Pricing Warrant by Using Binomial Model: Comparison between Historical and Implied Volatility

Khairu Azlan Abd Aziz[a] , Mohd Fazril Izhar Mohd Idris[a], Wan Suhana Wan Daud and Muhammad[b] Nasruddeen Bin Mazlan[a]

[a]Department of Mathematical Sciences and Statistics,Universiti Teknologi Mara (Perlis), Arau 02600,Malaysia

[b]Institute of Engineering Mathematics, Universiti Malaysia Perlis, Arau 02600, Malaysia

ABSTRACT

A warrant is a security that allows the holder to buy and sell the underlying share at a fixed price until expiry date. Warrant price will always fluctuates since the underlying share also fluctuates. Hence, determining the warrant price is the main problem among the investors in Malaysia. This research is focusing on pricing the warrant for five companies that were listed in Bursa Malaysia. The companies were chosen randomly from UiTM DataStream. The selected companies were Boon Koon Sdn Bhd, Hovid Bhd, Kelington Bhd, ML Global Bhd and Tropicana Corporation Bhd. The data contained underlying share, interest rate, exercise price and actual warrant price. This research aims to define the price of warrant by using Binomial model. Historical volatility and implied volatility were used in this research whereby volatility is the movement of the underlying share price. This research aims at comparing the actual warrant price with the calculated warrant price. The data were computed manually by using Microsoft Excel and the comparison was made between the two type of volatilities to give the nearest value of calculated warrant price to the actual warrant price. The nearest value was assumed the best value for this research. The result was made by analyzing the line graphs and comparing between historical volatility and implied volatility with actual warrant price. Mean Square Error was used to support the results that were obtained from the line graphs. In the end, implied volatility yielded better results compared to historical volatility.

Keywords: Warrant, Volatility, Binomial Model, Moneyness.

1. Introduction

A warrant is a security that allows the holder to buy and sell the underlying share at a fixed price until expiry date (Zhang et al., 2009). Exercising the warrant can be defined as the firm receiving cash equal to the exercise price and issuing a new share to the holder. A warrant holder can exercise the warrant if the current stock price is above the warrant's strike price. Thus, there are more outstanding shares. A warrant is categorized into two:a call warrant and a put warrant. A call warrant is the right to buy stocks at a fixed price but not at a specific time since it depends on the warrant's style. Usually, a call warrant is issued by the bank investment or shareholder of the company (Aziz et al., 2018). On the other hand, a put warrant can be defined as an agreement that gives the right to sell a share at a fixed price and fixed value. A put warrant is a company-issued option to sell back to the issuer a specified number of shares of the company's stock at a specific price and specific time. There are two types of warrant, namely American and European warrants. The American type of warrant has dividend that allows to be exercised at any time up to the expiry date while European style is non-dividend and only can be carried out on the day of the expiration. Sometimes,

Khairu Azlan Abd Aziz[a] , Mohd Fazril Izhar Mohd Idris[a], Wan Suhana Wan Daud And Muhammad[b] Nasruddeen Bin Mazlan[a]

warrants can be a mixture of American and European since it may be European up to a certain date and followed by American. The terms of the warrant series set out to how the investors can exercise the warrant.

Warrants and options are much likely the same since both represent a right and do not provide any control over the principal asset until the exercise process. Moreover, options and warrants offer their holders the chance to gain exposure to the rise and fall of the principal asset's price without possessing the asset. However, warrants are issued by a specific company, while options are issued by an options exchange. According to Aziz et al. (2018), warrants have longer time period than options. Usually, options only can go up to two years while warrants can last for fifteen years. Trading the warrant means that the firm receives cash equal to the exercise price and issues a new share to the holder (Conroy, 2008).

2. RELATED WORKS

i. Pricing Option by using Binomial Model

According to Conroy (2009), using Binomial model on pricing options looks simple, yet, it is a powerful method. It can be used to solve many complicated option pricing problems. It was developed in 1979 and it assumes a perfectly efficient market. By using this method, mathematical valuation of an option at each point in the timeframe specified can be provided. The Binomial model assumes that market share prices only can be increased or decreased during a specific period of time. The main idea of the Binomial model is to replace a continuous distribution of share prices by a simple two-point discrete distribution (Yuen et al., 2012). This model uses a "discrete time" model of varying price over time of underlying financial instrument. Binomial Options Pricing model approach has been widely used since it handles a variety of conditions for which other models cannot easily be applied.

ii. Pricing Warrant by using Binomial Model

In the current time, many researchers use Binomial model when it comes to solving the nonlinear problems such as the movement of the warrants price. According to Xiaoping et al. (2014), this model contains lots of mathematics which is easy for the investors since most of the investors have knowledge on mathematics. The Binomial model is selected to price the warrants since the changes of the warrants call price is nonaligned and precede the Brownian motion (Aziz et al., 2018). Certain assumptions need to be measured before the Brownian motion pricing formula can be derived. According to Londani (2010), many authors are avoiding Brownian motion pricing warrants. When the authors use the Brownian motion to price the warrants more than one time, they realized that the results were different. This is due to the long memory property. In conclusion, the Binomial model is better than Brownian motion since it is easier to understand and the result for the price warrants are more consistent.

3. METHODOLOGY

There are several processes need to be done in order to evaluate warrant price using Binomial Model. The processes include data collection from UiTM library's datastream, identifying the parameters of the model (initial market price (S), exercise price (X), interest rate (r) and maturity date (T-t)), calculate historical and implied volatility and apply the model. Historical volatility is one of the methods to calculate volatility (σ) or the price movement of the underlying shares. The formula is stated as below:

$$x_i = \ln\left(\frac{S_i}{S_{i-1}}\right), i = 1, 2, 0, \ldots, n.$$

(1)

$$\alpha = \frac{1}{n}\sum_{t=1}^{n} x_t$$

(2)

where

α = average day-to-day changes over n-day period.

An estimator of the standard deviation is given by:

$$S = \sqrt{\frac{\sum_{t=1}^{n}(x_i - \alpha)^2}{n-1}}$$

(3)

$$\sigma = \frac{S}{\sqrt{\frac{1}{365}}}$$

(4)

Calculation of the implied volatility is shown below:

$$f(\sigma) = SN(d1) - Xe^{-r(t-t)}N(d_2) - C = 0$$

(5)

$$\sigma_{n+1} = \sigma - \frac{(SN(d_1) - Xe^{-r(T-t)}N(d_2) - C)}{S\left(\frac{1}{2\pi}\right)e^{\frac{d_1^2}{2}}\sqrt{T-t}}\sigma_{n+1}$$

(6)

where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r + \sigma^2)(T - t)}{\sigma\sqrt{T-t}}$$

(7)

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

(8)

with

C = value of the call warrant,
S = price of the underlying stock,
X = exercise price of the call,
r = annualized risk-free interest rate,
T-t = time until expiration, and
N = probability from the cumulative standard normal distribution.

Khairu Azlan Abd Aziz[a] , Mohd Fazril Izhar Mohd Idris[a], Wan Suhana Wan Daud And Muhammad[b]
Nasruddeen Bin Mazlan[a]

Market share may increase and decrease in this process by denoting the current market share price at *S*. When *u* is greater than one, the market share increases to a level of *uS*. Besides, when *d* is lower than one, the market share decreases to *dS*.

$$A = \frac{1}{2}(e^{-r\Delta t} + e^{r\Delta t + \sigma^2 \Delta t}),$$

$$(9)$$

$$u = A + \sqrt{A^2 - 1},$$

$$(10)$$

$$d = \frac{1}{u},$$

$$(11)$$

$$p = \frac{e^{r\Delta t} - d}{u - d},$$

$$(12)$$

where
u = share price up,
d = share price down,
p = probability of price up,
Δt = maturity date.

Then we need to construct a binomial tree based on the time to maturity of a warrant.

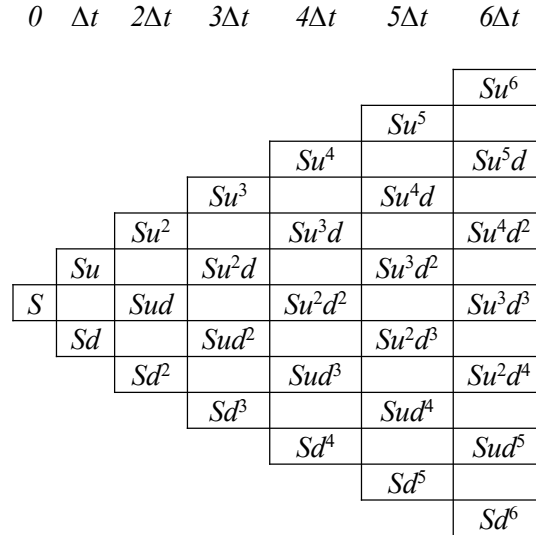| 0	Δt	$2\Delta t$	$3\Delta t$	$4\Delta t$	$5\Delta t$	$6\Delta t$
						Su^6
					Su^5	
				Su^4		Su^5d
			Su^3		Su^4d	
		Su^2		Su^3d		Su^4d^2
	Su		Su^2d		Su^3d^2	
S		Sud		Su^2d^2		Su^3d^3
	Sd		Sud^2		Su^2d^3	
		Sd^2		Sud^3		Su^2d^4
			Sd^3		Sud^4	
				Sd^4		Sud^5
					Sd^5	
						Sd^6

Fig. 1 Binomial Tree with 6 time period

Warrants payoff are calculated at the end of the date and at the end of each node. When the stock share price is calculated, maximum warrant payoffs at maturity date are also calculated.

$$W = \max(S - X, 0) \tag{13}$$

15

Final process is executing backward induction step and calculate price of warrant. This calculation is to discount the payoff of the warrant at expiration date.

$$C = e^{-r\Delta t}(pWu + (1-p)Wd) \tag{14}$$

Mean square error (MSE) between actual and model price was used to compare the accuracy of warrant price using binomial model.

$$MSE = \frac{\sum e_t^2}{n} \tag{15}$$

where
e_t = actual price of warrant – model price of warrant.

The intrinsic value or monetary value is calculated to identify either the underlying shares are in-the-money, at-the-money or out-of-money.

$$M = \frac{S_0 - Xe^{-rt}}{Xe^{-rt}} \tag{16}$$

4. RESULTS

The volatility of underlying share price is very important to identify the value warrant. It is because volatility of underlying share price can describe the potential of underlying share either goes up or down. Equation (4) and (6) were used to calculate historical and implied volatility.

Table 1. The historical and implied volatility for 5 companies

Company	Historical Volatility (%)	Implied Volatility (%)
Boon Koon Bhd	55.25	27.25
Hovid Bhd	30.42	19.01
Kelington Bhd	51.25	34.43
ML Global Bhd	44.15	90.87
Tropicana Corporation Bhd	18.78	9.00

After calculating the volatility of each warrant, then we apply the binomial model to obtain warrant price. The daily basis price calculated by using both volatility and the actual price were presented in the graph below.

Khairu Azlan Abd Aziz[a] , Mohd Fazril Izhar Mohd Idris[a], Wan Suhana Wan Daud And Muhammad[b] Nasruddeen Bin Mazlan[a]

Fig. 2 Actual Warrant Price vs Historical Warrant Price and Implied Warrant Price Boon Koon Sdn. Bhd.

Fig. 3 Actual Warrant Price vs Historical Warrant Price and Implied Warrant Price Hovid Bhd.

Fig. 4 Actual Warrant Price vs Historical Warrant Price and Implied Warrant Price Kellington Group Bhd.

Fig. 5 Actual Warrant Price vs Historical Warrant Price and Implied Warrant Price ML Global Bhd.
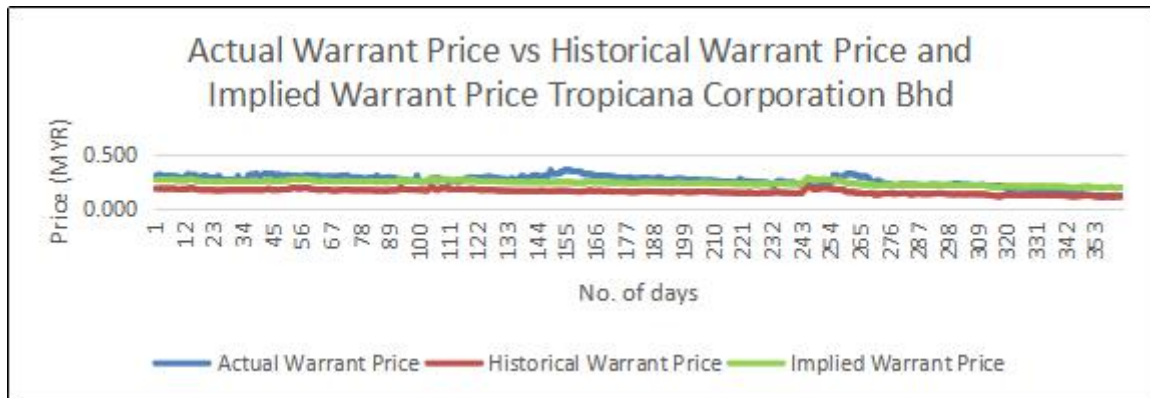

Fig. 6 Actual Warrant Price vs Historical Warrant Price and Implied Warrant Price Tropicana Corporation Bhd.

From the result above, we can compare the actual and model price. It has shown a similar pattern of warrant price and at certain time, the price of model price are equal to the actual price. In order to measure the accuracy of the model price, we have calculated the mean square error for each warrant using equation (15). Besides that, we have computed the value of moneyness of each warrant by using equation (16). If the warrants are indicated as in-the-money, then it has a good potential for investor to invest in that warrant and underlying shares. But if the warrants indicate out-of-money, then investors are advised to avoid investing in that warrants and underlying shares.

Table 2. The MSE of warrant price for 5 companies

Company	Historical Volatility	Implied Volatility
Boon Koon Bhd	0.005561	0.004302
Hovid Bhd	0.001170	0.001216
Kelington Bhd	0.011060	0.008855
ML Global Bhd	0.028638	0.026200
Tropicana Corporation Bhd	0.013879	0.001888

Khairu Azlan Abd Aziz[a] , Mohd Fazril Izhar Mohd Idris[a], Wan Suhana Wan Daud And Muhammad[b] Nasruddeen Bin Mazlan[a]

Table 3. The moneyness for 5 companies

Company	Moneyness
Boon Koon Bhd	1.396 (In the Money)
Hovid Bhd	1.288 (In the Money)
Kelington Bhd	1.049 (In the Money)
ML Global Bhd	1.800 (In the Money)
Tropicana Corporation Bhd	-0.0059 (Out of Money)

5. CONCLUSION

The study concluded that the implied volatility is a better way to calculate the warrant price compared to the historical volatility since implied volatility measures the current situation of the underlying share. The mean square error shows the percentage of error for implied volatility which is lower than historical volatility (see Table 2). Moreover, it is shown that binomial model is one of the techniques that can be adapted to price the warrant. Since the features of options and warrants have many similarities, any techniques in pricing option can be used to price the warrants.

REFERENCES

Aziz, K. A. A, Abdoh, W. M. Y. M, Idris, M. F. I. M, Saian, R. (2018). An Evaluation of Binomial Model with Implied Volatility in Pricing Warrant. *Jurnal Intelek. 13*(1): 37-43.

Conroy, R. M. (2008). The Pricing of Warrant. Retrieved from https://www.researchgate.net/publication/228181064

Conroy, R. M. (2009). Binomial Option Pricing. Retrieved from https://www.researchgate.net/publication/228180835

Londani, M. (2010). *Numerical Methods for Mathematical Models on Warrant Pricing*. University of the Western Cape.

Xiaoping, H., Jiafeng, G., Tao, D., Lihua, C., & Jie, C. (2014). Pricing Options Based on Trinomial Markov Tree. *Discrete Dynamics in Nature and Society*.

Yuen, F. L., & Yang, H. (2010). Pricing Asian Options and Equity-indexed Annuities with Regime Switching by The Trinomial Tree Method. *North American Actuarial Journal*, *14*(2), 256-272.

Zhang, W. G., Xiao, W. L., & He, C. X. (2009). Equity warrants pricing model and fractional Brownian motion and an empirical study. *Expert Systems with Applications, 36*, 3056–3065 (2009).

---

* Corresponding author