# Power transmission cost calculation in deregulated electricity market

Von der Fakultät für Ingenieurwissenschaften der Universität Duisburg-Essen zur Erlangung des akademischen Grades eines

Doktoringenieurs (Dr.-Ing.)

genehmigte Dissertation

von

**Georgios Stamtsis** 

aus Sidirokastro, Hellas

Referent: Prof. Dr.-Ing. habil. István Erlich Korreferent: Prof. Dr. Rainer Leisten Tag der mündlichen Prüfung: 03. Dezember 2003



Aber rühmen wir nicht nur den Weisen
Dessen Namen auf dem Buche prangt!
Denn man muß dem Weisen seine Weisheit erst entreißen.
Darum sei der Zöllner auch bedankt:
Er hat sie ihm abverlangt.

Bertold Brecht, Legende von der Entstehung des Buches Taoteking auf dem Weg des Laotse in die Emigration But the honour should not be restricted

To the sage whose name is clearly writ.

For a wise man's wisdom needs to be extracted.

So the customs man deserves his bit.

It was he who called for it.

Bertold Brecht, Legend of the origin of the book Tao-te-ching on Lao-tsu's road into exile

## Acknowledgement

First of all, I would like to express my gratitude to my supervisor, Prof. István Erlich. He gave me the opportunity to work on such an interesting theme and he has essentially contributed to this work through his guidance and encouragement. The discussions during the weekly meetings were also useful for my research work.

I am also grateful to my co-advisor, Prof. Rainer Leisten, for the discussions that we had and of course for bringing me in contact with the team of NHH. The last part of my research has been made during my stay in Bergen, visiting the Norwegian Business School (NHH). During that time, I had the opportunity to cooperate with economists and further understand the economic side of deregulated electricity market. Therefore, my deepest thanks go to Prof. Kurt Jörnsten as well as to Ass. Prof. Mette Bjørndal and Ass. Prof. Endre Bjørndal.

Special thanks go to my colleagues at the department. Except of their interesting comments during the weekly meetings they have also created a nice atmosphere in institute's daily life. I would also like to thank Jörn Christiansen for the cooperation during his diploma thesis.

This research work has been carried out under the patronage of the German Academic Exchange Service (DAAD) and I would like to express my appreciation for this support.

Last but not least, my gratitude goes to my family and good friends. They have stimulated me to begin the doctorate study and they have contributed to its completion through their support and encouragement.

Duisburg, December 2003 Georgios Stamtsis

## **Contents**

Con	tents		i
List	of Table	es	v
List of Figures		vii	
Cha	pter 1	Introduction	1
1.1	Motiv	vation for restructuring	1
1.2	Histor	rical evolution of market deregulation	3
	1.2.1	The deregulation process worldwide	3
	1.2.2	The deregulation process in Europe	7
1.3	Dereg	gulated electricity market structure	10
1.4	Objec	etives	13
1.5	Outlin	ne	14
Cha	pter 2	Congestion management in nodal pricing market	17
2.1	Conge	estions in electricity networks	17
2.2	Electr	ricity pool market	18
2.3	The o	ptimal power flow problem	20
2.4	Analy	vsis of nodal prices	26
	2.4.1	General formulation	27
	2.4.2	Nodal price analysis using the	
		Kuhn-Tucker Theorem	28
	2.4.3	Nodal price analysis using linearisation	
		and the dual problem	30
2.5	Congestion component analysis		32
2.6	Conge	estion management methods	36

*ii* Contents

2.7	Congestion component allocation		40
	2.7.1	Power flow tracing in terms of distribution factors	40
		2.7.1.1 On power flow distribution factors	40
		2.7.1.2 The Nodal Generation Distribution Factors	41
	2.7.2	Connecting congestion component	
		and market players	45
	2.7.3	Case studies	47
2.8	Mark	et participants' behaviour	51
	2.8.1	Pool model	51
	2.8.2	Third party access	52
	2.8.3	Producers' behaviour in a realistic network	54
	2.8.4	Consumers' behaviour in a realistic network	55
Cha	pter 3	Operation mode assessment of a real market	59
3.1	The N	Nord Pool	59
3.2	The N	Norwegian market operation mode	62
	3.2.1	Loss factors	62
	3.2.2	Zonal pricing and countertrade	64
3.3	Analysis of the Norwegian market using AC-OPF		
	3.3.1	Nodal price loss component	68
	3.3.2	Zonal pricing based on nodal pricing algorithm	69
3.4	Numerical results		73
	3.4.1	Unconstrained system	74
	3.4.2	Constrained system without zonal pricing	76
	3.4.3	Constrained system considering zonal pricing	78
	3.4.4	Constrained system. Intensive zonal pricing	79
Chaj	pter 4	Fixed cost allocation using game theory	83
4.1	Game	e theory	84
4.2	Cooperative game theory		87
	4.2.1	Terminology of cooperative game theory	88
	4.2.2	The core	91

Contents

	4.2.3	The nucleolus	93
	4.2.4	The Shapley Value	97
	4.2.5	The Solidarity Value	98
	4.2.6	The Owen Value	99
4.3	Usage	e based methods	101
	4.3.1	The Postage Stamp Method	101
	4.3.2	The MW-Mile Method	102
	4.3.3	The Counter Flow Method	103
	4.3.4	The Zero Counter Flow Method	103
4.4	The fi	ixed cost allocation game	104
	4.4.1	Game definition	104
	4.4.2	The game in the case of pool market	107
	4.4.3	The game in the case of bilateral transactions	110
	4.4.4	Characteristics of the game	113
		4.4.4.1 Superadditivity	113
		4.4.4.2 No convex game	113
		4.4.4.3 Absence of dummy player	114
	4.4.5	The nucleolus in fixed cost allocation game	114
	4.4.6	The Shapley value in fixed cost allocation game	115
	4.4.7	Comparison of different methods' results	119
4.5	Usage	e based methods and the core	120
4.5.1	The fi	ixed cost allocation as cost game	120
	4.5.2	A three players' paradigm	121
	4.5.3	Core points from usage based allocations	122
	4.5.4	Numerical results	125
4.6	Sensitivity based stand-alone usage		131
	4.6.1	The problem	131
	4.6.2	Methodology	132
	4.6.3	Numerical results	136
Chap	ter 5	Epilogue	141
5.1 Synopsis		141	

*iv* Contents

5.2	Future research work	142
Bibli	iography	145
App	endix A	155
App	Appendix B	
Appendix C		159
Acro	onyms and Symbols	163
List	of (some) Hellenic words	169
Publ	Publications	
Biog	graphic Data	173

List of Tables v

## **List of Tables**

		Page
Table 2.1	Lagrange multiplier $\mu$ and NGDFs in IEEE 14-bus	
	system	50
Table 2.2	Lagrange multiplier $\mu$ and NGDFs for the TPA cases	53
Table 2.3	Generators' share in congestion component for the	
	TPA cases	53
Table 2.4	NGDFs in a realistic high voltage network	55
Table 2.5	Reduction of objective function	56
Table 2.6	System situation and NLDFs of consumer at bus E	56
Table 3.1	Generation resources in Nordic countries, 2000	61
Table 3.2	On the countertrade explanation	67
Table 3.3	Unconstrained case. Loss components as loss factors	75
Table 3.4	Unconstrained case. Randomly chosen loss factors	76
Table 3.5	Constrained case without zonal pricing. Loss	
	components as loss factors	77
Table 3.6	Constrained case without zonal pricing. Randomly	
	chosen loss factors	78
Table 3.7	Constrained case considering zonal pricing. Loss	
	components as loss factors	79
Table 3.8	Constrained case considering zonal pricing. Randomly	
	chosen loss factors	79
Table 3.9	Constrained case, intensive zonal pricing. Loss	
	components as loss factors	80
<b>Table 3.10</b>	Constrained case, intensive zonal pricing. Randomly	
	chosen loss factors	80

Table 4.1	Consumers in the pool market as players of the game	108
Table 4.2	Use and savings for the game in pool market	109
Table 4.3	Initial and final use for each player in pool market	109
Table 4.4	Coalitions with negative characteristic function values	110
Table 4.5	Transaction patterns	111
Table 4.6	Use and characteristic function value for the 1st	
	session	111
Table 4.7	Initial and final use for the players concerning the 1st	
	session	112
Table 4.8	Applying the nucleolus solution in electricity market	115
Table 4.9	Difference between characteristic function value and	
	Shapley values sum	116
<b>Table 4.10</b>	On the acceptability of the Shapley value	118
<b>Table 4.11</b>	Results from different cooperative game theory	
	concepts	119
<b>Table 4.12</b>	Transactions in the 14-bus test system	127
<b>Table 4.13</b>	Weights of usage based methods. PS-MWM-CF	127
<b>Table 4.14</b>	Weights of usage based methods. PS-MWM-ZCF	128
<b>Table 4.15</b>	Weights of usage based methods. PS-CF-ZCF	128
<b>Table 4.16</b>	Performance of the smallest excess in the cost game	130
<b>Table 4.17</b>	Stand-alone power flow from different methods. IEEE	
	30-bus, line 1-3	138
<b>Table 4.18</b>	Stand-alone power flow from different methods. IEEE	
	30-bus, line 6-28	138

# **List of Figures**

		Page
Figure 1.1	Electricity market opening in Europe, July 2003	9
Figure 1.2	Deregulated electricity market	12
Figure 2.1	Electricity pool market	19
Figure 2.2	Supplier and consumer bid curves	20
Figure 2.3	Bid curve of fictitious generator	21
Figure 2.4	Proportional sharing principle	42
Figure 2.5	Search procedure for NGDFs' calculation	44
Figure 2.6	The IEEE 14-bus system	48
Figure 2.7	Share of line and voltage congestions in congestion component	48
Figure 2.8	Share of lines in congestion component	49
Figure 2.9	Allocation of congestion component to generators	50
Figure 2.10	Share of generators in active power production and congestion	
	component	51
Figure 2.11	Players' behaviour in a large network	54
Figure 3.1	The Nord Pool market	60
Figure 3.2	Impact of loss factors on participants' bids	63
Figure 3.3	Zonal pricing and countertrade	65
Figure 3.4	The 10-bus test system	74
Figure 4.1	Normal form of a game	85
Figure 4.2	The prisoners' dilemma	87
Figure 4.3	Geometrical representation of a three-person game	93
Figure 4.4	On nucleolus explanation	95
Figure 4.5	Counter flow	105

Figure 4.6	Differences between the characteristic function value and the sum		
	of Shapley values at each line	117	
Figure 4.7	Core of a three transactions game	122	
Figure 4.8	The restricted nucleolus placed in the core	124	
Figure 4.9	The 14-bus test system	126	
Figure 4.10	The IEEE 30-bus system	137	

## Chapter 1

#### Introduction

#### 1.1 Motivation for restructuring

Historically, the electricity industry was a monopoly industry with a vertical structure. In a vertically integrated environment, enterprises were responsible for the generation, transmission and distribution of electrical power in a given geographical area. Such companies could be state owned as well as private. But the last two decades, and especially during the 1990s, the electricity supply service has been undergoing a drastic reform all over the world. The old monopolist power markets are replaced with deregulated electricity markets open to the competition. Different forces have driven the power market towards the deregulation. Not all of them are behind the reform in all these countries. Furthermore, in each different country the same reason has to be studied taking into consideration the local circumstances. However, it is possible to categorize all these various causes in technical, economical and political.

The technological development of high voltage networks during the 1960s and 1970s made possible transmission of bulk power over long distances. This is a necessary condition in order the power market to be opened to producers that are located far from the main customers [1]. Despite this achievement the electricity industry remained a monopoly for the next twenty years.

So, there is another technical factor which has given a stronger impulse towards the deregulation. This factor is the improved power generation technologies. In 1960s and 1970s the typical size of thermal power plant units was between 600 and 1000 MW. The average construction time for such power plants was four to five years. In case of nuclear power plant this time was double. For this reason, the decisions of generation expansion could be taken only by a monopolist utility so as to make the necessary investments. The monopolist regime was also acting as protection against investment errors which could have dramatic consequences because of the investment size. The development of gas power plants, and especially of combined circle gas turbines, led to an optimal size of power production unit up to 300 MW. Besides the reduction in the investment cost, the construction time of such power plants is essentially shorter than it was before. Hence, it is now possible the generation expansion decisions to be taken by smaller enterprises [2]. The expansion of gas network is an additional reason that makes investments in gas power plants easily realisable.

Another mixed technical-ecological cause is the inclination of modern society for an increase in power produced by renewable sources. The emerging of independent producers who operate, mostly, wind power units gives a further competitive character to the power industry despite the fact that such producers survive still due to the subsidies.

The improvement of transmission technologies result in an efficient grid operated by the transmission companies. Devices such as FACTS enable a better control over the electrical features of the grid. Thus, the separation of generation and transmission decisions can be easier.

Beyond the technical improvements, a set of economical reasons may be considered as the main force behind the electricity market reform. The key economical idea, which led to the deregulation, was that a well operated competitive market can guarantee both cost minimization and average energy prices hold at a minimum level [1]. The economists believe that an open market provides stronger incentives to the supplier in order to apply cost-minimizing procedures than a regulated market. The second positive characteristic of a competitive market is its ability to drive the prices towards the marginal costs. Of course, in order this advantage to appear the market has to be well designed.

Another economical reason was the inability of countries with high national debt to meet the necessary investments in state owned power sector [3]. So, the only solution for these countries was the privatisation of the electricity industry. Through this process these countries achieved two objectives. Firstly, to free up public funds and make them available to serve the national debt or other demands. Secondly, the governments of these countries collected an essential amount from the sale of state owned utilities.

The third category of electricity industry restructure causes consists of political factors. The first deregulated power markets have acted as an example showing that the electricity sector can also be operated in a competitive environment [4]. Thus, it was easier for many countries to overcome hesitation and reform their electricity markets.

The liberalisation in USA of many economical sectors in 1970s such as transportation, gas and telecommunications was a sign that the power sector would also be reformed. The acceptance of open market mode in some economic sectors by the most governments world-wide was a further factor which enabled the restructuring. In 1980s, the social-democratic governments in many countries have accepted the introduction of competition in many economic sectors as a development tool. Among the political circles, the idea that the private companies apply more efficient practices than the public ones, in certain economic sectors, was getting more acceptance. Hence, the deregulation of power market was made possible in many countries. A further reason, which led to the deregulation, is the pressure of some multilateral organizations such as World Bank. These organizations set as a requirement the opening of markets including the power sector in order to support financially a country. Consequently, the electricity industry of many countries financed by the World Bank opened to the competition.

#### 1.2 Historical evolution of market deregulation

#### 1.2.1 The deregulation process worldwide

The first experiences of electricity market in USA, end of 19th century, were characterized by a competition without rules [1]. After this brachychronic phase, the power industry had been regulated in order to enforce the technology development and

to stabilize the market. The first step towards the opening of electricity market was the adoption of Public Utility Regulatory Policies Act by US government in 1978. This Act ordered the utilities to purchase power from particular independent producers. In 1992, the Energy Policy Act provided the power to Federal Energy Regulatory Commission (FERC) so as to require from the utilities transmission services for the wholesale customers. The real phase of deregulation in USA started in 1996 when FERC issued two directives. The first directive, "Promoting Wholesale Competition Through Open Access Nondiscriminatory Transmission Services by Public Utilities", demands all public utilities to provide non-discriminatory open access transmission services [5]. The second directive demands the utilities to develop an Internet-based system, that will enable the exchange of information about the transfer capacity on transmission lines. The power market deregulation has followed different paths in the numerous states which have their own separate markets. An additional reason for the different kind of development is the absence of jurisdiction by the side of FERC over the entire territory of USA. Generally, the wholesale market, in all states, is under the supervision of FERC while the competitive retail power markets remain a subject of the individual state regulatory commissions.

The electricity market of PJM with an installed capacity of more than 67000 MW is one of the largest fully liberalised markets in USA [6]. PJM came online in 1997 as a regional bid-based energy market. At that time its members were 89 while at the end of 2002 PJM counted more than 200 members. In spite of the successful operation of PJM, the most known case of power market liberalisation is the one of California. The initial euphoria of the first two years, after the market deregulation, was followed by problematic situations for all the market participants. The highlight of these problems was the blackouts that happened in California during 2001 and the bankruptcy of some wholesale companies which were involved in the market. A certain cause for this unsuccessful market performance was that the retail prices remained low due to regulatory orders while the wholesale prices were increasing dramatically. So, the wholesale distributors could not meet the demand. The transmission capacity shortage was another reason. That resulted in market power exercised by few sellers during network congestion situations [7, 8]. The paradigm of California teaches how a mediocre deregulation may result in significant problems. The epulosis process in

California's market started in short time, after the problems have been experienced, by adopting rules that facilitate a non- problematic market performance.

The most recent evolution in USA's electricity industry is the desire of FERC for the introduction of a standard market design (SMD) by all the deregulated markets [9]. The SMD should result in common transmission rules over all the states. Thereby the power trading by market participants who aim to transport power across different states will be simplified. Optimal economic performance should also be achieved through the adoption of SMD as a result of higher transmission efficiency.

Chronologically, the first deregulated electricity market world-wide was the one of Chile. As early as 1982 Chile introduced competition into power industry by giving the right to large end users to choose their supplier and negotiate the prices. Beyond this first step Chile realized later explicit market mechanisms in order to determine the generators' dispatch and the wholesale electricity price [10]. Thus, competition among the producers arose. In the case of Chile a non democratic regime imposed in a very short period such a drastic change as the reform of electricity sector. Comparatively, the most mature industrialized country, USA, needed almost two decades to transform the power sector. The experiment with Chile's deregulation was successful and so Argentina in 1992 opened its market in competition followed by Peru in 1993, Bolivia and Colombia in 1994 and the countries of Central America in 1997. Brazil is joining also the group of countries which have restructured their power industry. However, in the case of Brazil certain problems emerged concerning the privatisation of distribution companies. Generally, the deregulation in Latin America has led to an essential improve of power sector. The current trend in this area is the development of electricity markets covering large parts of the continent beyond the countries' borders [11].

The power sector restructuring in Oceania has also a long history. In 1987, the government of New Zealand began the reform of power sector by setting up the Electricity Corporation of New Zealand (ECNZ). The task of ECNZ was to own and operate the facilities of the Ministry of Energy. In 1988, the system operator, Transpower, was set up by ECNZ. After some years of initial restructuring, a voluntary wholesale electricity market was founded in New Zealand. Its performance to date brings the New Zealand's market among the most successful paradigms of power sector deregulation. The most present issue is the introduction of the financial transmission

rights as a tool for hedging transmission congestion costs and giving incentives for grid expansion investments [12]. In neighbouring Australia, the Industry Commission recommended reforms that included the state-owned electricity industry, in 1990. In 1994, in the state of Victoria a pool market was established. The same market form was introduced in New South Wales in 1996. These two markets were the founders of National Electricity Market of Australia in 1998. As key achievement of Australia's electricity market may be considered the implementation of a wholesale spot market. The next step of the reform process is the replacement of the present mixed federal and state regulatory structure with a national energy regulator [13].

The electricity market has undergone a reform in some countries of Asia too. The most economic powerful country of the region, Japan, started a restructuring process in 1995. The introduction of competition was achieved by promoting the entry of independent power producers into the wholesale market [14]. These producers were eligible to bid only in service areas outside from the area where they were located. Japan is divided in 10 zones operated by private vertically integrated companies. Since that time no many changes have taken place in Japan's power sector. However, in February 2003 the Electric Industry Committee has issued a directive demanding the establishment of a Power Exchange. Furthermore, consumers with demand more than 50 kW will be able to choose their supplier from April 2005 [15]. In China, despite the central controlled economy, the power sector has experienced a reform since mid-1980s. In first phase, private investment in generation has been allowed. In 2002, all the state-owned energy enterprises were transformed in commercial companies. However, there are no eligible consumers yet. The World Bank supports financially the government's five-year plan, from 2001 to 2005, in restructuring the electricity industry [16]. In India, some states have launched a power sector deregulation in mid-1990s. With more than 15% of population still not connected to the central grid the objectives of deregulation are rather different in India from the rest of the world. First of all, the reform targets to make electricity accessible for each household [17]. A further aim is the reduction of power supply cost and the increase of electricity services quality. The later should be achieved through improved operational efficiency and good governance of electricity industry.

In Africa, some countries have begun a restructure process concerning their electricity industry. In particular, the countries of northern Africa participate together with

members of European Union to build up the Mediterranean Electric Ring, an electric network around all the countries of Mediterranean region. In 1999, the government of Nigeria adopted a comprehensive privatisation program which should be completed in 2004. Deregulation of the state-owned electricity industry is planned for the third phase of this program [18]. In South Africa, the regulated electricity supply industry has performed well to date. However, the government has planned the deregulation of power sector. The main concern which drives the reform is to avoid the over-investment in capacity expansion, something that happened in the past [19].

#### 1.2.2 The deregulation process in Europe

In European continent, England started up the procedure of electricity industry restructure. In 1989 the parliament adopted the Electricity Act inaugurating a sweeping deregulation and privatisation of power sector. The following year, the new electricity industry came to being. Its operation mode was a mandatory pool market. In 1994 consumers with demand more than 100 MW became eligible to participate to the market. In 1998 the deregulation degree reached the full 100% by including all segments of the electricity market. The pool market was replaced in England and Wales in 2001 with a market based on the New Electricity Trading Arrangements (NETA). The pool market was criticized, despite its good performance, to be exposed in market power by side of large suppliers. Furthermore, the mandatory character of the pool was not giving the opportunity for bilateral contracts. The new market emerged by adoption of NETA tries to treat electricity as far as possible like any other commercial commodity [20]. Bilateral contracts are possible parallel to a voluntary pool market. A more flexible governance arrangement is also introduced in order to enable on time market changes whenever it is necessary.

At the same time, the second country next to England, which restructured its electricity market towards deregulation, is Norway. The beginning of deregulation was in 1990 by adopting the Energy Act. In 1995, the Swedish market was also reformed and together with the Norwegian electricity market established the Nord Pool which launched in early 1996 [21]. This is a power market, which includes both bilateral and voluntary pool modes. Thereby, it has avoided the non-flexibility of England's initial pool market.

Finland became a member of Nord Pool in 1998 followed by West Denmark in 1999 and finally the East Denmark in year 2000. The performance of Nord Pool brings it among the most successful paradigms of electricity sector deregulation.

In Russia public discussions about the power sector reform have started in the last years. A certain problem will be the rise of end consumer's price up to 300% when the subsidies of electricity price come to an end [22]. Moreover, the coordination of Russian giant network in a deregulated environment by itself is another challenge.

In European Union, with the exception of United Kingdom, the deregulation of electricity industry has been launched in 1996 by the adoption of Electricity Directive 96/92/EC [23]. This was the result of many years' negotiations between the member countries. The directive sets some thresholds for the progressive opening of the power sector. The final deadline is July 2007 when the electricity markets of all current member countries have to be fully deregulated. However, the directive does not define a common guideline for the electricity industry reform. Therefore, the restructure process has followed many different paths between the member countries.

In Germany, the adoption of Electrical Economy Right New Regulation Law signalled the power sector deregulation, in 1998. The German market was fully opened, in 100 %, i.e. the end-consumers are able to choose their supplier. The three Association Agreements between the energy producers and industrial consumers defined the framework for the calculation of transmission tariffs [24-26]. A particular characteristic of German electricity market is the absence of a regulator authority. The Cartel Office replaces some of the functions that a regulator would have. Taking the price reduction as criterion, one may describe the electricity industry deregulation as successful because both industrial and residential consumers have faced essential price reductions after the market opening.

In contrast to Germany, the power sector of France remains regulated and dominated in a high degree by the state-owned Electricité de France. In summer 2003 only a 35% of market volume was opened to competition. That corresponds to consumers with more than 7 GWh demand yearly. The situation in the rest countries of the European Union is a mirror of the two above paradigms. From the one side is Greece where the electricity market is opened up to 35% while the power market in Spain is already fully

deregulated. The present situation in European countries, regarding the electricity market opening, is illustrated in Figure 1.1 [34].

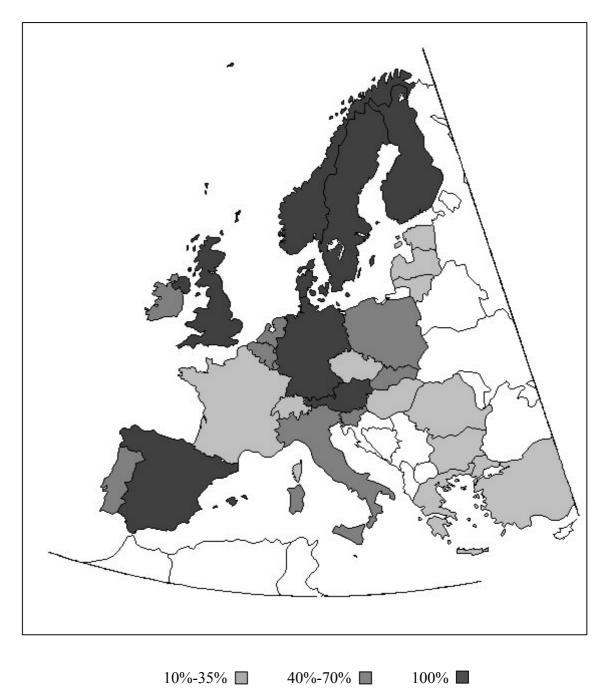


Figure 1.1 Electricity market opening in Europe, July 2003

Despite the different forms that the deregulation has taken in member countries, the final aim of European Union is to build up the Internal Market of Electricity (IEM) as a

Pan European single market for the commodity of electricity [27]. The IEM will contribute to the achievement of the aims that European Union has set concerning the electricity industry. The first aim is the increase of competitiveness by better service for consumers. The second aim, persuaded by European Union, is a better environmental protection and ultimately greater security for power supplies. In order to deal with the task of setting up the Internal Market, European Union has founded the Florence Regulatory Forum [28]. The Florence Forum focuses on three regulatory issues that are necessary for the development of IEM. The first point is the definition of a framework for the cross-border power trade. Furthermore, the Florence Forum has to set up rules for the use of transmission capacity in case of congestions. Finally, the development of procedures, which will lead to the increase of interconnections' capacity, is another important task of Florence Forum.

#### 1.3 Deregulated electricity market structure

The anatomy of deregulated power markets worldwide shows that the reform process has taken a number of different forms in various countries. Economic and political reasons, due to local conditions, have led to the adoption of different paradigms by the market restructure. However, there is a common basis and some similar characteristics that can be found to all of the competitive electricity markets.

A first characteristic is that, generally, the generation, transmission and distribution services are the responsibilities of different companies. This unbundling of services results in a deregulated electricity market. The transform process as well as the period after transformation is supervised by a regulatory authority. The task of this authority is to set up the general guidelines under which the market will operate. After the restructure, the task of the regulatory body focuses on the performance of the market. Although some markets, as in Germany, operate without regulatory authority, this is rather exception than a rule.

In the restructured market, the power generation is a competitive sector, which is qualified generation companies are able to take part in the market and sell their production. A very important principle of deregulated electricity market is the nondiscrimination. According to this principle, all producers and consumers, holding some certain conditions, must have open and fair access to the network.

While the generation is competitive the transmission remains a monopoly. The huge cost of investing in transmission network as well as ecological reasons does not allow the transform of this service into a competitive market. In order to ensure the open and fair access, the grid is operated by an Independent System Operator (ISO). The spectrum of ISO's responsibilities is quite wide [29]. The ISO has the responsibility to make available information regarding the network, such as the Available Transfer Capability, for all market participants. Moreover, its task comprises the real time operation of the network. This operation consists of adjusting network situation and, if necessary, ordering of the ancillary services so as to keep the system balance. Some of the ancillary service may be obtained through a balancing market where the participants submit their bids for increase/decrease their power in case of balancing operations.

Another characteristic, which is a case in matured deregulated markets, is the numerous contracts that hedge the risk resulting by the fluctuation of electricity prices. The market participants have the opportunity to purchase such contracts from ISO or from the regulatory authority. Furthermore, a secondary market can be organized where these contracts are traded as any other commodity.

The above described characteristics are common in the different types of deregulated market. Two basic market forms, the bilateral contract market and the pool market, are the common ground where the different types of markets are developed on. Besides these two types, a third alternative, based on multilateral agreements between the market participants, has been proposed [48].

In the bilateral market, producers and consumers directly negotiate the price and the quantity of traded energy. The transaction agents submit their schedule to the ISO requesting for permission to carry out the transaction. If the system balance is not endangered by this transaction then the ISO is committed to accept the schedule. If the transaction is accepted the ISO requires from the transaction agents to cover the associated transaction losses either by payment or by providing the necessary power. Furthermore, the ISO bills the transaction agents for using the network.

In a pool market, there is no direct contact between producers and consumers. The ISO, or a Pool Operator if one exists, collects one day in advance the bids for power sell

or purchase from the market participants. Then generators and loads are dispatched in such a way that leads to the economic optimum, i.e. the minimization of costs.

The majority of the deregulated markets are designed to include both two basic market forms. So, while a direct trading between consumer and producers is possible, the market participants can take part in the spot market where energy is traded as in a pool market [30]. The experience to date evidences that markets, that offer to their participants the flexibility to choose between bilateral contracts and spot market, have the best performance.

The entities that participate in the deregulated electricity markets include the generation companies or other commercial enterprises which can inject power into the network and they are described as suppliers. The side of consumers is represented by the distribution companies, the retail traders or directly by the end-consumers. In Figure 1.2 a general form of deregulated electricity market is illustrated.

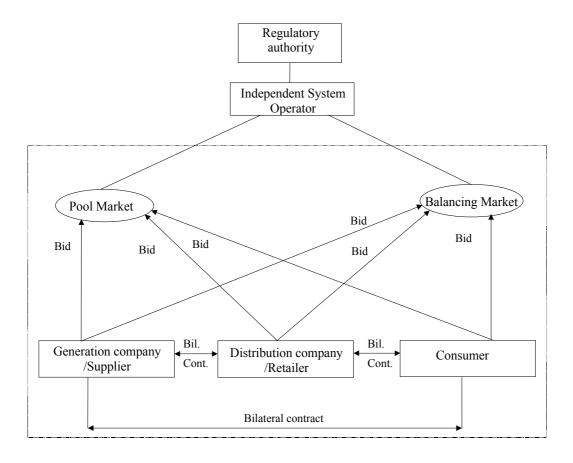


Figure 1.2 Deregulated electricity market

13

#### 1.4 Objectives

The scope of this work is the analysis of the costs that are associated with the power transfer as well as the realisation of new methods and tools concerning the calculation and the allocation of these costs.

The power transmission costs, which are charged to the market participants, are a central issue of the new cosmos of deregulated electricity markets. The increased requirement for fair and transparent pricing in the competitive environment as well as the complexity introduced by unbundling the services point out why this issue is of great importance [31].

In general, the costs associated with the power transfer may be categorized as follows:

- Cost associated with the power losses.
- Cost caused by system congestion.
- Fixed cost of the power system
- Cost of ancillary services.

In the deregulated electricity market, the participants are obliged to cover the power losses either by providing the necessary power or paying for the losses. The second category comprises the costs that are emerged when some technical features of the network reach their operational limits. In this case, the market equilibrium is different than the ideal optimum. The costs associated with this deviation are known as congestion costs. The fixed cost refers to the networks' investment and maintenance cost which is collected by the ISO.

The last category comprises the expenditures for the appropriate power system performance. In order to operate the network in a proper way, the ISO has to ensure the procurement of the so-called ancillary services [32, 33]. Although that these services vary between the different types of electricity markets, the following list illustrates the most common of them:

- Real power balance
- Voltage support
- Spinning reserves
- Non-spinning reserves

1.5 Outline

#### Back start

The real power balance is directly associated with the frequency control. In order to avoid significant problems, the network frequency must be maintained, with very small deviation, at the nominal value. For the voltage wider deviations from the nominal value are permitted. However, there are also some limits that have to be kept. So, in case of higher deviation, a voltage support action is necessary. The spinning reserves are resource capacities synchronized to the network that can supply energy or reduce demand within a certain time, usually 10 minutes. The non-spinning reserves are similar to the previous ones with the exception that they are not necessarily synchronized to the network. The black start is the capability of some generators to start up without being fed from the network. In case of large failures, generators possessing black start capability are necessary to start again the grid.

The largest part of power transmission cost consists of charges in order to recover the network fixed cost. The congestion cost may also be significant part of the power transmission cost depending on the nature of congestion. Therefore, this work is focused on these two components of transmission costs. Within the framework of this work, these two different kinds of costs have been analysed. New methods and tools concerning the calculation and allocation of these costs have also been realised.

#### 1.5 Outline

The organization of this work is as follows. In Chapter 2, first the definition of system congestions is presented as well as the analysis of different congestion management methods. Then, the analytical paradigm of pool market is described followed by the analysis of both nodal prices and their components that will be used as a framework in this dissertation. This followed by a new method which links the congestion costs to the market participants. In addition, some studies on the market participants' behaviour are presented and their influences on the network situation.

In Chapter 3, the effectiveness of this research work is demonstrated by analysing the operation mode of Norwegian electricity market. In particular, the market operation under the use of DC optimal power flow and zonal pricing is investigated. Comparisons

1.5 Outline 15

are made between the real AC situation and the DC market approximation. Moreover, a new method is presented for realisation of zonal pricing.

In Chapter 4, the problem of the network fixed cost is considered. In this framework, the game theory has been used to calculate and allocate these costs. Different game theoretical methods are analysed and the features of the cost allocation game are illustrated. Additionally, a new framework combining already known methods and the game theory is presented.

In Chapter 5, the conclusions of this research work can be found. Moreover, some suggestions on the extensions to potential topics for future research are presented.

1.5 Outline

### Chapter 2

# Congestion management in nodal pricing market

#### 2.1 Congestions in electricity networks

Electrical power networks are generally complicated systems consisting of numerous facilities and equipment. Generators, transmission as well as distribution lines and transformers are the main of them. All of these equipments are designed to operate between some certain limits. Additionally, the electrical features of a power system must be kept between the given values so as the system to operate without problem. Such features are the voltage magnitude of system buses and the difference of voltage angle at the beginning and the end of a transmission line. These electrical characteristics should not violate certain limits so as the power system to maintain a harmonious performance.

When any of the various system constraints reaches, or exceeds, its operational limits, then the resulting operational situation is defined as congestion. The most important constraints, regarding the economic performance of the system, are the thermal limits of

lines and transformers as well as the voltage magnitude constraints. The latter becomes rarely of great significance. Stability limits, based on the voltage angle difference, may become also economic critical in case of long transmission lines as they lead to reduction of allowable power flow.

Power flow causes energy losses in the form of heat through transmission lines and transformers. The material, from which lines are made of, has physical melt limit. When line temperature reaches this limit then the material melts and the line breaks. Hence, the power flow over a line must not increase the line temperature cross this limit. The different climate conditions that a line experiences during the seasons result in different power flow limits. Thus, the values of line power flow constraints should be considered in relation to the climate conditions. Transformers may be loaded over their nominal value for long periods without having operational problems. However, transformer power flow must hold certain limits too.

#### 2.2 Electricity pool market

One of the two basic types of deregulated power markets is the pool market. The pool model is based on a centralized arrangement in order to achieve the optimal economic performance of the market.

The history of central network dispatch aiming at an economic optimum dates back to early 1920s or even earlier when engineers already concerned themselves with the problem of economic allocation of generation [35]. Since 1930s the equal incremental cost method was favoured as the most efficient. Economic dispatch considers only real power generations and represents the electrical network by a single equality constraint, the power balance equation. In early 1960s the first methods of optimal power flow (OPF) were developed [36]. These methods treated the entire network in an exact manner. The objective function of the OPF methods was the minimization either of the generation costs or of the active power losses.

Nowadays the most well-known electricity pool markets are established in New Zealand, Australia, the Scandinavian countries (Nord Pool) and in eastern part of USA (PJM). The main characteristic of electricity pool market is that the power is traded through the market and not directly between producers and consumers. The market is

operated either by a separate Pool Operator or directly by the Independent System Operator. The task of market operator is to lead the pool market to a short-run economic optimum.

In order to achieve this aim, the market operator collects the electric power bids from suppliers as well as from consumers. The bids are related to a certain time interval, usually half or one hour, and they are submitted to the ISO a day before the applicability of the time [6]. Therefore, the modern pool markets are also known as a day ahead markets. When the bids are submitted, the market operator runs an OPF program taking into consideration the network constraints. The objective of this OPF program is to minimize the total costs also known as social welfare. The OPF calculates spot prices for each location (bus) of the grid as well as the quantity of power that is to be supplied or bought by each of the market participant. Consumers and suppliers are then billed to the spot price of their bus for the corresponding amount of power [37]. In some pool markets, such as the Nord Pool, there is no locational pricing and the bid mechanism is used to calculate a global, or sometimes zonal, market clearing price. In the following day if there is a difference between calculated schedule and real generation or demand then this difference is covered through the real time (balancing) market. A schematic description of pool market operation is given by Figure 2.1.

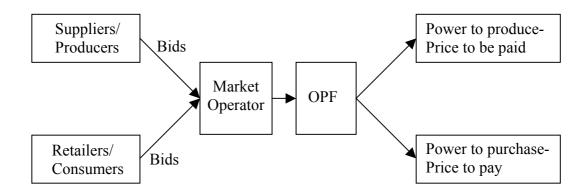


Figure 2.1 Electricity pool market

In the monopoly power markets the utility was performing an OPF knowing the real cost data of its generators. Furthermore, the load was also given and had to be fully covered. In the present deregulated market, the pool operator has no knowledge about the cost functions of power plants owned by the producers. Moreover, the wholesale

consumers are eligible to vary their demand according to the price that they face. Consequently, the market operator runs the OPF based on the bids collected from the market participants. In Figure 2.2, typical suppliers' as well as consumer's bid curves are illustrated. The supply bid curve shows the minimum price at which a supplier wants to sell a certain quantity of power. If the price is less than  $p_1$  then the supplier sells nothing to the pool market. In contrast, when the price is higher than  $p_3$ , the supplier has the willingness to offer up to his maximum capacity. On the other hand, the demand bid curve shows the maximum acceptable price at which the consumer is willing to buy a certain quantity of power. If the electricity price faced by the consumer is more than  $p_3$  then his demand is zero. When the price falls down from  $p_1$  then the consumer may purchase a power amount up to his maximal demand.

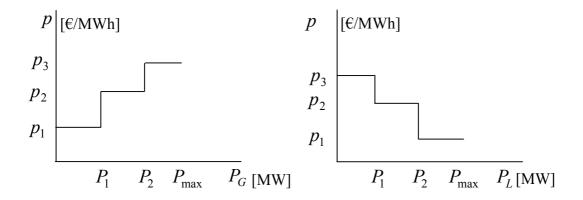


Figure 2.2 Supplier (left) and consumer (right) bid curves

#### 2.3 The optimal power flow problem

The Pool Operator, the day before the corresponding time interval, feeds an optimal power flow program with the bids collected from the market participants. Generally, optimisation problems aim to maximise or minimise a function while certain restrictions hold.

In the deregulated pool market, the optimisation problem has to serve a double task. First, it aims to minimise the power supply costs. At the same time the other objective of optimisation is to cover the load demand as much as possible. The more power the

consumers take the more profit they have through the use of power. So, if the consumers' full demand is not covered that results in profit losses which can be seen as a kind of cost. Thus, the objective function of the OPF is called social welfare because it aims to minimise the global system costs and thereby to maximise the profit of all market participants. Assuming for simplicity one step bid curves for both generators and loads, the objective function has the following form:

$$K(\mathbf{P}_G, \mathbf{P}_{UL}) = \mathbf{p}_{\min}^T \mathbf{P}_G + \mathbf{p}_{\max}^T \mathbf{P}_{UL}$$
 (2.1)

where

K: the social welfare

 $\mathbf{P}_G$ : vector of generation power

 $\mathbf{P}_{UL}$ : vector of uncovered load portion

 $\mathbf{p}_{min}$ : vector of minimum acceptable price(bid) from generators

 $\boldsymbol{p}_{\text{max}}$  : vector of maximum acceptable price(bid) from loads

A part of a particular load is not served if the load bid for this part is lower than the suppliers' bid or if system congestions do not allow the cover of this part of demand. Each uncovered load portion can be modelled through a fictitious generator [38]. From the consumer bid curve of Figure 2.2 the bid curve of fictitious generator can be defined as in Figure 2.3.

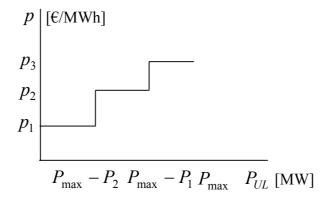


Figure 2.3 Bid curve of fictitious consumer generator

Figure 2.3 shows that a part of fictitious generator is dispatched if the corresponding bid price is lower than the suppliers' bid. It is also possible this generator to be dispatched if system congestions prevent the full cover of the load. For a load located at bus i it can be written:

$$0 \le P_{III,i} \le P_{\text{max},i} \tag{2.2a}$$

$$P_{Li} = P_{L\max i} - P_{ULi} \tag{2.2b}$$

where

 $P_{Li}$ : covered load portion at bus i

 $P_{L\max i}$ : maximum load demand at bus i

The objective function for the OPF in the Pool market can be now formulated as:

$$K(\mathbf{P}_G) = \mathbf{p}_{\min}^T \mathbf{P}_G \tag{2.3}$$

where the new vector  $\mathbf{P}_G$  includes the fictitious generators.

The restrictions of the OPF are with respect to the active and reactive power balance at each system node, the line flow constraints, the voltage magnitude limits and the active and reactive power generation limits. Thus, the OPF can be formulated as:

min 
$$K(\mathbf{P}_{G})$$
  
s.t.  $\mathbf{P}_{G} - \mathbf{P}_{L} + \mathbf{P}(\mathbf{V}, \mathbf{\theta}) = \mathbf{0}$   
 $\mathbf{Q}_{G} - \mathbf{Q}_{L} + \mathbf{Q}(\mathbf{V}, \mathbf{\theta}) = \mathbf{0}$   
 $\mathbf{S}(\mathbf{V}, \mathbf{\theta}) \leq \mathbf{S}_{\max}(\mathbf{V}, \mathbf{\theta})$   
 $\mathbf{V}_{\min} \leq \mathbf{V} \leq \mathbf{V}_{\max}$   
 $\mathbf{P}_{G \min} \leq \mathbf{P}_{G} \leq \mathbf{P}_{G \max}$   
 $\mathbf{Q}_{G \min} \leq \mathbf{Q}_{G} \leq \mathbf{Q}_{G \max}$ 

where

 $\mathbf{P}_L$ : vector of active power load

 $P(V, \theta)$ : vector of nodal active power flow

 $\mathbf{Q}_G$ : vector of reactive power generation

 $\mathbf{Q}_L$ : vector of reactive power load

 $Q(V,\theta)$ : vector of nodal reactive power flow

 $S(V, \theta)$ . vector of line power flow

V: vector of bus voltage magnitude

 $\theta$ : vector of bus voltage angle

It is now possible to give to the OPF a more compact form by grouping together all equality restrictions and putting up two groups of the inequality constraints:

min 
$$K(\mathbf{P}_{G})$$
  
s.t.  $\mathbf{f}(\mathbf{P}_{G}, \mathbf{Q}_{G}, \mathbf{V}, \mathbf{\theta}) = \mathbf{0}$   
 $\mathbf{g}(\mathbf{P}_{G}, \mathbf{Q}_{G}, \mathbf{V}, \mathbf{\theta}) \leq \mathbf{0}$   
 $\mathbf{h}(\mathbf{P}_{G}, \mathbf{Q}_{G}) \leq \mathbf{0}$  (2.5)

where

 $\mathbf{f}(\mathbf{P}_G, \mathbf{Q}_G, \mathbf{V}, \mathbf{\theta})$ : set of equality constraints with respect to active and reactive

power balance

 $\mathbf{g}(\mathbf{P}_G, \mathbf{Q}_G, \mathbf{V}, \mathbf{\theta})$ : set of inequality constraints with respect to bus voltage magnitude

and line power flow

 $\mathbf{h}(\mathbf{P}_G, \mathbf{Q}_G)$ : set of inequality constraints with respect to active and reactive

power generation

For the above optimisation problem, it is easy to formulate a function called Lagrange function. This Lagrange function is a sum of the original problem objective function and the original problem restriction which is now multiplied by factors called Lagrange multipliers. For the above described OPF the Lagrange function is as follows:

$$L(\mathbf{P}_{G}, \mathbf{Q}_{G}, \mathbf{V}, \mathbf{\theta}, \lambda, \mu, \sigma) = K(\mathbf{P}_{G}) + \mathbf{f}^{T}(\mathbf{P}_{G}, \mathbf{Q}_{G}, \mathbf{V}, \mathbf{\theta})\lambda + \mathbf{g}^{T}(\mathbf{P}_{G}, \mathbf{Q}_{G}, \mathbf{V}, \mathbf{\theta})\mu + \mathbf{h}^{T}(\mathbf{P}_{G}, \mathbf{Q}_{G})\sigma$$
(2.6)

where

 $\lambda$ : vector of Lagrange multipliers with respect to set of equality constraints  $\mathbf{f}(\mathbf{P}_G, \mathbf{Q}_G, \mathbf{V}, \mathbf{\theta})$ 

 $\mu$ : vector of Lagrange multipliers with respect to set of inequality constraints  $\mathbf{g}(\mathbf{P}_G, \mathbf{Q}_G, \mathbf{V}, \mathbf{\theta})$ 

 $\sigma$ : vector of Lagrange multipliers with respect to set of inequality constraints  $\mathbf{h}(\mathbf{P}_G, \mathbf{Q}_G)$ 

At the solution point of OPF the gradients of the Lagrange function with respect to any problem variable are equal to zero. Thus, solving sets of equations that obtained by the gradients with respect to the active and reactive generation one may find out that:

$$\lambda = -\left(\frac{\partial K}{\partial \mathbf{\tau}}\right)^{T} = -\left[\left(\frac{\partial K}{\partial \mathbf{P}_{G}}\right)^{T} \quad \left(\frac{\partial K}{\partial \mathbf{Q}_{G}}\right)^{T}\right]^{T}$$
(2.7)

where

 $\tau$ : vector including the active and reactive power

Consequently, the vector  $\lambda$  includes the marginal cost for additional active and reactive load at each bus of the power system. In the pool market these marginal costs are the nodal prices for active and reactive power at each bus. In order (2.7) to be valid for a system bus the only binding restriction related with the bus generation must be the bus power balance. In event of any other binding constraint that is related with the bus generation the Lagrange multiplier is not equal with the bus marginal cost. Consequently, the corresponding nodal price is also not equal to the marginal cost. Similarly to vector  $\lambda$ , the vectors  $\mu$  and  $\sigma$  contain the marginal change in cost with

respect to the corresponding constraints. The elements  $\mu_i$  and  $\sigma_j$  of vectors  $\mu$  and  $\sigma$  respectively are different than zero only in case that the corresponding constraints are active. By the solution of OPF the vectors  $\lambda$ ,  $\mu$  and  $\sigma$  are calculated and are available to the market operator. The nodal prices are then announced to the market participants so as to know which quantity of power and at which price they will trade. The Lagrange multipliers associated with the system constrains are used by the market operator in order to deal with any binding constraints.

For the solution of the OPF problem a great variety of methods has been applied. The first efficient OPF programs were based on the gradient method [36, 39, 40]. Although these methods treat the entire AC network in an exact way they have essential disadvantages. The gradients included derivatives, which had quite different magnitudes than the derivatives of the objective function. Thus, there was no high acriby by the achievement of optimum. The maintaining of constraints was also not satisfactory. In the next years methods of linear programming have been developed in order to approach closer the exact optimum [41, 42]. The quadratic programming based applications is another part of the OPF methods [43, 44]. In [45, 46] a different direction was followed by developing programs in the Newton's form.

The different OPF methods may be distinguished in two categories [47]. The first category consists of methods that begin the optimisation from an already solved power flow. In order to calculate the optimum these methods use the Jacobian as well as other sensitivity relations. This is an iterative process where the power flow problem is solved at each iteration. The methods that belong to the second category are based on the exact optimality conditions while the power flow relations are included as equality restrictions. By these methods there is no initial power flow solution. For the methods belonging to this category the process is also iterative and the intermediate solutions approach the power flow solution.

The majority of the first category methods use in their optimisation part either a linear programming or a quadratic programming. A significant advantage of these methods is the clear and systematic treatment of constraints. Particularly, the linear programming has performed a quite efficient operation concerning the active power dispatch. Another merit of this category is that the initial point is a solved power flow, which in most cases represents a feasible solution of the OPF. On the other hand the disadvantage of

the first category methods is that when linear programming is used in order to minimize the power losses there is no high acriby by the calculation of optimum. Additionally, a disadvantage of the linear programming is that when the generation cost functions are similar or identical this method shows very slow convergence rate.

The methods of second category based either on Newton algorithm or on a quadratic programming when the Lagrangian of the OPF is quadratic. The Quasi-Newton applications belong also to this category. These methods have a great benefit of solving the OPF in a global way. This fact makes them attractive because they are able to calculate exact the optimum. The convergence in the Newton approach is also very satisfactory. The major drawback that the methods of this category have is by the handling of the inequality constraints and particularly by the determination of the binding inequality constraints. The use of sparsity techniques as well as the application of ordinary power flow may cope with this problem.

## 2.4 Analysis of nodal prices

The nodal prices that are computed through the OPF are influenced by different factors. Generally, the basis of nodal prices is the generation or supply cost. Additionally, the system power losses result in different, dependent on the location, nodal prices. Furthermore, the system congestions are of great significance by the computation of nodal prices. The nodal prices are analysed into components because each of these component has an economical and physical interpretation. Knowing these components the pool operator may apply proper policies so as to achieve an optimal market performance. The numerous parts of nodal prices are not always independent from each other. The power losses component is associated with the generation component. Thus, in the following analysis, these two components are calculated as one. The decomposition of nodal prices has been illustrated as early as 1988 in [37]. More recent work on this issue may be found in [38, 49, 50]. In following sections the nodal prices are analysed into two components using two different methods.

#### 2.4.1 General formulation

The nodal prices, that are included in vector  $\lambda$  by the solution of OPF, are due to generation, losses and congestion. The aim is to divide these prices into a component due to generation-losses and a congestion component. For this purpose it is necessary to select a bus r in the system as reference bus. The OPF of (2.5) may now be formulated as follow:

min 
$$K(\mathbf{P}_{G})$$
  
s.t.  $\mathbf{f}_{r}(P_{Gr}, Q_{Gr}, V_{r}, \theta_{r}, \mathbf{V}, \mathbf{\theta}) = \mathbf{0}$   
 $\mathbf{f}(\mathbf{P}_{G}, \mathbf{Q}_{G}, V_{r}, \theta_{r}, \mathbf{V}, \mathbf{\theta}) = \mathbf{0}$   
 $\mathbf{g}(\mathbf{P}_{G}, \mathbf{Q}_{G}, V_{r}, \theta_{r}, \mathbf{V}, \mathbf{\theta}) \leq \mathbf{0}$   
 $\mathbf{h}(\mathbf{P}_{G}, \mathbf{Q}_{G}, P_{Gr}, Q_{Gr}) \leq \mathbf{0}$  (2.8)

where

 $P_{Gr}$ : active power generation at reference bus r

 $Q_{Gr}$ : reactive power generation at reference bus r

 $V_r$ : bus voltage magnitude at reference bus r

 $\theta_r$ : bus voltage angle at reference bus r

 $\mathbf{f}_r$ : set of equality constraints with respect to active and reactive power balance at reference bus r

It should be emphasized that the vector  $\mathbf{f}(\mathbf{P}_G, \mathbf{Q}_G, V_r, \theta_r, \mathbf{V}, \mathbf{\theta})$  in (2.8) does not include the equations of reference bus r. Similarly, the vectors  $\mathbf{P}_G$ ,  $\mathbf{Q}_G$ ,  $\mathbf{V}$  and  $\mathbf{\theta}$  do not contain the variables  $P_{Gr}$ ,  $Q_{Gr}$ ,  $V_r$  and  $\theta_r$  of the reference bus r any more. From (2.8) one may obtain the Lagrange function. The detailed form of this function is given in (2.9):

$$L(\mathbf{P}_{G}, \mathbf{Q}_{G}, \mathbf{V}, \mathbf{\theta}, P_{Gr}, Q_{Gr}, V_{r}, \theta_{r}, \lambda_{r}, \lambda, \mu, \sigma) =$$

$$K(\mathbf{P}_{G}) + \mathbf{f}_{r}^{T} (P_{Gr}, Q_{Gr}, V_{r}, \theta_{r}, \mathbf{V}, \mathbf{\theta}) \lambda_{r} + \mathbf{f}^{T} (\mathbf{P}_{G}, \mathbf{Q}_{G}, V_{r}, \theta_{r}, \mathbf{V}, \mathbf{\theta}) \lambda \qquad (2.9)$$

$$+ \mathbf{g}^{T} (\mathbf{P}_{G}, \mathbf{Q}_{G}, V_{r}, \theta_{r}, \mathbf{V}, \mathbf{\theta}) \mu + \mathbf{h}^{T} (\mathbf{P}_{G}, \mathbf{Q}_{G}, P_{Gr}, Q_{Gr}) \sigma$$

where

 $\lambda_r$ : vector of Lagrange multipliers with respect to active and reactive power balance at reference bus r

The vector  $\lambda$ , given in (2.9), does not include the Lagrange multipliers corresponding to the active and reactive power balance restrictions at reference bus r. From this point the nodal price analysis may follow two different ways. Both of them lead to the split up of nodal prices into a generation-losses and a congestion component. The first method takes in advance the Kuhn-Tucker theorem while the linearisation process as well as the duality feature of optimisation is used in the second method.

### 2.4.2 Nodal price analysis using the Kuhn-Tucker Theorem

The first step is the introduction of vector  $\mathbf{x} = (\mathbf{P}_{G_r} \mathbf{Q}_{G_r} \mathbf{V}, \mathbf{\theta}, P_{Gr_r} \mathbf{Q}_{Gr_r} \mathbf{V}_r, \theta_r)$ . Given the vector  $\mathbf{x}$ , (2.9) can be rewritten in a more compact form:

$$L(\mathbf{x}, \lambda_{\mathbf{r}}, \lambda, \boldsymbol{\mu}, \boldsymbol{\sigma}) = K(\mathbf{x}) + \mathbf{f}_{r}^{T}(\mathbf{x})\lambda_{\mathbf{r}} + \mathbf{f}^{T}(\mathbf{x})\lambda + \mathbf{g}^{T}(\mathbf{x})\boldsymbol{\mu} + \mathbf{h}^{T}(\mathbf{x})\boldsymbol{\sigma}$$
(2.10)

The Kuhn-Tucker theorem is a generalization of Lagrange multipliers. It is a theorem in nonlinear programming. The OPF is a typical nonlinear optimisation problem. According to the Kuhn-Tucker theorem at the solution point, where the objective function is minimized while all constraints are satisfied, the following condition is valid:

$$\nabla_{\mathbf{x}} L(\mathbf{x}, \lambda_{\mathbf{r}}, \lambda, \boldsymbol{\mu}, \boldsymbol{\sigma}) = \nabla_{\mathbf{x}} K(\mathbf{x}) + (\nabla_{\mathbf{x}} \mathbf{f}_{r}(\mathbf{x}))^{T} \lambda_{\mathbf{r}} + (\nabla_{\mathbf{x}} \mathbf{f}(\mathbf{x}))^{T} \lambda + (\nabla_{\mathbf{x}} \mathbf{g}(\mathbf{x}))^{T} \boldsymbol{\mu} + (\nabla_{\mathbf{x}} \mathbf{h}(\mathbf{x}))^{T} \boldsymbol{\sigma} = \mathbf{0}$$
(2.11)

where

 $\lambda_r$ : vector of nodal prices for active and reactive power at reference bus r.

 $\lambda$ : vector of nodal prices for active and reactive power at all the system buses except reference bus r.

It is essential to mention that the vectors  $\lambda_r$ ,  $\lambda$ ,  $\mu$  and  $\sigma$  as well as all the gradients are known from the solution of the OPF of (2.5). The OPF of (2.8) is a rewritten form of the (2.5) in order to deal with the task of nodal price analysis. Thus, the solution of (2.5) is exactly the solution of (2.8).

From (2.11) the gradients with respect to V and  $\theta$  are derived:

$$(\nabla_{\mathbf{V}}\mathbf{f}_r(\mathbf{x}))^T \lambda_r + (\nabla_{\mathbf{V}}\mathbf{f}(\mathbf{x}))^T \lambda + (\nabla_{\mathbf{V}}\mathbf{g}(\mathbf{x}))^T \mu = \mathbf{0}$$
 (2.12a)

$$(\nabla_{\theta} \mathbf{f}_r(\mathbf{x}))^T \lambda_r + (\nabla_{\theta} \mathbf{f}(\mathbf{x}))^T \lambda + (\nabla_{\theta} \mathbf{g}(\mathbf{x}))^T \mu = \mathbf{0}$$
 (2.12b)

In both (2.12a) and (2.12b) neither gradients of objective function K nor gradients of vector  $\mathbf{h}$  are appeared. This may be explained by taking a look at the following equation:

$$\nabla_{\mathbf{V}}K(\mathbf{x}) = \nabla_{\mathbf{\theta}}K(\mathbf{x}) = \nabla_{\mathbf{V}}\mathbf{h}(\mathbf{x}) = \nabla_{\mathbf{\theta}}\mathbf{h}(\mathbf{x}) = \mathbf{0}$$
 (2.13)

As can be seen the gradients of  $\mathbf{h}$  with respect to  $\mathbf{V}$  and  $\mathbf{\theta}$  are zero. Thus, vector  $\mathbf{h}$  can be neglected by the nodal price analysis. One may now solve the system of (2.12a) and (2.12b) for the vector  $\lambda$ . In this way it is possible to obtain the two nodal price components:

$$\lambda = \begin{bmatrix} (\nabla_{\mathbf{v}} \mathbf{f}(\mathbf{x}))^{T} \\ (\nabla_{\theta} \mathbf{f}(\mathbf{x}))^{T} \end{bmatrix}^{-1} \begin{bmatrix} (\nabla_{\mathbf{v}} \mathbf{f}_{r}(\mathbf{x}))^{T} \\ (\nabla_{\theta} \mathbf{f}_{r}(\mathbf{x}))^{T} \end{bmatrix} \lambda_{r}$$

$$+ \begin{bmatrix} (\nabla_{\mathbf{v}} \mathbf{f}(\mathbf{x}))^{T} \\ (\nabla_{\theta} \mathbf{f}(\mathbf{x}))^{T} \end{bmatrix}^{-1} \begin{bmatrix} (\nabla_{\mathbf{v}} \mathbf{g}(\mathbf{x}))^{T} \\ (\nabla_{\theta} \mathbf{g}(\mathbf{x}))^{T} \end{bmatrix} \mu$$
(2.14)

In (2.14) it is shown that the vector  $\lambda$ , which contains the nodal prices for active and reactive power for all the system buses except the reference bus, is divided into two components. The first part of the right side of (2.14) represents the component  $\lambda_{GL}$  for generation and losses. The system congestions' influence on nodal price is given by the second part of the right side of (2.14). This part is the nodal price congestion component  $\lambda_{C}$ . Hence, it is possible to write that:

$$\lambda_{GL} = \begin{bmatrix} (\nabla_{\mathbf{V}} \mathbf{f}(\mathbf{x}))^T \\ (\nabla_{\mathbf{\theta}} \mathbf{f}(\mathbf{x}))^T \end{bmatrix}^{-1} \begin{bmatrix} (\nabla_{\mathbf{V}} \mathbf{f}_r(\mathbf{x}))^T \\ (\nabla_{\mathbf{\theta}} \mathbf{f}_r(\mathbf{x}))^T \end{bmatrix} \lambda_r$$
 (2.15a)

$$\lambda_{C} = \begin{bmatrix} (\nabla_{\mathbf{V}} \mathbf{f}(\mathbf{x}))^{T} \\ (\nabla_{\theta} \mathbf{f}(\mathbf{x}))^{T} \end{bmatrix}^{-1} \begin{bmatrix} (\nabla_{\mathbf{V}} \mathbf{g}(\mathbf{x}))^{T} \\ (\nabla_{\theta} \mathbf{g}(\mathbf{x}))^{T} \end{bmatrix} \boldsymbol{\mu}$$
 (2.15b)

and:

$$\lambda = \lambda_{GL} + \lambda_C \tag{2.16}$$

It is essential to emphasize that the vector  $\lambda_r$  should contain nodal prices for active and reactive power that are only due to generation and losses and not due to congestion [38]. For this reason it is important to select a suitable reference bus. For the case of pool market the generation bus with the lowest bid and with available capacity is a practical choice. Thereby, the local additional load can be served by the generator of that bus.

# 2.4.3 Nodal price analysis using linearisation and the dual problem

The second method for the nodal price analysis follows a different approach up to a certain point. This method has been implemented in [38]. As in the previous method,

the vector  $\mathbf{x}$  is used in order to simplify the OPF form. It is now possible to rewrite the OPF of (2.8) as follows:

min 
$$K(\mathbf{x})$$
  
 $s.t.$   $\mathbf{f}_r(\mathbf{x}) = \mathbf{0}$   
 $\mathbf{f}(\mathbf{x}) = \mathbf{0}$   
 $\mathbf{g}(\mathbf{x}) \le \mathbf{0}$   
 $\mathbf{h}(\mathbf{x}) \le \mathbf{0}$ 

It is known from the optimisation theory, the solution of a nonlinear optimisation problem, such as OPF, may also be the solution of a linear program (LP). The problem of (2.17) can be linearised at the solution point  $\mathbf{x}_0$ . The resulting linear programming has the following form:

min 
$$\nabla_{\mathbf{x}} K(\mathbf{x}_{0}) \mathbf{x} + [-\nabla_{\mathbf{x}} K(\mathbf{x}_{0}) \mathbf{x}_{0} + K(\mathbf{x}_{0})]$$
s.t. 
$$\nabla_{\mathbf{x}} \mathbf{f}_{r}(\mathbf{x}_{0}) \mathbf{x} + [-\nabla_{\mathbf{x}} \mathbf{f}_{r}(\mathbf{x}_{0}) \mathbf{x}_{0} + \mathbf{f}_{r}(\mathbf{x}_{0})] = \mathbf{0}$$

$$\nabla_{\mathbf{x}} \mathbf{f}(\mathbf{x}_{0}) \mathbf{x} + [-\nabla_{\mathbf{x}} \mathbf{f}(\mathbf{x}_{0}) \mathbf{x}_{0} + \mathbf{f}(\mathbf{x}_{0})] = \mathbf{0}$$

$$\nabla_{\mathbf{x}} \mathbf{g}(\mathbf{x}_{0}) \mathbf{x} + [-\nabla_{\mathbf{x}} \mathbf{g}(\mathbf{x}_{0}) \mathbf{x}_{0} + \mathbf{g}(\mathbf{x}_{0})] \leq \mathbf{0}$$

$$\nabla_{\mathbf{x}} \mathbf{h}(\mathbf{x}_{0}) \mathbf{x} + [-\nabla_{\mathbf{x}} \mathbf{h}(\mathbf{x}_{0}) \mathbf{x}_{0} + \mathbf{h}(\mathbf{x}_{0})] \leq \mathbf{0}$$

$$(2.18)$$

The gradients at point  $\mathbf{x}_0$  have the same value with the gradients at solution point by the first method. In this case the symbol  $\mathbf{x}_0$  is used in order to underline the linearisation process and to distinguish the linearisation point from the vector of variables  $\mathbf{x}$ .

According to the optimisation theory each linear programming problem can be linked to a dual problem [51]. For the problem of (2.18) the variables of its dual problem are the Lagrange multipliers of the problem of (2.17). In the primal problem all the variables of vector  $\mathbf{x}$  are free. Therefore, the dual problem has only equality constraints. The detailed form of that dual problem is as follows:

max 
$$\mathbf{a}^{T} \boldsymbol{\lambda}_{r} + \mathbf{b}^{T} \boldsymbol{\lambda} + \mathbf{c}^{T} \boldsymbol{\mu} + \mathbf{d}^{T} \boldsymbol{\sigma}$$
  
s.t.  $(\nabla_{\mathbf{x}} \mathbf{f}_{r}(\mathbf{x}_{0}))^{T} \boldsymbol{\lambda}_{r} + (\nabla_{\mathbf{x}} \mathbf{f}(\mathbf{x}_{0}))^{T} \boldsymbol{\lambda}$  (2.19)  
 $+ (\nabla_{\mathbf{x}} \mathbf{g}(\mathbf{x}_{0}))^{T} \boldsymbol{\mu} + (\nabla_{\mathbf{x}} \mathbf{h}(\mathbf{x}_{0}))^{T} \boldsymbol{\sigma} = \mathbf{0}$ 

where the **a**, **b**, **c** and **d** are defined as follows:

$$\mathbf{a} = \left[ +\nabla_{\mathbf{x}} \mathbf{f}_r(\mathbf{x}_0) \mathbf{x}_0 - \mathbf{f}_r(\mathbf{x}_0) \right] \tag{2.20a}$$

$$\mathbf{b} = [+\nabla_{\mathbf{x}} \mathbf{f}(\mathbf{x}_0) \mathbf{x}_0 - \mathbf{f}(\mathbf{x}_0)] \tag{2.20b}$$

$$\mathbf{c} = [+\nabla_{\mathbf{x}} \mathbf{g}(\mathbf{x}_0) \mathbf{x}_0 - \mathbf{g}(\mathbf{x}_0)] \tag{2.20c}$$

$$\mathbf{d} = [+\nabla_{\mathbf{x}} \mathbf{h}(\mathbf{x}_0) \mathbf{x}_0 - \mathbf{h}(\mathbf{x}_0)] \tag{2.20d}$$

From the equality constraints of (2.19) the gradients with respect to V and  $\theta$  may be obtained. The derived equations are the same with (2.12a) and (2.12b). The further process, in order to achieve the split up of nodal prices in two components, is the same as in the first method.

# 2.5 Congestion component analysis

Once the nodal prices are decomposed, their component due to system congestions is available. For the system operator is of great importance to know how the different network restrictions influence the nodal prices. Having this knowledge the system operator may make the right decisions so as to cope with the congested situation. So, it is obviously that there is a necessity of decomposing further the congestion component. This decomposition should result in an allocation of nodal price congestion component to the single system constraints. The vector  $\lambda_C$ , given by (2.15b), is a column vector which embraces the congestion component of nodal prices for active and reactive power for all the system buses except the reference bus r. This bus has nodal prices only due to generation and losses. If  $m = n_b - 1$ , where  $n_b$  is the number of system buses, then it is:

$$\boldsymbol{\lambda}_{C} = \begin{bmatrix} \lambda_{CP1}, \dots, \lambda_{CPi}, \dots, \lambda_{CPm}, \lambda_{CO1}, \dots, \lambda_{COi}, \dots, \lambda_{COm} \end{bmatrix}^{T}$$
(2.21)

where

 $\lambda_{\mathit{CPi}}$ : congestion component of nodal price for active power at bus i .

 $\lambda_{\mathit{CQi}}$  : congestion component of nodal price for reactive power at bus i .

On the other hand, the column vector  $\mu$  comprises the Lagrange multipliers corresponding to the bus voltage magnitude constraints as well as for the line power flow limits. If the system has l lines then it can be written:

$$\mathbf{\mu} = [\mu_{V1\min}, ..., \mu_{Vi\min}, ..., \mu_{Vn\min}, \mu_{V1\max}, ..., \mu_{V1\max}, ..., \mu_{Ui\max}, ..., \mu_{Ui\max}, ..., \mu_{Ui}, ..., \mu_{Ui}]^T$$
(2.22)

where

 $\mu_{Vi\, {
m min}}$ : Lagrange multiplier with respect to the lower voltage magnitude limit at bus i .

 $\mu_{Vi\, 
m max}$  : Lagrange multiplier with respect to the upper voltage magnitude limit at bus i .

 $\mu_{Lj}$ : Lagrange multiplier with respect to the power flow limit on line j.

In order to simplify further the analysis, the matrix A is introduced. This matrix is defined as follows:

$$\mathbf{A} = \begin{bmatrix} (\nabla_{\mathbf{V}} \mathbf{f}(\mathbf{x}))^T \\ (\nabla_{\theta} \mathbf{f}(\mathbf{x}))^T \end{bmatrix}^{-1} \begin{bmatrix} (\nabla_{\mathbf{V}} \mathbf{g}(\mathbf{x}))^T \\ (\nabla_{\theta} \mathbf{g}(\mathbf{x}))^T \end{bmatrix}$$
(2.23)

where all the gradients are evaluated at the solution point  $\mathbf{x}_0$ .

Thus, (2.15b) can be rewritten as:

$$\lambda_C = \mathbf{A}\mathbf{\mu} \tag{2.24}$$

A more detailed form of (2.24) can be given:

$$\begin{bmatrix} \lambda_{CP1} \\ \vdots \\ \lambda_{CPi} \\ \vdots \\ \lambda_{CPm} \\ \lambda_{CQ1} \\ \vdots \\ \lambda_{CQm} \end{bmatrix} = \begin{bmatrix} a_{11}, a_{12}, \dots, a_{1(2n_b + n_t)} \\ \vdots \\ a_{i1}, a_{i2}, \dots, a_{i(2n_b + n_t)} \\ \vdots \\ a_{m1}, a_{m2}, \dots, a_{m(2n_b + n_t)} \\ \vdots \\ a_{(m+1)1}, a_{(m+1)2}, \dots, a_{(m+1)(2n_b + n_t)} \\ \vdots \\ a_{(m+i)1}, a_{(m+i)2}, \dots, a_{(m+i)(2n_b + n_t)} \\ \vdots \\ a_{2m1}, a_{2m2}, \dots, a_{2m(2n_b + n_t)} \end{bmatrix} \begin{bmatrix} \mu_{V1 \text{min}} \\ \vdots \\ \mu_{Vn \text{min}} \\ \vdots \\ \mu_{VI \text{max}} \end{bmatrix}$$

From (2.25) one may find out that:

$$\lambda_{CPi} = \sum_{k=1}^{2n_b + n_l} a_{ik} \mu_k \tag{2.26}$$

where

 $a_{ik}\mu_k$ : congestion component part of nodal price for active power at bus i due to constraint k

 $2n_b + n_l$ : total number of bus voltage magnitude and line power flow constraints

The aim is to estimate the contribution of each congestion to the nodal price congestion component. For this purpose it is useful to replace the vector  $\boldsymbol{\mu}$  in (2.24) with the matrix  $\boldsymbol{M}$ .

$$\mathbf{M} = diag(\mathbf{\mu}) \tag{2.27}$$

Through this replacement a new matrix  $\Lambda$  can be obtained:

$$\mathbf{\Lambda} = \mathbf{A}\mathbf{M} = \begin{bmatrix} \mathbf{\Lambda}_{PV \, \text{min}} & \mathbf{\Lambda}_{PV \, \text{max}} & \mathbf{\Lambda}_{PL} \\ \mathbf{\Lambda}_{QV \, \text{min}} & \mathbf{\Lambda}_{QV \, \text{max}} & \mathbf{\Lambda}_{QL} \end{bmatrix}$$
(2.28)

where

 $\Lambda_{PV\, \mathrm{min}}$ : matrix which contains in an arbitrary line i the contribution of the lower voltage magnitude limits to the  $\lambda_{CPi}$ 

 $\Lambda_{PV\, {
m max}}$ : matrix which contains in an arbitrary line i the contribution of the upper voltage magnitude limits to the  $\lambda_{CPi}$ 

 $\Lambda_{PL}$ : matrix which contains in an arbitrary line i the contribution of the line power flow limits to the  $\lambda_{CPi}$ 

 $\Lambda_{QV\, {
m min}}$ : matrix which contains in an arbitrary line i the contribution of the lower voltage magnitude limits to the  $\lambda_{CQi}$ 

 $\Lambda_{QV\, {
m max}}$ : matrix which contains in an arbitrary line i the contribution of the upper voltage magnitude limits to the  $\lambda_{COi}$ 

 $\Lambda_{QL}$ : matrix which contains in an arbitrary line i the contribution of the line power flow limits to the  $\lambda_{COi}$ 

So, each line i from the first m lines of matrix  $\Lambda$  comprises the system constraints' contribution to the nodal price congestion component for active power of bus i.

Moreover, each m+i line includes the influence of system constraints on the congestion component for reactive power at bus i.

Let the constraints at columns a, b and c of  $\Lambda$  be the upper and lower voltage limit at bus s and the power flow limit on line j, respectively. Then it is easy to find out from (2.28) that:

$$\lambda_{VCPi.s} = \lambda_{i.a} + \lambda_{i.b} \tag{2.29a}$$

$$\lambda_{LCPi,j} = \lambda_{i,c} \tag{2.29b}$$

where

 $\lambda_{VCPi.s}$ : part of congestion component for active power nodal price at bus

i due to voltage magnitude constraints at bus s

 $\lambda_{LCPi,j}$ : part of congestion component for active power nodal price at bus

i due to power flow constraint on line j

 $\lambda_{i,a}, \lambda_{i,b}, \lambda_{i,c}$ : elements of the *i* row of matrix  $\Lambda$ 

From (2.29a) and (2.29b), it is obvious that matrix  $\Lambda$  enables the estimation of system constraints' contribution to nodal price congestion component. Thus, the aim of allocating the congestion component to each single system congestion is straight forward.

# 2.6 Congestion management methods

One of the most challenging tasks in the deregulated electricity market is the management of congestion situations. There are three basic reasons that justify this fact. The first is the existence of numerous participants in the marketplace. Suppliers, consumers and the system operator have sometimes conflicting interests and the variety of their aims results in a complexity by the market operation. This complexity is further aggravated in the event of scarce resources, in other words in case of system

congestions. The second reason is based on the electricity physics. The power flows according to Kirchhoff's laws through all the available paths. Thus, the power of any transaction agent or market participant cannot be restricted to flow only over certain lines. Modern devices such as FACTS permit a better control of power flow. However, the phenomenon of power flow remains a not fully controllable issue. Last but not least it should be mentioned the inability to store electrical energy in significant quantities. Batteries' capacity is neglected in comparison to the power system size. The most important storage method to date is the use of pump storage hydroelectric power plants. Water is pumped up to a reservoir during the off-peak period. Then, this quantity of water is used at peak time to produce power. In spite of their importance, the pump storage power plants cannot solve the problem of electrical energy storage. New technology in the field of materials enables the manufacturing of huge capacitors where electrical energy can be stored in form of electromagnetic field in quantities up to 1 GWh. However, the economic efficient use of such technologies, in order to solve the storage problem, remains a future scenario.

The above mentioned reasons highlight the importance of an efficient congestion management. Different methods, depending on the electricity market structure, have been implemented so as to deal with this task.

In a bilateral market the common practice is the capacity auctions. The system operator periodically auctions the capacity, partly or fully, over certain lines. These are the lines, which are usually congested. The capacity auction can be considered as a purchase of physical rights to transfer power over certain path. The interesting market participants offer their bids in the same way as for each other commodity. The cross border lines connecting Germany and the Netherlands is a typical example of capacity auctions in a bilateral market. The capacity of submarine high voltage direct current wire which links Italy to Greece is another case where the power transfer rights are allocated by auctions. It should be underlined that the capacity auctions concern each direction in a separate way. That is, the right to trade power over the certain line to one direction is different than the right to transfer power to the other direction. Thus, separate auctions take place for the capacity at each direction. In the English market capacity auctions are also a common practice [52].

The congestion management in pool market may be more detailed and efficient. The reason is that there is available economic information about the scarce resources. This information is obtained as a by-product of optimal power flow mechanism. The congestion management methods of pool market may also be applied to a mixed pool/bilateral market. The economic information is in this case available through the pool market and can be connected to the bilateral transactions. Actually, the combined electricity markets are the most ideal case of deregulated power market. These markets offer to their participants the flexibility to choose between firm contracts and the purchase of power at the spot market. Such markets are seen as the final state of power sector deregulation. Therefore, it is of important to see how the congestion management takes place in these markets.

When the pool market operates without nodal pricing the methods which are usually used are the zonal pricing and the countertrade. These methods will be analysed in detail in the next Chapter in the context of Nord Pool.

In the pool markets that are based on locational marginal pricing (LMP) the congestion management is associated with the congestion component of nodal prices. So, it becomes now obvious the importance of nodal prices analysis which has been done before. Some works on congestion management with OPF implementation can be found in [53, 54]. Because of nodal price variation, it has been proposed the introduction of Financial Transmission Rights (FTR) that helps consumers and producers to hedge the risk caused by the price variation [55]. The FTRs are contracts that give the right to their holders to transfer power from point-to-point without taking into consideration the system performance. Hence, the possible existence of congestion does not affect the price that the power is delivered. The holder of such a contract pays only for the production and the losses. What really happens is that the contract holder pays for the delivered power the full nodal price, which is calculated for his location. Then, he receives a compensation, for the contracted amount of power, from system operator equal to the difference between the nodal price at the injection and take-over bus. This is valid when the holder is a transaction agent. If the holder is a single consumer then he receives the difference between the spot price at his bus and the reference bus. As mentioned above, the nodal price at reference bus is said to have only the component

due to generation and losses. These two cases of FTRs implementation are mathematically described as follows:

$$CP_i = P_i * (\lambda_i - \lambda_r) \tag{2.30a}$$

$$CP_i = P_i * (\lambda_1 - \lambda_2) \tag{2.30b}$$

where

 $CP_i$ : compensation payment to market participant i

 $P_i$ : contracted power associated with the market participant i

 $\lambda_i$ : nodal price at bus of i

 $\lambda_1, \lambda_2$ : nodal price at injection and takeoff bus respectively

The former, i.e. (2.30a), is related with a single consumer while the latter corresponds to a power transaction in the context of a bilateral/pool market. The FTRs are considered as a very useful tool in order to achieve an optimal market performance. First of all, they give the chance to market participants to protect themselves from the nodal price variations. Thus, the FTRs operate as a risk hedging tool. The second major benefit of FTRs is that they can stimulate the investment in transmission facilities. The system operator may assign FTRs to investors. The market participants are looking for purchase the FTRs, which are now a property of investors, and protect themselves from system congestions. When the price of FTRs becomes higher than the cost of build-up the new facilities the investors start to realise their investment plans. For these two reasons the FTRs are so valuable. Such financial contracts are already implemented in Australia, the markets of PJM and New England in USA and there is an ongoing process aiming to the introduction of FTRs in the market of New Zealand.

The Flowgate Rights (FGR) make up an alternative to the FTRs [56]. In a market, which has adopted the FGRs, the transactions are charged for the use of the congested lines. The charge is based on the usage of the congested line from the side of each transaction, measured by the Power Transfer Distribution Factors, and the shadow price

of the congested line, i.e. the Lagrange multiplier  $\mu$ . If a transaction agent wants to avoid this charge then the agent has to purchase FGRs for the congested lines.

Another approach of calculating the congestions charges allocated to the different market parties is presented in [57]. In this case the impact of congestion to nodal prices is calculated through an Aumann-Shapley procedure. The benefit of this method is that the total congestion charge equals the cost increase caused by the congestion.

The congestion management is, generally, a complicated and highly demanding task. The role of system operator is very important towards systemwide efficiency. However, the ISO is not able to solve all the problems that appear because of congestions [58]. Proper design of electricity market is a prerequisite for a harmonious market performance that cannot be replaced by any ISO's action.

## 2.7 Congestion component allocation

As it has already been seen, the nodal price congestion component is of great significance considering the congestion management. In the framework of the present work, a novel method which links the congestion component to the players of electricity marketplace has been developed. The method's basis is a combined use of congestion component analysis and distribution factors.

## 2.7.1 Power flow tracing in terms of distribution factors

#### 2.7.1.1 On power flow distribution factors

The concept behind the use of distribution factors is to find out how a particular generator or load influences the power flow over particular network lines. Until some years ago the interest of tracing the electricity was limited because the vertically integrated utilities had little interest to find their own generators impact on their own lines. Therefore, the scientific work on this field has not been so extended.

The Generation Shift Distribution Factors (GSDF) had been used in 1970s in order to deal with the task of power flow tracing. Those factors give the change of power flow

over a particular line after a generation shift. At that time, when the load flow calculation time was still an essential issue, the GSDFs were a good alternative to the complete power flow calculation.

In 1981 the Generalized Generation Distribution Factors (GGDF) have been proposed in [59]. Those factors were an improvement in comparison to GSDFs because their results were not dependent on a reference bus. Moreover, the total generation should not remain constant, which was a prerequisite in case of GSDFs. The basis of GGDFs is the equations of a DC load flow. So, their use is limited to the active power flow. The GGDFs show the impact of a particular generator on the power flow over a particular line. This impact may also be negative.

Recently, the need of power flow tracing has been increased. The reason is the deregulated environment of modern power markets with the unbundled services and the numerous participants. The detailed calculation of the different parties' impact on the network situation has lead to the development of a variety of distribution factors. In [60] the Topological Generation Distribution Factors (TGDF) have been presented. The TGDFs calculate the share of a particular generator on the power flow over any line. Their basis is a lossesless power network and consequently, they do not take into consideration the role of reactive power.

Another category of well-known factors are the Power Transfer Distribution Factors (PTDF) [61]. The PTDFs are calculated using a DC load flow. The difference to the previous factors is that the PTDFs determine the part of a power transaction that flow over a particular line. For a given network topology it is easy to calculate a matrix containing the PTDFs for all the possible power transactions. This matrix can be determined on the basis of a hypothetical 1 MW transaction. Then, the impact of each real transaction on the power flow over a line may be found by multiplying the transaction's PTDF corresponding to this line with the transaction's power amount.

#### 2.7.1.2 The Nodal Generation Distribution Factors

The main drawback of the above mentioned methods is that they use a DC approximation and so the calculated factors neglect the impact of reactive in power losses. In the context of the present work, the aim was to achieve an approach to system

situation with possible high acriby. Therefore, the factors that have been used to trace the power flow are the Nodal Generation Distribution Factors (NGDF) [62].

Basis for the calculation of the NGDFs is the proportional sharing principle. The principle is illustrated in Figure 2.4.

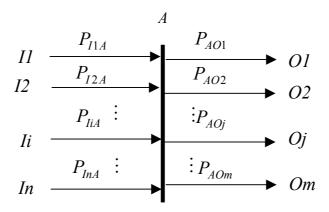


Figure 2.4 Proportional sharing principle

Since electricity cannot be distinguished, the principle suggests that the share of a particular line IiA, which supplies bus A, in power flow over a particular line AOj which is supplied from bus A, is equal to the share of line IiA in the sum of power inflows into the bus A:

$$\frac{P_{IiA,AOj}}{P_{AOj}} = \frac{P_{IiA}}{\sum_{r \in \Psi_A} P_{IrA}}$$
 (2.31)

where

 $\Psi_A$ : set of lines supplying bus A

 $P_{IiA}$ ,  $P_{AOj}$ : active power flows over the lines IiA and AOj

 $P_{IiA,AOj}$ : share of line IiA in the power flow over the line AOj

In (2.31) the power inflows may be replaced from the total contribution of the system generators to feeding of bus A. Through this replacement one can obtain from the right side of (2.31) the nodal generation distribution factor  $L_{A,k}$  of a particular generator k.

$$L_{A,k} = \frac{P_{A,k}}{\sum_{s=1}^{n_G} P_{A,s}}$$
 (2.32)

where

 $P_{A,k}$ : contribution of generator k to the active power flow feeding bus A

 $n_G$ : number of system generators

Using the NGDFs it is possible to calculate the share  $P_{AOj,k}$  of generator k in power flow over a certain line AOj, which is supplied by bus A:

$$P_{AOi\ k} = L_{A\ k} P_{AOi} \tag{2.33}$$

For the computation of  $P_{A,k}$  a direction search algorithm is used. This algorithm searches for power flow directions given the results of an AC power flow. Thus, the method takes into consideration the reactive power too. Distribution factors tracing the flow of both active and reactive power may be determined. Therefore, it is possible to generalize (2.32). Instead of P the symbol M can be used. Now, M represents either the active or the reactive power. So, one may calculate  $L_{A,k}$  for both active and reactive power.

Now the task is to determine the quantity  $M_{A,k}$ , which shows the impact of generator k on the power inflows, active or reactive power, at bus A. The power inflows and the generation at bus A have to be taken into account. That is:

$$M_{A,k} = \begin{cases} -\sum_{iA \in \Psi_A} L_{i,k} M_{iA} + M_A & \text{if } k = A \\ -\sum_{iA \in \Psi_A} L_{i,k} M_{iA} & \text{if } k \neq A \end{cases}$$
 (2.34)

where

 $\boldsymbol{M}_{A,k}$ : the share of generator located at bus k to power inflows at bus A

 $M_A$ : active or reactive generation at bus A

 $M_{iA}$ : active or reactive power flow over line iA

 $L_{i,k}$ : NGDF of generator k for the bus i which is the start point of line iA

As can be seen from (2.34), the order of calculations is important. First of all, the NGDFs of all the lines, which supply the bus A, have to be found. Therefore, the calculation of NGDFs must start from the source buses. A source bus is characterized as a bus, which is not supplied by any line. For the source buses the set  $\Psi_A$  is equal to the empty set. Of course, the sources buses are a subset of generator buses. In Figure 2.5 the search algorithm is illustrated.

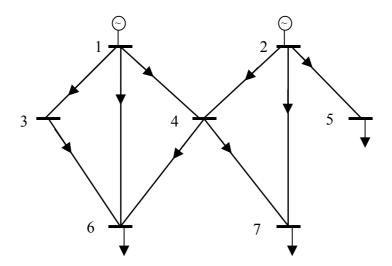


Figure 2.5 Search procedure for NGDFs' calculation

The buses 1 and 2 are not supplied by any line so the calculation of NGDFs begins from these two buses. For these two buses the quantity  $M_{A,k}$  is equal to the bus generation if k=A and zero at any other case. Consequently, from (2.32) it is obvious that the  $L_{1,1}=1$  and zero for all the other buses. Similarly,  $L_{2,2}=1$  and zero for any other bus. Thus, the NGDFs at the beginning of lines 1-3, 1-4, 2-4 and 2-5 are known and so the quantities  $M_{A,k}$  can now be calculated for the buses 3, 4 and 5. The procedure continues until the NGDFs are calculated for each system bus.

An extension of the Nodal Generation Distribution Factors has been realised in the context of this work. Distribution factors for loads have been calculated too. In this case the source buses of the search algorithm are those that are only supplied. These source buses neither have generation nor supply any line. For the example of Figure 2.5 the source buses are the buses 5, 6 and 7. Thus, the calculation of Nodal Load Distribution Factors (NLDF) is realised. The NLDFs facilitate the calculation of load share in the power flow over a particular line in the same way as the NGDFs.

## 2.7.2 Connecting congestion component and market players

It is of great importance in the modern deregulated electricity markets the congestion pricing to be a competitive pricing. This is necessary in order to establish a proper market operation. If the system operator wants to achieve this aim then the operator has to provide the participants in marketplace with a transparent pricing. Moreover, the ISO has to send to suppliers and consumers, through the congestion pricing, the right economical signals [31]. These signals concern the locational advantages for investment in generation and demand. Furthermore, the signals should indicate the need for investment in new transmission facilities.

In this work, a novel method for the connection of congestion component and market participants has been proposed. This connection aims to make more transparent the congestion situation. The second objective is to allocate the congestion components to the congestion causers. The power system restrictions, such as voltage and power flow limits, may become active in order to serve the producers' and consumers' patterns. Consequently, a connection between the market participants and the nodal price

congestion component has to be determined. Investigating the impact of system congestions on the nodal price for active power the role of voltage restrictions turned out to be insignificant in most of the cases. Also in [63, 64], by studying the influence of the congestion on the nodal prices, only power flow limits are taken into consideration. A comparison between the two components shows that the line power flow congestions have an influence on the nodal prices, which is from 10 to 1000 times higher than the voltage congestions' influence. Of course, there are exceptions but those cases rather confirm the rule that voltage congestions' influence can be neglected. Consequently, for a bus *i* the part of congestion component caused by active line constraints can be considered equal to the whole congestion component, i.e.:

$$\lambda_{LCPi} \cong \lambda_{CPi} \tag{2.35}$$

The NGD-Factors can now be used for achieving the allocation of nodal price congestion component caused by power flow constraints to particular generators. As it has been shown, these factors estimate the share of each generator in the power flow on a particular line. Assume that all suppliers are treated equally concerning the line congestions. That is, the share of a supplier in power flow over a congested line mirrored to linearly to the supplier's share in congestion creation. In this case the NGDFs can be used for calculation of generators' participation in the congestion nodal price component. From the previous congestion component analysis, the impact of each single congestion on the nodal price congestion component of each bus has been calculated. So, this impact incorporated with the NGDFs enables the computation of the total share of a generator k to the congestion component of nodal price at bus i:

$$\lambda_{LCPi,k} = \sum_{jm=1}^{l_C} \lambda_{LCPi,jm} L_{j,k}$$
 (2.36)

where

 $\lambda_{LCPi,k}$ : part of congestion component due to line congestions at bus i which is allocated to generator k.

 $l_C$ : number of system congested lines

Summing the share of all generators in the congestion component of bus i one can obtain the whole congestion component of this bus:

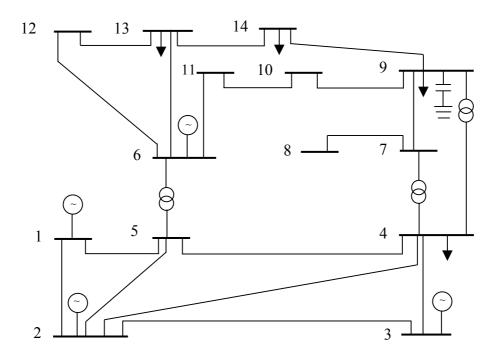
$$\lambda_{LCPi} = \sum_{k=1}^{n_G} \lambda_{LCPi,k}$$
 (2.37)

The conclusion drawn by (2.37) is that the above presented method results in an *ex post* reallocation of the nodal price congestion components to suppliers. The sense of this reallocation is the connection, through the NGDF, between the congestion costs and their origins. The  $\lambda_{LCPi,k}$  may be treated, in the most cases, as the total congestion component of bus i.

The same process can be used in order to allocate the congestion component to the set of consumers. The only difference is that in this case the NLDFs should be used instead the NGDFs.

#### 2.7.3 Case studies

In order to highlight the usefulness of the proposed method a pool market case has been investigated. The calculations have been carried out by a modified version of MATPOWER program [65]. The pool market network is the IEEE 14-bus system, which is shown in Figure 2.6. In this case study, the system comprises four suppliers and four consumers. The network data as well as the participants' economic offers can be found in Appendix A. Transfer capability limits on lines 2-4, 4-5 and 6-13 have been reduced to 50%, 80% and 50% of their original value, which are given in Table A.1, respectively. In the basic case the power flow on lines 2-4, 4-5 and 6-13 has reached the transfer limit. The upper voltage limit reached at buses 3 and 12. The congestion component analysis shows that the voltage congestions' impact on the nodal prices is insignificant. Figure 2.7 illustrates the different size of the two congestion component parts.



**Figure 2.6** The IEEE 14-bus system

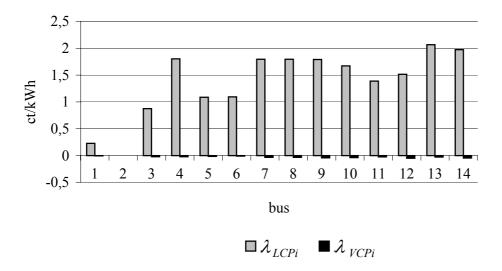


Figure 2.7 Share of line and voltage congestions in congestion components of nodal prices

Thus, the essential task is the estimation of active line power flow constraints' influence on the nodal prices. The share in the congestion component of nodal prices, each of the three active line power flow constraints, is presented in Figure 2.8. All the lines are not congested with the same intensity. The Lagrange multiplier  $\mu$ , of the corresponding

line power transfer constraint, is a measure for the congestion intensity. For the lines 2-4, 4-5 and 6-13 the multiplier  $\mu$  is 4.253, 0.269 and 1.179, respectively. The variation of  $\mu$  is the reason for the different impact, which has each congested line on the congestion component of nodal price.

Furthermore, a congested line does not affect in the same way the nodal price at all the buses as in Figure 2.8 shown. From (2.36), it is obvious that for the calculation of generator impact on nodal price congestion component the NGDFs can be used. These factors are listed in Table 2.1, in the left columns under the generator name for the basic case. The columns in Table 2.1 with the caption 'shifted bid' refer to the case where a market participant has changed his bid prices. This case will be analysed below. The impact of each generator of the nodal price of each bus is presented in Figure 2.9. It is evident that the generators have different impact on congestion component and in different way at each system bus.

The generator's share in congestion component of nodal prices (here weighted by the power of each bus and given as percentage of the sum of all generators) is not related in a proportional way with the generation contribution to active power production. As Figure 2.10 shows, the generator 2 has a high share in congestion component despite the fact that it produces only 12% of the total active power. The explanation is that it has also a high share in active power flow over line 2-4 which is congested in more intensive degree than the other two congested lines. Thus, a congestion pricing method should not be based only on power production of generators.

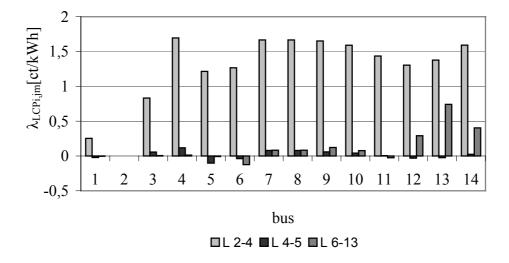


Figure 2.8 Share of lines in congestion component of nodal prices

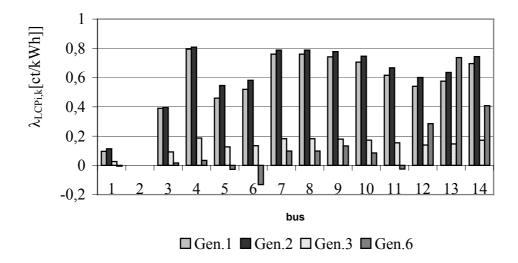
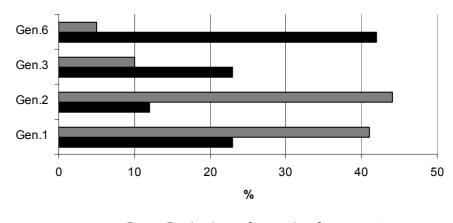


Figure 2.9 Allocation of congestion component of nodal prices to generators

Table 2.1 Lagrange multiplier  $\mu$  and NGDFs in IEEE 14-bus system

	μ		NGDF Gen.1		NGDF Gen.2		NGDF Gen.3		NGDF Gen.6	
Line	Basic	Shifted	Basic	Shifted	Basic	Shifted	Basic	Shifted	Basic	Shifted
	case	bid	case	bid	case	bid	case	bid	case	bid
2-4	4.253	2.982	0.43	0.43	0.46	0.04	0.11	0.53	0.00	0.00
4-5	0.269	0.136	0.59	0.60	0.18	0.01	0.04	0.21	0.19	0.18
6-13	1.179	1.194	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
3-4	0.000	2.537	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00

This method for allocating the congestion component to the generators may be used by the pool operator in order to apply an efficient and fair congestion pricing. A possible application could be the incorporation of the proposed method with the Financial Transmission Rights. Assume that the system operator has to pay a holder of FTR contract. Applying the proposed method the system operator may find out who caused the congestion components that the operator has to pay for. It is not sure that the system operator will then shift the FTR cost to the congestion causers. But in any case the ISO may have a detailed knowledge of the system's economic situation. Except the pure pool markets the above presented allocation can also be used in case of third party access (TPA) market as the next section shows.



 $\blacksquare$  Power Production  $\blacksquare$  Congestion Component

Figure 2.10 Share of generators in active power production and congestion component of nodal prices

# 2.8 Market participants' behaviour

Since the electricity market has been deregulated the participants have a variety of choices to improve their standing in the market. A set of different bidding strategies may be adopted by the participants in order to maximize their profit [66]. The consequences that such strategies have on the network operation are the subject of the following analysis.

#### 2.8.1 Pool model

In a pool market, the main strategy that the players have towards the individual profit maximization is the change of their bid patterns. For the pool operator, it is very interesting to have knowledge of the consequences on the system situation because of the bid pattern changes.

Such a case has been studied in the context of pool market, located in the IEEE 14-bus system, which has been presented in the previous section. The supplier who owns the generator at bus 3 aims to increase its share in power production. For this reason the supplier submit to the pool operator a bid pattern with reduced bid prices. After this bid change a new network situation emerges. In this new situation the power flow on line 3-

4 has reached the transfer limit while all the three congestions of basic case remain too. The Lagrange multiplier  $\mu$ , corresponding to the new situation, is given in Table 2.1 in the right columns, under the caption 'shifted bid'. As this index shows, the congestion intensity on lines 2-4, 4-5 is decreased while it is slightly increased on line 6-13.

In Table 2.1, the NGDFs of all generators, for the case that emerged by the bid change, are listed in the right columns. As can be observed, the NGDFs of generator 3 on all the lines, where the congestion situation is now different, have been either increased or remained at the same level. Generator 3 has not reduced NGDFs over any of these four lines. That is, the bid price reduction of supplier owning generator 3 is associated with the changes on the status of congested lines. During this research work, a number of various systems has been investigated. The results obtained at all of these systems have shown that only in event of zero NGDF, before and after the bid reduction, the corresponding producer has not increased share in the congestions. It is indifferent if the congestions in the new situation are relaxed or have been become more intensive. What has been observed is that the suppliers who have reduced their bid price have increased share in the power flow over those lines. These remarks have been confirmed not only for pure pool operation model.

## 2.8.2 Third party access

The case of direct purchase, within a combined pool/bilateral market, can also be considered as a bid reduction of the corresponding supplier. For the part of power that corresponds to the dimerous trade the supplier gives a very low bid price to the system operator. Consequently, the power resource associated with this bid will be surely used by the OPF program. On the other hand the consumer who participates at the direct purchase submits an offer, for the same amount of power, with a very high bid price. Thus, the full consumer's demand will be surely covered, if the system situation does not prevent the full cover, by the OPF program. Thus, in this way a direct purchase is incorporated in the OPF. Two Third Party Access (TPA) cases have been realised in the IEEE 14-bus system. The results of these two cases are illustrated in Table 2.2.

NGDF Gen.2 NGDF Gen.6 TPA **TPA TPA Basic** Basic Basic **TPA** Line  $2\rightarrow 9$  $6\rightarrow9$  $2\rightarrow 9$  $6\rightarrow 9$ case case case 2-4 4.253 5.213 2.478 0.86 0.00 0.00 0.46 4-5 0.269 0.138 1.867 0.18 0.33 0.19 0.28 6-13 1.179 1.183 2.422 0.00 0.00 1.00 1.00

Table 2.2 Lagrange multiplier  $\mu$  and NGDFs for the TPA cases

In the first case the consumer at bus 9 purchases 70 MW directly from producer at node 2. The realisation of this purchase results in no new congestion. As the multiplier  $\mu$  shows, two of the existing congestions are now stronger while the third one has reduced its intensity. For the generator at bus 2, the NGDFs on congested lines, after the direct selling, have been either increased or remained unchanged at zero.

The second TPA case that has been realised considers a direct purchase of 70 MW. The buyer is located at bus 9 and the seller at bus 6. In this case the congestions on lines 4-5 and 6-13 are now stronger while the line 2-4 is less intensive congested and no new congestion appears. The NGDFs of generator at bus 6, concerning these three lines, either remain unchanged or they are increased. This fact confirms that the above mentioned remarks are valid also in a combined pool/bilateral market.

As can be observed from Table 2.3, the objective function of OPF is reduced in both TPA cases. Nevertheless, applying the congestion component allocation method it is evident that the share of the corresponding producer in congestion component is, in both cases, increased. In this case the share has been computed in absolute values rather than in percentage as in Fig. 2.10.

Table 2.3
Generators' share in congestion component for the TPA cases

2002	Objective	Share	Share	
case	function [€/h]	of Gen. 2 [€/h]	of Gen. 6 [€/h]	
Basic	19396	3296	344	
TPA 2→9	17913	7451	-	
TPA 6→9	17204	-	398	

If the pool operator applies a wheeling pricing regarding only the nodal price difference then the producers, who participates at the TPA, may not be charged for it or they even may get a credit. In case that the pool operator regards the congestion situation, and especially the transmission congestion contracts, that have to be paid, then the producers may be charged though the reduction of nodal prices.

#### 2.8.3 Producers' behaviour in a realistic network

The same investigation, as of IEEE 14-bus system, has been carried out on a realistic high voltage network. This system, which is illustrated in Figure 2.11, is part of the European network and comprises more than 400 transmission lines that operate at voltage levels of 380 kV, 220 kV and 110 kV. In the presented investigations, 55 suppliers and 20 consumers have been located in the network area. The congestions presented here have been created artificially and they do not match the real network situation.

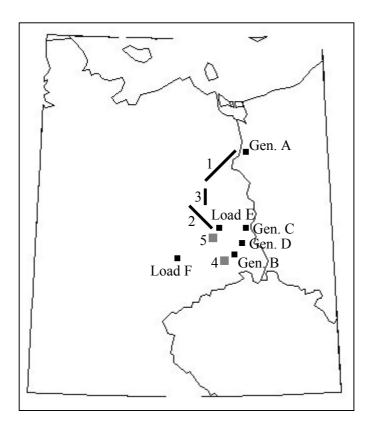


Figure 2.11 Illustration of the investigated large network

In the initial situation, the lines 1 and 2 are congested as can be seen, by multiplier  $\mu$ , in Table 2.4. The NGDFs of four selected suppliers, who own four generators, are also listed in Table 2.4. For the basic case the NGDFs are given in the left columns.

In order to increase its market share, the producer who owns generator A shifts down the bid price. This bid shift results in a new system situation, which is shown in Table 2.4, in the right columns. In the new network situation, the lines 1 and 2 are congested in a more intensive way while no new congestions appear.

**Table 2.4**NGDFs in a realistic high voltage network

	μ		NGDF		NGDF		NGDF		NGDF	
			Gen. A		Gen. B		Gen. C		Gen. D	
Line	Basic	Shifted	Basic	Shifted	Basic	Shifted	Basic	Shifted	Basic	Shifted
	case	bid	case	bid	case	bid	case	bid	case	bid
1	0.482	0.511	0.44	0.56	0.00	0.00	0.00	0.00	0.00	0.00
2	9.617	9.866	0.00	0.00	0.27	0.27	0.22	0.23	0.38	0.38

Table 2.4 shows that the share of generator A in power flow over line 1 is increased. On the other hand, the share of this generator in power flow on line 2 remains at zero, as it was before the bid change. This fact confirms the remarks that have been made in case of IEEE 14-bus system. In this part of European network, there is a large demand between line 1 and line 2. Therefore, the producer who owns the generator A cannot impact the power flow on line 2. The power flow on that line is affected by the production of suppliers who own the generators B, C and D.

#### 2.8.4 Consumers' behaviour in a realistic network

In the deregulated pool electricity markets not only the producers but also the consumers can influence the network situation by changing their bid patterns. An interesting case with negative nodal prices has been emerged in the part of European network, which has been presented in the previous section. That case is resulted through a pandemic increase of demand. The nodal price at buses E and F is now negative and equal to -1.72 ct/kWh and -0.33 ct/kWh, respectively.

According to the spot pricing theory of electricity a negative nodal price is an economical signal for either increase of demand or decrease of production at the corresponding bus. This change should lead to a lower value of the total costs, which is the objective function in the pool market. Neither at bus E nor at bus F is located any load. In order to decrease the objective function, a consumer offers a bid for purchasing energy at bus E. Increasing the bid price, the consumer receives more energy. Table 2.5 shows that the total costs are indeed reduced by increasing the demand at bus E. Except the economical aspect also the system situation should be investigated in this case. For this purpose the NLDFs can been used. Table 2.6 shows that after the location of 200 MW load at bus E there are two congested lines.

**Table 2.5**Reduction of objective function

	Load at bus E [MW]				
	0	200	700		
Objective function [€/h]	477260	473360	465690		
Nodal price at bus E [ct/kWh]	-1.72	-1.79	7.40		

**Table 2.6**System situation and NLDFs of consumer at bus E

		μ		NLDFs of consumer at bus E			
Load at bus E [MW]	0	200	700	0	200	700	
Branch 1	1.470	1.591	1.431	0.00	0.00	0.00	
Branch 2	19.242	19.194	18.724	0.00	0.00	0.00	
Branch 3	0	0	0.138	0.00	0.00	0.00	
Branch 4 (trans.)	0	0	0.016	0.00	0.00	0.03	
Branch 5 (trans.)	0	0	6.033	0.00	1.00	1.00	

The power transfer restrictions at those lines were binding at the basic case too. When the consumer shifts up the bid price and the load increases to 700 MW then 3 new congestions appear. The NLDFs of consumer at bus E, for all the five branches, either

remain unchanged or they are increased. This remark has been observed in all the investigated cases with consumer bid increase and is comparable to the remarks, which have been noticed by producer bid decrease. Consequently, the bid change, which leads to increase of produced or purchased power has the same kind of influences on the network situation.

# Chapter 3

# Operation mode assessment of a

## real market

#### 3.1 The Nord Pool

The pool model has not been realized in the same way in the numerous countries that have already deregulated their power sector. The mathematical background of the optimisation, which is the basis of the pool model, is implemented using different options. Thus, there are marketplaces where locational marginal prices are used while the operation mode of other markets is consistent with a global market clearing price. In addition, the choice between DC and AC power flow leads to quite different market performance. In the framework of this research work, the operation mode of a real market, the Norwegian power market, has been investigated.

The Norwegian power market is part of a larger international deregulated marketplace, the well-known Nord Pool. After the eastern part of Denmark joined Nord Pool in 2000, the common marketplace comprises all four Scandinavian countries. The Nordic marketplace has five system operators and a pool operator whose responsibility is the

operation of the Pool Exchange. The system operator in Norway is the company Statnett. In Sweden the grid is operated by Kraftnät and in Finland by the company Fingrid. In Eastern Denmark the company Elkraft has the responsibility for the transmission system whereas in Western Denmark the system operator is the company Eltra. These five regional system operators have cooperation in the framework of the Nordel organization, which is established as early as in 1960s in order to support the power trade between the Scandinavian countries. The Nordic countries, as well as the system operators' territories, are shown in Figure 3.1. Another characteristic of Nord Pool is that the electricity industry of each country is subject to a separate national regulatory authority.

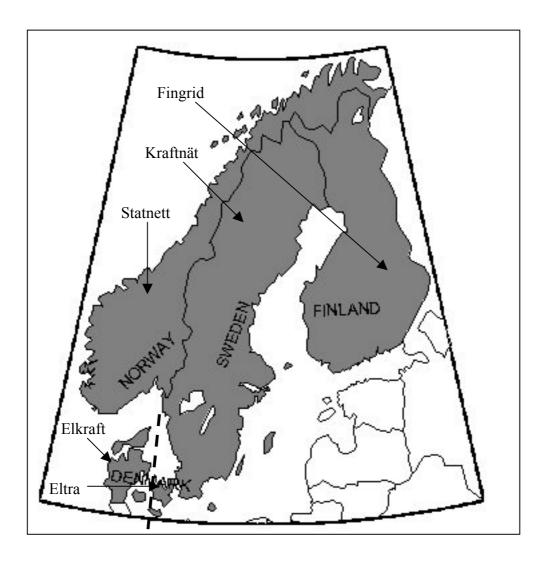


Figure 3.1 The Nord Pool market

The electrical energy is produced by different resources in each of the four countries. Almost 100% of energy in Norway is produced by hydroelectric power plants while the thermal power plants account to the great majority of the energy which is produced in Denmark. In addition, Finland and Sweden have nuclear plants, the latter with an essentially higher capacity in nuclear power generation. The total electrical energy consumption in year 2001 was about 393 TWh. The capacity of the diverse generation resources in Nord Pool countries is given in Table 3.1.

Table 3.1
Generation resources in Nordic countries, 2000

	Norway	Sweden	Finland	Denmark
Hydro+Wind	27.5 GW	16.4 GW	2.9 GW	2.4 GW
Nuclear	-	9.4 GW	2.6 GW	-
Thermal	0.3 GW	5.0 GW	11.0 GW	9.5 GW
Total	27.8 GW	30.8 GW	16.5 GW	11.9 GW

The 400 kV lines form the backbone of transmission network in Nordic countries. The secondary high voltage level varies between 132 kV in Eastern Denmark and 300 kV in Norway. Furthermore, the DC interconnections are of great importance for the network performance in Nordic region. There are High Voltage Direct Current (HVDC) cables between Sweden and Finland and also between Sweden and the European mainland. Other HVDC cables link Sweden to Denmark as well as Norway to Denmark. Additionally, the building of DC interconnections linking Norway to the Netherlands and England has been planned. However, the plan of building the cable to the Netherlands has major difficulties to be completed. On the other hand, the interconnection to England is expected to be in operation in 2006.

The common element among all the Nordic countries is the Pool Exchange, also known as Nord Pool [67]. The shareholders of Nord Pool are the companies Statnett and Kraftnät. Each of these two companies owns 50% of the shares in Nord Pool. Market players bid their offers in a day-ahead basis. The pool operator clears, in the afternoon, the market of the next day. Power unbalances, resulting by differences between schedule and real dispatch, are treated within the regulating real time market. In

addition, there is a future market where contracts are traded in a weekly basis. The 30% of total consumption is traded in the Pool Exchange while the 70% is based on bilateral transactions.

Transmission pricing issues as well as congestion management methods are subject to the five network operators. This fact yields in a variety of different pricing schemes. In event of congestion two major methods are used. These methods are the zonal pricing in Norway and the countertrade in the other countries. These two mechanisms will be analytically presented in the next section.

The most actual topic of public discussions in Nordic countries is the price rise during dry years. Because the hydroelectric power plants make up a large part of installed generation capacity, a dry season affects explicitly the prices. The winter 2002/2003 was such a period. The prices were essentially increased. Nevertheless, the market exhibited a pretty good performance. The reason is that, since the end consumers price are changed periodically, the end consumers faced this price increase and so they declined their demand. An analysis of data contained in [71] points that in Norway the consumption during the period 01.11.2002-28.02.2003 was 46.125 TWh. The electric energy which had been consumed in the period 01.11.2001-28.02.2002 was 47.437 TWh. Furthermore, the peak demand in Norway, measured in hourly basis, in winter 2002/2003 was 19.9 GWh while the maximal demand, for the same period, one year before was 20.7 GWh. If these differences do not seem to be large, one should also take into consideration that the power sectors in industrialized countries, usually, have to serve every year an increasing demand. Another fact, which has facilitated the cover of demand during the dry season, was the dispatching of mothballed thermal power plants.

For the near future, new interconnections to the European mainland as well as the increase of installed capacity are the two basic tools that the Nordic countries are going to use so as to cope with the dry season problem.

#### 3.2 The Norwegian market operation mode

#### 3.2.1 Loss factors

The Norwegian players, as every member of Nord Pool, are eligible to participate in the spot market. The procedure is common for all participants, no matter the country where they are located. However, the different transmission and power loss pricing methods, that the national system operators apply, influence the participants' offers.

The Norwegian high voltage network consists of 166 nodes. Periodically, the company Statnett, which is the Norwegian system operator, announces loss factors for each system node. Usually, the time interval, when these factors are valid, is 6-10 weeks. The objective of these factors is to cover the expenditure, which is associated with the power losses. The factors are given as percentage of the energy price at each system node. Each participant has to pay, for each traded MWh, to the Statnett an amount equal to the loss factor of the node where the participant is located.

Of course, this fact will affect the bids that the players submit to Pool Exchange. Generally, the pool operator accepts bids in two forms. Either the stepwise bids illustrated in Figure 2.2 or bids in form of linear functions which is presented in Figure 3.2.

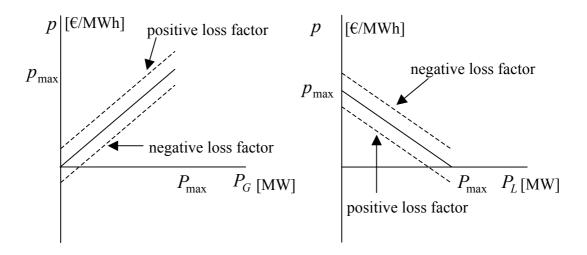


Figure 3.2 Continuous bid curves of supplier (left) and consumer (right) considering loss factor

The left side of Figure 3.2 shows the bid curve of a supplier. The original bid curve of this supplier is given by the solid line. As can be seen, the supplier has the willingness to inject power into the network up to its  $P_{\rm max}$  when the price is  $p_{\rm max}$ . Let hypothesize, that the system operator assigns a positive loss factor for the node of this supplier. The supplier will shift the resulting costs to its bid. So, the bid curve, which

will be given to the system operator by that supplier, will be the upper dashed line. That is, for the same amount of power the supplier demands a higher price in order to cover the loss charge. It is also possible that the loss factor has a negative value. In this case, the supplier will get a credit for each injected MWh equal to the loss factor. So, the player may reduce its bid price so as to be more competitive. The resulting bid curve is illustrated by the lower dashed line.

In Figure 3.2, the right side presents the bid curve of a consumer. If there were no loss factors, the consumer would purchase energy only if the price would be lower than a certain value  $p_{\rm max}$ . In case of positive loss factor, the consumer will offer less money for the same amount of power. Hence, its bid curve will be the lower dashed line. On the contrary, if the loss factor is negative, that is the consumer is granted an amount, then the bid curve will be represented by the upper dashed line. Now, the consumer is willing to pay more for the same amount of power. In any case, for both suppliers and consumers, the shift of the original bid curve along the y-axis is equal to the loss factor.

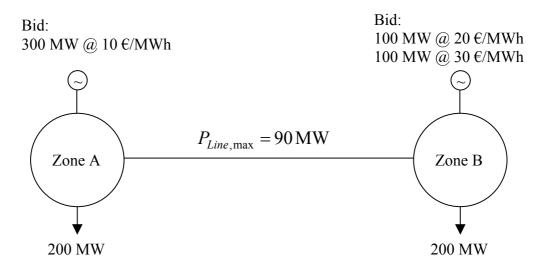
#### 3.2.2 Zonal pricing and countertrade

In Nord Pool, the congestion management is the responsibility of each system operator. This fact has resulted in two different congestion management methods that are used in the Nordic countries. Norway has adopted from the beginning on the zonal pricing scheme. In the other Nordic countries, the countertrade is used in order to deal with congestions.

The operator of the Pool Exchange, after the receipt of the participants' bids, clears the market of the next day without taking into consideration the line power flow limits. Thus, a unique price for the whole system is determined. If the resulting dispatch violates any power flow limit, the Norwegian system operator splits up the country into two or more zones. The objective is to define such prices for each zone that will yield to a relief of the congestion. Each zone is treated as a node and the zonal prices are calculated through a DC-OPF. The transfer capacity between the different zones is the sum of capacities of the lines that connect the zones. After OPF is run the new prices

are announced to the market participants. This congestion method is described as zonal pricing [61].

In Figure 3.3 the zonal pricing principle is illustrated. The generator in Zone A submits a bid of 20 €/MW for its whole production spectrum while generator in Zone B demands 20 €/MW for the first 100 MW and 30 €/MW for the rest 100 MW. In the presented case the consumers are must-run so they have submitted bids high enough to be fully covered. The pool operator clears the market and calculates the global price, considering the loads that are 200 MW in each zone. In this case the global price is 20 €/MWh. This price is attained by balancing demand and supply interest without taking into consideration the network. The generator in Zone A produces 300 MW and the generator in Zone B 100 MW. Consequently, the power flow over the line is 100 MW. If the transfer capacity of that line is only 90 MW then the system is constrained. In this case the system operator defines the Zones A and B and now runs an OPF considering the power flow limit over the lines that connect Zones A and B. The result of OPF is a price of 10 €/MWh in Zone A and 30 €/MWh in Zone B. These prices are actually the marginal cost for the corresponding production level of each generator. Now, the generator in Zone A produces 290 MW and the generator in Zone B is dispatched for 110 MW. Thus, the flow over the line is only 90 MW which is consistent with line's power flow limit.



**Figure 3.3** Example for demonstration Zonal Pricing and Countertrade

The remark, that could be done, is that the price at zone with excess generation is reduced while the price at the zone with excess demand is increased.

The definition of the different zones is of great importance in zonal pricing. As it has been shown, different splitting results in different surplus for the participants and the system operator [68]. The question why zonal pricing is used instead of nodal pricing dates back to the foundation of Nord Pool. At that time, the year 1993, the nodal pricing in electricity was not so wide known. The first deregulated market which introduced nodal pricing was New Zealand in 1996. It is claimed that zonal pricing is not as complex as nodal pricing. Therefore it is still in use in the Norwegian market. However, the experience proves that some markets that started using zonal pricing turned into nodal pricing. The opposite has not happened yet. A characteristic example is the case of PJM, which changed from zonal pricing to nodal pricing after only one year of implementation, in 1998. Further flaw of zonal pricing is the lack of economical signal for new investments since the prices do not always reflect the real cost of each node.

The other congestion management method that is used in the Nordic countries is the countertrade. By the countertrade, the participants submit bids for upwards or downwards deviation from the preferred schedule [55, 70]. The first step of countertrade is the calculation of the market clearing price. The suppliers are paid and the consumers pay for the power that has been assigned to them at the market clearing price. If there is any congestion the system operator seeks the schedules that should be increased or reduced. If a generator is called to increase its production then it will be compensated by the adjusting bid that it has submitted. In case of reduction the generator will pay the system operator its downwards bid. The market equilibrium is maintained when consumers and producers/suppliers have after the implementation of countertrade earnings equal to the ones of the unconstrained case. The choice of participants who will take part in the countertrade is made through an optimization, which has as objective function the minimization of the countertrade costs for the network operator. In the example of Figure 3.3 the generator in Zone A should decrease its output while the generator in Zone B has to increase its production. Assume, for simplicity, that the adjusting bids of both generators are the same as the normal bids. The Table 3.2, which follows, illustrates how the countertrade mechanism operates and gives the payment system in both unconstrained and countertrade case.

Unconstrained case Generator A Generator B 300\*200=6000 €/h 100\*20=2000 €/h Payment 300\*10=3000 €/h 10\*20=2000 €/h Cost **Earnings** 6000-3000=3000 €/h 2000-2000=0 €/h Constrained case using countertrade Generator A Generator B Payment 300\*20=6000 €/h 100\*20=2000 €/h 290\*10=2900 €/h Cost 100\*20+10\*30=2300 €/h -10\*10=-100 €/h 10\*30=300 €/h Credit by countertrade 6000-2900+(-100)=3000 €/h 2000-2300+300=3000 €/h **Earnings** -100+300=200 €/h Additional cost for the network operator

**Table 3.2**On the countertrade explanation

The cost illustrated in Table 3.2 is taken equal to the generators' bids that are given in Figure 3.2. As can be seen from Table 3.2, the earnings of the two generators remain unchanged. However, through the countertrade arises extra cost for the network operator. Usually, the system operator covers the additional cost through the transmission pricing [61].

# 3.3 Analysis of the Norwegian market using AC- OPF

In the context of this research work, the operation mode of the Norwegian electricity market has been investigated using a complete AC-OPF. Both loss factors and zonal pricing have been analysed and interesting conclusions have been drawn. This section presents the necessary theoretical background in order to realise this investigation.

#### 3.3.1 Nodal price loss component

In the case of loss factors, the objective of the investigation is to identify in which degree the use of loss factors, incorporating a DC-OPF, reflects the real network situation. In order to achieve this aim the following steps are necessary:

- Step 1: A complete AC-OPF is used. Nodal prices are calculated for each node.
- Step 2: The loss component of nodal prices is computed.
- Step 3: This loss component is used as nodal loss factor.
- Step 4: The participants' bids are modified according to the loss component.
- Step 5: The modified bids are given as input in a DC-OPF.
- Step 6: New nodal prices are calculated.
- Step 7: The net price of each participant is defined by adding/subtracting the loss factor from the new nodal prices.
- Step 8: Comparisons between the original nodal prices, obtained by AC-OPF, and the new net nodal prices may be done.

The above described procedure indicates the need of computing the loss component. In the nodal price analysis of Chapter 2, the nodal prices have been partitioned into two component, as (2.16) shows. One component which is due to generation and losses and another component which corresponds to system congestions. The former may be further decomposed into a component for generation and a loss component. This has as prerequisite that the nodal prices at reference bus are only due to generation. The assumption of nodal price analysis in Chapter 2 was that, at the reference bus a marginal increase of demand can be locally covered by the bus generator. If this hypothesis holds then the marginal increase of demand at the reference bus does not cause additional power losses. Thus, it is:

$$\frac{\partial P_{Losses}}{\partial P_{L,r}} = 0 \tag{3.1}$$

Assuming that (3.1) is satisfied one may write that:

$$\lambda_{GL,r} = \lambda_{G,r} \tag{3.2a}$$

$$\lambda_{Los\ r} = 0 \tag{3.2b}$$

where

 $\lambda_G$ : nodal price component due to generation

 $\lambda_{Los}$ : nodal price component due to losses

Consequently, the loss component of nodal price, at any system bus i, is given from the following equation:

$$\lambda_{Los,i} = \lambda_{GL,i} - \lambda_{G,r} \tag{3.3}$$

The (3.3) is valid for all the nodes because it is known that the deviation of nodal prices is caused by losses and congestions. Thus, if there are no losses and congestion one global price exists. Assuming that there is no congestion the deviations are only due to losses. That is what (3.3) shows. The deviation from the global price, here this global price would be equal to  $\lambda_{G,r}$ , is defined as the loss component of nodal price.

#### 3.3.2 Zonal pricing based on nodal pricing algorithm

The major characteristic of the Norwegian market operation mode is the use of zonal pricing as congestion management tool. In case that zonal pricing is active, all the nodes that belong to a zone face the same price. The objective of this research work is to simulate a zonal pricing situation by means of a complete AC-OPF (nodal price mechanism). The zonal pricing situation leads to same nodal prices for all the nodes that belong to a zone.

Usually, the nodal price at a bus is equal to the marginal cost of this bus, which is given by its bid curve. That is:

$$\lambda_i = -\frac{\partial K(\mathbf{P}_G)}{\partial P_{G,i}} \tag{3.4}$$

Only in case that a generation limit is reached the nodal price is different than the marginal cost. Thus, if the scope is to obtain same nodal prices for all the nodes that participate in a zone an additional restriction must be put in the OPF in order to achieve this goal. This restriction demands that the generators of the buses, which participate in a zone, should operate with the same marginal cost.

The market operator may treat the supplier bid curves, shown in Figure 3.2, as marginal cost curves resulting from polynomial cost functions. Additionally, the consumer bid curves may be simulated through bid curve of a fictitious generator. The production of this fictitious generator would represent the uncovered part of demand. Thus, the bid curves of any participant may be deduced from a polynomial cost function. If the bid curves of Figure 3.2 are treated as marginal cost curves then the corresponding cost function has the following form:

$$K(P_G) = aP_G^2 + c (3.5)$$

where a, and c are constants. It should be underlined, that the existence of the first degree term in (3.5) would not distort the generality of the following analysis. Hence, the analysis is also valid for the bid curves that are given by the dashed lines in Figure 3.2.

For any generator i the marginal cost, resulting from (3.5), is:

marginal 
$$cost(P_{G,i}) = 2a_i P_{G,i}$$
 (3.6)

From Figure 3.2 one may find out that the factor  $a_i$  is given by the relationship:

$$a_i = \frac{p_i}{2P_i} \tag{3.7}$$

where  $(p_i, P_i)$  is any corresponding pair of the bid curve. Consider now that in case of zonal pricing, there is a zone consisting of bus 1 and bus 2. Both of them are generation buses. If the generation limit at these two buses is not reached then the two generators should operate with the same marginal cost in order to face same nodal price. Thus, it is:

$$(\text{marginal cost})_1 = (\text{marginal cost})_2 \Rightarrow$$

$$\Rightarrow 2a_1 P_{G,1} = 2a_2 P_{G,2} \Rightarrow P_{G,1} - \frac{a_2}{a_1} P_{G,2} = 0$$
(3.8)

The (3.8) is the additional restriction which has to be incorporated in the OPF so as to obtain the same nodal prices for the buses 1 and 2. Consequently, the OPF will be formulated as:

$$\min K(\mathbf{P}_{G})$$

$$s.t \quad \mathbf{f}(\mathbf{x}) = \mathbf{0}$$

$$\mathbf{g}(\mathbf{x}) < \mathbf{0}$$

$$z(P_{G,1}, P_{G,2}) = 0$$
(3.9)

where

**f**: equality restrictions for nodal power balance

**g**: inequality restrictions of the power system

z: zonal pricing restriction for the buses 1 and 2

The Lagrange function, which corresponds to (3.9), is as follows:

$$L(\mathbf{x}) = K(\mathbf{P}_G) + \lambda \mathbf{f}(\mathbf{x}) + \mu \mathbf{g}(\mathbf{x}) + \xi z(P_{G,1}, P_{G,2})$$
(3.10)

where

 $\xi$ : Lagrange multiplier for the zonal pricing restriction

At the optimal point, according to the Kuhn-Tucker theorem, it is:

$$0 = \nabla_{\mathbf{x}} K(\mathbf{P}_G) + \lambda \nabla_{\mathbf{x}} \mathbf{f}(\mathbf{x}) + \mu \nabla_{\mathbf{x}} \mathbf{g}(\mathbf{x}) + \xi \nabla_{\mathbf{x}} z(P_{G,1}, P_{G,2})$$
(3.11)

The (3.11) facilitates the determination of a market equilibrium given the restriction of same nodal prices for all generators participating in a zone. This common nodal price can be found by taking the derivatives of (3.11) with respect to  $P_{G1}$  and  $P_{G2}$ . Thus, it is:

$$0 = \frac{\partial K(\mathbf{P}_{G})}{\partial P_{G,1}} - \lambda \frac{\partial \mathbf{f}(\mathbf{x})}{\partial P_{G,1}} - \xi \frac{\partial z(P_{G,1}, P_{G,2})}{\partial P_{G,1}} \Longrightarrow$$

$$\Rightarrow \frac{\partial K(\mathbf{P})}{\partial P_{G,1}} = \lambda_{1} + \xi \Rightarrow \lambda_{\text{expected}} = \lambda_{1} + \xi$$
(3.12a)

$$0 = \frac{\partial K(\mathbf{P}_{G})}{\partial P_{G,2}} - \lambda \frac{\partial \mathbf{f}(\mathbf{x})}{\partial P_{G,2}} - \xi \frac{\partial z(P_{G,1}, P_{G,2})}{\partial P_{G,2}} \Rightarrow$$

$$\Rightarrow \frac{\partial K(\mathbf{P})}{\partial P_{G,2}} = \lambda_2 - \xi \frac{a_2}{a_1} \Rightarrow \lambda_{\text{expected}} = \lambda_2 - \xi \frac{a_2}{a_1}$$
(3.12b)

From both (3.12a) and (3.12b) it is obvious that the price which has to be adopted as common nodal price for the buses 1 and 2 is the  $\lambda_{\text{expected}}$ . This price is equal to their common marginal cost and so it will be accepted from both participants.

It is important to underline that now the Lagrange multipliers  $\lambda_1$  and  $\lambda_2$  cannot be used as nodal prices for the buses 1 and 2. Both (3.12a) and (3.12b) highlight that  $\lambda_1, \lambda_2$  are different than the corresponding bid of the two producers, which is equal to the  $\lambda_{\text{expected}}$ . However, it can be shown that there is a fixed relationship between  $\lambda_1, \lambda_2$  and  $\lambda_{\text{expected}}$ . By multiplying (3.12a) by  $a_2/a_1$  and then adding it into the (3.12b) it is:

$$\frac{a_2}{a_1}\lambda_{\text{expected}} + \lambda_{\text{expected}} = \frac{a_2}{a_1}\lambda_1 + \frac{a_2}{a_1}\xi + \lambda_2 - \frac{a_2}{a_1}\xi \Longrightarrow$$

$$\Rightarrow (\frac{a_2}{a_1} + 1)\lambda_{\text{expected}} = \frac{a_2}{a_1}\lambda_1 + \lambda_2 \Rightarrow \lambda_{\text{expected}} = \frac{\frac{a_2}{a_1}\lambda_1 + \lambda_2}{\frac{a_2}{a_1} + 1} \Rightarrow$$

$$\Rightarrow \lambda_{\text{expected}} = \frac{\frac{a_2}{a_1} \lambda_1 + \frac{a_2}{a_2} \lambda_2}{\frac{a_2}{a_1} + \frac{a_2}{a_2}}$$
(3.13)

The (3.13) can be generalized for n generators participating in a zone as follows:

$$\lambda_{\text{expected}} = \frac{\frac{a_n}{a_1} \lambda_1 + \frac{a_n}{a_2} \lambda_2 + ... + \frac{a_n}{a_n} \lambda_n}{\frac{a_n}{a_1} + \frac{a_n}{a_2} + ... + \frac{a_n}{a_n}}$$
(3.14)

Consequently, the common marginal cost  $\lambda_{\rm expected}$  is the weighted average of the Lagrange multipliers  $\lambda_i$ . At this point it should be underlined that (3.14) holds if the production limits of generators participating in a zone are not reached. If such limits are reached then (3.14) is affected by the corresponding Lagrange multiplier  $\mu$ . However, it is necessary to mention that (3.14) is not needed in order to calculate the common marginal cost. This aim is served by the additional restriction introduced into the OPF. Once the power output of a generator participating into a zone is obtained, as co-product of OPF, the marginal cost can be estimated from the bid curve of this generator. The usefulness of (3.14) is consisting of showing that there is a standard relationship

between the Lagrange multipliers, of the buses belonging to a zone, and their common marginal cost given that the production constraints are not active.

#### 3.4 Numerical results

The analysis which is presented in Section 3.3 provides the necessary methods in order to assess the Norwegian market operation mode. The use of these methods will be highlighted using a 10-bus test system which is illustrated in Figure 3.4. The market, which is presented by this system, consists of four suppliers and four consumers. The participants' bids have the form which is shown in Figure 3.2. The network data as well as the market players' bids are given in Appendix B. Four different cases will be studied.

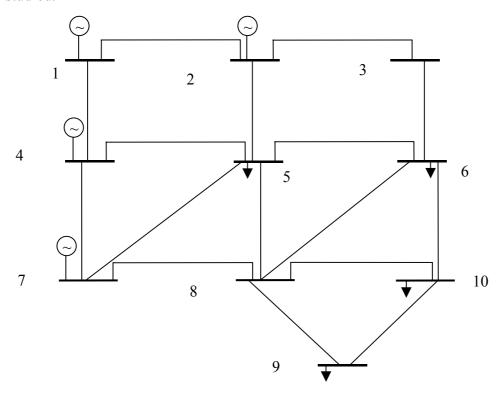


Figure 3.4 The 10-bus test system

#### 3.4.1 Unconstrained system

The assessment of the operation mode consists of calculating the deviations between the nodal prices obtained by AC-OPF and the prices that would be obtained if the system would operate as the Norwegian market.

Firstly, as the procedure shown in 3.3.1 indicates, the nodal prices are calculated. The market operator runs an AC-OPF using the bids of Table B.2. These bids result in an unconstrained system situation. In Table 3.3 the calculated nodal prices are presented. In this phase it is necessary to define a set of loss factors so as to simulate the Norwegian market. Assume that the market operator adopts the loss components of nodal prices as loss factors.

Actually, that would happen in Norway if the loss factors were changed at each time interval and they were not kept constant, as in the reality, for a period of 6-10 weeks.

The bus 7 is used as reference bus for the calculation of the nodal price loss components. The nodal price of this bus is at the middle of the price spectrum. So, the choice of this bus as reference bus leads in positive as well as negative loss components. This fact reflects the situation in Norway where both positive and negative loss factors may be announced. Moreover, at bus 7 a generator with large capacity is located. Therefore, the nodal price of bus 7 is chosen as reference price. The calculated loss components are included in the third column of Table 3.3. These components are now used as loss factors.

Table 3.3
Unconstrained case. Loss components as loss factors.
(All values in ct/kWh)

Bus	Nodal price AC	Loss factor	Nodal price DC	Net price DC	Difference: Net price DC- Nodal price AC
1	2.9296	-0.0234	2.9174	2.9408	0.0112
2	2.9527	-0.0003	2.9174	2.9177	-0.0350
3	2.9427	-0.0103	2.9174	2.9277	-0.0150
4	2.9565	0.0035	2.9174	2.9139	-0.0426
5	2.9633	0.0103	2.9174	2.9277	-0.0356
6	2.9526	-0.0004	2.9174	2.9170	-0.0356
7	2.9530	0.0000	2.9174	2.9174	-0.0356
8	2.9609	0.0079	2.9174	2.9408	-0.0201
9	2.9683	0.0153	2.9174	2.9177	-0.0506
10	2.9680	0.0150	2.9174	2.9277	-0.0403

The next stage is the modification of the original bid curves of the players according to the loss factors. After this point the procedure simulates the Norwegian market. The market operator receives the ultimate bid curves and runs a DC-OPF. The resulting nodal prices are shown in Table 3.3 in the fourth column. Since there is no congestion these prices are equal. However, the final prices that the market participants face come up after the subtraction of loss factors from the nodal DC prices.

The net DC prices represent the market situation as it would be formed by the Norwegian market operation mode. The deviations of those prices from the nodal prices, resulting by AC-OPF, are given in the last column of Table 3.3.

The differences are small but it should be considered that there is no congestion and the loss factors are based on the real AC situation. In the real world the same factors are used for more than 2000 time intervals during the ten-week period. Thus, it is reasonable to conclude that, in the most cases, the constant loss factors are not associated with the changing real AC situation. Therefore, the operation mode assessment is made again.

Table 3.4
Unconstrained case. Randomly chosen loss factors
(All values in ct/kWh)

Bus	Nodal price AC	Loss factor	Loss factor as % of nodal price	Nodal price DC	Net price DC	Difference: Net price DC- Nodal price AC
1	2.9296	0.1465	+5	2.9707	2.8242	-0.1054
2	2.9527	0.0591	+2	2.9707	2.9116	-0.0411
3	2.9427	0	0	2.9707	2.9707	0.0280
4	2.9565	-0.0887	-3	2.9707	3.0594	0.1029
5	2.9633	0.0593	+2	2.9707	3.0300	0.0667
6	2.9526	0.1181	+4	2.9707	3.0888	0.1362
7	2.9530	0	0	2.9707	2.9707	0.0177
8	2.9609	0	0	2.9707	2.9707	0.0098
9	2.9683	-0.1781	-6	2.9707	2.7926	-0.1757
10	2.9680	0.1187	+4	2.9707	3.0894	0.1214

This time, the loss factors are randomly chosen. In Norwegian market the loss factors are up to 10% of the nodal price. Such factors may be positive as well as negative. Table 3.4 illustrates, in the third column, the randomly chosen factors. The market operator runs again the DC-OPF and calculates the nodal DC prices. Then the net prices are announced to the market players. As can be seen, in the last column of Table 3.4, the

deviations from the AC nodal prices are now larger. This case study reveals that even in the absence of congestion and zonal pricing, a deviation from the AC nodal prices may be expected.

#### 3.4.2 Constrained system without zonal pricing

Consider a reduction of the power transfer limit of line 7-8 to 94 MVA, the participants' bids that are given in Table B.2 result in a congested situation. The power flow restriction, considering the line 7-8, is binding.

In Norway the existence of congestion leads to a splitting in different zones. For the research purpose, it is interesting to compare the congested AC nodal prices with the prices obtained by the Norwegian operation mode but without using zonal pricing.

So, assume that each node is a different zone. The calculated AC nodal prices as well as their loss components are given in Table 3.5. These components are used as loss factors and they modify the participants' bids. As discussed before, the market operator runs a DC-OPF, calculates the DC nodal prices and announces the net nodal prices. The differences between AC nodal prices and the DC net prices are given in the last column of Table 3.5. A comparison of these deviations to the ones given in Table 3.3 indicates that in case of congestion the differences are larger.

Table 3.5

Constrained case, without zonal pricing. Loss components as loss factors

(All values in ct/kWh)

Bus	Nodal price AC	Loss factor	Nodal price DC	Net price DC	Difference: Net price DC- Nodal price AC
1	2.9340	-0.0242	2.9160	2.9402	0.0062
2	3.1140	-0.0098	3.0620	3.0718	-0.0422
3	3.2570	0.0026	3.1770	3.1796	-0.0774
4	2.7890	-0.0113	2.7700	2.7813	-0.0077
5	3.1680	0.0056	3.0920	3.0976	-0.0704
6	3.4030	0.0144	3.2930	3.3074	-0.0956
7	2.3800	0.0000	2.3950	2.3950	0.0150
8	3.6250	0.0120	3.4950	3.5070	-0.1180
9	3.5590	0.0200	3.4610	3.4810	-0.0780
10	3.5450	0.0198	3.4130	3.4328	-0.1122

Table 3.6

Constrained case, without zonal pricing. Randomly chosen loss factors

(All values in ct/kWh)

Bus	Nodal price AC	Loss factor	Loss factor as % of nodal price	Nodal price DC	Net price DC	Difference: Net price DC- Nodal price AC
1	2.9340	0.0880	+3	2.9391	2.8511	-0.0829
2	3.1140	-0.1246	-4	2.9909	3.1155	0.0015
3	3.2570	0	0	3.0318	3.0318	-0.2252
4	2.7890	0.0558	+2	2.8816	2.8258	0.0368
5	3.1680	-0.0950	-3	3.0025	2.9075	-0.2605
6	3.4030	0.1702	+5	3.0775	3.2477	-0.1553
7	2.3800	0	0	2.7327	2.7327	0.3527
8	3.6250	0	0	3.1579	3.1579	-0.4671
9	3.5590	0.2135	+6	3.1432	3.3567	-0.2023
10	3.5450	0.0355	+1	3.1247	3.1602	-0.3848

Following the same procedure as in section 3.4.1, instead of the nodal price loss components, randomly chosen loss factors may be used. The corresponding loss factors as well as the comparison results are given in Table 3.6. It is evident that, in this case, the differences are essentially higher. Such differences are more likely to appear because the loss factors are not associated with the real AC situation.

#### 3.4.3 Constrained system considering zonal pricing

The third case consists of applying zonal pricing in the previous congested case. Assume that buses 1 and 2 form a zone while all the other buses remain as single-bus zones. Firstly, the AC nodal prices, shown in Table 3.7, are calculated.

As can be seen, the prices for buses 1 and 2 are equal because the zonal pricing restriction has been incorporated in the AC-OPF. The loss components of those prices are given in the second column. These components are used as loss factors by the pool operator. The pool operator runs a DC OPF demanding same nodal price for bus 1 and 2. The resulting prices as well as the net prices are also given in Table 3.7. The deviations are, generally, higher than in the previous case where the zonal pricing was not incorporated.

Table 3.7

Constrained case considering, zonal pricing. Loss components as loss factors

(All values in ct/kWh)

Bus	Nodal price	Loss	Nodal price	Net price	Difference:
	AC	factor	DC	DC	Net price DC-
					Nodal price AC
1	2.9956	0.0248	2.9670	2.9422	-0.0534
2	2.9956	-0.1010	2.9670	3.0680	0.0724
3	3.2620	0.0030	3.1740	3.1770	-0.0850
4	2.7870	-0.0115	2.7770	2.7885	0.0015
5	3.1720	0.0059	3.0920	3.0979	-0.0741
6	3.4100	0.0148	3.2860	3.3008	-0.1092
7	2.3720	0.0000	2.4090	2.4090	0.0370
8	3.6360	0.0124	3.4850	3.4974	-0.1386
9	3.6090	0.0204	3.4510	3.4714	-0.1376
10	3.5540	0.0202	3.4040	3.4242	-0.1298

Table 3.8

Constrained case considering, zonal pricing. Randomly chosen loss factors

(All values in ct/kWh)

Bus	Nodal	Loss	Loss	Nodal	Net price	Difference:
	price	factor	factor as %	price	DC	Net price DC-
	AC		of nodal	DC		Nodal price
			price			AC
1	2.9956	0.2097	+7	3.0150	2.8053	-0.1903
2	2.9956	0.2396	+8	3.0924	2.8528	-0.1428
3	3.2620	0	0	3.1533	3.1533	-0.1087
4	2.7870	-0.0836	-3	2.9292	3.0128	0.2258
5	3.1720	0.1586	+5	3.1095	3.2681	0.0961
6	3.4100	0.2046	+6	3.2215	3.4261	0.0161
7	2.3720	0	0	2.7071	2.7071	0.3351
8	3.6360	0	0	3.3415	3.3415	-0.2945
9	3.6090	0.2526	+7	3.3211	3.5737	-0.0353
10	3.5540	0.0355	+1	3.2919	3.3274	-0.2266

By choosing loss factors different from the loss components, these deviations are increased, as it is indicated in Table 3.8.

#### 3.4.4 Constrained system. Intensive zonal pricing

The last case describes a situation where a more intensive zonal pricing is applied, i.e. more buses participate in some zones. In particular, the four generator buses form one

zone. A second zone consists of the four consumer buses. The buses 3 and 8 remain single-bus zones. The results of this case are presented in Table 3.9. The loss components have been used as loss factors. It is obvious, that the differences, considering the great majority of the buses, are essentially higher than in the case described in Section 3.4.3.

Table 3.9

Constrained case, intensive zonal pricing. Loss components as loss factors

(All values in ct/kWh)

Bus	Nodal price	Loss	Nodal price	Net price	Difference:
	AC	factor	DC	DC	Net price DC-
					Nodal price AC
1	2.7780	0.0026	2.8380	2.8354	0.0574
2	2.7780	0.0117	2.8380	2.8263	0.0483
3	3.5860	0.0221	3.2590	3.2811	-0.3049
4	2.7780	0.0059	2.8380	2.8321	0.0541
5	3.9320	0.0205	3.4480	3.4685	-0.4635
6	3.9320	0.0257	3.4480	3.4737	-0.4583
7	2.7780	0	2.8380	2.8380	0.0600
8	4.3500	0.0121	3.6900	3.7021	-0.6479
9	3.9320	0.0269	3.4480	3.4749	-0.4571
10	3.9320	0.0258	3.4480	3.4738	-0.4582

Table 3.10

Constrained case, intensive zonal pricing. Randomly chosen loss factors

(All values in ct/kWh)

Bus	Nodal	Loss	Loss	Nodal	Net price	Difference:
	price	factor	factor as %	price	DC	Net price DC-
	AC		of nodal	DC		Nodal price AC
			price			
1	2.7780	0.1111	+4	2.8427	2.7316	-0.0464
2	2.7780	0.0556	+6	2.8427	2.7871	0.0091
3	3.5860	0	0	3.1960	3.1960	-0.3900
4	2.7780	-0.1389	-5	2.8427	2.9816	0.2036
5	3.9320	0.3146	+8	3.3554	3.6700	-0.2620
6	3.9320	0.2359	+6	3.3554	3.5913	-0.3407
7	2.7780	0	0	2.8427	2.8427	0.0647
8	4.3500	0	0	3.5570	3.5570	-0.7930
9	3.9320	0.0786	+2	3.3554	3.4340	-0.4980
10	3.9320	0.3146	+8	3.3554	3.6700	-0.2620

In case of more intensive zonal pricing, a set of randomly chosen loss factors may be applied. Both loss factors and resulting differences are given in Table 3.10. Some buses have increased differences while the deviations at other buses are reduced. The comparison is made between the differences that are given in Table 3.9 and Table 3.10.

A general comparison between the four cases, when randomly chosen loss factors are used, is not proper. In this case, an average deviation resulting by many different set of random factors would be a more appropriate approach. However, it is reasonable to compare the deviations in the case that the loss components have been used as loss factors. Such a comparison leads to the conclusion that the more administrative rules, such as loss factors and zonal pricing, are incorporated the higher the deviations from the AC nodal prices are.

It is also possible to make a further statement for the influence that the different rules have on the deviation of net prices from the AC nodal prices. For this purpose the differences presented in the last column of Tables 3.7, 3.9 and 3.10 are considered. The Table 3.7 corresponds to a not intensive zonal pricing. On the other hand Table 3.9 illustrates an intensive zonal pricing case. Table 3.10 corresponds also to intensive zonal pricing but in this case, differently as in Tables 3.7 and 3.9, randomly chosen loss factors have been used. It is obvious that the change from not intensive to intensive zonal pricing has larger influence on the deviations of net prices from AC nodal prices (shown as differences in Tables 3.7 and 3.9) than the change from the use of loss components as loss factors to the use of randomly chosen loss factors (shown as differences in Tables 3.9 and 3.10). Thus, the conclusion that one may draw is that the deviations of net prices in Norway from the AC nodal prices are mostly caused by the use of zonal pricing. The way that the loss factors are defined, same factors for a period of 6-10 weeks, has a secondary role to these deviations.

# Chapter 4

# Fixed cost allocation using game theory

The term *fixed costs*, generally, embraces the capital invested to build the network as well as the network maintenance costs. In a monopoly market, the utility covers those costs through the tariff policy. In the modern deregulated electricity markets, the network operation is the responsibility of the ISO. However, the company which is the network owner must still be compensated for those fixed costs. Hence, the ISO has to charge the market participants so as to collect the necessary amount.

In the liberalized power markets, the issue of charging the participants, regarding the fixed costs, is of great significance. The reason is that the fixed costs make up the largest part of transmission charges. Hence, it is easy to explain the demand for a fair and effective allocation of those costs to the market participants [31]. Discrimination policies, by assigning unreasonable high use-of-network charges, could be applied in order to prevent some market participants to access a part or even the whole network. Such policies cancel the isonomy, introduced by law in all deregulated markets, regarding the network access.

Several methods have been proposed aiming at a proper allocation of fixed costs [72]. These methods are well established from an engineering point of view but some of them may fail to send the right economical signals.

The allocation of embedded costs, i.e., the fixed costs, is a typical case where the cooperation between some agents produces economies of scale. Consequently, the resulting benefits have to be shared among the participating agents. The cooperative game theory concepts, taking into account the economies of scale, suggest reasonable allocations that may be economically efficient. The analysis in this research will illustrate the use of game theory in the fixed cost allocation. The incorporation of game theory under different market types will be investigated. Furthermore, new methods, considering the application of game theory, will be suggested.

#### 4.1 Game theory

Game theory is the study of multiperson decision problems. In these problems there is conflict of interests between people or group of people. Extending the use of game theory, participant in such a situation may also be considered any single individual. In this case, the participant is not necessary to be a human being. The term *game* corresponds to the theoretical models that describe such conflicts of interests. Game theory consists of analysing such conflict situations. In the most cases, the reality is too complicate to be described with acriby by a game, but a game could still be useful in order to describe the main types of movements that the participants could do and the various results that could come up.

The first paper on game theory was published in 1928 [73]. This was an investigation in the field of applied mathematic. But it was later in 1944 that the game theory was established as an autonomous field in mathematic [74]. In general, the situations investigated in the game theory may be categorized in two groups. The first group embraces the noncooperative games while the other group consists of the games where the participants may cooperate with each other.

The participants in a noncooperative game, as well as in a cooperative one, are called players. A basic assumption of game theory is that the players behave in a rational way. A participant is said to be rational if his aim is to maximize his payment from the game

taking into consideration the moves of the other players. The different decisions that a player may make to, during a game, are the strategies of each player. At the end of the game the payment received by each player is called *payoff*.

There are two alternatives to describe a noncooperative game. The first called extensive form and it describes, by means of a tree diagram, all the possible directions that a game can follow. The second alternative is called normal form. In the normal form representation of a game, each player, simultaneously, chooses a strategy and the combination of the strategies chosen by the players determines a payoff for each player. The normal form can be described through the paradigm shown in Figure 4.1.

	Player H2					
		$h2_1$	$h2_2$	$h2_3$		
Player H1	$h1_1$	0, 4	4, 0	5, 3		
	$h1_2$	4, 0	0, 4	5, 3		
	$h1_3$	3, 5	3, 5	6, 6		

Figure 4.1 Normal form of a game

The numbers in the left of each cell, in payoff matrix of Figure 4.1, represent the payment that player H1 will receive by choosing the corresponding strategy, given that player H2 will choose the strategy which corresponds to this column. If the players choose the pair of strategies  $(h1_1, h2_1)$  this will lead to the pair of payoffs (0, 4).

A solution to this game can be given by the maximin criterion. According to this criterion, the players are naturally pessimistic and so they try to find the best defence against their opponent. The player H1 looks, for each of his three strategies, which is the minimum payoff that he can expect. For the strategy  $h1_i$ , this task is formulated as follows:

$$\min_{j} u_{h1}(h1_{i}, h2_{j}), \quad j = 1, 2, 3 \tag{4.1}$$

where

$$u_{h1}(h1_i, h2_j)$$
: payoff to player  $H1$  for the pair of strategies  $(i, j)$ 

Between all of these minima, player H1 chooses the strategy i which guarantees him the highest of these minima.

$$\max_{i} \min_{j} u_{h1}(h1_{i}, h2_{j}), \quad i, j = 1, 2, 3$$
(4.2)

So, the player H1 maximizes his minimum expectation. For the game described above, the maximin criterion leads player H1 to adopt the strategy  $h1_3$  (expecting a minimal payoff of 3) while the player H2 chooses the strategy  $h2_3$  (expecting also a minimal payoff of 3). However, the payoff to both of them (6) is higher than the expected payment which is guaranteed by the maximin criterion (3).

Another solution for a noncooperative game is the Nash equilibrium [75]. According to this solution concept, each player will choose as strategy the best response to the optimal strategies of the other players. Consider a game  $G = \{ST_1, ..., ST_n; u_1, ..., u_n\}$  where  $ST_i$  is the set of strategies of the *i*-th player and  $u_i$  is the payoff to the *i*-th player. A group of strategies  $(st_1^*, ..., st_n^*)$  is a Nash equilibrium if for each player *i* the strategy  $st_i^*$  is the best response of *i* to the optimal strategies of the *n*-1 other players  $(st_1^*, ..., st_{i-1}^*, st_{i+1}^*, ..., st_n^*)$ .

Thus, it is:

$$u_{i}(st_{1}^{*},...,st_{i-1}^{*},st_{i}^{*},st_{i+1}^{*},...,st_{n}^{*}) \ge u_{i}(st_{1}^{*},...st_{i-1}^{*},st_{i}^{*},st_{i+1}^{*},...,st_{n}^{*}), \ \forall st_{i} \in ST_{i}$$
 (4.3)

Hence, strategy  $st_i^*$  is the solution to the following optimization problem:

$$\max_{st_i \in ST_i} u_i(st_1^*, ..., st_{i-1}^*, st_i^*, st_{i+1}^*, ..., st_n^*)$$
(4.4)

The Nash equilibrium may be further explained using the well-known game of the prisoners' dilemma. The payoff matrix of these game is given in Figure 4.2.

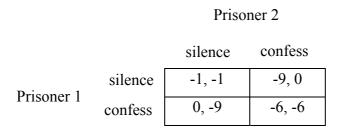


Figure 4.2 The prisoners' dilemma

In this game, two prisoners are interrogated for the same crime. They are held in different rooms so as to have no contact with each other. If one of them confesses while the other remains silent then the former is released and the latter is getting a sentence of 9 months. If both of them confess then they should stay in jail for 6 months and in case that they both remain silent then they are getting a sentence of 1 month. Investigating this game, in order to find the Nash equilibrium, it is obvious that for each player the best strategy is to confess, no matter what the opponent will do. Consequently, the Nash equilibrium of this game is the pair of strategies (confess, confess). The corresponding pair of payoffs is (-6, -6). However, it is clear from the payoff matrix that this pair of payoff matrix that the optimal that the two prisoners could achieve. It is obvious from the payoff matrix that the best solution for both of them would be to remain silent and get a sentence of just 1 month. This solution would be achieved if there were a contact, a cooperation, between the two prisoners. Such situations are investigated within the framework of cooperative game theory.

### 4.2 Cooperative game theory

The players participating in a cooperative game have the possibility to contact the other participants, so as to ensure a higher payoff than if they would act alone. The noncooperative game theory aims to describe the behaviour of players. On the other

hand, the cooperative game theory has rather a prescriptive character. Cooperative game theory does not target to describe the players' behaviour. Rather, it sets reasonable rules of allocation, or suggests indices in order to measure power. Therefore it is a challenge to use this theory in order to deal with the power systems' fixed cost allocation problem, especially because it can provide single-point solutions.

#### 4.2.1 Terminology of cooperative game theory

The set of players participating in a cooperative game is given as  $N = \{1, 2, ..., n\}$ . A cooperation between some players is possible to be established and so a coalition will form. A coalition S is then any subset of the set of players N,  $S \subset N$ .

When some players form a coalition S, it is assumed that they act as they were one player. The aim of the coalition members is to play a jointly set of strategies, aiming at maximizing the sum of payoffs to the players of the coalition. The next step is to allocate this sum between the members of the coalition.

The worst case for the players participating in a coalition S is, the rest of the players to form a coalition N-S and act against the players of coalition S. In this case the initial cooperative game with n players is transformed to a noncooperative game with two players, the S and the N-S. Using the maximin criterion it is possible to calculate the maximum payoff that the coalition S can now ensure itself. One may now define the characteristic function v of a cooperative game as the function which assigns to each coalition S the largest payoff that the coalition S can guarantee itself. Coalition S may obtain this payoff by coordinating the strategies which are available to its members. So if  $\mathbf{st}_S$  is a vector of strategies played by the players of coalition S, then it is:

$$v(S) = \max_{\mathbf{st}_S} \min_{\mathbf{st}_{N-S}} \sum_{i=1}^{n_S} u_i(\mathbf{st}_S, \mathbf{st}_{N-S}), \quad \mathbf{st}_S \in ST_S, \quad \mathbf{st}_{N-S} \in ST_{N-S}$$
(4.5)

where:

v(S): value of characteristic function for the coalition S

 $n_S$ : the number of players participating in coalition S

 $u_i$ : the payoff to player i

 $ST_S$ : the set of all strategies available to players of coalition S

 $ST_{N-S}$ : the set of all strategies available to players of coalition N-S

A game is called superadditive if its characteristic function holds the following condition:

$$v(S \cup \Theta) \ge v(S) + v(\Theta), \quad \forall S, \Theta \subset N, \text{ if } S \cap \Theta = \emptyset$$
 (4.6)

where:

 $S,\Theta$ : disjoint subsets of the set of players N

The (4.6) states, that the payoff which the union of S and  $\Theta$  can guarantee itself is at least equal to the sum of payoffs obtained by S and  $\Theta$  playing alone. For all the subsets of N if the equality holds in (4.6), then it is indifferent for the players to form any coalition and the game is called inessential:

$$v(S \cup \Theta) = v(S) + v(\Theta), \ \forall S, \Theta \in N, if \ S \cap \Theta = \emptyset$$
(4.7)

In this case the characteristic function  $\nu$  is just additive. For the inessential games it is:

$$v(N) = \sum_{i=1}^{n} v(i)$$
 (4.8)

where:

v(N): the value of characteristic function for the grand coalition N

v(i): the value of the characteristic function for the player I

A game which is not inessential and satisfies (4.6) is called essential.

It is important to investigate which coalitions are likely to form and how a coalition shares its payoff between the participating players. It is assumed that the important part of a cooperative game are the negotiations before the game begins, where coalitions form and the payoffs from the game are shared out. It is clear, that the way the payoffs are distributed influences also the formation of the coalitions. This happens because some players may try to attract some other players, in order to join their coalition, by promising them some extra amount. Any single player prefers to join the coalition which can guarantee to this player the highest payoff. Thus, it is not possible to make a prognosis of which coalitions are likely to form without knowing the way the payoffs are distributed.

The players are going to accept only reasonable payoffs. These payoffs should satisfy some certain conditions. The set of reasonable payoffs which can be rewarded to the players of a cooperative game are called *imputations*. A vector  $\mathbf{y} = (y_1, y_2, ...., y_n)$ , representing payoffs to the single players, is an imputation if it holds the following two conditions:

$$\sum_{i=1}^{n} y_i = v(N)$$
 (4.9a)

$$y_i \ge v(i), \text{ for } i = 1, 2, ..., n$$
 (4.9b)

The condition in (4.9a) indicates that the sum of all payoffs should be equal to the value that the grand coalition can guarantee itself. This is called global rationality and it is also known as a Pareto optimality condition. Pareto optimality means that it is impossible to move from  $\mathbf{y}$  to another vector of payoffs  $\mathbf{k}$  with all the players having an increased payoff. The second condition is called individual rationality. Its explanation is that each player would accept as payoff from the game an amount which is at least equal to the amount that the player can guarantee itself by playing alone.

Since the imputations satisfy the global rationality condition, it is not possible that an imputation  $\mathbf{y}$  can give a higher payment to each player  $i \in N$  than any other imputation  $\mathbf{k}$ . It is trivial to see that if for some players is  $y_i > k_i$  then it must be at least one player j with  $y_j < k_j$ . The reason is that the following condition holds:

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} k_i = v(N)$$
(4.10)

For a coalition S it might be possible that  $\mathbf{y}$  gives higher payoff to each member of the coalition than  $\mathbf{k}$ . In order to say that  $\mathbf{y}$  dominates  $\mathbf{k}$  over a coalition S two conditions must be satisfied:

$$y_i > k_i, \quad \forall i \in S$$
 (4.11a)

$$\sum_{i=1}^{n_S} y_i \le v(S) \tag{4.11b}$$

While (4.11a) is simple to understand, (4.11b) calls for further explanation. Assume that a coalition S wants to dominate over other coalitions by promising high payoffs to its members. In this case coalition S should be able to ensure to each of its members the payoff  $y_i$  that it promises.

#### **4.2.2** The core

One of the first solutions suggested for cooperative games is the core concept [76]. The core is based on the concept of domination of imputations. According to the core concept, an imputation will be favourable in the negotiations' phase if it is not dominated. The core of a game with characteristic function v, denoted by CR(v), is the set of all the imputations that are not dominated over any coalition. An imputation y, in order to belong to the core, must satisfy the following two conditions:

$$\sum_{i=1}^{n} y_i = v(N) \tag{4.12a}$$

$$\sum_{j=1}^{n_S} y_j \ge \nu(S), \quad \forall S \subset N \tag{4.12b}$$

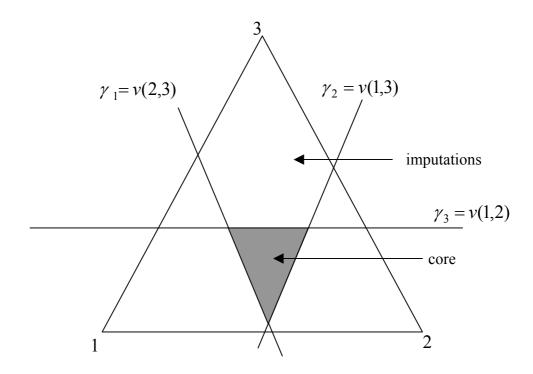
The former is the global rationality while the latter is the coalitional rationality. Thus, if an imputation  $\mathbf{y}$  belongs to the core then there are two possibilities for any coalition S which forms. Either coalition S concerns the  $\mathbf{y}$  as the best solution or if it prefers another imputation  $\mathbf{k}$  it has not the strength to enforce the change. The explanation of this statement is that since  $\mathbf{y}$  belongs to the core then it is not dominated over any coalition. Hence,  $\mathbf{y}$  is also not dominated over S. Thus, if an imputation  $\mathbf{k}$  would dominate  $\mathbf{y}$  over S then it should be:

$$k_i > y_i, \ \forall \ i \in S \Rightarrow \sum_{i=1}^{n_S} k_i \ge \sum_{i=1}^{n_S} y_i \ge v(S)$$
 (4.13)

But the condition in (4.13) conflicts the condition in (4.11b), that means  $\mathbf{k}$  exceeds the v(S), and so the imputation  $\mathbf{k}$  cannot dominate  $\mathbf{y}$  over the coalition S. The core of a game may include more than one imputations. A disadvantage of the core concept is that some games have an empty core.

Suppose that there is a cooperative game with three players, N=(1,2,3). Geometrically, the imputations can be represented as triples of barycentric coordinates of points in a triangle, as Figure 4.3 illustrates. For any point in triangle  $\Delta 123$ , its barycentric coordinates are the payoffs to the players of the game. That is, the closer a point is to a vertex the larger the payoff is to the corresponding player. Hence, an imputation must satisfy the inequalities of (4.12b) in order to be in the core. The lines  $\gamma_i$  represent the characteristic function values of the corresponding coalitions, i.e. the points belonging to these lines have a constant sum regarding the payoffs of the corresponding players. This sum is equal to the characteristic function value of the coalition which consists of those players. Then the imputations belonging to the core, in order to satisfy (4.12b), must be closer to the sides of the triangle than the corresponding lines. Thus, assuming

that for any player i it is v(i) = 0, geometrically the core of the game is the area which is bounded from the three lines  $\gamma_i$ .



**Figure 4.3** Geometrical representation of a three-person game

#### 4.2.3 The nucleolus

From Figure 4.3, it is clear that the core may include more than one imputation. Actually, when the core is not empty then either it has only one imputation or it embraces an infinite number of imputations. The former occurs when the interaction of the lines  $\gamma_i$  is a single point and the latter in all the other cases.

The nucleolus concept was introduced in order to choose a single solution among all the imputations belonging to the core [77, 82]. Every game has only one nucleolus and if the core exists the nucleolus is part of it. The nucleolus is based on the idea of making the most unhappy coalition under it happier than the most unhappy coalition under any other imputation. For a coalition S measure of its unhappiness is the excess e(S):

$$e(S) = v(S) - y(S), \quad with \ y(S) = \sum_{i=1}^{n_S} y_i$$
 (4.14)

Thus, y(S) is the sum of payoffs that the imputation y shares out to the members of coalition S. Since y is an imputation which belongs to the core then from (4.12b) one finds out that the excess e(S) is either zero or negative. This excess shows how much more could expect a coalition S from the imputation y in comparison to what it can guarantee itself. Thus, the more near to zero, means larger, the excess e(S) is, the more unhappy the coalition is with this imputation.

Define  $\theta(\mathbf{y})$  to be the  $2^n$  values v(S)- y(S) for all coalitions S, including the grand coalition N and the empty set  $\mathcal{O}$ , sorted in decreasing numerical order. Two imputations  $\mathbf{y}$  and  $\mathbf{k}$  may be compared by looking at the coalition which is unhappiest under each of them. Assume that they are the coalitions S and  $\Theta$  respectively. Then calculating the excesses v(S)- y(S), as well as  $v(\Theta)$ -  $k(\Theta)$ , the smaller the excess is the better the imputation. If these two excesses are equal then the next pair of the most unhappy coalitions is compared. The two vectors  $\theta(\mathbf{y})$  and  $\theta(\mathbf{k})$  can be ordered lexicographically, means as in a dictionary, so as to be:

$$\theta(\mathbf{y}) = (\theta(y)_1, \theta(y)_2, ..., \theta(y)_{2^n})$$
(4.15)

Then it is:

$$\theta(\mathbf{y}) < \theta(\mathbf{k})$$
if  $a$ )  $\theta(y)_1 < \theta(k)_1$ 
or
$$if \quad b$$
)  $\theta(y)_j = \theta(k)_j$ , for  $j = 1, 2, ..., i - 1$ 

$$and \quad \theta(y)_i = \theta(k)_i$$

$$(4.16)$$

95

According to (4.16), the nucleolus, denoted by N(v), is the smallest imputation from all the imputations which belong to the core.

$$N(v) = \{ \mathbf{y} \in CR(v) : \theta(\mathbf{y}) < \theta(\mathbf{k}), \forall \mathbf{k} \in CR(v) \}$$
(4.17)

A hypothetical game, which has just four imputations  $\varepsilon$ , k, y, and  $\zeta$  belonging to the core, will better clarify the concept of nucleolus. Assume that the most unhappy coalitions under each of these four imputations are the S,  $\Theta$ , E, Z respectively. The excesses corresponding to these coalitions are v(S)- $\varepsilon(S)$ =-10,  $v(\Theta)$ - $k(\Theta)$ =-11, v(E)-y(E)=-2 and v(Z)- $\zeta(Z)$ =-8. Figure 4.4 shows these four differences on the axis of the real numbers.

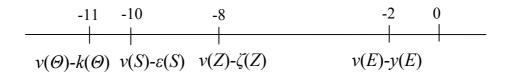


Figure 4.4 On nucleolus explanation

From Figure 4.4, one finds out that the nucleolus of the game is the imputation  $\mathbf{k}$ . As can be seen, this imputation makes the most unhappy coalition under it, in this case the coalition  $\Theta$ , happier than the most unhappy coalition under any other imputation.

A disadvantage of the nucleolus is that it is not monotonic. Monotonic means that when the characteristic function value v(S) of a coalition increases then the payoff to the members of this coalition is getting larger.

Although that nucleolus has been defined through the lexicography ordering of imputations, this procedure cannot be used to calculate the nucleolus. This happens because of the infinite number of imputations belonging to the core when it contains more than one imputation. For that reason, an iterative procedure based on linear programming has been proposed in order to compute the nucleolus of a game [78].

The first stage of this procedure has as objective to minimize the largest, considering all the coalitions, of the excess values. From (4.14) it is obvious that the excesses are less than zero since (4.12b) holds.

min 
$$e$$
  

$$s.t. y(S) + e \ge v(S) \qquad \forall \ \Sigma^0$$

$$y(N) = v(N)$$
(4.18)

where  $\Sigma^0 = \{S \subset N : S \neq \emptyset\}$ . The value resulted by solving the first iteration is  $e_1$ . The set of imputation resulting in  $e_1$  is denoted by  $Y^1$ . In case that the solution is not unique, that is  $Y^1$  includes more than one imputations, the problem of (4.18) is solved again. In this case the active inequality constraints are fixed into equality constraints. Denote by  $\Sigma_1 = \{S \in \Sigma^0 : y(S) + e_1 = v(S) \ \forall y \in Y^1\}$  the set of coalitions where the constraint is active. Then , the second stage is formed as:

min 
$$e$$
  
 $s.t.$   $y(S) + e_1 = v(S)$   $\forall S \in \Sigma_1$   
 $y(S) + e \ge v(S)$   $\forall S \in \Sigma^1$   
 $y(N) = v(N)$  (4.19)

where  $\Sigma^1 = \{\Sigma^0 - \Sigma_1\}$ . The optimal value of the second stage is  $e_2$  and the set of optimal solutions is  $Y^2$ . If there is no unique solution the linear programming is repeated. Generally, by the j-th iteration it will be:

min 
$$e$$
  
 $s.t.$   $y(S) + e_i = v(S)$   $\forall S \in \Sigma_i \ i = 1, 2, ..., j - 1$   
 $y(S) + e \ge v(S)$   $\forall S \in \Sigma^{j-1}$   
 $v(N) = v(N)$  (4.20)

97

This procedure is terminated when  $Y^{j}$  contains a unique solution and enables the calculation of nucleolus.

## 4.2.4 The Shapley Value

The core concept, as well as the nucleolus one, shows the payoff vectors that are likely to persist during the phase of negotiations, before the game begins. Earlier than these two concepts, the Shapley value has been proposed as solution of a cooperative game [79]. This value, denoted by  $\varphi_i(v)$  for the *i*-th player, calculates what a player could reasonably expect before the game has begun. For the foundation of this value, three axioms has been settled:

A1-Symmetry:  $\varphi_i(v)$  is independent of the labelling of the players. So, for each permutation  $\pi$  of the n game players if  $\pi v$  is the characteristic function of the permuted game, with the players numbers permuted by  $\pi$ , then it is:

$$\varphi_{\pi(i)}(\pi v) = \varphi_i(v) \tag{4.21}$$

A2-Efficiency: The sum of the expectations must be equal to the characteristic function value for the grand coalition N:

$$\sum_{i=1}^{n} \varphi_i(v) = v(N) \tag{4.22}$$

A3-Additivity: Suppose there are two games with characteristic functions v1 and v2 respectively. Then the sum of expectations, by these two games, for each player must be equal to the value which would be calculated if both the games would be played together:

$$\varphi_{i}(v1+v2) = \varphi_{i}(v1) + \varphi_{i}(v2)$$
 (4.23)

In [79] it has been proved that the only function which satisfies these three axioms is the following one:

$$\varphi_i(v) = \sum_{S \ i \in S} \frac{(n_S - 1)!(n - n_S)!}{n!} (v(S) - v(S - \{i\}))$$
(4.24)

The value  $\varphi_i(v)$  in (4.24) is known as the Shapley value. In order to interpret (4.24) assume that the players participate in the game one after another in random sequence. Additionally, assume that each player receives a payoff equal to his contribution to the increase of characteristic function value of the coalition which he joins. Then the Shapley value for a player i is the sum of these payoffs taking into account all the coalitions which include this player. The fraction in (4.24) represents the probability that first participate in the game the  $n_S - 1$  players of a coalition S, followed by the player i and then participate the rest  $n - n_S$  players.

Another form of the Shapley value is:

$$\varphi_i(v) = \sum_{S, i \notin S} \frac{n_S!(n - n_S - 1)!}{n!} (v(S \cup \{i\}) - v(S))$$
(4.25)

This second form is used in the literature by the definition of some other values while the Shapley value is usually described by (4.24).

In contrast to the nucleolus solution, the Shapley value exhibits monotonicity.

## 4.2.5 The Solidarity Value

The Shapley value concept seems to be a very attractive solution for the cooperative games because it gives a single solution and it is axiomatically founded. Therefore, a number of other values, based on the Shapley value, have been developed during the last decades. The Solidarity value attempts to support the weaker participants of a game [80]. Looking at (4.24) one finds out that the dummy players have a zero Shapley value.

99

Dummy is a characterisation for the players who contribute nothing to any of the coalitions where these players participate. Hence, a dummy player will be described by:

$$v(S \cup \{i\}) = v(S), \ \forall S \subset N \tag{4.26}$$

In order to support such dummy players and, in general, the weaker players of a game, the solidarity value uses the average marginal contribution  $A^{\nu}(S)$  of a coalition S instead of the marginal contribution of a player i:

$$A^{\nu}(S) = \frac{1}{n_S} \sum_{i=1}^{n_S} (\nu(S) - \nu(S - \{i\}))$$
 (4.27)

Using (4.27) the Solidarity value  $\psi_i(v)$  of a player *i* is defined as follows:

$$\psi_i(v) = \sum_{S, i \in S} \frac{(n_S - 1)!(n - n_S)!}{n!} A^v(S)$$
 (4.28)

The solidarity value satisfies also the axioms of symmetry, efficiency and additivity.

#### 4.2.6 The Owen Value

The Owen value has been introduced in order to take into consideration games with coalitional structure [81]. That is, before the game begins there are *a priori* coalitions between some players. Assume that there is a game with  $N=\{1,2,...,n\}$  players and that there is a set J of *a priori* unions between the players,  $J=\{T_1,T_2,...,T_m\}$ . Furthermore, let  $\Gamma=\{1,2,...,m\}$  be the set of the union numbers.

The game is played in two phases. In the former the payoff to the unions is calculated through the Shapley value solution. In the second phase this payoff is allocated to the members of the union using again a Shapley value process. However, there is a

difference in this step. That is, the amount that a coalition S, which belongs to a union  $T_j$ , could achieve by defecting the union  $T_j$  and joining any permutation of the rest unions it is also taken into account. The Owen value, for a player i participating in this game, is denoted by  $y_i(v;J)$ . This value is calculated as follows:

$$y_{i}(v;J) = \sum_{\substack{\pi \subset \Gamma \\ j \notin \pi}} \sum_{\substack{i \notin S}} \frac{n_{s}!(n_{Tj} - n_{S} - 1)!n_{\pi}!(m - n_{\pi} - 1)!}{n_{Tj}!m!} E$$
with  $E = [v(B \cup S \cup \{i\}) - v(B \cup S)]$  (4.29)

where:

 $\pi$ : permutation in  $\Gamma$ 

*j*: the number of the *a priori* union where the player *i* belongs

 $n_S$ : the number of players participating in coalition S

 $n_{T_j}$ : the number of players participating in the *a priori* union  $T_j$ 

 $n_{\pi}$ : the number of the unions belonging to the permutation  $\pi$ 

B: the union of all the *a priori* unions belonging to the permutation  $\pi$  ,  $B = \bigcup_{q \in \pi} \pi_q$ 

From (4.29) it is obvious that the probability  $\frac{n_{\pi}!(m-n_{\pi}-1)!}{m!}$  corresponds to the phase when the game is between the *a priori* unions. Similarly, the probability  $\frac{n_{s}!(n_{Tj}-n_{S}-1)!}{n_{Tj}!}$  is according to the second phase of the game, between the players

of a union. As can be seen, the marginal contribution in (4.29) reflects the possibility of the players to leave their *a priori* union and join a permutation  $\pi$  of the rest unions. The Owen value holds the axioms of symmetry, efficiency and additivity.

101

# 4.3 Usage based methods

The application of game theory in fixed cost allocation aims to overcome the lack in economic efficiency that some already known methods have. Some of these are based on the marginal cost and others on the measurement of the network usage [72, 83]. The problem with the marginal cost based methods is that, in general, they do not fully cover the fixed costs. Consequently, a supplementary charge is necessary.

Several usage-based methods have been developed in order to deal with the task of allocating the fixed cost of a power system among the market participants. The difference between these methods is how the network usage is measured.

## 4.3.1 The Postage Stamp Method

One of the traditional methods is the postage stamp method (PS), also known as the rolled-in method [84]. According to this method, the network usage from the side of a transaction is measured by the magnitude of the transaction  $P_i$ , without taking into account how the transaction affects the power flows over the various lines in the network. The amount to be paid by transaction i is:

$$PS_i = K \frac{P_i}{\sum_{j=1}^{n} P_j} \tag{4.30}$$

where

K: the total cost to be covered by the market participants

 $PS_i$ : the amount charged to participant i according to the postage stamp method

Obviously, since the postage stamp method does not take distances into account, it leads to cross-subsidization of long-distance transactions by short-distance transactions. Despite this fact, this method is widely implemented because of its simplicity.

#### 4.3.2 The MW-Mile Method

In order to achieve a more precise measurement of network usage, numerous methods based on power flow data have been developed. The MW-mile method (MWM) was the first such method to be introduced [85]. In order to determine the cost allocation, the network operator runs a power flow program for each single transaction and calculates the power flow due to this transaction over each system line. These power flows are then weighted by the specific transfer cost  $C_l$  of each branch l which is expressed in  $\ell$ /MW. The role of  $C_l$ , in the case that a pre-defined amount k must be proportionally allocated to the system users, is to differentiate the use of facilities with various costs. Thus, in this case k0 should not be confused with a direct payment, per MW, to the system operator. However, k1 may be indeed interpreted as direct, per MW, payment when other, than proportional share of a pre-defined amount k1, allocation form is adapted. This case will be illustrated in a following section. The usage of any branch k1 by transaction k1 will be:

$$f_{i,l} = C_l |P_{i,l}| (4.31)$$

where

 $f_{i,l}$ : the usage of branch l by the market participant i

The absolute value in (4.31) denotes that the power flow direction is disregarded. The total system usage  $f_i$  by transaction i is given by summing over all lines:

$$f_i = \sum_{l=1}^{n_l} f_{i,l} \tag{4.32}$$

By allocating proportionally the total system cost, the contribution of transaction i will be:

103

$$MWM_{i} = K \frac{f_{i}}{\sum_{j=1}^{n} f_{j}}$$

$$(4.33)$$

where

 $MWM_i$ : the amount charged to participant i according to the MW-mile method

#### 4.3.3 The Counter Flow Method

As already stated, the MW-mile method does not consider the direction of power flow that each transaction causes. However, it is often argued that power flows having opposite direction from the net flow, which is the power flow due to all transactions, contribute positive in the system situation by relieving congestions and increasing the available transfer capacity. In order to take this fact into account, a version of MWM has been developed. In this version the branch usage is calculated by the following equation:

$$f_{i,l} = C_l P_{i,l} \tag{4.34}$$

Then (4.34) is used in (4.32). This allocation procedure is called the counter flow method and results in the payment  $CF_i$ , for a participant i, using (4.33).

#### 4.3.4 The Zero Counter Flow Method

According to the counter flow method, the contribution of a transaction may be negative, i.e., the network operator has to pay agent i for carrying out his transaction. For various reasons this may not be acceptable to the network owner and/or the other market participants. A compromise that avoids negative contribution is the zero counter flow (ZCF) method. According to this method, the usage of a line by a particular transaction is set to zero if the power flow due to the transaction goes in the opposite

direction of the net flow for the line. Thus, instead of (4.31) the branch usage is calculated as follows:

$$f_{i,l} = \begin{cases} C_l P_{i,l} & P_{i,l} \ge 0\\ 0 & P_{i,l} < 0 \end{cases}$$
 (4.35)

The amount  $ZCF_i$ , to be paid by transaction i, is then found by using (4.35) as a basis for (4.32) and (4.33).

# 4.4 The fixed cost allocation game

The transit from monopoly to competitive market increased the need for economic efficiency by the power sector operation. In this context, the cooperative game theory has been applied in order to achieve efficient allocations of the power system fixed costs. In [86, 87], an introduction in use of cooperative game theory is presented. The nucleolus, as well as the Shapley value, is used as solution to the corresponding game. Furthermore, in [88] the allocation of network expansion cost is investigated by means of Kernel concept. In [89], the fixed cost allocation in a pool marketplace is addressed.

One of the main reasons that allocations based on the cooperative game theory methods are attractive is that they, in many cases, belong to the core. Thereby, the problem of cross-subsidization, as in postage stamp, is avoided.

#### 4.4.1 Game definition

A general equation, which represents the usage-based cost allocations shown in previous section, has the form:

$$R_i = K \frac{f_i}{\sum_{j=1}^n f_j} \tag{4.36}$$

Another alternative is to bill the participants directly for each MW which they transfer. In this case, the payment  $R_i$  of each market participant to the ISO is given by:

$$R_i = \sum_{l=1}^{n_l} f_{i,l}$$
 with  $f_{i,l} = |P_{i,l}| C_l$  (4.37)

It is noticeable, that in (4.37) the role of specific transfer cost  $C_l$  is to directly charge the participants for each MW that they transfer over the branch l.

In both (4.36) and (4.37), the network operator calculates the amount  $f_i$  as if i were the only participant in marketplace using either power flow or optimal power flow program. Thus, the power flows  $P_{i,l}$  are determined considering only player i. This usage is called *stand-alone* usage. The motivation for the participants to cooperate is the existence of counter flows.

Assume that some participants agree to cooperate. Then they could benefit from possible counter flows. Figure 4.5 illustrates this idea. By setting the specific cost at unit the usage is equal to the power flow and so it can be expressed in MW. This assumption will hold for the whole section 4.4. It should be underlined, that the general validity of the following analysis is not affected by this assumption. In a following section it will be illustrated the use of cooperative game theory also with different specific costs for each branch.

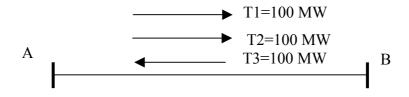


Figure 4.5 Counter flow

As can be seen, for each transaction i it is  $f_i = 100$  MW. If there is a cooperation of the three transactions then the measured use from the side of the coalition it will be  $f_{coalition} = 100$  MW. Hence, it is:

$$f_{i} = 100 \,\text{MW}, i = 1,2,3$$

$$\sum_{i=1}^{3} f_{i} = 300 \,\text{MW}$$

$$f_{coalition} = 100 \,\text{MW}$$

$$\sum_{i=1}^{3} f_{i} - f_{coalition} = 200 \,\text{MW}$$
(4.38)

From (4.38) one finds out that the three participants in case of cooperation would be billed for 200 MW less. This is the explanation why the market participants would have the incentive to cooperate.

The next step is to allocate this benefit among the coalition members. At this point the incorporation of the cooperative game theory can result in a fair and acceptable allocation. Considering the game of the power system fixed cost allocation, the characteristic function v can be defined as follows:

$$v(S) = \sum_{i=1}^{n_S} f_i - f_S$$
 (4.39)

where:

 $f_S$ : usage of the network by coalition S

From (4.39) is explicit that the characteristic function represents the savings that can be achieved in case of cooperation. It is obvious that for each player i it is v(i) = 0. Once the game is defined solutions may be found using the cooperative game theory concepts which have been described in section 4.2. Assume that the vector  $\mathbf{y}$  represents the

107

payoffs to the players arose from the solution of the game. These payoffs are resulting in a reduction of  $f_i$  for each player:

$$f_{i}^{'} = f_{i} - y_{i}, \quad if \quad f_{i} \ge y_{i}$$

$$f_{i}^{'} = 0, \quad if \quad f_{i} < y_{i}$$

$$(4.40)$$

where  $f_i^{'}$  is the new use of network or facility by player i. If the savings assigned to player i are larger than the original  $f_i$  then the  $f_i^{'}$  is set at zero. Thus, a player does not have the opportunity to receive money back from the network operator. The reason of making this adjustment is to prevent the misuse of game from the side of players. The  $f_i^{'}$  is now used by (4.36) or (4.37) in order to calculate the amount that player i has to pay. In case of (4.37), there is an explicit reduction of the amount  $R_i$  for all the players, if  $y_i$  is not zero. In case of (4.36), the use of cooperative game theory may bring an acceptable solution concerning the different ways the players can cooperate. This point will be further explained in the next sections.

## 4.4.2 The game in the case of pool market

The IEEE 14 bus system, shown already in Figure 2.6, is used as a pool marketplace. For this paradigm the loads have no elasticity, i.e. they have to be fully covered. The generators' cost data are given in Table C.1 in Appendix C. The competition takes place only in generation level. Such a market may appear in the beginning of deregulation phase in countries where, during the monopoly period, there was only one utility serving the whole country. That is, in the beginning of restructuring new players must come in and the domination power of the ex-monopoly must be controlled. Thus, bilateral contracts are not allowed and the whole power is traded in a mandatory pool with the pool operator having a wide knowledge of the generators' data. Greece belongs in this category. According to the Greek Regulatory Authority for Energy the mandatory pool should be online within 2004.

Table 4.1 presents the consumers as well as their inelastic demand. The fixed costs will be allocated to the consumers because of their price inelasticity [89].

Assume that the loads are going to act individually. Then the pool operator determines, separately for each player, through an optimal power flow (OPF) which generators have to be dispatched to match the demand. In Table 4.2 the first four rows correspond to the single-player coalitions. Thus, these rows include, in the second column, the use of power system by each player when this player acts alone.

Table 4.1
Consumers in the pool market as players of the game

Player	Bus	Demand,	
1 layer	Dus	[MW]	
1	4	40	
2	9	40	
3	13	30	
4	14	50	

If the four loads are going to cooperate with each other then the possible coalitions are 15, including the single-player coalitions. For each coalition Table 4.2 comprises its use of the system, as well as the characteristic function value. The characteristic function value of the grand coalition  $\{1, 2, 3, 4\}$ , i.e. the largest savings, has to be allocated to the four players. For this aim the Shapley value may be used. Table 4.3 presents the  $f_i$  and  $f_i$  for each player as well as the Shapley value  $\varphi_i(v)$ .

The allocation of fixed cost may also take place at the level of each single system branch. In this case  $f_i$  denotes the power flow over a particular branch caused by player i. The problem by realising the single branch game in a pool market is that negative characteristic function values may arise for some coalitions. The explanation of these negative values is that when a coalition forms the dispatched generators may be different from the generators dispatched to meet the demand of single players. Thus, the power flow over some branches may be also larger than the sum of  $f_i$ . Considering the whole system, the formation of each coalition results in savings.

Table 4.2
Use and savings for the game in pool market

coalition	$f_S$ [MW]	ν(S) [MW]
1	82.53	0
2	158.99	0
3	133.81	0
4	229.09	0
3,4	332.45	30.44
2,4	379.32	8.75
2,3	246.13	46.66
1,4	305.85	5.77
1,3	201.60	14.75
1,2	236.67	4.85
1,2,3	321.89	53.44
1,2,4	456.58	14.02
1,3,4	404.43	41.00
2,3,4	472.58	49.30
1,2,3,4	505.53	98.89

Table 4.3
Initial and final use for each player in pool market

player	$f_i$	$\varphi_i$	$f_i^{'} = f_i - \varphi_i$
piuyer	[MW]	[MW]	[MW]
1	82.53	16.40	66.13
2	158.99	24.98	134.01
3	133.81	39.23	94.58
4	229.09	18.28	210.81

However, the same cannot be said if the game is played at each system branch. In Table 4.4 coalitions with negative v(S) at some branches are shown. A negative function value for a coalition indicates that this coalition is not likely to form since the players

try to establish meaningful coalitions only. Thus, the fixed cost allocation game, as described above, it is not so likely to be played at each single system branch in a pool market.

Table 4.4
Coalitions with negative characteristic function values

C 1'4'	line 5-6	line 6-11	line 6-12
Coalition	v(S) [MW]	v(S) [MW]	v(S) [MW]
3,4	-0.34	4.96	0.05
2,4	-0.29	-0.07	0.06
1,2,4	-0.29	-0.06	0.16
1,3,4	-0.34	6.62	0.14
2,3,4	-0.80	7.96	0.09
1,2,3,4	17.96	2.93	-0.43

The same problem emerges even if a DC optimal power flow is used. The reason is the non-linear difference at the dispatch of the system's generators in order to serve the single players and in order to serve the coalition that these players form.

## 4.4.3 The game in the case of bilateral transactions

When the electricity market operates in an environment of dimerous trade then each transaction agent is responsible to pay a part of the power system fixed cost. Similarly to the case of pool market, the form of a coalition between some players can be profitable by the existence of counter flows. Note that the allocation of fixed cost is made for each time interval and not at a peak load moment. Hence, power flows in opposite direction are the motivation for the cooperation between players rather than the difference between players' peak loads and coalition peak loads as in [87]. Using again the IEEE 14-bus network of Figure 2.6 consider the transactions of Table 4.5. The different sessions are incorporated just to investigate the game under various load patterns.

In the beginning the embedded cost allocation of the entire system is investigated by means of an AC power flow program. In Table 4.6 the network usage, as well as the characteristic function value for each coalition, regarding the first time period, are presented.

**Table 4.5**Transaction patterns

Transaction	From	То	Power [MW]					
Transaction	bus	bus	Session 1	Session 2	Session 3	Session 4		
1	1	4	30	50	50	60		
2	2	13	30	50	55	65		
3	3	14	45	55	55	65		
4	6	9	50	60	80	70		

Table 4.6
Use and characteristic function value for the 1st session

coalition	$f_S$ [MW]	v(S) [MW]
1	74.68	0
2	135.43	0
3	206.38	0
4	169.71	0
3,4	290.24	85.85
2,4	200.80	104.34
2,3	312.28	29.54
1,4	233.73	10.65
1,3	245.71	35.35
1,2	185.48	24.63
1,2,3	331.23	85.26
1,2,4	261.55	118.27
1,3,4	345.24	105.53
2,3,4	372.08	139.44
1,2,3,4	414.48	171.72

Similarly to the pool market case, the savings achieved by the grand coalition {1,2,3,4} should be shared by the four transaction agents. A Shapley value approach to this task is given in Table 4.7 together with the initial and final network usage for each agent.

An investigation of the game played at each single system branch may follow. Now,  $f_i$  is the power flow over a system branch caused by agent i. In this case it is possible that some coalitions will have negative values at some branches. Actually, a cooperation between a set of transactions results in a superposition of the single transaction patterns. Thus, in the worst case at a branch for a coalition S it should be v(S)=0. This would happen if no counter flows exist. The explanation for the negative v(S) is that, since an AC power flow is used, a generator located at the reference bus must cover the losses. The dispatch of this generator leads to a deviation between the power flow caused by a coalition S and the perfect superposition of its members' patterns. Thus, for some branches the sum of power flows caused by some players may be smaller than the power flow caused when these players form a coalition, i.e.  $f_S > \sum_{i=1}^{n_S} f_i$ . Consequently, negative characteristic function values will emerge. However, when the electricity market is organized according to a bilateral transaction model the fixed cost allocation game can be played at each single system branch. To cope with the problem of negative  $\nu(S)$  a DC power flow program should be used instead of an AC one. Thereby, the losses are neglected and the power flow of a coalition S over each system branch is the

**Table 4.7**Initial and final use for the players concerning the 1st session

superposition of its members' patterns.

player	$f_i$ [MW]	$\varphi_i$ [MW]	$f_i' = f_i - \varphi_i \text{ [MW]}$
1	74.68	21.40	53.28
2	135.43	47.35	88.08
3	206.38	41.81	164.57
4	169.71	61.16	108.55

113

## 4.4.4 Characteristics of the game

Up to this point the analysis has focused on the implementation of cooperative game theory in different types of electricity markets. It is interesting to extend the analysis by examining which of the cooperative game theory characteristics are applied in this case.

#### 4.4.4.1 Superadditivity

It is more likely to emerge coalitions when a game is superadditive. The mathematical expression of this feature has been presented in (4.6).

Consider the example of Figure 4.5. For each singe player coalition S it is v(S)=0. If any of the transactions T1 or T2 cooperates with transaction T3 then it is v(j,3)=200 MW, j=1,2. If T1 and T3 cooperate with each other then it is v(1,2)=0. Thus, for any single player coalitions S and  $\Theta$ , it is  $v(S \cup \Theta) \ge v(S) + v(\Theta)$ . Hence, (4.6) is satisfied. This can be generalized for any coalition. If a coalition  $\Theta$  joins a coalition S and  $\Theta$  exhibits counter flow in comparison to the power flow of S then in (4.6) the inequality holds. Otherwise, (4.6) is satisfied through the equality.

#### 4.4.4.2 No convex game

A game is said to be convex by satisfying the following condition:

$$v(\Theta \cup \{i\}) - v(\Theta) \ge v(S \cup \{i\}) - v(S), \ \forall i, \Theta, S \in N \ if \ S \subseteq \Theta$$
 (4.41)

In other words, (4.41) states that assume coalition S is a subgroup of coalition  $\Theta$ . Then the profit that any player i produces by joining coalition S should be larger or equal to the profit that i produces by joining coalition S. This condition may hold in case of power system fixed cost allocation game but not always. Consider the case which is presented in Table 4.6. For i={1}, S={3} and  $\Theta$  ={3,4} one finds out from Table 4.6 that  $v(\Theta \cup \{i\}) - v(\Theta)$ =19.68 MW while  $v(S \cup \{i\}) - v(S)$ =35.43 MW. Hence, (4.41) is not satisfied and therefore the game is not convex.

#### 4.4.4.3 Absence of dummy player

Dummy player, as described in (4.26), is the one who brings no benefit to any coalition which he joins. It is interesting to discuss if the fixed cost allocation game has dummy players. Assume that at over a line there exists at least one counter flow, as in Figure 4.5, so that the game is essential. For any single-player coalition it is v(i)=0. If any of T1, T2 cooperates with T3 then it is v(j,3)>0, j=1,2. Hence, all three transactions bring profit to at least one coalition. This conclusion is valid for any number of transactions. The only prerequisite is the existence of at least one counter flow. If there is no any counter flow over a particular line, then for any coalition S it will be v(S)=0. In this case, it is impossible for any player to bring profit and so the term of dummy player is not meaningful for this case. Consequently, in the power system fixed cost allocation game there is no dummy player.

### 4.4.5 The nucleolus in fixed cost allocation game

At this point it is worthwhile to discuss the performance of some basic solution methods of cooperative game theory in the context of fixed cost allocation game.

First, the nucleolus concept is considered. The advantage of the nucleolus solution is that it is part of the core. Thus, no other payoff vector can dominate the nucleolus over any coalition. When a payoff vector is not dominated then it is more likely for the players to accept it.

As already stated, a drawback of nucleolus concept is that it does not satisfy the demand for monotonicity. In electricity market, this fact means that the players may not always receive the right economic signal regarding the cost minimization. This would occur in case that an increase of v(S) of a coalition S, i.e. decrease of network usage, would not result in an increase of their payoff. Hence, this decrease of network use would not be connected with a reduction of the amount that the players have to pay for the use of the network. In this case, the players would not have the motivation to decrease the network usage.

Another disadvantage of nucleolus concept is that it favours some players. This can be

highlighted by studying the example presented in Figure 4.5. The solution resulted by nucleolus is presented in Table 4.8.

Table 4.8
Applying the nucleolus solution in electricity market

$f_1 = 100 \text{ MW}$	$n_1=0 \text{ MW}$	$f'_{l} = f_{l} - n_{l} = 100 \text{ MW}$
$f_2 = 100 \text{ MW}$	$n_2 = 0 \text{ MW}$	$f'_2 = f_2 - n_2 = 100 \text{ MW}$
$f_3 = 100 \text{ MW}$	<i>n</i> <sub>3</sub> =200 MW	$f'_3 = f_1 - n_1 = -100 \text{ MW} \rightarrow 0 \text{ MW}$

It is obvious that only the third transaction profits from this solution. Although there is no dummy player and all the three transactions contribute positive to at least one coalition the nucleolus concept is not beneficial for the two of them. Of course, this is an extreme paradigm. Generally, the differences, by the benefit share, are not so high.

The nucleolus concept remains an attractive solution despite these two drawbacks. The reason is that it gives allocation belonging to the core, when the core is not empty. The advantage of stable solutions is large enough so as to overcome the nucleolus' idiosyncrasies.

## 4.4.6 The Shapley value in fixed cost allocation game

In contrast to nucleolus, the Shapley value is monotone and so it can always send the right signal to market participants regarding the minimization of the costs paid for the network use. One of the axioms that characterize the Shapley value is that of additivity. Considering the embedded cost allocation game the additivity means that the payoff assigned to a player for the entire system game is equal to the sum of payoffs when the game is played at each system branch. Thus, using Shapley value, the network operator may analyse the allocation process at each branch attaining at the end the same result.

Defining the Shapley value the coalitional rationality of (4.12b) is not a requirement, so the Shapley value does not always belong to the core. Examining the case of Figure 4.5 one finds out that  $\varphi_1(v) = \varphi_2(v) = 33.3$  MW and  $\varphi_3(v) = 133.4$  MW. So,  $\varphi_1(v) + \varphi_3(v) < v(1,3) = 200$  MW and consequently the allocation obtained by

Shapley value is not part of the core. However, depending on the network topology, the number of players and their transaction patterns, Shapley value may belong to the core as well.

For the transaction patterns presented in Table 4.5, the differences  $\sum_{i=1}^{n_S} \varphi_i - v(S)$  are illustrated in Table 4.9.

Table 4.9

Difference between characteristic function value and Shapley values sum

Coalition	$\sum_{i=1}^{n_S} \varphi_i - v(S) \text{ [MW]}$						
	Session 1	Session 2	Session 3	Session 4			
1	21.40	32.71	33.96	40.33			
2	47.35	75.22	88.34	96.34			
3	41.81	53.57	59.90	63.48			
4	61.16	81.80	1.21	99.52			
3,4	17.12	32.04	41.79	42.19			
2,4	4.17	3.46	6.75	3.64			
2,3	59.62	82.78	98.50	101.21			
1,4	71.91	99.50	116.80	122.09			
1,3	27.86	37.50	45.09	45.74			
1,2	44.12	66.87	78.40	84.56			
1,2,3	25.30	32.62	46.71	39.48			
1,2,4	11.65	13.06	16.42	14.06			
1,3,4	18.84	35.10	44.	47.05			
2,3,4	10.88	17.97	17.93	22.53			
1,2,3,4	0	0	0	0			

The positive values for all but the grand coalition indicate that in this case the Shapley value allocation is coalitionally rational. Furthermore, the zeros at the last row show that the requirement of global rationality is satisfied. Consequently, the allocation vectors, given by the Shapley value for all the four time periods, belong to the core.

Figure 4.6 gives an explanation for this conclusion. To each of the 15 coalitions correspond 20 columns. Each column shows the difference  $\sum_{i=1}^{n_S} \varphi_i - v(S)$  for one of the

20 system branches with respect to the fourth time session. As can be seen, many coalitions have negative differences for some branches. But the decisive point is that the sum of differences over all the lines is positive for any coalition. Hence, the coalitional rationality holds for the entire system game.

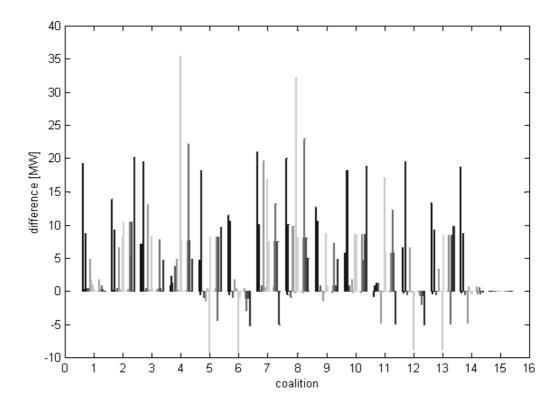


Figure 4.6 Differences between the characteristic function value and the sum of Shapley values at each line

If the fixed cost allocation takes place using (4.37) then the players faces in any case a cost reduction so it is probable that they will not object to the use of the Shapley value. Assume (4.36) is used and consider the first time period. In the beginning there is no set of players which submit a common schedule. Furthermore, it is made the assumption that the Shapley value is not incorporated in the allocation scheme. The corresponding amount that each player has to pay can be found in Table 4.10 in the second column. C is set at zero in order to denote the absence of common schedule submission by any set

of players. S equal to zero indicates that the Shapley value is not used by the network operator. If now the Shapley value is used the players should pay the amount given in the third column. Player 3 has to pay more and therefore may have an objection to the adoption of such an allocation method.

In an open deregulated electricity market player 3 cannot prevent other players to cooperate. The possible coalitions that can form, without player 3, are the  $\{1,2\}$ ,  $\{1,4\}$ ,  $\{2,4\}$  and  $\{1,2,4\}$ . Table 4.10 shows the cost allocation if any of these coalitions takes place. For each case the cost allocation is made first without using the Shapley value and then by incorporating the Shapley value in the allocation scheme. The difference  $D = \sum_{i \in S} R_i - R_S$  denotes how much each coalition saves within the framework of the

allocation scheme used at each time.

Table 4.10
On the acceptability of the Shapley value

		R and $D$ [% of $K$ ]										
Dlarran	C	S	C	S	С	S	С	S	C	S	C	S
Player	0	0	0	1	1	0	1	1	1	0	1	1
3	35.	.21	39.	71	36.	.75	39.	.61	35.	86	40.	49
1	12.	.74	12.	85	33.	.03	33.	.62	40.61	(1,4)	37.67	(1,4)
2	23.	.10	21.	25	D=2	2.74	D=(	).48	23.	53	21.	.84
4	28.	.95	26.	19	30.	.22	26.	.78	D=1	.08	D=1	1.37

	R and D [% of <i>K</i> ]							
Player	C	S	C	S	C	S	C	S
	1	0	1	1	1	0	1	1
3	42.83		42.66		44.10		43.34	
1	15.00		13.44		55.	.90	56.	.66
2	41.67		43.90					
4	D=1	0.38	D=3	3.54	D=8	3.89	D=3	3.63

Table 4.10 illustrates that in both cases, with and without the Shapley value, the coalitions {2,4} and {1,2,4} have the largest cost savings, as they are expressed by D. Thus, the probabilities these coalitions to form are higher than the probabilities for coalitions {1,2} and {1,4}.

As Table 4.10 indicates in the event of either {2,4} or {1,2,4} player 3 benefits from the use of the Shapley value. Hence, player 3 pays less in comparison to what this

player should pay if the Shapley value is not used and any of {2,4} or {1,2,4} will form. Thus, in this case the Shapley value protects player 3 against the most-probably-to-form coalitions. Hence, player 3 should not object to the use of the Shapley value.

## 4.4.7 Comparison of different methods' results

The network operator may have the intention to use some other solution methods rather than nucleolus and Shapley value. If political reasons demand the support of some weaker market participants then the Solidarity value will be a favourite candidate. In the event of a priori unions between some market participants, the network operator will rather use the Owen value than the Shapley value concept. Considering the first time session, results from the different solution methods have been calculated and they are presented in Table 4.11. The computation of Owen value is made under the assumption of an a priori union between the players 2 and 3.

Table 4.11
Results from different cooperative game theory concepts

	Shapley	Solidarity	Owen	Nucleolus
Player	Value	Value	Value	
	[MW]	[MW]	[MW]	[MW]
1	21.39	36.51	21.82	16.50
2	47.35	44.22	53.26	49.67
3	41.80	42.72	47.72	35.16
4	61.16	48.26	48.91	70.67

As can be seen, the Solidarity value results in a more 'flat' allocation of the savings between the market participants than the Shapley value. In contrast, by applying the nucleolus concept the player 4 is favoured. In case of nucleolus the spectrum of allocated payoffs, that is the difference between lowest and highest, is larger in comparison to any other solution method. However, the differences are not extreme. By

the implementation of Owen value the players 2 and 3, who have established the a priori union, profit.

In conclusion, it may be stated that the choice of the proper cooperative game theory concept depends on the particular aims, which have to be served by the network operator.

# 4.5 Usage based methods and the core

The use of game theory may be extended in order to assess the performance of the existing usage based methods. Such methods are widely used by the independent system operators so as to charge the market participants regarding the power system fixed cost. This section seeks to assess the usage based allocations regarding the core of the game. As already stated an allocation that belongs to the core is more likely to be accepted by the market participants. Furthermore, a new method which finds core points from usage based methods is presented.

## 4.5.1 The fixed cost allocation as cost game

In section 4.4, the fixed cost allocation game has been analysed as a savings game, indicated in (4.39). Generally, a cost allocation problem may be formed either as savings or as cost game. The latter means that the characteristic function value represents the costs caused by any coalition. In the case of fixed cost allocation game the characteristic function value of the cost game is given by:

$$c(S) = \sum_{l=1}^{n_l} |P_{S,l}| C_l$$
 (4.42)

where c(S) are the costs allocated to the coalition S. The form of (4.42) is same to the payment  $R_i$  given by (4.37). The only difference is that the solution of cost game is the amount that each player has to pay while the solution of a savings game is the amount

that is abstracted from the stand-alone payment of a player. In any case, the final payment for each player is the same no matter which kind of game, savings or cost, is applied. For the rest of section 4.5 the cost game form will be used.

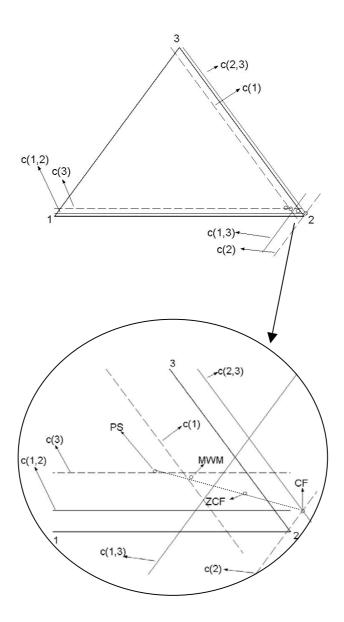
## 4.5.2 A three players' paradigm

In order to assess the performance of usage based allocations, regarding the core of the game, a paradigm with three transactions is considered. The marketplace is the AC 10-bus test system illustrated in Figure 3.4. The first transaction injects 15 MW at node 1 and withdraws them at node 2. The agent of the second transaction coordinates a purchase of 275 MW produced at node 3 and delivered at node 7. The third transaction involves the injection of 15 MW at node 10 and their take-over at node 4. This game, played as cost game, is geometrically illustrated in Figure 4.7. The core of the game embraces a limited space because of the large difference between the second transaction and the other two. This place is confined by all the lines corresponding to characteristic function value of one or two players' coalitions.

Using the equations of section 4.3 the cost allocations obtained by usage based methods may be determined. In order to make this allocations comparable to the game the total amount K, for the equations of section 4.3, is set equal to the cost of grand coalition c(N). These allocations are then located in the imputation space of the fixed cost allocation game. As can be seen, only the allocation resulted by zero counter flow method is placed into the core. The counter flow method allocation is located outside the triangle because player one is charged with a negative amount.

Although that some of the usage based methods do not yield core allocations such allocations may be obtained by combining several usage based methods. In paradigm of the three transactions, a convex combination of postage stamp and MW-mile method results in points those are located along the dashed line. A part of this line is inside the core.

Consequently, it is possible to obtain allocations that will be part of the game core by weighting properly the usage based methods. A systematic method in order to achieve this aim will be given below.



**Figure 4.7** Core of a three transactions game

# 4.5.3 Core points from usage based allocations

The empirical remark made at the end of section 4.5.2 gives the impulse to define a formal approach which will give core points from usage based methods. Let  $\Omega$  be the set of usage based allocations and  $\kappa_i^j$  the allocation to the *i*-th player by the *j*-th method. In order to find allocations that are as central to the core as possible, the

algorithm of (4.18) is used. This algorithm calculates the nucleolus of the game and so maximises the dissatisfaction of the most unhappy coalition. This will be also the criterion so as to find the optimal combination of the usage based methods. Each method j is weighted by a factor  $w_j$  which varies between zero and unit. The cost  $y_i$  allocated to any player i will be a convex combination of the weighted allocations obtained by the different usage methods. These prerequisites are summarized as follows:

$$y_{i} = \sum_{j \in \Omega} w_{j} \kappa_{i}^{j} \quad \forall i \in \mathbb{N}$$

$$\sum_{j \in \Omega} w_{j} = 1$$

$$w_{j} \geq 0 \quad \forall j \in \Omega$$

$$(4.43)$$

The conditions of (4.43) are incorporated in (4.18) and the linear program which calculates the optimal combination of the usage based methods is formulated as follows:

min 
$$e$$

$$s.t. y(S) + e \le c(S) \qquad \forall \ \Sigma^{0}$$

$$y_{i} = \sum_{j \in \Omega} w_{j} \kappa_{i}^{j} \qquad \forall i \in \mathbb{N}$$

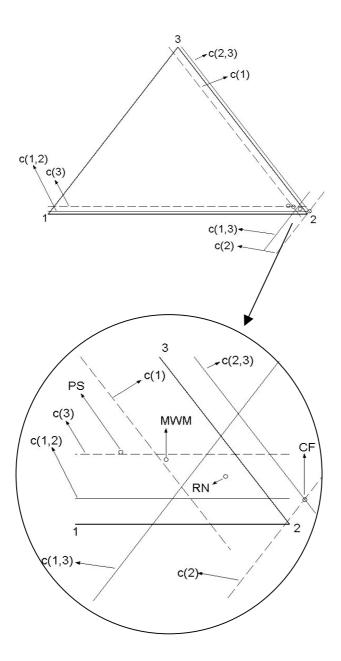
$$\sum_{j \in \Omega} w_{j} = 1$$

$$w_{j} \ge 0 \qquad \forall j \in \Omega$$

$$(4.44)$$

The problem of (4.44) is solved according to the process described in section 4.2.3. The resulting allocation vector  $\mathbf{y}$  is called restricted nucleolus (RN). This process of modifying the nucleolus algorithm is discussed in [90].

Regarding the three transactions paradigm of section 4.5.2 the set of the three methods that are outside the core is considered, i.e.  $\Omega = \{PS, MWM, CF\}$ . The new method presented in (4.44) results in the weights  $w_{PS} = 0$ ,  $w_{MWM} = 0.573$ ,  $w_{CF} = 0.427$ . Figure 4.8 illustrates the restricted nucleolus as well as the three usage based methods.



**Figure 4.8** The restricted nucleolus placed in the core.

The above presented procedure places the restricted nucleolus into the core of the game if the topology of the usage based methods enables this placement. Otherwise, this method brings the restricted nucleolus as close as possible to the core. In order to obtain a unique solution through (4.44) the allocations obtained by the usage based methods must be linearly independent. Furthermore, the number of players must be at least equal

to the number of methods in order to have enough equations to determine a unique solution.

In case that the set of methods consists of the MW-mile, counter flow and zero counter flow methods the allocations are not linearly independent. Assuming, for simplicity, that  $K = C_1 = 1$ , the usage measured by these three methods are:

$$f_i^{MWM} = \sum_{\ell} \max(P_{i,\ell}; -P_{i,\ell}), \ f_i^{CF} = \sum_{\ell} P_{i,\ell}, \ f_i^{ZCF} = \sum_{\ell} \max(P_{i,\ell}; 0)$$
 (4.45)

From (4.45) one easily finds out that  $f_i^{ZCF} = \frac{1}{2} \left( f_i^{MWM} + f_i^{CF} \right)$ . Thus, the cost allocated to a player i by the zero counter flow method can be written as:

$$ZCF_{i} = MWM_{i} \frac{\sum_{j \in N} f_{j}^{MWM}}{2\sum_{j \in N} f_{j}^{ZCF}} + CF_{i} \frac{\sum_{j \in N} f_{j}^{CF}}{2\sum_{j \in N} f_{j}^{ZCF}}$$
(4.46)

Hence, (4.46) states that the set of these three methods is not linearly independent. Therefore, in the framework of the proposed method, it is not possible the synchronous use of these three methods.

#### 4.5.4 Numerical results

The method outlined in section 4.5.3 could be used as a mechanism to support the design of a fixed cost allocation system. One of the central issues by such a system is the choice of the proper usage based method. The new method provides the flexibility to the network operator of using a synthesis of usage based methods by adjusting the optimal weights.

In order to illustrate further this method and investigate its possible application mode a case with seven transactions is considered. Each transaction corresponds to an agent and so there are seven players in the game. The marketplace is the 14-bus test system shown in Figure 4.9. The networks' data are given in Table C.2 of Appendix. The injection and delivery points of each transaction are presented in Table 4.12.

From the usage based methods, described in section 4.3, the following triples can be formed:  $\{PS, MWM, CF\}, \{PS, MWM, ZCF\}$  and  $\{PS, CF, ZCF\}$ . Neither the set  $\{PS, MWM, CF, ZCF\}$  nor the triple  $\{MWM, CF, ZCF\}$  can be incorporated in the method because of the linear dependency of MW-mile, counter flow and zero counter flow methods.

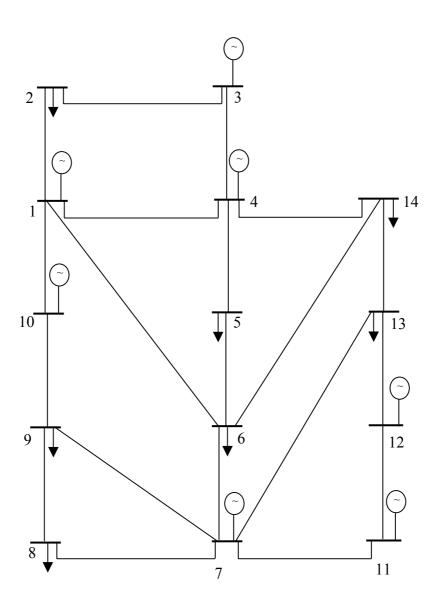


Figure 4.9 The 14-bus test system

Table 4.12
Transactions in the 14-bus test system

Transaction	Injection bus	Delivery bus
T1	1	14
T2	3	2
Т3	4	5
T4	7	6
Т5	10	13
T6	11	8
Т7	12	9

It is essential to examine the performance of the suggested method by different load patterns. Therefore, eight different cases are taken into account. In all the cases, the power traded by each of the first five transactions is fixed to 100 MW. The power of the sixth transaction varies from 150 MW to 300 MW keeping the seventh transaction fixed to 100 MW. The sixth transaction is fixed when it reaches the 300 MW and then the seventh transaction increases the delivering power, in steps of 50 MW, up to 300 MW.

Defining the eight different load patterns the fixed cost allocated to each agent by each usage based method is estimated.

Table 4.13
Weights of usage based methods. PS-MWM-CF

T1-T5 [MW]	T6 [MW]	T7 [MW]	PS	MWM	CF	RN
100	150	100	0.2829	0.0023	0.7148+	+
100	200	100	0.2160	0.0500	0.7340+	+
100	250	100	0.4673	0	0.5327+	+
100	300	100	0.4316	0	0.5684+	+
100	300	150	0.3818	0.1108	0.5074+	+
100	300	200	0.3265	0.0967	0.5768+	+
100	300	250	0.3996	0.0743	0.5261+	+
100	300	300	0.4138	0.1727	0.4135+	+

**Table 4.14**Weights of usage based methods. PS-MWM-ZCF

T1-T5 [MW]	T6 [MW]	T7 [MW]	PS	MWM	ZCF	RN
100	150	100	0.0299	0	0.9701	
100	200	100	0	0	1.0000	
100	250	100	0	0	1.0000	
100	300	100	0	0	1.0000	
100	300	150	0	0	1.0000	
100	300	200	0	0	1.0000	
100	300	250	0	0	1.0000	
100	300	300	0.0524	0	0.9476	

**Table 4.15**Weights of usage based methods. PS-ZCF-CF

T1-T5 [MW]	T6 [MW]	T7 [MW]	PS	ZCF	CF	RN
100	150	100	0.2829	0.0032	0.7138+	+
100	200	100	0.2160	0.0720	0.7120+	+
100	250	100	0.4673	0	0.5327+	+
100	300	100	0.4316	0	0.5684+	+
100	300	150	0.3818	0.1629	0.4553+	+
100	300	200	0.3265	0.1422	0.5313+	+
100	300	250	0.3996	0.1097	0.4907+	+
100	300	300	0.4138	0.2571	0.3292+	+

Then, for all the load patterns, the suggested method is applied using the three triples mentioned above. The total amount K is always equal to the corresponding cost of the grand coalition c(N). Tables 4.13, 4.14 and 4.15 show the weights of the usage based methods by applying the suggested method. The symbol + denotes that the corresponding allocation is part of the core. As can be seen, the weights are not stable. Their spectrum becomes quite wide by changing the load patterns. This remark is

applied to any triple with the exception of PS-MWM-ZCF where the zero counter flow method seems to have a dominating role. The weights' variation has also been noticed in other networks that have been investigated in the framework of this research work. However, from the Tables 4.13 and 4.15 it can be seen that for relative small changes in load patterns the weights may not drastically vary.

For an independent system operator it is of great significance to define a stable charging framework. For that reason, the weights should remain unchanged as much as possible. A solution to this problem is the allocation of fixed cost according to average weights. The ISO may calculate the weights corresponding to the usage based methods for any set of methods for an extended spectrum of load patterns. Then, the ISO can determine the average weight of each method. Using these average weights the ISO could charge the market participants for any load pattern. A regular updating of those weights, but not too often, taking into account the most current load patterns, would be necessary. This idea is implied in case of the three triples shown above. The average weights have been calculated and then the players have been charged for each case using those weights.

An essential criterion for the performance of any allocation vector  $\mathbf{y}$  is the excess e of the most dissatisfactory coalition. In case of cost game this excess is given as  $e = c(S) - \sum_{i \in S} y_i$ . Hence, the higher the excess the more satisfactory is the coalition.

This excess is presented in Table 4.16. For any load pattern the excess is calculated using the usage based methods, the suggested method based on the three triples as well as the average weights (Av) obtained for each of these three triples. A positive value indicates that the corresponding allocation is part of the core since the coalitional rationality holds even for the most dissatisfactory coalition. The indices 1, 2 and 3 refer to the triples PS-MWM-CF, PS-MWM-ZCF and PS-CF-ZCF respectively. The values in Table 4.16 are given as percentages of the grand coalition cost c(N) in order to be comparable. As can be seen, the triple of postage stamp, MW-mile and counter flow methods exhibits the highest excesses. This remark holds for both the restricted nucleolus and the average weights. In the most cases these excesses are equal to the ones of the triple of postage stamp, counter flow and zero counter flow. But in any case, the excesses of the first triple are at least as high as the excesses of the third triple.

Table 4.16
Performance of the smallest excess in the cost game

T1-T5	Т6	Т7	$e_{\min}[\inf \% \text{ of } c(N)]$					
[MW]	[MW]	[MW]	Av <sub>1</sub>	$RN_1$	Av <sub>2</sub>	RN <sub>2</sub>	Av <sub>3</sub>	RN <sub>3</sub>
100	150	100	2.04	3.20	-1.78	-1.59	2.01	3.20
100	200	100	0.49	2.93	-3.35	-3.30	0.47	2.93
100	250	100	2.89	3.25	-2.00	-1.92	2.89	3.25
100	300	100	2.20	2.40	-3.62	-3.53	2.20	2.40
100	300	150	1.86	2.17	-2.99	-2.91	1.86	2.17
100	300	200	1.57	1.88	-1.90	-1.82	1.57	1.88
100	300	250	0.96	1.02	-2.55	-2.51	0.96	1.02
100	300	300	0.67	1.15	-0.96	-0.79	0.67	1.15

T1-T5	Т6	T7	$e_{\min}$ [in % of $c(N)$ ]				
[MW]	[MW]	[MW]	PS	MWM	CF	ZCF	
100	150	100	-5.66	-5.42	0.20	-1.88	
100	200	100	-8.15	-7.79	1.52	-3.30	
100	250	100	-9.94	-9.53	0.32	-1.92	
100	300	100	-11.98	-11.54	0.54	-3.53	
100	300	150	-11.06	-11.36	0.09	-2.91	
100	300	200	-8.80	-9.68	0.07	-1.82	
100	300	250	-6.09	-7.78	0.21	-2.51	
100	300	300	-7.00	-5.39	0.31	-1.00	

The explanation of this phenomenon may be found from the remark made in section 4.5.3. Since ZCF is a convex combination of MWM and CF, any triple where ZCF substitutes one of MWM and CF cannot lead to a better placement of the combined allocation in comparison to the allocation obtained by the original triple. Thus, from all the triples investigated in the present work the most efficient is the set of postage stamp, MW-mile and counter flow methods.

A very important remark, made by observing Table 4.16, is that the allocations obtained from the average weights of the first and third triple are always part of the core. Hence, the network operator could adopt those average weights in order to provide a stable allocation scheme which yields core points.

### 4.6 Sensitivity based stand-alone usage

The last, chronologically, part of this research work has dealed with the issue of the stand-alone usage. In the short history of applying cooperative game theory in fixed cost allocation problem, the stand-alone usage has always been calculated considering each player alone in the network. The same philosophy has been followed in this research work. However, the increasing experience, obtained during the research, revealed some problems that appear because of this definition of stand-alone usage.

An alternative method, based on sensitivity calculations, which copes with these problems, has been developed. Despite the short time of working on this method, it should be useful to outline its principle and illustrate through a paradigm the performance of this method. The presentation of this method may be considered as an impulse for future work on this topic. It should be emphasized that the adoption of this method does not change at all the analysis described in the whole chapter 4. The only difference is that other values would be used as stand-alone usages.

### 4.6.1 The problem

When for selected players or coalitions the stand-alone network usage is calculated, taking into consideration by the load flow or OPF program only these players, several problems may arise.

First of all, if an AC program is used, indifferent if it is OPF or just a simple load flow program, it may be impossible to find a solution in large networks. This can happen in real interconnected networks where the AC will not converge if the player's load is just a small part of the total network capacity.

Furthermore, the generators' dispatch is much different at the operating point in comparison to the dispatch in order to serve the single players. This difference is large in case of pool market. For a bilateral market, using AC programs, this different is only due to dispatch of reference generator in order to cover the power losses.

In the case of DC load flow the convergence problem does not exist since DC load flow is a set of linear equations having always a solution. However, this algorithm provides only an approximation of the real network situation. Additionally, in case of pool market even using a DC-OPF program negative characteristic function values may arise when the game is played at each single system branch.

The above remarks point out that there is a demand of improving the way that the stand-alone costs are calculated.

### 4.6.2 Methodology

In the context of the research work, a new method has been developed in order to cope with the above problems. The method is numerical and it is based on the power flow sensitivities.

The philosophy of the new method is that the stand-alone usage should be determined considering the electricity market operating point. In this way the above outlined problems would not appear. The question is how could be estimated the separate usage of each player or coalition according to this philosophy. The power flow sensitivity seems to be a solution to this task. The use of sensitivities is a well-known tool among the engineers. In the investigated problem these sensitivities show the change in active power flow over a particular line with respect to change by the active and reactive load of each player. Thus, these sensitivities can be used as the basis for a new determination of the stand-alone usage.

The modern technology of computers, as well as the software development, enables the calculation of OPF or power flow in a very short time. Thus, there is no need for analytical estimation of the power flow sensitivities. Such sensitivities may be calculated by increasing a load in a marginal way and observing the change in power flow over a particular line.

Assume that there are n players participating in a marketplace. The symbol  $\tau_i$  denotes the active and the reactive power of player i.

133

The symbol  $P_{Line}$  denotes the active power flow over a particular line. In order to calculate the searched sensitivities, the power corresponding to each player is marginally increased and a new OPF or power flow is run. For each player this procedure consists of two phases. In the first phase the active power is increased while the reactive power remains unchanged. In the second phase the active power returns to its initial value and the reactive power is marginally increased. Hence, for the n players the number of the necessary calculations is 2n. The reactive power is taken into consideration because it affects the active power losses. The active power flow sensitivity, regarding the power change of player i, is:

$$\frac{\partial P_{Line}}{\partial \mathbf{\tau}_i} = \left[\frac{\partial P_{Line}}{\partial P_i} \frac{\partial P_{Line}}{\partial Q_i}\right] \tag{4.47}$$

where

 $\frac{\partial P_{Line}}{\partial P_i}$ : Change in power flow over a particular line caused by a marginal change in active power of player i

 $\frac{\partial P_{Line}}{\partial Q_i}$ : Change in power flow over a particular line caused by a marginal change in reactive power of player i

If all the loads, active and reactive, were simultaneously changed, the difference in power flow over a particular line would be:

$$\Delta P_{Line} = \frac{\partial P_{Line}}{\partial \mathbf{\tau}_1} \Delta \mathbf{\tau}_1^T + ... + \frac{\partial P_{Line}}{\partial \mathbf{\tau}_i} \Delta \mathbf{\tau}_i^T + ... + \frac{\partial P_{Line}}{\partial \underline{\mathbf{\tau}}_n} \Delta \mathbf{\tau}_n^T$$
(4.48)

where

 $\Delta P_{Line}$ : change in active power flow over a particular line

 $\Delta \tau_i$ : change in power of player *i* 

The vector of sensitivities, shown in (4.47), may be denoted as a factor  $\eta_i$ . If the power flow would represent a linear problem, then Equation (4.48) could be generalized. That is, the total active power flow over a particular line would be given as follows:

$$P'_{Line} = \mathbf{\eta}_1 \mathbf{\tau}_1^T + ... + \mathbf{\eta}_i \mathbf{\tau}_i^T + ... + \mathbf{\eta}_n \mathbf{\tau}_n^T$$
(4.49)

However, if AC is used, neither power flow nor optimal power flow is linear. In particular, the optimal power flow problem exhibits a strong non-linear character. Thus, it is always  $P'_{Line} \neq P_{Line}$ . In order to overcome this mismatch a diorthosis of the factor  $\eta_i$  is necessary. The right factors  $\eta_i^*$  should result in power flow equal to the real one. Thus, the diorthosis is made through the following equation:

$$\mathbf{\eta}_{i}^{*} = \mathbf{\eta}_{i} \frac{P_{Line}}{P_{Line}^{'}} \tag{4.50}$$

Using the new factors one may retrieve the real active power flow over a line:

$$P_{Line} = \mathbf{\eta}_{1}^{*} \mathbf{\tau}_{1}^{T} + ... + \mathbf{\eta}_{i}^{*} \mathbf{\tau}_{i}^{T} + ... + \mathbf{\eta}_{n}^{*} \mathbf{\tau}_{n}^{T}$$
(4.51)

From (4.51), it is obvious that the stand-alone usage of player i, considering a particular line, may be defined as:

$$f_{Line,i} = \left| P_{Line,i} \right| C_{Line} = \left| \mathbf{\eta}_{i}^{*} \mathbf{\tau}_{i}^{T} \right| C_{Line}$$
 (4.52)

For any coalition S its stand-alone usage is the absolute value of the sum of its members' power flows, weighted by the specific transfer cost:

$$f_{Line,S} = \left| C_{Line} \sum_{i=1}^{n_S} P_{Line,i} \right| \tag{4.53}$$

The new method is consistent with the philosophy of stand-alone usage since it calculates how each participant uses the network facilities. Moreover, it takes into consideration the idiosyncrasies of the electric networks.

In comparison to the existing definition of the stand-alone usage, the new method has the following advantages:

- The new method considers the operating point. Thus, the calculated usages refer to the real network situation.
- There is no convergence problem in any case. Full AC programs can be used in order to calculate the usages. Thereby, the exact situation is taken into consideration and not an approximation, as with the DC programs.
  - The new method has a great benefit considering the calculation time. The 2n calculations, needed for the sensitivities' estimation, can be carried out in a short time. After this estimation the proposed method needs  $2^n$  calculations in order to determine the usage for each coalition. This is the same number with the calculations needed by the existing method. However, there is a significant difference. The proposed method will execute  $2^n$  algebraic calculations, as (4.53) indicates. The existing stand-alone definition requires  $2^n$  power flow or OPF calculations. In case of OPF the best programs, running on the faster computers, needs about 0.1 second to calculate one case. So, assuming 20 players the required time is about 100000 seconds or more than a day. The same number of algebraic calculations can be executed within 1-2 minutes. Hence, the proposed method copes with the most serious, for the time being, problem of applying game theory in fixed cost allocation. This is the problem of time. Of course, the new method is also limited. But in a wholesale market consisting of 20-30 players it is possible to implement this method.
- The sum of stand-alone usages calculated by the suggested method matches exactly the real power flow over any line.

• Using the sensitivity based method the motivation of forming a coalition is only due to the possible counter flows, as (4.53) indicates. In the existing method the players may have the intention to form a coalition even in absence of counter flow. This could be possible if the different generators' dispatch, considering the coalition and the single players, results in lower line usage. But the motivation to the game should be based rather on realistic facts, such as the counter flows, than on fictitious situations such as the use of the whole network from one player.

One the other hand, the suggested method has a single drawback. The deviation between  $P'_{Line}$  and  $P_{Line}$  can be very large. Of course, the factor diorthosis eliminates this deviation.

### 4.6.3 Numerical results

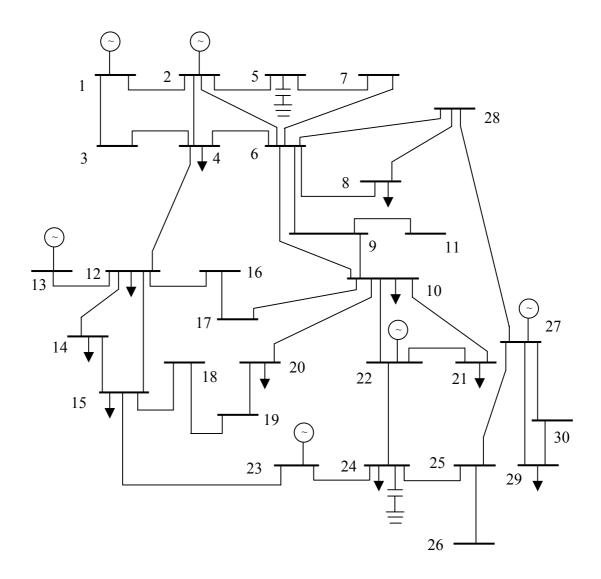
The sensitivity based method has been tested in the IEEE 30-bus network, Figure 4.10, in order to illustrate its performance. The network's data are given in Table C.3, in Appendix C.

The marketplace is a mandatory pool, as the one described in section 4.4.2, and consists of six suppliers and ten consumers. The consumers have inelastic demand and they are considered as the fixed cost allocation game player.

In order to highlight the different performance of the suggested method, its result will be compared to the usage obtained by the existing method. Furthermore, results from a version of the existing method will be presented.

The existing method consists of calculating the stand-alone usage as if the player, or coalition, were alone in the network. Thus, in the investigated case an OPF calculation will be executed for each single player. The results of this calculation will be the standalone usage.

Another way of calculating the stand-alone usage  $f_S$  of a coalition S is, if the usage is given as the difference  $f_N - f_{N-S}$  between the grand coalition usage and the usage of the coalition of the N-S players. In this case, the incremental change in line power flow is considered as the stand-alone usage.



**Figure 4.10** The IEEE 30-bus system

In the framework of this work, the existing method is called upstream method because it begins from single players and goes up to the grand coalition. The modified version of the existing method is likewise called downstream method.

Considering each of these three methods, the stand-alone usage of the single players has been calculated. At all the network lines the specific transfer cost  $C_l$  is set at 1  $\mbox{\colored}/MW$ . Thus, power flow and the line usage will be identical, regarding the absolute value. The number of players, ten, results in 1024 possible coalitions. For that reason, only the single-players usage is presented.

Table 4.17
Stand-alone power flow from different methods. IEEE 30-bus, line 1-3

	Upstrea	m method	Downstrea	am method	Sensitivit	ty method
D.I.	$P_{Line,i}$	$P_{Line,i}$	$P_{Line,i}$	$\frac{P_{Line,i}}{D}$	$P_{Line,i}$	$\frac{P_{Line,i}}{D}$
Player		$P_{Line,N}$		$P_{Line,N}$		$P_{Line,N}$
	[MW]	[%]	[MW]	[%]	[MW]	[%]
1 (bus 4)	3.23	11.49	2.51	8.93	3.70	13.16
2 (bus 8)	2.19	7.79	1.45	5.16	2.13	7.58
3 (bus 10)	2.42	8.61	1.82	6.47	2.68	9.53
4 (bus 12)	2.86	10.17	2.17	7.72	3.14	11.17
5 (bus 14)	2.93	10.42	2.14	7.61	3.26	11.60
6 (bus 15)	2.79	9.93	2.02	7.19	2.99	10.64
7 (bus 20)	2.60	9.25	1.93	6.87	2.91	10.35
8 (bus 21)	2.43	8.64	1.80	6.40	2.66	9.46
9 (bus 24)	2.55	9.07	1.31	4.66	2.56	9.11
10 (bus 29)	2.59	9.21	1.07	3.81	2.08	7.40
Sum 1-10	26.59	94.58	18.22	64.82	28.11	100.00

Table 4.18
Stand-alone power flow from different methods. IEEE 30-bus, line 6-28

	Upstream method		Downstrea	Downstream method		y method
	$P_{Line,i}$	$P_{Line,i}$	$P_{Line,i}$	$P_{Line,i}$	$P_{Line,i}$	$P_{Line,i}$
Player		$P_{Line,N}$		$P_{Line,N}$		$P_{Line,N}$
	[MW]	[%]	[MW]	[%]	[MW]	[%]
1 (bus 4)	-0.82	-106.49	-4.22	-548.05	0.19	24.68
2 (bus 8)	1.66	215.58	-1.93	-250.65	0.09	11.69
3 (bus 10)	0.67	87.01	-2.61	-338.96	0.12	15.58
4 (bus 12)	0.25	32.47	-2.98	-387.01	0.13	15.58
5 (bus 14)	0.45	58.44	-2.79	-362.34	0.13	16.88
6 (bus 15)	0.68	88.31	-2.57	-333.77	0.12	15.58
7 (bus 20)	0.63	81.82	-2.67	-346.75	0.12	15.58
8 (bus 21)	0.92	119.48	-2.27	-294.81	0.10	14.29
9 (bus 24)	2.73	354.55	-0.88	-114.29	0.03	3.91
10 (bus 29)	10.64	1381.82	3.78	490.91	-0.26	-33.77
Sum 1-10	17.81	2312.99	-19.84	-2485.71	0.77	100.00

139

Nevertheless, these usages provide enough information in order to assess the performance of the three methods. In Tables 4.17 and 4.18 the stand-alone power flows regarding two of the system lines are presented.

As can be seen, the suggested method is the only one which matches the real active power flow. In the case of line 1-3, the two other methods underestimate the power flow. A very impressive result is shown in Table 4.18. The existing method, as well as its modified version, yields deviation of more than 2300% from the real power flow. Of course, these large differences are an extra motivation, except of counter flows, for the players to cooperate but as it already stated it should be better if the motivation were based only on more realistic arguments, such as the counter flows. This paradigm highlights also the difference between generators' dispatch considering the grand coalition and the dispatch regarding the single players. Another remark, made by observing both Table 4.17 and Table 4.18, is the large difference between the upstream and the downstream method. Both of them are consistent with the narrow definition of the stand-alone usage. However, the large differences indicate that it is very important if the OPF calculation is carried out starting from each single player or by computing the incremental changes.

In conclusion, it may be stated that the proposed method overcomes some significant problems arising from the existing definition of stand-alone usage. Further work towards a more proper definition of stand-alone usage may result in an improved incorporation of cooperative game theory in fixed cost allocation problem.

## Chapter 5

# **Epilogue**

### 5.1 Synopsis

The electricity industry has undergone a dramatical change in many countries during the last years. The demand for higher economic efficiency as well as the political trends of market opening have been resulting in deregulation of the power sectors. The existing high voltage networks and the increasing efficiency of generation technology have facilitated this restructuring. The next phase of the deregulation process is the emerging of inter-regional markets, such as the Internal Market of Electricity in European Union, and the coordination of the existing liberalised markets, which can be faced in the FERC's directive requiring a Standard Market Design in USA.

This PhD thesis deals with the issue of congestion management in markets operating under the pool model. The congestion component of nodal prices has been analysed. Moreover, a method for the allocation of this component to the market participants has been suggested. The usefulness of this method is highlighted by the different share of suppliers in generation and congestion component. This difference points out that the ISO should not take into consideration only the suppliers' output by applying the congestion management. Additionally, in the context of this work, the issue of both

suppliers' and consumers' behaviour, regarding the bid prices, in a pool market has been addressed. Bid price reductions have been found to lead to an increased impact, from the side of the corresponding participant, on power flows over the lines with changed congestion situation. The ISO may take advance of this finding by the set up of an efficient congestion management framework.

Furthermore, the implementation of the pool model in a real market, such as the Nord Pool, has been investigated. Particularly, the incorporation of loss factors and zonal pricing has been compared to the performance of a nodal pricing mechanism. In order to facilitate this comparison, a theoretical model, which enables the calculations of zonal prices using nodal pricing algorithm, has been developed. The conclusion obtained by this comparison is that the more administrative rules are implied the higher the deviation from the nodal prices will be.

Beyond the costs caused by system congestions, the research work has focused on the allocation of the power system fixed cost to the market participants. For this purpose, the incorporation of cooperative game theory has been analysed. The methods of this theory seems to provide stable and economic efficient allocations. A conclusion drawn by examining different types of electricity market is that the fixed cost allocation should take place rather in the whole network than in single branch level. A further part of this research was the development of a new method based on the existing usage based allocation methods. The new method, using the nucleolus philosophy, provides allocations that are more likely to be accepted by the market participants. Additionally, a new approach to the calculation of the stand-alone usage has been presented. This approach overcomes problems arising by the traditional definition of stand-alone usage.

### 5.2 Future research work

The deregulation of the electricity market brings numerous challenges. The integration of local markets, operating under different modes, in an inter-regional market requires further investigation. The coordination of several congestion management mechanisms, as well as different fixed cost tariffs, is a field where the present work can be continued.

A further research topic is made up by the ancillary services that are necessary for the harmonious performance of the network. Among them the consideration of reactive power in congestion situations forms a special challenge.

A central aim of this work was to combine the technical side of the power systems with the economic nature of a market. In this context, the development of methods that will deal with the hedging of risk is a field, which calls for more research. The risk in a

143

deregulated market arises, mainly, through price volatility but also because of uncertainties regarding regulatory and technical issues. Therefore, all of these different kind of risks should be further investigated.

Finally, the use of game theory in the deregulated electricity market is a field where the future research will focus on. The investigation of marketplaces with a large number of players should be realised. An engineer, working on the electricity market, could take advance of game theoretical methods in order to cope with this problem. However, the idiosyncrasies of the electric networks should be taken into account. The definition of stand-alone usage is only one topic where improvements have to be done.

- [1] S. Stoft, *Power System Economics*. Piscataway: IEEE Press, 2002, pp. 6-16.
- [2] J.J. Alba Ríos, "The liberalisation of the electricity industry: Reasons and alternative structures, the organisation of wholesale electricity markets", in *Proc.* of "Liberalisation and modernization of power systems: Operation and control problems" Conference, 2000, pp. 6-18.
- [3] F.A. Wolak, R.H. Patrick, "Industry structure and regulation in the England and Wales electricity market", in *Pricing and regulatory innovations under increasing competition*, 3rd ed., M.A. Crew, Ed. Boston: Kluwer Academic Publ., 1999, pp. 65-90.
- [4] M. Huneault, F.D. Galiana, G. Gross, "A review of restructuring in the electricity business", in *Proc. 13th PSCC*, 1999, pp. 19-31.
- [5] Federal Energy Regulatory Commission (1996, April). Promoting wholesale competition through open access nondiscriminatory transmission services by public utilities. Available: http://www.ferc.gov/About/mission/anual96.htm
- [6] A.L. Ott, "Experience with PJM market operation, system design, and implementation", *IEEE Trans. on Power Systems*, vol. 18, pp. 528-534, May 2003.
- [7] C.K. Woo, D. Lloyd, A. Alberta, "Electricity market reform and failures: UK, Norway, Alberta and California", *Energy Policy*, vol. 31, pp. 1103-1115, September 2003.
- [8] T.J. Brennan, K.L. Palmer, S.A. Martinez, *Alternating currents: Electricity markets and public policy*. Washington, DC: RFF Press, 2002, pp. 46-58.

[9] N.S. Rau, "Issues in the path toward an RTO and standard markets", *IEEE Trans.* on *Power Systems*, vol. 18, pp. 435-443, May 2003.

- [10] C. Osana. (2002, Sept.). Regulatory reform in the electricity supply industry: An overview. International Energy Agency. Available: http://www.iea.org/about/divers.htm
- [11] H. Rudnick, J. Zolezzi, "Electric sector deregulation and restructuring in Latin America: Lessons to be learnt and possible ways forward", *IEE Proc. Gener. Transm. Distrib.*, vol. 148, pp. 180-184, March 2001.
- [12] B. Girdwood, "In New Zealand, LMP's rated a success not a problem", *The Electricity Journal*, vol. 16, pp. 2-4, Jan.-Feb. 2003.
- [13] H.R. Outhred, "Some strengths and weaknesses of electricity industry restructuring in Australia", in Proc. *Power Tech Conference*, 2003.
- [14] H. Asano, K. Okada, "Evaluation of cost based transmission pricing in Japan", International Journal of Global Energy Issues, vol. 11, pp. 146-154, 1998.
- [15] Power Line (2003, Apr.). Electric utility deregulation and the need for nuclear recycling. The Federation of Electric Power Companies of Japan. Available: http://www.fepc.or.jp/english/menu/public/newslette.html
- [16] East Asia and Pacific regional office (2003, Feb.). Project appraisal document for Yixing pumped storage project. World Bank. Available: http://www-wds.worldbank.org/
- [17] L. Monari (2002, Oct.). India power sector reform and the poor. World Bank. Available: http://www-wds.worldbank.org/
- [18] Africa regional office (2001, May). Project appraisal document to the federal government of Nigeria for a privatisation support project. World Bank. Available: http://www-wds.worldbank.org/
- [19] A. Eberhard. (2003, Feb.). The political, economic, institutional and legal dimensions of electricity supply industry reform in South Africa. Presented at 'The political economy of power market reform conference'. Available: http://pesd.stanford.edu/events/mrkt\_rfm\_2003.html

[20] S. Hesmondhalgh, "Is NETA the blueprint for wholesale electricity trading arrangements of the future?", *IEEE Trans. on Power Systems*, vol. 18, pp. 548-554, May 2003.

- [21] N. Flatabø, G. Doorman; O.S. Grande, H. Randen, I. Wangensteen, "Experience with the Nord Pool design and implementation", *IEEE Trans. on Power Systems*, vol. 18, pp. 541-547, May 2003.
- [22] RAO-UESR. (2003, June). RAO UES of Russia: Restructuring. Available: http://www.rao-ees.ru/en/reform/
- [23] Council and European Parliament, "Directive 96/92/EC of the European parliament and of the Council of December 19, concerning common rules for the internal market in electricity", Official Journal, L 027, 30.01.1997, pp. 0020-0029.
- [24] BDI, VIK, VDEW, "Verbändevereinbarung über Kriterien zur Bestimmung von Durchleitungsentgelten", 22.05.1998, Available: http://www.eta-energieberatung.de/aktuell/gesetze/VV1.pdf
- [25] BDI, VIK, VDEW, "Verbändevereinbarung über Kriterien zur Bestimmung von Netznutzungsentgelten für elektrische Energie", 13.12.1999, Available: http://www.vik-online.de/infocenter/default.htm
- [26] BDI, VIK, VDEW, "Verbändevereinbarung über Kriterien zur Bestimmung von Netznutzungsentgelten für elektrische Energie und über Prinzipien der Netznutzung", 13.12.2001, Available: http://www.vikonline.de/infocenter/default.htm
- [27] F.L.P. Montero, I.J. Pérez-Arriaga, F.J.R. Odériz, "Benchmark of electricity transmission tarrifs in the countries of the internal electricity market of the European Union", *Utilities Policy*, vol. 10, pp. 47-56, March 2001.
- [28] European Commission. (2003, Jul.). Electricity. European Union. Bruxelles. Available: http://europa.eu.int/comm/energy/electricity/index en.htm
- [29] D. Shirmohammadi, B. Wollenberg, A. Vojdani, P. Sandrin, M. Pereira, F. Rahimi, T. Schneider, B. Stott, "Transmission dispatch and congestion

- management in the emerging market structure", *IEEE Trans. on Power Systems*, vol. 13, pp. 1466-1474, November 1998.
- [30] F.D. Galiana, I. Kockar, P.C. Franco, "Combined pool/bilateral dispatch-Part I: Performance of trading strategies", *IEEE Trans. on Power Systems*, vol. 17, pp. 92-99, February 2002.
- [31] R. Green, "Electricity transmission pricing: an international comparison", *Utilities Policy*, vol. 6, pp. 177-184, September 1997.
- [32] K.W. Cheung, P. Shamsollahi, D. Sun, J. Milligan, "Energy and ancillary services dispatch for the interim ISO New England electricity market", *IEEE Trans. on Power Systems*, vol. 15, pp. 968-974, August 2000.
- [33] H. Singh, A. Papalexopoulos, "Competitive procurement of ancillary services by an Independent System Operator", *IEEE Trans. on Power Systems*, vol. 14, pp. 498-504, May 1999.
- [34] European Commission. (2003, Apr.). Second benchmarking report on the implementation of the internal electricity and gas market. European Union. Bruxelles. http://europa.eu.int/comm/energy/electricity/index en.htm
- [35] G.R. Davison, "Dividing load between units", *Electrical World*, December 23, pp. 1385-1387, 1922.
- [36] J. Carpentier, "Contribution á l'étude du dispatching économique", *Bulletin de la Société Française des Electriciens*, ser. 8, vol. 3, pp.431-447, 1962.
- [37] F.C. Sweppe, M.C. Caramanis, R.D. Tabors and R.E. Bohn, *Spot pricing of electricity*. Boston. Kluwer Academic Publishers, 1988, pp. 31-53.
- [38] J.D. Finney, H.A. Othman, W.L. Rutz, "Evaluating transmission congestion constraints in system planning", *IEEE Trans. on Power Systems*, vol. 12, pp. 1143-1150, August 1997.
- [39] H.W. Dommel, W.F. Tinney, "Optimal power flow solutions", *IEEE Trans. on PAS*, vol. 87, pp. 1866-1876, 1968.
- [40] J. Peschon, D.W. Bree, L.P. Hadju, "Optimal solutions involving system security", in *Proc. of PICA Conference*, May 1971, pp. 210-218.

[41] B. Stott, J.L. Marinho, "Linear programming for power system network security applications", *IEEE Tans. on PAS*, vol. 98, May/June 1979, pp. 837-848.

- [42] E. Hobson, D.L. Fletcher, W.O. Stadlin, "Network flow linear programming techniques and their application to fuel scheduling and contingency analysis", *IEEE Trans. on PAS*, vol. 103, July 1984, pp. 1684-1691.
- [43] H. Nicholson, M.J.H. Sterling, "Optimum dispatch of active and reactive generation by quadratic programming", *IEEE Trans. on PAS*, vol. 92, pp. 644-654, 1973.
- [44] R.C. Burchett, H.H. Happ, D.R. Vierath, "Quadratically convergent optimal power flow", *IEEE Trans. on PAS*, vol. 103, pp. 3267-3275, 1984.
- [45] D.I. Sun, B. Ashley, B. Brewer, A. Hughes, W.F. Tinney, "Optimal power flow by Newton method", *IEEE Trans. on PAS*, vol. 103, October 1984, pp. 2864-2880.
- [46] G.A. Maria, J.A. Findlay, "A Newton optimal power flow program for Ontario Hydro EMS", *IEEE Trans. on Power Systems*, vol. 2, August 1987, pp. 576-584.
- [47] H. Glavitsch, R. Bacher, "Optimal power flow algorithms", Swiss Federal Institute of Technology, Zürich, 1991.
- [48] F.F. Wu, P. Varaiya, "Coordinated multilateral trades for electric power networks: theory and implementation", *International Journal of Electrical Power & Energy Systems*, vol. 21, pp. 75-102, February 1999.
- [49] H.H. Wilfert, J. Fischer, "A program for the computation of locationally varying short-term marginal prices of active and reactive power in large scale transmission networks", in *Proc. of EPSOM'98*, 1998.
- [50] L. Chen, H. Suzuki, T. Wachi, Y. Shimura, "Components of nodal prices for electric power systems", *IEEE Trans. on Power Systems*, vol. 17, pp. 41-49, February 2002.
- [51] I.N. Bronstein, K.A. Semendjajew, G. Musiol, H. Muehlig, *Taschenbuch der Mathematik*. Frankfurt am Main. Verlag Harri Deutsch, 1999, pp. 841-877.

[52] G. Yarrow, "Capacity auctions in the UK energy sector", *Utilities Policy*, vol. 11, pp. 9-20, March 2003.

- [53] R.S. Fang, A.K. David, "Transmission Congestion management in an electricity market", *IEEE Trans. on Power Systems*, vol. 14, pp. 877-883, August 1999.
- [54] T.W. Gedra, "On transmission congestion and pricing", *IEEE Trans. on Power Systems*, vol. 14, pp. 241-248, February 1999.
- [55] W.W. Hogan, "Contract networks for electric power transmission", *Journal of regulatory economics*, vol. 4, pp. 211-242, September 1992.
- [56] R.P. O'Neill, U. Helman, B.F. Hobbs, W.R. Stewart, M.H. Rothkopf, "A joint energy and transmission rights auction: Proposal and properties", *IEEE Trans. on Power Systems*, vol. 17, pp. 1058-1067, November 2002.
- [57] A. Bakirtzis, "Aumann-Shapley transmission congestion pricing", *IEEE Power Engineering Review*, vol. 21, pp. 67-69, March 2001.
- [58] E. Allen, M. Ilic, Z. Younes, "Providing for transmission in times of scarcity: an ISO cannot do it all", *International Journal of Electrical Power and Energy Systems*, vol. 21, pp. 147-163, February 1999.
- [59] W.Y. Ng, "Generalized Generation Distribution Factors for power system security evaluations", *IEEE Trans. on PAS*, vol. 100, pp. 1001-1005, March 1981.
- [60] J. Bialek, "Tracing the flow of electricity", *IEE Proc. Gener. Transm. Distrib.*, vol. 143, pp. 313-320, 1996.
- [61] R.D. Christie, B.F. Wollenberg, I. Wangenstein, "Transmission management in the deregulated environment", *Proceedings of the IEEE*, vol. 88, pp. 170-194, 2000.
- [62] D. Grgic, F. Gubina, "New generation distribution factors for active and reactive powers transmission costing", in *Proc. of Power Tech'99 Conference*, 1999.
- [63] P. Marannino, R. Vailati, F. Zanellini, E. Bompard, G. Gross, "OPF tools for optimal pricing and congestion management in a two sided auction market structure", in *Proc. of the IEEE Power Tech 2001 Conference*, 2001.
- [64] S. Stoft, *Power System Economics*. Piscataway: IEEE Press, 2002, pp. 390-394.

[65] R.D. Zimmerman, D. Gan. (1997, Dec.). User's manual for MATPOWER. Cornell University. Available: http://blackbird.pserc.cornell.edu/matpower/

- [66] F. Wen, A. Kumar David, "Optimal bidding strategies and modeling of imperfect information among competitive generators", *IEEE Trans. on Power Systems*, vol. 16, pp. 15-21, February 2001.
- [67] I. Herguera, "Bilateral contracts and the spot market for electricity: some observations on the British and the NordPool experiences", *Utilities Policy*, vol. 9, pp. 73-80, June 2000.
- [68] M. Bjørndal, K. Jörnsten, "Zonal pricing in a deregulated energy market", *The Energy Journal*, vol. 22, pp. 51-73, January 2001.
- [69] W. Hogan (1999, Feb.). Transmission congestion: the nodal-zonal debate revisited. Kennedy School of Government, Harvard University. Available: http://ksghome.harvard.edu/~.whogan.cbg.Ksg/
- [70] H.Y. Yamina, S.M. Shahidehpour, "Congestion management coordination in the deregulated power market", *Electric Power System Research*, vol. 65, pp. 119-127, May 2003.
- [71] The Statnett Company. Powermarket, the power system. Available: www.statnett.no
- [72] J.W.M., Lima, "Allocation of transmission fixed charges: an overview", *IEEE Trans. on Power Systems*, vol. 11, pp. 1409-1418, November 1996.
- [73] J. von Neumann, "Zur Theorie der Gesellschaftsspiele", *Mathematische Annalen*, vol. 100, pp.295-320, 1928.
- [74] J. von Neumann, O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, 3rd Edition, 1963.
- [75] J. Nash, "Non-cooperative games", *Annals of Mathematics*, vol. 54, pp. 286-295, 1951.
- [76] D.B. Gillies, "Solutions to general non-zero-sum games", in *Contributions to the Theory of Games, Vol. IV (Ann. Math. Studies, No. 40)*, A.W. Tucker and D.R. Luce, Ed. Princeton: Princeton University Press, 1959, pp. 47-85.

[77] D. Schmeidler, "The nucleolus of a characteristic function game", *S.I.A.M. Journal of Applied Mathematics*, vol. 17, pp. 1163-1170, November 1969.

- [78] A. Kopelowitz, "Computation of the kernels of simple games and the nucleolus of n-person games", The Hebrew University of Jerusalem, Jerusalem, RM 31, Research program in game theory and mathematical economics, 1967.
- [79] L.S. Shapley, "A value for n-person games", in *Contributions to the Theory of Games II*. Princeton. Princeton University Press, pp. 307-317, 1953.
- [80] A.S. Nowak, T. Radzik, "A solidarity value for n-person transferable utility games", *International Journal of Game Theory*, vol. 23, pp. 43-48, 1994.
- [81] G. Owen, "Values of games with a priori unions", in *Essays in Mathematical Economics and Game Theory*, R. Henn and O. Moeschlin, Ed. Springer-Verlag, 1977, pp. 76-88.
- [82] L.C. Thomas, *Games, Theory and Applications*. Chichester: Ellis Horwood Limited, 1984, pp. 85-111.
- [83] J. Pan, Y. Teklu, S. Rahman, K. Jun, "Review of usage-based transmission cost allocation methods under open access", *IEEE Trans. on Power Systems*, vol. 15, pp. 1218-1224, November 2000.
- [84] H.H. Happ, "Cost of wheeling methodologies", *IEEE Trans. on Power Systems*, vol. 9, pp. 147-155, February 1994.
- [85] D. Shirmohammadi, P.R. Gribik, E.T.K. Law, J.H. Malinowski, R.E. O'Donnell, "Evaluation of transmission network capacity use for wheeling transactions", *IEEE Trans. on Power Systems*, vol. 4, pp. 1405-1413, October 1989.
- [86] H. Singh, S. Hao, A. Papalexopoulos, M. Obessis. [1996, Aug.). Cost allocation in electric power networks using cooperative game theory. Ecco Intl., CA. Available: http://www.eccointl.com/pub\_publications.html
- [87] Y. Tsukamoto, I. Iyoda, "Allocation of fixed transmission cost to wheeling transactions by cooperative game theory", *IEEE Trans. on Power Systems*, vol. 11, pp. 620-629, May 1996.

[88] J. Contreras, F.F. Wu, "A Kernel-oriented algorithm for transmission expansion planning", *IEEE Trans. on Power Systems*, vol. 15, pp. 1434-1440, November 2000.

- [89] J.M. Zolezzi, H. Rudnick, "Transmission cost allocation by cooperative games and coalition formation", *IEEE Trans. on Power Systems*, vol. 17, pp. 1008-1015, November 2002.
- [90] E. Bjørndal, *Cost allocation problems in network and production settings*. Bergen, Norway. Doctoral thesis, Norwegian School of Economics and Business Administration, 2002, pp. 123-171.

# **Appendix A**

**Table A.1**Data of the IEEE 14-bus system

From	То				Transfer
		r [p.u.]	x [p.u.]	b [p.u.]	capacity
bus	bus				[MVA]
1	2	0.01938	0.05917	0.05280	200
1	5	0.05403	0.22304	0.04920	100
2	3	0.04699	0.19797	0.04380	100
2	4	0.05811	0.17632	0	100
2	5	0.05695	0.17388	0.03400	100
3	4	0.06701	0.17103	0.03460	100
4	5	0.01335	0.04211	0.01280	100
4	7	0	0.20450	0	100
4	9	0	0.53890	0	100
5	6	0	0.23490	0	100
6	11	0.09498	0.19890	0	100
6	12	0.12291	0.25581	0	100
6	13	0.06615	0.13027	0	100
7	8	0	0.17615	0	100
7	9	0	0.11001	0	100
9	10	0.03181	0.08450	0	100
9	14	0.12711	0.27038	0	100
10	11	0.08205	0.19207	0	100
12	13	0.22092	0.19988	0	100
13	14	0.17093	0.34802	0	100

Shunt susceptance at bus 9: 19 MVAR injected at V = 1 p.u.

$$S_b = 100 \text{ MVA}$$

$$V_b = 138 \text{ kV}$$

The data of the IEEE 14-bus system are taken from the files included in [65].

Table A.2

Bid offers of market participants in the case of IEEE 14-bus system in Chapter 2

Bus	Art	Power	Price	Power	Price
Bus	AII	[MW]	[ct/kWh]	[MW]	[ct/kWh]
1	Supplier	70	2.24	200	3.0
2	Supplier	70	2.25	200	3.1
3	Supplier	70	2.26	200	3.2
6	Supplier	70	2.27	200	3.3
4	Consumer	140	4.2	ı	-
9	Consumer	140	4.3	-	-
13	Consumer	140	4.4	-	-
14	Consumer	140	4.5	-	-

# Appendix B

**Table B.1**Data of the 10-bus test system

From	To bus	r [p.u.]	x [p.u.]	b [p.u.]	Transfer capacity [MVA]	Specific transfer cost $C_l$ $[\in/MW]$
1	2	0.0034	0.0360	1.2696	800	4.0
1	4	0.0034	0.0360	1.2696	800	4.0
2	3	0.0034	0.0360	1.2696	800	4.0
2	5	0.0034	0.0360	1.2696	800	4.0
3	6	0.0034	0.0360	1.2696	800	4.0
4	5	0.0034	0.0360	1.2696	800	4.0
4	7	0.0028	0.0288	1.0156	800	3.2
5	6	0.0028	0.0288	1.0156	800	3.2
5	7	0.0034	0.0360	1.2696	800	4.0
5	8	0.0017	0.0180	0.6348	800	2.0
6	10	0.0024	0.0252	0.8888	800	2.8
6	8	0.0034	0.0360	1.2696	800	4.0
7	8	0.0017	0.0180	0.6348	800	2.0
8	9	0.0017	0.0180	0.6348	800	2.0
8	10	0.0028	0.0288	1.0156	800	3.2
9	10	0.0024	0.0252	0.8888	800	2.8

 $S_b = 100 \,\mathrm{MVA}$  $V_b = 380 \,\mathrm{kV}$ 

Table B.2

Bid offers of market participants in the case of 10-bus test system

Bus	Art	$P_{\max}$	$p_{\mathrm{max}}$
Dus	7111	[MW]	[ct/kWh]
1	Supplier	150	3
2	Supplier	150	6
4	Supplier	150	6
7	Supplier	250	9
5	Consumer	100	20
6	Consumer	100	20
9	Consumer	100	20
10	Consumer	100	20

# **Appendix C**

**Table C.1**Generation data for the IEEE 14-bus system in Chapter 4

			J	
		Cost data		$P_{\mathrm{max}}$
Bus	а	b	С	[MW]
1	0.01	10	100	70
2	0.01	10	100	70
3	0.02	20	100	80
6	0.02	15	100	90

 $\label{eq:c.2} {\bf Data~of~the~14\text{-}bus~test~system~(}~S_b=100\,{\rm MVA}, V_b=380\,{\rm kV}~{\bf )}$ 

	ı		$(\sim_b$		, 0	/
From bus	To bus	r [p.u.]	x [p.u.]	b [p.u.]	Transfer capacity [MVA]	Specific transfer cost $C_l$ [ $\in$ /MW]
1	2	0.0017	0.0180	0.6348	1250	4.0
2	3	0.0017	0.0180	0.6348	1250	4.0
3	4	0.0017	0.0180	0.6348	1250	4.0
1	4	0.0017	0.0180	0.6348	1250	4.0
1	10	0.0017	0.0180	0.6348	1250	4.0
10	9	0.0017	0.0180	0.6348	1250	4.0
9	8	0.0017	0.0180	0.6348	1250	4.0
8	7	0.0017	0.0180	0.6348	1250	4.0
7	6	0.0017	0.0180	0.6348	1250	4.0
6	5	0.0017	0.0180	0.6348	1250	4.0
5	4	0.0017	0.0180	0.6348	1250	4.0
1	6	0.0043	0.0450	1.5870	1250	10.0
9	7	0.0043	0.0450	1.5870	1250	10.0
7	11	0.0017	0.0180	0.6348	1250	4.0
11	12	0.0017	0.0180	0.6348	1250	4.0
12	13	0.0017	0.0180	0.6348	1250	4.0
13	14	0.0017	0.0180	0.6348	1250	4.0
14	4	0.0017	0.0180	0.6348	1250	4.0
6	14	0.0043	0.0450	1.5870	1250	10.0
7	13	0.0043	0.0450	1.5870	1250	10.0

**Table C.3**Data of the IEEE 30-bus system

From bus         To bus         r [p.u.]         x [p.u.]         b [p.u.]         Transfer capacity [MVA]           1         2         0.02         0.06         0.03         130           1         3         0.05         0.19         0.02         130           2         4         0.06         0.17         0.02         65           3         4         0.01         0.04         0         130           2         5         0.05         0.20         0.02         130           2         6         0.06         0.18         0.02         65           4         6         0.01         0.04         0         90           5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65		1	Jala OI lile	ILLL 30-00	is system	1
1         2         0.02         0.06         0.03         130           1         3         0.05         0.19         0.02         130           2         4         0.06         0.17         0.02         65           3         4         0.01         0.04         0         130           2         5         0.05         0.20         0.02         130           2         6         0.06         0.18         0.02         65           4         6         0.01         0.04         0         90           5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           4         12         0         0.26         0         65           12         13         0			r [p.u.]	x [p.u.]	b [p.u.]	capacity
1         3         0.05         0.19         0.02         130           2         4         0.06         0.17         0.02         65           3         4         0.01         0.04         0         130           2         5         0.05         0.20         0.02         130           2         6         0.06         0.18         0.02         65           4         6         0.01         0.04         0         90           5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           12         13         0         0.14         0         65           12         14         0.12	1	2	0.02	0.06	0.03	
2         4         0.06         0.17         0.02         65           3         4         0.01         0.04         0         130           2         5         0.05         0.20         0.02         130           2         6         0.06         0.18         0.02         65           4         6         0.01         0.04         0         90           5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           4         12         0         0.26         0         65           9         10         0         0.11         0         65           12         14         0.12         0.26         0         32           12         14         0.12						
3         4         0.01         0.04         0         130           2         5         0.05         0.20         0.02         130           2         6         0.06         0.18         0.02         65           4         6         0.01         0.04         0         90           5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           12         13         0         0.14         0         65           12         14         0.12						
2         5         0.05         0.20         0.02         130           2         6         0.06         0.18         0.02         65           4         6         0.01         0.04         0         90           5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           4         12         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09						
2         6         0.06         0.18         0.02         65           4         6         0.01         0.04         0         90           5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           6         10         0         0.11         0         65           9         10         0         0.11         0         65           9         10         0         0.14         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0	2					
4         6         0.01         0.04         0         90           5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           9         10         0         0.21         0         65           9         10         0         0.11         0         65           9         10         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20 </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>						
5         7         0.05         0.12         0.01         70           6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           9         10         0         0.26         0         65           9         10         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         14         0.12         0.26         0         32           12         16         0.09         0.20         0         32           12         16         0.09         0.20         0         32           14         15         0.22						
6         7         0.03         0.08         0.01         130           6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           9         10         0         0.11         0         65           4         12         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           12         16         0.09         0.20         0         16           15         18         0.11         0.22         0         16           15         18         0.11         0	5				0.01	70
6         8         0.01         0.04         0         32           6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           4         12         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           12         16         0.09         0.20         0         32           12         16         0.09         0.20         0         32           12         16         0.09         0.20         0         16           15         18         0.11         0.22         0         16           15         18         0.11 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td></td<>						
6         9         0         0.21         0         65           6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           4         12         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         21         0.03         <		8		0.04		32
6         10         0         0.56         0         32           9         11         0         0.21         0         65           9         10         0         0.11         0         65           4         12         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           18         19         0.06         0.13         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         21         0.03	6	9	0	0.21	0	
9         11         0         0.21         0         65           9         10         0         0.11         0         65           4         12         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         21         0.03		10			0	
9         10         0         0.11         0         65           4         12         0         0.26         0         65           12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         21         0.03         0.07         0         32           10         22         0.07 </td <td>9</td> <td></td> <td>0</td> <td></td> <td>0</td> <td></td>	9		0		0	
12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         21         0.03         0.07         0         32           10         21 <t< td=""><td>9</td><td>10</td><td>0</td><td></td><td>0</td><td></td></t<>	9	10	0		0	
12         13         0         0.14         0         65           12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         21         0.03         0.07         0         32           21         22 <t< td=""><td>4</td><td>12</td><td>0</td><td>0.26</td><td>0</td><td>65</td></t<>	4	12	0	0.26	0	65
12         14         0.12         0.26         0         32           12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         21         0.03         0.07         0         32           10         21         0.03         0.07         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           10         22         0.07         0.15         0         32           15         23	12	13	0		0	65
12         15         0.07         0.13         0         32           12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           10         22         0.07         0.15         0         32           15         23         0.10         0.20         0         16           22         24		14	0.12	0.26	0	
12         16         0.09         0.20         0         32           14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           10         22         0.07         0.15         0         32           10         22         0.07         0.15         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24	12	15			0	32
14         15         0.22         0.20         0         16           16         17         0.08         0.19         0         16           15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26	12	16	0.09		0	32
15         18         0.11         0.22         0         16           18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           10         22         0.07         0.15         0         32           21         22         0.01         0.02         0         32           21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26	14	15	0.22	0.20	0	16
18         19         0.06         0.13         0         16           19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29 <t< td=""><td>16</td><td>17</td><td>0.08</td><td>0.19</td><td>0</td><td>16</td></t<>	16	17	0.08	0.19	0	16
19         20         0.03         0.07         0         32           10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           29         30 <t< td=""><td>15</td><td>18</td><td>0.11</td><td>0.22</td><td>0</td><td>16</td></t<>	15	18	0.11	0.22	0	16
10         20         0.09         0.21         0         32           10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           29         30         0.24         0.45         0         16           29         30 <t< td=""><td>18</td><td>19</td><td>0.06</td><td>0.13</td><td>0</td><td>16</td></t<>	18	19	0.06	0.13	0	16
10         17         0.03         0.08         0         32           10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           29         30 <t< td=""><td>19</td><td>20</td><td>0.03</td><td>0.07</td><td>0</td><td>32</td></t<>	19	20	0.03	0.07	0	32
10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           29         30         0.24         0.45         0         16           8         28 <td< td=""><td>10</td><td>20</td><td>0.09</td><td>0.21</td><td>0</td><td>32</td></td<>	10	20	0.09	0.21	0	32
10         21         0.03         0.07         0         32           10         22         0.07         0.15         0         32           21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           29         30         0.24         0.45         0         16           8         28 <td< td=""><td>10</td><td>17</td><td>0.03</td><td>0.08</td><td>0</td><td>32</td></td<>	10	17	0.03	0.08	0	32
21         22         0.01         0.02         0         32           15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	10	21	0.03		0	32
15         23         0.10         0.20         0         16           22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	10	22	0.07	0.15	0	32
22         24         0.12         0.18         0         16           23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	21	22	0.01		0	32
23         24         0.13         0.27         0         16           24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	15	23	0.10	0.20	0	16
24         25         0.19         0.33         0         16           25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	22	24	0.12	0.18	0	16
25         26         0.25         0.38         0         16           25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	23	24	0.13	0.27	0	16
25         27         0.11         0.21         0         16           28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	24	25	0.19	0.33	0	16
28         27         0         0.40         0         65           27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	25	26	0.25	0.38	0	16
27         29         0.22         0.42         0         16           27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	25	27	0.11	0.21	0	16
27         30         0.32         0.60         0         16           29         30         0.24         0.45         0         16           8         28         0.06         0.20         0.02         32	28	27	0	0.40	0	65
29     30     0.24     0.45     0     16       8     28     0.06     0.20     0.02     32	27	29	0.22	0.42	0	16
8 28 0.06 0.20 0.02 32	27	30	0.32	0.60	0	16
	29	30	0.24	0.45	0	16
6 28 0.02 0.06 0.01 32	8	28	0.06	0.20	0.02	32
	6	28	0.02	0.06	0.01	32

Shunt susceptance at bus 5: 0.19 MVAR injected at V = 1 p.u.

Shunt susceptance at bus 24: 0.04 MVAR injected at V = 1 p.u.

$$S_b = 100 \text{ MVA}$$

$$V_b = 135 \text{ kV}$$

The data of the IEEE 30-bus system are taken from the files included in [65].

**Table C.4**Generation data for the IEEE 30-bus system

		Cost data	•	$P_{\max}$
Bus	а	b	c	[MW]
1	0.0200	2.00	0	80
2	0.0175	1.75	0	80
22	0.0625	1.00	0	50
27	0.0083	3.25	0	55
23	0.0250	3.00	0	30
13	0.0250	3.00	0	40

# **Acronyms and Symbols**

### **Acronyms:**

ATC Available Transfer Capability

CF Counter Flow Method

ECNZ Electricity Corporation New Zealand

FACTS Flexible AC Transmission System

FERC Federal Energy Regulatory Commission

GGDF Generalized Generation Distribution Factor

GSDF Generation Shift Distribution Factor

IEM Internal Market of ElectricityISO Independent System Operator

LMP Locational Marginal Pricing

LP Linear Programming

MW Megawatt

MWM MW-Mile Method

NETA New Electricity Trading Arrangements

NGDF Nodal Generation Distribution Factor

NLDF Nodal Load Distribution Factor

OPF Optimal Power Flow

PS Post Stamp Method

SMD Standard Market Design

TGDF Topological Generation Distribution Factor

TPA Third Party Access

ZCF Zero Counter Flow Method

### Latin symbols:

A	Matrix used in the compact form of congestion component equation
$A^{v}(S)$	Average marginal contribution to coalition $S$
<i>a, b, c</i>	Factors of polynomial cost function
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$	Vectors of variable factors of dual problem objective function
В	Union of all the <i>a priori</i> unions belonging to the permutation $\pi$
C	Common schedule submission
$C_l$	Specific transfer cost of branch $l$
$CF_i$	Amount charged to participant $i$ according to the Counter Flow method
$CP_i$	Compensation payment by FTRs to participant $i$
CR(v)	The core of a game
c(S)	Characteristic function value of coalition $S$ in the case of cost game
e	Excess value
f	Set of equality constraints in OPF
$f_i$	Usage of network by the market participant $i$
$f_{i,l}$	Usage of branch $l$ by the market participant $i$
$\mathbf{f}_r$	Set of equality constraints concerning the reference bus at OPF
g	Set of inequality constraints at OPF
h	Set of inequality constraints at OPF
<i>h</i> 1	Set of strategies of player H1
J K	Set of <i>a priori</i> unions Social welfare
1.	Power system branch
$L_{A,k}$	NGDF of generator at bus $k$ for the bus $A$
$egin{array}{c} \mathbf{M} \\ M \end{array}$	Matrix equal to diag( <b>μ</b> ) Symbol denoting either active or reactive power
	Amount charged to participant $i$ according to the MW-
$MWM_i$	Mile method
n	Number of players
$n_l$	Number of power system lines
$n_b$	Number of power system buses
· ·	•

Set of players N

N(v)The nucleolus of a game

Number of system generators  $n_G$ 

Number of players in coalition S $n_{S}$ 

p Price for electricity

Impact of generator at bus k to the power inflows of bus  $P_{A,k}$ 

 $\boldsymbol{A}$ 

Share of generator at bus k in the power flow over the  $P_{AOj,k}$ 

line AOi

 $P_{G}$ Real power generation

 $\mathbf{P}_{G}$ Vector of real power generation

 $\mathbf{P}_{G \max}$ Vector of maximal active generation  $\mathbf{P}_{G\min}$ Vector of minimal active generation

 $P_{Gr}$ Active power generation at reference bus

 $P_i$ Power associated with participant i

 $P_{IiA,AOj}$ Share of line *IiA* in power flow over the line *AOj* 

Real power demand  $P_{L}$ 

 $\mathbf{P}_{L}$ Vector of real power demand

 $P_{Line}$ Active power flow over a particular line

 $P_{\text{max}}$ Maximal generation or demand

Maximal accepted price  $p_{\rm max}$ 

Vector of maximal accepted prices  $\mathbf{p}_{\text{max}}$ 

Minimal accepted price  $p_{\min}$ 

Vector of minimal accepted prices  $\mathbf{p}_{\min}$ 

Amount charged to participant *i* according to the Postage  $PS_i$ 

Stamp method

 $P_{III}$ Uncovered portion of load

 $\mathbf{P}_{UL}$ Vector of uncovered portion of load

 $\mathbf{Q}_G$ Vector of reactive generation

 $\mathbf{Q}_{G\max}$ Vector of maximal reactive generation  $\mathbf{Q}_{G\min}$ Vector of minimal reactive generation

 $Q_{Gr}$ Reactive power generation at reference bus

 $\mathbf{Q}_L$ Vector of reactive load

Reference bus r

R Payment to the ISOS Use of Shapley value

S Vector of apparent power flow

 $S, \Theta, E, Z$  Coalitions

 $\mathbf{S}_{\text{max}}$  Vector of maximal apparent power flow

st Strategy in a game

 $ST_i$  Set of strategies of *i*-th player

 $T_j$  The *j*-th *a priori* union

*u* Payoff

V Vector of voltage magnitude v Characteristic function

v(i) Characteristic function value of player i

v(N) Characteristic function value of the grand coalition

V<sub>r</sub> Bus voltage magnitude at reference busx Vector including the variables of OPF

 $\mathbf{X}_0$  Linearisation point

 $y_i(v; J)$  Owen value **y**, **k**,  $\varepsilon$ ,  $\zeta$  Imputations

z Zonal pricing restriction

 $ZCF_i$  Amount charged to participant i according to the Zero

Counter Flow method

### **Hellenic symbols:**

 $\alpha_{ij}$  Element at the *i*-th row and *j*-th column of matrix **A** 

 $\Gamma$  Set of *a priori* unions' numbers

 $\gamma_i$  Line in core figure

**\empty** Factor based on power flow sensitivity

H1,H2 Players

 $\theta$  Vector of voltage angle  $\theta(\mathbf{y})$  The  $2^n$  values v(S)- y(S)

 $\theta_r$  Bus voltage angle at reference bus

 $K_i^j$  Cost allocated to *i*-th player by the *j*-th method

λ Vector of nodal price /Lagrange multipliers of bus power

balance

 $\Lambda$  Matrix giving the allocation of congestion component to

the single congestions.

 $\lambda_C$  Vector of nodal price component due to congestions

$\lambda_{\mathit{CPi}}$	Congestion component of nodal price for active power at bus $i$
$\lambda_{CQi}$	Congestion component of nodal price for reactive power at bus $i$
$\lambda_{ ext{expected}}$	Common marginal cost in case of zonal pricing by means of nodal pricing mechanism
$\lambda_G$	Nodal price component due to generation
$\lambda_{\mathit{GL}}$	Vector of nodal price component due to generation and losses
$\lambda_i$	Nodal price at bus <i>i</i>
$\lambda_{i,a}$	Element at the $i$ -th row and $a$ -th column of matrix $\Lambda$
$\lambda_{LCPi,j}$	Part of nodal price congestion component of active power at bus $i$ due to power flow congestion at line $j$
$\lambda_{\mathit{Los}}$	Nodal price component due to losses
$oldsymbol{\Lambda}_{PV ext{min}}$	
$oldsymbol{\Lambda}_{PV ext{max}}$	
$oldsymbol{\Lambda}_{PL}$ , $oldsymbol{\Lambda}_{QL}$	
$oldsymbol{\Lambda}_{QV ext{min}}$	Submatrices of matrix $\Lambda$
$oldsymbol{\Lambda}_{QV ext{max}}$	
$\lambda_r$	Nodal price vector for reference bus
$\lambda_{VCPi,s}$	Part of nodal price congestion component of active power at bus $i$ due to voltage congestion at bus $s$
μ	Vector of Lagrange multipliers of voltage magnitude and line flow constrains
$\mu_i$	The $i$ -th element of $\mu$
$\mu_{\mathit{Lj}}$	Lagrange multiplier with respect to the power flow limit over the line $j$
$\mu_{Vi  ext{max}}$	Lagrange multiplier with respect to the upper voltage magnitude limit at bus $i$
$\mu_{Vi ext{min}}$	Lagrange multiplier with respect to the lower voltage magnitude limit at bus $i$
ζ	Lagrange multiplier for zonal pricing restriction
$\pi$	Permutation
σ	Vector of Lagrange multipliers of active and reactive generation constrains
$\sigma_{j}$	The $j$ -th element of $\sigma$
τ	Vector of active and reactive power
$\varphi_i(v)$	Shapley value
$\Psi_A$	Set of lines supplying bus $A$

 $\psi_i(v)$  Solidarity value

 $\Omega$  Set of usage based methods

# (Some) Hellenic words

Acriby accuracy

Brachychronic Short term

Pandemic Universal

Isonomy Equality of legal rights

Idiosyncrasy Mental or physical peculiarity

Dimerous Arranged or divided in two parts

Epulosis Formation of scar

Diorthosis Correction

Publications 171

# Publications during the doctorate study

- G.S. Stamtsis, J. Christiansen, I. Erlich, "Evaluation of power system congestions using nodal price analysis", in *Proc. MEPS Conference* 2002, September 2002, pp. 25-30.
- G.S. Stamtsis, I. Erlich, "Nodal price congestion component analysis and market participants" bid behaviour", in *Proc. Med Power Conference* 2002, November 2002.
- G.S. Stamtsis, I. Erlich, "Congestion analysis and participants' behaviour in a pool market", *IEE Proceedings-Generation, Transmission and Distribution*, vol. 151, pp.127-132, January 2004.
- G.S. Stamtsis, I. Erlich, "On the use of cooperative game theory in power system fixed cost allocation", accepted for publication in *IEE Proceedings-Generation, Transmission and Distribution*.

Additionally, the two following papers will be submitted for publication to international peer review journals:

- E. Bjørndal, G.S. Stamtsis, I. Erlich, "Finding core allocations for fixed cost games in electricity networks"
- G.S. Stamtsis, M. Bjørndal, I. Erlich, K. Jörnsten, "Assessment of the Norwegian transmission pricing rules by using a modified AC-OPF"

# Biographic Data

Personal Data:

Name: Georgios Stamtsis

Born: 28th August 1974, Mömlingen, Bayern, Germany

Nationality: Hellenic

Marital status: Single

#### **School Education:**

1980-1986 3rd primary school of Sidirokastro, Hellas

1986-1989 Pallatidio Gymnasium, Sidirokastro

1989-1992 Pallatidio Lyceum, Sidirokastro

Study:

1992-1998 Department of Electrical Engineering, Aristotle University of

Thessaloniki, Hellas

Diploma of Electrical Engineering, July 1998

Work Experience:

1999-2003 Research Assistant at the Institute of Power Systems, Faculty of

Engineering, University Duisburg-Essen

Since 2000 as holder of a DAAD scholarship