Uncertainty in Reliability Evaluation

A Framework and Practical Case Studies

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Abstract

In modern society, human became dependent on sophisticated engineering systems. During the recent years, the major man-made disasters triggered more concerns on reliability and dependability of these systems. More public attention is paid to the presence of uncertainty and risk regarding reliability of engineering systems. This dissertation focuses on the identification of uncertainty and its effects, and on the determination of a framework of reliability evaluation under the presence of uncertainty.

This dissertation is based on the definitions and terminologies from renowned international standards and literature. The fundamentals and basic formulas are adequately explained. Existing works of similar topic have been compared and their advantages and limitations are mentioned. A novel framework is proposed to overcome the limitations of the existing works. The framework is designed to be compatible with the guidelines in risk management, and is composed of three approaches for different types of uncertainty. These approaches are: 1) approach for aleatory uncertainty, 2) approach for epistemic uncertainty, and 3) approach for early design stage. Practical case studies are demonstrated at the end of the chapter of each approach. The uncertainty information, which is obtained from this framework, can improve the confidence in the application of reliability studies to critical engineering systems.

Keywords: Availability, Reliability, Risk, Uncertainty

Zusammenfassung

Der technologische Fortschritt, das Zusammenfügen unterschiedlicher technologischer Bereiche und die steigende Komplexität von Systemen machen Zuverlässigkeits- und Verfügbarkeitsanalysen unverzichtbar und bereits in der Planungsphase zum integralen Bestandteil von Systemauslegung, um später finanzielle Schäden und hohe Strafzahlungen zu vermeiden. Bisher angewandte Verfahren zur Bestimmung von Zuverlässigkeits- und Verfügbarkeitskennwerten haben den Nachteil, dass Unsicherheiten nicht ausreichend berücksichtigt werden und deshalb auch keine Aussage über deren Einfluss auf die Kenngrößen gemacht werden können.

Die Dissertation basiert auf den Definitionen und Terminologien internationaler Normen und wissenschaftlicher Erkenntnisse. Divere Grundlagen und die grundlegenden Verfahren werden erklärt. Bestehende Werke zu Unsicherheiten werden verglichen und deren Vor- und Nachteile genannt. Ein neues Rahmenwerk wird vorgeschlagen, um die Beschränkungen der bestehenden Werke zu überwinden. Dieses orientiert sich an den Leitlinien des Risikomanagements und besteht aus drei Ansätzen zur Berücksichtigung verschiedener Arten von Unsicherheiten. Diese Ansätze sind: 1) Ansatz für aleatorische Unsicherheit, 2) Ansatz für epistemische Unsicherheit und 3) Ansatz von Unsicherheit in frühen Planungs- bzw. Entwicklungsphasen (Early Design Stage). Praktische Fallstudien zu Unsicherheiten werden am Ende der jeweiligen Kapitel aufgezeigt. Die aus den Analysen gewonnenen Informationen können zur Verbesserung des Vertrauens bei der Anwendung von Zuverlässigkeitsstudien dienen.

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List of Symbols and Abbreviations

Av	availability
C	component
CDF	cumulative distribution function
D, D_C, D_S	down state (failure, outage) in general, of a component C, and of a system S
E_i	event i
$E(X)$	expectation value of a random variable X
$\hat{E}(X)$	estimate of an expectation value of a random variable X
$erf(x)$	error function of any real variable x
$f(x)$	failure density function for any real variable x of a random variable X.
$F(x)$	failure function for any real variable x of a random variable$X$.
$fr(Z)_i$	i-th simulation sample of frequency of a state Z
$Fr(Z)$	$= Fr(Z)_{mean}$, mean frequency of a state Z
$Fr(Z)_{max}$	maximal frequency of a state Z
$Fr(Z)_{mean}$	mean frequency of a state Z
$Fr(Z)_{min}$	minimal frequency of a state Z
$Fr(Z)_{x\%}$	frequency of a state Z at x-th percentile
$h(X)$	relative frequency of a random variable X
$h(\theta)$	continuous function of a parameter θ
$\hat{h}(\theta)$	maximum likelihood estimate of a continuous function with parameter θ
HVAC	high-voltage alternating current
HVDC	high-voltage direct current
HVDC INT	HVDC with integrated power electronics
HVDC LCC	HVDC with line-commutated converter

HVDC LCC-1	HVDC with line-commutated converter, alternative structure
HVDC LCC-2	HVDC with line-commutated converter
HVDC VSC	HVDC with voltage-source converter
INT	HVDC with integrated power electronics
$imp_{i,j}$	dependency impact of component i to j
k	number of events in a specific time interval in a Poisson process
K_E	excess kurtosis
$L_i(\theta \mid t_i)$	probability of a single failure observation i, and this observation fails at time t_i
$L(\theta)$	likelihood function of an i-th observation of a parameter θ
LCC	HVDC with line-commutated converter
$LL(\theta)$	log-likelihood function of a parameter θ
$m(x)$	maintenance density function for any real variable x of a random variable X.
m_L	mean value of a lognormal distribution
m_N	mean value of a normal distribution
MC	minimal cut
MMC	Markov minimal cut (minimal cut modelled with MPM)
MLE	maximum likelihood estimate
MPM	Markov process model
$MTTF$	mean time to failure
$MTTSF$	mean time to system failure
$MTTR$	mean time to repair
$MTTSR$	mean time to system repair
n	total number of events/occurances/samples
NAv	non-availability
$p(x)$	probability density function or PDF for any real variable x of a random variable X.

$P(x)$	cumulative distribution function or CDF for any real variable x of a random variable X.
PDF	probability density function
pMp	probable Markov path
$pr(Z)_i$	i-th simulation sample of probability of a state Z
$Pr(Z)$	$= Pr(Z)_{mean}$, probability of an event Z
$Pr(Z)_{max}$	maximal probability of an event Z
$Pr(Z)_{mean}$	mean probability of an event Z
$Pr(Z)_{min}$	minimal probability of an event Z
$Pr(Z)_{x\%}$	probability of an event Z at x-th percentile
q	number of the observations of non-censored repairs
RBD	reliability block diagram
r	number of the observations of non-censored failures
$R(t)$	reliability function or survival function at time t
S	system
Δt	time interval
$t(Z)$	time (duration) of a state Z
$t(Z)_i$	i-th simulation sample of time (duration) of a state Z
$ti(Z)_i$	i-th simulation mean time (duration) of a state Z
$Ti(Z)$	$= Ti(Z)_{mean}$, mean time (duration) of a state Z
$Ti(Z)_{max}$	maximal time (duration) of a state Z
$Ti(Z)_{mean}$	mean time (duration) of a state Z
$Ti(Z)_{min}$	minimal time (duration) of a state Z
$Ti(Z)_{x\%}$	time (duration) of a state Z at x-th percentile
TTF	time to failure
TTR	time to repair

U, U_C, U_S	up state (operation, function, service) in general, of a component C, and of a system S
VSC	HVDC with voltage-source converter
X	random variable
Y	random variable
$[Z(t), t > 0]$	stochastic process, time dependent state of Z
$Z = [Z(t), t \rightarrow \infty]$	steady state of Z, e.g. $Z = \{U,D\}, \{U_C,D_C\}, \{U_S,D_S\}$
α	scale parameter of a Weibull distribution
β	shape parameter of a Weibull distribution
Γ	gamma function
λ	failure rate, parameter of an exponential function
$\hat{\lambda}$	estimate of a failure rate, or of parameter of exponential function
$\lambda(t)$	failure rate as a function of time
μ	repair rate of an exponential function
$\hat{\mu}$	estimate of the repair rate of an exponential function
θ	parameter of a likelihood function
$\underline{\theta}$	vector of parameters of a likelihood function
σ	standard deviation
σ^2	variance
$\hat{\sigma}_{\hat{\lambda}}^2$	estimate of asymptotic variance of an estimate of an exponentially distributed failure rate
$\hat{\sigma}_{\hat{\mu}}^2$	estimate of asymptotic variance of an estimate of an exponentially distributed repair rate
$\hat{\sigma}_{\hat{\theta}}^2$	estimate of asymptotic variance of an estimate of a likelihood function's parameter θ
τ	a real variable describing a function with random variable t
ϕ	a real variable describing a function with random variable x
Ω	sample space

1 Introduction

1.1 Motivation

Over the past decades, sophisticated engineering systems, such as control systems, power systems, production systems, automation systems, etc., have become a vital part of a human life. The modern society has become dependant on these complex systems. People rely on the transit system, such as trains, subway, and automobiles, in order to commute from their accommodations to their workplaces. The productivity of a workplace depends on the information technology infrastructure of the company. The telecommunication network is essential in business, and also in social activities.

Each of these engineering systems also depends on each other. The modern financial system relies on the availability of the computer network. The computer network is dependant on electricity, which is generated from power plants. A single outage of a system may lead to the domino effect. The importance of the reliability and availability of engineering system is unquestionable.

The major engineering-related disasters during the recent years, such as the Deepwater Horizon incident in 2010, the Fukushima Daiichi nuclear disaster in 2011, and the India power blackout in 2012, led to increasing concerns of the importance of reliability and availability studies of critical engineering systems. Even for the most reliable systems, the presence of risk and uncertainty has been more and more anticipated. This lead to the two important questions in this study:

I. What are the sources, types, and effects of uncertainty on reliability?
II. How to evaluate and measure the reliability under the presence of uncertainty?

The answers of these two questions will lead to the improvement on the confidence in the application of reliability studies to critical engineering systems.

1.2 Objectives

This study focuses on the consideration of uncertainty in the reliability evaluation of engineering systems. The sources and the effects of uncertainty will to be identified. A proper framework of uncertainty in the reliability evaluation will be sought and determined. This framework will be applied to practical case studies to illustrate the applicability of the framework in industrial systems.

1.3 Approaches

Because there are many aspects of reliability and uncertainty, therefore, all of the essential terminologies are defined according to renowned international standards and literature. The fundamentals in this study are based on these definitions.

The existing works of the similar subject are reviewed. Their advantages and limitations are compared, and because of these limitations, a novel framework is introduced to overcome these limitations.

The framework is applied to practical case studies, namely the reliability evaluation of high-voltage direct current (HVDC) converter stations (e.g. for offshore windfarms), and the reliability evaluation of a process control system. The sources and the effects of uncertainty are identified and evaluated in each case study.

1.4 Outline

Chapter 1 is the introductory chapter which contains the motivation, the objective, the approaches, and the outline of the dissertation.

Chapter 2 contains the definitions of the essential terminologies regarding reliability and uncertainty. The definitions are based on widely accepted standards and literature.

Chapter 3 describes the fundamentals of the subject. The important formulas are derived with sufficient explanations.

Chapter 4 reviews the existing works of the similar subject. A framework of uncertainty in reliability evaluation is proposed. This framework suggests three reliability evaluation approaches for the aleatory uncertainty, the epistemic uncertainty, and the uncertainty in an early design stage. These approaches are mentioned in details in Chapter 5, 6, and 7, respectively. The approaches are divided into simples steps, and practical case studies are given at the end of each chapter.

Chapter 8 concludes the study. The empirical findings, implications of the study, the limitation and the recommendations for future research are summarized.

2 Definitions

Terms related to reliability are used differently in various contexts. In a formal engineering context, reliability terminologies have been based on standards and languages where they are established. Among the most widely accepted standards there are some which have translations in different languages such as [IEC 60050-191 1990, ISO 31000 2009, ISO Guide 73 2009, Laprie 1992]. These selected works have established terminologies and definitions which are used in many modern standards and literature such as [Avizienis et al. 2004, DKE-IEV Chapter 191 2005, IEC 61078 2006, IEC 61165 2006]. Apart from these literature, there are a number of English language standards which are widely accepted such as [IEEE 90 1990, MIL-HDBK 217F 1991].

2.1 Regarding Systems and Components

First, it is important to define basic taxonomy from the entity with which the reliability is assessed. In [Avizienis et al. 2004], the successor of [Laprie 1992], this object or entity is called a system, which interacts with other systems in the environment.

Definition 2.3 System, Environment

A system is an entity that interacts with other entities, i.e., other systems, including hardware, software, humans, and the physical world with its natural phenomena. These other systems are the environment of the given system. The system boundary is the common frontier between the system and its environment [Avizienis et al. 2004].

From a structural point of view, a system can be sub-divided into subsystems. If the division of subsystems reaches an atomic level, the final entity is called a component.

Definition 2.4 Component, Atomic

A system is composed of a set of components bound together in order to interact, where each component is another system, etc. The recursion stops when a component is considered to be atomic: Any further internal structure cannot be discerned, or is not of interest and can be ignored [Avizienis et al. 2004].

In [IEC 60050-191 1990], systems, subsystems, and components are referred to as an item or an entity which can be individually considered.

Definition 2.5 Item, Entity

any part, component, device, subsystem, functional unit, equipment or system that can be individually considered [IEC 60050-191 1990].

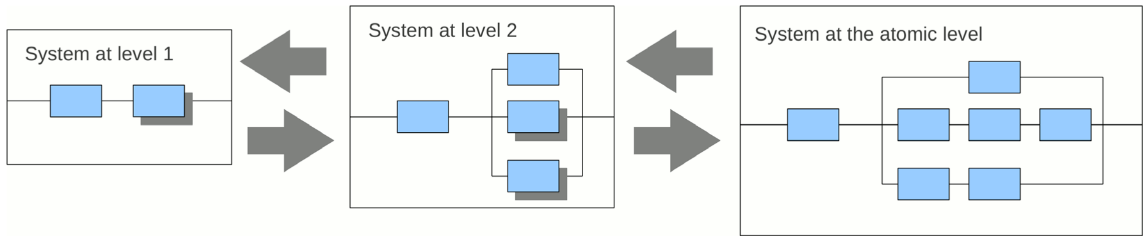


Fig. 2.1: A system can be subdivided until it reaches an atomic level, and items at atomic level are called components

The definition of an item is a very useful definition which generalizes reliability evaluation methods for either a component or a system. Further definitions in this study are established on these terms.

2.2 Regarding States of an Item

A scenario of an item is defined in different item states. Generally it is defined in a perspective of the behavior of an item, whether it can perform its intended function.

Definition 2.6 Function, Behavior

The function of such a system is what the system is intended to do and is described by the functional specification in terms of functionality and performance. The behavior of a system is what the system does to implement its function and is described by a sequence of states [IEC 60050-191 1990].

If an item can perform its function then it is said the system is in an operating state, otherwise it is in an outage state or a disabled state.

Definition 2.7 Operating state

The state when an item is performing a required function [IEC 60050-191 1990].

Definition 2.8 Outage state, Disabled state

A state of an item characterized by its inability to perform a required function, for any reason [IEC 60050-191 1990].

If the cause of an outage state of an item is looked closely, it can be seen that there are cases where an item is able to perform its function, but the lack of external resources prevent this item to operate. The up state and down state are defined with the assumption that these external resources are provided if needed.

Definition 2.9 Up state

A state of an item characterized by the fact that it can perform a required function, assuming that the external resources, if required, are provided [IEC 60050-191 1990].

Definition 2.10 Down state

A state of an item characterized either by a fault, or by a possible inability to perform a required function during preventive maintenance [IEC 60050-191 1990].

An item may not be operating, but are up and ready to be operated if external resources are available. The consideration of an external disabled state distinguishes up/down states from operating/outage states, as illustrated in Fig. 2.2.

Definition 2.11 External disabled state

That subset of the disabled state when the item is in an up state, but lacks required external resources or is disabled due to other planned actions than maintenance [IEC 60050-191 1990].

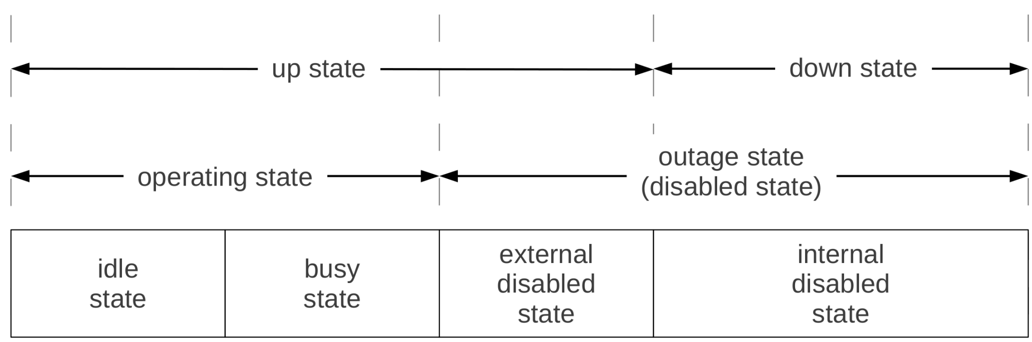


Fig. 2.2: Classification of item states, adapted from [IEC 60050-191 1990]

As mentioned earlier, the cause of an outage is caused by an inability to perform a required function. In the next section the causes of this inability are defined.

2.3 Regarding Failures

A failure is a termination from an operating state to an outage state, or from an up state to a down state if the required external resources are provided.

Definition 2.12 Failure

The termination of the ability of an item to perform a required function [IEC 60050-191 1990].

It is often necessary to distinguish the cause of an outage into three abstraction levels of impairments: failure, error, and fault.

Definition 2.13 Failure, Error, Fault

A failure means that at least one (or more) state of the system deviates from the correct service state. The deviation is called an error. The adjudged or hypothesized cause of an error is called a fault [Avizienis et al. 2004].

A failure is corresponded to the behavioral level of an item which transit from one state to another. In a logical level, this state movement is caused by an error in an item. This error is physically a fault, e.g. a damage caused by other systems in an environment, or a design failure of the item under consideration. Finally a failure in a component may lead to a fault in its subsystem. This relation between the three levels is called a fault pathology, which is illustrated in Fig. 2.3.

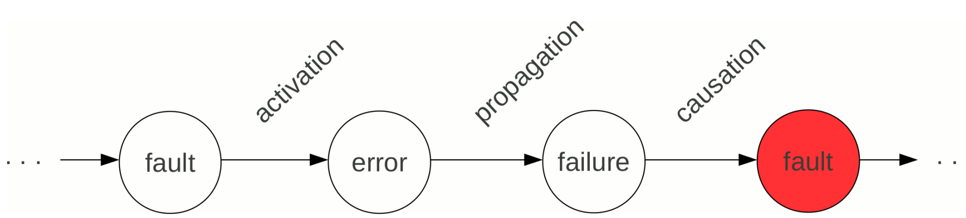


Fig. 2.3: Fault pathology

Special cases of failures called common-mode failures (or common-cause failures), are two or more failures which are caused by one similar error, illustrated in Fig. 2.4.

Definition 2.14 Common-mode failure, Common-cause failure

An event having a single external cause with multiple failure effects where the effects are not consequences of each other [Billinton et al. 1981].

Different classes of these impairments are mentioned in details [Avizienis et al. 2004, Laprie 1992]. In this work a sufficient amount of impairment terms is covered just to lead to the concept of reliability assessment.

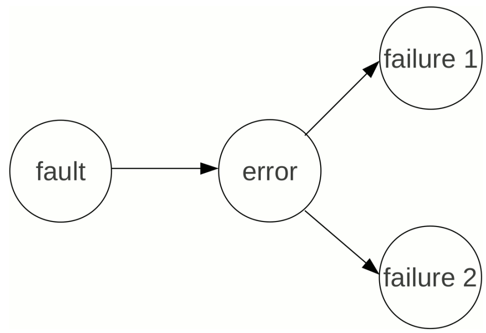


Fig. 2.4: Common-mode failure

2.4 Regarding Reliability

In the context of engineering, reliability is defined as an ability of an item to perform a function. The term unreliability is used if an item is not able to perform such function.

Definition 2.15 Reliability

The ability of an item to perform a required function under given conditions for a given time interval [IEC 60050-191 1990].

According to Definition 2.15, reliability is used as a general term regardless of maintenance or repair. To be more specific, the terms availability and non-availability (or unavailability) are used if maintainability of an item is considered.

Definition 2.16 Availability

The ability of an item to be in a state to perform a required function under given conditions at a given instant of time or over a given time interval, assuming that the required external resources are provided [IEC 60050-191 1990].

Nevertheless, availability and reliability are often used interchangeably, especially when the maintenance is not explicitly mentioned. Another important term to describe an item is dependability.

Definition 2.17 Dependability

The ability to avoid service failures that are more frequent and more severe than is acceptable [Avizienis et al. 2004].

While reliability and availability can be determined as a quantitative value, dependability is used only for general descriptions in qualitative terms [IEC 60050-191 1990]. Safety and security are among these qualitative terms and have been further explained in [Avizienis et al. 2004]. The relations between these terms are called as a dependability tree, which is summarized in Fig. 2.5.

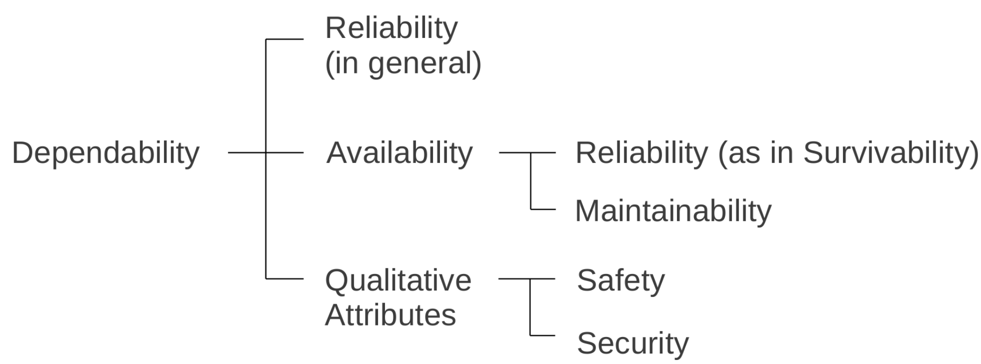


Fig. 2.5: Dependability tree with terms related to reliability, adapted from [Avizienis et al.2004, IEC 60050-191 1990]

This work focuses on the reliability assessment in quantitative terms. Lately in this work, it is illustrated how some of the qualitative terms can be quantified the qualitative amount into calculable values using a proposed method, called influencing factors and quantification functions.

2.5 Regarding Uncertainty and Risks

Uncertainty is usually defined from different perspectives. In a statistical perspective, uncertainty is classified by sources into two entities: incompleteness and indeterminacy.

Definition 2.18 Incompleteness

Caused by a simplifying representation which permits the usage of only a partial amount of information available [Walley 1991].

Definition 2.19 Indeterminacy

Reflects limitations of the available information [Walley 1991].

In practice it is hard to find the border between the incompleteness and the indeterminacy of uncertainty, however, the tendency toward either one can be noticed.

For instance, the source of uncertainty of a rainy day given knowledge of 10-year weather data mostly comes from incompleteness. Although a small portion of indeterminacy exists, one can say 10 years of data is not much different from those of 20 years or more. So an incomplete weather model is the dominating source of uncertainty.

On the other hand, if a relatively accurate weather model is developed, and if one tries to use this model with only two days of past weather data, it can be expected that an indeterminacy from a lack of data is the dominating source of uncertainty. Fig 2.6 illustrates the gray boundary between an incompleteness and an indeterminacy.

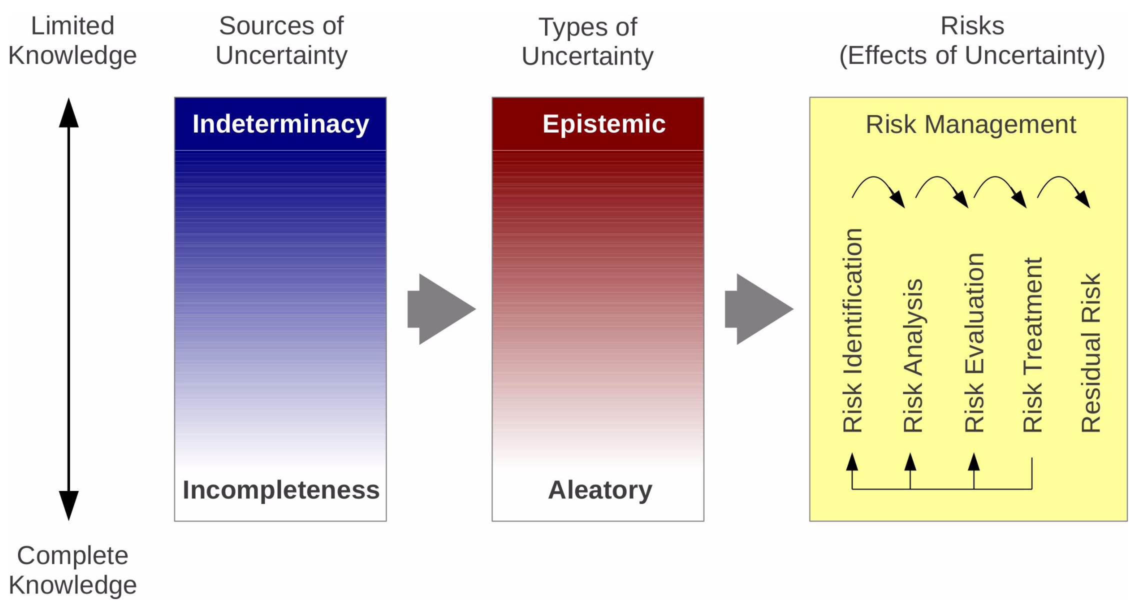


Fig. 2.6: Uncertainty by source, types, and effects (risks)

In the probabilistic perspective, uncertainty is classified into aleatory uncertainty and epistemic uncertainty.

Definition 2.20 Aleatory uncertainty, Irreducible uncertainty, Stochastic uncertainty

Aleatory uncertainty is also referred to in the literature as variability, irreducible uncertainty, inherent uncertainty, and stochastic uncertainty. We use the term aleatory uncertainty to describe the inherent variation associated with the physical system or the environment under consideration [Oberkampf et al. 2004].

Definition 2.21 Epistemic uncertainty, Reducible uncertainty

Epistemic uncertainty is also termed reducible uncertainty, subjective uncertainty, and model form uncertainty. Epistemic uncertainty derives from some level of ignorance, or incomplete information, of the system or the surrounding environment [Oberkampf et al. 2004].

Similar to the statistical perspective, uncertainty tends to be aleatory if a complete knowledge is given. However, if the knowledge is very limited, it tends to be epistemic. Still, there is no clear border between the aleatory uncertainty and the epistemic uncertainty as illustrated in Fig. 2.6.

In the management perspective, effects of uncertainty which may alter the results of a project are specially focused. These effects are called risks and the process that tries to control risks is called a risk management.

Definition 2.22 Risk

Effect of uncertainty on objectives [ISO 31000 2009, ISO Guide 73 2009].

Definition 2.23 Risk management

Coordinated activities to direct and control an organization with regard to risk [ISO 31000 2009, ISO Guide 73 2009].

The process of risk management can be subcategorized into risk identification, risk analysis, risk evaluation, and risk treatment. This process repeats until the concerned risks are eliminated and minimized. After the risk treatment, a level of risks still remains in the final results at a certain level. The remaining risks at this level are called residual risks.

Definition 2.24 Residual risk

Risk remaining after risk treatment [ISO 31000 2009, ISO Guide 73 2009].

The level of the residual risk is a subjective amount, which depends on the sources of uncertainty which are taken into account. If fewer sources of uncertainty are identified, a rise in residual risk can be expected. The residual risk can be either aleatory, epistemic, or both.

3 Fundamental Concepts and Approaches

3.1 Statistical Concepts

Statistical methods are the studies of data and turn into information. They also involve making estimates or forecasts of a larger group from the smaller group data. All members in this specified data group is called a **population**. Most of the time, it is impractical or even impossible to address all members of a population, thus, a subset of a population called a **sample** is observed instead.

Data is usually summarized and presented tabularly as a **frequency distribution**, or graphically as a **histogram**. The purpose of the data presentation is to illustrate the data in a more understandable way. In order to describe characteristics of data, it is commonly done by using three different types of measures: **measures of central tendency**, **measures of location**, and **measures of dispersion**.

3.1.1 Measures of Central Tendency

The most frequently used measure of central tendency is an **arithmetic mean**, or simply called a **mean**. One of the reasons of the popularity is the mathematical simplicity, which is the sum of the observations divided by the number of observations [DeFusco et al. 2004]. One drawback of the mean value is the sensitivity to extreme values. A few observations at extreme values may shift the mean significantly. In this case the use of the mean may mislead to describe characteristics of data.

The second measure of central tendency is the **median**, which is the middle value of a set of samples or of a population that has been sorted into ascending or descending order [DeFusco et al. 2004]. Median is not affected by extreme values. However, one drawback is that it does not describe the size and magnitude of the observations, only the relative position is concerned.

Another measure of central tendency is the **mode**, which is the most frequently occurred value in all observations [DeFusco et al. 2004]. In a histogram, the mode is always at the highest bar. One drawback of the mode is that in a population or a set of samples there can be more than one mode, or even no mode at all.

There are other important measures such as the **geometric mean**, which describes the growth rate of the observations, or the **harmonic mean**, which describes the averaging amount of the observations. More details of these measures can be found in [DeFusco et al. 2004, Kreyszig 2010, Ross 2010, Ryan 2007, Yates et al. 2004].

3.1.2 Measures of Location

It is often necessary to determine where the specified proportions of the data lie. The most general approach is to describe these locations as **quantiles**. One of the most commonly used quantiles is **percentiles**.

Percentiles divide the data distribution into hundredths and the x-th percentile is a value at x percent of the observations. For instance, the tenth percentile is the location of a distribution that ten percent of the distribution is below it. Sometimes it is necessary to approximate the value of the percentile as the actual data observations cannot be divided into exactly hundred.

In reliability engineering practice, quantiles and percentiles are usually used when a part of the data is highly concerned, e.g. the power stations below the fifth percentile of the entire power outage samples are inspected for the causes of the outages. The application of percentiles are mentioned further in this work.

3.1.3 Measures of Dispersion

Dispersion, or the scattering of data around the mean value, is another important measure to understand the data itself. The simplest measure of dispersion is the **range** of data, which is the difference between the minimum and the maximum values of the entire observations. However, the range can be affected by extreme values and may mislead to describe characteristics of data.

The most frequently used measure of dispersion is the **variance**, denoted by σ^2. Nevertheless, the variance is always measured in squared, therefore the **standard deviation**, which is the square root of the variance, is introduced as σ. The mathematical formula of the sample variance, the population variance, and respectively the sample standard deviation and the population standard deviation, can be found in [DeFusco et al. 2004, Kreyszig 2010, Ross 2010, Ryan 2007, Yates et al. 2004]. In this work, only the basic ideas are mentioned as a fundamental for the later chapters.

Sometimes only the conservative side of the dispersion is concerned, especially when the distribution of the observations is not symmetric. The **semivariance** and the **semi-standard deviation** are the measures which concerns only on the downside risk. The computation is similar to the variance and the standard deviation, except the observations which is greater than the mean value is not considered. If the distribution is not symmetric, it is called **skewed**. The **skewness** is a measure of asymmetry. The distribution is called positive skewed if it is skewed to the right, and called negative skewed if it is skewed to the left. This is illustrated in Figure 3.1.

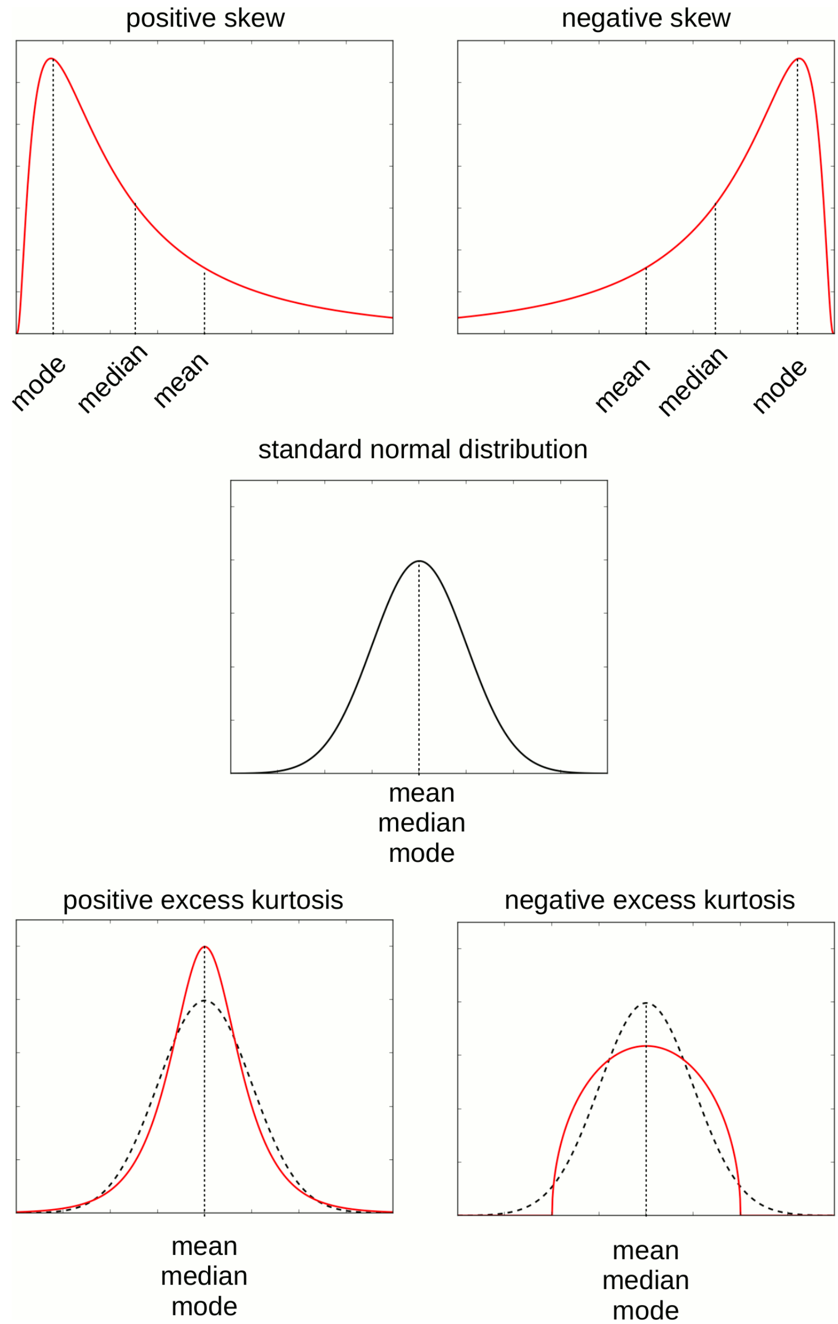


Fig. 3.1: Skewness and kurtosis in probability distributions

Kurtosis is a measure whether the data are peaked or flat relative to a normal distribution [NIST/SEMATECH 2014]. The normal distribution has the kurtosis equal to 3, therefore it is more practical to use an **excess kurtosis**, which equals to the kurtosis minus 3. Let X be a random variable with mean $\bar{x}$, the excess kurtosis K_E is

$$K_E = \frac{E\{(X-\bar{x})^4\}}{E\{(X-\bar{x})^2\}^2} - 3 \tag{3.1}$$

A distribution with a positive excess kurtosis is more peak and has more extreme values than the normal distribution, and a distribution with a negative excess kurtosis is less peaked and has fewer extreme values.

These measures and their usages are applied to reliability, uncertainty, and risks. There are many other measures of risks which are commonly used. For instance, a measure of risk called value-at-risk (VaR) has been extensively adopted especially in the financial sector [Szegö 2002]. Conditional value-at-risk (CVaR) is another measure which has been used to overcome the undesirable properties of VaR [Artzner 1997, Artzner 1999, Rockafellar 2002]. These measurements are recommended for further study of risks. In this work, it is concentrated only on the basic measures in the central tendency, the basic measures of location, and the basic measures of dispersion.

3.2 Probability Concepts

3.2.1 Basic Principles of Probability

According to [Kreyszig 2010], an **experiment** is a process of measurement or observation in a general sense. In experiments that involve **randomness**, a result cannot be predicted exactly. A **trial** is a single performance of an experiment. Its result is called an **outcome** or a **sample point**. n trials give a sample of **size** n consisting of n sample points. The **sample space** Ω of an experiment is the set of all possible outcomes. The subsets of sample space are called **events**.

A probability of an event is a measure of how frequently the event is likely to occur. From Kolmogorov's probability axioms [Kreyszig 2010], the probability of any event E_1, denoted as $Pr(E_1)$, is a number between 0 and 1.

$$0 \leq Pr(E_1) \leq 1 \tag{3.2}$$

For **mutually exclusive** events E_1 and E_2 the sum of the probabilities is

$$Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2) \tag{3.3}$$

The entire sample space Ω, or the sum of the probabilities of any set of mutually exclusive and exhaustive events, equals to 1.

$$Pr(\Omega) = 1 \tag{3.4}$$

An event E_1 has its compliment $\overline{E_1}$

$$Pr(\overline{E_1}) = 1 - Pr(E_1) \tag{3.5}$$

However, the events are usually not mutually exclusive in many cases, therefore, the **addition rule** for arbitrary events in the same space is introduced as

$$Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2) - Pr(E_1 \cap E_2) \tag{3.6}$$

where $Pr(E_1 \cap E_2)$ denotes the **joint probability** that the two events E_1 and E_2 will occur simultaneously.

Sometimes it is necessary to determine the probability of an event E_1 given that the event E_2 occurs. This probability is called the **conditional probability** of E_1 given E_2 and is denoted by $Pr(E_1|E_2)$, thus

$$Pr(E_1|E_2) = \frac{Pr(E_1 \cap E_2)}{Pr(E_2)} \tag{3.7}$$

The joint probability $Pr(E_1 \cap E_2)$ can be derived from the above equation, such that:

$$Pr(E_1 \cap E_2) = Pr(E_1|E_2) \cdot Pr(E_2) \tag{3.8}$$

which is called the **multiplication rule**. In case of **independent events**, where the occurrence of an event does not depend on the occurrence of the other, the multiplication rule is

$$Pr(E_1 \cap E_2) = Pr(E_1) \cdot Pr(E_2) \tag{3.9}$$

There are three approaches to estimate probabilities. The estimate of probability of an event based on the historical data is called **empirical probability.** This type of probability is the most common probability used in engineering. With a large number of samples or a large population size, empirical probability is relatively accurate. However, if the probability of an event is not in the historical record or only few observations are available, it is more useful to consider other types of probability.

Subjective probability is the probability based on a subjective, personal judgement. Subjective probability is important in reliability assessments. The third type of probability is **a priori probability** which is the probability based on a logical analysis rather than on observations of data. And because empirical probability and a priori probability does not vary from person to person, they can be classified as **objective probability**. These three types of reliability are summarized in Fig. 3.2.

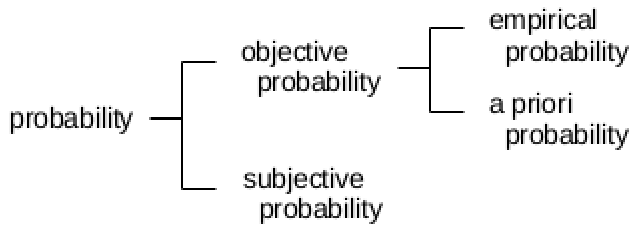


Fig. 3.2: Three types of probability

3.2.2 Random Variables and Probability Distributions

The quantity that one observes in an experiment is denoted by X, and called a **random variable** because the value it will assume in the next trial depends on randomness [Kreyszig 2010]. A **probability distribution** (also called a **distribution**) shows the probabilities of events in an experiment. The distribution of X is determined by the **distribution function** $P(x)$ (also called **cumulative distribution function**, **CDF**).

$$P(x) = Pr(X \le x) \tag{3.10}$$

which is the probability that X will assume any value not exceeding x [Kreyszig 2010]. The probability corresponding to an interval $x_1 < x \le x_2$ is

$$Pr(x_1 < X \le x_2) = P(x_2) - P(x_1) \tag{3.11}$$

A random variable X is **discrete** if X assumes countable values, otherwise it is **continuous**. If X and its distribution are continuous, then the distribution function can be given by

$$P(x) = \int_{-\infty}^{x} f(\phi)d\phi \tag{3.12}$$

The integrand $p(x)$ called the density of the distribution is continuous and non-negative. The **density function** (also called **probability density function**, **PDF**) is

$$p(x) = \frac{dP(x)}{dx} \tag{3.13}$$

From Eq. 3.11 and 3.12, it follows that

$$Pr(x_1 < X \le x_2) = P(x_2) - P(x_1) = \int_{x_1}^{x_2} p(\phi)d\phi \tag{3.14}$$

And from Eq. 3.4 and 3.12, it yields

$$\int_{-\infty}^{\infty} p(x)dx = 1 \tag{3.15}$$

More details and explanations of discrete and continuous distributions can be found in [Kreyszig 2010, Papoulis 1991]

The common probability distributions in the area of a reliability study are exponential distribution, Weibull distribution, lognormal distribution, and normal distribution. These distributions and their PDF and CDF are illustrated in Table 3.1.

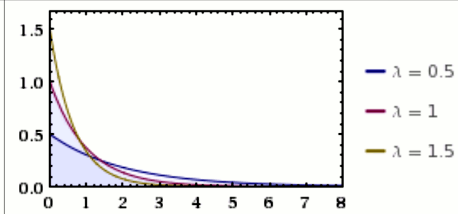
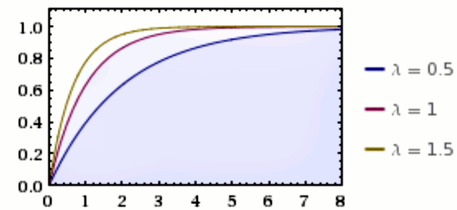
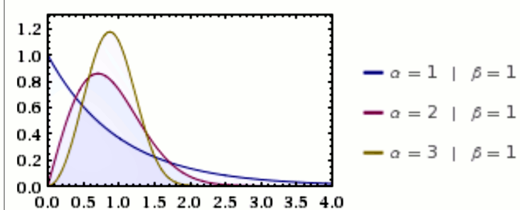
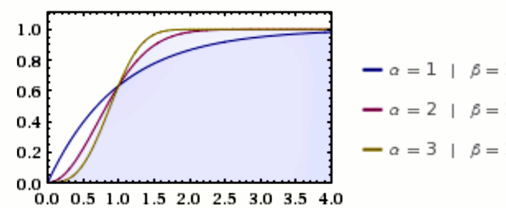
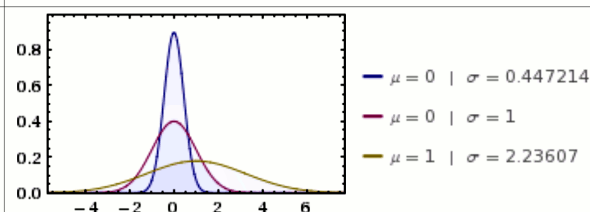
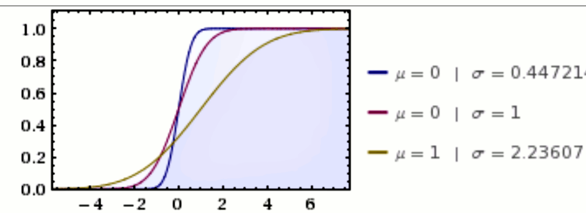
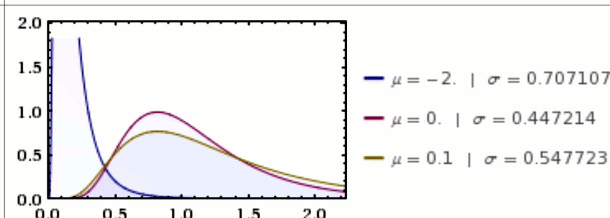
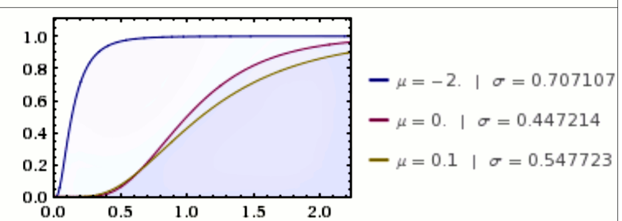
Distribution	PDF	CDF
Exponential $f(x)=\begin{cases}\lambda e^{-\lambda x} & ,x>0\\ 0 & ,x\leq 0\end{cases}$ $F(x)=1-e^{-\lambda x}$		
Weibull $f(x)=\begin{cases}\frac{\alpha}{\beta}(\frac{x}{\beta})^{k-1}e^{(\frac{x}{\beta})^{\alpha}} & ,x>0\\ 0 & ,x\leq 0\end{cases}$ $F(x)=1-e^{(\frac{x}{\beta})^{\alpha}}$		
Normal $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{(x-\mu)^2}{2\sigma^2}}$ $F(x)=\frac{1}{2}[1+erf(\frac{x-\mu}{\sqrt{2\sigma^2}})]$		
Lognormal $f(x)=\begin{cases}\frac{1}{x\sqrt{2\pi\sigma^2}}e^{\frac{(\ln(x)-\mu)^2}{2\sigma^2}} & ,x>0\\ 0 & ,x\leq 0\end{cases}$ $F(x)=\frac{1}{2}+\frac{1}{2}[1+erf(\frac{\ln(x)-\mu}{\sqrt{2\sigma^2}})]$		

Table 3.1: Common probability distributions

3.3 Reliability Evaluation Concepts

3.3.1 Reliability Function

Reliability is the ability of an item to perform a required function under given conditions for a given time interval [IEC 60050-191 1990].

An **unreliability function** $Q(t)$, also known as a **failure function** or an **outage function** $F(t)$, is the probability that an item under consideration will fail, or will switch from an up state to a down state from the origin (at time-zero, $t = 0$) until time t. Its integrand $f(t)$ is called a **failure density function**.

$$Q(t) = F(t) = \int_0^t f(\tau)d\tau \quad (3.16)$$

Reliability function $R(t)$, also called **survivability function** [Billinton 1992, Endrenyi 1978], is the probability that an item will not fail before (or at) time t.

$$R(t) = 1 - Q(t) \quad (3.17)$$

Eq. 3.16 and 3.17 can be applied to the evaluation of reliability for any simple and **non-repairable item**, given that the failure function is known. However, for a **repairable** item, the maintainability function has to be considered as well.

3.3.2 Repairable Items

In the case of repairable items (e.g. components, systems), when the item under consideration fails, the maintenance of that item will take place. The maintainability at time t is modelled as a density function $m(t)$ called **repair density function**.

During the real operation of an item, the item may fail and may have been repaired many times. The failure and repair of such an item can be modelled as distribution functions. This behavior can be modelled with states that corresponds to the objective of reliability analysis. This so-called objective model is illustrated in Fig. 3.3.

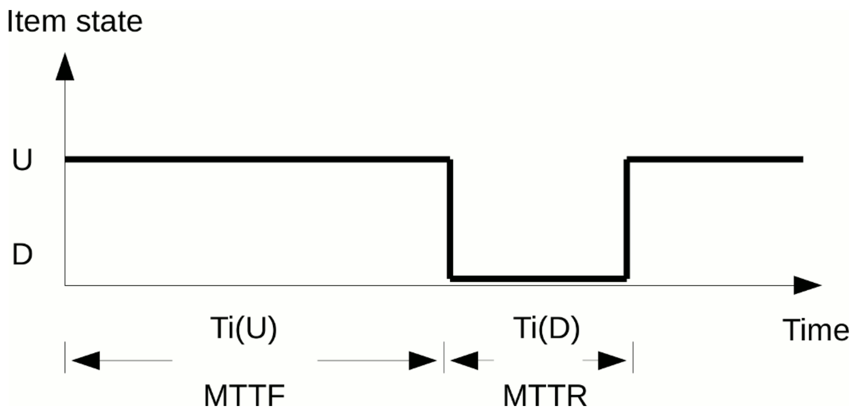


Fig. 3.3: Two-state objective model with objective reliability indices

In a simple objective model, the objective states can be classified into up state and down state [Kochs 1984].

U up state (or operating state) of an item
D down state (or outage state) of an item

The concepts of system reliability evaluation are based on this model and its objective states. According to this model, objective reliability indices can be determined.

3.3.3 Reliability Indices

From the objective model in Fig. 3.3, reliability is practically measured by its probability, frequency, and duration of each objective state. For the two-state objective model in Fig. 3.3, the following set of stationary reliability indices are considered [Kochs 1984, Kongniratsaikul 2009]:

$$\begin{bmatrix} Pr(U), Pr(D) \\ Fr(U), Fr(D) \\ Ti(U), Ti(D) \end{bmatrix} \quad \begin{matrix} \text{probabilities} \\ \text{frequencies} \\ \text{durations} \end{matrix}$$

The mean durations $Ti(U)$ and $Ti(D)$ can be determined from the expected values of random variables X and Y, which are associated to failure events, and repair events, respectively, of an item [Billinton 1992, Kochs 1984].

$$Ti(U) = E(X) = \int_0^\infty xf(x)dx \tag{3.18}$$

$$Ti(D) = E(Y) = \int_0^\infty ym(y)dy \tag{3.19}$$

Probability can be derived from the mean durations as follows:

$$Pr(U) = \frac{Ti(U)}{Ti(U) + Ti(D)} \tag{3.20}$$

$$Pr(D) = \frac{Ti(D)}{Ti(U) + Ti(D)} = 1 - Pr(U) \tag{3.21}$$

The mean frequencies are formulated as follows:

$$Fr(D) = Fr(U) = \frac{1}{Ti(U) + Ti(D)} \tag{3.22}$$

From these system indices the following often used indices are derived, which can added to the objective indices.

$Av = Pr(U)$	availability
$NAv = Pr(D)$	unavailability
$MTTF = Ti(U)$	Mean Time to Failure
$MTTR = Ti(D)$	Mean Time to Repair
$MTBF = Ti(U) + Ti(D)$	Mean Time to Between Failure

The availability and its other indicators of availability can be determined by using a Markov model [Billinton 1992, Kochs 1984].

The objective indices, Eq. 3.18 to 3.22, are independent from the shape of the density functions of up and down times.

If the failure and repair pdfs are exponential (Table 3.1), the distribution parameters will be commonly called the failure rate λ, and the repair rate μ, respectively. These failure rate λ and repair rate μ are kept constant for electrical/electronic items, one of the reasons is that the constant transition rates are simple to apply to some evaluation approaches such as Markov model [Billinton 1992, Endrenyi 1978, Kochs 1984, MIL-HDBK-217F 1991]. Eq. 3.18 and 3.19 can be simplified to the following.

$$Ti(U) = \frac{1}{\lambda} \tag{3.23}$$

$$Ti(D) = \frac{1}{\mu} \tag{3.24}$$

3.3.4 Reliability Block Diagram

In the structural viewpoint, an item or a system may be logically constructed from many other components. In most cases the determination of system reliability has to be calculated from the reliability of their components. The result depends on the reliability of each component and the interconnection between them, which is commonly represented by a reliability block diagram [Billinton 1992, Endrenyi 1978, Kochs 1984].

Given a system with *n* components, if a failure of any component would cause a system failure, that system is called a **serial system**. On the other hand, if the system fails only in the case that all of its components fail, the system is called a **parallel system**. The structure of these systems, including other complicated systems, are illustrated in Fig. 3.4.

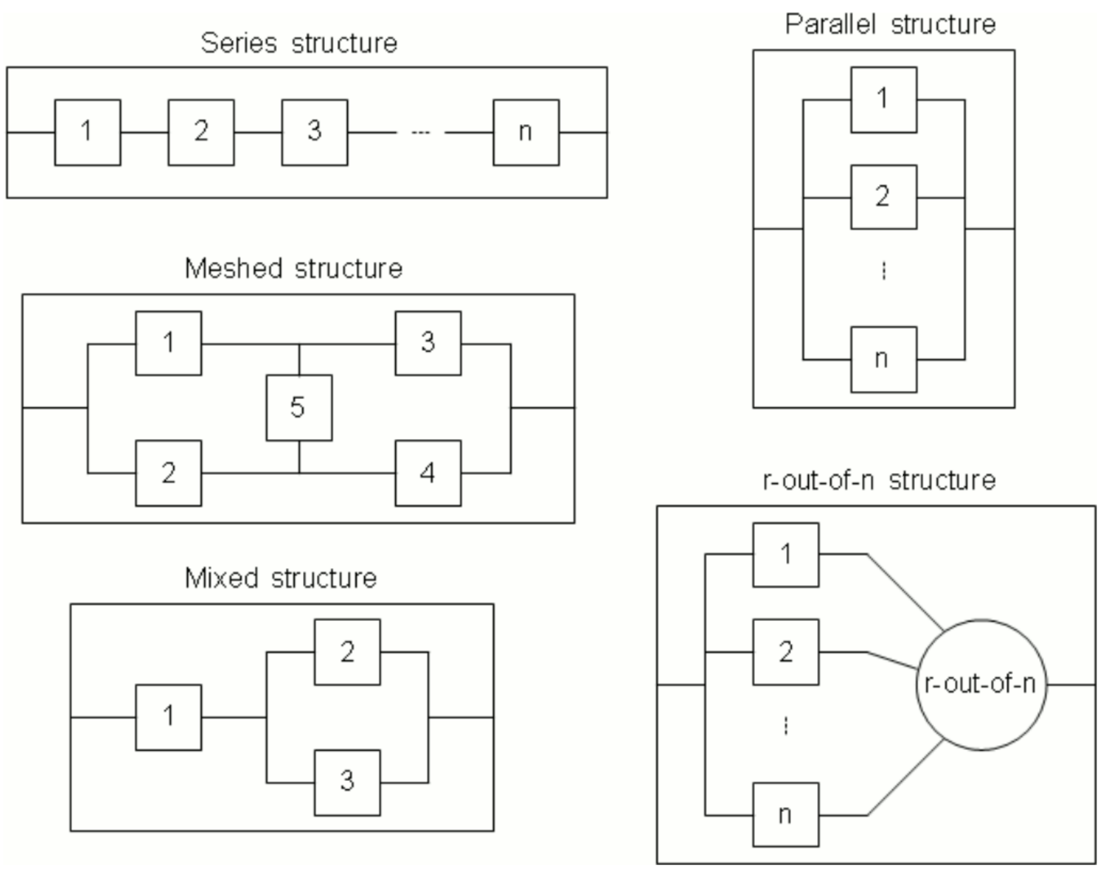


Fig. 3.4: Classification of basic logical structures

Highly complicated interconnections can be efficiently simplified by using the minimal cut method, which is briefly described in section 3.5. The details can be found in other literature e.g. [Kochs 1984].

Meshed and r-out-of-n structure can be converted to series, parallel, and mixed structures, e.g. with the minimal cut approach [Billinton 1992, Kochs 1984]. In this dissertation, only the formulas for serial systems and parallel systems are derived and used in further chapters.

All components in the following systems are assumed to be stochastically independent, which means the cause of an outage of a component is not the cause of another and **common-mode failures** do not occur. More details about the common-mode failure and its solutions can be found in [Kochs 1984].

Formulas for Serial Systems

As mentioned earlier, if any component of a system are down, the system will be down. That means the system will be up only if all components are up.

$$U_S = U_1 \wedge U_2 \wedge \ldots \wedge U_n \tag{3.25}$$

The probability of the system up state can be described as follows:

$$Pr(U_S) = Pr(U_1 \wedge U_2 \wedge \ldots \wedge U_n) \tag{3.26}$$

Regarding the probability concepts in Section 3.2, if the outage and repair of each component is independent, the probability of the system up state is equivalent to:

$$Pr(U_S) = Pr(U_1) \cdot Pr(U_2) \cdot \ldots \cdot Pr(U_n) \tag{3.27}$$

The probability of the system down state can be calculated relative to that of the up state.

$$Pr(D_S) = 1 - Pr(U_S) \tag{3.28}$$

The Mean Time To System Failure (MTTSF) can be calculated from the sum of an inverse of each component's MTTF [Kochs 1984].

$$\frac{1}{Ti(U_S)} = \frac{1}{Ti(U_1)} + \frac{1}{Ti(U_2)} + \ldots + \frac{1}{Ti(U_n)} \tag{3.29}$$

The mean frequencies can be derived from the probability and the mean durations.

$$Fr(U_S) = \frac{Pr(U_S)}{Ti(U_S)} \tag{3.30}$$

$$Fr(D_S) = Fr(U_S) \tag{3.31}$$

And the Mean Time To System Repair (MTTSR) can be derived from the mean frequencies and the probability of down state.

$$Ti(D_S) = \frac{Pr(D_S)}{Fr(D_S)} \tag{3.32}$$

Formulas for Parallel Systems

The parallel system will be up if any of its component is up.

$$U_S = U_1 \vee U_2 \vee \ldots \vee U_n \tag{3.33}$$

The calculation of this addition rule is complicated because the joint probability has to be considered (Section 3.2). To simplify the equation, it is more convenient to calculate from the inverse, or the down state of the system.

$$D_S = \overline{U_S} = \overline{U_1 \vee U_2 \vee \ldots \vee U_n} \tag{3.34}$$

By using the De Morgan's law, a simple formula is achieved.

$$D_S = D_1 \wedge D_2 \wedge \ldots \wedge D_n \tag{3.35}$$

With the similar approach as the serial systems, the parallel system reliability indices can be determined as follows:

$$Pr(D_S) = Pr(D_1) \cdot Pr(D_2) \cdot \ldots \cdot Pr(D_n) \tag{3.36}$$

$$Pr(U_S) = 1 - Pr(D_S) \tag{3.37}$$

$$\frac{1}{Ti(D_S)} = \frac{1}{Ti(D_1)} + \frac{1}{Ti(D_2)} + \ldots + \frac{1}{Ti(D_n)} \tag{3.38}$$

$$Fr(D_S) = \frac{Pr(D_S)}{Ti(D_S)} \tag{3.39}$$

$$Fr(U_S) = Fr(D_S) \tag{3.40}$$

$$Ti(U_S) = \frac{Pr(U)}{Fr(U_S)} \tag{3.41}$$

So far the reliability indices of an item (a component or a system) has been formulated using distribution functions. These functions are estimated from the lifetime data of each item. The analysis of this lifetime data is mentioned in the next section.

Reliability indices of other complex systems can be determined by the evaluation methods mentioned in section 3.5.

3.4 Lifetime Data Analysis

Statistical reliability data of a system can be taken from crucial test procedures, from real-world operation data of that system, or from expert estimates. The unequal amount of data (e.g. collection of different data sources), different aspects and assumptions of data, and complexity of reliability model effects the reliability evaluation more or less.

The analysis of lifetime data is customary based on the hazard function. The hazard function (or **hazard rate**, **failure rate**) is the ratio of the failure density function and the reliability function of an item.

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)} \tag{3.42}$$

The famous **bathtub curve** in Fig. 3.5 describes the common failure behavior of an item. In the early life of an item, the failure rate is relatively high but is decreasing rapidly through developments. At a certain point of time, an item is sent to operation. This period is called the useful life of an item, and, during the period, the item is fully developed so the failure rate becomes relatively constant. At the end life, the item begins to wear, and the failure rate increases because of the wear-out. More information about the bathtub curve can be found in [Kochs 1984, Nelson 2004].

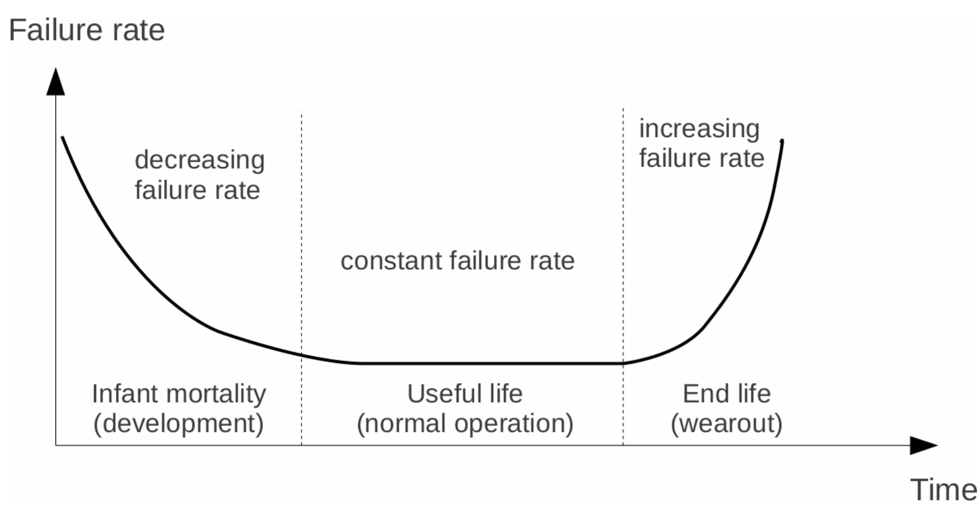


Fig. 3.5: The Bathtub Curve

In reliability evaluation, the customary consideration of an item is during its useful life which has a constant failure rate. The corresponding failure function and reliability function can be determined from a probability model.

Suppose that failure events of an item is random, and these events occur at a constant rate λ. The number of failure events in time interval Δt follows a homogenous Poisson process [Kreyszig 2010, Papoulis 1991].

$$P\{k \text{ in } \Delta t\} = e^{-\lambda \Delta t} \frac{(\lambda \Delta t)^k}{k!} \tag{3.43}$$

where k is the number of failure events in time interval Δt. The reliability function at time t equals to the probability that no failure occurs in time interval $\Delta t = [0, t]$.

$$R(t) = P\{0 \text{ in } (t-0)\} = e^{-\lambda(t-0)} \frac{(\lambda(t-0))^0}{0!} \tag{3.44}$$

$$R(t) = e^{-\lambda t} \tag{3.45}$$

The failure function and its density function are

$$F(t) = 1 - R(t) = 1 - e^{-\lambda t} \tag{3.46}$$

$$f(t) = \frac{dF(t)}{dt} = \lambda e^{-\lambda t} \tag{3.47}$$

which is an exponential function with parameter λ. It can be seen that the failure rate is constant if the failure distribution is exponential.

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda \tag{3.48}$$

This constant failure rate satisfies the condition in an item’s useful life, therefore the use of exponential distribution is common in reliability evaluation. However, in some applications, the failure rate may not be constant with respect to time, therefore, other distributions such as Weibull distribution and lognormal distribution are also widely used

Suppose that a distribution function has been selected for an application, the parameters of such distribution function have to be estimated from lifetime data. There are a number of estimation approaches such as Least Square, Linear Regressive, Maximum Likelihood, Minimum Variance, etc. [Nelson 2004, ReliaSoft 2013]. The Maximum Likelihood Estimate is focused in this work because of its advantages which are mentioned in the following subsection.

3.4.1 Maximum Likelihood Estimate (MLE)

The principle of maximum likelihood estimate (MLE) dated back to Laplace in 1774, and was popularized by Fisher during the beginning of 20th century. In contrast to the conventional ideas of Baysian probability, inverse probability was introduced to infer the probability backwards from the data to the parameter, or from effects to causes. For more details regarding the history of Bayesian probability and maximum likelihood estimate, see [Fienberg 2006, Fisher 1922, Lehmann et al. 1998]. In the modern reliability theory, the idea of maximum likelihood estimate in [Lehmann et al. 1998, Nelson 2004, Rupert 2010] is adopted in this dissertation.

The basic idea of the MLE is that among the countless sets of data only a small portion is measured and collected due to cost and time constraints. An unknown parameter to be estimated has some unknown mean and variance, and if such parameter exists then MLE must be a function of it. This is called the **sufficiency** property of MLE. With more available data, the more accurate the estimators are, and the variance of the estimates are asymptotically normal distributed. These properties are called the **consistency** of MLE and the **asymptotic normality** of MLE, respectively [Nelson 2004].

Later the estimators of parameters are used to determine reliability indices. These reliability indices are functions of parameters and cannot be estimated directly from the data itself, but can be estimated from the so-called induced likelihood. This property is called the **invariance** of MLE [Nelson 2004].

These properties are some of the major advantages suitable for the lifetime analysis where data uncertainty is considered. Sufficiency, consistency, and asymptotic normality ensure the accuracy of the estimators regardless of the amount of statistical data. Invariance makes it possible to find reliability indices and their variance from the estimates. In order to use MLE, one must define a likelihood function suitable for the given statistical data.

3.4.2 Likelihood Function

The likelihood function is a function of an unknown distribution parameter θ, given a collection of known lifetime data [Greene 2011, Lehmann et al. 1998, Nelson 2004]. If $L_i(\theta|t_i)$ is the probability of a single failure observation i, and this observation fails at time t_i, as shown in Fig. 3.6. The probability is equivalent to the density function on the failure time t_i of the observation i, depending on parameter θ. This relation can be formulated as Eq. 3.49.

$$L_i(\theta|t_i) = f(t_i;\theta) \tag{3.49}$$

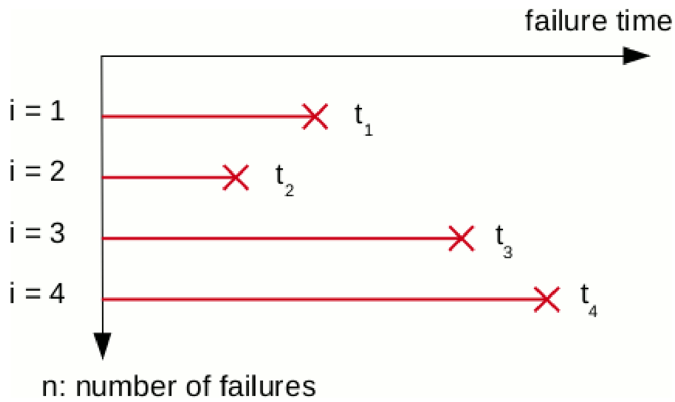


Fig. 3.6: Lifetime data with complete failure time

This probability becomes the contribution of an observation to the likelihood. For a sample of n failure observations, a likelihood function $L(\theta)$ is formulated as the following joint probability [Duckworth et al. 2002, Rodriguez 2007].

$$L(\theta) = \prod_{i=1}^{n} L_i(\theta|t_i) \tag{3.50}$$

$$L(\theta) = \prod_{i=1}^{n} f(t_i;\theta) \tag{3.51}$$

[Greene 2011] noted that, the joint density is written as a function of the data conditioned on the parameters, but the likelihood function is written in reverse, as a function of the parameters, conditioned on the data. Though the two functions are the same, it is to be emphasized that the likelihood function is written in this fashion to highlight the interest in the parameters and the information about them that is contained in the observed data.

Regarding to the bathtub curve of the hazard function in Fig. 3.5, the common consideration of system reliability is within an item's useful life, which has constant hazard rate (explained in Section 3.4). Therefore, the exponential function is generally used to describe the failure behavior. The joint probability and the likelihood function of an exponential distribution is as follows:

$$L_i(\lambda) = \lambda \cdot e^{-\lambda t_i} \tag{3.52}$$

$$L(\lambda) = \prod_{i=1}^{n} \lambda \cdot e^{-\lambda t_i} \tag{3.53}$$

The estimator of the parameter $\hat{\theta}$ can be determined by finding the maximum of the likelihood function.

$$\frac{\partial L(\theta)}{\partial \theta} = 0 \tag{3.54}$$

In many cases it is more convenient to find the maximum of a logarithm of the likelihood function instead. Because the logarithm yields the similar answer and often has a simpler form.

$$\frac{\partial LL(\theta)}{\partial \theta} = \frac{\partial \ln(L(\theta))}{\partial \theta} = 0 \tag{3.55}$$

For a likelihood function of an exponential distribution with one parameter, a logarithm of the likelihood function is the following.

$$\frac{\partial LL(\lambda)}{\partial \lambda} = \frac{\partial \ln\left(\prod_{i=1}^{n} \lambda \cdot e^{-\lambda t}\right)}{\partial \lambda} \tag{3.56}$$

$$\frac{\partial LL(\lambda)}{\partial \lambda} = \frac{\partial\left(n \cdot \ln(\lambda) - \lambda \sum_{i=1}^{n} t_i\right)}{\partial \lambda} \tag{3.57}$$

$$\frac{\partial LL(\lambda)}{\partial \lambda} = n \cdot \frac{1}{\lambda} - \sum_{i=1}^{n} t_i = 0 \tag{3.58}$$

From Eq. 3.58 the estimator $\hat{\lambda}$ of the only scale parameter λ is shown in Eq. 3.59.

$$\hat{\lambda} = \frac{n}{\sum_{i}^{n} t_i} \tag{3.59}$$

With high number of observations n the estimator $\hat{\lambda}$ is asymptotic and the distribution of $\hat{\lambda}$ is close to normal. The variance $\hat{\sigma}_{\hat{\lambda}}^2$ of the estimator $\hat{\lambda}$ can be obtained from an inverse of Fisher Information.

The use of Fisher Information yields the true asymptotic variance, given large sample [Nelson 2004]. Therefore, it is adopted in this work.

3.4.3 Fisher Information

Fisher information is the expectation of the negative of the 2nd partial derivative of the log-likelihood with respect to the parameter [Nelson 2004]. θ_0 denotes the "true" value (as in the value to be estimated) of the probability density function parameter. The Fisher information of the “true” value is the following.

$$I(\theta_0) = E\left(-\frac{\partial^2 LL(\theta)}{\partial\theta^2}\bigg|\theta = \theta_0\right) \tag{3.60}$$

The true asymptotic variance σ_θ^2 for the “true” value is an inverse of Fisher information.

$$\sigma_{\theta_0}^2 = \frac{1}{E\left(-\frac{\partial^2 LL(\theta)}{\partial\theta^2}\bigg|\theta = \theta_0\right)} \tag{3.61}$$

In the case of exponential distribution with large sample, the asymptotic variance $\hat{\sigma}_{\hat{\lambda}}^2$ of the MLE $\hat{\lambda}$ is equal to the true asymptotic variance, given life data with complete failure time [Nelson 2004].

$$\hat{\sigma}_{\hat{\lambda}}^2 = \frac{1}{E\left(-\frac{\partial^2 LL(\lambda)}{\partial\lambda^2}\bigg|\lambda = \hat{\lambda}\right)} \tag{3.62}$$

$$\hat{\sigma}_{\hat{\lambda}}^2 = \frac{\hat{\lambda}^2}{n} \tag{3.63}$$

3.4.4 Right-Censored Data

In the previous section, it is assumed that the complete failure time is given in the statistical data, or the data is not censored. However, in practice, the failure data is limited to the time constraint, and failures above a certain time are neglected, as shown in Fig. 3.7. This type of time-censored data is called right-censored as the information beyond the right part is censored.

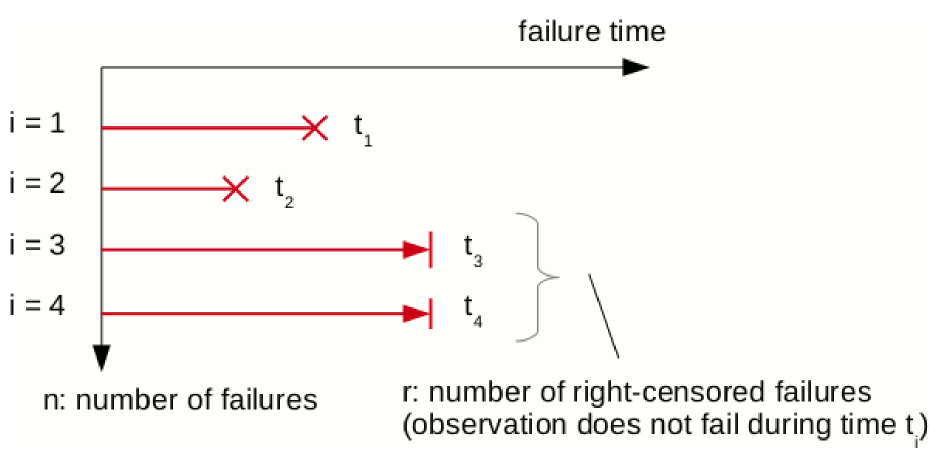


Fig. 3.7: Lifetime data with right-censored observations

In the case of right-censored data, the likelihood which contributes to the likelihood function is equal to the probability of survival of the observation until the censored time t_i.

$$L_i(\theta|t_i) = R(t_i) \tag{3.64}$$

$$L_i(\theta|t_i) = 1 - F(t_i) \tag{3.65}$$

Given a total of n observations with r non-censored failures and $n-r$ censored failures, the likelihood function of these n observations is the following.

$$L(\theta) = \prod_{i=1}^{r} f(t_i|\theta) \cdot \prod_{i=1}^{n-r} (1 - F(t_i)) \tag{3.66}$$

For a likelihood function based on an exponential distribution, its log-likelihood yields a rather simple form.

$$LL(\lambda) = \ln\left(\prod_{i=1}^{r} f(t_i|\lambda) \cdot \prod_{i=1}^{n-r} (1 - F(t_i))\right) \tag{3.67}$$

$$LL(\lambda) = r \cdot \ln(\lambda) - \lambda \sum_{i=1}^{r} t_i - \lambda \sum_{i=1}^{n-r} t_i \tag{3.68}$$

$$LL(\lambda) = r \cdot \ln(\lambda) - \lambda \sum_{i=1}^{n} t_i \tag{3.69}$$

The maximum likelihood is found from the first derivative of the likelihood function as shown in Eq. 3.70 to 3.71.

$$\frac{\partial LL(\lambda)}{\partial \lambda} = \frac{\partial\left(r \cdot \ln(\lambda) - \lambda \sum_{i=1}^{n} t_i\right)}{\partial \lambda} = 0 \tag{3.70}$$

$$\frac{\partial LL(\lambda)}{\partial \lambda} = r \cdot \frac{1}{\lambda} - \lambda \sum_{i=1}^{n} t_i = 0 \tag{3.71}$$

The maximum likelihood yields the estimator $\hat{\lambda}$ in Eq. 3.72.

$$\hat{\lambda} = \frac{r}{\sum_{i=1}^{n} t_i} \tag{3.72}$$

The asymptotic variance $\hat{\sigma}_{\hat{\lambda}}^2$ for the MLE λ may differ to the true asymptotic variance, but numerically close [Nelson 2004]. Therefore one can obtain the asymptotic variance from an inverse of Fisher information in Eq. 3.61, which gives the result in Eq. 3.73.

$$\hat{\sigma}_{\hat{\lambda}}^2 = \frac{\hat{\lambda}^2}{r} \tag{3.73}$$

3.4.5 Repairable Items

A more complicated case is when the lifetime of a component or a system under consideration is repairable. The lifetime data in Fig. 3.8 is composed of failure times, censored failures, repair times, and also censored repair.

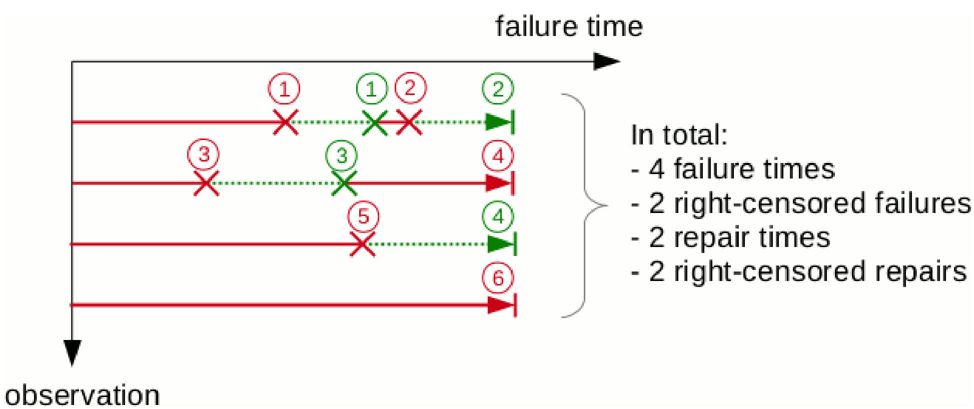


Fig. 3.8: Lifetime data of a repairable item

Assuming that each failure and repair is independent from each other, the lifetime data of a repairable component can be transformed into two sets of right-censored data: failure distribution and repair distribution, as shown in Fig. 3.9. The estimators of the parameters of the failure distribution and the repair distribution can be determined separately.

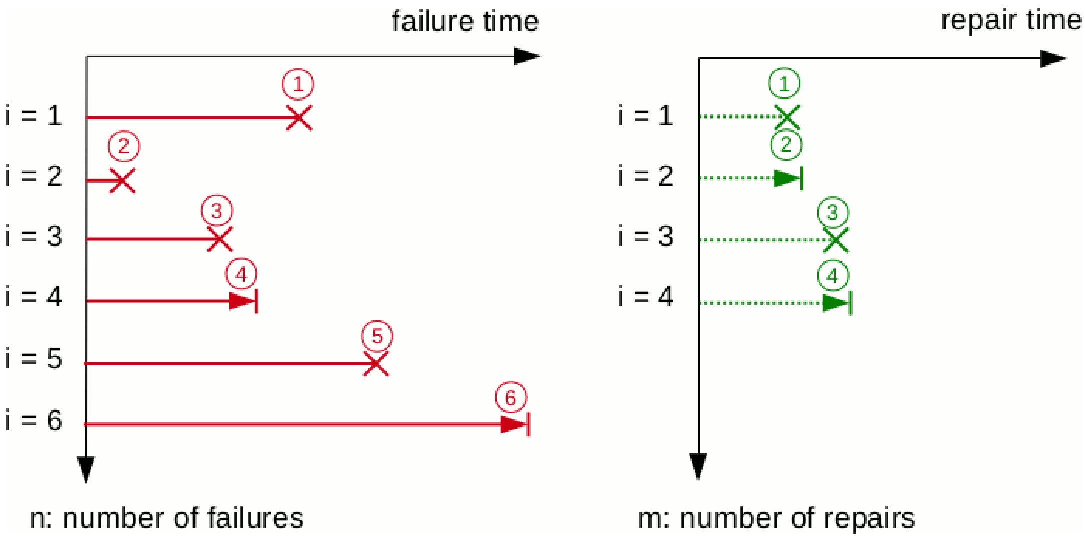


Fig. 3.9: Transformation of lifetime data in Fig. 3.8, assuming all failures and repairs are independent

Similar to right-censored data, MLE can be estimated from the log-likelihood function. The estimator $\hat{\lambda}$ and the asymptotic variance $\hat{\sigma}^2_{\hat{\lambda}}$ of an exponential failure distribution is as follows:

$$\hat{\lambda} = \frac{r}{\sum_{i=1}^{n} t_i} \tag{3.74}$$

$$\hat{\sigma}^2_{\hat{\lambda}} = \frac{\hat{\lambda}^2}{r} \tag{3.75}$$

The estimator $\hat{\mu}$ of the repair rate and the asymptotic variance $\hat{\sigma}^2_{\hat{\mu}}$ can be determined in the similar manner, given a total of m repair observations with q non-censored repairs. This results in the estimators in Eq. 3.76 and 3.77.

$$\hat{\mu} = \frac{r}{\sum_{i=1}^{m} t_i} \tag{3.76}$$

$$\hat{\sigma}^2_{\hat{\mu}} = \frac{\hat{\mu}^2}{q} \tag{3.77}$$

3.4.6 Pre-Estimated Data

In practice there are a great number of reliability data sources in which the distribution model and its corresponding parameters have been estimated. Since the model in this work is based on exponential distribution, a transformation of the pre-estimated data is required for consistency of data. If the raw observation data is available one may estimate the failure rate $\hat{\lambda}$ and the repair rate $\hat{\mu}$ directly. Nevertheless, in most cases the observation data is not published together with the pre-estimated data.

The reliability indices of an item are functions of MTTF and MTTR, regardless of their distribution model. Therefore, a consistent conversion of the pre-estimated data must retain its MTTF and MTTR. Under this condition the estimator of the exponential distribution parameters $\hat{\lambda}$ and $\hat{\mu}$, and their variance can be achieved.

The MTTF is an inverse of the failure rate. If the estimator $M\hat{T}TF$ is given, then $\hat{\lambda}$ can be obtained from the following.

$$\hat{\lambda} = \frac{1}{M\hat{T}TF} \tag{3.78}$$

Finding variance is a little more complicated. The maximum likelihood estimator as a very useful property called invariance property. If the pre-estimated distribution parameters are MLE, these parameters can be used to obtain other estimates indirectly by defining an induced likelihood.

Given a function $h(\theta)$ is a continuous function of a parameter θ. The MLE of this function $\hat{h}$ is equal to the function of the estimator $\hat{\theta}$.

$$\hat{h}(\theta) = h(\hat{\theta}) \tag{3.79}$$

This is called the invariance property of MLE. For large number of observations, the distribution of $\hat{h}$ is close to normal with asymptotic variance $\hat{\sigma}_{\hat{h}}^2$. Let $(\ )_0$ denotes that the partial derivative is evaluated at $\theta = \theta_0$,

$$\hat{\sigma}_{\hat{h}}^2 = \left(\frac{\partial h}{\partial \theta}\right)_0^2 \cdot \hat{\sigma}_{\hat{\theta}}^2 \tag{3.80}$$

In this dissertation, only the basic idea of MLE is used. More details regarding MLE can be found in [Greene 2011, Lehmann et al. 1998, Nelson 2004, Rodriguez 2007, Rupert 2010].

Some of the most popular distribution models are Weibull distribution, normal distribution, and lognormal distribution. With Eq. 3.80, the results of the estimator of the exponential distribution parameters and its variance are shown in Table 3.2.

Distribution	$\hat{MTTF}$	$Var\left(\hat{MTTF}\right)$	$\hat{\lambda}$	$\hat{\sigma}_{\hat{\mu}}$
Lognormal	$e^{\hat{m}+\frac{\hat{\sigma}^2}{2}}$	$\left(e^{\hat{\sigma}^2}-1\right)e^{2\hat{m}+\hat{\sigma}^2}$	$\frac{1}{e^{\hat{m}+\frac{\hat{\sigma}^2}{2}}}$	$\frac{e^{\hat{\sigma}^2}-1}{e^{2\hat{m}+\hat{\sigma}^2}}$
Normal	$\hat{m}$	$\hat{\sigma}^2$	$\frac{1}{\hat{m}}$	$\frac{\hat{\sigma}^2}{\hat{m}^4}$
Weibull	$\hat{\alpha}\Gamma\left(1+\frac{1}{\hat{\beta}}\right)$	$\hat{\alpha}^2\Gamma\left(1+\frac{2}{\hat{\beta}}\right)-\hat{\alpha}^2\Gamma^2\left(1+\frac{1}{\hat{\beta}}\right)$	$\frac{1}{\hat{\alpha}\Gamma\left(1+\frac{1}{\hat{\beta}}\right)}$	$\frac{\Gamma\left(1+\frac{2}{\hat{\beta}}\right)-\Gamma^2\left(1+\frac{1}{\hat{\beta}}\right)}{\hat{\alpha}^2\Gamma^4\left(1+\frac{1}{\hat{\beta}}\right)}$

Table 3.2: Conversion of pre-estimated data into exponential distribution

3.5 Set of Standard Reliability Approaches

Most reliability approaches are complex mathematical procedures, which allow exact evaluation of reliability only for a few special systems, provided that the input indices are well known. It is impossible to model and calculate large and complex systems exactly. A number of approximated procedures for large and complex systems which cover a wide range of engineering applications, have been developed [Billinton 1992, Endrenyi 1978, Kochs 1984, Ross 2010, Schneeweiss 1973].

Fig. 3.10 shows a comprised overview of reliability evaluation approaches with their assessment criteria. These approaches are briefly described according to their essential features in this section, and more details can be found in [Kochs 1984]. All these reliability approaches do not consider uncertainties.

assessment criteria:	large systems	intermeshed system structure	s-dependency (real-world conditions)	transparency (clarity, comprehensibility)	simplicity (cost efficiency)	(white box) documentation	
analytical approaches/results:							
network approaches:							easy calculation of simple structured systems (excluding s-dependency)
boolean algebra		x				x	
serial and parallel structures	x			x	x	x	
minimal path	$x^{1)}$	$x^{1)}$		x		x	
minimal cut (*MC*)	x	x		x		x	
probable minimal cut	x	x		x	x	x	
Markov minimal cut approach	**x**	**x**	**x**	**x**	**x**	**x**	**easy calculation of large <u>and</u> complex systems**
state space approaches:							
probable Markov path			x	x	x	x	for complex subsystems to embed in *MC*
Markov process			x	x		x	
semi/non Markov process			x				for theoretical studies
Petri networks		$x^{1)}$	x				for special subsystems
simulation approach/results:							
Monte Carlo simulation	$x^{1)}$	$x^{1)}$	x				for comprehensive analysis and studies

x constraints fulfilled to a high degree
$x^{1)}$ if the failure probability of the components are very low, then the calculation time can increase drastically

Fig. 3.10: An overview of standard reliability approaches

Boolean algebra

Boolean algebra is suitable and simple to use for small and meshed system structure [Schneeweiss 1973].

Approaches for serial and parallel system structure

This approaches, mentioned in details in Section 3.3.4, are easily applicable to unmeshed serial and parallel structure, which can be large. Dependency between components and intermeshed system structures cannot be taken into account, except for special cases.

Minimal path approach

The approach is based on networks based on component up states. Because the probabilities of the component up states are normally nearly one, or nearly 100%, the calculation is more elaborate than the minimal cut approach, especially for large systems without many parallel paths. Dependencies cannot be taken into account [Billinton 1992, Endrenyi 1978, Kochs 1984].

Minimal cut approach

The approach is based on networks based on component down states, which makes the calculation with the minimal cut approach much easier than the calculation with the minimal path approach. Dependencies cannot be taken into account. A difficulty is the identification of all minimal cuts, whose number can be very high and whose minimal cut structure can be difficult. A major advantage is the identification of minimal cuts direct from the functional structure of a system (taken into account operation and outage behavior), without need for developing reliability block diagrams as intermediate step. This make the application transparent [Billinton 1992, Endrenyi 1978, Kochs 1984].

Probable minimal cut approach

In practise only a few number of minimal cuts determine the result (objective indices). The significant minimal cuts can be determined “manually” from the functional system structure. “Manual” (as contrast to automatic) determination of minimal cuts is of advantage to get a deep insight of the operation and outage behavior of systems (transparent). Furthermore up till now no automatic determination procedure for complex (real-world) system structures is known. In any case the determination of all relevant minimal cuts needs careful recognition, but the significant minimal cuts are as a rule not difficult to define [Kochs 1984].

Markov process approach

A Markov process approach is a fundamental and powerful reliability tool for relative small systems with dependent components. A Markov process is based on Markov states (component or system states) and the transition between them that are characterized by constant transition rates (which means exponential pdfs). Even for small systems which are not simple to model and to calculate [Billinton 1992, Endrenyi 1978, Kochs 1984, Ross 2010].

Probable Markov path approach

In order to overcome the difficulty in modelling and calculation of Markov processes (or to fulfill the criteria simplicity) the approximated probable Markov path approach has been developed. The idea is very simple, namely to model and calculate only the probable paths, which reduce the modelling and calculation effort considerably or makes an evaluation possible at all [Dib 1978, Kochs 1984].

Markov minimal cut approach

Markov process models (or probable Markov path models) fill the gap of minimal cut models, namely to consider dependency. Thus, the idea is to integrate Markov process approach in minimal cut approach to benefit the advantages and avoid the disadvantages [Dib 1978, Singh 1980]. The combined approach is called Markov minimal cut approach. The approach fulfills all criteria to a high degree. A full description of the Markov minimal cut approach is first published in [Kochs 1984], and further applications are published in [Kochs et al. 1999, Kochs 2002].

Semi/non Markov process approach

Such approaches are characterized by stochastic processes with non constant transition rates (or non exponential pdfs). Because of the extreme difficulties to model and calculate Semi-Markov or non-Markov processes, they only have a niche role for small system studies, e.g. [Edwin et al. 1979-1, Edwin et al. 1979-2].

Petri networks

Petri networks belong to the state space approaches. The advantage is the comprehensive modelling capability of small and complex systems and the determination of time dependent events. Problems with the analysis of statistical data (slow convergence, long calculation time, calculation of estimates) are the same as for simulation. The application area are primarily software and transport networks [Schneeweiss 1973].

Monte Carlo simulation approach

Monte Carlo simulations allows simple calculation of complex (meshed structure, dependency, pdfs) systems, but they have the disadvantage that they normally need extensive calculation time due to slow convergence depending mainly on component reliability indices. The higher component reliability (lower the failure rate), the higher is the calculation time. The result are estimated object indices, which accuracy depend on the number of simulation steps. The way of calculation cannot be verified (black box approach), thus it is not transparent and cannot be documented in a verifiable way. More information about Monte Carlo can be found in [Hazewinkel 2001].

3.6 Application Areas of Reliability Evaluation

The approaches listed and assessed in Fig. 3.10 are applicable for several technical engineering systems, both for large scale and complex systems, e.g.:

- power generation, transmission, distribution systems,
- transformer stations,
- switching stations,
- computer systems,
- automation and control systems (e.g. process, environment, energy. traffic),
- mechatronic systems,
- automotive systems.

In this work, the application in automation and control systems, and in power transmission systems, are given as case studies.

4 Consideration of Uncertainty and Risks in Reliability

In Chapter 3, the conventional reliability evaluation procedure has been introduced. This procedure is bounded under two assumptions:

I) There is no uncertainty in the statistical data
II) It is sufficient to represent reliability indices with mean values

In reality, statistical data is gathered under restricted conditions. For instance, the restricted data measurement time frame, restricted number of samples, restricted data representation models. These restrictions are sources of uncertainty. According to the definitions in Chapter 2, **uncertainty** is classified by its sources into two classes: **indeterminacy** and **incompleteness**. The border between the two classes is not exact, rather smooth or fuzzy.

Uncertainty is often studied with respect to its properties which is either **aleatory** or **epistemic**. The two types of uncertainty affect the determination of the reliability and its attributes. These effects of uncertainty are called **risks**. Thus, the inconsideration of uncertainty of statistical data sources would cause unaccounted, unrepresented risks in reliability assessment.

The main objective of this work is to establish a reliability evaluation framework, in which uncertainty and risks are accounted for, and are practically represented.

Furthermore, as mentioned in Chapter 3, an arithmetic mean value can be used as a **measure of central tendency.** If the reliability indices are based only on mean values, as being used in conventional reliability evaluation procedures, the effects of its uncertain behavior cannot be shown. To study the effects of uncertainty, **measures of location** and **measures of dispersion** have to be considered apart of the central tendency. Therefore, the second objective of this work is to adapt these measurements for reliability indices.

A number of literature regarding the consideration of uncertainty in reliability evaluation (Table 4.1) has been surveyed and reviewed regarding its approach and limitations. Later in this chapter a reliability framework is introduced to overcome these limitations of the existing approaches.

4.1 Literature Review

The classification of uncertainty into aleatory and epistemic is commonly accepted, and many literature try to cope with either by using different classes of probability, namely **empirical probability**, **subjective probability**, and **a priori probability**. A number of works and literature have been surveyed and compared according to this classification. Table 4.1 summarizes and compares characteristics of all reviewed frameworks and the proposed framework of uncertainty.

Literature	Uncertainty		Probability			Reliability Indices	Statistical Measures	
	Aleatory	Epistemic	Empirical	Subjective	A Priori		Central Tendency	Location or Dispersion
[Coit et al. 2004, Coit et al. 2009, Tekiner et al. 2011]	✔		✔			partial	✔	✔
[Limbourg 2008, Limbourg et al. 2006, Limbourg et al. 2007, Rocquigny 2008]		✔	✔	✔		partial	✔	
[Heard et al. 2006]		✔	✔		✔	partial	✔	✔
[Kochs et al. 2012, Kongniratsaikul 2009]		✔	✔	✔	✔	full	✔	
[Frank 1995, Frank 1996]		✔	✔		✔	partial	✔	
Proposed Framework	✔	✔	✔	✔	✔	full	✔	✔

Table 4.1: Comparison of the surveyed uncertainty frameworks and the proposed framework

Median, quantiles, and dispersion of the unreliability distribution have been considered in [Coit et al. 2004, Coit et al. 2009, Tekiner et al. 2011]. Apart from mean values, reliability is also measured in median, in distribution such as percentiles, and in dispersion such as variance, semivariance, and coefficient of variation. The approach was based on empirical probability, and tried to estimate reliability indices from statistical data, which was assumed to be creditable. Indeterminacy, or a lack of knowledge, was not taken into consideration and epistemic uncertainty still remains. Equations of some reliability indices, not all, have been formulated. In these works partial reliability indices, compared to the full reliability indices mentioned in Chapter 3, are represented by a variety of statistical measures.

Dempster-Shafer theory has been used to evaluate epistemic uncertainty in [Limbourg 2008, Limbourg et al. 2006, Limbourg et al. 2007, Rocquigny 2008]. Expert knowledge or a form of subjective probability has been used to find the “upper bound" and "lower bound" of the probable reliability and unreliability functions. The usage of empirical probability was not explicitly evaluated and the aleatory uncertainty remains. Most of the reliability indices except the frequency were evaluated by its central tendency.

In [Heard et al. 2006] and similarly in [Kongniratsaikul 2009], the system under consideration was in an early design stage. The parameters of the system unreliability function were unknown, thus, computer simulations were built to find a distribution or a confidence interval of the unreliability function. These approaches showed that a priori probability or a logical analysis can be used with an empirical probability to overcome epistemic uncertainty. A number of statistical measures of partial reliability indices are calculated in [Heard et al. 2006]. The reliability evaluation can be co-operated with a subjective probability in the form of expert knowledge [Kochs et al. 2012, Kongniratsaikul 2009].

It is important to note that the reliability evaluation in an early design stage is usually incorporated with a priori probability or a logical analysis of system design itself. This is consistent to [Frank 1995, Frank 1996] which considered systems in design stage. The calculations were based on both empirical probability and a priori probability.

So far the existing frameworks of reliability evaluation are either focused on aleatory uncertainty or epistemic uncertainty, but not both. This is rooted from the different perspective of uncertainty, whether the given knowledge is sufficient. If the knowledge is sufficient, one may treat the uncertainty due to the evaluation as aleatory. But if the knowledge is not sufficient, subjective probability and a priori probability has to be supplied to solve the epistemic uncertainty caused by the limited knowledge.

Unlike the existing works in the subject of uncertainty in reliabiltiy evaluation, both types of uncertainty are identified and taken into consideration in the proposed framework in this work.

4.2 State of the Art and Proposed Framework of Uncertainty

As already mentioned in Chapter 2, there is an unclear border between aleatory and epistemic uncertainty. Such unclear border diverses the uncertainty in reliability evaluation into two different ways. Either treat the uncertainty as aleatory, under the perspective that the given knowledge is sufficient, or treat it as epistemic under the perspective that the given knowledge is not sufficient.

A framework incorporating both types of uncertainty is proposed, and illustrated in Fig. 4.1. Three reliability evaluation approaches have been proposed; one for aleatory uncertainty and two for epistemic uncertainty. The use of different types of probability (empirical, subjective, or a priori) distinguishes among the three approaches, leading to a convenient selection of the approach in practice.

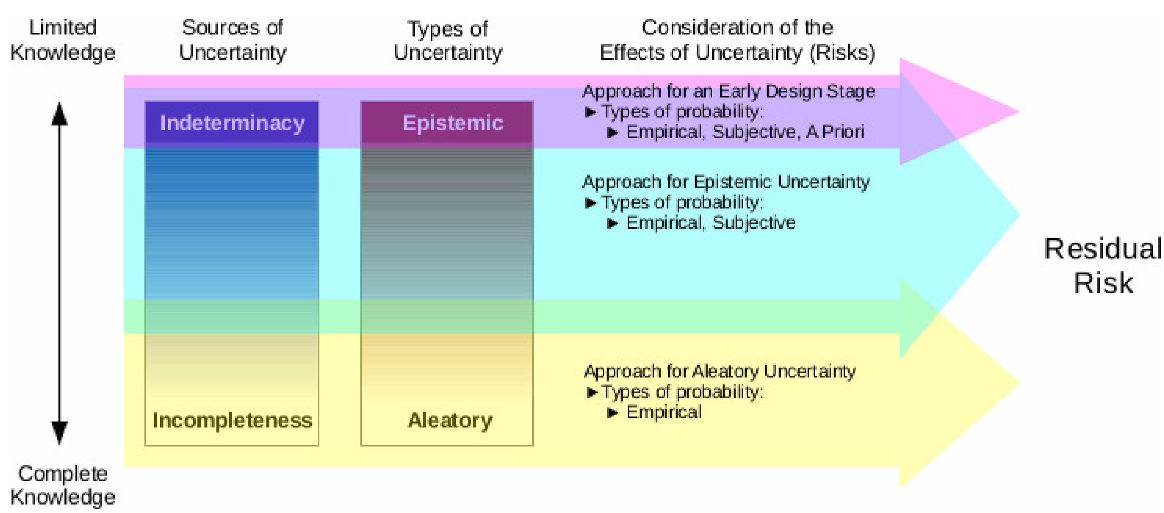


Fig. 4.1: Proposed uncertainty and risks framework containing three reliability evaluation approaches, which results in a residual risk

That is, **(A)** if only an empirical probability is focused, the approach dealing with aleatory uncertainty is a most suitable approach. **(B)** If a subjective probability is required to solve a problem, then an epistemic approach is more appropriate. **(C)** In the case where subjective probability is not enough, or subjective probability may lead to controversial results, which occurs frequently in an early stage of system design, the third approach dealing incorporating with subjective and a priori probability is taken into consideration.

In Chapter 5, the approach for aleatory uncertainty (A) is mentioned and measures of uncertainty in reliability evaluation are derived from empirical probability. In Chapter 6, the approach for epistemic uncertainty (B) which uses subjective probability is described. And the approach for an early design stage (C) is mentioned in Chapter 7.

5 Approach for Aleatory Uncertainty

The fundamentals of the reliability evaluation and its mathematical statements have been formulated in Chapter 3. In this chapter the consideration of the aleatory uncertainty, defined in Chapter 2, is investigated.

The process of reliability assessment under aleatory uncertainty can be divided into 4 steps, illustrated in Fig. 5.1.

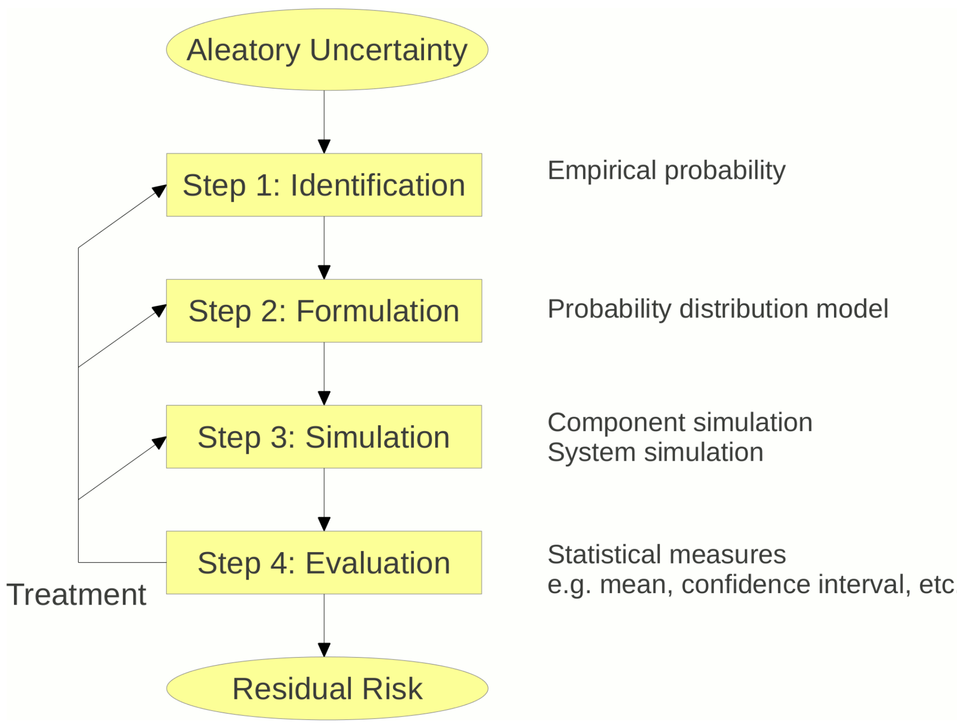


Fig. 5.1: The process of reliability assessment under aleatory uncertainty

5.1 Step 1: Identification

Aleatory uncertainty occurs by the statistic/random behavior of component and system and cannot be suppressed by more accurate measurements. Aleatory uncertainty can be expressed by well known pdfs of statistically distributed up and down times of components and systems.

In the conventional reliability evaluation method, where the aleatory uncertainty is not regarded, the evaluation of reliability indices is generally focused on the mean up time and mean down time. However, one can perceive that two systems with similar mean up time and mean down time may have different reliability profile, which is reflected by their system up and down time pdfs.

With the help of the system reliability block diagram analysis, the system up and down time pdfs can be determined from the component up and down time pdfs.

5.2 Step 2: Formulation

With representative statistics of $t(U)_i$, $t(D)_i$ (Fig 5.2 and 5.3) density functions and their indices can be estimated by means of probability theory.

Weibull distribution is widely used in reliability evaluation, and, with specific parameters, it is reducible to exponential distribution [Nelson 2004]. Therefore, it is selected in this work to demonstrate the effect of aleatory uncertainty.

To analyze the influence of pdfs of component up and down times to system indices Weibull density functions (Fig. 5.1) are assumed to be exactly given for each component. Given α and β a scale parameter and shape parameter of a Weibull pdf, respectively.

$$f(t) = \frac{\beta}{\alpha} \cdot \left(\frac{t}{\alpha}\right)^{\beta - 1} \cdot e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \tag{5.1}$$

With $\beta = 1$ the Weibull distribution is equivalent to exponential distribution. The MTTF can be described with a gamma function as follows:

$$MTTF = \alpha \cdot \Gamma\left(1 + \frac{1}{\beta}\right) \tag{5.2}$$

If $\frac{1}{\beta}$ is a natural number, then gamma function can be replaced with a factorial function.

$$MTTF = \alpha \cdot \left(\frac{1}{\beta}!\right) \tag{5.3}$$

Fig. 5.2. shows that under similar MTTF and MTTR, the shape of pdfs is vary depending on the selection of β. If the condition of aleatory uncertainty is met, the parameters of Weibull distribution can be estimated from sufficient information.

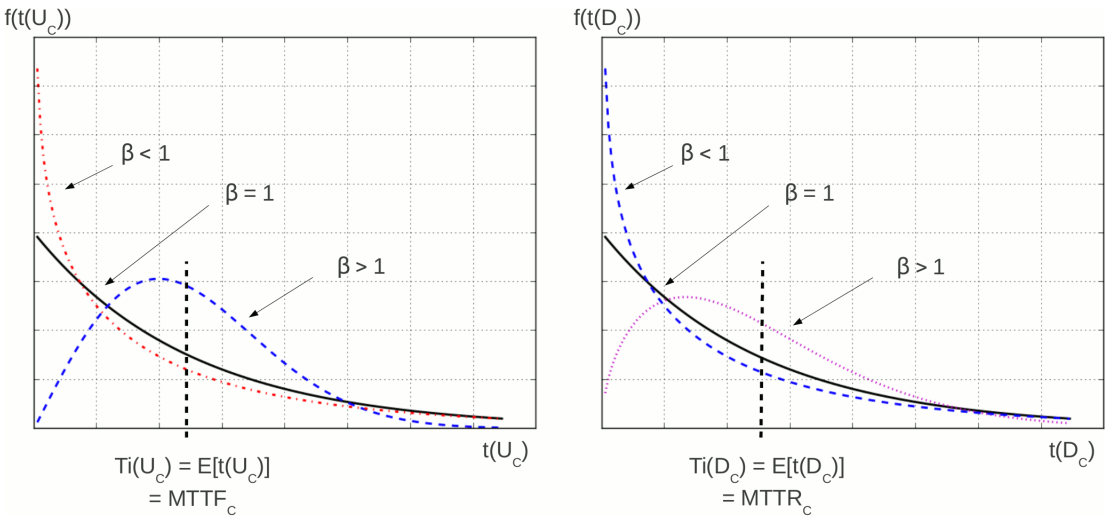


Fig. 5.2: Underlying pdfs of weibull distributed up and down times

Up to this point the component pdfs of up and down time have been determined. Like a component, system reliability indices are derived from its pdfs of up and down time. However, system pdfs of up and down time are usually much more complicated than those of a component. Because of the complication, it is highly impractical to determine system pdfs analytically. Alternatively, with the state space approach the system pdfs can also be determined numerically by computer simulation.

5.3 Step 3: Simulation

The aleatory uncertainty simulation is based on the simulation of up and down time of each system component. With the help of the state space approach, the system under consideration is modelled as an interconnection of its components. This network of interconnected components are usually constructed on a serial network or a parallel network.

If the network is more complex than a serial network or a parallel network, the minimal-cut method can be applied to reduce the complex network into a simpler network of serial and parallel connections.

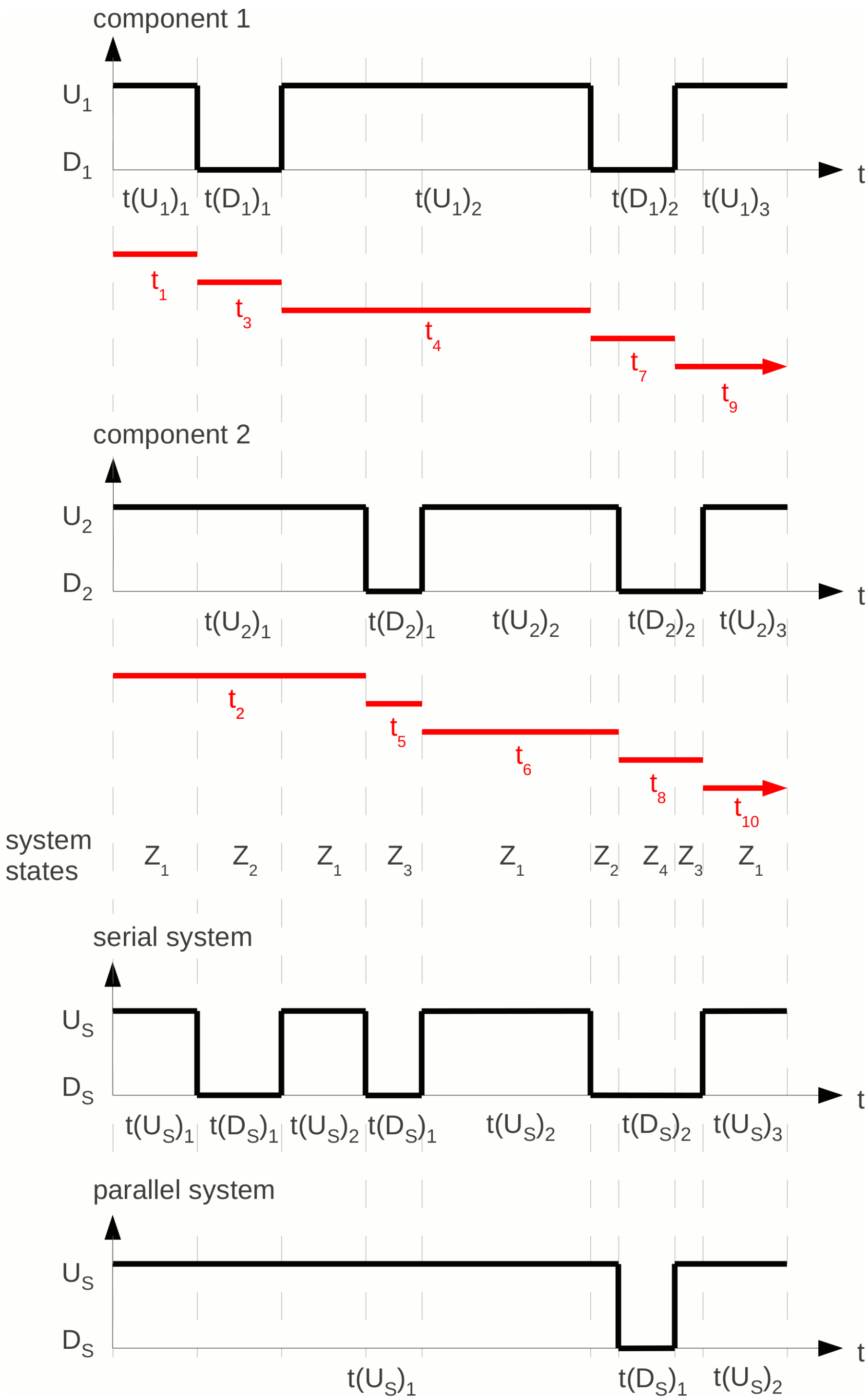


Fig. 5.3: Simulation framework to evaluate aleatory system uncertainty, exemplarily demonstrated at a two-component system with arbitrary pdfs (Fig. 5.2), where t_1, t_2, t_3, t_4, and so on denoting the simulation sequence

5.3.1 Algorithm

The basic idea of the simulation is the imitation of component up and down time. For a system of 2 stochastically-independent components, as in Fig. 5.3. the up and down states of both components are generated from the parameters of their pdfs. Depending on the interconnection, if the system is serial-connected, the system state will be up if both components are up. Otherwise the system will be down. This algorithm is similar to the hazard rate method in [Ross 2010].

If the system is parallel-connected, the system state will be down if both components are down. Otherwise the system will be up. The same procedure is also valid for a simulation of more than two components. It can be seen that the simulation sequence of up and down time of each component is ordered by simulation time t_i, which is illustrated (in red color) in Fig. 5.3.

Both components are assumed to be at up state at the beginning of the simulation. The up time, or the time to the next outage of both components are generated as t_1 and t_2. In this example, t_1 is less then t_2, which means that component 1 is switched to down state before t_2. After component 1 is down, its up time is then generated as t_3. But still t_1+t_3 is less than t_2, which means component 1 is switched back to up state while component 2 does not change state at all.

At this point of time, t_4 is generated for component 1, while component 2 is still covered by t_2. Now the next state transition finally happens to component 2 at time $t_2 < t_1+t_3+t_4$, and component 2 is switched to down state. Then t_5 is generated for component 2 and the same procedure continues until the simulation is terminated.

For arbitrary non-exponential pdf, the system state space process in Fig. 5.3 is a non-Markov process, where the simulation is capable of. Fig. 5.4 and Fig. 5.5 show the core variable and the detailed core algorithm of this simulation, respectively.

```
## Core Variable ##

n                       # number of components
comp_state = []        # store components' states (up or down)
comp_t = []            # store all components failure/repair time

system_state_last      # last system state
system_state_now       # current system state

t_buffer = []          # system time buffer

sys_t_avail = []       # Up-time of system
sys_t_unavail = []     # Down-time of system

r                      # residual range
maxErr                 # maximal relative error acctepted

mean_avail             # temporary variable of mean value
flag_terminate         # while-loop termination flag
```

Fig. 5.4: Core variables of the aleatory uncertainty simulation

```
## Core Algorithm ##

# Initializing
for i in range(n):
   # Initialize all components and the system on "up" state
   # Generate failure time of each component from their failure distribution function
   comp_state[i] = "up";
   comp_t[i] = "random variate of failure-distribution(component i)";


# Simulation Loop
while ( True ):
   # Find which and when the next component's state changes
   t_change = min(comp_t)
   i_change = comp_t.index(t_change)

   # Subtract the elapsed time from all components' time
   for i in range(n):
      comp_t[i] = comp_t[i] - t_change

   # store elapsed time to system time buffer
   t_buffer.append(t_change)

   # Generate the failure/repair time of the component which changes state
   if comp_state[i_change] == True:
      comp_t[i_change] = "random variate of repair-distribution(component i)";
   else:
      comp_t[i_change] = "random variate of failure-distribution(component i)";
   # Toggle the state from up to down, and vice versa
   comp_state[i_change] = not(comp_state[i_change])

   # Update system state according to the interconnection of the components
   if "series connected":
      return reduce(lambda x, y: x and y, comp_state)
   elif "parallel connected":
      return reduce(lambda x, y: x or y, comp_state)

   # If system state changes, sum up system time buffer and assign it to failure/repair time of last system state
   if system_state_now != system_state_last:
      if system_state_now == "up":
         sys_t_unavail.append(math.fsum(t_buffer))
      else:
         sys_t_avail.append(math.fsum(t_buffer))
      t_buffer = []
      system_state_last = system_state_now

      # Simulation terminates at the steady state, which is determined by system availablity at each iteration. System availablity is determined from the total elased time.
      sys_avail_list.append( math.fsum(sys_t_unavail) / (math.fsum(sys_t_avail) + math.fsum(sys_t_unavail)) )

      # If the range of system failure time during the r last iterations are within the maximal error, the state transition is assumed to be steady.
      if len(sys_unavail_list) > r:
         mean_avail = math.fsum(sample_sys_avail_list[-r:]) / r
         flag_terminate = True
         for i in sys_avail_list[-r:]:
            if abs((mean_avail - i) / mean_avail) >= maxErr:
               flag_terminate = False
               break

      if flag_terminate:
         break

   # System failure time and repair time are now generated. Distribution plots can be plotted from their histogram.
```

Fig. 5.5: Algorithm of the aleatory uncertainty simulation

By a computer simulation the system pdfs of up and down time can be achieved. These pdfs are used to determine MTTF and MTTR, as well as other measures, of a system.

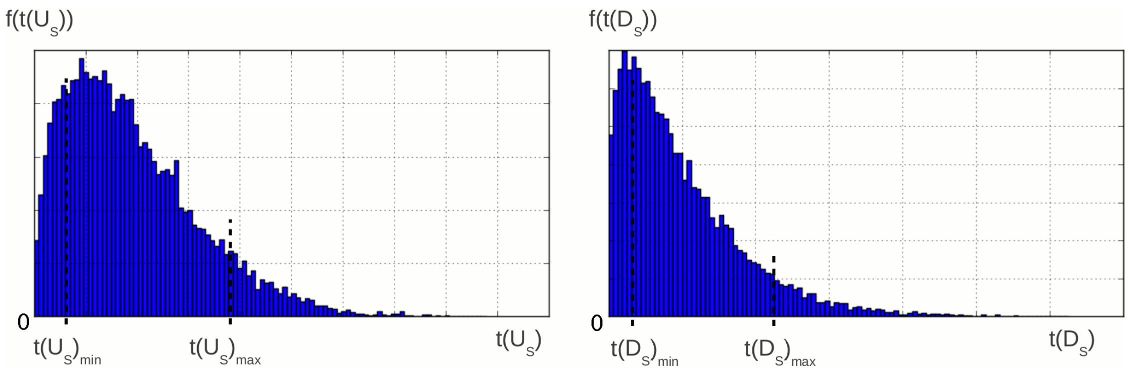


Fig. 5.6: Example of histograms produced by the simulation

The conventional system reliabiltiy indices are derived from the mean value of up and down time. However, under aleatory uncertainty, not only the mean value but all possible outcomes (e.g. pdf) have to be considered. The main challenge is to find a simple but practical form to represent the aleatory uncertainty. In the next section, the min-max boundary of the system pdf is defined to evaluate the interval tendency of reliability indices. The percentile of pdf can be used to determine the interval of up and down time, namely $t(U_S)_{min}$, $t(U_S)_{max}$, $t(D_S)_{min}$, and $t(D_S)_{max}$ (Fig. 5.6). All min-max boundary reliability indices are calculated from these values.

Alternatively one may try to approximate the complex system pdfs with simpler pdf, such as an exponential distribution function. Later on in this chapter, it will be discussed whether this method is rational or not.

5.3.2 Interval Estimation by Min-Max Boundary

The calculation of min-max boundary indices are derived from the min-max boundary of up and down time (Fig. 5.7). The index, $Pr(U_S)$ for instance, lie in the boundary of $Pr(U_S)_{min}$ and $Pr(U_S)_{max}$, where min and max are defined according to the confidence interval of each application. For example, if in one application the minimum reliability of a planned system must be at least 80% confident, according to all possible reliability outcomes, then the min boundary will be set to 10% and the max boundary 90%, such that 80% of the outcomes are bounded.

Mathematically, this example can be formulated as:

$$Pr(Pr(U_S)_{10\%} \le Pr(U_S) \le Pr(U_S)_{90\%}) = 0.8 \qquad (5.4)$$

To generalize this equation, all the numbers are replaced with min, max, and confidence level, where confidence level is a value between 0 to 1.0.

$$Pr(Pr(U_S)_{min} \le Pr(U_S) \le Pr(U_S)_{max}) = \text{confidence level} \tag{5.5}$$

Non-reliability function is also in the same manner.

$$Pr(Pr(D_S)_{min} \le Pr(D_S) \le Pr(D_S)_{max}) = \text{confidence level} \tag{5.6}$$

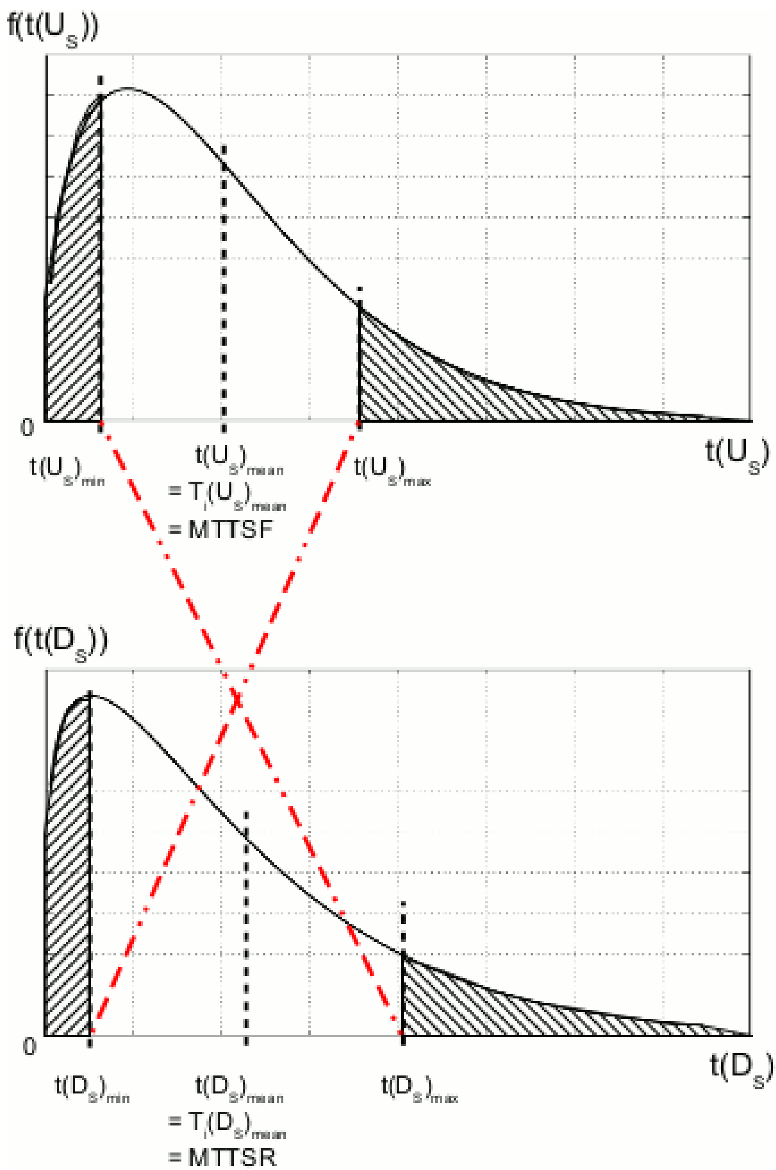


Fig. 5.7: Calculation of min-max boundary indices

The boundary of system reliability (or probability) is determined from that of the system up and down time.

$$Pr(U_S)_{max} = \frac{t(U_S)_{\max}}{t(U_S)_{\max} + t(D_S)_{\min}} \tag{5.7}$$

$$Pr(U_S)_{min} = \frac{t(U_S)_{\min}}{t(U_S)_{\min} + t(D_S)_{\max}} \tag{5.8}$$

$$Pr(D_S)_{max} = \frac{t(D_S)_{\max}}{t(U_S)_{\min} + t(D_S)_{\max}} \tag{5.9}$$

$$Pr(D_S)_{min} = \frac{t(D_S)_{\min}}{t(U_S)_{\max} + t(D_S)_{\min}} \tag{5.10}$$

Now it is obvious that one can calculate the min of non-reliability by one minus the max of reliability, and the max of non-reliability by one minus the min of reliability, and vice versa.

$$Pr(D_S)_{min} = 1 - Pr(U_S)_{max} \tag{5.11}$$

$$Pr(D_S)_{max} = 1 - Pr(U_S)_{min} \tag{5.12}$$

The frequency indices can be calculated from the same sense as well.

$$Fr(U_S)_{max} = \frac{1}{t(U_S)_{\min} + t(D_S)_{\max}} \tag{5.13}$$

$$Fr(U_S)_{min} = \frac{1}{t(U_S)_{\max} + t(D_S)_{\min}} \tag{5.14}$$

$$Fr(D_S)_{max} = Fr(U_S)_{min} \tag{5.15}$$

$$Fr(D_S)_{min} = Fr(U_S)_{max} \tag{5.16}$$

It is important to note that, in the case of frequency, the definition of min and max is loosely followed. The system pdf of up and down time will affect whether $Fr(U_S)_{max} > Fr(U_S)_{min}$ or the other way around. The unusual case that $Fr(U_S)_{max} < Fr(U_S)_{min}$ occurs when the system down time is greater than the system up time, thus results in the reverse order. Fortunately, however, the ascending or descending order of the frequency has no effects on the correctness of the results.

The last indices are the min-max boundary of up and down time themselves, where it has been shown that they are equivalent to the defined boundary from the simulation result.

$$Ti(U_S)_{max} = \frac{Pr(U_S)_{max}}{Fr(U_S)_{min}} = t(U_S)_{\mathrm{max}} \tag{5.17}$$

$$Ti(U_S)_{min} = \frac{Pr(U_S)_{min}}{Fr(U_S)_{max}} = t(U_S)_{\mathrm{min}} \tag{5.18}$$

$$Ti(D_S)_{max} = \frac{Pr(D_S)_{max}}{Fr(D_S)_{min}} = t(D_S)_{\mathrm{max}} \tag{5.19}$$

$$Ti(D_S)_{min} = \frac{Pr(D_S)_{min}}{Fr(D_S)_{max}} = t(D_S)_{\mathrm{min}} \tag{5.20}$$

5.4 Step 4: Evaluation

In the conventional reliability evaluation, only the central tendency of the reliability indices is estimated. However, the uncertainty of reliability of indices is hidden, but it can be uncovered when measured by the use of the interval tendency.

The min-max boundary estimates that interval tendency and illustrates the effects of the aleatory uncertainty on the reliability indices. After the evaluation, the aleatory uncertainty is minimized, with some remaining risks and uncertainty called residual risks. If the level of these residual risks is not satisfied, the treatment of the process has to be done by revising steps 1 to 3.

Two example systems are given to illustrate the evaluation of reliability under the aleatory uncertainty.

5.4.1 Example 1: Serial Systems

Three examples of serial systems are investigated. Each example contains 10, 20, and 100 components, respectively, as illustrated in Fig. 5.8.

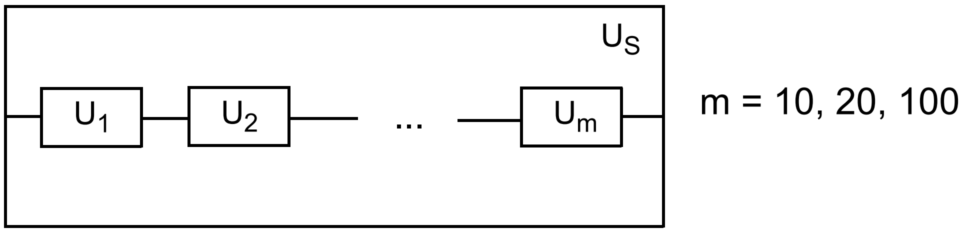


Fig. 5.8: Example serial systems

To simplify the examples, it is assumed that each system is composed of a number of similar components. These components have weibull distributed up time with $MTTF_C = 100{,}000\ \mathrm{h}$, which is a little more than 10 years, and log-normal distributed down time with $MTTR_C = 10\ \mathrm{h}$.

In order to observe the behavior of the system with different shape of pdfs, different shape parameters β of Weibull distribution are selected. The scale parameter α is adjusted to β, such that each pdf would get the similar $MTTF_C$. Parameters for the lognormal down time are fixed to $\mu = 1.80259$ and $\sigma = 1$ to simplify the examples.

The conventional reliability indices, where point estimates of assumed exponential distribution are evaluated, are provided in a separate table for comparison.

It may be of interest to start with 10 components, which is the lowest number. Three simulations have been executed. In each simulation the shape and scale of Weibull distributed up time has been adjusted accordingly. In the end of each simulation, the reliability indices are determined by min-max boundary approach. The resulted reliability indices are shown in Table 5.1. And the reliability indices from the conventional method are shown in Table 5.2.

Weibull Parameter	Label	$t(U_s)$ (h)	$t(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
β = 0.5	$min_{10\%}$	3.00E+02	1.73E+00	9.32E-01	6.71E-05	3.89E-05	3.11E-03
	mean	1.00E+04	9.93E+00	9.99E-01	9.90E-04	9.97E-05	9.97E-05
	$max_{90\%}$	2.57E+04	2.17E+01	1.00E+00	6.75E-02	3.11E-03	3.89E-05
β = 1.0	$min_{10\%}$	1.03E+03	1.05E+00	9.78E-01	4.56E-05	4.34E-05	9.52E-04
(Exponential)	mean	9.99E+03	1.01E+01	9.99E-01	1.01E-03	1.00E-04	1.00E-04
	$max_{90\%}$	2.30E+04	2.33E+01	1.00E+00	2.21E-02	9.52E-04	4.34E-05
β = 3.0	$min_{10\%}$	1.17E+03	1.71E+00	9.82E-01	7.62E-05	4.45E-05	8.36E-04
	mean	9.98E+03	1.00E+01	9.99E-01	1.00E-03	1.00E-04	1.00E-04
	$max_{90\%}$	2.25E+04	2.20E+01	1.00E+00	1.85E-02	8.36E-04	4.45E-05
		simulation		min-max calculation			

Table 5.1: Reliability indices of the example serial system with 10 components

Label	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
mean	1.00E+04	1.00E+01	9.99E-01	9.99E-04	9.99E-05	9.99E-05
	conventional calculation method (approximated)					

Table 5.2: Conventional reliability indices of the example serial system with 10 components

The component reliability indices (mean values) $Pr(U_C)$, $Pr(D_C)$, $Ti(U_C)$, $Ti(D_C)$, $Fr(U_C)$, and $Fr(D_C)$ are independent from the shape of the pdfs $f(t(U_C))$ and $f(t(D_C))$ [Kochs 1984]. Thus, the system reliability indices (mean values in Table 5.1, 5.3, and 5.5) $Pr(U_S)$, $Pr(D_S)$, $Fr(U_S)$, and $Fr(D_S)$ are also independent from the shape of the component and system pdfs. Minor deviations are resulted from numerical errors of the simulation. The main differences lie in the min-max boundary of the reliability indices.

It can be seen that if β is smaller, the min-max boundary will be wider. Thus, the mean reliability indices when β is small can be interpreted as less precise than that of the case when β is large.

The same tendency occurs when the number of components are increased to 20 in Table 5.3. and Table 5.4. Also similarly when it is increased to 100 in Table 5.5. and Table 5.6.

Weibull Parameter	Label	$t(U_S)$ (h)	$t(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
β = 0.5	$min_{10\%}$	2.14E+02	1.67E+00	9.08E-01	1.35E-04	8.04E-05	4.24E-03
	mean	4.98E+03	1.00E+01	9.98E-01	2.00E-03	2.01E-04	2.01E-04
	$max_{90\%}$	1.24E+04	2.17E+01	1.00E+00	9.21E-02	4.24E-03	8.04E-05
β = 1.0 (Exponential)	$min_{10\%}$	5.31E+02	1.01E+00	9.59E-01	8.72E-05	8.63E-05	1.81E-03
	mean	5.03E+03	9.93E+00	9.98E-01	1.97E-03	1.98E-04	1.98E-04
	$max_{90\%}$	1.16E+04	2.29E+01	1.00E+00	4.13E-02	1.81E-03	8.63E-05
β = 3.0	$min_{10\%}$	5.73E+02	1.69E+00	9.63E-01	1.49E-04	8.82E-05	1.68E-03
	mean	4.99E+03	9.91E+00	9.98E-01	1.98E-03	2.00E-04	2.00E-04
	$max_{90\%}$	1.13E+04	2.17E+01	1.00E+00	3.65E-02	1.68E-03	8.82E-05
		simulation		min-max calculation			

Table 5.3: Reliability indices of the example serial system with 20 components

Label	$Ti(U_S)$ (h)	$Ti(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
mean	5.00E+03	1.00E+01	9.98E-01	2.00E-03	2.00E-04	2.00E-04
	conventional calculation method (approximated)					

Table 5.4: Conventional reliability indices of the example serial system with 20 components

When the number of components in a serial system doubles, the mean probability of down time $Pr(U_S)_{mean}$ also doubles. More interestingly the min and max boundary $Pr(U_S)_{min}$ and $Pr(U_S)_{max}$ also double.

Weibull Parameter	Label	$t(U_S)$ (h)	$t(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
β = 0.5	$min_{10\%}$	6.73E+01	1.70E+00	7.52E-01	7.22E-04	4.25E-04	1.12E-02
	mean	9.81E+02	1.01E+01	9.90E-01	1.02E-02	1.01E-03	1.01E-03
	$max_{90\%}$	2.35E+03	2.22E+01	9.99E-01	2.48E-01	1.12E-02	4.25E-04
β = 1.0 (Exponential)	$min_{10\%}$	1.06E+02	1.07E+00	8.20E-01	4.67E-04	4.39E-04	7.77E-03
	mean	1.00E+03	1.01E+01	9.90E-01	9.96E-03	9.87E-04	9.87E-04
	$max_{90\%}$	2.28E+03	2.31E+01	1.00E+00	1.79E-01	7.77E-03	4.39E-04
β = 3.0	$min_{10\%}$	1.05E+02	1.67E+00	8.27E-01	7.26E-04	4.34E-04	7.88E-03
	mean	1.00E+03	1.00E+01	9.90E-01	9.91E-03	9.87E-04	9.87E-04
	$max_{90\%}$	2.30E+03	2.19E+01	9.99E-01	1.73E-01	7.88E-03	4.34E-04
		simulation		min-max calculation			

Table 5.5: Reliability indices of the example serial system with 100 components

Label	$Ti(U_S)$ (h)	$Ti(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
mean	1.00E+03	1.00E+01	9.90E-01	9.95E-03	9.90E-04	9.90E-04
	conventional calculation method (approximated)					

Table 5.6: Conventional reliability indices of the example serial system with 100 components

Analogously, when the number of components increases from 10 to 100, the mean, min, and max probability of down time increase ten times. It is important to note that the effects on min reliability indices are greater than that on mean reliability indices because of the difference in exponent.

At $\beta = 1$, when the number of components increases from 10 to 100, $Pr(U_S)_{mean}$ decreases from 99.9% to 99.0%, but $Pr(U_S)_{min}$ remarkably decreases from 93.2% to 75.2%. This reflects virtually that the aleatory uncertainty in a serial system can play an important role at a high number of system components.

5.4.2 Example 2: Parallel Systems

Parallel systems are commonly found especially when the high system reliability is required, but the system itself is constructed from components with relatively low reliability. A parallel system will fail if all component fails, thus the system reliability is improved if the number of components increases, given that the components are stochastically independent. Such parallel system occur in mostly in minimal cuts of second and third order.

In this section two parallel systems with 2 and 3 components are illustrated, respectively. These parallel systems use inferior components than the serial systems in the previous examples. Each component in a parallel system has $MTTF_C = 10{,}000\ \text{h}$, or about 1 year, which is ten times shorter than the serial examples. Other variables remain unchanged.

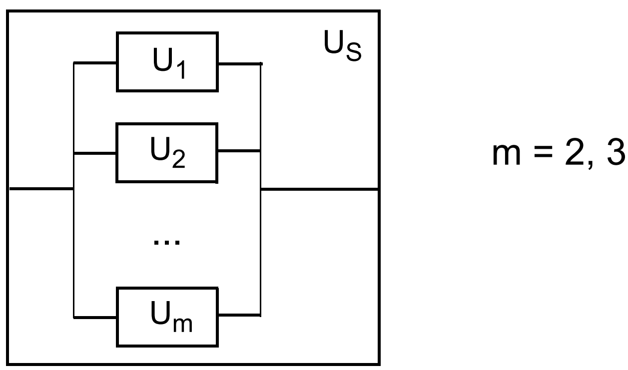


Fig. 5.9: Example parallel systems

Weibull Parameter	Label	$t(U_s)$ (h)	$t(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
β = 0.5	$min_{10\%}$	2.64E+05	8.02E-01	1.00E+00	6.80E-08	8.48E-08	3.79E-06
	mean	4.99E+06	4.96E+00	1.00E+00	9.96E-07	2.01E-07	2.01E-07
	$max_{90\%}$	1.18E+07	1.08E+01	1.00E+00	4.09E-05	3.79E-06	8.48E-08
β = 1.0 (Exponential)	$min_{10\%}$	5.32E+05	5.24E-01	1.00E+00	4.54E-08	8.67E-08	1.88E-06
	mean	5.00E+06	4.99E+00	1.00E+00	9.98E-07	2.00E-07	2.00E-07
	$max_{90\%}$	1.15E+07	1.15E+01	1.00E+00	2.16E-05	1.88E-06	8.67E-08
β = 3.0	$min_{10\%}$	5.25E+05	8.10E-01	1.00E+00	7.10E-08	8.76E-08	1.90E-06
	mean	4.96E+06	5.00E+00	1.00E+00	1.01E-06	2.02E-07	2.02E-07
	$max_{90\%}$	1.14E+07	1.08E+01	1.00E+00	2.06E-05	1.90E-06	8.76E-08
		simulation		min-max calculation			

Table 5.7: Reliability indices of the example parallel network with 2 components

β = 1.0	$Ti(U_S)$ (h)	$Ti(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
mean	5.01E+06	5.00E+00	1.00E+00	9.98E-07	2.00E-07	2.00E-07

conventional calculation method (approximated)

Table 5.8: Conventional reliability indices of the example parallel network with 2 components

Reliability indices in the case of two components are shown in Table. 5.7 and Table 5.8. Notice that the parallel-connected system improves the *MTTSF* drastically, despite the low $MTTF_C$.

The same reliability improvement occurs when the number of components increases to 3, shown in Table. 5.9 and Table 5.10. At a point, the system is so reliable that there is no virtually difference between the min or max reliability indices. For instance, at a very high reliability, the investment costs and risks dominate the consideration of min-max reliability indices [Bollen et al. 2006]. Therefore, the effects of min-max boundary are weaker (in the viewpoint of a decision maker) when the number of components grows.

Weibull Parameter	Label	$t(U_S)$ (h)	$t(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
β = 0.5	$min_{10\%}$	3.21E+08	4.45E-01	1.00E+00	6.24E-11	1.40E-10	3.11E-09
	mean	3.05E+09	3.24E+00	1.00E+00	1.06E-09	3.28E-10	3.28E-10
	$max_{90\%}$	7.13E+09	6.61E+00	1.00E+00	2.06E-08	3.11E-09	1.40E-10
β = 1.0	$min_{10\%}$	3.47E+08	4.88E-01	1.00E+00	6.24E-11	1.28E-10	2.88E-09
(Exponential)	mean	3.27E+09	3.13E+00	1.00E+00	9.60E-10	3.06E-10	3.06E-10
	$max_{90\%}$	7.82E+09	6.76E+00	1.00E+00	1.95E-08	2.88E-09	1.28E-10
β = 3.0	$min_{10\%}$	2.15E+08	3.62E-01	1.00E+00	3.94E-11	1.09E-10	4.65E-09
	mean	3.31E+09	3.25E+00	1.00E+00	9.82E-10	3.02E-10	3.02E-10
	$max_{90\%}$	9.19E+09	6.97E+00	1.00E+00	3.24E-08	4.65E-09	1.09E-10

simulation — min-max calculation

Table 5.9: Reliability indices of the example parallel network with 3 components

β = 1.0	$Ti(U_S)$ (h)	$Ti(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
mean	3.34E+09	3.33E+00	1.00E+00	9.97E-10	2.99E-10	2.99E-10

conventional calculation method (approximated)

Table 5.10: Conventional reliability indices of the example parallel network with 3 components

5.5 Summary

The reliability under the aleatory uncertainty can be evaluated by the simulation method based on the up and down time of system components. Reliability indices have to be determined as intervals instead of points. With min-max boundary, one can find min and max reliability indices under the required confidence level.

Despite different shapes of pdfs of a system's up and down time, the mean reliability indices are equivalent. The main differences lie in the min and max reliability indices.

The min reliability indices of an exponential pdf are more conservative than those of a weibull pdf with $\beta > 1$. In other words, the system up and down time pdfs can be conservatively approximated to exponential pdfs, if the pdfs are not "steeper" than an exponential pdf.

In a serial system, the aleatory uncertainty has more influence on the consideration of reliability when the number of components increase. In a parallel system, however, it has more influence when the number of components decreases.

In an application to the minimal cut set approach, a complex system is reduced into a set of serial-parallel systems. The higher number of minimal cuts in a system, the more serial connections exists. In this case, the aleatory uncertainty should not be ignored.

5.6 Case Study: HVDC Converter Station

Electrical power system is a network of electrical components used to generate, transmit, and distribute electric power (See Fig. 5.10). The reliability of an existing HVDC converter station, which is a part of an electrical power system and is actually discussed for large windfarms [Lutz et al. 2007], is used as a realistic example to illustrate the aleatory uncertainty and a practical approach to deal with this uncertainty.

5.6.1 Overview of HVDC Transmission

Electrical power system can be classified into three parts: power generation, power transmission, and power distribution (Fig. 5.10). As the name suggests, the power generation is responsible to generate electrical power to an electrical system. Electrical power can be generated from power plants such as fossil energy, renewable energy, nuclear energy, etc. After the generation, electrical power is transmitted from the power plants to the distribution centers. Each distribution center is called a distribution station, and is responsible to distribute electrical power to, for instance, factories, department stores, supermarkets, and household.

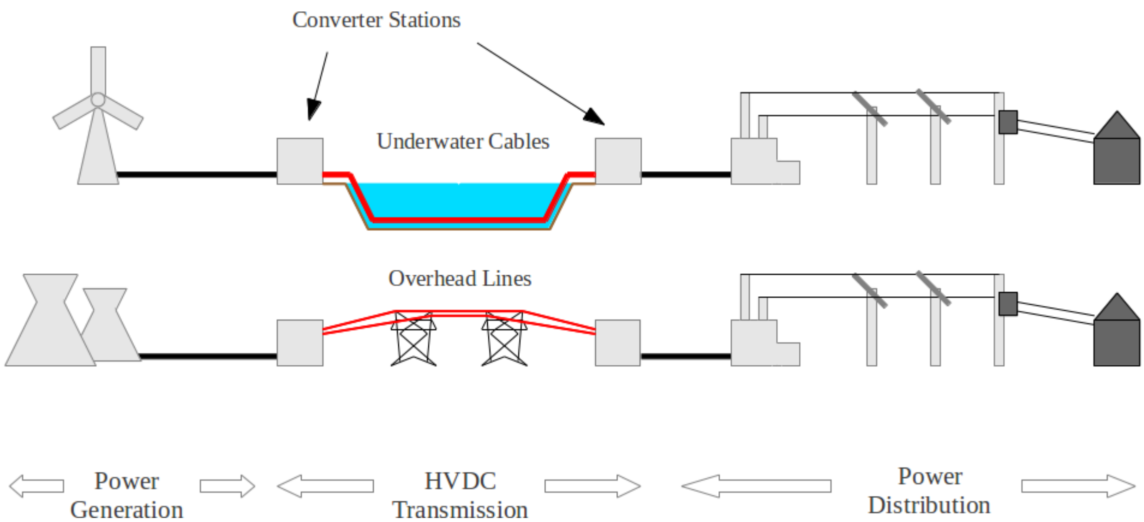


Fig. 5.10: Overview of an electrical power system and HVDC transmission

Between power generation and power distribution, electrical power is transmitted at a high-voltage in order to reduce electrical losses. This transmission can be categorized into two classes: High-Voltage Alternate-Current (HVAC) transmission and High-Voltage Direct-Current (HVDC) transmission. These two classes of power transmission have their own advantages and disadvantages, nevertheless, HVDC transmission is more attractive in long-distance electrical power transmission because of the lower loss in transmission lines, and is also an attractive solution for connecting offshore windfarms with long-distance to shore. HVDC transmission can use either overhead transmission lines or underground/underwater transmission cables.

5.6.2 Step 1: Identification

Square Butte HVDC is an existing HVDC converter station in USA [The Square Butte HVDC Scheme 2005]. It has been chosen as a realistic example, and the simplified diagram of Square Butte HVDC is illustrated in Fig. 5.11. Its statistical data from 2001 until 2006 [Vancers et al. 2004, Vancers et al. 2006, Vancers et al. 2008] is adopted from CIGRE and is shown in Table. 5.11. According to this data, the HVDC converter station is classified into 5 components as follows:

1. **AC-E** or AC-Equipments which contain/s all sub-components on the AC side of the converter station
2. **V** or Valves which mean/s the converter values
3. **C&P** or Control and Protection of the entire converter station
4. **DC-E** or DC-Equipments which contain/s all sub-components on the DC side of the converter station
5. **O** or Others, other sub-components which cannot be categorized into the above classes

This statistical data contains an observation of failures and repairs that happened though-out six years of system operation. It can be seen that the number of failures and repair time of each year has some uncertainty. The uncertainty is related directly to the nature of failure and repair, which are random processes, thus this uncertainty can be considered as an aleatory uncertainty.

By the statistical data of failures and repairs, this system can be categorized as a repairable system (See Section 3.4), therefore, Eq. 3.74 - 3.77 are used. Exponential distribution of failures and repairs are assumed.

Data Year	AC-E		V		C&P		DC-E		O		Total Operating Time (h) *
	No. of Failures	Time to Repair (h)	No. of Failures	Time to Repair (h)	No. of Failures	Time to Repair (h)	No. of Failures	Time to Repair (h)	No. of Failures	Time to Repair (h)	
2001	5	25.6	0	0	1	0.1	3	7.8	1	5.6	8720.9
2002	5	5.9	1	13.2	1	0	1	33.9	0	0	8707
2003	2	1.7	3	5.8	4	0.1	0	0	1	0.2	8752.2
2004	9	38.7	0	0	7	6.8	3	4.3	0	0	8710.2
2005	2	23.8	3	8.6	2	2.7	0	0	1	40.3	8684.6
2006	0	0	1	0.6	1	2.5	2	5.6	1	2	8749.3
Total	23	95.7	8	28.2	16	12.2	9	51.6	4	48.1	52324.2

* 24-hour operation in 365 days minus the total of time to repair

Table 5.11: Reliability data of Square Butte HVDC converter station [Vancers et al. 2004, Vancers et al. 2006, Vancers et al. 2008]

5.6.3 Step 2: Formulation

If the statistical data of each year is processed separately, the parameter of each failure rate and repair rate can be uncertain. This aleatory uncertainty is transformed into an estimate and a variance using MLE and Fisher Information Matrix in Eq. 3.74 - 3.77, as shown in the last row of Table. 5.12.

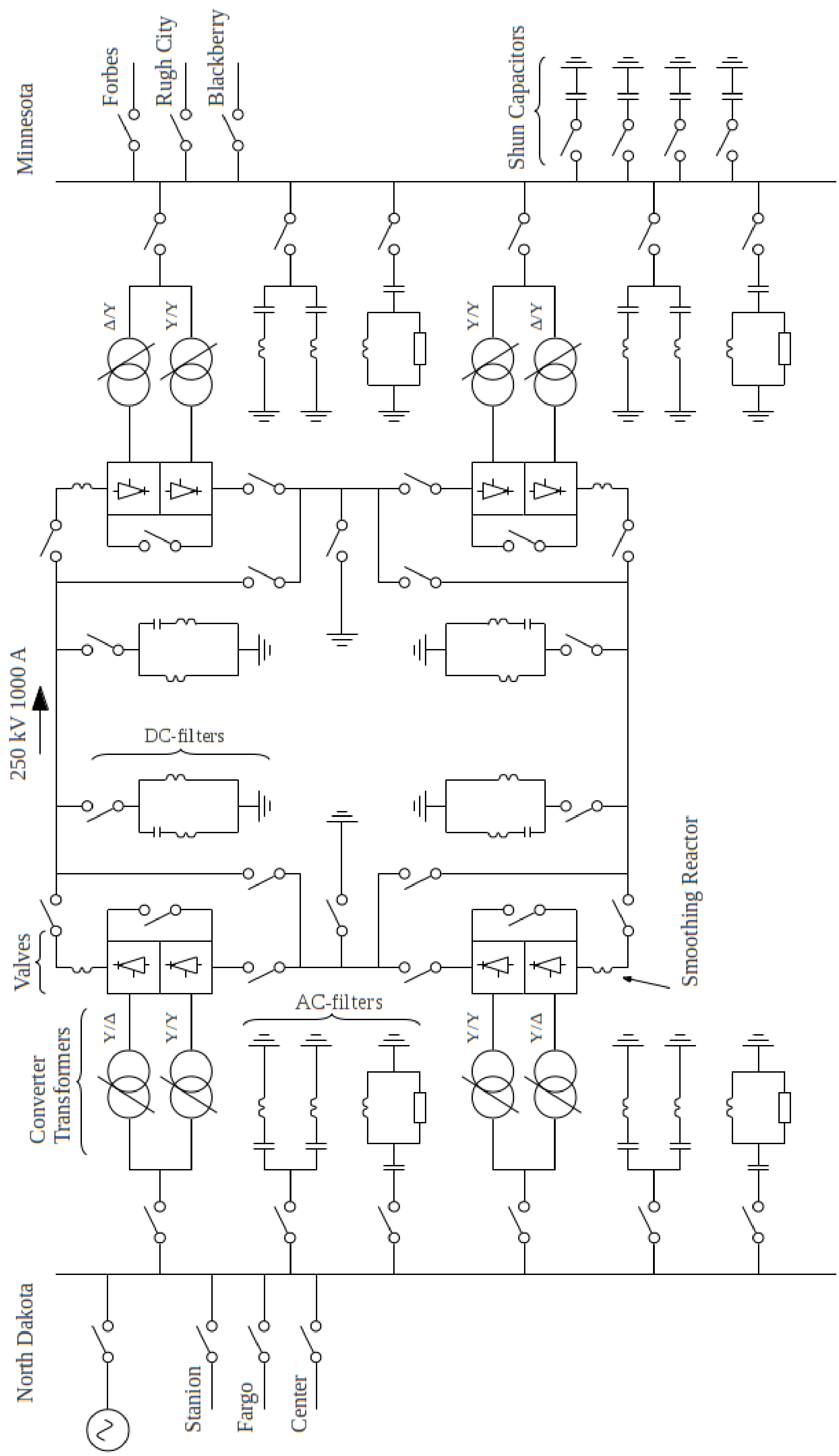


Fig. 5.11: Simplified single line diagram of Square Butte HVDC converter station, adapted from [The Square Butte HVDC Scheme 2005]

Data Year		AC-E		V		C&P		DC-E		O	
		λ	μ	λ	μ	λ	μ	λ	μ	λ	μ
2001	Estimate	5.73E-04	1.95E-01	N/A	N/A	1.15E-04	1.00E+01	3.44E-04	3.85E-01	1.15E-04	1.79E-01
2002	Estimate	5.74E-04	8.47E-01	1.15E-04	7.58E-02	1.15E-04	N/A	1.15E-04	2.95E-02	N/A	N/A
2003	Estimate	2.29E-04	1.18E+00	3.43E-04	5.17E-01	4.57E-04	4.00E+01	N/A	N/A	1.14E-04	5.00E+00
2004	Estimate	1.03E-03	2.33E-01	N/A	N/A	8.04E-04	1.03E+00	3.44E-04	6.98E-01	N/A	N/A
2005	Estimate	2.30E-04	8.40E-02	3.45E-04	3.49E-01	2.30E-04	7.41E-01	N/A	N/A	1.15E-04	2.48E-02
2006	Estimate	N/A	N/A	1.14E-04	1.67E+00	1.14E-04	4.00E-01	2.29E-04	3.57E-01	1.14E-04	5.00E-01
Total	Estimate	4.40E-04	2.40E-01	1.53E-04	2.84E-01	3.06E-04	1.31E+00	1.72E-04	1.74E-01	7.64E-05	8.32E-02
	Variance	8.40E-09	2.51E-03	2.92E-09	1.01E-02	5.84E-09	1.07E-01	3.29E-09	3.38E-03	1.46E-09	1.73E-03

Table 5.12: Parameters of failure and repair distribution of Square Butte HVDC converter station

5.6.4 Step 3: Simulation

The computer-based method (See Section 5.3) is selected and the up time and the down time of each component are simulated. In each failure cycle, reliability indices are determined by the min-max boundary approach.

5.6.5 Step 4: Evaluation

The reliability indices of all simulation cycles are collected and measured by the central tendency and the confidence interval, as shown in Table 5.13.

β = 1.0 (exponential)	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
mean	8.72E+02	3.94E+00	9.96E-01	4.50E-03	1.14E-03	1.14E-03

conventional calculation method (approximated)

β = 1.0 (exponential)	t(U_s) (h)	t(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
$min_{10\%}$	9.21E+01	2.17E-01	9.03E-01	1.08E-04	4.97E-04	9.80E-03
mean	8.75E+02	3.94E+00	9.96E-01	4.48E-03	1.14E-03	1.14E-03
$max_{90\%}$	2.01E+03	9.94E+00	1.00E+00	9.74E-02	9.80E-03	4.97E-04

simulation min-max calculation

Table 5.13: Reliability indices of Square Butte HVDC converter station

From the table above, it can be seen that the confidence interval of the probability of the system down time is relatively wide.

If the estimates of central tendency, or the mean values, are used like in the conventional method, the indices from the mean values will not reflect the behavior of failures, where most of the failures lie in an interval below the mean values, while only few occurs above the interval. Therefore, the mean value is not sufficient to indicate the reliability under aleatory uncertainty. The confidence interval from the proposed method uncovers this aleatory uncertainty, and finally can reflect more on the failure behavior of the HVDC converter station.

6 Approach for Epistemic Uncertainty

From the previous chapters we have dealt with the reliability evaluation in general, and the reliability evaluation under aleatory uncertainty. In this chapter the other type of uncertainty, namely epistemic uncertainty, is investigated. The definitions of aleatory and epistemic uncertainties are given in Chapter 2.

The process of reliability assessment under epistemic uncertainty can be divided into 4 steps, illustrated in Fig. 6.1.

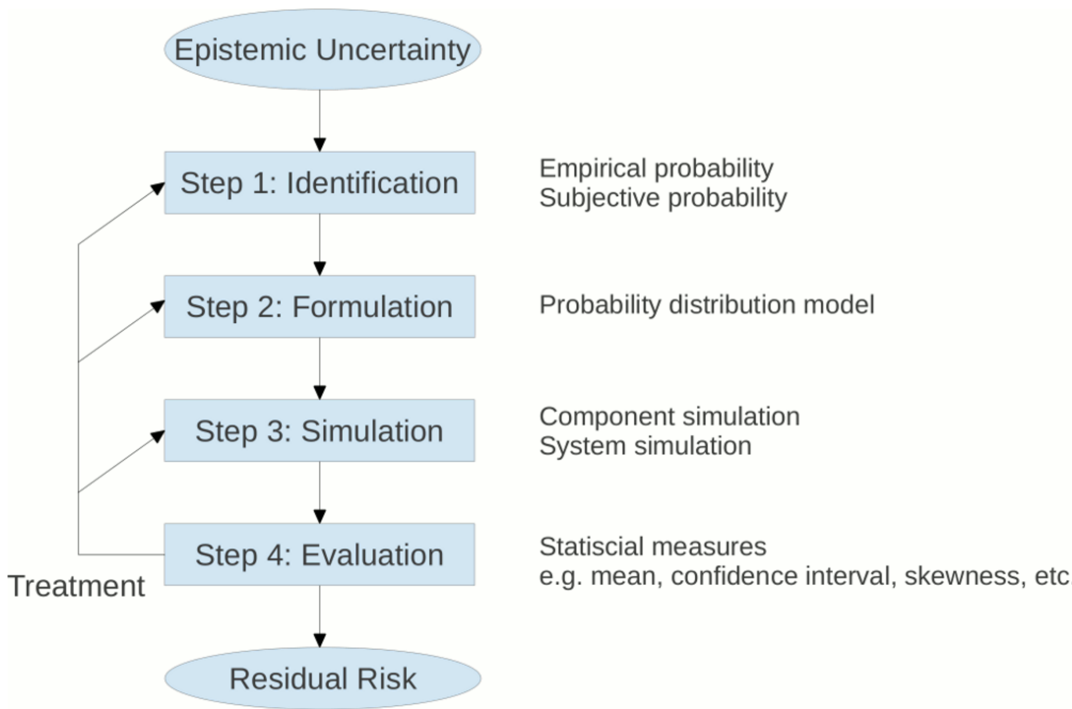


Fig. 6.1: The process of reliability assessment under epistemic uncertainty

6.1 Step 1: Identification

The sources of the epistemic uncertainty are, unlike the aleatory uncertainty, usually not statistically interpreted and are commonly in a qualitative form. The use of the empirical probability depends on the quantitative information, thus it is not sufficient for the evaluation of epistemic uncertainty. The subjective probability and the probability can compensate the lack of this quantitative information.

In practice, the border between the aleatory and epistemic uncertainty is not very clear, however, the tendency toward either one can be noticed. The most common form of the epistemic uncertainty is when two or more reliability data sources give inconsistence conclusions. The use of either one data source may lead to a fallacy. These data have to be analyzed to show how the epistemic uncertainty could affect the reliability results.

6.2 Step 2: Formulation

Recalling from Chapter 3, the mean values $Ti(U_C)$ and $Ti(D_C)$ are *MTTF* and *MTTR* of a component *C*. The presence of the epistemic uncertainty makes the determination of the true mean value very difficult.

In a simplified case of two inconsistent data sources, each of the data source yields its own conclusion of mean values. The larger mean values are indicated as $Ti(U_C)_{max}$ and $Ti(D_C)_{max}$, and the smaller are $Ti(U_C)_{min}$ and $Ti(D_C)_{min}$. If the two given data sources have the same degree of trustworthiness, the most likely estimate of the true value must lie somewhere between the min and max values.

The evaluation of reliability can be based up on the consideration of all possible outcomes of the selection of a value between min and max values. If the probability of the selected value is uniform distributed, illustrated in Fig. 6.2, the density functions are described as Equation 6.1 - 6.2.

$$f(Ti(U_C)) = \begin{cases} \dfrac{1}{Ti(U_C)_{max} - Ti(U_C)_{min}} & , Ti(U_C)_{min} \le Ti(U_C) \le Ti(U_C)_{max} \\ 0 & \text{, otherwise} \end{cases} \quad (6.1)$$

$$f(Ti(D_C)) = \begin{cases} \dfrac{1}{Ti(D_C)_{max} - Ti(U_C)_{min}} & , Ti(U_C)_{min} \le Ti(U_C) \le Ti(U_C)_{max} \\ 0 & \text{, otherwise} \end{cases} \quad (6.2)$$

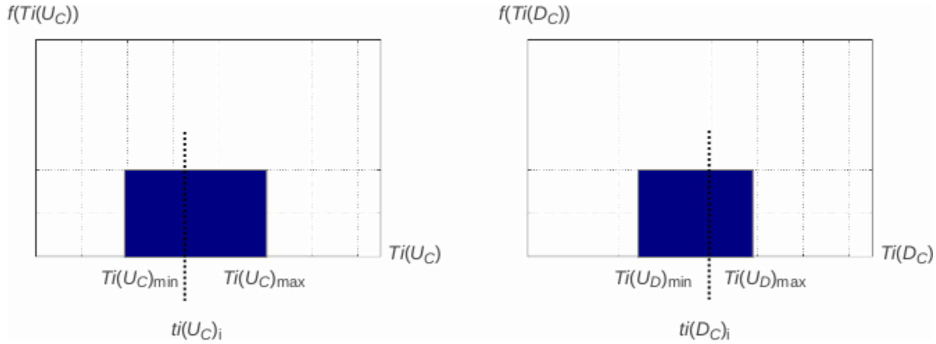


Fig. 6.2: Reliability data under epistemic uncertainty modelled as uniform distributions

In conventional reliability evaluations, arithmetic or geometric mean values of the min and max values are used for the calculation of the final reliability indices. In order to see how the epistemic uncertainty affects the reliability indices, a computer simulation is constructed to determine all possible outcomes of the reliability indices in step 3. These indices are compared with the conventional indices in step 4.

6.3 Step 3: Simulation

6.3.1 Component Simulation

In the previous section the epistemic uncertainty of $Ti(U_C)$ and $Ti(D_C)$ are modelled as uniform distributions between min and max values. Simulation samples are generated from these distributions. Suppose that $ti(U_C)_i$ and $ti(D_C)_i$ indicate each sample of the uniform distribution in Eq. 6.1 - Eq. 6.2, respectively. As these $ti(U_C)_i$ and $ti(D_C)_i$ are assumed to be the likely mean values of an uncertain component C, the following component reliability indices can be calculated (see Chapter 3).

$$pr(U_C)_i = \frac{ti(U_C)_i}{ti(U_C)_i + ti(D_C)_i} \tag{6.3}$$

$$pr(D_C)_i = \frac{ti(D_C)_i}{ti(U_C)_i + ti(D_C)_i} \tag{6.4}$$

$$fr(U_C)_i = fr(D_C)_i = \frac{1}{ti(U_C)_i + ti(D_C)_i} \tag{6.5}$$

where i indicates the index of simulation samples. With high number of simulation samples, the characteristics of the population, or all possible outcomes of component reliabiltiy indices, can be estimated.

6.3.2 System Simulation

A system is an entity composed of interconnected components. By using the reliability block diagram method the interconnections between components can be categorized as serial, parallel, or others.

For a **serial system** with stochastically independent components, the system reliability indices can be calculated directly from the equations in Chapter 3 as follows:

$$pr(U_S)_i = \prod_{C=1}^{n} pr(U_C)_i \tag{6.6}$$

$$ti(U_S)_i = \frac{1}{\sum_{C=1}^{n} \frac{1}{ti(U_C)_i}} \tag{6.7}$$

$$fr(U_S)_i = \frac{pr(U_S)_i}{ti(U_S)_i} \tag{6.8}$$

$$fr(D_S)_i = fr(U_S)_i \tag{6.9}$$

$$pr(D_S)_i = 1 - pr(U_S)_i \tag{6.10}$$

$$ti(D_S)_i = \frac{pr(D_S)_i}{fr(D_S)_i} \tag{6.11}$$

where i and n indicate the index of simulation samples and the total number of samples, respectively. In the case of a **parallel system** with stochastically independent components, system reliability indices can be calculated directly as follow:

$$pr(D_S)_i = \prod_{C=1}^{n} pr(D_C)_i \tag{6.12}$$

$$ti(D_S)_i = \frac{1}{\sum_{C=1}^{n} \frac{1}{ti(D_C)_i}} \tag{6.13}$$

$$fr(D_S)_i = \frac{pr(D_S)_i}{ti(D_S)_i} \tag{6.14}$$

$$fr(U_S)_i = fr(D_S)_i \tag{6.15}$$

$$pr(U_S)_i = 1 - pr(D_S)_i \tag{6.16}$$

$$ti(U_S)_i = \frac{pr(U_S)_i}{fr(U_S)_i} \tag{6.17}$$

If a system contains more complicated interconnection, the minimal cut method can be applied to reduce the interconnection into a set of parallel and serial systems.

After all simulation samples have been determined, the histogram of each reliability index can be plotted, as illustrated as an example in Fig. 6.3. These samples can be used to find mean values, confidence intervals, and other statistical measures.

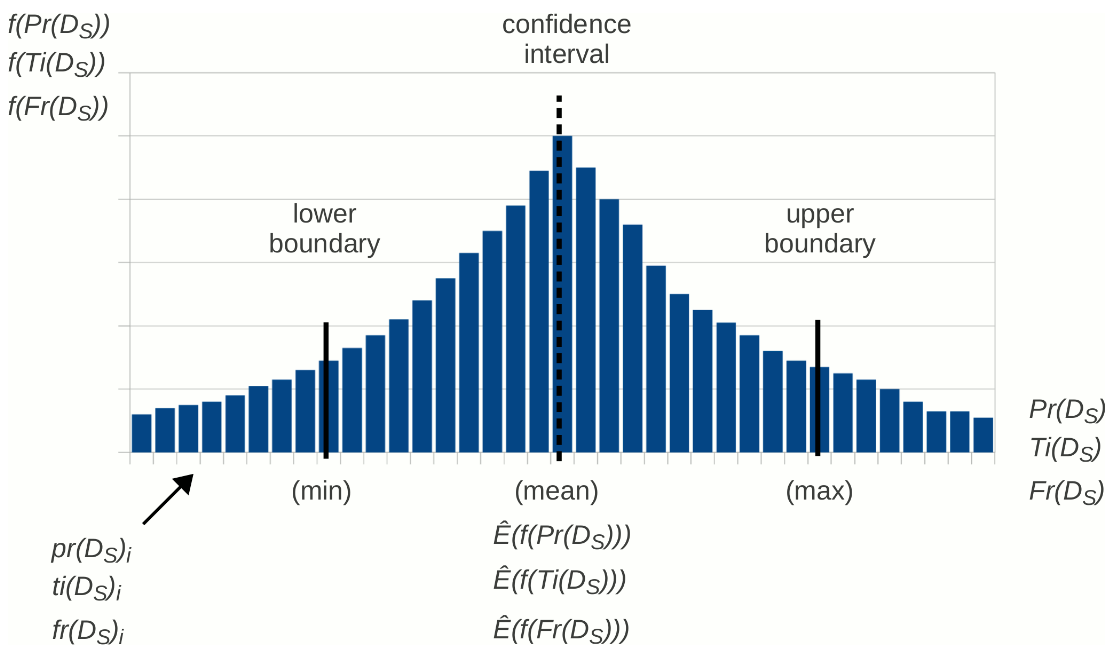


Fig. 6.3: Example histogram of reliability indices for the down state

6.4 Step 4: Evaluation

In conventional reliability evaluations, only mean values are usually estimated. This, however, may cause an issue when the uncertainty of reliability indices is concerned. The reliability evaluation approach proposed in this chapter tries to overcome this issue by a determination of other statistical measures such as median, ranges, confidence intervals, skewness, etc.

After the evaluation, the epistemic uncertainty is minimized. The remaining risks and uncertainty are quantified as residual risks. If the level of this residual risks is not satisfied, the treatment of the process has to be done by revising steps 1 to 3.

Six example systems have been given to illustrate and compare the two different approaches.

6.4.1 Example 1: Component, Low Uncertainty

Suppose that a component under consideration is measured by two experts. The first expert gives an estimate of $MTTF_C = 50{,}000\ \mathrm{h}$ and $MTTR_C = 5\ \mathrm{h}$. The second expert gives a different estimate $MTTF_C = 150{,}000\ \mathrm{h}$ and $MTTR_C = 20\ \mathrm{h}$. There could be many reasons behind the dissimilarity, for instance, two experts may use different sources of reliability data.

In the conventional calculation method, when dissimilarity between two sets of information occurs, mean values of these sets of information will be used. In the proposed method, the use of both the arithmetic mean and the geometric mean are introduced. The results of the calculation are shown in Table 6.1. It can be seen that the use of geometric mean values gives a more conservative estimate of the reliability indices comparing to the arithmetic mean. The conventional calculation shows only the measures of central tendency of the reliability indices. The information about the underlying shape or the dispersion of each index is not known. This information can be obtained by the simulation method proposed earlier.

The results from the simulation method using the uniform distribution model with 1,000,000 sample size, shown in Table 6.1, are composed of measures of central tendency such as mean values and medians, measures of locations such as confidence intervals, and measures of dispersion such as variance and skewness. From the simulation it is possible to plot the histogram of each reliability index. With a high number of samples, the histogram is close to the density function of each reliability index. Some of the histograms are illustrated in Fig. 6.4.

1 Component		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	50,000	150,000	100,000	86,603
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	1.00E+05	1.25E+01	1.00E+00	1.25E-04	1.00E-05	1.00E-05
	- Geometric Mean	8.66E+04	1.00E+01	1.00E+00	1.15E-04	1.15E-05	1.15E-05

Simulation Results							
Measure	Index	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
Central Tendency	Arithmetic Mean	9.99E+04	1.25E+01	1.00E+00	1.37E-04	1.10E-05	1.10E-05
	Median	9.99E+04	1.25E+01	1.00E+00	1.25E-04	1.00E-05	1.00E-05
Location	80% Confidence						
	- $Min_{10\%}$	6.00E+04	6.50E+00	1.00E+00	6.21E-05	7.15E-06	7.15E-06
	- $Max_{90\%}$	1.40E+05	1.85E+01	1.00E+00	2.33E-04	1.67E-05	1.67E-05
Dipersion	Variance	8.32E+08	1.87E+01	4.47E-09	4.47E-09	1.26E-11	1.26E-11
	S.D.	2.89E+04	4.33E+00	6.69E-05	6.69E-05	3.56E-06	3.56E-06
	Skewness	2.22E-03	-3.22E-04	-9.63E-01	9.63E-01	7.85E-01	7.85E-01
	Excess Kurtosis	1.80E+00	1.80E+00	3.68E+00	3.68E+00	2.57E+00	2.57E+00

Table 6.1: Reliability indices of the example component with low uncertainty

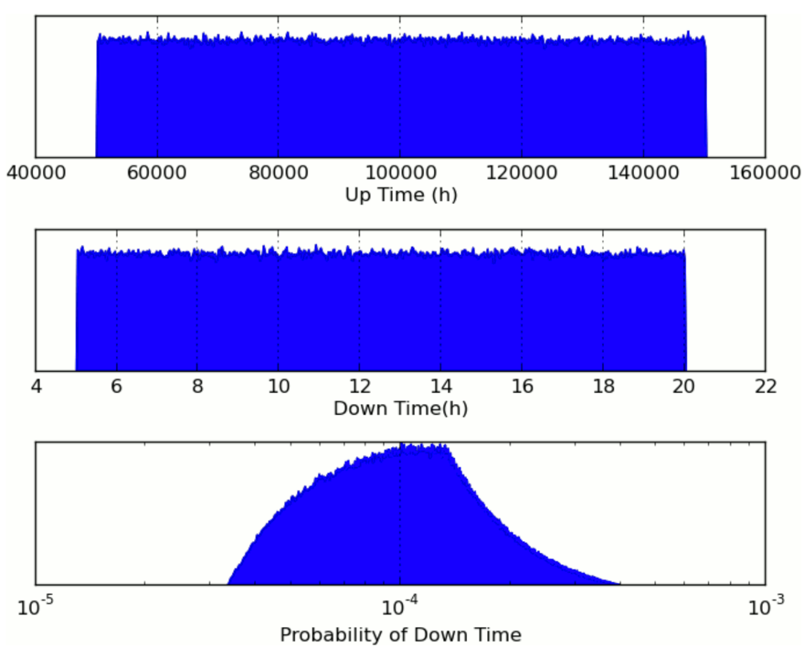


Fig. 6.4: Histograms of the up time, the down time, and the probability of down time of the component with low uncertainty

The results from the conventional calculation and the simulation are compared. First, the results from the input arithmetic mean give the most optimistic estimate and the input geometric mean has the most conservative estimate. The simulation arithmetic mean lies in between.

A downside of mean value is its sensitivity to extreme values, for instance, in Fig. 6.4 it is shown that some samples of $Pr(D_C)$ occur at the extreme right. These extreme values shift the mean significantly. Therefore the median, which is not affected from extreme values, is also given in the results.

The confidence interval gives the idea how scatter the simulated reliability indices are. Generally it is useful to ensure that the reliability of the component designed is greater than the endpoints of the confidence interval. The use of the 80% confidence interval is focused in this work and more details will be discussed further in this chapter.

Measures of dispersion can be used to determine how the results are scattered. Skewness is another important measure which numerically indicates the efficiency of confident intervals. For example, $Pr(D_C)$ in Fig. 6.4 is positive skew, which means most of the extreme values lie to the right side of the distribution and thus make the upper endpoint of the confidence interval very scattered. That is, even though most of the probability of down time lie inside the confidence interval, there are some extreme cases which the probability is extremely high, which is an unneeded character for reliability.

Excess kurtosis is another useful measure of dispersion. Positive excess kurtosis means the distribution has more extreme values than a normal distribution. Negative excess kurtosis means there are fewer extreme values.

More applications of these measures will be discussed in the examples of serial and parallel systems.

6.4.2 Example 2: Component, High Uncertainty

In the examples with high uncertainty, it is given a component which is measured by two experts. The first expert gives an estimate of $MTTF_C = 10{,}000\ \text{h}$ and $MTTR_C = 5\ \text{h}$. The second expert gives a different estimate $MTTF_C = 1{,}000{,}000\ \text{h}$ and $MTTR_C = 20\ \text{h}$. The results from the conventional calculation and the simulation are shown and illustrated in Table 6.2 and Fig. 6.5.

1 Component		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	10,000	1,000,000	505,000	100,000
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	$Ti(U_c)$ (h)	$Ti(D_c)$ (h)	$Pr(U_c)$	$Pr(D_c)$	$Fr(U_c)$ (1/h)	$Fr(D_c)$ (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	5.05E+05	1.25E+01	1.00E+00	2.48E-05	1.98E-06	1.98E-06
	- Geometric Mean	1.00E+05	1.00E+01	1.00E+00	1.00E-04	1.00E-05	1.00E-05

Simulation Results							
Measure	Index	$Ti(U_c)$ (h)	$Ti(D_c)$ (h)	$Pr(U_c)$	$Pr(D_c)$	$Fr(U_c)$ (1/h)	$Fr(D_c)$ (1/h)
Central Tendency	Arithmetic Mean	5.05E+05	1.25E+01	1.00E+00	5.81E-05	4.65E-06	4.65E-06
	Median	5.05E+05	1.25E+01	1.00E+00	2.48E-05	1.98E-06	1.98E-06
Location	80% Confidence						
	- $Min_{10\%}$	1.09E+05	6.51E+00	1.00E+00	1.06E-05	1.11E-06	1.11E-06
	- $Max_{90\%}$	9.01E+05	1.85E+01	1.00E+00	1.15E-04	9.19E-06	9.19E-06
Dipersion	Variance	8.18E+10	1.87e+01	1.42E-08	1.42E-08	7.92E-11	7.92E-11
	S.D.	2.86E+05	4.33E+00	1.19E-04	1.19E-04	8.90E-06	8.90E-06
	Skewness	-2.30E-03	-3.06E-04	-6.38E+00	6.38E+00	5.56E+00	5.56E+00
	Excess Kurtosis	1.80E+00	1.80E+00	5.72E+01	5.72E+01	4.14E+01	4.14E+01

Table 6.2: Reliability indices of the example component with high uncertainty

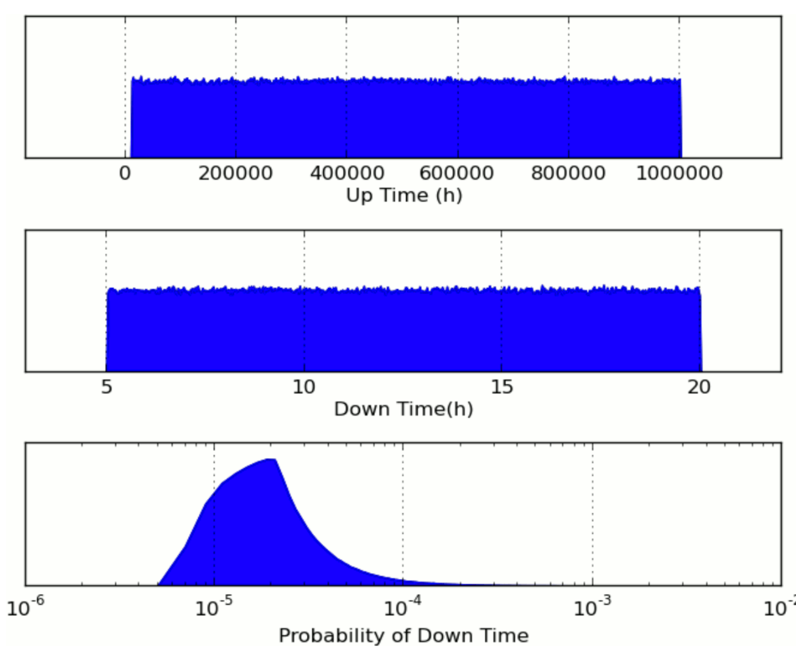


Fig. 6.5: Histograms of the up time, the down time, and the probability of down time of the component with high uncertainty

By looking at the conventional calculation (Table 6.2), the difference between the input arithmetic mean and input geometric mean of $Pr(D_C)$ is greater than the previous example with low uncertainty (Table 6.1). This indicates that the performance of the measure of central tendency decreases when the uncertainty increases.

In order to see how much the reliability has been influenced by uncertainty, the simulation results give important measures such as the confidence interval and the skewness. These measures determine whether the mean values properly represent the underlying reliability or not. The benefits are more obvious in the examples of serial systems.

6.4.3 Example 3: Serial System, Low Uncertainty

This section studies the effect of epistemic uncertainty if 10 and 100 components are connected in series. It is assumed that the components have the same reliability indices as Example 1.

Table 6.3 and Fig 6.6 show the simulation results of the serial system with 10 components. It is noted that the simulated arithmetic mean and median of $Ti(U_S)$ now lie between the input arithmetic mean and input geometric mean.

It can also be noticed that some measures, such as the mean values and variance, are ten times larger compared to the results of a single component. The main measures which differ are the confidence interval, skewness, and excess kurtosis. This can be interpreted that the central tendency of reliability indices are usually proportional to the number of components. The dissimilarity lies in the shape of distributions, presented by measures of location, measures of dispersion, and histograms.

The information in Table 6.3 describes characteristics of the reliability indices. For example, $Pr(D_S)$ is less positive skew than $Pr(D_C)$ in Table 6.1, which means there are fewer extreme values on the upper end. The excess kurtosis is lower but still positive, which means the extreme values have been decreased in general. The decreases in extreme values improve the certainty of the confidence interval, especially on the upper end. These characters of $Pr(D_S)$ are illustrated in Fig. 6.6.

10 Components in Serial		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	50,000	150,000	100,000	86,603
	MTTR	5	20	12.50	10

Measure	Index	Conventional Calculation					
		$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	1.00E+04	1.25E+01	9.99E-01	1.25E-03	9.99E-05	9.99E-05
	- Geometric Mean	8.66E+03	1.00E+01	9.99E-01	1.15E-03	1.15E-04	1.15E-04

Measure	Index	Simulation Results					
		$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Arithmetic Mean	9.20E+03	1.25E+01	9.99E-01	1.37E-03	1.10E-04	1.10E-04
	Median	9.14E+03	1.25E+01	9.99E-01	1.36E-03	1.09E-04	1.09E-04
Location	80% Confidence						
	- $Min_{10\%}$	8.02E+03	1.07E+01	9.98E-01	1.11E-03	9.56E-05	9.56E-05
	- $Max_{90\%}$	1.05E+04	1.44E+01	9.99E-01	1.65E-03	1.25E-04	1.25E-04
Dipersion	Variance	8.94E+05	2.06E+00	4.46E-08	4.47E-08	1.26E-10	1.26E-10
	S.D.	9.45E+02	1.44E+00	2.11E-04	2.12E-04	1.12E-05	1.12E-05
	Skewness	3.22E-01	1.77E-03	-3.03E-01	3.04E-01	2.47E-01	2.47E-01
	Excess Kurtosis	3.01E+00	2.83E+00	3.06E+00	3.06E+00	2.96E+00	2.96E+00

Table 6.3: Reliability indices of the serial system of 10 components with low uncertainty

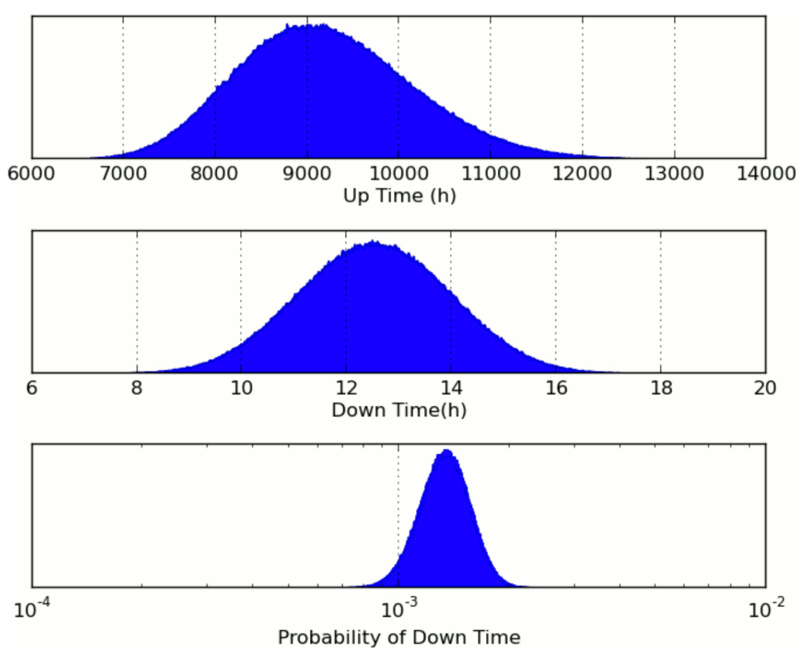


Fig. 6.6: Histograms of the up time, the down time, and the probability of down time of the serial system of 10 components with low uncertainty

If the number of components in the serial system is increased to 100, similar effects occur. Most measures of central tendency increase by ten times, and the main differences are the measures of location and dispersion, as illustrated in Table 6.4 and Fig. 6.7.

In the case of $Pr(D_S)$, the positive skewness and the positive excess kurtosis have been reduced compared to the serial system with 10 components. This indicates that the uncertainty of $Pr(D_S)$ does not decrease if the number of components in the example serial system increases.

It was mentioned earlier that the conventional calculation determines reliability indices using mean values of the input. The usage of the arithmetic mean gives relative optimistic results and the geometric mean relative conservative results. The conservativeness of the results from the input geometric mean is very high, and this may cause an error in the estimation if not considered carefully. For instance, the input geometric mean of $Pr(D_S)$ is higher than its upper confidence interval from the simulation. This shows that the conventional calculation gives the estimates of reliability indices that is unlikely to occur.

On the other hand, the simulation gives the results which describe various characters of the reliability indices, compared to the conventional method. If the epistemic uncertainty is taken into considered, the proposed simulation method is preferred for the evaluation.

100 Components in Serial		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	50,000	150,000	100,000	86,603
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	1.00E+03	1.27E+01	9.88E-01	1.25E-02	9.88E-04	9.88E-04
	- Geometric Mean	8.66E+02	1.01E+01	9.89E-01	1.15E-02	1.14E-03	1.14E-03

Simulation Results							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Arithmetic Mean	9.11E+02	1.27E+01	9.86E-01	1.37E-02	1.08E-03	1.08E-03
	Median	9.10E+02	1.27E+01	9.86E-01	1.37E-02	1.08E-03	1.08E-03
Location	80% Confidence						
	- $Min_{10\%}$	8.74E+02	1.21E+01	9.86E-01	1.29E-02	1.04E-03	1.04E-03
	- $Max_{90\%}$	9.49E+02	1.33E+01	9.87E-01	1.46E-02	1.13E-03	1.13E-03
Dipersion	Variance	8.70E+02	2.19E-01	4.36E-07	4.48E-07	1.20E-09	1.20E-09
	S.D.	2.95E+01	4.67E-01	6.60E-04	6.69E-04	3.46E-05	3.46E-05
	Skewness	1.12E-01	7.08E-03	-9.64E-02	9.84E-02	7.88E-02	7.88E-02
	Excess Kurtosis	3.01E+00	2.98E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00

Table 6.4: Reliability indices of the serial system of 100 components with low uncertainty

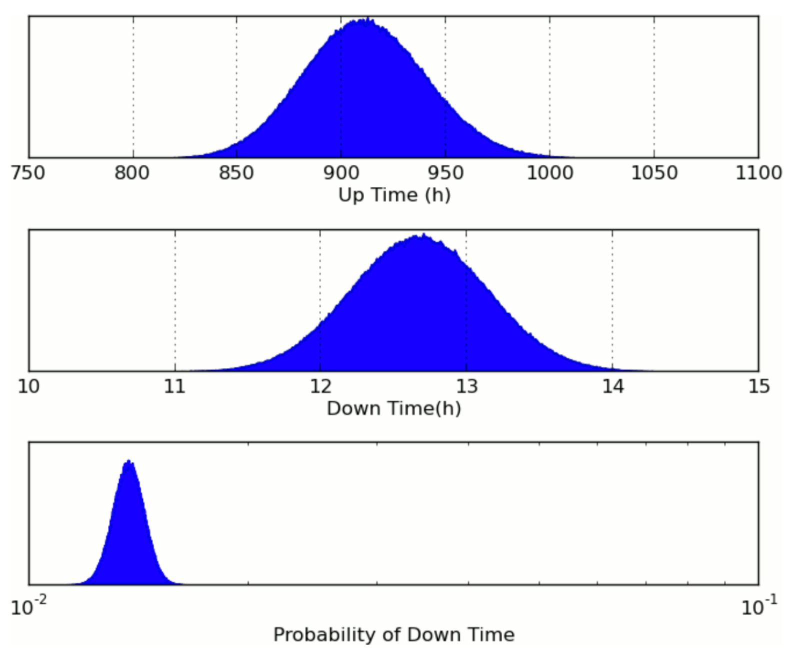


Fig. 6.7: Histograms of the up time, the down time, and the probability of down time of the serial system of 100 components with low uncertainty

6.4.4 Example 4: Serial System, High Uncertainty

This section studies the effect of epistemic uncertainty if 10 and 100 components are connected in series. It is assumed that the components have the same reliability indices as Example 2.

For a serial system with 10 components in Table 6.5 and Fig. 6.8, it can be seen that the difference of the input arithmetic mean and the input geometric mean of probability $Pr(D_S)$ has been increased comparing the Table 6.3. The values of these input means are affected by extreme values and are less accurate. It is noted that the input geometric mean of $Pr(D_S)$ is even not in the confidence interval.

The same result is confirmed when the number of components increases to 100 in Table 6.6 and Fig. 6.9. The input geometric mean of $Pr(D_S)$ is smaller than the lower-end of the confidence interval, and the arithmetic mean of $Pr(D_S)$ is greater than the greater-end of the confidence interval. These input mean values cannot measure the reliability of the serial systems accurately when the uncertainty is high.

10 Components in Serial		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	10,000	1,000,000	505,000	100,000
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	5.05E+04	1.25E+01	1.00E+00	2.48E-04	1.98E-05	1.98E-05
	- Geometric Mean	1.00E+04	1.00E+01	9.99E-01	1.00E-03	9.99E-05	9.99E-05

Simulation Results							
Measure	Index	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
Central Tendency	Arithmetic Mean	2.81E+04	1.25E+01	9.99E-01	5.81E-04	4.65E-05	4.65E-05
	Median	2.66E+04	1.25E+01	1.00E+00	4.62E-04	3.75E-05	3.75E-05
Location	80% Confidence						
	- $Min_{10\%}$	1.17E+04	9.74E+00	9.99E-01	2.58E-04	2.14E-05	2.14E-05
	- $Max_{90\%}$	4.67E+04	1.53E+01	1.00E+00	1.07E-03	8.57E-05	8.57E-05
Dipersion	Variance	1.77E+08	4.46E+00	1.41E-07	1.41E-07	7.82E-10	7.82E-10
	S.D.	1.33E+04	2.11E+00	3.75E-04	3.75E-04	2.80E-05	2.80E-05
	Skewness	5.39E-01	-2.99E-04	-2.02E+00	2.02E+00	1.75E+00	1.75E+00
	Excess Kurtosis	2.76E+00	2.69E+00	8.44E+00	8.44E+00	6.80E+00	6.80E+00

Table 6.5: Reliability indices of the serial system of 10 components with high uncertainty

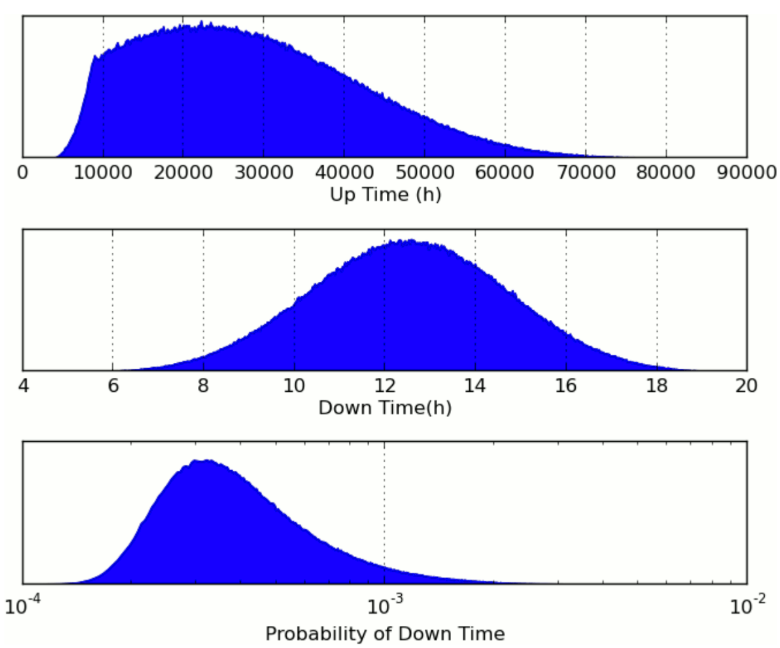


Fig. 6.8: Histograms of the up time, the down time, and the probability of down time of the serial system of 10 components with high uncertainty

100 Components in Serial		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	10,000	1,000,000	505,000	100,000
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	5.05E+03	1.25E+01	9.98E-01	2.48E-03	1.98E-04	1.98E-04
	- Geometric Mean	1.00E+03	1.01E+01	9.90E-01	1.00E-02	9.90E-04	9.90E-04

Simulation Results							
Measure	Index	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
Central Tendency	Arithmetic Mean	2.23E+03	1.26E+01	9.94E-01	5.81E-03	4.63E-04	4.63E-04
	Median	2.19E+03	1.26E+01	9.94E-01	5.68E-03	4.54E-04	4.54E-04
Location	80% Confidence						
	- $Min_{10\%}$	1.71E+03	1.14E+01	9.93E-01	4.40E-03	3.57E-04	3.57E-04
	- $Max_{90\%}$	2.79E+03	1.37E+01	9.96E-01	7.40E-03	5.79E-04	5.79E-04
Dipersion	Variance	1.77E+05	8.10E-01	1.39E-06	1.41E-06	7.65E-09	7.65E-09
	S.D.	4.21E+02	9.00E-01	1.18E-03	1.19E-03	8.74E-05	8.74E-05
	Skewness	4.93E-01	9.05E-03	-6.37E-01	6.40E-01	5.50E-01	5.50E-01
	Excess Kurtosis	3.20E+00	2.90E+00	3.53E+00	3.54E+00	3.37E+00	3.37E+00

Table 6.6: Reliability indices of the serial system of 100 components with high uncertainty

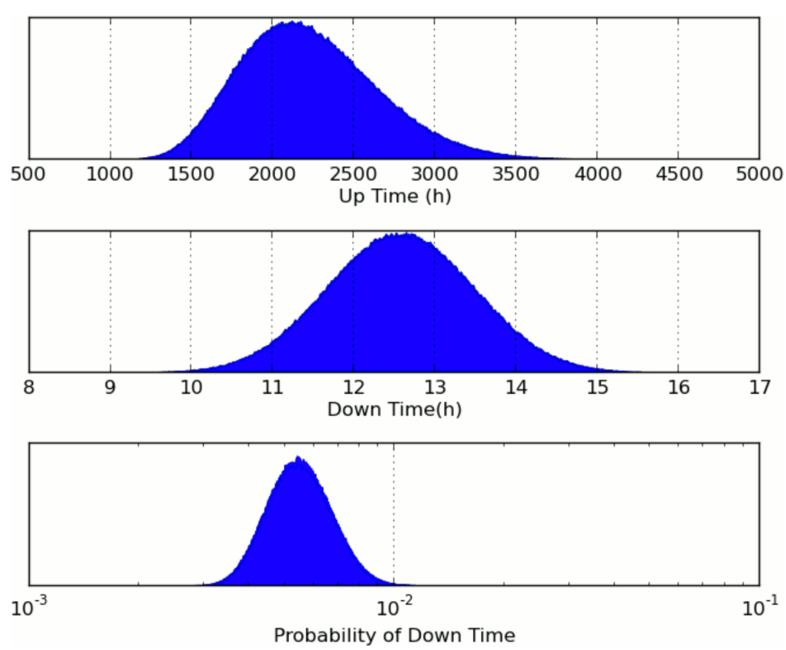


Fig. 6.9: Histograms of the up time, the down time, and the probability of down time of the serial system of 100 components with high uncertainty

6.4.5 Example 5: Parallel System, Low Uncertainty

This section studies the effect of epistemic uncertainty if 2 and 3 components are connected in parallel. It is assumed that the components have the same reliability indices as Example 1 and 3.

Like the example serial systems, the conventional calculation gives two measures. The input arithmetic mean gives a relative optimistic results and the input geometric mean give conservative results. The simulation results are divided into three measure types: the measures of central tendency, the measures of location, and the measures of dispersion. All measures are shown in Table 6.7.

Fig 6.10 illustrates the histogram of the up time, the down time, and the probability of down time of the parallel system with 2 components. The distribution of $Ti(U_S)$ is skew to the right which is consistent to the positive skewness in Table 6.7. The extreme values from skewness make the use of mean values less creditable, as the mean values tend to shift to the extreme values.

2 Components in Parallel		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	50,000	150,000	100,000	86,603
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	4.00E+08	6.25E+00	1.00E+00	1.56E-08	2.50E-09	2.50E-09
	- Geometric Mean	3.75E+08	5.00E+00	1.00E+00	1.33E-08	2.67E-09	2.67E-09

Simulation Results							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Arithmetic Mean	4.28E+08	5.86E+00	1.00E+00	1.88E-08	3.02E-09	3.02E-09
	Median	3.81E+08	5.69E+00	1.00E+00	1.51E-08	2.62E-09	2.62E-09
Location	80% Confidence						
	- $Min_{10\%}$	1.93E+08	3.85E+00	1.00E+00	5.99E-09	1.38E-09	1.38E-09
	- $Max_{90\%}$	7.22E+08	8.22E+00	1.00E+00	3.64E-08	5.18E-09	5.18E-09
Dipersion	Variance	4.92E+16	2.67E+00	1.88E-16	1.88E-16	2.66E-18	2.66E-18
	S.D.	2.22E+08	1.63E+00	1.37E-08	1.37E-08	1.63E-09	1.63E-09
	Skewness	1.30E+00	3.16E-01	-1.89E+00	1.89E+00	1.51E+00	1.51E+00
	Excess Kurtosis	5.50E+00	2.21E+00	8.32E+00	8.32E+00	6.13E+00	6.13E+00

Table 6.7: Reliability indices of the parallel system of 2 components with low uncertainty

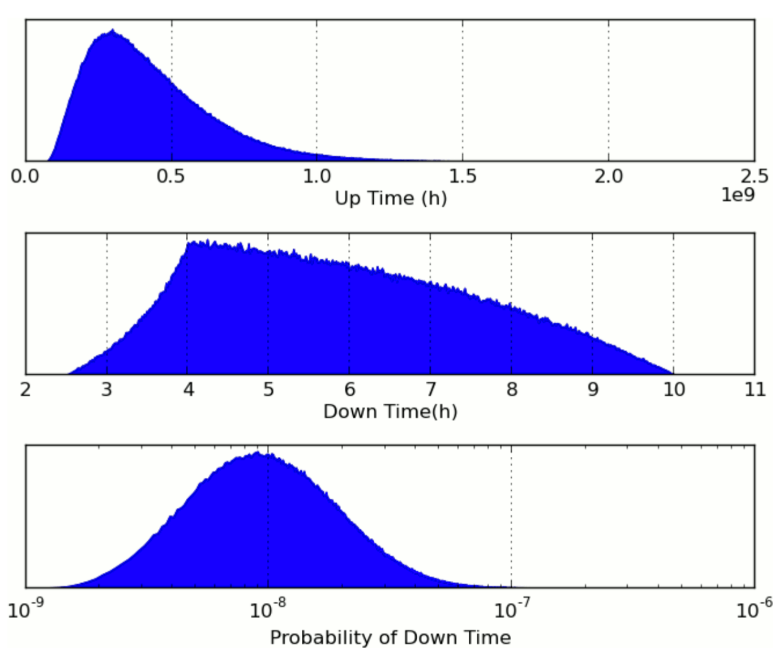


Fig. 6.10: Histograms of the up time, the down time, and the probability of down time of the parallel system of 2 components with low uncertainty

If the number of components increases from 2 to 3, the arithmetic mean, median, and the confidence interval $Ti(U_S)$ will be improved significantly, as shown in Table 6.8. It is to say that the parallel system has an overall better reliability indices. However, any conclusion has to be made with care, because the skewness and the excess kurtosis also increase by a good margin. This gives the higher chance that the extreme values would occur.

When the histogram of $Pr(D_S)$ of 2 and 3 components (Fig. 6.10 - 6.11) are compared, it can be seen that the uncertainty does not decrease if the number of components increases.

3 Components in Parallel		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	50,000	150,000	100,000	86,603
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	Ti(U$_s$) (h)	Ti(D$_s$) (h)	Pr(U$_s$)	Pr(D$_s$)	Fr(U$_s$) (1/h)	Fr(D$_s$) (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	2.13E+12	4.17E+00	1.00E+00	1.95E-12	4.69E-13	4.69E-13
	- Geometric Mean	2.17E+12	3.33E+00	1.00E+00	1.54E-12	4.62E-13	4.62E-13

Simulation Results							
Measure	Index	Ti(U$_s$) (h)	Ti(D$_s$) (h)	Pr(U$_s$)	Pr(D$_s$)	Fr(U$_s$) (1/h)	Fr(D$_s$) (1/h)
Central Tendency	Arithmetic Mean	2.57E+12	3.81E+00	1.00E+00	2.58E-12	6.21E-13	6.21E-13
	Median	2.04E+12	3.70E+00	1.00E+00	1.84E-12	4.90E-13	4.90E-13
Location	80% Confidence						
	- Min$_{10\%}$	8.39E+11	2.72E+00	1.00E+00	6.03E-13	2.04E-13	2.04E-13
	- Max$_{90\%}$	4.91E+12	5.09E+00	1.00E+00	5.42E-12	1.19E-12	1.19E-12
Dipersion	Variance	3.77E+24	8.02E-01	5.96E-24	5.96E-24	2.20E-25	2.20E-25
	S.D.	1.94E+12	8.96E-01	2.44E-12	2.44E-12	4.69E-13	4.69E-13
	Skewness	2.37E+00	4.22E-01	-2.81E+00	2.81E+00	2.29E+00	2.29E+00
	Excess Kurtosis	1.29E+01	2.54E+00	1.63E+01	1.63E+01	1.17E+01	1.17E+01

Table 6.8: Reliability indices of the parallel system of 3 components with low uncertainty

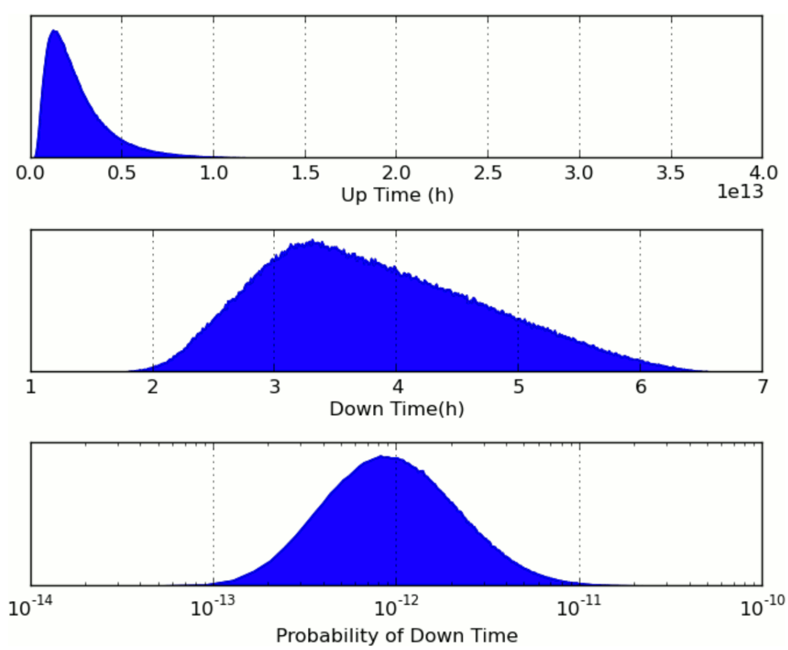


Fig. 6.11: Histograms of the up time, the down time, and the probability of down time of the parallel system of 3 components with low uncertainty

6.4.6 Example 6: Parallel System, High Uncertainty

This section studies the effect of epistemic uncertainty if 2 and 3 components are connected in parallel. It is assumed that the components have the same reliability indices as Example 2 and 4. Table 6.9 - 6.10 and Fig. 6.12 - 6.13 give the same conclusion as the previous examples.

2 Components in Parallel		Inputs			
		min	max	arithmatic mean	geometric mean
	MTTF	10,000	1,000,000	505,000	100,000
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	$Ti(U_S)$ (h)	$Ti(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	1.02E+10	6.25E+00	1.00E+00	6.13E-10	9.80E-11	9.80E-11
	- Geometric Mean	5.00E+08	5.00E+00	1.00E+00	1.00E-08	2.00E-09	2.00E-09

Simulation Results							
Measure	Index	$Ti(U_S)$ (h)	$Ti(D_S)$ (h)	$Pr(U_S)$	$Pr(D_S)$	$Fr(U_S)$ (1/h)	$Fr(D_S)$ (1/h)
Central Tendency	Arithmetic Mean	1.09E+10	5.86E+00	1.00E+00	3.38E-09	5.41E-10	5.41E-10
	Median	7.87E+09	5.69E+00	1.00E+00	7.46E-10	1.27E-10	1.27E-10
Location	80% Confidence						
	- $Min_{10\%}$	1.02E+09	3.85E+00	1.00E+00	1.92E-10	3.97E-11	3.97E-11
	- $Max_{90\%}$	2.52E+10	8.22E+00	1.00E+00	5.99E-09	9.85E-10	9.85E-10
Dipersion	Variance	1.06E+20	2.67E+00	2.96E-16	2.96E-16	6.31E-18	6.31E-18
	S.D.	1.03E+10	1.63E+00	1.72E-08	1.72E-08	2.51E-09	2.51E-09
	Skewness	1.49E+00	3.17E-01	-3.64E+01	3.64E+01	2.93E+01	2.93E+01
	Excess Kurtosis	5.77E+00	2.21E+00	2.57E+03	2.57E+03	1.54E+03	1.54E+03

Table 6.9: Reliability indices of the parallel system of 2 components with high uncertainty

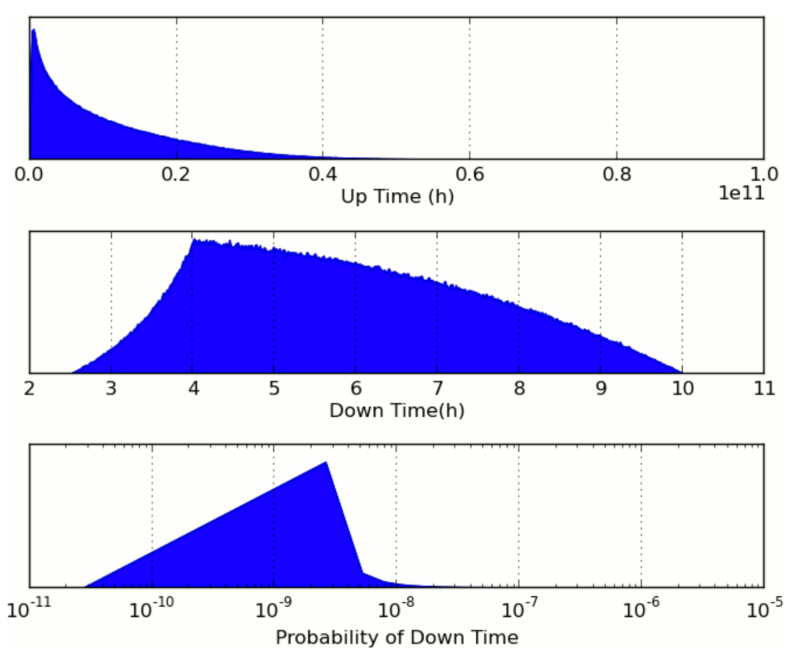


Fig. 6.12: Histograms of the up time, the down time, and the probability of down time of the parallel system of 2 components with high uncertainty

3 Components in Parallel	Inputs				
		min	max	arithmatic mean	geometric mean
	MTTF	10,000	1,000,000	505,000	100,000
	MTTR	5	20	12.50	10

Conventional Calculation							
Measure	Index	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	2.75E+14	4.17E+00	1.00E+00	1.52E-14	3.64E-15	3.64E-15
	- Geometric Mean	3.33E+12	3.33E+00	1.00E+00	1.00E-12	3.00E-13	3.00E-13

Simulation Results							
Measure	Index	Ti(U_s) (h)	Ti(D_s) (h)	Pr(U_s)	Pr(D_s)	Fr(U_s) (1/h)	Fr(D_s) (1/h)
Central Tendency	Arithmetic Mean	3.31E+14	3.81E+00	1.00E+00	1.96E-13	4.69E-14	4.69E-14
	Median	1.67E+14	3.70E+00	1.00E+00	2.28E-14	5.99E-15	5.99E-15
Location	80% Confidence						
	- $Min_{10\%}$	1.51E+13	2.72E+00	1.00E+00	3.87E-15	1.18E-15	1.18E-15
	- $Max_{90\%}$	8.50E+14	5.09E+00	1.00E+00	2.67E-13	6.63E-14	6.63E-14
Dipersion	Variance	2.02E+29	8.03E-01	6.96E-24	6.96E-24	2.98E-25	2.98E-25
	S.D.	4.49E+14	8.96E-01	2.64E-12	2.64E-12	5.46E-13	5.46E-13
	Skewness	3.13E+00	4.22E-01	-4.52E+02	4.52E+02	3.24E+02	3.24E+02
	Excess Kurtosis	1.92E+01	2.55E+00	3.93E+05	3.93E+05	2.32E+05	2.32E+05

Table 6.10: Reliability indices of the parallel system of 3 components with high uncertainty

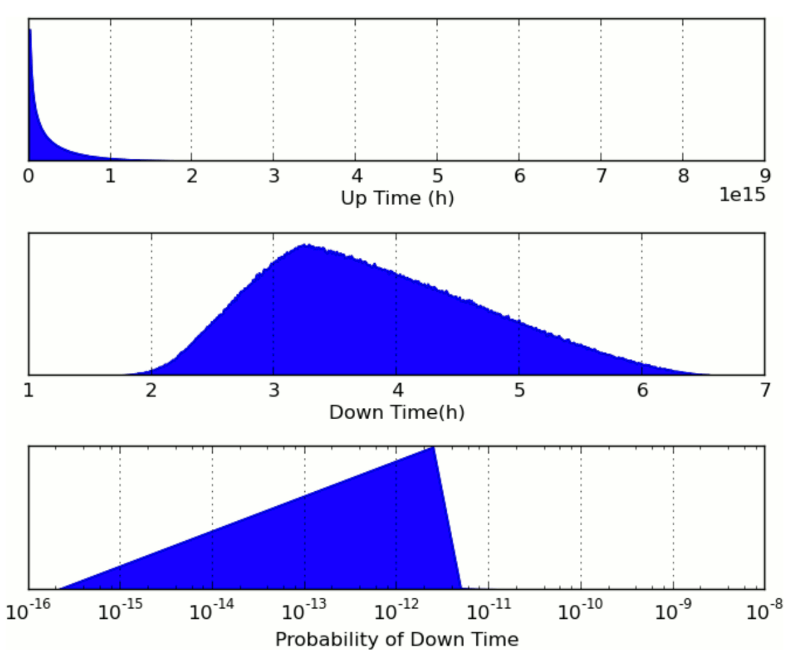


Fig. 6.13: Histograms of the up time, the down time, and the probability of down time of the parallel system of 3 components with high uncertainty

6.5 Summary

If the epistemic uncertainty is low, the conventional calculation gives acceptable reliability indices. The input arithmetic mean gives the relative optimistic estimates and the input geometric mean gives the relative conservative estimates.

However, if the epistemic uncertainty is high, using the conventional calculation may lead to inaccurate reliability indices. Mean values can be greatly affected by extreme values, therefore other statistical measures such as the confidence interval or the skewness have to be considered as well. These measures can be obtained by the proposed simulation method in 4 steps.

Nevertheless, the conventional method still has an advantage on the simplicity of the computation. Therefore, it depends on the application and the objective, whether the deviation of the result is acceptable or not.

6.6 Case Study: HVDC Converter Station

The reliability evaluation of HVDC converter station in section 5.6 is once again taken as a case study. In the previous chapter it is focused on the aleatory uncertainty of the reliability of Square Butte HVDC converter station, given 6 years of statistical data. A new HVDC converter station is planned, and its reliability has to be estimated. The structure, technology, and the operating environment of the new HVDC project is similar to the existing systems, therefore, the project engineer may adopt the reliability data of the existing systems and use it as the expected reliability of the new system.

Since the data is not measured from the new system itself, there exist an amount of data uncertainty. With the approach proposed in this chapter, the uncertainty can be measured by the measurement of location and dispersion.

6.6.1 Step 1: Identification

A new HVDC converter station is planned and its reliability has to be estimated. There is no reliability information of this planed converter station, but the experts estimate that this new converter station uses thyristor valves, and the structure is very similar to the converter stations in Square Butte and Skagerrak 1 & 2. If the reliability of the planned converter station is estimated from the statistical data of the existing systems [Vancers et al. 2004, Vancers et al. 2006, Vancers et al. 2008], shown in Table 6.11, epistemic uncertainty will occur due to the limitation of reliability information.

This system is categorized as a repairable system (See Section 3.4), therefore, Eq. 3.57 - 3.60 are used. Exponential distribution of failures and repairs are assumed.

System	AC-E		V		C&P		DC-E		O		Total Operating Time (h) *
	No. of Failures	Time to Repair (h)	No. of Failures	Time to Repair (h)	No. of Failures	Time to Repair (h)	No. of Failures	Time to Repair (h)	No. of Failures	Time to Repair (h)	
Square Butte	18	70.1	8	28.2	15	12.1	6	43.8	3	42.5	52324.2
Vancouver Island Pole 2	16	51.7	4	31.2	8	45.3	4	8.5	8	12.4	52410.9

* 24-hour operation in 365 days minus the total of time to repair

Table 6.11: Reliability data of existing systems

It can be noticed that, according to the data, C&P of Vancouver Island failed less often than Square Butte, but the repair time is higher. The limitation of detailed data makes the exact estimate of reliability less likely. This is an example of the epistemic uncertainty.

6.6.2 Step 2: Formulation

The statistical data of each existing HVDC converter station is processed according to Eq. 3.16, 3.17, 3.74 - 3.77, and the result is shown in Table 6.12. $Ti(U_C)_{min}$, $Ti(U_C)_{max}$, $Ti(D_C)_{min}$, and $Ti(D_C)_{max}$ are used as inputs of the simulation.

System	AC-E		V		C&P		DC-E		O	
	$Ti(U_c)$ (h)	$Ti(D_c)$ (h)	$Ti(U_c)$ (h)	$Ti(D_c)$ (h)	$Ti(U_c)$ (h)	$Ti(D_c)$ (h)	$Ti(U_c)$ (h)	$Ti(D_c)$ (h)	$Ti(U_c)$ (h)	$Ti(D_c)$ (h)
Square Butte	2.91E+03	3.89E+00	6.54E+03	3.53E+00	3.49E+03	8.07E-01	8.72E+03	7.30E+00	1.74E+04	1.42E+01
Vancouver Island Pole 2	3.28E+03	3.23E+00	1.31E+04	7.80E+00	6.55E+03	5.66E+00	1.31E+04	2.13E+00	6.55E+03	1.55E+00
min	2.91E+03	3.23E+00	6.54E+03	3.53E+00	3.49E+03	8.07E-01	8.72E+03	2.13E+00	6.55E+03	1.55E+00
max	3.28E+03	3.89E+00	1.31E+04	7.80E+00	6.55E+03	5.66E+00	1.31E+04	7.30E+00	1.74E+04	1.42E+01

Table 6.12: Parameters of failure and repair distribution of existing HVDC converter stations

6.6.3 Step 3: Simulation

Eq. 6.3 to 6.17 are implemented and simulated with 1,000,000 simulation samples. The reliability indices with various measures have been determined.

6.6.4 Step 4: Evaluation

The simulation results and the reliability indices are shown in Table 6.13. The epistemic uncertainty has been revealed by the measures of location and dispersion.

Conventional Calculation							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Input Mean						
	- Arithmetic Mean	1.25E+03	4.33E+00	9.97E-01	3.45E-03	7.97E-04	7.97E-04
	- Geometric Mean	1.21E+03	3.59E+00	9.97E-01	2.96E-03	8.26E-04	8.26E-04

Simulation Results							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Arithmetic Mean	1.23E+03	4.36E+00	9.96E-01	3.56E-03	8.16E-04	8.16E-04
	Median	1.23E+03	4.35E+00	9.96E-01	3.52E-03	8.13E-04	8.13E-04
Location	80% Confidence						
	- $Min_{10\%}$	1.12E+03	3.57E+00	9.96E-01	2.86E-03	7.49E-04	7.49E-04
	- $Max_{90\%}$	1.33E+03	5.17E+00	9.97E-01	4.31E-03	8.87E-04	8.87E-04
Dipersion	Variance	6.18E+03	3.77E-01	3.16E-07	3.18E-07	2.76E-09	2.76E-09
	S.D.	7.86E+01	6.14E-01	5.63E-04	5.64E-04	5.25E-05	5.25E-05
	Skewness	2.30E-02	1.06E-01	-3.56E-01	3.57E-01	2.79E-01	2.79E-01
	Excess Kurtosis	2.55E+00	2.69E+00	2.99E+00	2.99E+00	2.69E+00	2.69E+00

Table 6.13: Reliability indices of the new HVDC converter station

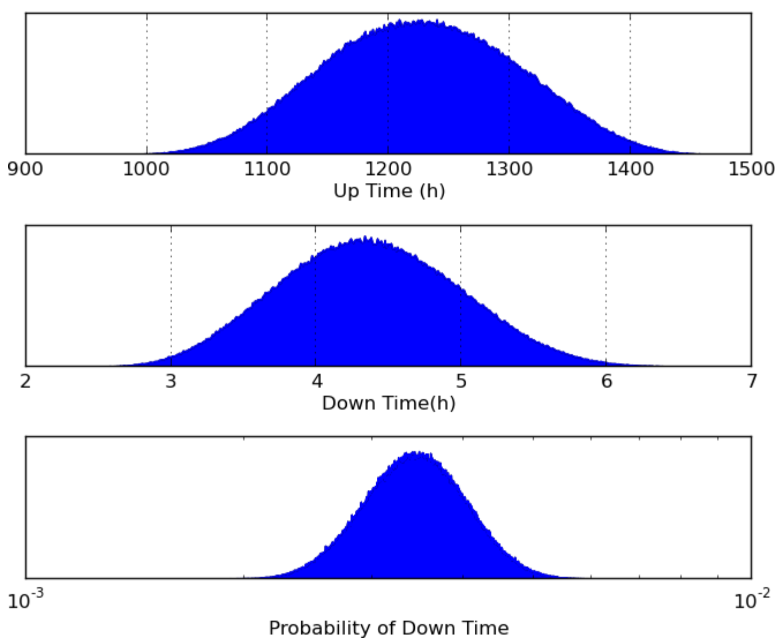


Fig. 6.14: Histograms of the up time, the down time, and the probability of down time of the reliability of the new HVDC converter station

6.7 Case Study: Process Control System

A process control system is extensively used in industry. It monitors an operation, and enables an automation, which reduces human intervention in regulating a complex process. In many areas the process control system reliability is a significant factor in determining the success of the process. A process control system can be very complicated, and sophisticated reliability approaches, such as the Markov minimal cut approach, have been applied to an industrial system [Kochs 2012]. Uncertainty and its effects to reliability of this system is taken as a case study in this section.

6.7.1 Overview of Process Control System

Reliability of a complex process control system has been evaluated in [Kochs 2012]. In this system, a combination of many reliabiltiy approaches, such as the Markov process, the minimal cut approach, and the reliability block diagram, have been applied. The minimal cut set of the system is modelled in Fig. 6.15 to 6.18. More details of this system can be found in [Kochs 2012].

6.7.2 Step 1: Identification

The source of reliability information has been taken from the manufacturer, third parties, and the system experts. Some of these informations are subjective due to the limitation of data, which lead to epistemic uncertainty in reliability data. This uncertainty has not been considered in the original work. With the framework of uncertainty in this work, the uncertainty and its effects on reliability is illustrated.

6.7.3 Step 2: Formulation

In the original work, the outage rate λ and the repair rate μ of each component and each minimal cut are determined. Both parameters are uncertain. If these parameters are expected to be in the range of 0.5 and 1.5 times of their originally estimated values, the MTTF and MTTR of each minimal cut can be formulated by Eq. 6.1 and 6.2.

For example, the minimal cut MS1 is composed of two components; the service station BS and the engineering station ES. The outage rate of BS and ES depends on the outage rate of their components, and can be formulated as Eq. 6.18 - 6.19.

$$\lambda_{ES} = 6 \cdot (\lambda_1 / \mu)^2 \cdot 2 \cdot \mu_1 \quad (6.18)$$

$$\lambda_{BS} = (\lambda_2 / \mu)^2 \cdot 2 \cdot \mu_2 \quad (6.19)$$

The parameters λ_1, λ_2, μ_1, and μ_2 were estimated as fixed values [Kochs 2012]. With the approach for epistemic uncertainty proposed in the framework, these parameters are modelled as uniform distributions with a range from 0.5 to 1.5 times the original value. The same procedure is applied to other minimal cuts. A computer simulation is used to evaluate the reliability indices of the system.

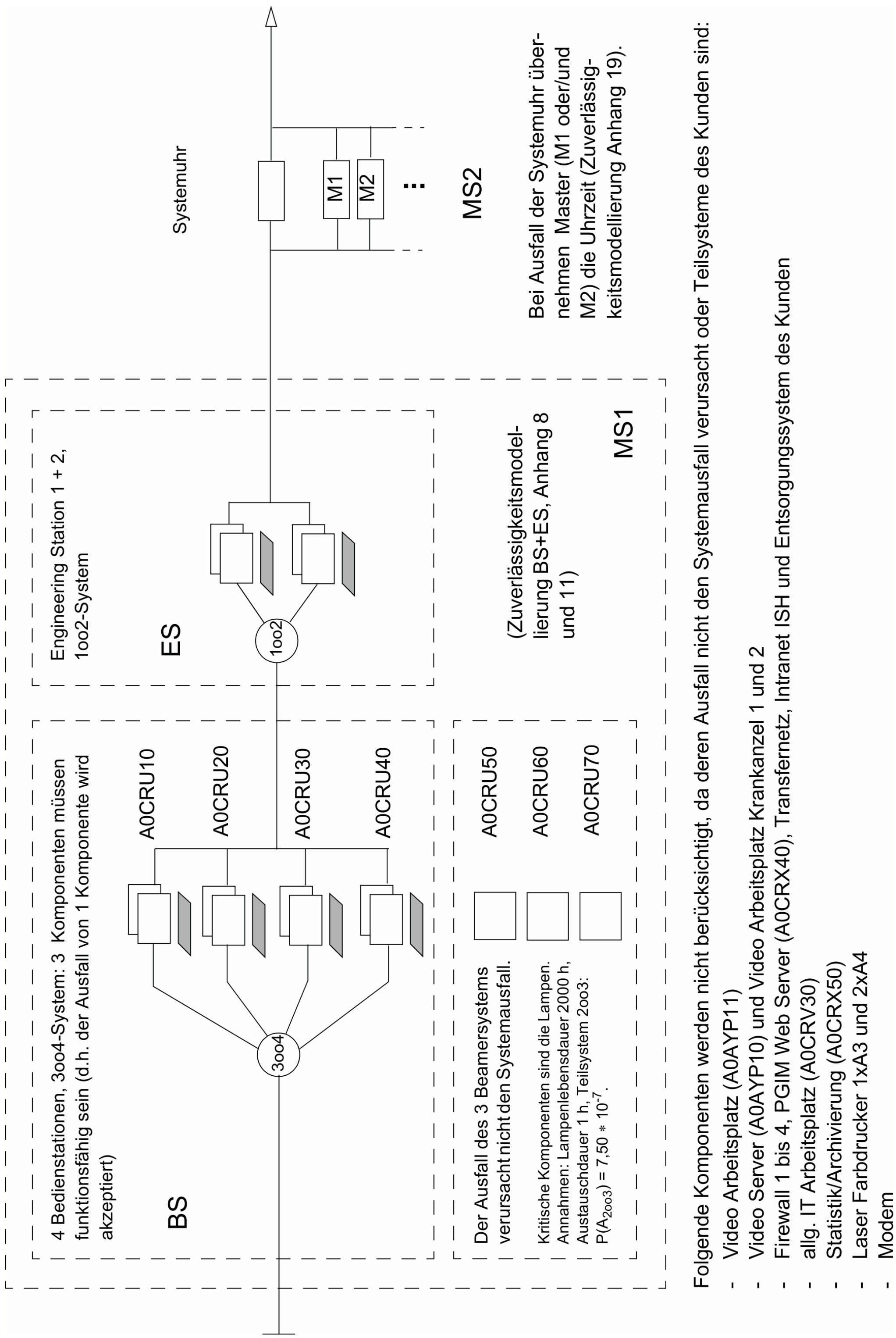


Fig. 6.15: Minimal cuts of a process control system from [Kochs 2012], page 1 of 4

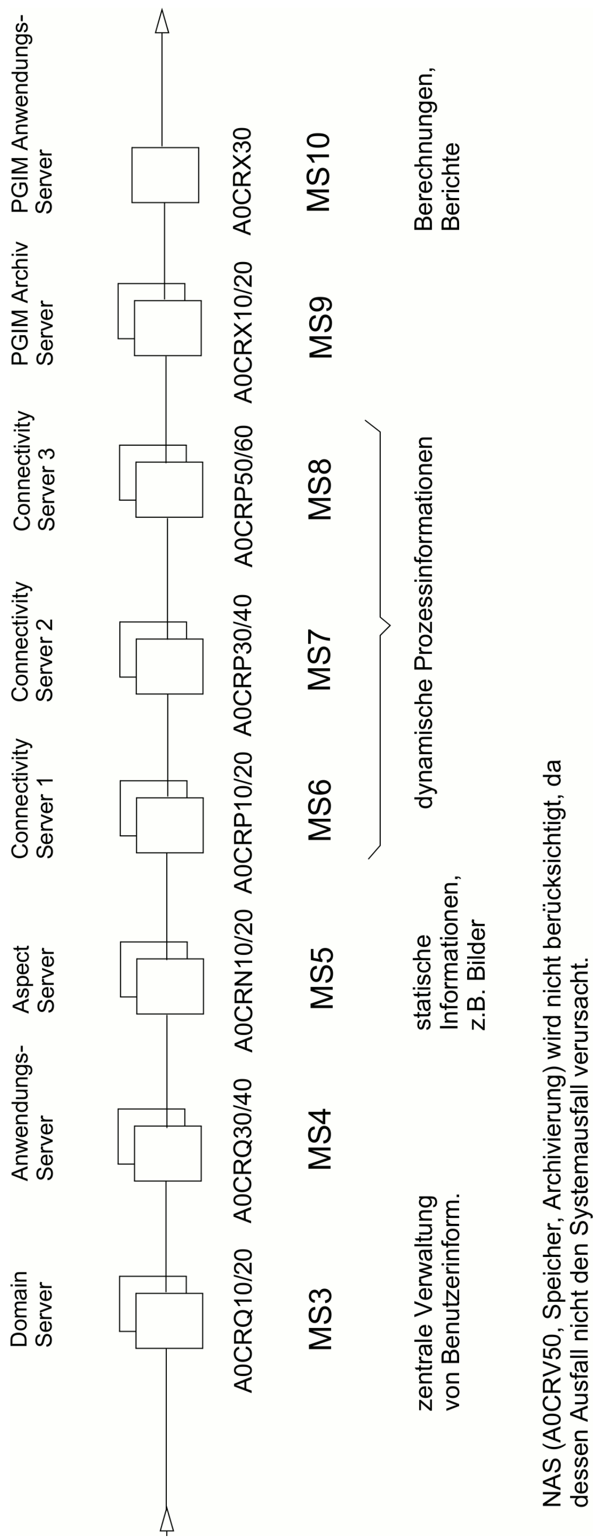


Fig. 6.16: Minimal cuts of a process control system from [Kochs 2012], page 2 of 4

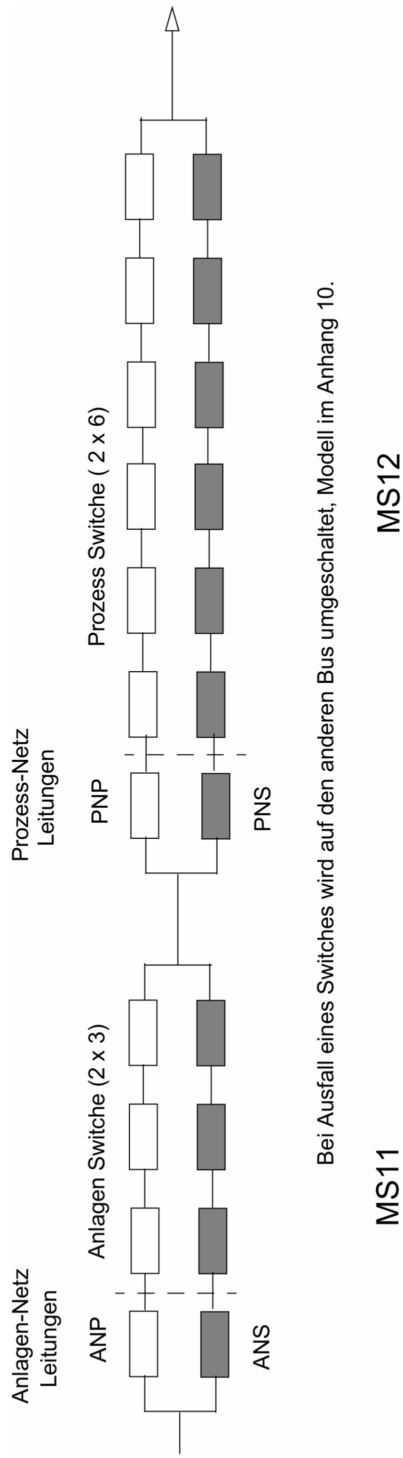


Fig. 6.17: Minimal cuts of a process control system from [Kochs 2012], page 3 of 4

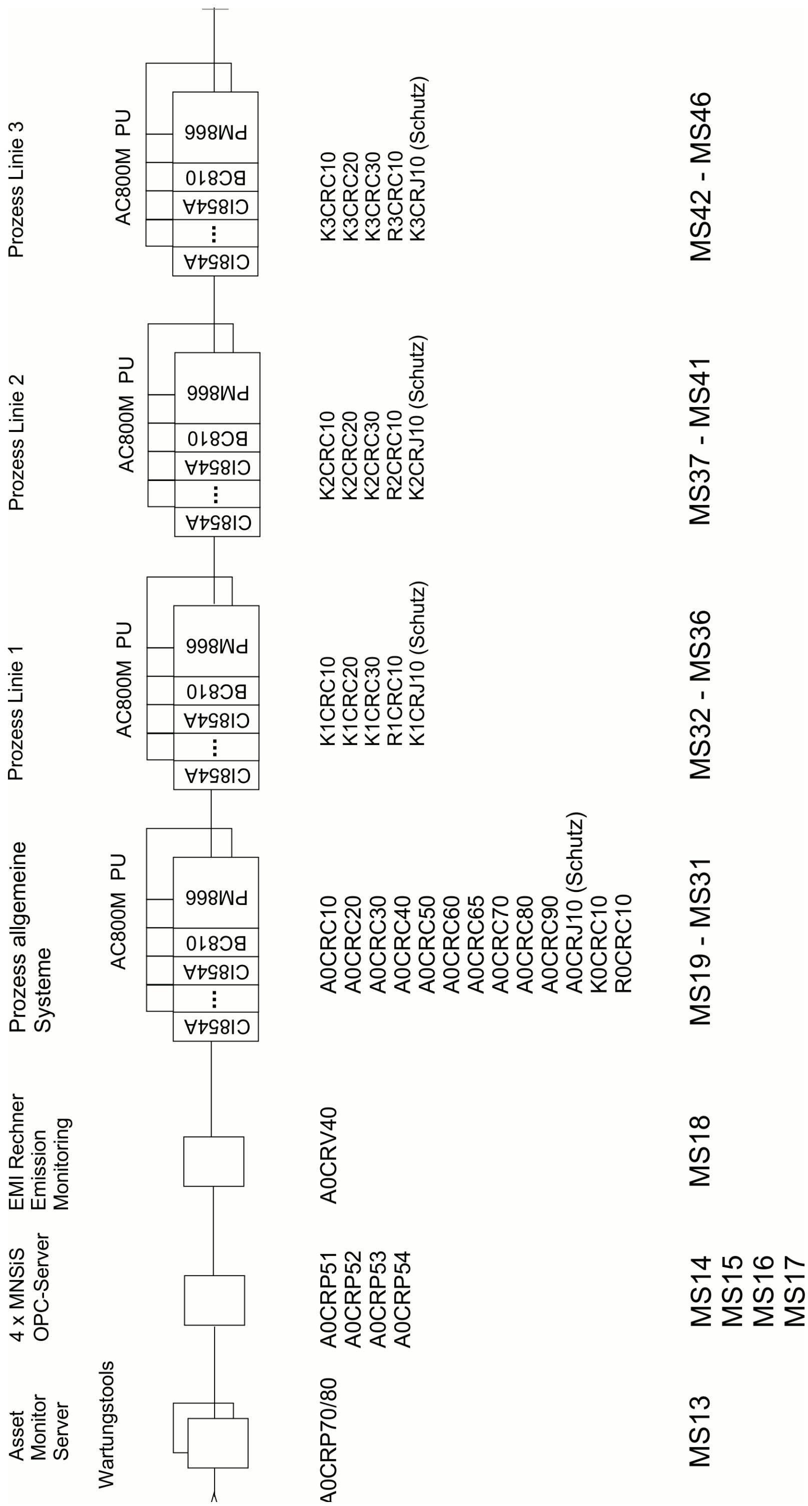


Fig. 6.18: Minimal cuts of a process control system from [Kochs 2012], page 4 of 4

6.7.4 Step 3: Simulation

The formulated model is simulated using Eq. 6.3 to 6.17. With 1,000,000 simulation samples, the resulted reliability indices and the histograms can be measured and presented.

6.7.5 Step 4: Evaluation

The results from the simulation, shown in Table 6.14, are composed of measures of central tendency such as mean values and medians, measures of locations such as the confidence intervals, and measures of dispersion such as variance and skewness. The histogram of some reliability indices are illustrated in Fig. 6.4.

From the confidence interval of the probability of system down time, It can be seen that, although the range of reliability parameters are from 0.5 times to 1.5 times, the resulted effects to the system reliability is still within ±15% from the original estimation. The effect of uncertainty to the system reliability is relatively small.

Conventional Calculation							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Geometric Mean	7.41E+03	6.73E+00	9.99E-01	9.08E-04	1.35E-04	1.35E-04

Simulation Results							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Arithmetic Mean	7.42E+03	7.36E+00	9.99E-01	9.93E-04	1.35E-04	1.35E-04
	Median	7.41E+03	7.33E+00	9.99E-01	9.88E-04	1.35E-04	1.35E-04
Location	80% Confidence						
	- $Min_{10\%}$	6.98E+03	6.76E+00	9.99E-01	8.89E-04	1.27E-04	1.27E-04
	- $Max_{90\%}$	7.89E+03	7.99E+00	9.99E-01	1.10E-03	1.43E-04	1.43E-04
Dipersion	Variance	1.24E+05	2.28E-01	6.97E-09	6.98E-09	4.05E-11	4.05E-11
	S.D.	3.52E+02	4.78E-01	8.35E-05	8.35E-05	6.36E-06	6.36E-06
	Skewness	2.54E-01	2.89E-01	-3.01E-01	3.01E-01	7.23E-03	7.23E-03
	Excess Kurtosis	2.95E+00	3.00E+00	3.10E+00	3.10E+00	2.82E+00	2.82E+00

Table 6.14: Reliability indices of the process control system

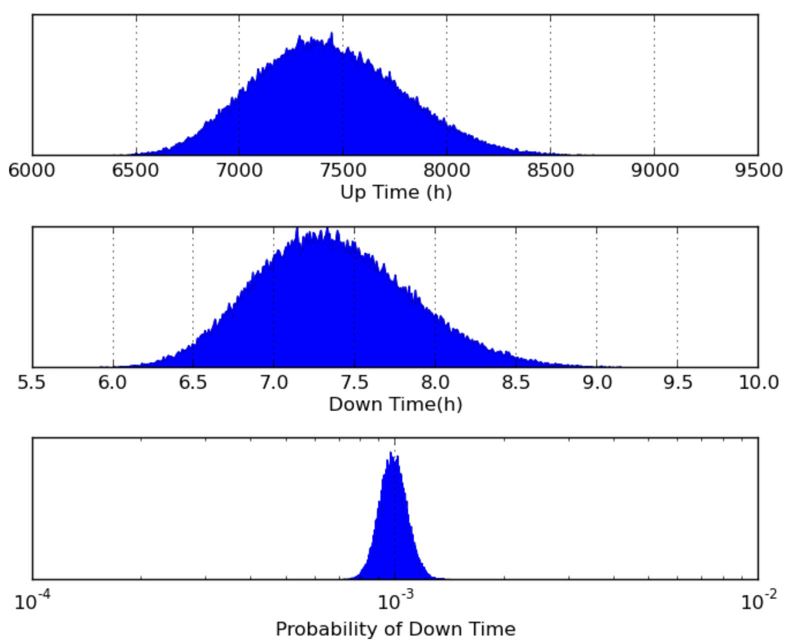


Fig. 6.19: Histograms of the up time, the down time, and the probability of down time of the reliability of the process control system

7 Approach for Early Design Stage

This section will concentrate on the viewpoint of systems, where no or insufficient knowledge according to the described features is available. This is especially in the case of comparing well established and well working systems with new concepts during an early design and development stage.

Regarding the definitions in Chapter 2, the insufficient knowledge is a character of the epistemic uncertainty. The general approach dealing with the epistemic uncertainty is described in Chapter 6, however, the insufficient knowledge limits the use of the empirical probability, and the use of subjective probability may lead to controversial results, the approach incorporating with a priori probability is taken into consideration in the assessment of reliability.

In this chapter, the special case of the epistemic uncertainty, namely the uncertainty at an early stage of system design, is intentionally focused. This approach has been adapted from [Kochs et al. 2012] with an extension in regard to the measurement of uncertainty.

The process of reliability assessment under an early design stage can be divided into 4 steps, illustrated in Fig. 7.1. The approach is applied to three examples of HVDC converter stations in section 7.6.

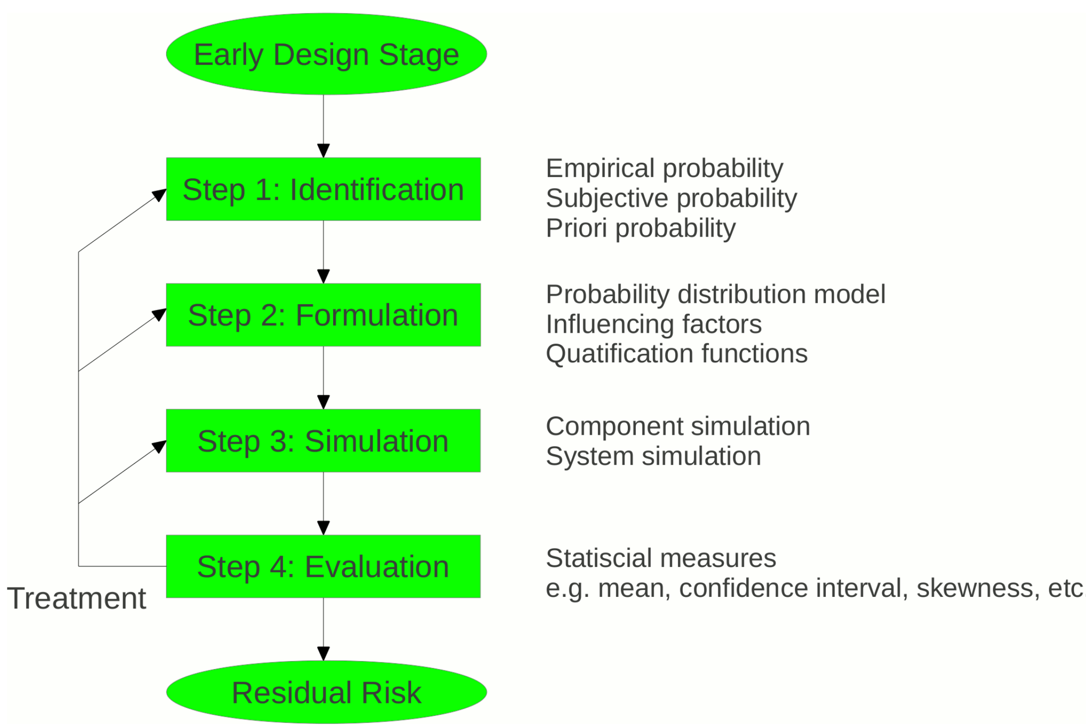


Fig. 7.1: The process of reliability assessment in an early design stage

7.1 Step 1: Identification

In Chapter 6, a general approach dealing with the epistemic uncertainty is mentioned in details. Nevertheless, in the early stage of system design, the reliability knowledge of the system is very limited and cannot be assessed effectively by the standard reliability approaches. The use of the subjective probability may lead to unclear results, and sometimes controversial issues depending on the level of acceptance of the subjective information, such as estimates from experts.

In order to solve this issue, the use of a priori probability is introduced to minimize the doubtful subjective probability.

In this step the system at an early design stage has to be described and compared with the existing systems. The evaluation will be determined in the following steps.

7.2 Step 2: Formulation

In the simple model of component reliability, the failure function and the maintainability distribution function are assumed to be exponential, as mentioned in Chapter 3.3. From this assumption, Eq. 3.23 and 3.24 are derived and can be used to determine other reliability indices in Eq. 3.20 to 3.22. Therefore, the basic reliability parameters of the components are the outage rate λ and the repair rate μ.

Suppose that there are two comparable components. The determination of each component's reliability indices can be done either by using statistical data, i.e. by the empirical probability; or it can be done by expert estimates, i.e. the subjective probability; or by a priori knowledge of the differences between those components. This a priori knowledge or a priori probability can be formulated by **influencing factors** and **quantification functions**.

7.2.1 Influencing Factors

Theoretically, the reliability of each functional-comparable component may differ because of the differences in structure, operating environment, technology, etc. An crucial first step is to identify important factors which may influence the reliability. These factors are called influencing factors.

The influencing factors π can be evaluated qualitatively, e.g. by text description, and quantitatively such as quantities, measurements, or scores.

7.2.2 Quantification Functions

After the identification of influencing factors, each factor has to be quantified by defined functions, called quantification functions. The reliability of all components can be determined from the basic parameters: the outage rate λ, the repair rate μ, and two corresponding quantification functions $f(\ldots)$ and $g(\ldots)$.

Suppose that there are n influencing factors that affect the base outage rate and m influencing factors that affect the base repair rate, the two quantification functions (where C denotes a component of a system S) can be written as the following:

$$\lambda_{S,C} = \lambda_{Base,C} \cdot f(\pi_{\lambda,S,C,1}, \pi_{\lambda,S,C,2}, \ldots, \pi_{\lambda,S,C,n}) \quad (7.1)$$

$$\mu_{S,C} = \mu_{Base,C} \cdot g(\pi_{\mu,S,C,1}, \pi_{\mu,S,C,2}, \ldots, \pi_{\mu,S,C,m}) \quad (7.2)$$

With the mathematical conditions that all influencing factors are independent from each other, the multivariate quantification functions on Eq. 7.1 and 7.2 can be approximated by multiplication of single-variable quantification functions (homomorphism) as the following.

$$\lambda_{S,C} = \lambda_{Base,C} \cdot \prod_{i=1}^{n} f_i(\pi_{\lambda,S,C,i}) \quad (7.3)$$

$$\mu_{S,C} = \mu_{Base,C} \cdot \prod_{i=1}^{m} g_i(\pi_{\mu,S,C,i}) \quad (7.4)$$

Similar conditions are widely used in reliability standards and data sources [CEA 2005, MIL-HDBK-217F 1991, IEC 61709 1996]. If reliability indices of a component C of a system S_1 exists, then the reliability indices of the component of other systems can be estimated by substitution of the base system in Eq. 7.3 and 7.4 with a new system S_2.

$$\lambda_{S2,C} = \lambda_{S1,C} \cdot \prod_{i=1}^{n} \frac{f_i(\pi_{\lambda,S2,C,i})}{f_i(\pi_{\lambda,S1,C,i})} \quad (7.5)$$

$$\mu_{S2,C} = \mu_{S1,C} \cdot \prod_{i=1}^{m} \frac{g_i(\pi_{\mu,S2,C,i})}{g_i(\pi_{\mu,S1,C,i})} \quad (7.6)$$

Therefore, it is not necessary to explicitly calculate the basic parameters λ_{Base} and μ_{Base}, which are not known. Reliability indices can be calculated from these basic parameters as shown in Eq. 3.16 - 3.41.

7.3 Step 3: Simulation

The simulation is not required if all quantification functions can be defined with the knowledge, or the prior reliability information. Nevertheless, if the prior reliability is not sufficient, the subjective probability can be added to the calculation.

Recalling Eq. 7.3 and 7.4, the basic reliability parameters depend on the definition of $f_i(\pi_{\lambda, S, C, i})$ and $g_i(\pi_{\mu, S, C, i})$. If the subjective probability is used, each quantification function may yield an uncertain range due to the epistemic uncertainty. A Monte Carlo simulation, in which each simulation sample emulates a possible nonlinear quantification function, estimates the reliability indices of each sample. The uncertainty can be evaluated from the measures of location and dispersion.

7.4 Step 4: Evaluation

The estimation of reliability indices can be done in the same manner as the conventional reliability approaches. If the subjective probability is used, the underlying uncertainty can be revealed by the measure of location and dispersion. A case study from a real engineering practice is shown in section 7.6. as an example for the simulation and the evaluation steps.

7.5 Summary

The system reliability at an early design stage is modelled as a special case of the epistemic uncertainty. Unlike the reliability approach for epistemic uncertainty in Chapter 6, a priori probability has been adopted for the identification of influencing factors and quantification functions.

With the assumption of exponential distribution, the reliability parameters can be simplified into the outage rate λ and the repair rate μ. A priori probability adjust these parameters for each particular system in an early design stage. The modified parameters are used to determine the reliability indices.

There is still a limit in availability of a priori probability. In practice, it has to be incorporated with the subjective probability. The uncertainty may occur from the nature of the subjective probability, and has to be uncovered by the measurements of location and dispersion, similarly to the approach for epistemic uncertainty in Chapter 6. These measurements can uncover the uncertainty and are useful information for project planning.

In the following section, the case study of reliability evaluation of HVDC converter stations have been extended from [Kochs et al. 2012] in regard to uncertainty.

7.6 Case Study: HVDC Converter Stations

High Voltage Direct-Current (HVDC) transmission is used in long-distance electrical power transmission, and is also an attractive solution for connecting offshore wind-farms with long-distance to shore. Recently, new concepts of HVDC converter stations are developed which have to compete with “conventional” or “classical” HVDC converter stations. In this case study, it is focused on the estimation and the comparison of the reliability of different HVDC converter stations.

7.6.1 Overview of Modern HVDC Converter Stations

There are a number of HVDC converter stations which are focused in this case study, namely:

- HVDC LCC, converter station with Line-Commutated Converter
- HVDC VSC, converter station with Voltage-Source Converter
- HVDC INT, converter station with integrated power electronics.

HVDC LCC is well established and has been operated since many decades and its statistical reliability data is widely available, and HVDC VSC has been constructed and operated in recent years and has relatively limited statistical data. HVDC INT, on the contrary, is in an early stage of system design, where only prototypes exist and no statistical data is available. One crucial step in this paper is to develop common HVDC reliability structures in order to compare the reliability of the different HVDC converter stations.

7.6.2 Step 1: Identification

7.6.2.1 HVDC LCC

HVDC LCC or the “classical” HVDC is the most common approach to HVDC transmission system [Arrillaga 1998, Astrom et al. 2005, Gönen 2008, Kundur 1993, Rendina et al. 2008]. Regardless of different configurations, they share similar basic components. A typical functional structure of HVDC LCC converter station concept is illustrated in Fig. 5.11, a simplified structure is shown in Fig. 7.2a and labeled as LCC-1.

If protection from hazardous environment, such as humidity, salt (e.g. in off-shore area), and sand, is a major concern of an HVDC LCC converter station, components are put indoors or are encapsulated in the control environment. One alternative is using encapsulated AC switchgear [Rendina et al. 2008] and use indoor DC

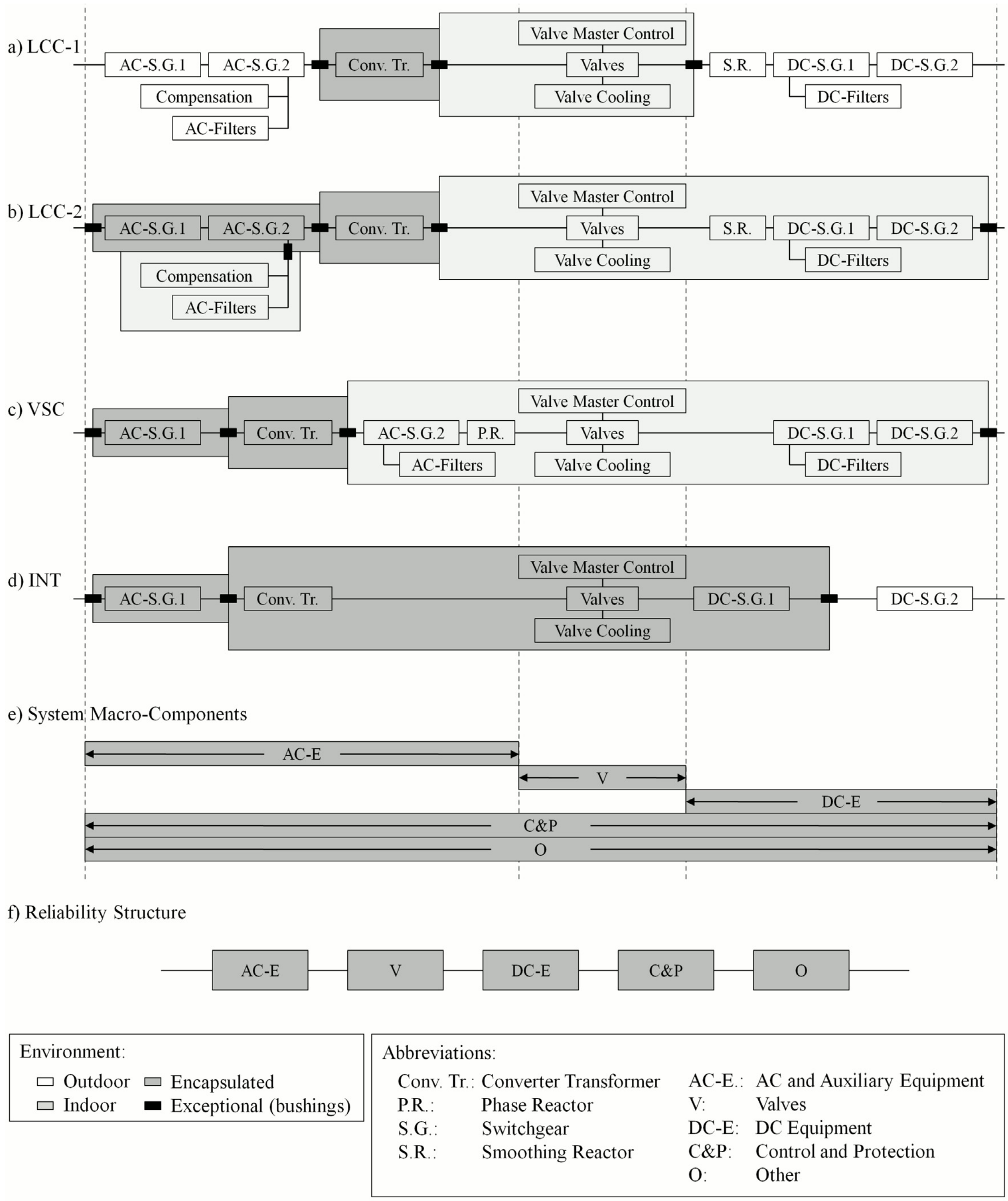


Fig. 7.2: System structure and components
a) HVDC LCC-1 converter station
b) HVDC LCC-2 converter station
c) HVDC VSC converter station
d) HVDC INT converter station
e) System macro-components based on CIGRE
f) Serial reliability structure based on CIGRE

switchgear [Astrom et al. 2005]. This alternative structure is illustrated in Fig.7.2b and labeled as LCC-2.

For HVDC LCC statistical data exist from CIGRE. CIGRE defines the following four macro-components: AC-E, V, DC-E and C&P (see Fig. 7.2e), for which statistical data are available. These components are the common basis for the other converter stations.

7.6.2.2 HVDC VSC

In the mid 1990s, another HVDC concept called HVDC VSC has been developed [Andersen 2005, Arrillaga 1998, Bahrman 2007, Jacobson et al. 2005, Linden et al. 2010]. The voltage level is generally divided into two to three levels and the concept is named differently among the manufacturers. A simplified diagram of HVDC VSC converter station is illustrated in Fig. 7.2c labeled as VSC.

HVDC VSC is relatively new comparing to HVDC LCC and little information is available, especially the reliability data which will be further mentioned in this paper.

Voltage-source converters are also used in another new HVDC concept called HVDC MMC (Modular Multilevel Converter), which enables the higher number of voltage levels [Friedlich 2010, Li et al. 2010]. The reliability estimation of HVDC MMC converter station is not included in this work.

7.6.2.3 HVDC INT

HVDC INT (or HVDC Integrated) is an HVDC concept which is intended to reduce the requirement of filters, as well as to reduce the size of HVDC converter station and to integrate power electronics [Lutz et al. 2007]. The structure of HVDC INT is illustrated in Fig.7.2d. The main differences are mentioned briefly: Compensation or reactive power compensation, AC-Filters and DC-Filters are not required. Valves are based on IGBT like HVDC VSC. The valves are encapsulated in the same container as converter transformers, together with the DC components. Valve Cooling is integrated into the same container as converter transformers. Worth mentioning is the complete encapsulation of the components, outlined in Fig. 7.2d.

Whereas HVDC LCC and HVDC VSC are well established converter stations in operation over many years, HVDC INT is a concept (and prototypical realized) at an early design stage.

7.6.3 Step 2: Formulation

For reliability evaluation of the different HVDC substations, the five macro-components (Fig. 7.2f) are taken as a common basis for all three substations, which differ in their influencing factors.

7.6.3.1 Influencing Factors

The factors which influence the HVDC converter station's reliability are categorized into the following two types of influencing factors:

1. The structure factor which denotes influences of complexity-features or structural/physical differences of a component.
2. The protection factor which denotes influences of the protection mechanisms from hazardous environment of a component. The level of protection is classified into four classes: outdoor, indoor, encapsulated, and exceptional (bushings) (see Fig. 7.2).

Because each HVDC converter station has established a set of procedure for repair, replacement, and maintenance, which takes approximately the similar amount of time, therefore the same repair rates are used for all systems and only the influencing factors of the outage rates are considered.

Six years of statistical operating data of HVDC LCC [Vancers et al. 2004, Vancers et al. 2006, Vancers et al. 2008] are used for the macro-components (Fig. 7.2): AC-E, V, DC-E, C&P, and O, which are connected in series in a logical diagram (Fig. 7.2f).

The system reliability model with the corresponding macro-components regarding Eq. 7.5 and 7.6 is illustrated in Fig. 7.3. The influencing factors in this HVDC substation comparison are classified by scores from "lower/worse" to "higher/better", which is numerically described from -2 to +2 for simplicity, i.e. $\pi \in \{-2, -1, 0, +1, +2\}$. A simple set of rules is used to define the score of each influencing factor. This set of rules is as follows:

- The scores of LCC-1 will be considered as a reference, where the statistical data is taken (CIGRE data), and the score is set to 0 by default.
- The scores will be in a range of -2 (worse/lower) to +2 (better/higher), respectively any graduation between them depending on the level of knowledge/information.
- If the macro-component under consideration is available in the reference structure but not in the other structure, the score +2 is given to the latter structure (missing components cannot fail).
- If the macro-component under consideration is not available in the reference structure but is available in the other structure, the score -2 is given to the latter structure.

With the numerical scores of the influencing factors from -2 (worse) to +2 (better), respectively any graduation between them depending on the level of knowledge/information, it is generally possible to consider and compare different complexity characteristics of the components with respect to reliability.

Because of the defined influencing factors, the scores designate the improvements or deterioration in reliability/availability of a component. A higher positive score means a lower outage rate, and vice versa. Therefore, the quantification functions in Eq. 7.1 to 7.6 are monotonically decreasing, see Fig. 7.4 in the next section. All influencing factors and corresponding scores are shown in Table 7.1, which is the result of a priori information and the analysis of literature.

The systems under consideration are LCC-1, LCC-2, VSC, and INT. With all identified influencing factors, the reliability indices can be estimated with the quantification functions, as illustrated in Fig. 7.3.

Component	Influencing Factor	LCC1		LCC2		VSC		INT	
		Structure	Protection	Structure	Protection	Structure	Protection	Structure	Protection
AC-E	AC-Switchgear 1	0	0	0	+1	0	+1	0	+1
	AC-Switchgear 2	0	0	0	+2	+1	+1	+2	+2
	Compensation	0	0	0	+1	+2	+2	+2	+2
	AC-Filters	0	0	0	+1	+1	+1	+2	+2
	Bushings (AC)	0	0	-2	0	-1	0	0	0
	Converter Transformer	0	0	0	0	+1	0	+1	0
	Phase Reactor	0	0	0	0	-2	-2	0	0
V	Valves	0	0	0	0	0	0	0	+1
	Valves Master Control	0	0	0	0	0	0	+1	+1
	Valve Cooling	0	0	0	0	0	0	+1	+1
DC-E	Bushings (DC)	0	0	0	0	0	0	0	0
	Smoothing Reactor	0	0	0	+1	+2	+2	+2	+2
	DC-Switchgear 1	0	0	0	+1	0	+1	+1	+2
	DC-Switchgear 2	0	0	0	+1	0	+1	0	0
	DC-Filters	0	0	0	+1	0	+1	+2	+2
C&P	Control and Monitoring Wiring	0	0	0	0	0	0	+1	0
	SCADA System	0	0	0	0	0	0	0	0
O	*no influencing factor is found*	-	-	-	-	-	-	-	-

Table 7.1: Summary of influencing factors and scores

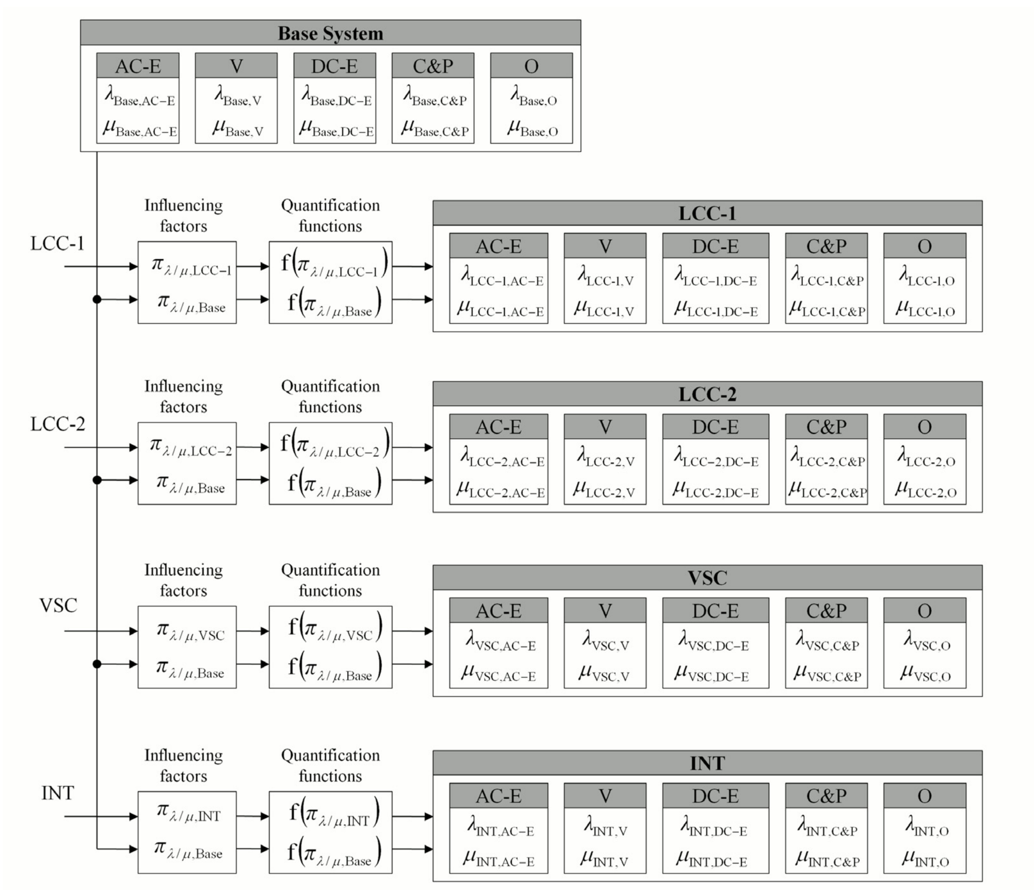


Fig. 7.3: Model of comparable macro-components

7.6.3.2 Quantification Functions

The influencing factors are quantified and assembled with quantification functions of each component. Little information is found whether how much each influencing factor would affect the reliability of the corresponding component, e.g. how much an AC-filter affects the reliability of AC-E. In the previous work, only a particular linear quantification function based on an existing evidence [CIGRE Work Group 23-02 2000] is used as an example, and all quantification functions are assumed to be linear.

In this case study, however, it is focused as if the quantification functions are nonlinear, and has a level of uncertainty which is caused by subjective estimates. It is assumed that a component with +2 score can be at most 2 times more reliable than the comparable component with +0 score. And the component with -2 score is 2 times less reliable than the comparable score. Each quantification function $f_i(\lambda)$ can be dissimilar. The possible range of each nonlinear quantification is illustrated in Fig. 7.5.

The influencing factors and the quantification functions are used in the reliability simulation.

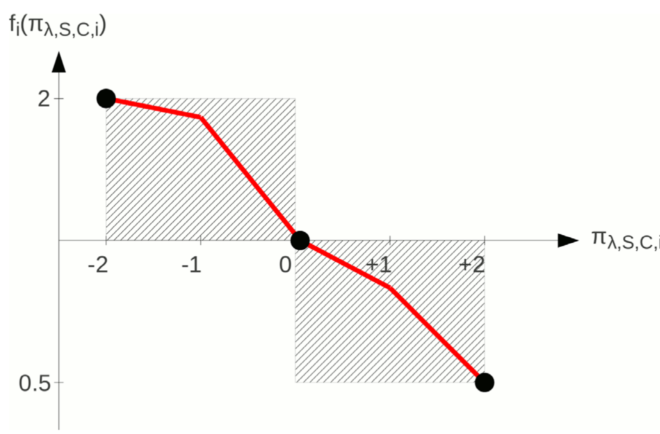


Fig. 7.4: Range of the nonlinear quantification function

7.6.4 Step 3: Simulation

A Monte Carlo simulation [Hazewinkel 2001, Russell et al. 2003] simulates system reliability indices under different possibilities of quantification functions. In each simulation sample, a nonlinear quantification function is generated from the possible range. The reliability indices of each simulation sample are estimated according to Eq. 3.27 to 3.31. The results of all simulation samples are compiled and measured by location and dispersion, as shown in Table 7.2 and Fig. 7.5. The rest of the simulation procedure is similar to the approach for epistemic uncertainty in Chapter 6.3.

7.6.5 Step 4: Evaluation

With 1,000,000 Monte Carlo samples, the measurements of different HVDC converter stations' reliability indices are estimated.

In the case of LCC-1, the reliability indices are calculated according to the reliability data source, and the epistemic uncertainty of LCC-1 is not considered. For LCC-2, VSC, and INT, the subjective identification of quantification functions causes the epistemic uncertainty. This epistemic uncertainty can be illustrated in a similar manner as Chapter 6. The reliability results are shown in Table 7.2.

In Fig. 7.5, the probability of system down time of each HVDC converter station are compared. It can be seen that, in average, INT is the most reliable regarding the probability of down time. LCC2 and LCC1 are the second and the third most reliable, respectively. VSC is the least reliable with very high uncertainty. However, in the optimistic case of 80% confidence interval, VSC is more reliable than LCC2. In the pessimistic case, LCC2 is even less reliable than LCC1. The measure of location reveals this uncertainty information to the decision maker in order to determine the proper converter station for any specific application.

The uncertainties can be diminished if more reliability information is found and quantification functions are revised according to the information.

LCC-1							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Mean = Median	2.16E+03	2.61E+01	9.88E-01	1.19E-02	4.57E-04	4.57E-04

LCC-2							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Mean	2.54E+03	2.34E+01	9.90E-01	9.98E-03	4.06E-04	4.06E-04
	Median	2.46E+03	2.39E+01	9.90E-01	9.65E-03	4.03E-04	4.03E-04
Location	80% Confidence						
	- $Min_{10\%}$	1.89E+03	1.53E+01	9.84E-01	4.53E-03	2.96E-04	2.96E-04
	- $Max_{90\%}$	3.36E+03	3.06E+01	9.95E-01	1.60E-02	5.21E-04	5.21E-04
Dipersion	Variance	2.99E+05	3.22E+01	1.79E-05	1.80E-05	6.91E-09	6.91E-09
	S.D.	5.47E+02	5.67E+00	4.24E-03	4.24E-03	8.31E-05	8.31E-05
	Skewness	5.16E-01	-3.01E-01	-3.54E-01	3.54E-01	2.26E-01	2.26E-01
	Excess Kurtosis	2.48E+00	2.35E+00	2.41E+00	2.41E+00	2.30E+00	2.30E+00

VSC							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Mean	2.61E+03	2.40E+01	9.87E-01	1.26E-02	4.44E-04	4.44E-04
	Median	2.59E+03	2.39E+01	9.91E-01	9.14E-03	3.83E-04	3.83E-04
Location	80% Confidence						
	- $Min_{10\%}$	1.34E+03	9.68E+00	9.72E-01	2.48E-03	2.56E-04	2.56E-04
	- $Max_{90\%}$	3.90E+03	3.87E+01	9.98E-01	2.82E-02	7.28E-04	7.28E-04
Dipersion	Variance	8.67E+05	1.10E+02	1.21E-04	1.21E-04	4.10E-08	4.10E-08
	S.D.	9.31E+02	1.05E+01	1.10E-02	1.10E-02	2.02E-04	2.02E-04
	Skewness	-2.39E-02	1.05E-01	-1.60E+00	1.60E+00	1.59E+00	1.59E+00
	Excess Kurtosis	1.95E+00	1.94E+00	5.73E+00	5.73E+00	5.75E+00	5.75E+00

INT							
Measure	Index	$Ti(U_s)$ (h)	$Ti(D_s)$ (h)	$Pr(U_s)$	$Pr(D_s)$	$Fr(U_s)$ (1/h)	$Fr(D_s)$ (1/h)
Central Tendency	Mean	4.43E+03	1.22E+01	9.96E-01	3.61E-03	2.51E-04	2.51E-04
	Median	4.30E+03	1.06E+01	9.98E-01	2.40E-03	2.32E-04	2.32E-04
Location	80% Confidence						
	- $Min_{10\%}$	2.59E+03	4.67E+00	9.91E-01	7.30E-04	1.54E-04	1.54E-04
	- $Max_{90\%}$	6.50E+03	2.27E+01	9.99E-01	8.70E-03	3.83E-04	3.83E-04
Dipersion	Variance	2.09E+06	4.45E+01	9.29E-06	9.31E-06	7.01E-09	7.01E-09
	S.D.	1.45E+03	6.67E+00	3.05E-03	3.05E-03	8.38E-05	8.38E-05
	Skewness	4.90E-01	5.15E-01	-1.07E+00	1.07E+00	6.56E-01	6.56E-01
	Excess Kurtosis	2.58E+00	2.00E+00	3.04E+00	3.04E+00	2.54E+00	2.54E+00

Table 7.2: Reliability indices of different HVDC converter stations

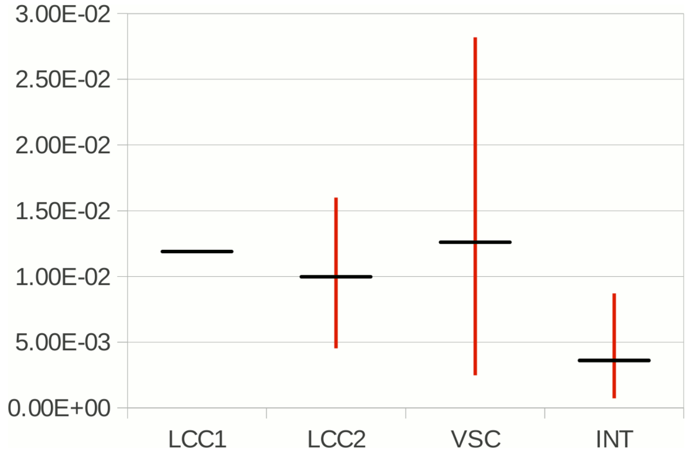


Fig. 7.5: Probability of system down state with mean values and confidence intervals

8 Summary

The study was set to explore the uncertainty and its effects in the reliability evaluation. Different types of uncertainty have been identified. The study has also sought to estimate the reliability under the presence of uncertainty. Advantages and disadvantages of the standard reliability approaches have been discussed. The general literature on this subject is inconclusive on several vital questions. The study sought to answer two of these questions:

I. What are the sources, types, and effects of uncertainty on reliability?
II. How to evaluate and measure the reliability under the presence of uncertainty?

8.1 Empirical Findings

The main empirical findings are divided into chapters and are synthesized in this section to answer the study's two research questions.

I. What are the sources, types, and effects of uncertainty on reliability?

A. The sources of uncertainty is generally defined into two sources: indeterminacy and incompleteness. There is an unclear border between these two sources (see Section 2.4).

B. The effects of uncertainty, also called risks, is the perspective of uncertainty from the project management. The presence of risks has to be managed and, if possible, minimized.

C. The classification of uncertainty by sources and effects are goal-oriented, and practical in reality. In the theoretical viewpoint, uncertainty is usually categorized by its probabilistic properties as the aleatory uncertainty, and the epistemic uncertainty.

In Chapter 2, definitions regarding uncertainty, as well as reliability and many related terms, are mentioned in details.

II. How to evaluate and measure the reliability under the presence of uncertainty?

A. The standard reliability approaches usually focus only on the determination of the mean values of the reliability indices. The uncertainty is often unidentified, thus is left unrevealed by the evaluation. The measures of location and dispersion are often unregarded.

B. Statistical and probability theories are powerful mathematical tools. Uncertainty in general can be modelled by these theories.

In Chapter 3, the fundamentals of statistical concept, probability concept, and reliability concept are mentioned. The existing views and approaches of the subject are mentioned in Chapter 4.

8.2 Theoretical Implication

The approaches of reliability evaluation need to be revised in order to apply to the consideration of uncertainty. Although there are a number of existing works in the subject, the approaches of these works and the aspect of uncertainty are not fully categorized.

A series of reliability study [Coit et al. 2004, Coit et al. 2009, Tekiner et al. 2011] mentioned the use of the measures of location and the measures of dispersion. The usage of empirical probability has been extensively applied. However, the studies focused on the aleatory uncertainty. The epistemic uncertainty, and the use of subjective probability and a priori probability to overcome limited reliability data, are not mentioned.

On the other hand, epistemic uncertainty has been mentioned in [Limbourg 2008, Limbourg et al. 2006, Limbourg et al. 2007, Rocquigny 2008] in the form of a subjective probability. Still, the consideration of aleatory uncertainty and other types of probabilities have been neglected. The results are evaluated by the central tendency of reliability indices. The measure of central tendency can show the existence of uncertainty, but it can hardly represent the level or amount of uncertainty in the results.

In [Heard et al. 2006, Kochs et al 2012, Kongniratsaikul 2009], a priori probability and subjective probability have been used with empirical probability to evaluate the reliability with epistemic uncertainty. The measure of central tendency and the measure of locations have been calculated. Nevertheless, the consideration of aleatory uncertainty is not mentioned.

Because of the inconclusive and incomplete of the literature of this subject, a framework has been proposed in Chapter 4 to classify and to evaluate the reliability under the presence of each type of uncertainty. In Chapter 5 and 6, the reliability approach for aleatory uncertainty and epistemic uncertainty have been mentioned in details.

Furthermore, in an early stage of system design, the reliability evaluation is usually incorporated with a priori probability. [Frank 1995, Frank 1996] uses a logical analysis of system design to estimate the possible reliability. This special case of the reliability evaluation is common in practice. Therefore, the approach for reliability evaluation under the presence of uncertainty in an early design stage has been introduced in Chapter 7.

Under the proposed framework, the uncertainty in the reliability evaluation can be evaluated. This important information can improve the degree of confidence in the utilization of reliability studies for critical engineering systems, such as power systems, computer systems, automation systems, etc.

8.3 Policy Implication

This work points the fact that uncertainty has a considerable amount of impacts to the system reliability. A number of recognizable standards have defined the basic terms regarding uncertainty and reliability in [IEC 60050-191 1990, Laprie 1992, MIL-HDBK 217F 1991], and the general guidelines suggesting the process of uncertainty/risks management are available [ISO 31000 2009, ISO Guide 73 2009]. This study have realized the guidelines into a set of practical approaches, as illustrated in Fig. 8.1. The basis on standards and guidelines ensures the quality and compatibility of this study for business and industrial application.

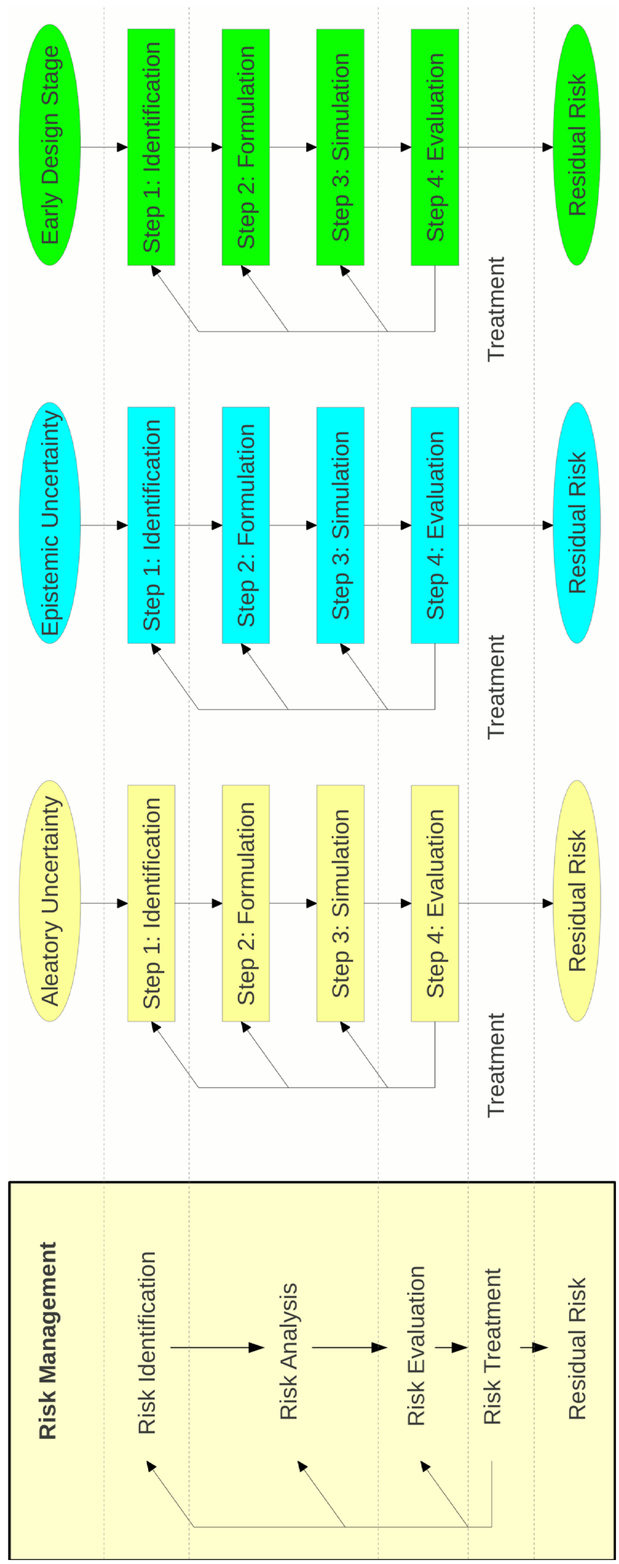


Fig. 8.1: Comparison of the proposed framework with the guideline of risk management process [ISO 31000 2009, ISO Guide 73 2009]

8.4 Limitation of the Study and Recommendation for Future Research

The study has offered a reliability evaluation framework which considers and represents the presence of uncertainty. The study has encountered a number of limitations, which need to be considered.

First, because there is no clear distinction between the aleatory uncertainty and the epistemic uncertainty in practice, the consideration of either one uncertainty type may lead to the presence of another type. Although the awareness of the remaining uncertainty is explicitly mentioned as the residual risk in the framework, it is possible to minimize the residual risk if both the aleatory uncertainty and the epistemic uncertainty are considered. It is recommended for the future research to consider both the aleatory uncertainty and the epistemic uncertainty in the reliability evaluation.

Second, in this study, the measures of system reliability are determined by computer simulations. The use of simulations may lead to numerical errors and this may be counted as another source of uncertainty. This uncertainty is remained as the residual risk of the reliability evaluation. In the future research, the analytical solution of the measures of system reliability, such as confidence interval, variance, and skewness, should be determined and be simplified for a practical implementation.

Lastly, the proposed framework have been applied in a number of reliability evaluation approaches such as the reliability block diagram, the series/parallel approach, and the minimal cut approach. In the future research, the framework can be adapted for other reliability evaluation approaches, such as the Markov process approach, and the probable minimal cut approach, the semi/non Markov process approach, etc.

8.5 Conclusion

In contrast to the conventional reliability evaluation approaches, where the presence of uncertainty is usually not mentioned, this study explicitly identifies the types of uncertainty and proposes a framework to measure and represent each type of uncertainty. This framework is designed to be compatible with the risk management guidelines, and has been applied to a number of case studies from the real engineering practices. The framework have revealed and represented the uncertainty of these case studies, which have been previously hidden from the assessment of reliability. The information of uncertainty can improve the degree of confidence in the utilization of reliability studies for critical engineering systems.

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