Two Constants in Carnap's View on Scientific Theories

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Abstract

The received view on the development of the correspondence rules in Carnap's philosophy of science is that at first, Carnap assumed the explicit definability of all theoretical terms in observational terms and later weakened this assumption. In the end, he conjectured that all observational terms can be explicitly defined in in theoretical terms, but not vice versa. I argue that from the very beginning, Carnap implicitly held this last view, albeit at times in contradiction to his professed position. To establish this point I argue that, first, Carnap's 'Über die Aufgabe der Physik' is a contribution to the philosophy of science of logical empiricism, contrary to Thomas Mormann and in agreement with Herbert Feigl. Second, Michael Friedman misunderstands the 'Aufgabe' with his claim that it describes a method for arriving at explicit definitions for theoretical terms.

Another received view on Carnap's philosophy of science is that it assumed a formalization of physical theories that was too detached from actual physics and thus justly disavowed in favor of the semantic view as, for example, developed by van Fraassen. But the 'Aufgabe' and related works including the *Aufbau* show that from the very beginning to his last works, Carnap suggested formalizing physical theories as restrictions in mathematical spaces, as in the state-space conception of scientific theories favored by van Fraassen.

Keywords: Rudolf Carnap, logical empiricism, logical positivism, received view, theory structure, correspondence rules, theoretical terms, Bas van Fraassen, constructive empiricism, state-space, phase space

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1 Introduction

The received view on the history of Rudolf Carnap's logical empiricist view on the structure of scientific theories is an almost pitiful one. Initially, he assumed that theories can be formalized in predicate logic, with all theoretical terms defined in observational terms. The latter assumption, Carnap himself had to weaken almost immediately and repeatedly until he had completely abandoned the demand that all concepts be introduced successively, starting from observational terms. What was left was the hope that observational terms could be defined in theoretical terms. Carnap's assumption that theories could be formalized in predicate logic was given up by almost everyone else, because it would render any actual formalization of theories inconvenient and far removed from actual science. Almost nothing of Carnap's original approach remains, as it has now been effectively replaced by the semantic view of, for instance, Bas van Fraassen. This story is very briefly recounted in §2.

Against this received view, I argue that the history of Carnap's view on the structure of scientific theories is not as tragic as it appears. First, from his earliest writings on the topic (which, I argue in §3, should be considered contributions to the logical empiricist view on theories), Carnap advocated for a description of physical theories as restrictions on mathematical spaces (§4.1). This is essentially the state-space approach advocated by van Fraassen (§4.2). Second, Carnap's conception of correspondence rules was not weakened in repeated concessions. Rather, his final conception, the possibility of defining maybe all observational terms in theoretical terms and the impossibility of the converse, was already present in his earliest writings on the general structure of scientific theories, albeit at times in contradiction to his professed position (§5.1). The same conception is also assumed by van Fraassen in his state-space approach (section 5.2).

Carnap's view on the structure of scientific theories thus emerges as surprisingly resilient, surviving, first, as an undercurrent beneath his overly positivistic tendencies during the time of the Vienna Circle, and later as a core idea of one of its alleged replacements.

2 The decline and fall of Carnap's early view on the structure of scientific theories

With respect to the interpretation of scientific terms, Carnap (1963, §9) himself describes in his 'Intellectual Autobiography' the development of his view on scientific theories as a gradual liberalization. Initially, in *Der logische Aufbau der Welt* (Carnap 1928, §35, *Aufbau* from now on), every meaningful sentence is supposed to be translatable into a sentence about experiences, and this means that "the concepts of science are explicitly definable on the basis of observation concepts" (Carnap 1963, 59).

In 'Testability and Meaning', Carnap (1936, 1937) relaxes this claim because he has come to the conclusion that it is impossible to define disposition terms. Instead, he suggests that a new term should be introduced by a pair of reduction sentences, that is, one necessary and one sufficient condition for the new term.

In the *Foundations of Logic and Mathematics*, Carnap (1939) describes two ways of relating theoretical terms (he calls them "abstract") and observational ("elementary") terms. There is, first, the construction of theoretical terms from observational terms by reduction sentences and, second, the construction of observational terms from theoretical terms. In both methods, only the observable terms are directly interpreted. Carnap (1939, 206) conjectures that in the second method, "so it seems at present, explicit definitions will do" while the first method does not allow for such a strict relations.

In two later works, Carnap does not even mention the first method of interpreting theoretical terms. In 'The Methodological Character of Theoretical Concepts', Carnap (1956, 47) states that the correspondence rules (C-rules)

> specify the relation R which [...] relates to an observable space-time region u, e. g., an observable event or thing, a class u' of coordinate quadruples which may be specified by intervals around the coordinate values x, y, z, t.
>
> On the basis of these C-rules for space-time designations, other C-rules are given for [theoretical terms]. These rules [...] hold for any space-time location. They will usually connect only very special kinds of value distributions of the theoretical magnitudes in question with an observable event. For example, a rule might refer to two material bodies [...] observable at locations u and v [...]. Another rule may connect the theoretical term "temperature" with the observable predicate "warmer than" in this way: "If u is warmer than v, then the temperature of u' is higher than that of v'."

Thus observable space-time regions can be assigned to specific space-time coordinates, and observable terms are assigned to specific values of theoretical magnitudes. Carnap also emphasizes the asymmetry between observable terms and theoretical magnitudes when noting that some, but not all, distributions of theoretical magnitudes are assigned an observable event.[1] For instance, Carnap (1962,

1. Note that his example of a correspondence rule does not, contrary to Carnap's conjecture about the definability of observational terms, provide an explicit definition of 'warmer than' in

12–13) notes that it is not required that "if x is not warmer than y (in the prescientific sense), then the temperature of x is not higher than that of y", because "[w]hen the difference between the temperatures of x and y is small, then, as a rule, we notice no difference in our heat sensations."

In 'Beobachtungssprache und theoretische Sprache', Carnap (1958, 81) repeats the construction from 'The Methodological Character of Theoretical Concepts' and notes that there is an asymmetry not only between theoretical magnitudes and observable predicates, but also between space-time coordinates and space-time regions:

> We can [. . .] introduce a space-time coordinate system in which each small body at any time point can be assigned an ordered quadruple of real numbers. Then by generalization, every such quadruple can be regarded as representative of a space-time point (therefore as an unobservable theoretical object.) Then physical magnitudes, such as mass-density, may be introduced, which have a value for every space-time point e. g., a real number. A function of this sort is construed in our system by a function F of quadruples of real numbers.

This passage is very much in keeping with the one above (Carnap 1956, 47). The space-time point u (rather than the region) is given by a small body, and the theoretical magnitudes are the physical magnitudes. In this passage, however, Carnap is even clearer about the impossibility of translating every theoretical statement into an observation statement.

Carnap's *Philosophical Foundations of Physics: An Introduction to the Philosophy of Science* (Carnap 1966), based on a seminar taught in 1958 but updated to include later results, serves as a stopping point for this brief history of the decline of Carnap's view on scientific theories. In this introductory textbook, he repeats the claim that, while it may be possible to define all observational terms in theoretical terms, the reverse is impossible (234). In connection with measurement and with the correspondence rules between theoretical and observational terms, Carnap (1966, 57, 72, 266) also references a monograph on concept formation by Hempel (1952, 684, §7), who repeatedly stresses the importance of

> *theoretical constructs*, i. e., the often highly abstract terms used in the advanced stages of scientific theory formation, such as 'mass, 'mass point' [. . .], 'volume', 'Carnot process' [. . .], 'proton', 'ψ-function', etc [. . .]. Terms of this kind are not introduced by definitions or reduction chains based on observables; in fact, they are not introduced

theoretical terms. If the conjecture is right, the relation is at best *entailed* by the complete correspondence rule.

> by any piecemeal process of assigning meaning to them individually. Rather, the constructs used in a theory are introduced jointly, as it were, by setting up a theoretical system formulated in terms of them and by giving this system an experiential interpretation.

One way of giving an experiential interpretation to the theoretical system consists in defining further concepts with the help of the theoretical constructs and interpreting those further concepts directly (Hempel 1952, 686–87, §7). This is Carnap's second method of giving empirical meaning to theoretical terms (Carnap 1939); Hempel's treatment of 'mass' as such a theoretical term (Hempel 1952, §12) was even suggested by Carnap himself (738, n. 72). Hempel (1952, §§6–7) considers the use of theoretical constructs another step in the liberalization of empiricism: The claim that all terms are explicitly definable in observational terms is the narrower thesis of empiricism (676), the claim that all terms are reducible to observational terms (Carnap 1936) is the liberalized thesis of empiricism (Hempel 1952, 683). The need for reduction sentences shows that the narrower thesis is false, and the need for theoretical constructs shows that the liberalized thesis is false.

While Carnap saw value in the concept of correspondence rules until the end, Hempel (1970, §6) extended his critique of explicit definitions and reduction sentences to theoretical constructs and dismissed the whole concept of correspondence rules as misguided. Further criticisms of correspondence rules (Suppe 1972, §II) and the use of axiomatizations in general (Hempel 1970, §3, Suppe 1974, §IV.f) contributed to the downfall of Carnap's view on the structure of scientific theories. Hempel (1969, 1970), for example, abandoned it completely. In its stead, the semantic view on scientific theories became an important framework for the reconstruction of theories, possibly the dominant one (Suppe 1989, 3). The semantic view assumes set theoretical descriptions of scientific theories (Suppes 1967) or descriptions in phase space (van Fraassen 1970), both of which are considered closer to actual science than Carnap's view.

So much for the received view on Carnap's view. To arrive at a more positive assessment, it suffices to look at Carnap's earliest works on the structure of scientific theories.

3 Three early works of Carnap

Both Suppe (1974, 12) and Feigl (1970, 3) cite 'Über die Aufgabe der Physik und die Anwendung des Grundsatzes der Einfachstheit'[2] (Carnap 1923, 'Aufgabe'

2. 'On the Task of Physics and the Application of the Principle of Maximal Simplicity'

from now on) as Carnap's earliest exposition of the logical empiricist's view on scientific theories (Feigl also mentions Campbell (1920)). Mormann (2007, 159, n. 13) criticizes this classification (in a footnote, without elaborating further):

> Feigl once went so far to trace back the essentials of the Logical Empiricist account of empirical theories to an early (pre-Vienna) paper of Carnap that may well be classified as belonging to his neo-Kantian period [. . .]. This stance betrays, to put it mildly, that Feigl did not pay too much attention to the amendments that had taken place since then.

However, even without consulting the 'Aufgabe' itself, there are some reasons to consider it a contribution to logical empiricism's view of theories. For one, Mormann's claim of a discontinuity rests on the text's belonging to Carnap's neo-Kantian period, but even though Carnap wrote the article while holding neo-Kantian views (Carus 2019a, note b), this does not mean that they are manifest in the article. Carnap himself, for example, did not seem to think so. When discussing the influence of Kant's views on his own work in his autobiography, Carnap (1963, 12) mentions his doctoral dissertation (Carnap 1922), specifically the chapter on intuitive space (Carnap 1963, 4), but not the 'Aufgabe'. As influences for this article, Carnap (1963, §13 and 77–78) rather lists Poincaré and Hugo Dingler, and at another point Hilbert, Poincaré, and Duhem.

Furthermore, Carnap (1963, 15, §2) classifies the 'Aufgabe' as written in the same "period" as the monograph *Physikalische Begriffsbildung*[3] (Carnap 1926), which was written in Vienna after the main work on the *Aufbau* had concluded (Carus and Friedman 2019, lxi), and whose main points were much later taken up by both Hempel (1952) and Carnap (1966).

Perhaps most importantly, Carnap begins §13 of his autobiography, entitled 'The Theoretical Language', with the 'Aufgabe' (followed by his monograph on concept formation) and ends it with his article on the methodological character of theoretical terms (Carnap 1956). This is relevant because §13 occurs in Part II of Carnap's autobiography, entitled 'Philosophical Problems', where "[i]n each section, a certain problem or complex of problems [is being] dealt with". So Carnap himself considers the 'Aufgabe' a starting point of the development that led to one of his core articles on scientific theories.

It is revealing that Carnap ends the overview that begins with the 'Aufgabe' with his article on theoretical terms, which is a discussion of the formal structures of correspondence rules and the formal requirements they must fulfill so that theoretical terms and sentences count as empirically significant. Such formal

3. *Physical Concept Formation*

aspects can survive radical changes in meta-theoretical perspective. For instance, Carnap's discussion of empirical significance showed no significant break when he included explicitly semantic considerations in his analysis (cf. Lutz 2017), and neither did his reliance on higher order logic (Lutz 2012, §2). Thus even if Carnap wrote the 'Aufgabe' from a Kantian perspective, all other aspects, and especially the technical ones on the structure of scientific theories, can already anticipate the logical empiricist view.

In his autobiography, Carnap (1963, 15) summarizes the 'Aufgabe' already in his later terminology:

> I imagined the ideal system of physics as consisting of three volumes: The first was to contain the basic physical laws, represented as a formal axiom system; the second to contain the phenomenal-physical dictionary, that is to say, the rules of correspondence between observable qualities and physical magnitudes; the third to contain descriptions of the physical state of the universe for two arbitrary time points. From these descriptions, together with the laws contained in the first volume, the state of the world for any other time-point would be deducible (Laplace's form of determinism), and from this result, with the help of the rules of correspondence, the qualities could be derived which are observable at any position in space and time. The distinction between the laws represented as formal axioms and the correlations to observables was resumed and further developed many years later in connection with the theoretical language.

Here, then, Carnap also points out the continuity between the 'Aufgabe' and later discussions of theoretical terms.

In the 'Aufgabe' itself, Carnap (1923, 90, emphases removed) introduces his main point with a reference to Poincaré and Dingler, but not Kant:

> It is the main thesis of the conventionalism expounded by Poincaré and further developed by Dingler that in the construction of physics we have to make stipulations that are subject to our free choice. [. . .] But the choice among these stipulations ought not therefore to be made arbitrarily, rather it should follow certain methodological principles—and in the end the principle of maximal simplicity has to decide.

The laws of physics in the first volume can be chosen according to the principle of maximal simplicity, Carnap (1923, 90) states, because they are not determined by experience. Whether, for example, the world has a Euclidean or non-Euclidean geometry depends on the objects that are chosen to be rigid bodies (an example that Carnap will repeat in his introductory textbook (Carnap 1966) when discussing the conventional elements in the concept of length). Therefore, Carnap (1923, 97) concludes, the laws of physics are

> *synthetic a priori propositions*, although not exactly in the Kantian transcendental–critical sense. For that would mean that they expressed necessary conditions of the objects of experience, themselves conditioned by the forms of intuition and thought. [...] Actually, though, [this volume's] construction is in many ways left to our choice. [...] As an identifying description of the first volume, the concept of a "hypothetico-deductive system" [...] is therefore to be preferred to the Kantian concept of the a priori.

This is the only passage in the 'Aufgabe' that contains an explicit reference to Kant, and it is far from an endorsement of the Kantian doctrine of the synthetic a priori, but rather, just as Carnap (1963) states in his autobiography, an endorsement of Poincaré's conventionalism. One could of course argue that Poincaré's conventionalism is neo-Kantian, but then Carnap never gave up neo-Kantianism in this sense, since he endorsed the conventionality of theoretical constructions to the end.

The connections between the 'Aufgabe'and Carnap's later view on scientific theories are obvious. He makes a strict distinction between, first, the domain of perception and, second, the domain of physical theories, a distinction that "cannot be emphasized sharply enough". The contents of perception do not occur at all in theoretical physics, which is not obvious only because terms like 'pressure' and 'heat' are used in both domains (Carnap 1923, 99). The connection between the two domains is given, "in a way, through a kind of dictionary that indicates which objects (elements, complexes, processes) in the second domain correspond to the particular ones of the first". This dictionary is just a metaphor for correspondence rules. Carnap (1923, 98–99) gives the following examples for dictionary entries:

> "Such and such a shade of blue (designated, for example, according to Ostwald's color system) corresponds to a certain periodical movement of electrons (denoted by the frequency of oscillation)" [...] "Such and such a pungent smell (the smell of chlorine; [smells] lack a classificatory system) corresponds to a certain mixture of peculiarly structured electron complexes (Cl-atoms)"; "A certain temperature sensation ([these also] lack a classificatory system) corresponds to a certain average kinetic energy of a number of electron complexes (atoms or molecules)"

The two examples for correspondence rules that Carnap (1966, 233) would later give in his introductory textbook read almost like translations of these dictionary entries:

> An example for such a rule is: "If there is an electromagnetic oscillation of a specified frequency, then there is a visible greenish-blue color of a certain hue". Here something observable is connected with a nonobservable microprocess.
>
> Another example is: "The temperature (measured by a thermometer, and therefore, an observable in the wider sense explained earlier) of a gas is proportional to the mean kinetic energy of its molecules". This rule connects a nonobservable in molecular theory, the kinetic energy of molecules, with an observable, the temperature of a gas.

Here, the Ostwald color-system is substituted by the hue of the colors, and the missing system of designation for heat experiences is circumvented by using an observational term in the wider sense, the temperature according to a thermometer. The connection to heat experiences has to be given through the further correspondence rules between temperature and the observational relation 'warmer than' already noted above Carnap (1956, 12–13).

In an article published shortly after the 'Aufgabe', entitled the 'Dreidimensionalität des Raumes und Kausalität. Eine Untersuchung über den logischen Zusammenhang zweier Fiktionen',[4] Carnap (1924, 107–8) relies, as in the 'Aufgabe', on the distinction between the domain of perception and the domain of physical theories. He speaks of experiences of the first level, which are the phenomena, and experiences of the second level, which can be those of physics or, since this level is subject to conventional choices, also the ordinary experiences involving everyday concepts. The contents of the experiences on the first level are called the "primary", those of the second level the "secondary world", which can again be the "world of physics" or the "ordinary world" (Carnap 1924, 108-9). The connection between the secondary and the primary world is, with a reference to the 'Aufgabe', given through a "relation of coordination" (Carnap 1924, 108).

In his monograph on concept formation, Carnap (1926, 60) also gives another example of correspondence rules: Specific electron configurations are assigned specific atoms or crystals, say, chloride and sodium or sodium-chloride. These configurations are then assigned the qualities 'white' and 'salty'. These correspon-

4. 'Three-Dimensionality of Space and Causality: An Investigation of the Logical Connection Between Two Fictions'

dence rules are very close to the ones in his introductory textbook,[5] but they are even closer to an example that Carnap (1939, 207) gives somewhat earlier:

> [L]et us imagine a calculus of physics is constructed, according to the second method, on the basis of primitive specific signs like 'electromagnetic field', 'gravitational field', 'electron', 'proton', etc. The system of definitions will then lead to elementary terms, e. g. to 'Fe', defined as a class of regions in which the configuration of particles fulfils certain conditions, an 'Na-yellow' as a class of space-time regions in which the temporal distribution of the electromagnetic field fulfils certain conditions. The semantical rules are laid down stating that 'Fe' designates iron and 'Na-yellow' designates a specific yellow color. (If 'iron' is not accepted as sufficiently elementary, the rules can be stated for more elementary terms.)

When it comes to the detailed description of the relation of the observational and theoretical terms, the 'Aufgabe', the monograph on concept formation, this much later text, and the introductory textbook show barely a change at all.

Finally, the *Aufbau* puts the discussions of the three early works in a wider perspective. Here, Carnap aims at developing a constitution system in which all statements are translatable into statements that contain only one primitive relation (although more are possible) (Carnap 1928, §§1–2, §35, §156). The lower levels of the system constitute the autopsychological objects from the primitive relation, the middle level the physical objects from the autopsychological objects, and the upper levels the heteropsychological and cultural objects from the physical objects. The correspondence rules of the 'Aufgabe' and the monograph on concept formation are found in the middle level.

The autopsychological objects are first used to constitute "the entire space-time world, with the assignment of sense qualities to the individual world points, [which] we call the perceptual world" (Carnap 1928, §133). The perceptual world then allows the constitution of the "physical world"(§136).[6] When constructing

5. Here is an abbreviated list of further correspondences in content between the 'Aufgabe', the monograph on concept formation, and the introductory textbook, given with the respective page numbers: The role of simplicity in choosing scientific laws and correspondence rules: §§IV–V/ 16–17, 27, 30–31/69, 84, 145, 168. The possibility of choosing what counts as rigid body: 91/25/ 91. The possibility of choosing what counts as periodic process: 91–92/39/80. Time dependence of counting: —/15/60. Five rules for measurement scales: —/22–23/63–65. Standardization of temperature and length through successive approximation: —/35–36/98–99. Explicit endorsement of Kantian doctrines: —/—/—.

6. George translates the title of the paragraph as "The World of Physics". The original uses the same phrase ("Die physikalische Welt") as the article on three-dimensionality and causality.

the physical world, Carnap refers to his three earlier workds for further elaboration.[7]

In the construction of the world as described in the *Aufbau*, then, the method of interpreting theoretical terms that Carnap (1923) describes in the 'Aufgabe' is not involved in the the assignment of sensory qualities to space-time points, but covers the whole of the second step, the correlation of numbers as values of state magnitudes to these sensory qualities. The Kantian notions of Carnap's doctoral dissertation would occur somewhere in the construction leading to the assignment of sensory qualities to space-time points. So whatever Kantian notions about synthetic a priori statements Carnap had at the time of writing, they would not have been relevant for the construction developed in the 'Aufgabe'. This is the final reason why Mormann's claim that the 'Aufgabe' cannot be considered a contribution to the logical empiricist's theory of science is incorrect.

In fact, a bigger break in Carnap's reasoning about the structure of scientific theories seems to have come after he had written his monograph on concept formation: The finitism of Wittgenstein's *Tractatus* seemed to make a theoretical language impossible (Carus and Friedman 2019, xli). Accordingly, there is a long gap of Carnap's writings on the philosophy of science until after the *Logische Syntax der Sprache* (Carnap 1934), which deals with this problem (and others).

4 The formalization of scientific theories

In one of his criticism of the logical empiricists, van Fraassen (1989, 225) writes:

> The scholastic logistical distinctions that the logical positivist tradition produced—observational and theoretical vocabulary, Craig reductions, Ramsey sentences, first-order axiomatizable theories, and also projectible predicates, reduction sentences, disposition terms, and all the unholy rest of it—had moved us mille milles de toute habitation scientifique, isolated in our own abstract dreams.

I will discuss in the next section how much worse Carnap's logical empiricist view of the relation between theory and observation is than van Fraassen's semantic view. In this section, I will focus on how much worse Carnap's view on the formalization of theories is. Before I do so, it bears repeating that the alleged fixation of logical empiricists on first-order logic was invented from whole cloth by critics of logical empiricism (Lutz 2012, §2), and that Carnap saw higher order logic as

7. All (and only) these three works of Carnap's are listed in the references to §136 of the *Aufbau*, contrary to Carus (2019b, note h).

including all of mathematics (Carnap 1939, §14), which is a quite plausible position (Andrews 2002, vi–xiv). The question is then how logic and mathematics were supposed to be used for describing theories according to Carnap.

4.1 Carnap's purely numerical structures

In his article on three-dimensionality and causality, Carnap (1924, 107) repeats the point of the 'Aufgabe' that the physical world is free from perceptual qualities and contains

> only spatial and temporal magnitudes, together with certain nonsensible state-magnitudes. In its purest form, moreover, these three types of magnitudes have no character comparable with spatiality, temporality, or sensible qualities, but are rather mere numerical determinations, i. e., relational terms.

The laws in this pure physical world are also free from the concept of causation as it occurs in the ordinary world (Carnap 1924, 108, emphasis in the original):

> The processes in the physical world *do not act* on one another; rather they are governed by a dependency that is to be conceived of as a purely mathematical, functional relation [. . .].

The use of "mere numerical determinations" with a dependency that is considered "a purely mathematical, functional relation" to describe scientific theories is the first major constant in Carnap's view on scientific theories.

Carnap (1924, 120) notes that instead of describing the value of each magnitude at each point in space for two different times, one can equivalently describe the value and the derivative of each magnitude at each point in space for just one time. In the 'Aufgabe', Carnap (1923, 101) notes that for logical reasons, using two different times is more correct (Carnap refers to Russell and Hausdorff (Carus 2019a, note f) for the proof that derivatives cannot be considered instantaneous magnitudes); the equivalence however enables the use of the more expedient description at one time:

> If it is possible to calculate, from the coordinate values of the n elements at time t_0 and the $3n$ components of their velocities, their $3n$ coordinates at time t_1, then we only need to think of the $3n$ equations specifying these coordinates as solved for the $3n$ velocity components in order to see that these components are then also determined by their coordinates at times t_0 and t_1.

While Carnap in general in the 'Aufgabe' speaks of the laws of scientific theories without further specification as a set of formal axioms,[8] this more concrete elaboration is a description of phase space that is only missing the explicit classification as a geometrical space.[9] (This classification, however, brings with it substantial formal gains (Scheck 2010, §1.18).)

The explicit classification as a geometrical space is first mentioned in the article on three-dimensionality and causality, where Carnap (1924, II.c) claims that the physical world is four-dimensional with the qualification that this "concerns only spatial and temporal dimensions, not qualitative dimensions" (113). Here Carnap is conceptualizing physical values as fixing one dimension of an abstract space.[10] Laws of theories are described as restrictions on abstract numerical functions (119):

> If any element of a class depends on other elements in such a way that it is uniquely determined as soon as a certain subclass of the remainder is fixed, then we call the relation of dependency a "determining law" and the class "determined".[. . .]
>
> We call laws of dependency that do not result in unique determinacy for any element, even if all the rest are determined, but still limit the possibilities for this element, "*constraining laws*".

This abstract view on scientific theories is elaborated in chapter III of Carnap's monograph on concept formation, entitled "Abstract Stage: the Four-Dimensional Universe". Here, Carnap first suggests identifying space-time points by four-tuples so that a physical description of the world just consists in assigning the values of the basic physical magnitudes to each point in four-dimensional space-time. But, Carnap notes, it is also possible to go beyond this geometrical description and let descriptions of the world consist of sets of tuples with the first four values being the space-time points, the rest being the values of the physical magnitudes. This then is the purely numerical description of his article on three-dimensionality and causality. As in that article, laws of nature are in both cases restrictions on the possible descriptions: restrictions on the possible assignments of values to space-time points in the geometrical case, and restrictions on the possible sets of tuples in the numerical case (Carnap 1926, 58).

8. Cat describes in section 2.3.4 of this volume the specific scientific theories that Carnap considers for axiomatization.

9. Thus this description is missing the conceptual step from the formalism to the interpretation as a space, corresponding to the step from Liouville, Jacobi, and Boltzmann to Gibbs, Poincaré, and Paul Ehrenfest (Nolte 2010).

10. By treating time as another geometric dimension rather than a parameter, Carnap technically does not use phase space, but rather the more general extended phase space (Scheck 2010, §1.20).

In §136 of the *Aufbau*, finally, the "physical world" is accordinlgy considered a "(purely numerical) structure".

In the monograph on concept formation, Carnap (1926, 55–56) is explicit that he is thinking of field theories,[11] noting that it is at this point not yet clear whether all of physics can be so described. A classical field theory is the infinite dimensional analogue to the finite dimensional phase space (Scheck 2010, §§7.1, 7.4, 7.6).

Carnap (1963, 15–16) himself considered this aspect of his monograph to be closely connected to his later work:

> I described the world of physics as an abstract system of ordered quadruples of real numbers to which values of certain functions are co-ordinated; the quadruples represent space-time points, and the functions represent the state-magnitudes of physics. This abstract conception of the system of physics was later elaborated in my work on the theoretical language.

Of course, these points of the monograph essentially recapitulate the same points in the article on three-dimensionality and causality. The formalism suggested in these two works would later be used again (Carnap 1956, 1958), with the same stress on the purely mathematical character of the space-time tuples and the assignment of other numbers to those tuples as physical magnitudes at those space-time points.

4.2 Van Fraassen's state-space

In the semantic view, theories are not formalized as sets of postulates in predicate logic, but rather through the structures that are models of such sets (Suppe 1989, 4), or through restrictions on an abstract state-space (van Fraassen 1980, 44). When they rely on structures, the formalizations of the semantic view can be expressed in higher order logic (Halvorson 2013; Lutz 2014b; Da Costa and Chuaqui 1988; Hudetz 2017).[12] This leaves van Fraassen's state-space approach as a possible competitor to Carnap's view.

In an outline of his semantic view that is endorsed, for example, by Suppe (1974, 222–21), van Fraassen (1970, 328–29) focuses on

> the formal structure of *nonrelativistic* theories in physics [. . .]. A physical system is conceived of as capable of a certain set of *states*,

11. In this focus on field theories, Carnap is in the company of Schlick: See Cat's section 2.3.1 in this volume.

12. This was first mentioned by Montague (1965, 143).

> and these states are represented by elements of a certain mathematical space, the *state-space*. [...] To give the simplest example, a classical particle['s] [...] state-space can be taken to be Euclidean 6-space, whose points are the 6-tuples of real numbers $(q_x, q_y, q_z, p_x, p_y, p_z)$.

Van Fraassen (1970, 330) then distinguishes between laws of coexistence, laws of succession, and laws of interaction. In the non-statistical case (§5.1), laws of coexistence select the physically possible subset of the state-space (330), laws of succession select, in the instantaneous state picture, the physically possible trajectories in the state-space (331), and laws of interaction at least in principle reduce to the above (332).

Van Fraassen's phase space example recalls Carnap's description of an *n*-particle system in the 'Aufgabe'. And van Fraassen's schema for the formalization of scientific theories is essentially that described in Carnap's article on three-dimensionality and causality. Both assume a purely mathematical description of the possible states of a system and consider scientific theories as restrictions on this space. The main difference is that Carnap focuses on systems that contain continuous fields and thus have infinitely many degrees of freedom, while van Fraassen focuses on systems that contain a finite number of particles with accordingly finitely many degrees of freedom.[13]

Formally, the main difference between Carnap and van Fraassen is that the former assumes that the description of the theory is given in higher order logic, the latter assumes that it is given in the semi-formal meta-language of an elementary language to be discussed below. But since both Carnap and van Fraassen take their respective languages to include all of mathematics and allow as much or as little explicit formalization as expedient (Lutz 2012, §§2–3), this difference is merely verbal (Lutz 2014b, 1489).

5 The relation between observational and theoretical terms

The formalizations assumed in Carnap's view on theories did not move us "mille milles de toute habitation scientifique", at least not more so than van Fraassen's view. But how about the "observational and theoretical vocabulary [...] and all the unholy rest of it"? Specifically, is Carnap's view, and especially his early view, on the rules connecting theory and observations indeed too far removed from science?

13. A minor difference is that Carnap, when discussing finite dimensional systems, assumes a formalization in extended phase space (see n. 10), while van Fraassen assumes normal phase space. I thank Bobby Vos for pointing this out to me.

5.1 Carnap's correspondence rules

In his introductory textbook, Carnap (1966, 233–34, footnote removed) writes:

> Different writers have different names for these rules. I call them "correspondence rules". P. W. Bridgman calls them operational rules. Norman R. Campbell speaks of them as the "Dictionary". Since the rules connect a term in one terminology with a term in another terminology, the use of the rules is analogous to the use of a [. . .] dictionary. [. . .]
>
> There is a temptation at times to think that the set of [correspondence] rules provides a means for defining theoretical terms, whereas just the opposite is really true. A theoretical term can never be explicitly defined on the basis of observable terms, although sometimes an observable can be defined in theoretical terms.

This is Carnap's final view on correspondence rules: Observational terms can sometimes be explicitly defined in theoretical terms, but not vice versa. As I will argue, it is the second constant in Carnap's view on scientific theories, identifiable in all of his major works on the structure of scientific theories, starting from the 'Aufgabe'. There, Carnap (1923, 100, emphasis in the original) states that any statement in observational terms can be described in theoretical terms, but not vice versa:

> The dictionary can be used in both directions: it serves to translate a phenomenal state of affairs to the corresponding physical one and vice versa. However, we have to note that *the correlation is unique only in the second case*; a particular sensory content corresponds not to just a single particular physical state of affairs but to an infinite number of them.

Carnap's first reason for the lack of (unique) translatability into the phenomenal language are the multiple microscopic realizations of physical macrostates in, for example, thermodynamics. His second reason is the "psycho-physical fact of the threshold of sensitivity" (100). As recapitulated above, Carnap (1962, 13) will later argue that because of this threshold of sensitivity, it cannot be required that there is a difference in temperatures only if there is a difference in heat sensations. Carnap (1924, 126) also adduces the perception threshold in his article on three-dimensionality and causality and adds two more reasons: First, a sensation does not uniquely identify the location of its physical source. Second, a sensation, say, of a color, corresponds to a multitude of physical states, say, a multitude of frequency distributions of electromagnetic waves. Thus phenomenal states are multiply realized by physical states, which, if they are physical macrostates (as in thermodynamics), are themselves multiply realized by physical microstates.

In his monograph on concept formation, Carnap (1926, 60) repeats the claim that observational states are uniquely determined by physical states:

> The retranslation of the pure number statements of abstract physics into qualitative statements is possible because a particular distribution of values of particular state-magnitudes is uniquely co-ordinated with certain physical qualities, ultimately sensory qualities. A certain character of a set of 14-tuples of numbers is e.g. to be interpreted as a certain motion of electrons within a certain spatial configuration. And this in turn is to be interpreted e. g. as a chlorine atom or a sodium atom or a sodium chloride crystal, i. e. table salt. A set of 14-tuples so constituted is then co-ordinated with the qualities white and salty. Another such set is to be interpreted as, say, another motion of a set of electrons, and this in turn as a particular periodic distribution of air molecules, hence a sound wave of a particular frequency; a set of 14-tuples of numbers so constituted is then associated with a tone of particular pitch, timbre and volume, or a precisely characterized sound.

Another example would be the translation of the quadruples of space-time numbers with their assigned momenta into average kinetic energies, from there into temperature, and finally into heat experience (Carnap 1958, 1966, 1962).

Note that 'uniquely co-ordinated' here means that each state-magnitude is assigned exactly one sensory quality, so that 'retranslation' means that each description of state-magnitudes is assigned exactly one description of physical qualities, but not necessarily *vice versa*; this is the converse of the modern use of 'translation' in logic and definition theory (cf. Creath and Richardson 2019, note dd).

In the monograph, Carnap does not explicitly state that physical qualities cannot be uniquely co-ordinated with state-magnitudes, but repeats his claim from the 'Aufgabe' and his article on three-dimensionality and causality that there is a perception threshold (Carnap 1926, 17):

> As a first attempt, we could [. . .] assign a higher temperature to one of the bodies if it evokes a stronger sensation of warmth [. . .]. This method of assignment, however, would turn out not to work, owing to the fact of "heat transfer". For if two bodies are brought into contact with each other [and] only one has a perceptible change, then we ascribe to the other a complementary change of imperceptible

> magnitude (for the sake of later laws of nature in connection with the concept of specific heat).[14]

This illustration of the perception threshold led Carnap in the earlier two articles to conclude that the theoretical states are not uniquely determined by the observational states, and Carnap does nothing to counter this conclusion here. However, he does not give a perfectly clear endorsement of the thesis; this clear endorsement is given in the *Aufbau*, of all places.

In §136 of the *Aufbau*, the "physicoqualitative correlation" consists, first, of a one-to-one correspondence between the world points of physics (the space-time points from the monograph on concept formation) and the world points of the perceptual world and, second, a many-one relation between the physical magnitudes and the perceptual qualities as in the 'Aufgabe'. While each physical state can be assigned a perceptual state,

> in the opposite direction, the correlation is not unique; the assignment of a quality to a world point in the perceptual world does not determine which structure of state magnitudes is to be assigned to the neighborhood of the corresponding physical world point of the world of physics; the assignment merely determines a class to which this structure must belong.

Implicitly, this position entails that observational terms can be determined in theoretical terms, but not *vice versa*, because, as in the 'Aufgabe' (Carnap 1923, 99), Carnap assumes a strict distinction between the concepts applying to the physical states (which are named by theoretical terms) and the concepts applying to the perceptual states (which are named by observational terms).

In modern terminology, the relation between phenomenal and physical states can be phrased syntactically or semantically. (Carnap could not easily have made that distinction, since it did not become clear to him until 1930 (Reck 2007, 189–191).) Syntactically, Carnap claims that every sentence in observational terms can be translated into a sentence in theoretical terms with the help of the correspondence rules but not vice versa. From the theory of definition it is known that then the correspondence rules entail an explicit definition for each observational term in theoretical terms, while the converse does not hold. Semantically, Carnap claims that a structure for the theoretical vocabulary can be expanded at most in one way to a model of the correspondence rules, while at least some structures for the observational vocabulary can be expanded to more than one

14. The translation of the parenthetical remark ("späteren Naturgesetzen zuliebe, die bei dem Begriff der spezifischen Wärme auftreten") is changed from the translation by Dean and Richardson.

model of the correspondence rules. This means that the correspondence rules do not entail an explicit definition for each theoretical term (according to Padoa's theorem), and that the correspondence rules do entail an explicit definition for each observational term (according to Beth's theorem) for correspondence rules in first order logic and (according to theorem 3 by Tarski (1935)) for a finite number of correspondence rules in finite type theory (cf. Leivant 1994, §5.1).

In other words, already in the 'Aufgabe', his very first publication on the structure of theories, as well as in his article on three-dimensionality and causality, his monograph on concept formation, and in the *Aufbau*, Carnap indirectly claims that theoretical terms are interpreted through observational terms according to the second method described in the *Foundations of Logic and Mathematics* (Carnap 1939), although without the later qualification that explicit definability *seems* possible given the contemporary state of science. Of course, up to the 1920s, definition theory and formal semantics were not yet developed enough to phrase this consequence so clearly. Nonetheless, this is what Carnap's position entails.[15]

In his autobiography, Carnap (1963, 19) notes how the part of the *Aufbau* in which he claims the undefinability of theoretical terms relates to his later work:

> For the construction of the world of physics on the basis of the temporal sequence of sensory qualities, I used the following method. A system of ordered quadruples of real numbers serves as the system of co-ordinates of space-time points. To these quadruples, sensory qualities, e. g., colors, are assigned first, and then numbers as values of physical state magnitudes. [. . .] In general, I introduced concepts by explicit definitions, but here the physical concepts were introduced instead on the basis of general principles of correspondence, simplicity, and analogy. It seems to me that the procedure which is used in the construction of the physical world, anticipates the method which I recognized explicitly much later, namely the method of introducing theoretical terms by postulates and rules of correspondence.

Carnap's discussion of his attempt to define all theoretical terms in observational terms here follows Quine (1969, 76–77), and focuses on the first step at which the attempt fails: The construction of the phenomenal world through simplicity and analogy (cf. Friedman 1992, n. 9; Friedman 1999, 160–62). But Carnap discusses the step to the phenomenal world together with the step to the physical world, and thereby brushes over the point I have made here: Even if Carnap had succeeded in explicitly defining the concepts of the perceptual world, the physical

15. Carnap (1936, 168) referred to Tarski's article soon after its publication, and so at least could have seen the relation then.

magnitudes and anything that relies on physical magnitudes for its constitution would not be explicitly definable in perceptual terms, according to Carnap's own position in his three earlier works and the *Aufbau* itself.

Again it seems that the technical details of Carnap's view on the structure of scientific theories survived changes in his meta-theoretic views: Carnap (1928) clearly thinks of his technical analysis in his three early works as correct, since he refers to them for the details of his construction and explicitly restates their central point about correspondence rules. It is just that he also has meta-theoretic views (here: overly positivistic ones) that in this case conflict with his technical analysis.

Contrary to this result, Friedman (1992), in his discussion of §136 of the *Aufbau*, claims that Carnap does give a method for arriving at explicit definitions in spite of the one-to-many relation between phenomenal and physical facts. Specifically, Friedman (1992, 21–22) states:

> Although Carnap [claims] that the coordination between "phenomenal facts" and corresponding state-magnitudes is only unique [. . .] in the direction from the latter to the former, he [. . .] outlines a procedure for nonetheless approximating to a unique assignment of physical state-magnitudes by focusing on a small neighborhood of a given phenomenally characterized space-time point and working back and forth using the laws of physics (1923, pp. 102–03). The crucial point is that the laws of physics, together with an unambiguous determination of phenomenal qualities from physical state-magnitudes, provide a methodological procedure for narrowing down the ambiguity in the assignment of physical state-magnitudes: in principle, a unique assignment is thereby constructed after all.

This is not Carnap's claim. The method to which Friedman refers is described in a passage in the 'Aufgabe' in which Carnap relaxes the idealizing assumptions about the third volume of an ideal physics, the complete knowledge of the state of the world. "Then the task is to calculate from the observed state of a restricted region, namely our own spatio-temporal neighborhood, the state of another space–time region" (102). As a technical problem of this task, Carnap notes that to calculate even an arbitrarily small area just for one second would demand knowledge of the state of the world in a 300 000 km radius. The bigger problem is that in principle the physical state of an area cannot be uniquely determined from observations, because the dictionary contains only one-many relations.

The method to come to predictions is given in the following passage (Carnap 1923, 102–3):

> The reason why a physics that is as yet far from even this more modest fiction is nevertheless able to make predictive calculations on the basis of observations is the following. To be sure, an infinite set of physical states of the region corresponds to a given observational result, and thus also an equipollent set of such states for the future moment to be calculated [. . .]. But with the inverse translation of this infinite set of physical states back into sensory contents, there often turns out to be a relatively small set of sensory contents that in favorable cases forms a continuous domain of qualities (e.g., a domain of similar shades of color). What we try to do is, first of all, to carry out the observations in such a way that several unconnected qualitative domains do not result for the future point in time and, second, to narrow the boundaries of the one qualitative domain. The two defects of prediction, ambiguity and imprecision, can thus be reduced more and more as science progresses. In special cases, for time intervals that are not too long, they can be entirely eliminated, i.e., unambiguous prediction can be achieved. [. . .] In contradistinction, science always remains infinitely far removed from the unambiguous prediction of physical states, even for arbitrarily small time intervals.[16]

So, contrary to Friedman's claim, Carnap does not give a method for explicitly defining the terms of physics (and there is no working back-and-forth). Rather, he points out that for some cases and small regions of space-time, exact prediction of a future observation, but not of a future physical state, is possible. His argument against the prediction of physical states rests on the one-many relation between observations and physical states, the assumptions that physical states can be determined only through observations (this is implicit), and that for each current physical state, there is exactly one physical state in the future (a set of physical states evolves over time into a set of physical states of the same cardinality). Since at any point in time, one can only determine an infinite set of physical states, any prediction can therefore also only determine an infinite set of physical states.[17] (Carnap does not consider the possibility of using observations at more than one point in time to narrow down the set of physical states.)

Friedman was probably led astray because he assumed that Carnap's exposition in §136 is compatible with the central claims of the *Aufbau*. Clearly, Friedman is correct about Carnap's intentions regarding the definability of theoretical

16. The translation of the last sentence ("Von der eindeutigen Voraussage physikalischer Zustände dagegen bleibt die Wissenschaft auch bei noch so kleinen Zeitabständen immer unendlich weit entfernt.") is modified from the one given Friedman et al.

17. I thank Christopher French for extensive discussions of this passage, although we still disagree on both its content and its relevance.

terms; but he is mistaken about the implications of Carnap's three early works and their recapitulation in §136. Just as Carnap was mistaken when he referenced and recapitulated them.

5.2 Van Fraassen's satisfaction function

In Carnap's view, then, theories are restrictions on mathematical spaces, and the correspondence rules "uniquely co-ordinate[. . .] with certain physical qualities, ultimately sensory qualities" (Carnap 1926, 60). Thus according to Carnap, volumes of the mathematical spaces are mapped to physical qualities, which in turn are mapped to phenomenal qualities. Basically this account of theories was suggested about 45 years later by van Fraassen.

In his current account, van Fraassen (1980, 64) describes the connection through model theory, specifically through embeddings. This account can be expressed directly in Carnap's view on scientific theories (Lutz 2014a). In van Fraassen's earlier account (cf. van Fraassen 1989, 365, n. 34), the relation between physical theories and observations relies on "satisfaction functions" (van Fraassen 1970, 328–29):

> Besides the state-space, the theory uses a certain set of *measurable physical magnitudes* to characterize the physical system. This yields the set of *elementary statements* about the system (of the theory): each elementary statement U formulates a proposition to the effect that a certain such physical magnitude m has a certain value r at a certain time t. (Thus we write $U = U(m, r, t)$ [. . .].)
>
> [. . .] For each elementary statement U there is a region $h(U)$ of the state-space H such that U is true if and only if the system's actual state is represented by an element of $h(U)$. [. . .] The mapping h (the *satisfaction function*) is the third characteristic feature of the theory. [. . .] The exact relation between $U(m, r, t)$ and the outcome of an actual experiment is the subject of an auxiliary theory of measurement, of which the notion of "correspondence rule" gives only the shallowest characterization.

The disparaging remark about measurement and correspondence rules references an article by Suppes (1967), which is odd considering that both Hempel (1952, §§11–14) and Carnap (1966, §§6–10) extensively discuss the representational theory of measurement championed by Suppes.

Be this as it may, van Fraassen's account thus assumes a purely mathematical description of the physical system, whose states are mapped to values of physical quantities, which are in turn related to measurement results and finally observations. That is exactly what happens in Carnap's account.

6 Conclusion

The structure of correspondence rules as it was presented in Carnap's three early works and in §136 of the *Aufbau* did not change much: Specifically, the one-many relation between observational states and theoretical states that Carnap first described in the 'Aufgabe' was still present in his last works on the the structure of scientific theories. In his later works, however, Carnap (e. g., 1939) phrased it in terms of the explicit definability of observational terms in theoretical terms. The description of the world as a mathematical space and the conceptualization of scientific theories as a restrictions on this mathematical space as conceived in Carnap's article on three-dimensionality and causality, his monograph on concept formation, and in §136 of the *Aufbau* also remained in Carnap's view on scientific theories to the end.

That Carnap uses the results of his three earlier works in the *Aufbau* and rediscovers them in his later works (Carnap 1956, 1958) shows that these works belong to Carnap's logical empiricism. That the results survived Carnap's changes on more meta-theoretical views on the status of scientific theories, logic, and semantics, and even as undercurrents to Carnap's incompatible official position of the *Aufbau*, only to resurface later on and become Carnap's final view, shows how resilient they are. So resilient, in fact, that they are at the core of the account of scientific theories by one of Carnap's staunchest opponents.

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