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Mathematical Models for Screening

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1. Introduction

The need for preliminary screening techniques in the design of water resource systems was discussed in our earlier Report (May 1973). Two screening methods were suggested. The first is based on the concept of storage-yield contours, previously applied to a single reservoir (Fiering, 1967). Joint storage-yield contour maps can be constructed for systems of reservoirs by considering the performance of various storage configurations, and such maps used successively to eliminate sites which do not contribute significantly to system performance. Several problems with this method remained unresolved, and the method has not been further investigated.

An improved method, termed the Equivalent Reservoir Method, or Method I, uses the concept of equivalent capacity, defined as the downstream capacity which could give the same level of performance as a particular reservoir system. Using this concept, reservoirs are successively combined, starting upstream, into downstream equivalent reservoirs, each of which represents the entire system of reservoirs above it. Successive solutions are tabulated in the manner of dynamic programming, enabling optimal system configurations to be read by working back through the tables. An example of this method was presented in our Report.

In this Report, another screening method, termed the Effective Capacity Method, or Method II, is suggested. This again uses the equivalent capacity concept, but here the equivalent capacities downstream of the entire system, termed effective capacities, are calculated for each potential reservoir individually. The cost-capacity functions for each site are modified to cost-effective capacity functions and the required system storage is allocated on the basis of these modified cost functions.

A numerical example is given to illustrate and compare the Equivalent Reservoir (I) and the Effective Capacity (II) methods. Then in the following section, potential difficulties and errors of the two methods are discussed.

These developments, including the numerical examples presented, maintain the simplifying assumptions described in the earlier Report. Annual streamflow values are used so that only over-year storage is considered, and a simple single-purpose benefit function is assumed. The final section of this report discusses extending the scope of application of these proposed screening methods by developing standard tables of equivalent capacities for various combinations of design parameters and by allowing multi-season and multi-purpose models.

2. Effective Capacity (Method II)

This technique is related to the Equivalent Reservoir (Method I) in that the concept of equivalent capacity is again used. However, in this case, each potential reservoir site is evaluated individually in terms of its contribution to system performance. The performance of each reservoir is determined for a range of capacities, assuming in each case that this reservoir is the only one in the system and that it is operated in conjunction with all the unregulated streamflows throughout the system. The corresponding equivalent capacities downstream of the entire system, that is, the downstream capacities required to give the same levels of performance, are determined. The same computational techniques are available as those described in the Equivalent Reservoir Method. Denoting these downstream equivalent capacities as effective capacities, the cost-capacity function for the site may be modified to give a cost-effective capacity function. After performing the analysis for each site, for any given system performance level the required system storage is allocated on the basis of these cost-effective capacity functions.

3. Example

Consider a river basin with 4 potential reservoir sites, R1, R2, R3, R4 as shown in Figure 1. The cost functions for each site are assumed to be of the form

$$C_i = k_i \; x_i^{0.5}$$

where x_i = storage capacity at i^{th} site

and values of k_i are

i	1	2	3	4
k_i	2·3	2·5	1·8	2·2

The maximal storage capacity of each site is 1 unit. The system target output is 100% of the mean annual total flow past the downstream use point at E3 (6 units), and no unregulated intermediate runoff occurs. The reliability index, R, is defined as

$$R = \frac{\text{total water delivered (excluding amounts in excess of target)}}{\text{total target}}$$

We want to determine the optimal system configurations for given levels of system reliability using the Equivalent Reservoir (I) and Effective Capacity (II) Methods. To simplify calculations

Year	I_1	I_2	I_3	I_4	$\sum I_i$
1	0	3	1	2	6
2	2	0	1	2	5
3	1	2	1	3	7
4	0	4	1	2	7
5	0	2	1	1	4
6	1	2	0	5	8
7	0	2	2	1	5
8	3	4	0	0	7
9	1	0	2	1	4
10	2	1	1	3	7
Mean	1.0	2.0	1.0	2.0	
Std. deviation*	1.0	1.34	0.63	1.34	
Coeff of var.	1.0	0.67	0.63	0.67	

* Correction for bias is not made

Total target = 10 x 6 = 60
Delivered: 6+5+6+6+4+6+5+6+4+6 = 54
R (with no structure) = 54/60 = 27/30 for the whole system

R (with no structure) = (3+2+3+3+2+3+2+3+1+3)/30
= 25/30 for R1,R2

= (3+3+3+3+2+3+3+0+3+3)/30
= 26/30 for R3,R4

Note: System Reliability = 0.9>0.867, 0.833 = Subsystem Reliabilities

Table 1 Streamflow Sequences Used in Example we use simulation with a multivariate sequence of only 10 annual streamflows, although longer runs would normally be used. Each simulation run assumes all reservoirs initially empty. The 10-year streamflow sequences are given in Table 1.

a) Equivalent Reservoir (Method I)

This section summarizes work developed at an earlier time and included here to contrast the techniques. Two reservoirs in parallel, R1 and R2, are represented by an equivalent reservoir E1; the two parallel reservoirs, R3 and R4, are represented by the equivalent reservoir E2; and the two parallel reservoirs, E1 and E2, are represented by the equivalent reservoir E3. Cost curves for the equivalent reservoirs are calculated by applying a simple Z-shaped rule to both reservoirs; given the linearity of the loss function (i.e., the way in which reliability is measured), more sophisticated rules are unwarranted. We have, for the R1, R2 system:

Capacity	Capacity	Reliability	Equivalent capacity	Cost (Quadratic)		
R1	R2	R	E	R_1	R_2	Total
0	0	25/30	0	0	0	0
0	0.5	26/30	0.5	0	0.625	0.625
0	1.0	27/30	1.0	0	2.5	2.5
0.5	0	25.5/30	0.25	0.55	0	0.55
0.5	0.5	26.5/30	0.75	0.55	0.625	1.175
0.5	1.0	27.5/30	1.5	0.55	2.5	3.05
1.0	0	26/30	0.5	2.2	0	2.2
1.0	0.5	27/30	1.0	2.2	0.625	2.825
1.0	1.0	28/30	2.0	2.2	2.5	4.7

Capacity	Reliabilty
E1	R
0	25/30
0.5	26/30
1.0	27/30
1.5	27.5/30
2.0	28/30

The equivalent capacity E is the capacity required at E1 to give the same reliability as that provided by the combined R1,R2 system. The table shows that E1 = 1.0 is attained for (R1 = 1.0, R2 = 0.5), with a cost of 2.825, and for (R1 = 0.0, R2 = 1.0), with a cost of 2.5; clearly the former is dominated by the cheaper combination (R1 = 0.0, R2 = 0.5).

The smaller table shows the relationship between the equivalent capacity E1 and R, from which E1-values are derived by linear interpolation for combinations (R1 = 0.5, R2 = 0.0) and (R1 = 0.5, R2 = 0.5).

The cost curves in Figure 2a are generated for the equivalent reservoir E1. For capacity E1 ≤ 1.0, the reservoir R1 = 0 and the (minimal) cost is a quadratic function of R2 alone. For E1 > 1.0, the least-cost combination is achieved for R2 = 1.0 and R1 meeting the remaining capacity, or R1 = E1 - 1.0.

Similarly, we derive in Figure 2b a cost curve for E2, the equivalent reservoir for the R3, R4 system. The relevant calculations are shown in the following abbreviated table, similar to that for E1:

Capacity R3	Capacity R4	Reliability R	Equivalent capacity E
0	0	26/30	0
0	0.5	27/30	0.5
0	1.0	28/30	1.0
0.5	0	26.5/30	0.25
0.5	0.5	27.5/30	0.75
0.5	1.0	28/30	1.0
1.0	0	27/30	0.5
1.0	0.5	27.5/30	0.75
1.0	1.0	28/30	1.0

Capacity E2	Reliability R
0	26/30
0.5	27/30
1.0	28/30

Finally, the equivalent reservoirs are combined to produce E3, the equivalent reservoir for the E1, E2 system. The tables are given below, and the cost curve is shown in Figure 2c.

Capacity E1	Capacity E2	Reliability R	Equivalent capacity E
0	0	54/60	0
0	1.0	56/60	0.67
1.0	0	57/60	1.0
1.0	1.0	59/60	2.0
2.0	0	59/60	2.0

Capacity E3	Reliability R
0	54/60
1.0	57/60
2.0	59/60

Consider the following example for these tables. Let the specified downstream reliability be 59/60 (0.983). The required capacity at E3 is 2.0, giving a cost of 4.7. Working back this corresponds to a configuration (E1 = 1.0, E2 = 1.0), which in turn corresponds to a system configuration (R1 = 0, R2 = 1.0, R3 = 0, R4 = 1.0). One should check that the final configuration does in fact give the specified reliability.

b) Effective Capacity (Method II)

The performance of each reservoir is calculated, each being operated in conjunction with the other (three) unregulated streamflows in the system. The Effective Capacity is the capacity downstream of the entire system, i.e. at E3, required to give the same reliability as a particular reservoir capacity. Thus, for example, if R1 = 1.0 and R2 = R3 = R4 = 0, the system reliability = 57/60; consulting the previous table of R vs. E3, and interpolating linearly between E3 = 0 and E3 = 1, we have the following results:

R1	R	Eff. cap	R2	R	Eff. cap	R3	R	Eff. cap	R4	R	Eff. cap
0	54/60	0	0	54/60	0	0	54/60	0	0	54/60	0
0.5	55.5/60	0.5	0.5	55.5/60	0.5	0.5	54.5/60	0.167	0.5	55/60	0.333
1.0	57/60	1.0	1.0	57/60	1.0	1.0	55/60	0.333	1.0	56/60	0.667

In this example, the relationship between effective capacity and actual capacity is linear for each site, with coefficients (1, 1, $\frac{1}{3}$, $\frac{2}{3}$) for all sites. Clearly no coefficients can exceed unity because the effective capacity cannot exceed the actual capacity it replaces.

Cost-effective functions for each site can therefore be derived by substitution; for example, for R3, the Effective Capacity is $\frac{1}{3}$(R1) so that the cost coefficient on E is $1.8\sqrt{3} = 3.12$; the set of coefficients is (2.3, 2.5, 3.12, 2.69).

Again, let the specified downstream reliability be 59/60 (0.983). The effective capacity required is 2.0. Assigning the required capacity on the basis of effective capacity-cost functions gives the system configuration (R1 = 1.0, R2 = 1.0, R3 = 0, R4 = 0) with a total cost of $2.3\sqrt{1} + 2.5\sqrt{1} + 3.12(0) + 2.69(0) = 4.8$.

4. Discussion of Equivalent Reservoir and Effective Capacity Methods

a) The essence of both methods is that they attempt to evaluate each reservoir site on the basis of its storage cost and its hydrologic impact or contribution to system performance. Determination of the hydrologic impact of a particular unit of capacity in a multi-reservoir system is extremely difficult because of the complex interrelationships among the several reservoirs and streamflow sources. Neither of the two methods captures the full richness of such a system; however, the Equivalent Capacity (II) does represent an approximate measure of the hydrologic impact of each site.

We can contrast this approach to the method of allocating storage capacity so as to minimize the cost per unit of capacity. This traditional procedure overlooks the property that the hydrologic impact of a unit of capacity will generally vary according to the location of that capacity; in effect, it assumes equality of hydrologic impact for each unit of capacity in the system. This can lead to appreciable errors, as illustrated in subsection (b) below.

However, equality of hydrologic impact does not hold in certain cases; in particular, it holds when all capacity ratios are unity (see the example in our earlier Report). It is therefore of some interest to determine those conditions under which the capacity ratio is equal to (or close to) one because this can help to evaluate the applicability of the proposed screening methods.

b) Comparison with the optimal solution of the results obtained by the two screening methods in the preceding example illustrates potential weaknesses in each method. By considering all possible reservoir configurations, it can be established that the test-cost combination which meets a downstream reliability of 59/60 is (R1 = 1.0, R4 = 1.0), which gives an equivalent capacity of 2.0 at a cost of 4.5. Note that neither of the two proposed screening methods locates this optimal configuration.

The Equivalent Capacity Method (I) gives R2 = 1.0, R4 = 1.0 (cost = 4.7). Because this method considers R1 only in conjunction with R2, and in the (R1,R2) subsystem R2 is preferred, the potential of site R1 in conjunction with other sites in the system is necessarily bypassed.

The Effective Capacity Method (II) indicates (R1 = 1.0, R2 = 1.0), with system cost = 4.8. This method fails to rank R4 highly enough because sites are considered individually and the potential benefit of having R4 operating in conjunction with any other reservoirs is overlooked.

Despite these weaknesses, the results obtained are considerably better than those given by minimizing cost per unit capacity. This method indicates R3 and R4 as the most favorable sites. However, the configuration (R3 = 1.0, R4 = 1.0) has equivalent capacity of 1.0 whence additional capacity is required elsewhere to meet the specified target reliability. This illustrates the desirability of considering differences in hydrologic impact at different sites.

c) The observations in (b) can be generalized as follows. The Equivalent Reservoir Method (I) apparently discriminates against reservoir sites which perform poorly relative to immediately adjacent sites, overlooking the fact that such apparently unpromising sites may perform much better in conjunction with other sites in the system.

The Equivalent Capacity Method may overrate certain sites which are good individually, while it tends to underrate sites which perform poorly individually but well in conjunction with other reservoirs in the system. If the final system consists of only a single reservoir, the Effective Capacity Method does evaluate each site correctly.

d) In view of the major drawbacks in each of the two methods, it may be advisable to analyze the system by both methods and to retain for further consideration those sites favored by both.

e) Because virgin or unregulated inflows are assumed at each site, one or more additional passes through the system will be required to account for regulation imposed by potential upstream reservoirs.

f) With respect to the Equivalent Reservoir Method (I), it should be noted that although the equivalent reservoirs at each stage are calculated to duplicate exactly the performance of the upstream reservoirs which they represent, after several stages there may be a discrepancy between the performance of the equivalent reservoir and that of its upstream system. This is because the analysis at each stage considers only the two reservoirs (in parallel) immediately upstream, and therefore does not consider the actual range of options available for jointly operating the systems which these two reservoirs represent. The consequence is that the downstream equivalent reservoir may incorrectly estimate the performance actually available from the system.

An illustration of this effect is evident in the example. Reservoir R1 = 1.0 gives an equivalent capacity at E1 of 0.5, which gives and equivalent capacity at E3 of 0.5. However, the system consisting of R1 = 1.0 (in addition to the other unregulated streamflows) gives an equivalent capacity at E3 of 1.0.

However, the system configuration (NOT the equivalent capacity) given by the Equivalent Reservoir Method (I) will be correct in spite of the potential error in performance evaluation. That is, we are primarily concerned with specifying the proper system, and this may happen despite systematic errors or bias in the calculations. The performance value itself is easily corrected by simulation runs incorporating a more sophisticated joint system operating rule; such more detailed operation studies will, in any case, follow the preliminary screening stage of the system design.

g) With respect to the Effective Capacity Method (II), errors are introduced because the effective capacity of a reservoir system is not necessarily the sum of the effective capacities of the component reservoirs.

5. Further Development of the Two Screening Methods

a) Standard Tables

Application of the Equivalent Reservoir Method (I) involves first decomposing the system into subsystems of two reservoirs in parallel and two (or more) reservoirs in series, then successively determining equivalent capacities and hence equivalent reservoirs for these subsystems. As described earlier, under certain simplifying assumptions the analysis is easy for the series subsystems. For the parallel systems, it is proposed that a set of standard results could be constructed, either in tabulator or graphical form, giving the capacity ratio for a range of system parameters. The relevant parameters would be those describing the system inflows, the system configuration, and the system target output. This would eliminate the need to carry out, or reduce the extent of, the equivalent capacity calculations in each particular case. it is likely that similar tables could be used for the Effective Capacity Method (II).

Note that if simulation techniques are used to construct tables of results, the question of the length of simulation required to sufficiently reduce the variability of the results must be considered. Generally the variability is reduced by increasing the length of simulation and a trade-off between computing cost and resulting accuracy must be made.

b) Application to Multi-season Models

Initial investigations assume an annual or single-season model for streamflows, so that only over-year storage in the reservoir system is considered. For many systems, the provision of within-year storage may be equally or more important, and analysis of this aspect of system performance requires a multi-season streamflow model.

Conceptually there appears to be no reason why the Equivalent Reservoir Method should not be equally applicable to a multi-season model. The basic premise of the method is that for a given reservoir system, operated according to some specified operating rule, there exists a capacity termed the Equivalent Capacity, which if located downstream of the system and operated according to some rule, would give the same level of performance as the system. If simulation is to be used in the final stage of the analysis, flow generation models for the multi-season case have been widely used and documented.

The main complication introduced with a multi-season model is the need to combine seasonal system outputs into a single measure of performance. The definition of Equivalent Capacity is the downstream capacity which matches the performance of the upstream system; in general, it is not possible to match a multi-variable performance measure by varying a single parameter, the downstream capacity.

c) Application to Multi-purpose Models

Earlier parts of the analysis assume downstream water supply as the only system output; that is, benefits are related only to the amount of water released for downstream utilization. This assumption is nearly satisfied by such uses as irrigation, domestic and industrial water supply, low flow augmentation and navigation. It is not satisfied by hydroelectric power generation, flood control and recreational uses of reservoirs.

Benefits from power generation depends on the product of flow rate and head. Recreation and flood benefits depend intimately on the volume of stored water, the relationship being different for each reservoir site. It is readily seen that the equivalent reservoir concept does not apply directly to either of these cases. For power generation, the required downstream capacity to match the upstream system performance would depend on the capacity-head relationship for the downstream reservoir. For recreation, the required downstream capacity would depend on the recreation benefit-storage level relationship for the downstream reservoir. It is not clear how these relationships could be appropriately specified.

The Equivalent Reservoir Method then is applicable mainly to systems in which benefits are directly related to water releases from the system. It can be used when several types of release-dependent benefits are present, provided that a combined benefit function can be constructed.

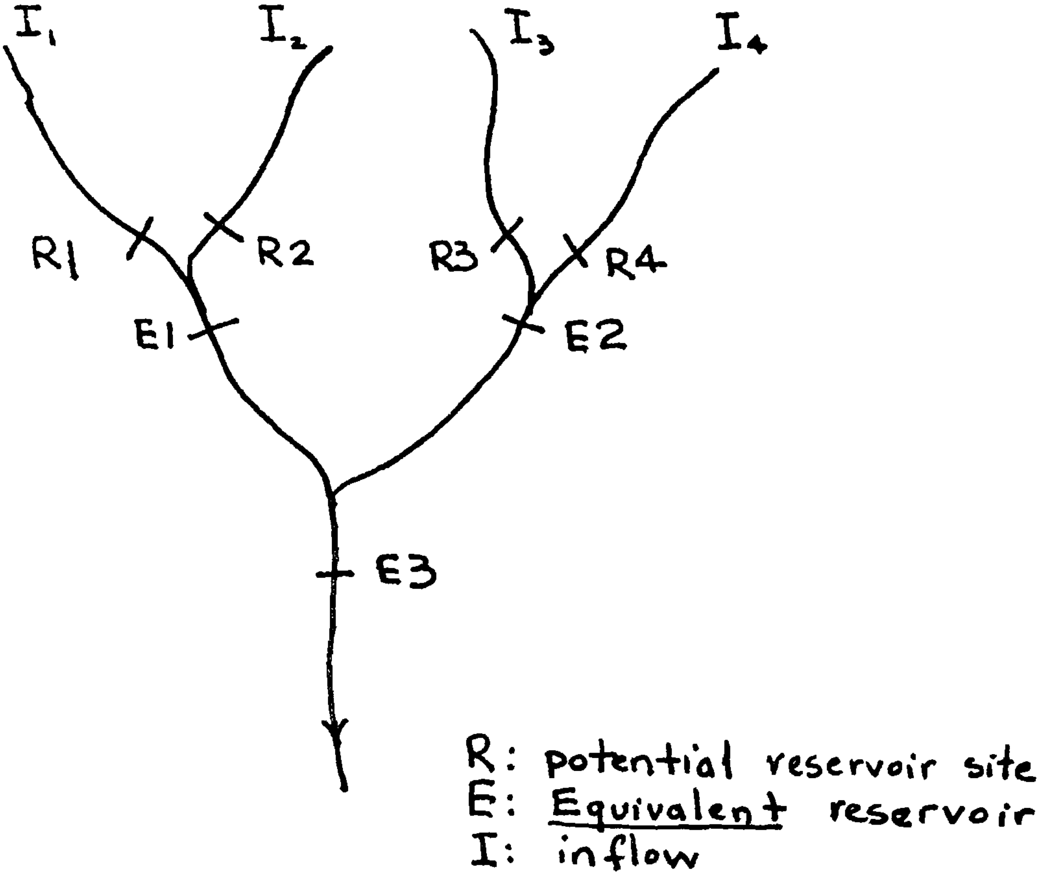
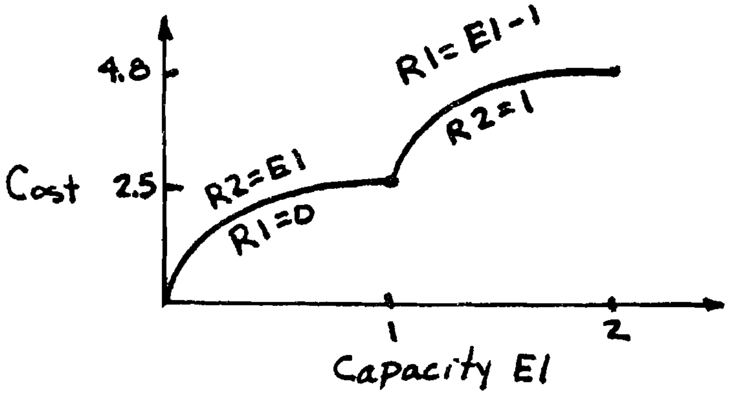
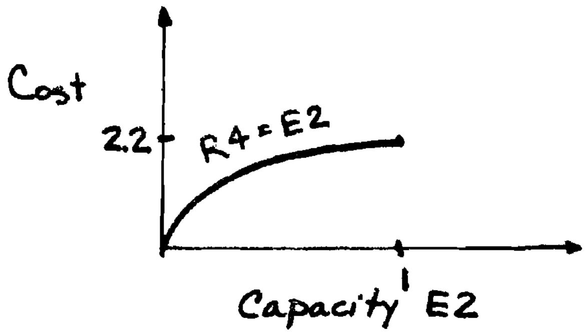


Figure 1- River Basin Used in Example

(a) Equivalent Reservoir E1



(b) Equivalent Reservoir E2



(c) Equivalent Reservoir E3

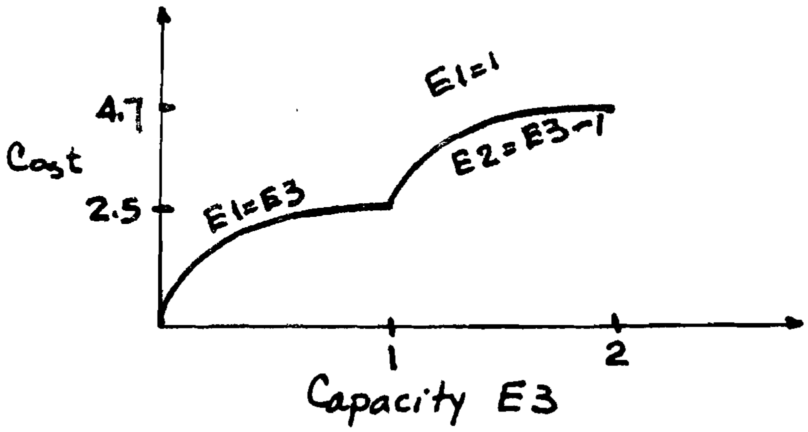


FIGURE 2 - Cost Curves for Equivalent Reservoirs