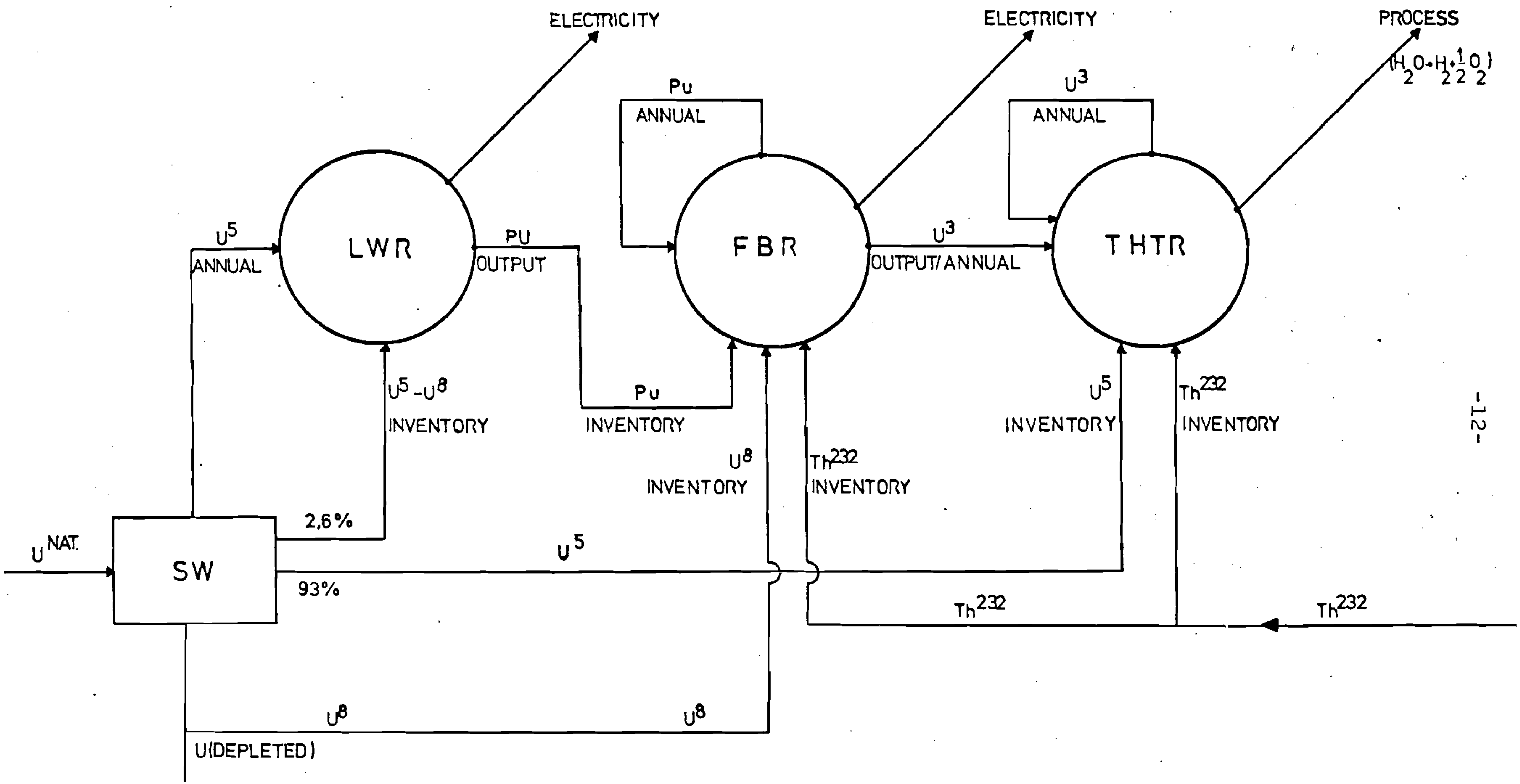


FIG. 2 TRANSIENT REACTOR SYSTEM

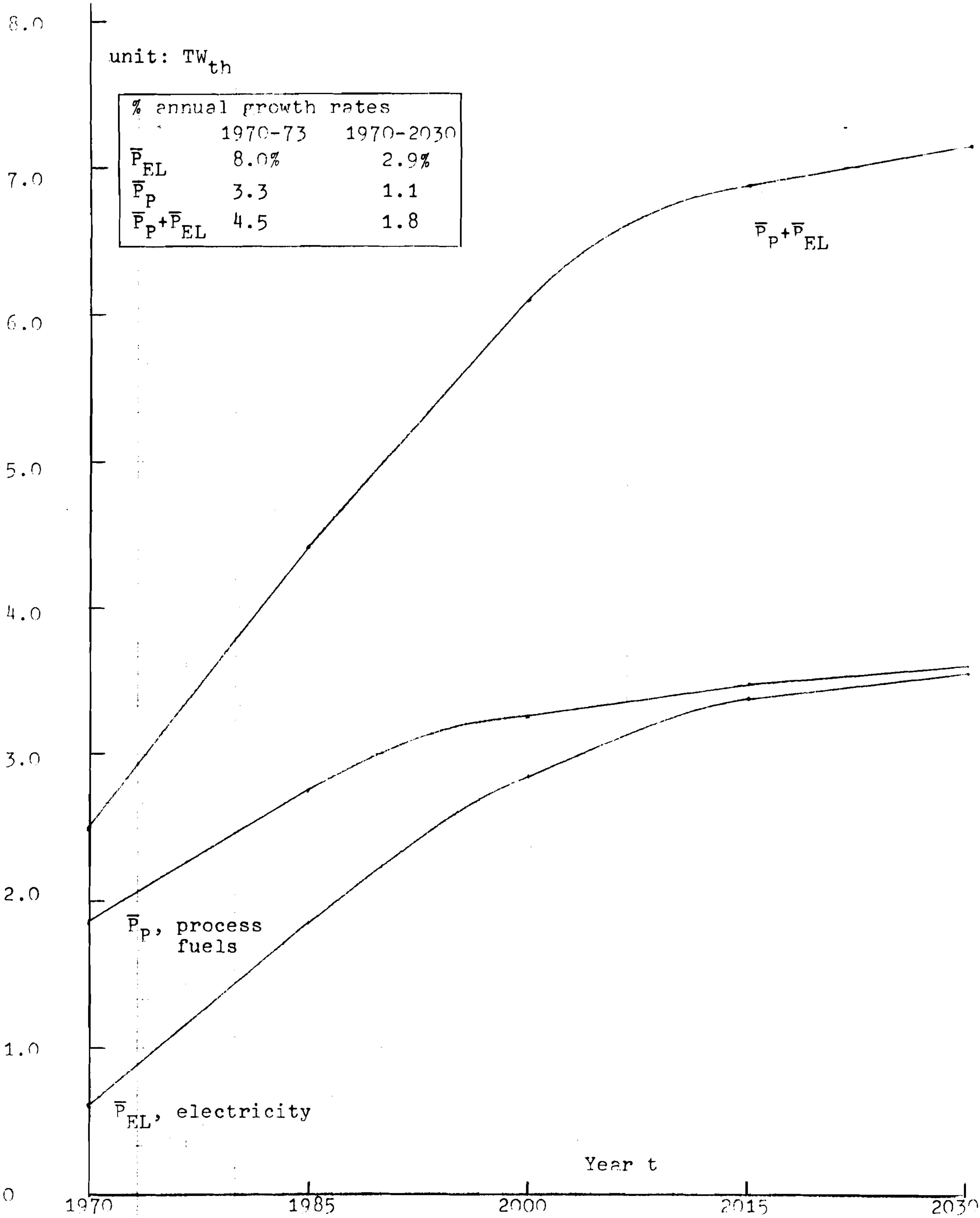


Figure 3. Power demands.

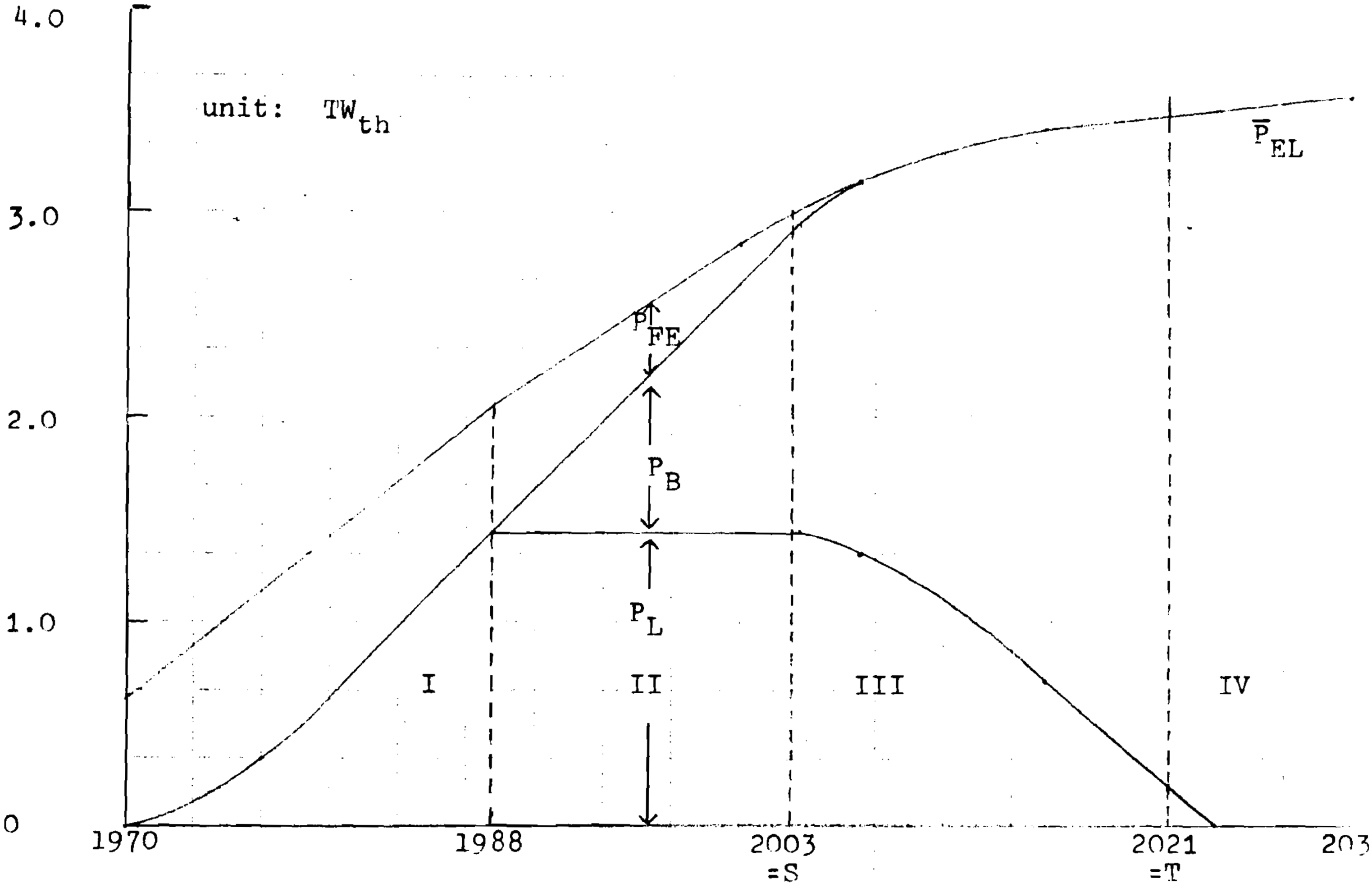


Figure 4. Electric power demands and sources of supply.

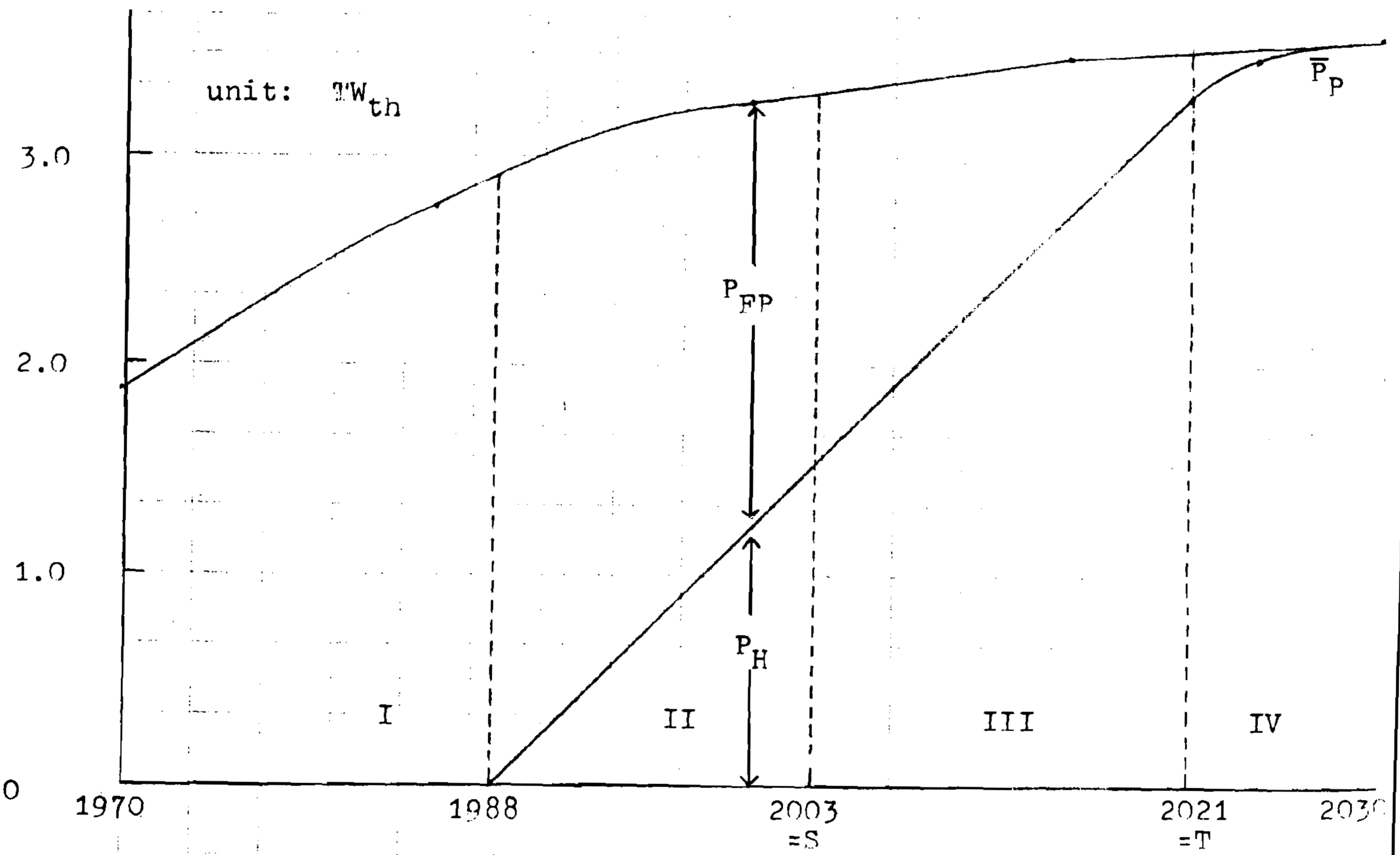


Figure 5. Process fuel demands and sources of supply.

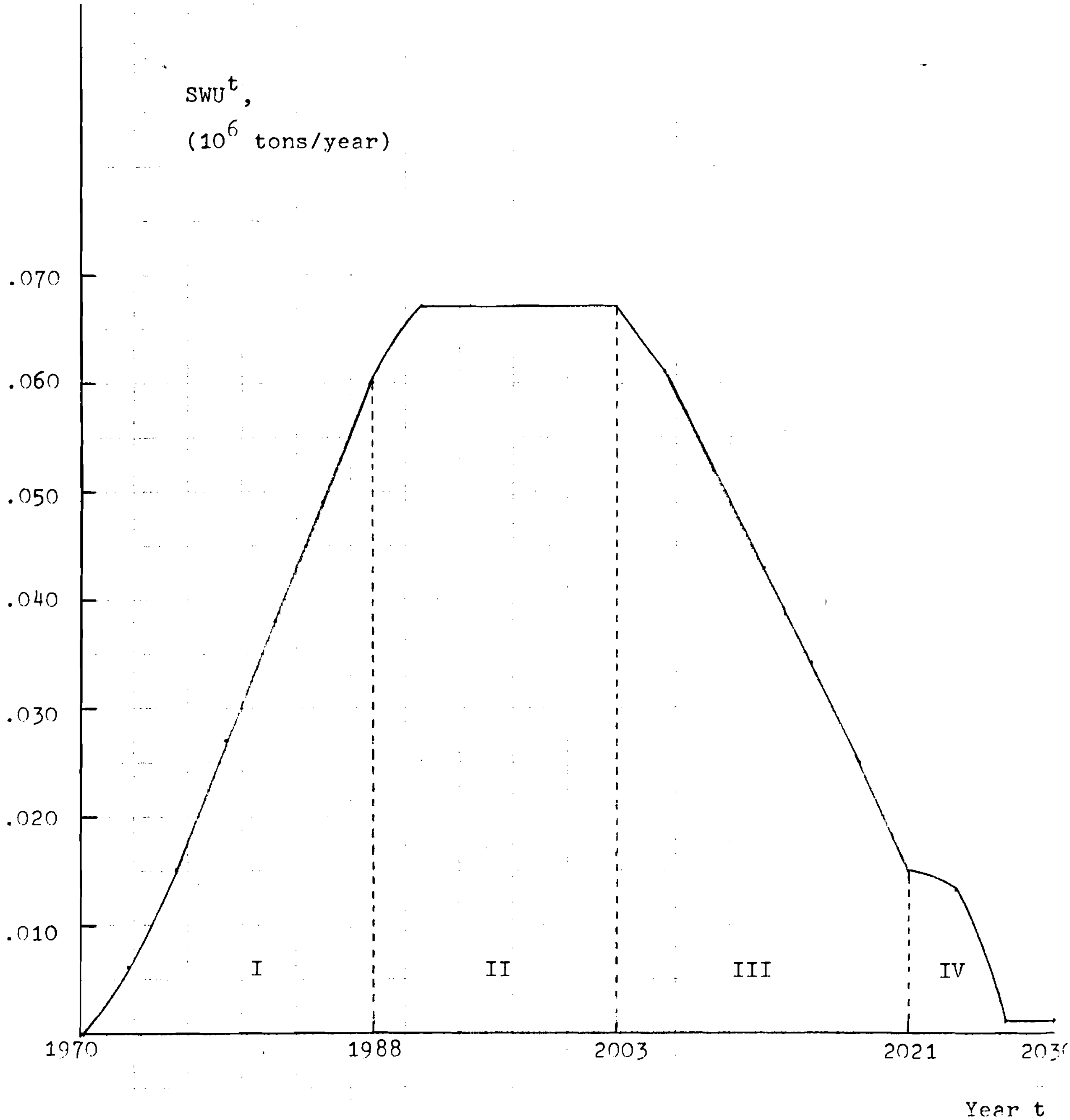


Figure 6. Annual separative work.

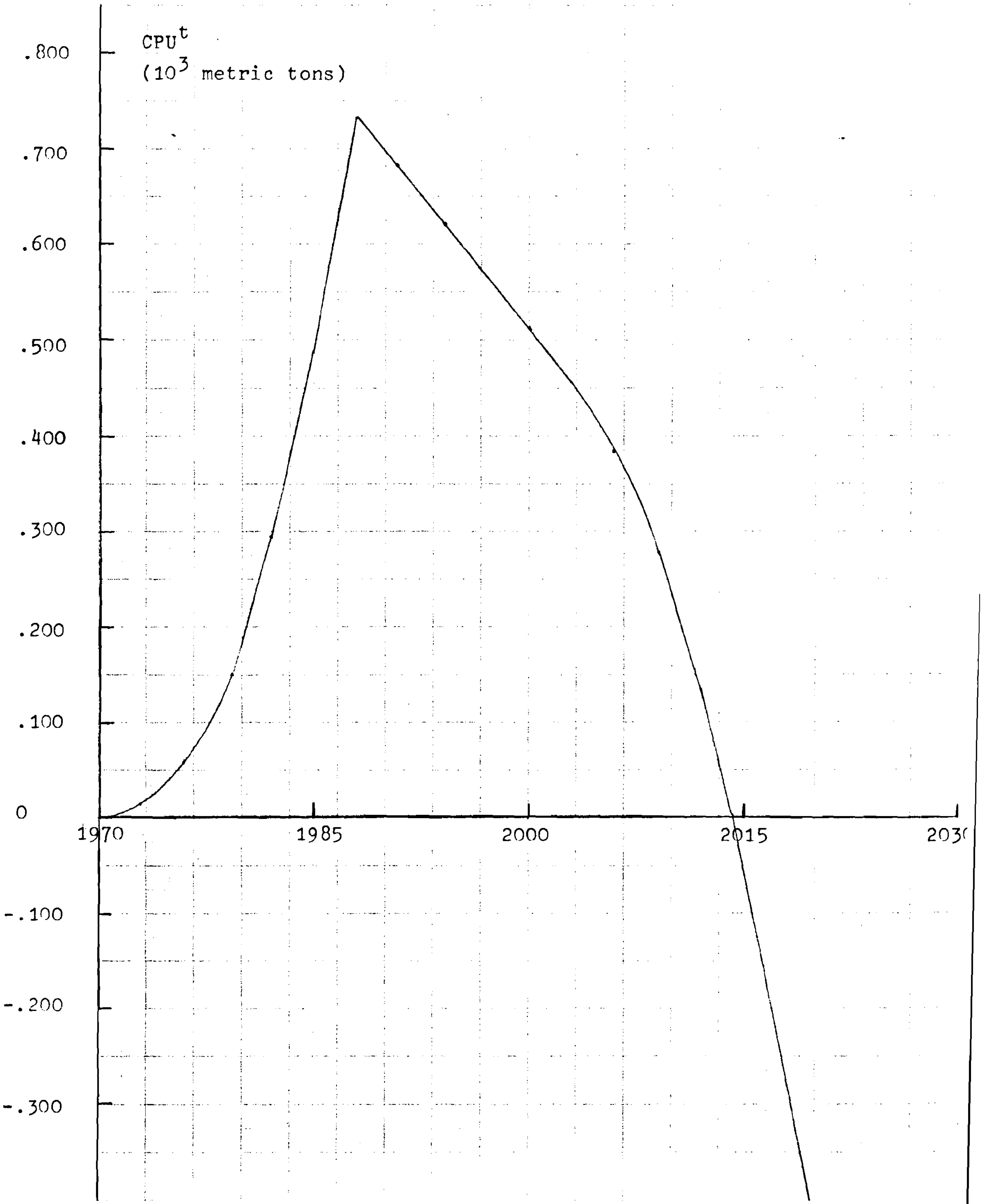
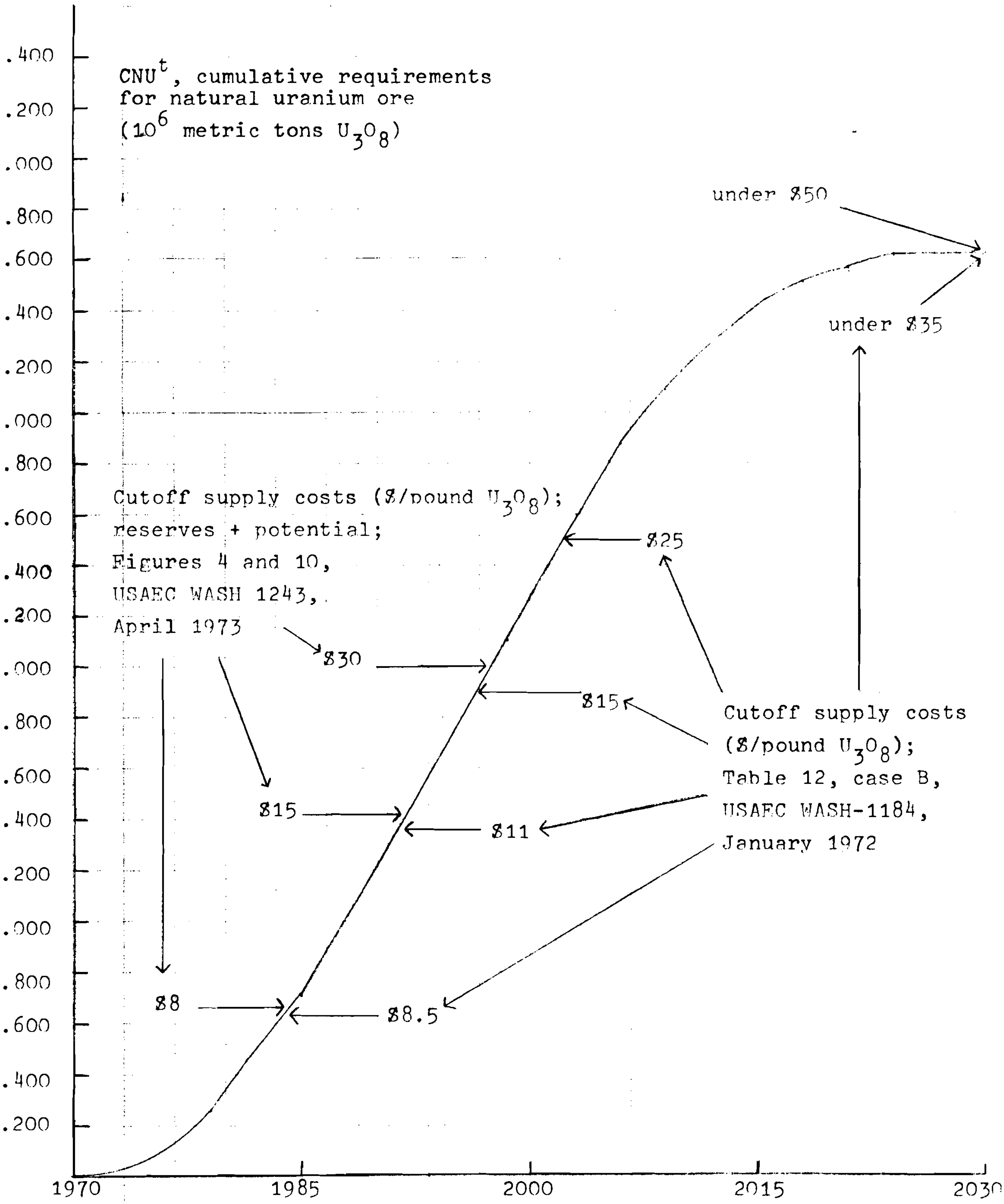


Figure 7. Plutonium stockpile.


.400
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.200
.000
.800
.600
.400
.200
CNU^t, cumulative requirements for natural uranium ore (10^6 metric tons U3O8)
under $50
under $35
Cutoff supply costs ($/pound U3O8); reserves + potential; Figures 4 and 10, USAEC WASH 1243, April 1973
$25
$30
$15
$15
$11
$8
$8.5
Cutoff supply costs ($/pound U3O8); Table 12, case B, USAEC WASH-1184, January 1972
1970
1985
2000
2015
2030

Figure 8.

References

[1] Haefele, W., and Schikorr, W. "Reactor Strategies and the Energy Crisis." IIASA, November 1973.

[2] Marchetti, C. "Hydrogen and Energy." Chemical Economy and Engineering Review, Tokyo, January 1973.

December 1973

Comments on the Notes that were made by A.S.Manne on the Häfele-Schikorr Model of Reactor Strategies, (November 1973)

by W.Häfele

The main purpose of these comments is to introduce a few details that make at little expense the model in question more realistic.

1.) I continue to feel that we should employ both, the polynomial and the logistic curve that describe the energy demand. The polynomial allows explicitly to play with the length of the transition period, that is t_1, while all other factors remain constant. The logistic curve may be more familiar to econometricians and was probably designed for more near term considerations, I feel. It does not allow to change the length of the transition period with everything else being constant (P_o,R_o,P_1).

2.) The introduction of the various shifts that express first order properties of the fuel cycle starts on page 4, equation 4.
It should read as follows:

$$P_H^{\ t} = b\ P_B^{\ t-3}$$

This backward time shift describes the fact that getting the U^{233} out of the breeder and putting it into the HTGR takes roughly 3 years.

3.) The switch from producing LWR's to producing FBR's should be more realistic.
I propose the following (see (5) on page 5):

$$\left.\begin{aligned} \dot{P}_L &\leq A^t \\ \dot{P}_B &= 0 \end{aligned}\right\} \quad \text{for } t \leq R-3 \quad (\text{recall: } R = t^*)$$

$$\left.\begin{aligned} \dot{P}_L &= \frac{A}{2} \\ \dot{P}_B &= \frac{A}{2} \end{aligned}\right\} \quad t = R$$

$$\left.\begin{aligned} \dot{P}_L &= 0 \\ \dot{P}_B &\leq A^t \end{aligned}\right\} \quad t \geq R+3$$

4.) The formulae describing the implications for the natural uranium industries should be modified as follows (page 7):

a.) $$CNU^t = CNU^{t-3} + 3.0{,}5.\eta_L \dot{P}_L^{t+3} + 3.0{,}18.\eta_L P_L^t + 3.0{,}23 . \dot{P}_H^{t+3} , \quad t \leq S - 3$$

The factor of 3 should be made explicit on page 7 as we deal with the cumulative demands. Simultaneously, the thermal efficiencies should be made explicit to allow for better approximation to reality. Note: 0,23 = 0,54·0,425, that is, 0,425 is the thermal efficiency if the HTGR were an electrical plant for which the value of 0,54 holds. But it is meant to produce hydrogen and therefore the thermal efficiency for producing electrical power should not enter.

The forward time shifts in the demands for the inventories describe the fact that this fuel has to be provided for 3 years prior to the start of the operation.

b.) $$CNU^S = CNU^{S-3} + 3\cdot 0{,}18\cdot\eta_L\cdot P_L^S + 3\cdot 0{,}23\cdot \dot{P}_H^{S+3}$$

This formulation takes care of the issue of discontinuities.

c.) $CNU^{t} = CNU^{t-3} + 3 \cdot 0{,}18 \cdot \eta_L \cdot P_L{}^{S} + 3 \cdot 0{,}23\ \dot{P}_H{}^{t+3}$

$- 3 \cdot 0{,}5 \cdot \eta_L\ \dot{P}_L{}^{t-3}$

$S+3 \leq t \leq T-3$

d.) $CNU^{T} = CNU^{T-3}$

e.) $CPU^{t} = CPU^{t-3} + 0{,}17\,\eta_L\ P_L{}^{t-3} - 3{,}0\,\eta_B\ \dot{P}_B{}^{t}$

The time lag in 4.) again describes the step of reprocessing.

The term $3 \cdot 0{,}5\eta_L\ \dot{P}_L{}^{t-3}$ in c.) is a negative requirement. It describes the availability of the inventories of the LWR's upon decommissioning. Reprocessing and refabrication takes roughly 3 years. This term corresponds with the term for the first core inventories during the build up phase.
This interplay between fuel requirements for the remaining LWR's and the availability of old cores holds for $t \leq T-3$. This can be seen by the following consideration:

During the decommissioning phase the mass balance is as follows:

$$\frac{\delta M}{\delta t} = 0{,}18\,\eta_L P_L{}^{t} - 0{,}5\,\eta_L\ \dot{P}_L{}^{(t-3)} \quad , \quad \dot{P}_L = A$$

at $t = t^{X}$ should $\frac{\delta M}{\delta t} = 0$, that is

$$P_L^{X}(t) = \frac{0{,}5}{0{,}18}\ A(t^{X}-3) = 3A$$

3A is the decommissioning rate over 3 years, that is one interval and that means:

$$\frac{\delta M}{\delta t} = 0 \quad \text{at} \quad t^{X} = T-3$$

I propose not to take into account any request for natural uranium for $t > T-3$. These are negligible effects. That implies equation d.)

5.) The formulae describing the implications for the separative work industries should be modified as follows (on page 8):

a.) $SWU^t = 0{,}23\ \eta_L\ \dot{P}_L^{\,t+3} + 0{,}110\eta_L\ P_L^{\,t} + 0{,}186\cdot\dot{P}_H^{\,t+3}$

$t \leq S-3$

Similarly to above one notes that $0{,}186 = 0{,}438\cdot 0{,}425$

b.) $\int_{S-3}^{S}(SWU)dt = 3.0{,}110\cdot\eta_L\ P_L^{\,S} + 3\cdot 0{,}186\cdot\dot{P}_H^{\,S+3}$

c.) $SWU^t = 0{,}110\cdot\eta_L\ P_L^{\,t}\ (1-\dfrac{0{,}5\ \dot{P}_L^{\,t-3}}{0{,}18\ P_L^{\,t}}) + 0{,}186\cdot\dot{P}_H^{\,t+3}$

$S+3 \leq t \leq T-3$

6.) In table 2 the column for the HTGR should read as follows:

separative work requirement, initial loading $\left[\frac{t_o}{GWth}\right]$ (attention: delete "cum"	0,186
ore requirements (U_3O_8), initial loading $\left[\frac{t_o}{GWth}\right]$	0,23

5.) Delete in table 2 the suffix "cum" at the separative work row in general.