Stability-Oriented Dynamics and Control of Complex Rigid-Flexible Mechanical Systems Using the Example of a Bucket-Wheel Excavator

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Kurzfassung

Der Schwerpunkt dieser Arbeit liegt auf der Modellbildung und der Regelung von Schaufelradbaggerauslegern. Die Schaufelradbagger stellen eine besondere Art komplexer Maschinensysteme dar, die im Braunkohletagebau eingesetzt werden. Der Schaufelradausleger ist hierbei als dreidimensionaler elastischer Balken nach der EULER-BERNOULLI Balken-Hypothese modelliert. Durch den Erhalt von Termen höherer Ordnung in den nichtlinearen Relationen zwischen Verschiebung und Verzerrungen, sind Kopplungseffekte höherer Ordnung der gesamtheitlichen Verschiebung und der flexiblen Deformation mitberücksichtigt. Bei der Modellierung der geometrischen Nichtlinearität des dreidimensionalen elastischen Balkens wurde weiterhin die zusätzliche Elastizität von Hebekabeln miteinbezogen. Komplexere Bewegungen, speziell die geführte Bewegung in Kombination mit Grabkräften wurden aufgezeigt und diskutiert. Die Elastizität des Auslegers wurde in Bezug auf die Interaktion zwischen Schneidewerkzeug (Baggerschaufel) und Oberflächenmaterial berücksichtigt. Einflüsse von Kopplungen höherer Ordnung zwischen flexiblen Deformationen, Förderseilen und Grabgegenkräften auf das dynamische Verhalten des Schaufelradauslegers werden mithilfe intensiver Simulationsstudien dargestellt. Dynamische Phänomene, die sich aus den geometrischen und dynamischen Kopplungen höherer Ordnung ergeben, die der geführten Bewegung und den Grabgegenkräften ausgesetzt sind, wurden im Detail analysiert. Die destabilisierenden Einflüsse, die zu großen Deformationen des Systems führen, beruhen auf den oben genannten Kopplungen, werden in den Simulationsergebnissen gezeigt. Das entwickelte Model sowie die damit verbundene Abbildung des dynamischen Systems liefert somit eine gute Basis für weitere Untersuchungen der Systemstabilität in Zusammenhang mit den Grabgegenkräften.

Das nichtlineare dynamische System des Schaufelradauslegers wird durch ein erweitertes lineares System mit Nichtlinearitäten eines passenden fiktiven Modells für die Ansteuerungsanalyse und Designzwecke approximiert. Ein PI-Beobachter wird basierend auf diesem erweiterten linearen System eingesetzt, der alle Zustände des Systems schätzen und das Zeitverhalten der Nichtlinearitten rekonstruiert. Von diesem Standpunkt aus ist die beobachtergestützte PI-Zustandsregelung in Kombination mit einer Störungs- Kompensationsregelung realisiert. Drei Störungs- Kompensationsregelungsansätze bestehen aus dem statischen Ansatz, dem Davison Ansatz und dem erweiterten Ansatz nach dem Davison wurden zur Kompensation der Nichtlinearitäten diskutiert. Anhand von Simulationsbeispielen wird die effiziente Unterdrückung von Vibrationen und der Systemstabilisierung des Schaufelradbaggers während des Grabprozesses gezeigt. Die Ergebnisse zeigen, dass der Davison Ansatz und der erweiterte Ansatz nach dem Davison die dynamische Verbesserung des Schwingungsverhaltens sowie Stabilisierung des Schaufelradbaggers gewähren können. Demnach kann die Produktivität und somit die Ertrag des Schaufelradbaggers erhöht werden.

Abstract

The focus of this thesis is the modeling and control of the boom of the Bucket-Wheel excavator, which represents a specific type of complex machine systems used in mining technology. Hereby the Bucket-Wheel boom is modeled as the three-dimensional flexible beam using the Euler-Bernoulli beam theory. Retaining higher-order terms in the nonlinear strain-displacement relationship, higher-order coupling effects between the overall motion and flexible deformations are considered in the modeling. Furthermore, the nonlinear modeling of the three-dimensional elastic boom is also considered with the additional elasticity of hoisting cables. More complex motions, especially the guided motion in combination with digging resistance forces, are mentioned and discussed. So far, the elasticity of the boom along with the interaction between the cutting head and the face material is taken into account. The effects of higher-order couplings between flexible deformations, hoisting cables, and digging resistance forces on dynamical responses of the Bucket-Wheel boom are illustrated by intensive simulation studies. Dynamic phenomena resulting from higher-order geometrical and dynamical couplings undergoing the guided motion and digging resistance forces are therefore analyzed in detail. The destabilizing effects leading to large deformations (may be critical) of the system due to the above mentioned couplings are shown in simulation results. Thus, the developed model as well as the related dynamic system representation gives a good base for the advanced study of the stability of the system in combination with the digging resistance forces.

For control analysis and design purposes, the nonlinear dynamical system of the Bucket-Wheel boom is approximated by the extended linear system with nonlinearities modeled by a suitable fictitious model. Based on this extended linear system, a high-gain PI-Observer is applied to estimate all states of the system and to reconstruct the time behavior of the nonlinearities. From this point of view, a high-gain PI-Observer-based state feedback control is realized in combination with disturbance rejection control approaches. Three disturbance rejection control approaches including the static disturbance rejection control approach, Davison approach, and the extended approach of Davison are discussed for compensating nonlinearities. Simulation examples are included to illustrate the efficient suppression of vibrations as well as the stabilization of the system during the digging process of the Bucket-Wheel Excavator. The results show that the static disturbance rejection control approach cannot stabilize the system, while Davison approach and the extended approach of Davison can stabilize successfully the system with the suitable dynamic feedback terms. Consequently, application of these approaches can improve operating ranges of the Bucket-Wheel excavator. Therefore, an exploitation productivity of the Bucket-Wheel excavators can be increased.

Contents

Symbols and Abbreviations

List of Symbols

e	Vector of the nodal coordinates of the element
f_x, f_y, f_z	Vector of shape functions
$f_{g1},\ f_{g2}$	Vector of external forces and moments
f_{gg}	Vector of gravitational effects
$l_{GE'}^{(G)}$	Joint oriented boundaries of the considered finite element
$n(x(t), t)$	Nonlinearities of the dynamic system
$u_{EP'}^{(G)}$	Vector of the elastic deformations
$u_{EP'}^{(G)}$	Vector of the elastic deformations
u_x, u_y, u_z	Logitudinal, bending deformations
$u_x^{'}, u_y^{'}, u_z^{'}, \alpha'$	differentiation with respect to the x-coordinate of flexible deformations
$u(t), \dot{u}(t), \ddot{u}(t)$	Vector of displacements, deflections, and its derivatives of the elastic motion
r	Position vector of a finite mass point
$r_{OG}^{(I)}$	Position vector of the joint
$r_{GP}^{(I)}$	Position vector of the mass point from the joint
$\delta r_{OP}^{(I)}$	Virtual displacement
$\ddot{r}_{OP}^{(I)}$	Accelerator vector of the virtual displacement
$s_{E'E}^{(G)}$	Position of the undeformed disc in the element coordinate system
x, y, z	Physical coordinates of the element
$y(t)$	Vector of measurements
$z(t)$	Control input
$x(t)$ and $v(t)$	States of the extended linear system
$\hat{x}(t),\ \hat{v}(t)$	Estimation states of the extended linear system
α	Torsional deformation
ϵ	Longitudinal strain
$\epsilon_x,\ \epsilon_y,\ \epsilon_z$	Extension of the beam
$\epsilon_{1..6}$	Six independent strain components of the beam
$\delta\epsilon, \delta\kappa_1, \delta\kappa_2, \delta\kappa_3$	Virtual displacements of the elastic variables
γ_{yz}	Deformation of the cross section
γ_{xy}, γ_{xz}	Transverse shear deformations
$\kappa_1, \kappa_2, \kappa_3$	Curvatures of a 3D curve
$\omega_1,\ \omega_2,\ \omega_3$	Angular velocities
$\tilde{\omega}$	Skew symmetric matrix

Φ, Θ, Ψ	Three Tail-Bryan angles
T, $\hat{T}$	Rotation matrices
F^C	Tension force vector of the hoisting cable
F_T	Tangential digging force
F_N	Lateral digging force component
F_B	Side digging force component
$Hv(t)$	Additional variables representing fictitious approximations of nonlinearities

Abbreviations

3D	Three Dimensional
ANCF	Absolute Nodal Coordinate Formulation
BWE	Bucket-Wheel Excavator
CAD	Computer Aided Design
FFRF	Floating Frame of Reference Formulation
ICADA	Integration of Computer Aided Design and Analysis
LQR	Linear Quadratic Regulator
NF1	First Nonlinear Modeling
NF2	Second Nonlinear Modeling
NF3	Third Nonlinear Modeling
NURBS	Non-Uniform Rational B-Splines
PI-Observer	Proportional-Integral-Observer

1 Introduction

1.1 Motivation

Flexible structures are used widely in mechanical systems such as space manipulators, helicopter propellers, turbine blades, booms of cranes, etc. With the increasing demand for an exploitative productivity, developmental trends of machines are a gain in size, mass, strength, and velocity or a combination of some motion mechanisms in order to shorten the cycle time. During the operation of machines it can cause vibrations and undesirable deformations. Vibrations caused by elastic deformations are a serious problem in mechanical systems. They can affect normal working conditions, operational effectiveness, and lead to fatigue, failure, and the instability of the structure. Therefore, the vibration analysis is necessary to predict the dynamic behavior of flexible structures under real working conditions. They have received special attention in structural dynamics.

In order to eliminate these vibrations or reduce them as much as possible, many control and design approaches for flexible structures are successfully proposed. However, the obtained accuracy resulting from the vibration analysis depends on a mathematical model of the structure. A simplified mathematical model cannot capture the significant dynamical phenomena as well as related effects. Thus, it leads to inexact dynamic responses of a real system undergoing different working conditions. On the other hand, a fully mathematical modeling usually results in the complexity to analyze and synthesize controller for the above system. Consequently, an adequate modeling to represent the dynamic behavior of a real structure is necessary. For flexible structures operating at high angular velocities, effects geometric stiffness will become significant, causing changes of stiffness, possibly resulting in unstable situations and also changing the natural frequencies of the system. In addition, new structures used in practical applications of flexible structures become larger, lighter, and slender. The problem of large elastic deformations undergoing overall motion in these structures has to be considered in the dynamic modeling. Effects of higher-order couplings of flexible deformations combining nonlinear kinematics and kinetics need to be also investigated. Therefore, it is important to take into account all above mentioned terms in order to get an adequate modeling for describing the dynamic behavior of flexible structures.

By changing the stiffness of the structure or adding damping, the conventional approaches, known as a passive structural vibrations control, have been successfully applied in reduction of vibrations of flexible structures to external excitations. However, these approaches usually lead to significant increase in mass and therefore affect the cost of building structures. With rapid development in the mechatronic field, nowadays, advanced control devices can suitably be placed on the system to supply forces for controlling vibrations of the structure or to change critical parameters, inputs for adapting with changing characteristics of the system to different working conditions. From this point of view, obviously, advanced control systems, known as an active structural vibrations control, will be widely applied controlling the dynamics of the flexible structures.

Due to the advantages of a high excavation productivity (from 420 to 22700 m^3 of overburden per hour) and low production costs, large mining machines like Bucket-Wheel excavators (BWE), which represent a specific type of complex machine systems, are widely used in coal mining industry and become one of the most important heavy machines in opencast mining. The BWE is a favorable combination of complex spatial structures, like the Bucket-Wheel, slewing, hoisting mechanism, crawlers, and electric equipment. The operation of the BWE is carried out by combining continuous rotation of the Bucket-Wheel in the vertical plane with the main slewing mechanism in the horizontal plane. During operation, the system is exposed to a number of external forces and disturbances like digging resistances on the Bucket-Wheel that cause transverse, longitudinal, and torsional vibrations. All vibrations will affect to normal working conditions, operational effectiveness, and may under specific conditions also affect the stability of the BWE. The dynamic behavior of this system is complicated due to coupling effects of flexible deformations as well as combined effects of rotating motion undergoing digging process. Thus, the analysis of the dynamic behavior of the BWE during operating is necessary in order to understand effects of related dynamic phenomena as well as reasons leading to the resonance. Based on the modeling of the BWE, advanced control approaches can be proposed controlling the vibrations of the BWE. As a result, the efficient suppression of vibrations as well as the stabilization of the system are obtained during the digging process of the Bucket-Wheel Excavator. Therefore, operating ranges of the BWE can be improved.

1.2 Aims of this work

This work focuses on modeling and control of a boom of the Bucket-Wheel excavator. The Bucket-Wheel boom can be modeled as a flexible beam using the Euler-Bernoulli beam theory. Additionally, it is assumed that the boom is attached to the excavator turning platform. The nonlinear modeling of the three-dimensional elastic boom considering the elasticity of suspending cables and also couplings resulting from geometrical nonlinear deformations is presented. Here, the known modeling approach of higher order is used and extended to model the Bucket-Wheel boom of a Bucket-Wheel Excavator including guided rotating motion in combination with digging resistance forces.

Based on the nonlinear modeling of the Bucket-Wheel boom, the time behavior of nonlinearities (as additive effects in relation to the linearized system) is estimated in combination with related system states using a high-gain PI-observer. Then, the well-known disturbance rejection control approaches, often used for control of nonlinear mechanical systems, such as static and dynamic disturbance rejection approaches are applied controlling vibrations of the Bucket-Wheel boom. Efficiency of these approach in suppressing vibrations as well as stabilization of the system during the digging process of Bucket-Wheel Excavator is discussed and compared through intensive simulation studies.

1.3 Organization of this thesis

This thesis is arranged into five chapters.

The focus of this thesis is to model and control complex flexible structures representing the Bucket-Wheel Excavators. Therefore, fundamentals about modeling and control of flexible structures are essential to be described. The second chapter reviews briefly the most two widely used formulations to study the dynamics of the flexible structures with the defined coordinate systems, the derivation of the elastic forces using an elastic line approach, and a continuum mechanics approach, etc. Based on these formulations, problems in the dynamical analysis of flexible structures are also reviewed as the dynamic coupling problem of flexible structures undergoing overall large motions and arbitrary elastic deformations, the large deformation problem, and the choice of shape functions with contributions published in recent years. In addition, the comparison and relationship among formulations is presented. Applications and limitations of them in dynamical analysis of flexible structures will be shown from a mechanics/dynamics point of view for engineering applications. Finally, the floating frame of reference formulation is selected to model the Bucket-Wheel boom of the BWEs. Three approaches controlling vibrations of flexible structures, such as active structural vibration control, passive structural vibration control, and hybrid structural vibration control, are reviewed. As a result, the active vibration control approach is applied damping vibrations and stabilizing the Bucket-Wheel Excavator.

The contents of this thesis are developed based on results of the predecessor research, shown in [Söf96], where length-variable, elastic robot arms are modeled as a finite elastic beam with nonlinear kinematics up to terms of second order. Matrix methods combined with beam theory of 3rd order are used to generate the equations of motion for describing the dynamic behavior of the nonlinear elastic beam. Thus, nonlinear effects of higher order as well as related dynamics are represented in terms of mass, damping, and stiffness matrices. By doing this, it is easy to apply standard programs to simulate the dynamics. An indirect measurement method using a PI-observer is applied to reconstruct all states of a system as well as nonlinearities assumed as external disturbances acting on certain elastic variables. From this point of view, observer-based controllers using different disturbance compensation techniques are realized to control vibrations and to stabilize the system. Based on this work, in the third chapter, the nonlinear modeling of the three-dimensional elastic boom considering the elasticity of suspending cables and also couplings resulting from geometrical nonlinear deformations is presented using a matrix method combined with beam theory of 3rd order. More complex motions, especially the effects of additional guided motions in combination with digging resistance forces coupled with state as well as state dependences in the nonlinear matrices, are discussed and considered. The significant dynamic phenomena due to higher-order geometrical couplings undergoing guided motion and external excitations on the dynamic behavior of the Bucket-Wheel boom are analyzed in detail.

In the fourth chapter, the extended linear model, in which nonlinearities (here higher-order couplings between flexible deformations effected by complex motions) are assumed as additive unknown inputs affecting the linearized/linear system, will be used to describe the dynamical behavior of the Bucket-Wheel boom during exploitation. Then, a high-gain PI-observer is applied to estimate states of system and to reconstruct the time behavior of the nonlinearities. From this point of view, the PI-Observer-based control design combinated with disturbance rejection approaches are discussed and compared.

In the fifth chapter, intensive simulation studies show demonstrating effects of higher-order couplings resulting to large deformations and instability of the system undergoing the guided motion and external forces. The application of the high-gain PI-Observer to estimate system states as well as unknown disturbances is illustrated by the simulation results. Moreover, the efficient suppression of vibrations as well as stabilization of the system during the digging process of BWE using different disturbance rejection control approaches are also given.

The last chapter summarizes contributions and main results of the thesis. Then, some recommendations for the future work are suggested.

2 Fundamentals: Modeling and control of flexible structures

2.1 Dynamics of flexible structures

2.1.1 Introduction

In practical structure and machine applications, flexible structures are used widely due to advantages of lightweight and stiffening. Nowadays, development trends of these structures become larger, lighter, and slender and thus the dynamic behavior of them also becomes very complex due to geometrically nonlinear coupling effects between elastic deformations and large overall motions. The flexibility of structural components needs to be considered in dynamic modeling. Until now there are a lot of investigations focused on this field. In general, these systems or parts of them are always based on idealized flexible beams attached to moving or guided base or mechanisms. They are modeled using the Bernoulli-Euler and Timoshenko beam theory. With beams having a long length in comparison with its cross-sectional dimension, the Bernoulli-Euler beam theory is efficient to describe the dynamic behavior of the elastic beam. Alternatively, for short beams or beams subjected to high-frequency excitation effects of the rotary inertia of a cross-section along with shear deformations can be solved using the Timoshenko beam theory. In early studies, dynamic models of flexible structures are often obtained with simple equations based on the linear elasticity theory neglecting coupling effects resulting from elastic deformations and overall large motions. Obviously, the obtained results for these cases will lead to the error for predicting dynamic responses of flexible structures. In the last two decades, modelling of the dynamic behavior of flexible structures considering the nonlinear dynamic effects related with the coupling dynamics are established and extended more and more. Furthermore, approaches for kinematic and dynamical analysis of them are also developed correspondently. It is necessary to classify and review all approaches for modelling dynamics of flexible structures in order to get a overall overview in this field. From this, applications and limitations of approaches with different aspects in the dynamic analysis of flexible structures can be pointed out.

A review of the flexible multibody dynamics is carried out in [Sha97] with basic approaches used in the dynamic analysis of flexible mechanical systems such as the floating frame of reference formulation (FFRF), large rotation vector formulation, the finite element incremental methods, and the absolute nodal coordinate formulation (ANCF). Furthermore, analytical methods based on these formulations for deriving the equations of motion of flexible multibody systems are also reviewed. The future research directions for flexible multibody systems such as coupling problems between elastic deformations, large deformation problems, the relationship and combination of different formulations, controlling flexible structures are outlined.

In addition, a more general view in kinematic and dynamic analysis of flexible multibody systems is reviewed in detailed with 887 references cited in [WN02]. Based on these references, different aspects of the flexible multibody dynamics are addressed as follow: modeling of flexible components, couplings of flexible deformations, the area of control for the flexible mutibody system, etc. Three approaches widely used in in kinematic and dynamic analysis related to elastic deformations undergoing overall large motion, namely floating frame reference, corotational frame reference, and inertial frame reference formulation, are presented. The advantages and limitations of them are discussed for modeling flexible multibody systems. New applications, strategies for controlling and increasing the computational efficiency in this field are identified in the future direction of research.

In recent two decades, investigations related to the dynamics of flexible structures have been published in a large number of papers. In this chapter, two widely used formulations (FFRF and ANCF) in which nodal coordinates are introduced using a finite element method, are briefly reviewed with defined coordinate systems, the derivation of the elastic forces using an elastic line approach, and a continuum mechanics approach in modeling of flexible structures. By introducing this, different aspects in analyzing kinetics and dynamics of flexible structures including coupled problems of flexible deformations, large deformations, and the choice of shape functions for describing displacements of flexible deformations are reviewed with investigations published in recent years. In addition, the comparison and relationship among formulations are presented. Applications and limitations of them in dynamic analysis of flexible structures will be shown from a mechanics/dynamics point of view for the engineering applications. Finally, the choice of the floating frame of reference formulation is given to model the Bucket-Wheel boom of the BWEs.

2.1.2 Approximation methods for formulating equations of motion

In this section, the description of coordinates used with FFRF and ACNF is briefly reviewed. Elastic beam kinematics under large overall motions and small/large flexible deformations can be described using a set of these different coordinates. From this kinematic description, basic principles in mechanics, such as the principle of virtual work, D'Alembert's principle, Hamilton's principle, etc., are used for deriving the equations of motion of flexible structures.

Floating frame of reference formulation

Using the floating frame of reference formulation, the mixed sets of coordinates are used to describe dynamics of flexible structures: a global and a local reference. Large overall motions are defined in the absolute Cartesian and elastic deformations of elements are described with respect to the joint coordinate system of the finite elements. The dynamical behavior of flexible structures undergoing large overall

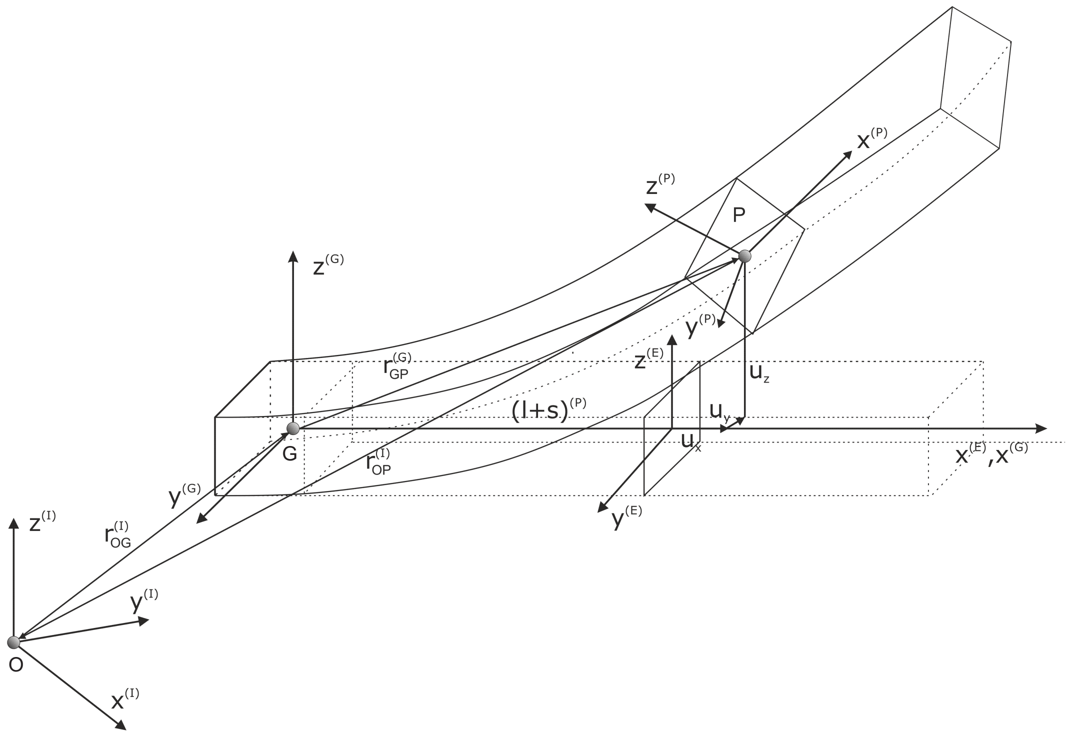

Figure 2.1: Modeling of the 3D elastic beam: coordinate system - used with FFRF [Söf96]

motions using this approach can be described by the equations of motions. Many contributions based on the floating frame of reference formulation to analyze dynamics of flexible structures are presented with different techniquies. A brief description of this formulation based on the work of Söffker [Söf96] is introduced in this section.

The used coordinate systems of the beam are illustrated in Fig. 2.1. Frame $O - X^I Y^I Z^I$ is the inertial reference frame. The moveable joint coordinate system $G - X^G Y^G Z^G$ is a beam-fixed frame with its origin located at the centroid of the cross-section. The frame $E' - X^E Y^E Z^E$ is the reference frame defined for the cross-section to describe the elastic variables u_x, u_y, u_z in the moveable joint coordinate system. The frame $P - X^P Y^P Z^P$ is defined for the position and orientation of the considered infinite section.

The position vector of a finite mass point dm (Fig. 2.1) in the finite disc (P) with respect to the inertial reference is given by

$$\begin{aligned} r(t) &= r^{(I)}_{OP}(t) = r^{(I)}_{OG}(t) + r^{(I)}_{GP}(t) \\ &= r^{(I)}_{OG}(t) + T(l^{(G)}_{GE'} + s^{(G)}_{E'E} + u^{(G)}_{EP'} + \widehat{T} t^{(E)}_{P'P}) \end{aligned} \tag{2.1}$$

with $r_{OG}^{(I)}, r_{GP}^{(I)}$ as the vectors of the joint and the mass point from the joint respectively, in the inertial reference frame, the deformation of the centroidal axis of the considered cross-section is described by the vector of the elastic deformations $u_{EP'}^{(G)}$. The vector $r_{GP}^{(I)}(t)$ (described in the moveable joint coordinate system) also includes the vectors $l_{GE'}^{(G)} s_{E'E}^{(G)}$ describing the joint oriented boundaries of the considered finite element and the position of the undeformed disc in the element coordinate system, and the vector $\widehat{T} t_{P'P}^{(E)}$ describing the finite mass point of the stiff disc in the deformed position. The orientation of the coordinates can be identified using the three independent Tail-Bryan anglers, Rodriguez parameters, or the four dependent Euler parameters. In this section, the rotation matrices T and $\hat{T}$ with three independent Tail-Bryan angles are used to define the element and joint coordinate system based on the variables of $r(t)$, which are given in the inertial reference frame.

The transverse, longitudinal, and torsional displacements are related to the node variables and are approximated by using the usual and well-known interpolation functions for a finite element of length $\xi = x_i/l_i$. The Hermite polynomial functions as shape functions are selected for bending variables

$$\begin{aligned} g_1(\xi) &= 1 - 3\xi^2 + 2\xi^3, \\ g_2(\xi) &= -\xi(1-\xi)^2 l_i, \\ g_3(\xi) &= 3\xi^2 - 2\xi^3, \\ g_4(\xi) &= -\xi^2(1-\xi) l_i, \end{aligned} \tag{2.2}$$

and for longitudinal and torsion variables

$$\begin{aligned} h_1(\xi) &= 1 - \xi, \\ h_2(\xi) &= \xi. \end{aligned} \tag{2.3}$$

Using a coupled set of the inertial reference and the elastic coordinates to define the global position of a finite mass point on the deformable body, the kinetic energy and the virtual work of elastic forces are defined. Thus, the equations of motion of the rotating flexible beam are obtained using one of the methods like the principal of virtual work, Lagrange's equations, Hamilton's principal, Newton-Euler equations.

The virtual work of inertia forces is given by

$$\delta W_m = -\rho \int\int\int \delta r_{OP}^{(I)T} \ddot{r}_{OP}^{(I)} dm, \tag{2.4}$$

where ρ is the density of the material (ρ=constant), $\delta r_{OP}^{(I)}$ and $\ddot{r}_{OP}^{(I)}$ are the virtual displacement and its second time derivative, respectively.

Applying the principle of virtual work,

$$\delta V_e = \delta W_a + \delta W_m, \tag{2.5}$$

the equations of motion of the elastic beam are finally expressed in the specified element matrix form as follow

$$M_g \ddot{u} + D_g \dot{u} + K_g u = f_{g1} + f_{g2} + f_{gg}, \tag{2.6}$$

with the virtual potential energy V_e which is calculated in the next part using different approaches (elastic line approach, a continuum mechanics approach) for formulating elastic forces, the virtual work of inertia forces W_a, the virtual work of external forces W_m, the global mass matrix M_g, the global damping matrix $D_g(u, \omega)$, the global stiffness matrix K_g, the vector of external forces and moments f_{g1},f_{g2} resulting from translational and rotational motion of the joint, and the vector of gravitational effects f_{gg}.

Using the floating frame of reference formulation, the dynamical behavior of the flexible beam undergoing the overall large motion and elastic deformations is described.

Absolute Nodal Coordinate Formulation

Due to the use of the local coordinate for describing elastic deformations, the floating of reference formulation leads to the high nonlinearity expression of the mass matrix as result of the coupling between the reference motion and elastic deformations. Dealing with this disadvantage of FFRF, an absolute nodal coordinate formulation, in which only the inertia coordinate system is used to define the global position vector of an arbitrary point on the element, is proposed by Shabana [Sha97] to analyze dynamics of flexible bodies in case of large overall motions and elastic deformations undergoing arbitrary displacements. A brief review of the absolute nodal coordinate formulation applied to three-dimensional flexibe beam elements based on [NHMS09] is presented in this section.

Figure 2.2 shows the coordinate system used to define the global position and orientation of a three-dimensional beam element with ANCF. The global position vector of a finite mass point P on the centroid axis can be defined using the global shape function and the absolute nodal coordinate

$$r = S(x, y, z).e, \tag{2.7}$$

where S is the global shape function, which can be obtained using the Hermite polynomial functions as given in Equations (2.2) and (2.3), x, y, and z are the local coordinates of the element, e is the vector of the nodal coordinates of the element given by

$$e = [e_I^T \quad e_J^T], \tag{2.8}$$

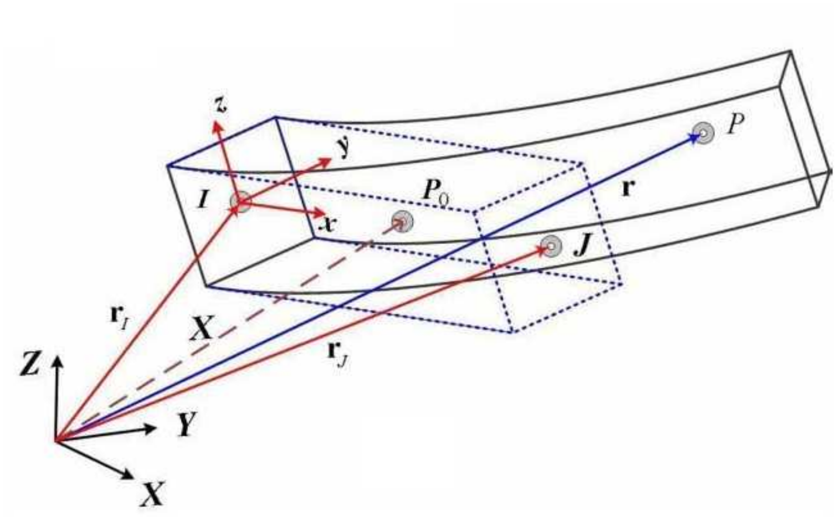

Figure 2.2: Modeling of the 3D elastic beam: coordinate system - used with ANCF [NHMS09]

where e_I^T and e_J^T denote the coordinates of nodes I and J, written in the form

$$e_I = \begin{bmatrix} r_I^T & \frac{\delta r_I^T}{\delta x} & \frac{\delta r_I^T}{\delta y} & \frac{\delta r_I^T}{\delta z} \end{bmatrix}^T, \tag{2.9}$$

$$e_J = \begin{bmatrix} r_J^T & \frac{\delta r_J^T}{\delta x} & \frac{\delta r_J^T}{\delta y} & \frac{\delta r_J^T}{\delta z} \end{bmatrix}^T. \tag{2.10}$$

As shown in Equations (2.9) and (2.10), the nodal coordinates of the element can be described by the vector e, which is defined in terms of the global position of nodes as well as the slopes at nodes.

With this description, the kinetic energy for deriving the mass matrix of the flexible multibody system can be determined as

$$\begin{aligned} T =& \frac{1}{2}\int_V \rho \dot{\mathrm{r}}^{\mathrm{T}}\dot{\mathrm{r}} dV = \frac{1}{2}\int_L \rho A \dot{\mathrm{r}}^{\mathrm{T}}\dot{\mathrm{r}} dx \\ &= \dot{\mathrm{e}}^{\mathrm{T}} \int_L^0 \rho A \mathrm{S}^{\mathrm{T}} \mathrm{S} dx \dot{\mathrm{e}} \\ &= \dot{\mathrm{e}}^{\mathrm{T}} \mathrm{M}_a \dot{\mathrm{e}}, \end{aligned} \tag{2.11}$$

where ρ is the density of the material, A is the cross-sectional area of the beam element, and M_a is mass matrix that can be defined as

$$\mathrm{M}_a = \int_L^0 \rho A S^T S dx. \tag{2.12}$$

The expression for the strain energy using the ANCF can be obtained using the different approaches such as an elastic line approach and a general continuum mechanics approach. The strain energy of the flexible multibody system is written as

$$U = \frac{1}{2}\mathrm{e}^{\mathrm{T}}\mathrm{K}_a\mathrm{e}, \tag{2.13}$$

where K_a is the stiffness matrix of the element.

Applying one of the above mentioned methods, such as the principle of virtual work, Lagrange's equations, Hamilton's principle, Newton-Euler equations, the equations of motion of the flexible multibody systems undergoing large motion and the elastic deformations are derived from the principle of virtual work in the matrix form as

$$\mathrm{M}_a\ddot{\mathrm{e}} + \mathrm{D}_a\dot{\mathrm{e}} + \mathrm{K}_a\mathrm{e} = \mathrm{Q}_a, \tag{2.14}$$

where Q_a is the vector of generalized nodal forces.

With a simple description of the global position vector of an arbitrary point on the element in absolute coordinates, the obtained mass matrix is constant and Coriolis as well as centrifugal forces are zero with ANCF. Many contributions based on this formulation are developed in recent investigations to solve all practical problems of dynamics of multibody systems [HS98, BS00, OS01, YS01a, YS01b, Dom02, SM03, Ger03, VEMD03, VSS05b, YKK07, MBS08, GI08, GMM08].

2.1.3 Different approaches for formulating elastic forces

As previously pointed out, using a coupled set of different coordinate systems, the global position and orientation of a three-dimensional elastic beam element can be defined. Based on these descriptions, the virtual work of internal elastic forces for determining the stiffness matrix of the deformable body is evaluated using nonlinear strain displacement relationship, the Cauchy strain tensor, Green-Lagrange strain tensor, etc. Thus, the accuracy of modeling the three-dimensional elastic beam depends on representation of elastic forces, which are associated with longitudinal, transverse, torsional deformations as well as couplings between them. In this section, an elastic line approach and a continuum mechanics approach for formulating the elastic forces of the three-dimensional elastic beam under large reference motions and arbitrary flexible deformations will be reviewed with the floating frame of reference formulation and the absolute nodal coordinate formulation.

Elastic line approach (EL)

Using an elastic line approach with the floating frame of reference formulation [Söf96], the deformed curve of the flexible beam is defined as a curved 2D or 3D line with the vectors of position gradients along the beam centre-line. Therefore, the deformations of them are expressed in terms of the longitudinal strain (ϵ) as well as the curvatures ($\kappa_1, \kappa_2, \kappa_3$) of a 3D curve using the coordinate system as given in Figure 2.1.

$$\begin{aligned}
\epsilon =& \quad u_x^{'} + \frac{1}{2}(u_y^{'2} + u_z^{'2}), \\
\kappa_1 =& - \alpha^{'} + u_y^{''} u_z^{'}, \\
\kappa_2 =& - u_y^{''} + u_y^{'} u_x^{''} + u_y^{''} u_x^{'} - \alpha^{'} u_y^{'}, \text{ and} \\
\kappa_3 =& - u_z^{''} + u_x^{'} u_z^{''} + u_x^{''} u_z^{'} + \alpha^{'} u_y^{'}.
\end{aligned} \tag{2.15}$$

Here, the results are expressed as functions of the elastic variables, where u_x, u_y, u_z, α denote the logitudinal, transverse according to y-direction, transverse according to z-direction, and torsional deformation in the local coordinate system, respectively. Here the apostrophe denotes differentiation with respect to the x-coordinate, κ_1 represents the torsional deformation, and κ_2 and κ_3 represent the bending deformations.

The elastic forces are determined by the virtual potential energy of the deformed beam as sum of the axial, torsional, and bending elastic deformation energies using four variables representing the axial strain and the curvatures

$$\delta V_e = \int_l^0 (\delta \epsilon N + \delta \kappa_1 M_x + \delta \kappa_2 M_y + \delta \kappa_3 M_z) dx, \tag{2.16}$$

where $N = EA\epsilon$, $M_x = GI_t\kappa_1$, $M_y = EI_y\kappa_2$, $M_z = EI_z\kappa_3$ (assuming constant material behavior for Youngs modulus and the shear coefficient with E, G).

Equation (2.16) shows that all deformable components as well as coupled effects between them are taken into account in the strain energy function.

Using an elastic line approach with the absolute nodal coordinate formulation [SM10], the elastic curve of the beam element is defined in terms of the axial, shear, torsional, bending deformations. The vectors of position gradients along the beam centre-line are written as follows

$$r_x = r_{,x}(x, 0, 0), \quad r_y = r_{,y}(x, 0, 0), \quad r_z = r_{,z}(x, 0, 0), \tag{2.17}$$

where the vector r_x is tangent to the beam centerline, the vectors r_y, r_z denote the shear plane at an arbitrary point x on the beam centerline.

With this description, the strain components along the beam centre-line can be defined using the Lagrange strain tensor and the curvatures for the element as

$$\begin{aligned}
\epsilon_x =& \frac{1}{2}(r_x^T r_x - 1), & \epsilon_y =& \frac{1}{2}(r_y^T r_y - 1), & \epsilon_z =& \frac{1}{2}(r_z^T r_z - 1), \\
\gamma_{yz} =& r_y^T r_z, & \gamma_{xy} =& r_x^T r_y, & \gamma_{xz} =& r_x^T r_z, \\
\kappa_1 =& \frac{1}{2}(r_z^T r_y^{'} - r_y^T r_z^{'}), & \kappa_2 =& -r_z^T r_x^{'}, & \kappa_3 =& r_y^T r_x^{'},
\end{aligned} \tag{2.18}$$

where ϵ_x, ϵ_y, and ϵ_z denote the extension of the beam according to x-, y-, z-direction, γ_{yz} denotes the deformation of the cross section, γ_{xy}, γ_{xz} denote the transverse shear deformations.

Based on the strain components, the strain energy U for deriving elastic forces can be determined as sum of the longitudinal U_L, shear U_S, torsional U_T, and bending U_B deformation energies as

$$U = U_L + U_S + U_T + U_B, \tag{2.19}$$

$$U = \frac{1}{2} l \int_0^1 (A\bar{\epsilon}^T \bar{E}\bar{\epsilon} + GA\kappa_2\gamma_{xy}^2 + GA\kappa_3\gamma_{xz}^2 + S_t\kappa_1^2 + EI_y\kappa_2^2 + EI_z\kappa_3^2)d\xi, \tag{2.20}$$

where $\bar{\epsilon} = (\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{yz})^T$ denote strains and $\bar{E}$ denote the elasticity coefficients, and G is the modulus of rigidity.

In comparison with Equation (2.16), this strain energy function of the elastic forces (Equation 2.20) with respect to the nodal coordinates is extended with an addition of the axial deformations in three different directions, shear deformations, and the deformation of the cross section. Therefore, the assumption about the rigid of the cross-section using the Euler-Bernoulli and Timoshenko beam theory can be relaxed. Obviously, exact solutions for large and very large deformation problems of flexible structures can be obtained.

A continuum mechanics approach

Using a continuum mechanics approach with the absolute nodal coordinate formulation [SM10], the strain energy for deriving the elastic forces is evaluated regardless of elements which are straight or not in the reference configuration. The position vector u of a finite mass point P on the beam is descirbed in the inertia coordinate as shown in Figure 2.2

$$u = r - r|_0, \tag{2.21}$$
$$u = S(e - e|_0), \tag{2.22}$$

where $r|_0$ and $e|_0$ are the initial undeformed configuration, and the initial nodal coordinate, respectively.

Using Green-Lagrange strain tensor, the strain-displacement relationship is expressed as

$$\epsilon_{i,j} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}), \quad i, j, k = x, y, z \tag{2.23}$$

where partial derivatives are denoted by $u_{x,y} = \partial u_x/\partial y$, etc. From this description, six independent strain components are determined as

$$\begin{aligned}
\epsilon_1 &= \frac{1}{2}(r_{,x}^T r_x - 1), \qquad & \epsilon_4 &= r_{,x}^T r_{,y}, \\
\epsilon_2 &= \frac{1}{2}(r_{,y}^T r_y - 1), \qquad & \epsilon_5 &= r_{,y}^T r_{,z}, \\
\epsilon_3 &= \frac{1}{2}(r_{,z}^T r_z - 1), \qquad & \epsilon_6 &= r_{,z}^T r_{,x}.
\end{aligned} \tag{2.24}$$

From the above definition of the strain components, the strain energy for deriving elastic forces can be determined as

$$U = \int_V \sigma^T \delta\epsilon dV, \tag{2.25}$$

where σ is the stress vector determined under assumption of a Saint-Venant-Kirchhoff material model.

Three approaches, which are widely used for formulating the elastic forces of spatial finite beam elements, are briefly reviewed. Based on these derivations, geometrical nonlinear stiffening effects relating to couplings between large overall motions and arbitrary elastic deformations can be considered for solving real problems in aerospace, civil, and mechanical engineering applications [Sha99, SMH01, BCS01, MVD04, LNP08, GI08, MBS08, NHMS09, DBM10, SM10].

2.1.4 Coupling problem

In the early researches [Sch97], assumptions in which flexible displacements of an elastic beam such as elongations, transverses, and torsions are independent undergoing large overall motions and arbitrary flexible deformations are usually used to model a rotating elastic beam. Actually, these assumptions are only suitable in cases of considering small flexible deformations. Nowadays, with the increasing demand for an exploitative productivity the speed of machines, such as flexible robot arms, turbine blades, turbo-engine blades, and the helicopter blades, etc., is also increased respectively. The effect of geometric stiffness on dynamic characteristics, in case of operation with such high speed, will become significant because it causes the change of the stiffness. This may result in unstable situations and changes the natural frequencies of the system [BS02, VSS05a, VSS05b, MBS08]. In addition, the dynamic behavior of flexible structures undergoing large overall motions and external excitations, like the Bucket-Wheel excavator considering the impact and digging resistance forces, etc., become very complex. For large complex mechanical systems, in order to obtain the consistent and stable solution for all above mentioned problems, geometric and dynamic coupling effects between overall large motions and elastic deformations must be taken into account in the model of flexible structures. In this section, the coupling problem with different aspects in dynamic analysis of flexible structures using the floating frame of reference formulation and the absolute nodal coordinate formulation is reviewed with contributions published in recent years.

Coupling problem using FFRF

The dynamical behavior of a rotating elastic beam using the floating frame of reference formulation is present considering the coupling effects of the flexible deformations in [KRB87, YRS95, MD96, AS97, Söf99, LH04, YJC04, CHY05].

Considering coupling terms of longitudinal and bending deformations: For rotating elastic beams at high angular velocities, dynamic couplings due to the longitudinal displacement caused by bending deformations in combination with inertia forces may have significant effects on dynamic characteristics. Without considering the effects of such couplings, incorrect and unstable solution is always obtained in this case. These coupling terms are expressed in terms of the strain displacement relationship and entered in the equations of motion of the system using the strain energy function, as shown in Equation (2.15). The importance of dynamic stiffening effects due to the couplings is in [KRB87] mentioned firstly. However, due to the use of the linear beam theory, quadratic terms of a longitudinal deformation are not considered in the modeling. The results lead to a uncouple between the longitudinal and bending deformations. Along with technical development processes, many approaches for modeling of complex flexible systems have been extended to capture coupling effects between flexible deformations.

Based on an elastic line approach in FFRF, the nonlinear coupling terms due to the axial displacement caused by the transverse deformations, known as the stiffening effect, have been demonstrated to prevent the system from going unstable as compared with the linear case in many investigations [LH04, MVD04, CHY05, LNP08]. This effect accounted in the model will stiffen the system and yield the correct results for the transient response characteristics of the rotating elastic beam. In contrast with the stiffening effect, the nonlinear coupling term due to the transverse displacement caused by the longitudinal deformation, known as the softening effect, causes larger deformations in comparison with the linearized case, if considered in the modeling. All the above mentioned coupling terms are known as the geometrical nonlinearities and must be taken into account in the modeling of flexible structures. In order to capture these coupling effects of the elastic deformations, the nonlinear strain-displacement relationship must be considered at least the second-order term for deriving the dynamic equations as demonstrated in [MD96]. The quadratic terms in elastic deformations lead to the higher-order deformations in the mass and stiffening matrix of the equations of motion. The general equation describing the dynamical behavior of a rotating elastic beam considering the corresponding matrix representation of a model 3rd order as well as coupling effects of elastic deformations is discussed detailed in [Söf96]. The efficiency of consideration of the second-order coupling term of the longitudinal displacement caused by the transverse displacements on the dynamic characteristics of the high-speed rotating flexible structure is verified in [LH04, CHY05] in comparison with three computer programs with the higher-order terms of flexible deformations, the approximation to first order, and the neglect of coupling effects in the dynamic equations. The effects of flexible coupling dynamics due to the interaction between the large motion of the system and elastic deformations of the flexible beam is considered in this contribution. With the additional tip mass on the flexible beam, the effects of the elastic vibrations of the flexible beam on the law of the large motion of the system are shown by simulation examples. This problem is also extended for the three-dimensional Euler-Bernoulli beam element in [SMH01] with consideration of the second-order terms in the deformation variables for describing the elastic rotation matrix of the cross section with respect to the body frame.

From the description of the geometric nonlinearity of the flexible beam in [Sha96, MVD04] and the use of the approximation of them at the different level for expression of the strain energy, the influence of the geometric stiffening due to the geometric nonlinearities as well as foreshortening effects due to the deflection of the beam on the dynamic response of the system is discussed and compared in [LNP08] using the floating frame of reference formulation in natural coordinate. Once again, the higher order formulation is proved to be necessary to obtain the correct results.

Considering the coupling terms of torsional and bending deformations: Based on an elastic line approach as well as the quadratic terms considered in the nonlinear strain-displacement relationship, as previously mentioned, a general modeling including all the nonlinear coupling terms is presented in [Söf96]. The effects of dynamic couplings between bending and torsional deformations can be easily considered with this approach. This modeling can be applied for flexible structures such as wind turbine rotor blades, an helicopter rotor blades, etc.

Coupling problem using ANCF

The floating frame of reference formulation is only limited in the Euler-Bernoulli beam theory, in which the cross section of a beam is infinitely rigid in its own plane and remains plane after deformation and neglect of a shear deformation. Considering coupling effects of elastic deformations as well as the deformation of the cross-section of flexible structures, the absolute nodal coordinate formulation has been demonstrated efficiently for dynamic analysis of flexible structures in case of very large deformations as compared with FFRF using an elastic line approach and a continuum mechanics approach [BS00, BS02, Dom02, VEMD03, VSS05b, MBS08, GMM08].

Considering the coupling terms of longitudinal and bending deformations: Using the position vector in the inertia coordinate system and the slope vector instead of other orientation parameters in ANCF, the stiffening matrices considering the dynamic couplings between longitudinal and bending deformations are usually expressed in the highly nonlinear form. In order to solve this problem, the implementation of a continuum mechanics approach in ANCF leads to the simplification in the vector of the elastic forces. Several longitudinal and bending force models for analyzing the coupling effects of the flexible deformations are presented in [BS00]. In comparison with the FFRF, the results obtained using this approach are proved to be more accurate as well as a significant saving in the computer time.

As previously pointed out in FFRF, the effects of the inertia centrifugal and Coriolis forces become significant in case of a high-speed motion of the flexible structures. The dynamic couplings due to the bending deformations caused by the axial displacement, known as a geometric stiffening effect, are necessary to maintain the stability of the system. The dynamic behavior of the rotating elastic beam at high angular velocity with ANCF using a continuum mechanics and an elastic line approach is discussed in [BS02, VSS05a, VSS05b, MBS08]. Using the description of the nonlinear Green-Lagrange strain-displacement relationship, the equations of motion of the elastic beam undergoing overall large motion using ANCF can capture the coupling effects of the longitudinal and bending deformations. With this approach the results lead to stable solutions for any values of the angular velocities. Thus, the centrifugal stiffening effects on dynamic responses of the system are proved. This can be automatically taken into account in ANCF.

Due to the use of the finite element method in conjunction with different formulations, the effect of the number of the finite elements to model the dynamics of the rotating elastic beam at high speed is also necessary to be examined. In [VSS05a, VSS05b], the dependence of the number of finite elements on the critical speeds is discussed using linear and nonlinear elasticity theory. The simulation results demonstrate that critical speeds increase with the change of the number of finite elements when linear elasticity theory is used. On the other hand, the stable solution, regardless of the number of the finite elements used, is always obtained using a nonlinear elasticity theory. Once again, the use of the nonlinear elastic theory based on ANCF is proved to be efficient for analyzing dynamics of the three-dimensional rotating elastic beam including the change of the number of finite elements.

Considering the coupling terms between bending deformations and the cross-section deformation: For flexible structures or plasticity problems, the dynamic coupling between the bending deformations and the cross-section deformation, along with couplings of flexible deformations as above mentioned, becomes significantly and must be taken into account. Using a continuum mechanics approach as well as the Green-Lagrange strain components, this coupling effect is considered in formulation of the elastic forces as shown in [YS01a, SM03, MBS08]. The assumption of a rigid of the cross-section is relaxed. On the contrary, an elastic line approach, in which the deformation of the flexible beam is defined as a curved 2D or 3D, has been demonstrated that it can not capture dynamic couplings between bending deformations and the cross-section deformation.

2.1.5 Deformation problem

New structures, for the practical engineering applications, become larger, lighter, and slender. Large elastic deformations, in which the flexibility of structures allows them to have large elastic deformations relative to a rigid body, undergoing overall large motions must be considered in dynamic modeling of flexible structures. For example, due to requirement of minimum weight, light polymer materials are usually used in design of helicopter blades. Therefore, they can be subjected to large flexible deformations without exceeding elastic limitations during rotation at high angular velocities. The floating frame of reference formulation and the absolute nodal coordinate formulation in combination with an elastic linear apporach and a continuum mechanics approach can be solved with small/large deformation problems of flexible structures

Deformation problem using FFRF

The floating frame of reference formulation has been demonstrated as been efficient with respect to small deformation problems, when using the linear elasticity theory, in which the axial and bending deformations assumed are uncoupled. However, in case of large elastic deformations the dynamic coupling effects of the elastic beam undergoing large reference displacements must be considered in the modeling. In early investigations, the substructuring technique, as shown in [WH88], is applied to analyze large elastic deformations of flexible structures using the FFRF. With this technique, the large deformation is assumed to sum of the small deformations and the structure can be divided into substructures that are connected at the nodes as well as the reference frame applied for each of them. The equations of motion for each substructure are obtained with the basic approaches. By applying the suitable constraints at the nodes among the substructures, the equations of motion for the flexible multibody system considering the geometric nonlinearity are expressed in matrix form. As pointed out in [VEMD03, MVD04, DBM10], this technique is not efficient for the large deformation analysis because of the large number of the moving reference as well as the variables used in the modeling.

Based on an elastic line approach and description of the nonlinear strain-displacement relationships, the FFRF can be used in the large deformation analysis of the elastic beam undergoing large reference displacements, as presented in [Söf99, NHMS09, DBM10]. The validation of this formulation in the large deformation problem is confirmed by comparing the ANCF with an elastic line approach and the experimental results in [NHMS09]. However, large deformations considering the shear and cross-section deformation cannot capture with this formulation due to the use of the Euler-Bernoulli beam theory.

Deformation problem using ANCF

An absolute nodal coordinate formulation proposed by Shabana [Sha97], using the slope vectors along with the element nodal coordinates defined in the inertia coordinate system, can be efficiently applied for the dynamic analysis of flexible structures exhibiting large deformations.

Based on the classical beam theory for deriving the elastic forces in ANCF, the geometrically nonlinear effects due to dynamic couplings of flexible deformations are automatically captured in the modeling, as shown in [HS98, DSM06]. Large deformation problems can be easily solved. The efficiency of the use of the absolute nodal coordinate formulation to analyze the large deformation of flexible structures in comparison with the floating frame of reference formulation is demonstrated through the simulation examples. Applying both FFRF and ANCF for the small deformation problem leads to good convergence of the obtained results. On the other hand, the limitation of FFRF for analyzing the very large deformation remains.

As pointed out in ANCF, the geometric stiffening matrices considering coupling effects between the overall large motion and flexible deformations, based on the classical beam theory, are always expressed in highly nonlinear form. Overcoming this disadvantage, a continuum mechanics approach in ANCF is proposed in order to get the simplification of the elastic forces. Using this approach in ANCF, an efficient implementation for the large deformation problem is presented in many investigations [BS00, YS01a, BS02, SM03, MBS08, GMM08, SM10].

The large deformation analysis of the flexible beam using the absolute nodal coordinate formulation is extended for the three-dimensional beam element with an elastic line approach in [YS01a, Dom02, MBS08] and with a continuum mechanics approach in [YS01b, MBS08, SM03]. Along with the longitudinal, bending deformations, the effects of the inertia of the cross section, the shear, and the torsion deformations are additionally considered for the three-dimensional beam element using the developed absolute nodal coordinate formulation.

The absolute nodal coordinate formulation with an elastic line approach, based on the Euler-Bernoulli beam theory, is investigated in [Dom02, MBS08] for the large spatial deformation problem of the three-dimensional elastic beam. Using the nonlinear strain-displacement relationship, in which all nodal variables are defined in the inertia coordinate system, the geometrically nonlinear description of the stiffening matrices is obtained and the spatial flexible deformations like the axial, bending, and torsion deformation are taken into account. The results of static and dynamic analysis of the rotating elastic beam are carried out through numerical simulation examples to assess the accuracy of the modeling and application of the method for the large deformation problem. However, for the three-dimensional elastic beam element undergoing large overall motions and arbitrary flexible deformations, the effects of shear deformations and the inertia of the cross section must be considered but this problem can not be solved by an elastic line approach with ANCF. A non-incremental absolute nodal coordinate formulation is presented in [YS01a] to capture effects of a rotary inertia and shear deformation.

The absolute nodal coordinate formulation with a continuum mechanics approach, compared with an elastic line approach, can be efficiently used for large deformation problems of the three-dimensional beam elements [YS01b, SM03, MBS08]. Using the Green-Lagrange strain tensor as well as the second Piola-Kirchoff stress for deriving the elastic forces, the effects of the torsion, shear deformation, and the inertia of the cross section are automatically taken into account. When the elastic structures undergo very large deformation, the cross-section of them will deform and change their shape. With a continuum mechanics approach, the assumption about a rigid of a cross-section using the Euler-Bernoulli and Timoshenko beam theory, is also relaxed and the solution for this problem can be easily obtained.

2.1.6 Element shape functions

In FFRF and ANCF, the deformations of flexible structures undergoing the overall rotation are defined with respect to the reference configuration and expressed in terms of the nodal values of the elements as well as the shape functions. The Rayleigh-Ritz approximation method is usually employed with different-order interpolating polynomials to describe displacements of flexible deformations. The dynamical behavior of flexible structures will obviously be affected when changing the order of the interpolating polynomials. The accuracy of the obtained results becomes better if the higher-order interpolating polynomials used. On the contrary, the solution for this problem will also become more complex along with a significant computational cost. Actually, effects of the selection of the order for the interpolating polynomials on the dynamic response of flexible structures are not mentioned in many investigations. Instead of this standard shape functions (e.g., the linear interpolation function describing the longitudinal and torsional displacement and the cubic interpolation function for describing transverse displacements as shown in Equations (2.2) and (2.3)) are applied for a beam element.

The problem of the choice of the shape functions on the solution of the equations of motion describing displacements due to flexible deformations of the structure is presented in [Sha99, SWS99]. The new beam element using the cubic interpolation function, instead of the linear function, for describing the axial displacement is presented in [Sha99] to study the dynamic response of the flexible multibody system. The simulation results, in case of the beam spin-up problem, reveal that the tip response of the bending deformation leads to the good convergence with the proposed new element in comparison with the standard element, when increasing the number of the elements.

Actually, the accuracy of the displacement due to the deformations depends on the choice of the order for the interpolating polynomials. So far, it is difficult to describe exactly the complex geometry of the flexible structures, which are modeled using FFRF, ANCF, or other formulations. In order to increase the exact definition of the complex geometry, the Bernstein, Boor polynomials used in a Computer Aided Design (CAD) packages to describe the geometry can be employed in combination with the absolute nodal coordinate formulation [SS09a]. From this point of view, the disadvantage of the ANCF for describing the complex geometry is improved in framework of CAD. In addition, the efficiency of the dynamic analysis of flexible structures is easily obtained using ANCF. The new direction based on this idea for studying dynamics of flexible structures is published in some recent investigations [SS09a, SS09b, LS10]. The combination between the absolute nodal coordinate formulation and the computer aided design software will briefly reviewed in the next section.

2.1.7 Comparison, relationship, and combination among formulations

As pointed out in the previous sections, the number of formulations for studying dynamics of flexible structures in conjunction with different approaches is presented. Each of them has not only advantages but also disadvantages. The comparison among different formulations to verify the accuracy and equivalence of them with respect to practical engineering applications is presented. Based on this comparison, the mathematical relationships among the different methods can be pointed out and the advantage of them, obviously, can also be combined together in order to increase the accuracy and reduce the computation time for different problems in this field. Furthermore, it is very useful to understand and select the best approach to model the dynamics of flexible structures in their special applications as well as additional aspects regarding the usability for model-based control design standard.

Comparison and relationship among formulations is presented in [Sha97, BS00, OS01, BCS01, MVD04, VSS05a, MBS08, NHMS09] to verify an accuracy of them. Using the absolute nodal coordinate formulation proposed by Shabana, it is possible to establish the relationships between different formulations as pointed out in the future research directions of dynamics of flexible multibody system [Sha97]. Due to the use of the vector of the element nodal coordinates defined in the inertia coordinate system, in general, the global stiffness matrix, in contrast with the FFRF, is always expressed in a highly nonlinear function. In order to get a simple expression of the elastic forces in ANCF, the nodal coordinate relationship between the FFRF and ANCF as well as a coordinate transformation is presented in [BCS01]. With this coordinate transformation, the definition of the strain energy for deriving the elastic forces in FFRF can easily applied in ANCF.

For flexible structures undergoing overall large motions and arbitrary elastic deformations, geometric stiffening effects due to dynamic couplings are very important and need to be accounted in models. As pointed out in [BS00], the floating frame of reference formulation using the linear elastic theory cannot capture the dynamic couplings of elastic deformations. So far, it leads to incorrect and unstable solution in case large deformations. On the other hand, using the absolute nodal coordinate formulation with the nonlinear continuum mechanics approach, the geometric stiffening effects always are accounted in models. By comparing the stiffness matrices derived by the ANCF and FFRF, the modification in the FFRF, in which the floating frame is replaced by the local frame, is suggested in [VSS05a] in order to account the geometric stiffening effects. A correct stable solution at any angular velocities of the rotating flexible beam can be obtained by the modified floating frame of reference formulation in comparison with ANCF. Another comparison between FFRF and ANCF is also implemented in [MVD04] for the large deformation problem. This problem is solved using FFRF with the substructuring technique, the nonlinear beam theory and ANCF with a continuum mechanics approach. Based on the comparison in this investigation, the authors show that the geometric stiffening effects can be easily accounted in the FFRF with the substructuring technique. For analyzing very large deformations of flexible structures, the FFRF with the nonlinear beam theory is limited due to use the Euler-Bernoulli beam theory. On the contrary, the ANCF is concluded to be efficient to the very large deformation analysis.

As shown in [GMM08], the derivation of the elastic forces, based on utilizing the St. Venant-Kirchhoff material and the Piola-Kirchhoff stress tensor, considering the effect of the shear deformations using ANCF with a continuum mechanics approach in [OS01] leads to diverged results in comparison with the large rotation vector formulation. By implementing the strain energy defined in the large rotation vector formulation into the frame work of the ANCF, the numerical results for large deformation problems of flexible structures obtain good convergence of two formulations.

The comparison between the geometrically exact beam theory and the ANCF with an elastic line, and continuum mechanics approach for studying the effect of the centrifugal forces on the eigenvalue solution of the rotating beam is presented in [MBS08]. The obtained results are in good agreement in case of the geometrically exact beam theory and the ANCF with an elastic line approach, in which the coupling effects of the bending and the cross-section deformation are neglected. Once again, the ANCF with a continuum mechanics approach is proved for being able to be efficiently applicable to very flexible structures and plasticity problems.

Combination among formulations is present in [VEMD03, YPS04, LL07, LHC07, VEMD08]. The efficiency of the ANCF in the dynamic analysis of flexible structures such as spatial beams, plates, and solids undergoing large overall motions and arbitrary elastic deformations is proved in many publications. Using the inertia frame for describing the nodal coordinates and global slopes for describing the orientation of flexible bodies, obtained mass matrices of the finite elements as well as the force matrices are constant matrices and the geometric stiffening effects are automatically taken into account in the modeling. However, the elastic forces based on the strain energy function are expressed in terms of the high nonlinear form. Taking advantage and eliminating disadvantage of the absolute nodal coordinate formulation are ideas for combination between the ANCF and other formulations.

The efficiency of this combination, known as a new hybrid coordinate formulation, in the dynamic analysis of the flexible multibody system is proved in [YPS04] with the combination between the ANCF and the modal coordinate formulation and in [LL07] with the combination between the ANCF and the linear hybrid coordinate formulation. The expression of the nonlinear elastic forces based on ANCF is eliminated with the use of the modal coordinate formulation, as shown in [YPS04]. Furthermore, the effects of the rigid-flexible coupling dynamics of the three-dimensional hub-beams system undergoing a large overall motion are easily considered with a proposed formulation in [LL07].

Another combination of the different formulations, the natural coordinate for describing the rigid motion and the absolute nodal coordinate for describing the flexible deformations, used to model dynamics of complex flexible-rigid structures is presented in [VEMD03, VEMD08] in case of a planar and spatial motion. Using this mixed formulation, advantages of the absolute nodal coordinate in the large deformation analysis are retained like the constant mass matrix. In addition, the number of coordinates as well as constraints can be reduced significantly because of a systematic procedure used for a coordinate reduction.

Due to the use of the simple shape function to describe the displacements undergoing flexible deformations, as above pointed out, the absolute nodal coordinate formulation does not lead to the exact definition for representing the geometry of the complex structures. On contrary, this problem can be easily solved using a computer aided design (CAD) systems with Bezier, B-splines, or Non-Uniform Rational B-Splines (NURBS) curves, which employ basic functions and control points to define the geometry. In order to integrate the Bezier curves and B-splines into the ANCF finite element, the coordinate transformation based on the relationship between the ANCF kinematic description and the geometric description with the Bezier curves and B-splines is established in [SS09a]. From this idea, the displacement due to the deformations in ANCF, defined by the shape functions, can be described in terms of the control points, which relate to the order of the shape functions. Based on this combination, an exact geometric description for complex structures can be applied in the ANCF, in which all advantages, such as the constant mass matrix, zero Coriolis and centrifugal forces, etc., are still maintained. Integration of the geometry based on computer aided design (CAD) systems and ANCF finite element analysis, called as the integration of computer aided design and analysis (ICADA), is the new direction in dynamic analysis of flexible structures [SS09a, SS09b, LS10].

2.1.8 Applications and limitations of approaches

In previous sections, different aspects in dynamic analysis of flexible structures, such as the derivation of elastic forces, dynamic coupling problem, and deformation problem, are pointed out using FFRF and ANCF. Comparison between formulations for modelling flexible structures is also discussed. In addition, combination of them can be reproduced in order to capture advantages. In this section, applications and limitations of the formulations used in dynamical analysis of flexible structures will be systematized based on the investigations published in recent years.

• The FFRF using an elastic line approach [Sha99, SMH01, BCS01, MVD04, LNP08, NHMS09, DBM10] is limited in case of the small deformations if the linear elasticity theory is used for deriving the strain energy. For the large deformation problem of flexible structures, the nonlinear strain-displacement relationship must be accounted in models to capture the coupling effects due to flexible deformations. However, it is impossible to solve for the practical engineering applications considering a shear and cross-section deformations with this formulation due to the use of the Euler-Bernoulli beam theory.

• The FFRF using the sub-structuring techniques [WH88] can apply in case of large and very large elastic deformations but the computational efficiency of this method is decreased because of the large number of the reference system and variables used.

• The FFRF in conjunction with an update Lagrange formulation [DBM10] is developed for solid and plate elements. This formulation is applied for the structure analysis of rotor blades, micro air vehicles, and fluid-structure interaction problems.

• The ANCF using an elastic line approach [Dom02, GI08, NHMS09, SM10] can avoid the locking phenomenon for the thin and stiff beams and yields better rigidities. It can be used to model beam and shell elements. However, the effects of shear deformations and inertia of the cross section cannot be solved when using an elastic line approach with ANCF.

• The ANCF using a basic continuum mechanics approach [OS01, DSM06, GS06, MBS08, NHMS09, SM10] yields simple expressions of the elastic forces in the large deformation analysis. The effects of the couplings between elastic deformations and the deformation of the cross-section are considered. Furthermore, the assumptions of the Euler-Bernoulli beam theory and Timoshenko beam theory can be relaxed with ANCF using a continuum mechanics approach. In general, all flexible deformations of flexible strucutres undergoing an overall large motion, such as the longitudinal, bending, torsional, shear deformations are described. So far, this formulation can be applied efficiently for very flexible structures and extended for cable elements. However, it has several shortcomings such as the inaccurate solution with high-frequency coupled deformation modes, a source of numerical and locking problems in case of slender stiff structures, and difficulties to describe the warping of the cross-section.

• The ANCF in combination with a computer aided design (CAD) systems with Bezier, B-splines, or Non-Uniform Rational B-Splines (NURBS) curves [SS09a,SS09b, LS10] can be used for modeling the complex industrial, biomechanics systems, etc.

Two formulations, the FFRF and ANCF, used widely to model flexible structures have been reviewed with contributions published in recent years. Limitations and applications of them have also been pointed out. In this thesis, the floating frame of reference formation is used to model the boom of the BWE as the elastic beam because

- Advantages of using local coordinates to define flexible deformations with respect to it coordinates result in simple expressions of geometric stiffening matrices.

- Higher-order coupling effects of flexible deformations can be taken into account in the modeling with an elastic line approach.

- Matrix methods combined with beam theory of 3rd order can be applied to generate the equations of motion in terms of mass, damping, and stiffness matrices. Thus, standard structural dynamic programs can be used to simulate the dynamic behavior of the Bucket-Wheel boom during operation.

- Observer-based controllers can be easily realized to control the system.

2.2 Vibration control of flexible structures

2.2.1 Introduction

Due to practical applications of flexible structures, like turbine blades, turbo-engine blades, and helicopter blades, etc., problems of vibration suppresion, precise position control, structural stability, noise reduction for this structures are neccessary. Thus, unwanted vibrations, which appear during operation, need to be eliminated or reduced as much as possible. The dynamic behavior as well as related dynamic effects in structural dynamics and machinery dynamics can be analyzed and predicted in detail using approximation approaches as mentioned in the previous section. From a mechanics/dynamics point of view for the practical engineering applications, many design and control approaches for suppressing undesirable vibrations are proposed and developed continuously over the last two decades. In this section, three approaches controlling vibrations of flexible structures will be reviewed such as active structural vibration control, passive structural vibration control, and hybrid structural vibration control.

2.2.2 Passive vibration control

Using an adequate modeling for describing a dynamic behavior of flexible structures, obviously, dynamic responses can be predicted accurately under different excitations. From a theoretical viewpoint, dynamic phenomena, parameters, and causes leading to vibrations of flexible structures, therefore, will be explained in detail. Understanding system characteristics is very useful for design engineers to modify the plant's properties, such as geometrical parameters (thickness, length, diameter, etc.) or material properties (Young modulus, density, etc.) expressed in terms of stiffness, mass, and damping. Moreover, the sufficiently extra elements, such as ball balancers, discontinuos springs, spring-dampers, etc., can be suitably added to structural or mechanical systems in order to obtain desired responses. This approach is known as passive vibration control of flexible structures and it will be reviewed with different aspects as vibration isolation, design modification.

Vibration isolation

In most practical applications of structures and machines, vibration isolators are widely applied. In general, vibration isolation is aimed using sufficiently soft elements, such as springs, spring-dampers, flexible materials, etc., which are installed between parts of structures to dissipate energy causing to vibrations. Therefore, a controlled system can, for example, significantly reduce oscillations transmitted to other parts of the structures; prevent them from resonance frequencies; protect equipments of the system from damage during operation, etc. In [HP02] the classification of types as well as characteristics of the passive vibration isolators are presented in detail. By doing this, the selection and combination of elements in the passive vibration isolators can be carried out for different applications of structures and machines.

In order to obtain desired responses with suitable cost, the optimization problem of elements along with positions of them in the passive vibration isolators is necessary and it is also a significant subject in many investigations [Kea95, KB96, AJG04, DMJ06]. The genetic algorithm for global optimization of complex systems is efficiently applied to optimize parameters of a car suspension system, which are modeled as a multi-degrees-of-freedom vibration isolator in [AJG04, DMJ06]. The optimal curves of a car suspension system depending on the design parameters will be achieved under various conditions. Thus, the design parameter values of the passive vibration isolators can be suitably selected for the best vibration reduction. For lightweight aerospace structures, the genetic algorithms are also applied in combination with unusual geometric configurations to optimize the isolation characteristics of the structures [Kea95, KB96]. The efficiency of this approach in the vibration energy isolation transmitted from the ends of a network of beams is verified by theoretical and experimental results.

Due to requirements from practical applications, structural and mechanical systems must be subjected to severe working conditions such as shocks, impact loads, earthquakes, etc. Considering such conditions result to large deflections of isolators and can cause damages to the system. Therefore, nonlinearities in design of vibration isolators are significant and need to be considered. Due to the high damping characteristics of viscoelastic materials, strategy using a material damping, such as metals, polymers, cement and composites, is wieldy used in structures and mechanical devices [SSR90, Chu01, HP02, Lak02]. A comprehensive assessment about recent advances in nonlinear passive vibration isolators is reviewed in [Ibr08]. Basic characteristics of nonlinear isolators as well as influence of them on the response of systems including the shift of resonance frequency, internal resonance, chaotic motion, etc. are investigated systematically. Applications of nonlinear vibration isolators, such as metallic nonlinear and viscoelastic nonlinear isolators, for vibration isolation in aerospace, building, bridge structures, automotive systems are also presented in detail. Furthermore, the concept of using highly deformed spring characteristics or discontinuous spring characteristics in design of isolators is also pointed out [VSP08, Ish11, Ish12]. A highly deformed slender beam attached to a vertically oscillating base can produce a negative stiffness for suppressing vibrations or forcing frequencies beyond the first natural frequency in [VSP08]. With automatic ball balancers, leaf springs added to rotating mechanical systems, the efficiency of suppressing harmonic resonances due to rotor unbalance, unstable vibrations at critical velocities, self-excited oscillations due to internal friction, or torsion vibrations are analyzed for in [Ish11, Ish12] for rotor system.

Design modification

Structural modification in design of structural and mechanical systems is technique to change geometrical parameters (thickness, length, diameter, etc.) or material properties (Young modulus, density, etc.). Based on this modification, dynamical responses of the system undergoing various working conditions can be simulated for finding the best parameters. Thus, reduction of vibrations, improvement of dynamic stability, and optimization in weight and cost of systems can obtain obviously. Generally, structural dynamic modification methods can mainly be divided into two groups, model-based structural modification and experimental data-based structural modification. In framework of model-based on structural modification, the dynamic characteristics of structural and mechanical systems can be modified by changing geometrical parameters or material properties in terms of mass, stiffness, damping matrices [Kun00, Bra01, Nad07, Tri07]. From this theoretical viewpoint, this method can be combined with optimal dynamic design techniques to improve the dynamic characteristics of the system. Actually, an accurate modeling for describing dynamical behavior can be difficult or impossible to integrate increasing demands on higher performance of complex mechanical and structural systems, such as machine tools, robots, aerospace systems, etc. Solving this problem, the identification of such systems can be defined using a modal testing (experimental data). In [SD89, DG08], experimentally determined frequency response functions will be used to describe the dynamic behavior of the system. Model adjustments based on this approach are accounted to elements of the system (lumped mass, axial spring, viscous damper, and dynamic absorber, etc.) for obtaining desired dynamic characteristics such as reduced vibration levels, dynamic stability, and mode shapes.

Passive vibration control method of mechanical and structural systems including vibration isolation and design modification has been reviewed in this section. Advances and developments of this method are also pointed out with contributions published in recent years. However, these methods are unsuitable for applying to Bucket-Wheel excavators because

- Extra elements to dissipate energy causing vibrations result to significantly increasing mass of BWEs.

- An accurate modeling exactly describing dynamic behavior of BWEs is impossible due to complex spatial structures of BWEs.

- This method is unable to adapt to changes in structural properties and external excitations when operating in severe conditions of BWEs.

2.2.3 Active vibration control

As above mentioned, by modifying physical and geometrical parameters of mechanical and structural systems, desired responses can be obtained with a passive vibration control method. Additionally, energy causing vibrations can be dissipated using extra elements to the system. Thus, unwanted vibrations affecting to normal working conditions of the system have been efficiently suppressed. This method depends on the accuracy of the selected mathematical modeling for describing the dynamic behavior of the system. Besides, due to addition of extra elements (single mass, spring, etc.) it also leads to significant increase in mass and therefore affecting costs of flexible structures. Overcoming above mentioned disadvantages, an active vibration control method has been suggested and applied to flexible structures. In principle, an active vibration control system consists of three basic components as follows:

- sensors (strain, force, velocity, etc.) which are located on the system to measure response characteristics like state variables of the system or uncertainties, etc.,
- controllers which receive signals coming from the sensors, analyze them and then calculate necessary control forces based on conventional control algorithms, and
- actuators which receive information sent from controllers to supply required external energy (forces, torques, acceleration, etc.) to system for reacting external disturbances.

In such a way, obviously, the properties of the system can be easily modified to reach the desired responses for structural and mechanical applications. Furthermore, advance controllers can be applied to change critical parameters or inputs for adapting with changing characteristics of the system to different working conditions. Therefore, this method has been widely used to control vibrations of flexible structures. A large number of researches focused on this problem have made over many decades. Different aspects affecting to the efficiency, robustness, and reliability of a controlled system, such as control algorithms, actuator/sensor configurations, will be briefly reviewed in this section with contributions published since 2000. Moreover, a review of a tendon control system will be also presented.

Control algorithms

Traditionally, tasks including motion control, vibration control, etc. in structural and mechanical systems are handled using feedforward techniques. Theoretically, desired responses as well as stability of the system can be guaranteed. However, in most case, controlled systems usually expose uncertainties due to difference between the actual plant and the mathematical modeling or changes of environmental and working conditions, etc. Therefore, feedforward controllers cannot perfectly adapt in these cases. Using suitable feedback signals (acceleration, velocity, or position) in combination with different control laws (the proportional, integral, derivative control) feedback controllers can easily adjust the system to obtain desired responses. Nowadays, tasks controlling structural and mechanical systems become more complicated due to practical requirements of engineering applications, i.e. machine tools, robots required with faster and more accurate motions. Thus, controllers must also be developed correspondingly to adapt with these complicated tasks. With significant developments in the mechatronic field, distributed piezoelectric sensors/actuators and advanced composite materials are integrated in structural systems to produce appropriate electric potential for controlling vibrations, shapes, and stability of the system. Such structures are so-called smart structural systems and used for advanced real engineering applications such as aerospace, civil, robots, vehicles, etc. Conventional feedback control algorithms, such as velocity feedback in [YBEF10, CL10], the acceleration feedback in [SHJ12], or the optimal control, linear quadratic regulator in [NAR+13], are performed to suppress vibrations in flexible structures using distributed piezoelectric sensors/actuators.

In practical applications of flexible structures, such as robot arms, spacecrafts, helicopter blades, etc., couplings between flexible deformations and rigid-body motions result in very complex dynamic behavior when operating at high-speed motions or external excitations. Strategies of using conventional control methods (PD-controller, PID-controller, etc.) seem to be difficult or impossible for solving control problems. To overcome these challenges, some different control methods can be combined in the system to improve the performance of the control system, such as the efficiency, capacity, and durability [YJC04, HM05, QHZ+09, WQHW10, LGZ12]. In [YJC04] positive position feedback control and the momentum exchange feedback control are combined to control vibrations of a uniform Euler-Bernoulli beam attached to a rotating rigid hub. Positive position feedback control in which a second-order scalar system subjects to second-order actuator is applied to the root of the flexible beam for suppressing the vibrations. Additionally, in order to improve the performance of vibration suppression of a flexible beam, momentum exchange feedback control is applied simultaneously to rigid hub. Dramatic results in suppressing vibrations of the rotating elastic beam can be obtained using this control strategy. In [HM05] integration of variable structure control and positive position feedback control using the smart materials is successfully applied for reducing vibrations of the flexible spacecraft during attitude maneuver. A large number of degrees of freedom used to model flexible structures results to increase in a computational effort and difficulty in control design. Instead of this, a reduced model is used to design controllers. However, due to effects of the residual vibration modes controllers based on a reduced model lead to observation and spillover problems. Solving the problems, two control algorithms, a sliding mode variable structure control with phase shifting technology for eliminating control spillover problem, high frequency noise and an acceleration proportional feedback control for attenuating both the larger and lower amplitude vibrations, are combined efficiently in [QHZ+09]. The suppression of vibrations including higher and lower amplitude vibration in flexible structures is also obtained in [WQHW10] using the dual-mode controllers, in which the adopted fuzzy control and proportional integral control algorithms are integrated in the system using the collocated piezoelectric sensors/actuators. Problems controlling the high precision attitude for flexible spacecrafts under general uncertainties due to the unwanted vibration modes, external disturbances, and differences between the actual plant and the mathematical modeling are discussed in [LGZ12] using a composite controller with hierarchical architecture combination of two controls (a disturbance observer-based control and PD- control). The effects of the uncertainties (vibrations resulting from the flexible appendages) are compensated with the disturbance observer-based controller and the attitude of the flexible spacecraft can be accurately controlled with PD-controller. The simulation results prove the efficiency of the composite control strategy for stabilizing the system and improving the precision attitude of flexible spacecrafts.

For aerospace, civil, and mechanical engineering applications operating in real conditions, robustness is an important characteristic of controlled systems. The different perspectives of the robust control under highly nonlinear effects, uncertainties due to modeling errors, external disturbances, etc., are discussed in many contributions. In [HN05] the robust control method to actively suppress vibrations of flexible structures in case of considering the model parameter uncertainty due to the structured and unstructured uncertainty is presented. The robustness as well as stability of the controlled system is analyzed using the Lyapunov stability theory. The efficiency of this method in active vibration suppression is proved by the simulation and experiment results for a flexible circular plate which is integrated with piezoelectric sensors/actuators. For a class of general external disturbances, a robust H_2 linear feedback controller is proposed in [SFH$^+$05] for controlling vibrations of flexible structures. Moreover, the problem of the uncertain contact due to interaction between flexible structures with external bodies resulting to unstable behavior or poor control performance of the system is discussed in [CWP08]. The Popov stability criterion-based controller is proposed for solving a class of system with uncertain contact characteristics modeled by a sector-bounded non-linear function. The vibration attenuation and system stability of the proposed method are verified through theoretical and experimental results. In [PX14] a novel robust control strategy in the framework of a nonlinear optimal control combined with θ-D method is implemented controlling flexible nonlinear robot manipulators under external unknown loads or large changing loads. In such strategy, not only robust stability of the system under uncertainties can be guaranteed but also performance optimization can be obtained.

In addition to above mentioned control algorithms, the fast and precise control performance as well as the stability of flexible structures considering nonlinear effects, modeling uncertainties, and external disturbances, etc., can be obtained using the advanced control algorithms such as fuzzy control, adaptive control, sliding mode control algorithms in [SKA06, HDLW07, Hu08, WQHW10].

Actuator and sensor placement

During several decades, sensors/actuators using piezoelectric materials have been used successfully to suppress unwanted vibrations of flexible structures. In general, piezoelectric sensors/actuators are distributed on the system to produce required external energy for reacting to external disturbances. Therefore, controllers using such smart materials can adapt to system changes. For the real applications of flexible structures, the number and placement of sensors/actuators can be arranged and combined in many different ways. Thus, the arrangement will affect to controllability and observability of system and obviously it will also affect the performance of the control system. The problem of the optimal configuration as well as the location of piezoelectric sensors/actuators are investigated in many contributions. The sensors/actuators location and orientation optimization on the structure can be obtained using different algorithms. In [MYK11] the invasive weed optimization algorithm, in which the objective function is approximated based on the maximization of frequency response function peaks of the system, is applied for finding the optimal placement of the piezoelectric actuators on the smart fin with the best vibration attenuation. Using H_2 norm in [QZWZ07], the particle swarm optimization technique in [ZSS12] , and the genetic algorithms in [BGN10, LH13], the optimization problems, such as size, location, and orientation optimization of sensors/actuators, have been solved in order to obtain the best efficiency in suppressing vibrations.

Tendon control system

In flexible structures, due to advantages of lightweight stiffening, ability for shape control, cable-structure systems have widely been used not only in aerospace engineering applications but also in civil engineering applications. Cable-structure systems consist of groups of cables connected with beam, area, and solid elements to guarantee equilibrium of the whole system. By measuring cable tension forces and changing the tip displacement of the tendon, instead of using piezoelectric actuators as the above mentioned conventional approaches, the stiffness of structures will increase correspondently. From this point of view, challenging problems, such as vibration attenuation as well as stabilization of the system, in the active vibration control of flexible structures can be easily solved in a simple way.

In [PAB00, PB00] the inertia, mass, and dynamics of suspended cables are assumed to be neglected. Its interaction with the structures is limited by the tension of the cables. The active damping of vibrations in space truss structures is guaranteed using the (positive) integral force feedback of active tendons with displacement actuators and force sensors. The simulation and experimental results confirm the efficiency of this control approach, including stability properties, robust properties with respect to uncertainties of the actuators and sensors (the nonlinearity of the actuator, noise in the sensor, etc.), and ability for controlling microvibrations. In the same framework of a force feedback control algorithm for suppressing vibration of the cable-structures, control algorithms are improved in [GLL08] with a proportional integral force feedback control law and in [GLC12] with a differential force feedback control law. With the added feedback terms, the active damping efficiency of vibrations in space structures can be significantly improved when compared with the integral feedback control.

Controlling vibrations of a flexible cantilever beam, in [NMS06], a cable is connected at the tip of the beam to produce the forces for removing the vibration energy from the structure. The controller, in which the control input (here the tension in the cable) is suitably switched below the bucking load, is efficiently designed for obtaining the vibration suppression. The effects of the bandwidth of the actuator, the attendant characteristics of the filter, and change of the control force, bias force on the control performance are analyzed in detail. Based on the work of [NMS06], the active vibration control for the general class of frame structures including dynamics of structures and effects of interactions between the cable and structure is developed in [IMS10]. Two effects of cable tensions, a parametric effect causing the change of structural stiffness and a direct effect providing the external forces to deform the configuration, on the structures are considered to synthesize a controller for vibration suppression of the frame structures. In addition, the optimization of cable placements on the structure in order to obtain the best vibration attenuation is also presented.

2.2.4 Hybrid vibration control

As pointed out in the previous sections, unwanted vibrations, which appear during operation, can be eliminated or reduced using the passive and active vibration control methods. However, each of them has not only advantages but also disadvantages. For passive control methods, extra elements embedded to systems lead to the complexity in the structures along with increase in cost. With respect to active control methods, in some cases of real structural and mechanical applications, optimization of sensors/ actuators is difficult or impossible. Thus, it can degrade performance of control system. To overcome these challenges, both the passive and the active control method are combined together for controlling the system. This approach is denoted as hybrid control of structures, resulting in effective structural vibration suppression. A detailed review of advantages in hybrid structural vibrations control is presented in [Ben01]. The high damping characteristics of viscoelastic materials are combined with external energy sources produced from the piezoelectric materials to synthesize the hybrid damping configuration. The different aspects in hybrid vibration control of beams, plates, and shells are reviewed in detail, such as modeling of hybrid damping, piezoelectric effects on the response of the structure, response analysis methods, control strategies, etc. The current and developing trends in field of hybrid active-passive damping treatments are also discussed. The experimental studies of combining active piezoelectric materials and passive viscoelastic materials (both separately and together) with different configurations for controlling vibrations of beam structures are presented in [Lan05,Tri10]. Results confirm that a hybrid passive-active damping configuration provides the efficient damping method including high performance, reliability, robust, etc. In [NLLG08] active force actuators are installed parallel with passive spring isolators (passive dampers) on the coupled source-mount-receiver or source-path-receiver systems for reducing the vibration transmission between the machine and foundation. In addition, vibrations of the flexible foundation are cancelled using torque actuators such as piezoelectric patches. An optimal control strategy is applied for controlling the power flows transmitted into the foundation. Efficiency of this hybrid control method for significant vibration damping is proved by simulation results for complex flexible coupled systems. In [MHK13] the passive technique, in which viscoelastic layers are placed between the base structure and a constrained layer to generate shear stresses for passive damping, is proved to be efficient in high-frequency ranges. On the other hand, the semi-active pulse switching technique is only efficient to damp vibrations without external energy sources in low-frequency ranges. Combining these two control methods result to significant damping of vibrations in all frequency ranges.

Recent research on active vibration control methods of flexible structures performed since 2000 is surveyed. With ability to adapt to a wide range of different operating conditions, these methods can be used to control Bucket-Wheel excavators. However, active vibration control methods using smart materials like piezoelectric sensors/ actuators are difficult to apply to BWEs due to the very large, heavy weight structure of them. For large space structures like BWEs, due to advantages of lightweight stiffening, ability for shape control the method of using tension cables combined with control algorithms is efficient to stiffen and control such systems.

2.3 Summary

In this chapter, the most two widely used formulations, FFRF and ANCF, to model dynamics of flexible structures are briefly reviewed with the defined coordinate systems, the derivation of the elastic forces using an elastic line approach, and a continuum mechanics approach, etc. Based on these two formulations, problems in dynamical analysis of flexible structures are also reviewed from papers published in recent years. Dynamic coupling problems undergoing overall large motion and arbitrary elastic deformations, large deformations, and the choice of shape functions are discussed. Comparisons and limitations of them in dynamical analysis of flexible structures are pointed out for mechanical and civil engineering applications. From these theoretical points of view, the floating frame of reference formation is selected to model the boom of Bucket-Wheel excavator. Moreover, methods for vibration control of flexible structures, including passive vibration control, active vibration control, and hybrid vibration control, are briefly reviewed. From analysis and comparison of above mentioned methods, the method of using tension cables combined with control algorithms is selected to control the Bucket-Wheel boom of the BWEs. In next sections, modeling and control of the Bucket-Wheel excavator using selected methods will be discussed.

3 Modeling of the Bucket-Wheel excavator

3.1 Introduction

Due to the advantages of a high excavation productivity (from 420 to 22700 m^3 of overburden per hour) and low production costs, large mining machines like Bucket-Wheel Excavators (BWEs) are widely used in open cast coal mining industry and become one of the most important heavy machines in opencast mining. The BWE is a favorable combination of complex spatial structures, like the Bucket-Wheel, slewing, hoisting mechanism, crawlers, and electric equipment. The operation of the BWE is carried out by combining continuous rotation of the Bucket-Wheel in the vertical plane with the main slewing mechanism in the horizontal plane. During digging process the BWE is exposed to a number of external forces and disturbances like digging resistances on the Bucket-Wheel that cause transverse, longitudinal, and torsional vibrations. All vibrations can be affected by normal working conditions, operational effectiveness, and may effect the stability of the BWE. To increase working conditions advanced control systems can be applied controlling dynamics, especially induced structural vibrations. In order to analyze and synthesize a controller for the above mentioned system, adequate modeling to exactly describe the dynamical behavior of the system under operating conditions is necessary.

The dynamic behavior of the BWE is presented in [BZO06, SS10, SS12] using various dynamic models of the structure, mechanisms, and excitations. By analyzing these dynamic models in [BZO06] several practical problems of dynamics, which occur during operation of BWE as defining loads on the Bucket-Wheel boom, analysis of impacts of counterbalance weights on dynamic parameters and system response, are solved. During exploitation of the BWE characteristics of external excitations and resistances to the motion of the Bucket-Wheel are changeable and random. It is necessary to describe exactly these forces, in order to define the adequate dynamic behavior of the mechanisms depending on external excitations and disturbances caused by various components of resistances to digging. Detailed research work is investigated in [VKK66], [GGK+77], and [PJMM03]. External effects from the electric motor as well as resistances to the digging process of the BWE are modeled in [PJMM03] as non-linear functions of the angular velocity and random functions of time, respectively. Horizontal and vertical oscillations of the working unit of the BWE are studied in [Chu07, Chu08]. A simplified modeling with one degree of freedom for studying the dynamic system of working unit oscillations is presented. Parameters of the digging process have been determined suitably in order to obtain effective suppression of vibrations during operation of the BWE. However, due to the use of simplified models actual researches focusing on this problem are not satisfying with respect to the control of the system. Due to the very large structure the task is to maintain the static and dynamic stability of the BWE or other components

Figure 3.1: Bucket-Wheel Excavator [Ima14]

of the system during operation. The static stability of the BWE on an inclined ground surface is given in [Ber79]. Two methods for determining stability as well as defining conditions related to the prevention of slipping on a sloping surface are discussed. Investigations of the dynamic stability during the operational process of the BWE are reported in [Chu09]. Equations of motion of the system using d'Alembert's principle are established and the Routh-Hurwitz criterion is used to analyze conditions of stability of the system. Areas of instability including points of operating parameters of machines as well as parameters of the digging process are determined. However, actual researches focused on this problem are not satisfying with respect to the control of the system.

In this chapter, it is assumed that the Bucket-Wheel boom can be modeled as a flexible beam using the Euler-Bernoulli beam theory. Additionally it is assumed that the boom is attached to the excavator turning platform. The nonlinear modeling of the three-dimensional elastic boom considering the elasticity of suspending cables and also couplings resulting from geometrical nonlinear deformations is presented. Based on the work of Söffker [Söf96], here the known modeling approach of higher order is used and extended to model the Bucket-Wheel boom of a Bucket-Wheel-Excavator including guided rotating motion in combination with digging resistance forces. Therefore, dynamic phenomena resulting from the higher-order modeling including higher-order geometrical couplings as well as the external excitations on the dynamic behavior of the Bucket-Wheel boom can be analyzed.

3.2 Assumptions

The Bucket-Wheel boom is attached with the excavator turning platform (assumed as the rigid body) which is driven by the main slewing mechanism of the BWE. The front part of the boom is suspended by cables. Basic assumptions used in this model are:

- Space trusses of the boom are assumed as an equivalent continuum elastic beam.
- The cross section of a beam is infinitely rigid in its own plane and remains plane after deformation.
- The beam's cross section remains perpendicular to the deformed axis of the beam.
- Shear due to bending and warping due to torsion are neglected.
- The inertia and dynamics of the hoisting cables of the boom are neglected. Thus, the hoisting cables are modeled as linear springs and attached only to the nodal point of the boom.
- External forces to digging process are acting simultaneously on the boom-tip according to the three directions (x-, y, and z- direction).
- The height of the spoil pile is assumed to be equal with height of the joint of the Bucket-Wheel boom. Therefore, the Bucket-Wheel boom is approximately in its horizontal position.

3.3 Modeling of Bucket-Wheel boom

In the following parts of this chapter, approach for modeling the Bucket-Wheel Boom is presented based on the contributions in [Söf96, LS12b, LS13a, LS13b, LS14b]. The dynamic model and used coordinate systems of the Bucket-Wheel boom during operation are illustrated in Figure 3.2. The frame $O - X^I Y^I Z^I$ is the inertial reference frame. The moveable joint coordinate system $G - X^G Y^G Z^G$ is a beam-fixed frame with its origin located at the center of the cross-section. The frame $E' - X^E Y^E Z^E$ is the reference frame defined for the cross-section to describe elastic variables u_x, u_y, u_z in the moveable joint coordinate system. The frame $P - X^P Y^P Z^P$ is defined for the position and orientation of the considered infinite section.

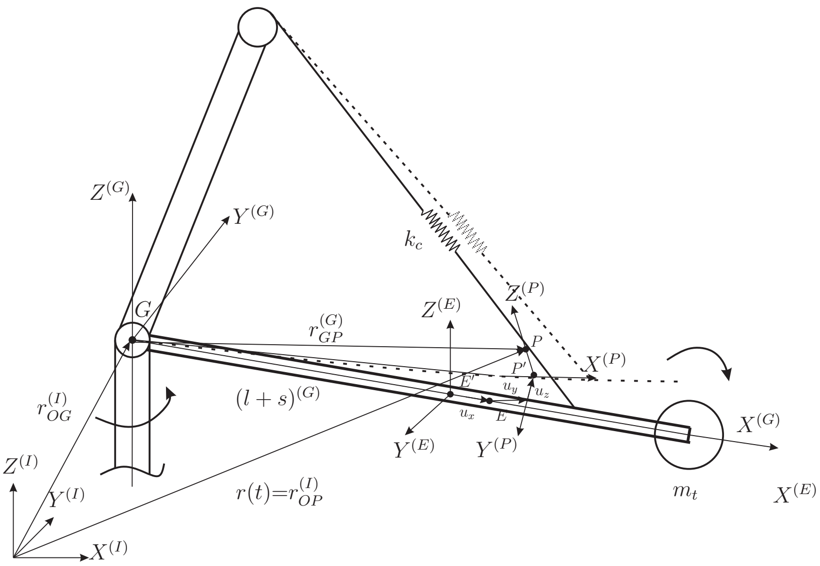

Figure 3.2: Sketch of the Bucket-Wheel boom

3.4 Equations of motion of Bucket-Wheel boom

3.4.1 Dynamic equations of the flexible boom

The position vector of a finite mass point dm (Fig.1) in the finite disc (P) with respect to the inertial reference is given by

$$\begin{aligned} r(t) &= r_{OP}^{(I)}(t) = r_{OG}^{(I)}(t) + r_{GP}^{(I)}(t) \\ &= r_{OG}^{(I)}(t) + T(l_{GE'}^{(G)} + s_{E'E}^{(G)} + u_{EP'}^{(G)} + \widehat{T}t_{P'P}^{(E)}), \end{aligned} \tag{3.1}$$

with $r_{OG}^{(I)}(t), r_{GP}^{(I)}$ as the vectors of the joint and the mass point from the joint respectively, in the inertial reference frame, the deformation of the centroidal axis of the considered cross-section is described by the vector of the elastic deformations $u_{EP'}^{(G)}$. The vector $r_{GP}^{(I)}(t)$ (described in the moveable joint coordinate system) also includes the vectors $l_{GE'}^{(G)} s_{E'E}^{(G)}$ describing the joint oriented boundaries of the considered finite element and the position of the undeformed disc in the element coordinate system, and the vector $\widehat{T}t_{P'P}^{(E)}$ describing the finite mass point of the stiff disc in the deformed position. The rotation matrices T and $\hat{T}$ are needed to define the element and joint coordinate respectively based on the variables of $r(t)$, which are given in the inertial reference frame. The rotation matrix T using three Tail-Bryan angles (Φ, Θ, and Ψ) and the corresponding order of rotations is obtained in [Söf96] by

$$T = \left(\begin{array}{c|c|c} c\Theta c\Psi & c\Theta s\Psi & -s\Theta \\ \hline s\Phi s\Theta c\Psi & -c\phi s\psi s\Phi s\Theta s\Psi + c\Phi s\Psi & s\Phi c\Theta \\ \hline c\Phi s\Theta c\Psi + s\phi s\psi & c\Phi s\Theta s\Psi - s\Phi c\Psi & c\Phi c\Theta \end{array} \right) \tag{3.2}$$

with $c\Theta = cos\Theta$, $s\Theta = sin\Theta$, and can be expressed defining the operation point Φ=Θ=Ψ=0, by the linearized expression

$$\widehat{T} = \left(\begin{array}{c|c|c} 1 - \frac{\beta^2}{2} - \frac{\alpha^2}{2} & \gamma & -\beta \\ \hline \alpha\beta - \gamma & 1 - \frac{\alpha^2}{2} - \frac{\gamma^2}{2} & \alpha \\ \hline \beta + \alpha\gamma & \beta\gamma - \alpha & 1 - \frac{\alpha^2}{2} - \frac{\beta^2}{2} \end{array} \right), \tag{3.3}$$

using the results given in [Söf96]

$$\beta = u_y^{'} - u_x^{'} u_z^{'}, \tag{3.4}$$
$$\gamma = u_z^{'} - u_x^{'} u_z^{'}, \text{and} \tag{3.5}$$
$$\epsilon = \sqrt{(1 + u_x^{'})^2 + u_y^{'2} + u_z^{'2}} - 1. \tag{3.6}$$

Here the result is expressed as functions of the elastic variables, where u_x, u_y, u_z denote the elastic motion components of u in the joint coordinate system, with the assumption $u_x^{'} = u_y^{'2} + u_z^{'2}$, $(u_x^{'2} \approx 0)$. The apostrophe ($u_x^{'}, u_y^{'}$, and $u_z^{'}$) denotes differentiation with respect to the x-coordinate. The transverse, longitudinal, and torsional displacements are related to the node variables and are approximated by using the usual and well-known interpolation functions for a finite element of length $=x_i/l_i$. Using Hermite polynomials the known separation principle (here shown the bending displacement $u_y(x,t)$) can be applied. Applying a vector of shape functions $f_y(x)(f_y(x) = g(x))$, the relations for the elastic coordinates result to

$$u_y(x,t) = f_y^T(x) u_y(t), \tag{3.7}$$
$$u_y^{'}(x,t) = f_y^{'T}(x) u_y(t), \tag{3.8}$$
$$u_y^{''}(x,t) = f_y^{''T}(x) u_y(t). \tag{3.9}$$

The virtual displacements for bending are obtained as

$$\delta u_y(x,t) = \delta u_y^T(t) f_y, \tag{3.10}$$
$$\delta u_y^{'}(x,t) = \delta u_y^T(t) f_y^{'}(x), \text{and} \tag{3.11}$$
$$\delta u_y^{''}(x,t) = \delta u_y^T(t) f_y^{''}(x). \tag{3.12}$$

The elastic longitudinal strain ϵ and the curvatures κ_1, κ_2, κ_3 are

$$\epsilon = u_x^{'} + \frac{1}{2}(u_y^{'2} + u_z^{'2}), \tag{3.13}$$
$$\kappa_1 = -\alpha^{'} + u_y^{''}u_z^{'}, \tag{3.14}$$
$$\kappa_2 = -u_y^{''} + u_y^{'}u_x^{''} + u_y^{''}u_x^{'} - \alpha^{'}u_y^{'}, \text{ and} \tag{3.15}$$
$$\kappa_3 = -u_z^{''} + u_x^{'}u_z^{''} + u_x^{''}u_z^{'} + \alpha^{'}u_y^{'}. \tag{3.16}$$

and determined as functions of the elastic variables. The virtual displacements of the elastic variables are obtained by

$$\delta\epsilon = \left[1 - \frac{1}{2}(u_y^{'2} + u_z^{'2})\right]\delta u_x^{'} + \left[u_y^{'} - u_x^{'}u_y^{'}\right]\delta u_y^{'} + \left[u_z^{'} - u_x^{'}u_z^{'}\right]\delta u_z^{'}, \tag{3.17}$$
$$\delta\kappa_1 = -\delta\alpha^{'} + u_y^{''}\delta u_z^{'} + u_z^{'}\delta u_y^{''}, \tag{3.18}$$
$$\delta\kappa_2 = -u_y^{''}\delta u_x^{'} + u_y^{'}\delta u_x^{''} - u_z^{'}\delta\alpha^{'} - \alpha^{'}\delta u_z^{'} + u_x^{''}\delta u_y^{'} + (1 + u_x^{'})\delta u_y^{''}, \text{ and} \tag{3.19}$$
$$\delta\kappa_3 = (u_x^{'} - 1)\delta u_z^{''} + u_x^{''}\delta u_z^{'} + u_z^{'}\delta u_x^{''} + u_z^{''}\delta u_x^{'} + u_z^{''}\delta u_x^{'} + u_y^{'}\delta\alpha^{'} - \alpha^{'}\delta u_y^{'}. \tag{3.20}$$

The stiffness matrix is determined by the virtual potential energy of the deformed beam using four variables representing the axial strain and the curvatures

$$\delta V_e = \int_l^0 (\delta\epsilon N + \delta\kappa_1 M_x + \delta\kappa_2 M_y + \delta\kappa_3 M_z)dx, \tag{3.21}$$

where $N = EA\epsilon$, $M_x = GI_t\kappa_1$, $M_y = EI_y\kappa_2$, $M_z = EI_z\kappa_3$ (assuming constant material behavior for Young's modulus and the shear coefficient with E, G). Effects of different order couplings of the elastic boom undergoing the overall motion and arbitrary flexible deformations are described in the form of the stiffness matrix, which is determined by the virtual potential energy of the deformed beam, as given in Equation (3.21), using the elastic line approach (EL). With this approach, the deformed curve of the flexible beam is defined as a curved 2D or 3D line with the vectors of position gradients along the beam centreline. The deformations are expressed in terms of the longitudinal strain (ϵ) as well as the curvatures ($\kappa_1, \kappa_2, \kappa_3$) of a 3D curve and determined as functions of the elastic variables (u_x, α, u_y, u_z). Neglecting or retaining the second-order (and higher) in the strain energy function, the model with different order couplings can be easily considered [Söf96, LS13a].

Linear modeling (LF)

$$\delta V_e = \int_0^l (\delta u_x^{'}EAu_x^{'} + \delta\alpha^{'}GI_t\alpha^{'} + \delta u_y^{''}EI_yu_y^{''} + \delta u_z^{''}EI_zu_z^{''})dx \tag{3.22}$$

The linear modeling described by the Equation (3.22) contains the first-order term in the strain-displacement relationship, and therefore neglects the coupling effects of flexible deformations.

First nonlinear modeling (NF1)

$$\delta V_e = \int_0^l (\delta u_x^{'} EA u_x^{'} + \delta \alpha^{'} GI_t \alpha^{'} + \delta u_y^{''} EI_y u_y^{''} + \delta u_z^{''} EI_z u_z^{''} + u_y^{'} \delta u_y^{'} EA u_x^{'} + u_z^{'} \delta u_z^{'} EA u_x^{'}) dx \tag{3.23}$$

Considering the second-order term in the the strain-displacement relationship, the coupling effects of the bending deformations due to axial displacement are expressed in last two terms of the Equation (3.23).

Second nonlinear modeling (NF2)

$$\delta V_e = \int_0^l (\delta u_x^{'} EA u_x^{'} + \delta \alpha^{'} GI_t \alpha^{'} + \delta u_y^{''} EI_y u_y^{''} + \delta u_z^{''} EI_z u_z^{''} + \delta u_x^{'} EA \frac{1}{2}(u_y^{'2} + u_z^{'2})) dx \tag{3.24}$$

The bending deformations caused by longitudinal displacement (the axial foreshortening effect) are expressed in last two terms of the Equation (3.24).

Third nonlinear modeling (NF3)

$$\delta V_e = \int_0^l (\delta u_x^{'} EA u_x^{'} + \delta \alpha^{'} GI_t \alpha^{'} + \delta u_y^{''} EI_y u_y^{''} + \delta u_z^{''} EI_z u_z^{''} - u_y^{''} \delta u_z^{'} GI_t \alpha^{'} - u_z^{''} \delta u_y^{'} GI_t \alpha^{'}) dx \tag{3.25}$$

The bending deformations caused by the torsion displacement are expressed in last two terms of the Equation (3.25).

In this study, the coupling effects are limited between longitudinal and bending deformations (neglecting the torsional deformation). Therefore, the nonlinear modeling is considered with the first and second nonlinear modeling (NF1 + NF2). This allows capturing the geometrically nonlinear stiffening effect in the modeling of the BWE.

The mass matrix of the elastic boom is determined by the virtual work of the inertia forces

$$\delta W_m = -\rho \int \int \int \delta r_{OP}^{(I)T} \ddot{r}_{OP}^{(I)} dV, \tag{3.26}$$

where ρ, $\delta r^{(I)}_{OP}$, and $\ddot{r}^{(I)}_{OP}$ denote the density of the material (ρ=constant), the virtual displacement, and second time derivative, respectively. From Equation (3.1) the virtual displacement $r^{(I)}_{OP}$ and second time derivative $\ddot{r}^{(I)}_{OP}$) can be calculated as

$$\begin{aligned} \delta r^{(I)}_{OP} &= \delta r^{(I)}_{OG} + T\left(\delta l^{(G)}_{GE'} + \delta s^{(G)}_{E'E} + \delta u^{(P)}_{EP'} + \hat{T}\delta t^{(E)}_{P'P}\right) \qquad (3.27) \\ \ddot{r}^{(I)}_{OP} &= \ddot{r}^{(I)}_{OG} + \ddot{T}\left(l^{(G)}_{GE'} + s^{(G)}_{E'E} + u^{(P)}_{EP'} + \hat{T}t^{(E)}_{P'P}\right) + 2\dot{T}\left(\dot{l}^{(G)}_{GE'} + \dot{s}^{(G)}_{E'E} + \dot{u}^{(P)}_{EP'} + \dot{\hat{T}}t^{(E)}_{P'P}\right) \\ &\quad + T\left(\ddot{l}^{(G)}_{GE'} + \ddot{s}^{(G)}_{E'E} + \ddot{u}^{(P)}_{EP'} + \ddot{\hat{T}}t^{(E)}_{P'P}\right). \qquad (3.28) \end{aligned}$$

The virtual work of the inertial terms follows a complex series of terms and is given in detail in [Söf96]. Applying the principle of virtual work, the following dynamical equations of motion is finally obtained for the flexible boom as follows

$$M^B\ddot{u} + D^B\dot{u} + K^B u = f_{g1} + f_{g2} + f_{gg}, \qquad (3.29)$$

using the mass matrix M^B, the damping matrix D^B, the stiffness matrix $K^B = K^{B1} + K^*(u,t)$ of the elastic boom of the BWE, the nonlinear stiffness matrix $K^*(u,t)$ due to couplings of flexible deformations, the vector of external forces and moments f_{g1}, f_{g2} respectively resulting from translational and rotational motion of the joint, and the vector of gravitational effects f_{gg}. The matrices and the vector of external forces and torques are given in detail in [Söf96].

Contribution of hoisting cables to elastic boom dynamics

Considering the cable structure system of the BWE (Figure 3.1), groups of hoisting cables are connected at the structure nodes between the Bucket-Wheel boom and the tower of the BWE. As previously pointed out in tendon control system, advantages of using the hoisting cables added in the structure are to stiffen a truss in terms of weight, to eliminate of the geometric uncertainties due to gaps, and to bring active damping into the system. However, nonlinear dynamic effects of the hoisting cables interacting with the structure, such as cable sag, stretching due to cable vibration, reaction forces due to transverse motion of the cable, etc. on the dynamic response of the system are very complicated. In [PAB00, GLL08, Pre11] these effects are discussed in detail.

In this section, the effect of the hoisting cable stiffness entering into the global stiffness matrix of the system will be investigated for the cable-structure interaction. The inertia and dynamics of the hoisting cables of the Bucket-Wheel boom are assumed to be neglected in the vibration control. Its interaction with the structure is limited by the tension of the hoisting cables (F^C), which are modeled as linear springs and attached only to the nodal point of the boom. Using the Equation

(15.3) given in [Pre11], the governing equations of the flexible boom considering there effects can be expressed in the form as

$$M^B\ddot{u} + D^B\dot{u} + K^B u = -B^* F^C + f_{g1} + f_{g2} + f_{gg}, \tag{3.30}$$

where B^* denotes the influence matrix of the cable forces projected in the global coordinate system and F^C denotes the tension force vector of the hoisting cable, which is given as

$$\begin{aligned} F^C &= [F_x^C \;\; 0 \;\; F_y^C \;\; F_z^C]^T \\ &= K_c[u_x sin(\phi) \;\; 0 \;\; u_y cos(\phi) \;\; u_z cos(\phi)]^T \\ &= K_c B^{*T} u, \end{aligned} \tag{3.31}$$

where $B^{*T}u$ represents the relative displacements of the hoisting cables on the structure, ϕ is the inclination angle of the hoisting cables with the longitudinal axis of the boom, K_c is the stiffness matrix of the cables defined as

$$K_c = diag(k_c), \tag{3.32}$$

with

$$k_c = \frac{E_c A_c}{L_c}, \tag{3.33}$$

where k_c, E_c, A_c, and L_c denote the axial stiffness of the cables, Young's modulus, the cross-section, and length of the hoisting cables, respectively.

Replacing Equation (3.31) into Equation (3.30), the equations of motion of the elastic boom including the hoisting cables is expressed in the matrix form as

$$\begin{aligned} M^B\ddot{u} + D^B\dot{u} + K^B u &= -B^* K_c B^{*T} u + f_{g1} + f_{g2} + f_{gg}, \\ M^B\ddot{u} + D^B\dot{u} + \left(K^B + B^* K_c B^{*T}\right) u &= f_{g1} + f_{g2} + f_{gg}, \\ M^B\ddot{u} + D^B\dot{u} + \left(K^B + K^C\right) u &= f_{g1} + f_{g2} + f_{gg}, \end{aligned} \tag{3.34}$$

with $K^C = B^* K_c B^{*T}$. The effects of the hoisting cable entering into the equation of motion of the elastic boom can be described using Equation (3.34) in which $K^B + B^* K_c B^{*T}$ is the stiffness matrix of the elastic boom including the hoisting cables.

3.4.2 Dynamic equations of the tip mass of the Bucket-Wheel

With the heavy weight of the Bucket-Wheel attached to the tip of the boom for digging deposits in coal-mines, the effect of this mass on the dynamical response of the boom is important and need to be taken into account in the modeling of the Bucket-Wheel boom.

The position vector $r_m(t)$ of the tip mass m_t in the inertia system $O - X^I Y^I Z^I$ is given by

$$r_m(t) = r_{OG}^{(I)}(t) + T(l + u_{EP'}^{(G)}(t)). \tag{3.35}$$

According to [GKL12] the virtual work of the tip mass is given by

$$\delta W_m^M = -\delta r_m(t) m_t \ddot{r}_m(t). \tag{3.36}$$

The virtual displacement of the tip mass δr_m and the time derivatives $\dot{r}_m$, $\ddot{r}_m$ are calculated as

$$\dot{r}_m(t) = \dot{r}_{OG}^{(I)} + \dot{T}(l + u_{EP'}^{(G)}) + T\dot{u}_{EP'}^{(G)}, \tag{3.37}$$

$$\ddot{r}_m(t) = \ddot{r}_{OG}^{(I)} + \ddot{T}(l + u_{EP'}^{(G)}) + 2\dot{T}\dot{u}_{EP'}^{(G)} + T\ddot{u}_{EP'}^{(G)}, \text{ and} \tag{3.38}$$

$$\delta r_m(t) = \delta r_{OG}^{(I)} + T^T \delta u_{EP'}^{(G)}, \tag{3.39}$$

so Equation (3.36) results in

$$\delta W_m^M = - m_t \left(\delta r_{OG}^{(I)} + T^T \delta u_{EP'}\right) \left(\ddot{r}_{OG}^{(I)} + \ddot{T}(l + u_{EP'}^{(G)}) + 2\dot{T}\dot{u}_{EP'}^{(G)} + T\ddot{u}_{EP'}^{(G)}\right), \tag{3.40}$$

$$\delta W_m^M = - m_t T^T \delta u_{EP'} \left(\ddot{r}_{OG}^{(I)} + \ddot{T}(l + u_{EP'}^{(G)}) + 2\dot{T}\dot{u}_{EP'}^{(G)} + T\ddot{u}_{EP'}^{(G)}\right), \tag{3.41}$$

$$\begin{aligned}\delta W_m^M = -m_t(&\delta u_{EP'} T^T \ddot{r}_{OG}^{(I)} + \delta u_{EP'} T^T \ddot{T} l + \delta u_{EP'} T^T \ddot{T} u_{EP'}^{(G)} + \\ &2\delta u_{EP'} T^T \dot{T} \dot{u}_{EP'}^{(G)} + \delta u_{EP'}^{(G)} \ddot{u}_{EP'}^{(G)}).\end{aligned} \tag{3.42}$$

In connection between the rotation angles Φ, Θ, Ψ of the moveable joint coordinate system (G) and the angular velocities ω_1, ω_2, ω_3, the skew symmetric matrix $\tilde{\omega}$ is obtained as shown in [Söf96]

$$T^T \dot{T} = \tilde{\omega} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}, \tag{3.43}$$

and

$$T^T \ddot{T} = \tilde{\omega}^2 + \dot{\tilde{\omega}}, \tag{3.44}$$

where

$$\dot{\tilde{\omega}} = \begin{pmatrix} 0 & -\dot{\omega}_3 & \dot{\omega}_2 \\ \dot{\omega}_3 & 0 & -\dot{\omega}_1 \\ -\dot{\omega}_2 & \dot{\omega}_1 & 0 \end{pmatrix}, \tag{3.45}$$

$$\tilde{\omega}^2 = \begin{pmatrix} -\omega_2^2 - \omega_3^2 & \omega_1\omega_2 & \omega_1\omega_3 \\ \omega_1\omega_2 & -\omega_1^2 - \omega_3^2 & \omega_2\omega_3 \\ \omega_1\omega_3 & \omega_2\omega_3 & -\omega_1^2 - \omega_2^2 \end{pmatrix}. \tag{3.46}$$

Replacing Equations (3.43) and (3.44) into Equation 3.42 results to

$$\begin{aligned}\delta W_m^M = -m_t(&\delta u_{EP'} T^T \ddot{r}_{OG}^{(I)} + \delta u_{EP'}(\tilde{\omega}^2 + \dot{\tilde{\omega}})l + \delta u_{EP'}(\tilde{\omega}^2 + \dot{\tilde{\omega}})u_{EP'}^{(G)} + \\ &2\delta u_{EP'}\tilde{\omega}\dot{u}_{EP'}^{(G)} + \delta u_{EP'}^{(G)}\ddot{u}_{EP'}^{(G)}),\end{aligned} \tag{3.47}$$

$$\delta W_m^M = -\delta u_{EP'}^T m_t(T^T \ddot{r}_{OG}^{(I)} + (\tilde{\omega}^2 + \dot{\tilde{\omega}})l + (\tilde{\omega}^2 + \dot{\tilde{\omega}})u_{EP'}^{(G)} + 2\tilde{\omega}\dot{u}_{EP'}^{(G)} + \ddot{u}_{EP'}^{(G)}). \tag{3.48}$$

Using Equations (3.7) to (3.12) as well as a vector of shape functions, the virtual work of the tip mass is expressed as

$$\begin{aligned}\delta W_m^M = &-(\delta u_x^T \;\; \delta\beta_\alpha^T \;\; \delta u_y^T \;\; \delta u_z^T)^T [\\ &F_1^M \begin{pmatrix} \ddot{r}_{x,OG}^{(I)} \\ 0 \\ \ddot{r}_{y,OG}^{(I)} \\ \ddot{r}_{z,OG}^{(I)} \end{pmatrix} + F_2^M + K^M \begin{pmatrix} u_x \\ 0 \\ u_y \\ u_z \end{pmatrix} + D^M \begin{pmatrix} \dot{u}_x \\ 0 \\ \dot{u}_y \\ \dot{u}_z \end{pmatrix} + M^M \begin{pmatrix} \ddot{u}_x \\ 0 \\ \ddot{u}_y \\ \ddot{u}_z \end{pmatrix}].\end{aligned} \tag{3.49}$$

Applying the principle of virtual work, the contribution of the mass of the Bucket-Wheel to the system can be described in the vector form as

$$M^M \ddot{u} + D^M \dot{u} + K^M u = F^M, \tag{3.50}$$

with the mass matrix M^M, the damping matrix D^M, the stiffness matrix K^M, the vector force F^M of the mass of the Bucket-Wheel to the system.

3.4.3 Determination of cutting resistance forces

During digging process the BWE, cutting resistance forces exerted by the cutting knife of a bucket are main reason causing vibrations of the Bucket-Wheel excavator. It leads to large deformations of structures, resonant oscillations, appearance of fatigue cracks at connection parts of the BWE, etc. Therefore, dynamical responses of the BWE under these forces are important. Actually, determination of the digging force is complicated because the values and directions of this force are usually random and unknown. It depends on many factors such as the construction of the Bucket-Wheel, the characteristics of excavated soil, operating parameters of BWE, cutting methods, etc. In general, as shown in [Ras75, PJMM03], the digging force consists of three components:

- the tangential force acting on the cutting edge in the rotor's plane (F_T),
- the lateral force acting normal to the connecting line between Bucket-Wheel axis and the excavator slewing axis in the rotor's plane (F_N),
- the forward thrust force acting radially to the knife (F_B).

The tangential cutting force is the largest component compared with other components of the digging force. It plays an important role in determining the torque of the Bucket-Wheel drive as well as the digging power. However, a small number of contributions focused in determination of the tangential cutting force has been published in many recent years.

In early studies [Ras75], the cutting force is expressed in the following formulations.

- The tangential force is calculated using the specific cutting resistance referred to a cutting length of 1 cm [kg(f)/cm]

$$F_T = \frac{100}{2\pi} z f_L \sqrt{\frac{\alpha}{c}} \left(k_m c + \frac{\widehat{\varphi}_H}{\alpha} \right) \sqrt{\frac{I}{Rf}} \quad [\mathrm{kg}(f)]. \tag{3.51}$$

- The tangential force is calculated using the specific cutting resistance referred to a slice cross-sectional area A_m [kg(f)/cm^2] as

$$F_T = \frac{f_A c A}{\psi} \quad [\mathrm{kg}(f)], \tag{3.52}$$

with

f_L : Specific cutting resistance referred to a cutting length of 1 cm [kg(f)/cm],
z : Number of buckets,
α : Slice depth ratio,
c : Ratio of cutting height to wheel radius,
k_m : Correction factor for rounded knife corners,
I : Bucket capacity [m^3],
R : Wheel cutting circle radius [m],
f : Swell ($1.3 < f < 1.65$),
f_A : Specific cutting resistance referred to a slice cross-section areas A_m [kg(f)/cm^2] ,
A : Slice cross-section [cm^2], and
ψ : Angle in radians between buckets.

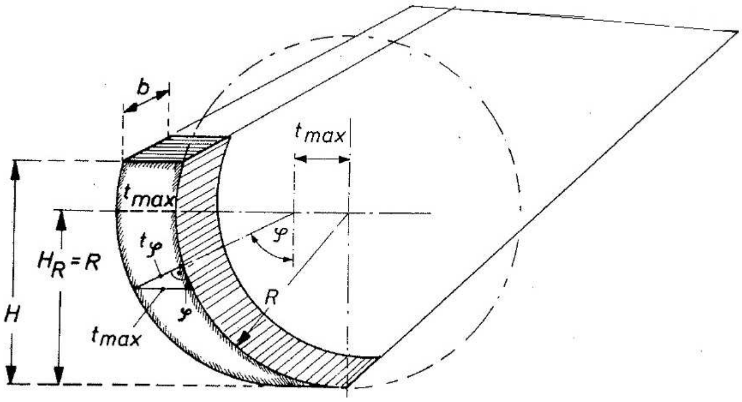

Figure 3.3: Change of the tangential force during exploitation [Ras75]

Calculation methods based on the specific cutting resistance referred to a cutting knife length or a slice cross-sectional area result to constant values of the tangential force. Actually, the direction and amplitude of it will change according to time and depend on the periodic contact of the spoons with the soil, as shown in Figure 3.3. Therefore, the dynamical behavior of the BWE exhibits inadequate responses undergoing the tangential force using the above formulations. In [PJMM03] digging resistance force is assumed to be changed on the spoons when digging non-homogenous massifs. It is described as nonlinear and random functions and is modeled as

$$F_T(\psi) = k_l l_{sr} f_o(\psi), \tag{3.53}$$

$$f_o(\psi) = \begin{cases} \sin\psi, & \text{if} \quad 0 \leq \psi \leq \pi/2, \\ \frac{\alpha-\psi}{\alpha-\pi/2}, & \text{if} \quad \pi/2 < \psi \leq \alpha, \\ 0, & \text{if} \quad \psi > \alpha, \end{cases} \tag{3.54}$$

with
k_l : specific resistance to digging with a random value which changes depending on the digging environment,
l_{sr} : active length of cutting edges of spoons, and
ψ : the angular coordinate of the rotor's revolution by using angular velocity.

As pointed out in Equation (3.53) , the value of the tangent force depends on the specific cutting resistance (k_l), which will change with different digging soils or different working methods (terrace cut, dropping cut) of BWE. Furthermore, the change of the tangent force relating to the change of positions of the spoon when exploitation is described by the function of time $f_o(\psi)$ depending angular velocity of the rotor of Bucket-Wheel.

The lateral digging force component F_N and side digging force component F_B can be determined based on the tangential force F_T using the following relations [Ras75]

$$F_N = k_N F_T, \tag{3.55}$$

$$F_B = k_B F_T, \tag{3.56}$$

where k_N and k_B are proportional coefficients of the normal and side components of the resistance to digging.

Besides, the general mathematical approaches for determining the resistance forces to the digging process, the experimental approaches using measured signals on the upper steel structure of the BWE are also applied to analyze the dynamical properties of the BWE undergoing the different operation conditions [Got12, OCJ13]. However, accuracy of these approaches depends mostly on the number of sensors and their positions installed on the structure. Therefore, they are only applied to the detailed case of the BWEs.

With advantages of using the nonlinear and random function for describing the digging force, the resistance force components to digging process causing the deformations of the BWE in this study are calculated using the approach given in [PJMM03]. These forces are assumed to be simultaneously acting on the tip of the boom according to the longitudinal, vertical, and horizontal direction.

3.4.4 Total dynamic equations of the Bucket-Wheel boom

From the Equations (3.34), (3.50), (3.53), (3.55), and (3.56), the total equations of motion of the Bucket-Wheel boom are expressed in the matrix form as follows

$$M\ddot{u}(t) + D\ddot{u}(t) + Ku(t) = f(t) + B^* z(t) + N^* n(u(t), t), \tag{3.57}$$

where M, G, and K are the global mass, damping, stiffness matricies, respectively, and the force vectors determined as

$$\begin{aligned} M =& M^B + M^M, \\ K =& K^{B1} + K^C + K^M, \\ N^* n(u,t) =& - K^*(u,t)u(t), \\ f(t) =& f_{g1} + f_{g2} + f_{gg} + F^M + F_T + F_N + F_B. \end{aligned}$$

From the Equation (3.57), it can be observered that all effects of the elasticity of the boom, the hoisting cables, and the mass-tip are considered to describe the dynamic behavior of the Bucket-Wheel boom undergoing the guided motion in combination with digging resistance forces.

3.5 Summary

In this chapter, it was assumed that the Bucket-Wheel boom can be modeled as a flexible beam using the Euler-Bernoulli beam theory and attached on the excavator turning platform. As a result the elastic and coupled effects of the boom along with the interaction between the cutting head and the face material during vibrations were taken into account. The nonlinear modeling of the three-dimensional elastic boom considering the elasticity of hoisting cables and couplings between flexible deformations was presented. The effects of the hoisting cables, resistance forces, and the tip mass on the dynamical responses of the Bucket-Wheel boom were considered. In addition, the modeling aprroach of higher-order is used and extended to model the Bucket-Wheel boom of a Bucket-Wheel-Excavator including guided rotating motion in combination with digging resistance forces. The approach includes especially geometrical nonlinearities. Therefore, dynamic phenomena resulting from the higher order modeling including higher-order geometrical couplings as well as the external excitations on the dynamic behavior of the Bucket-Wheel boom can be analyzed in detail. The developed model and the related dynamic system representation give a good base for the advanced study of the stability of the guided system in combination with external process forces resulting from the digging process of large excavator systems with long and slender booms.

4 Control system design for the Bucket-Wheel Excavator

4.1 Introduction

Bucket-Wheel excavators (BWE) represent a specific type of complex machine system used in mining technology. During operation, it is exposed to a number of external forces and disturbances like digging resistances on the Bucket-Wheel that cause transverse, longitudinal, and torsional vibrations. All vibrations will be affected during regular working conditions. This also may also effect the stability of the BWE. The task controlling dynamics of structures against vibrations is necessary and received special attention in structural dynamics. In the last two decades, many control approaches applied to complex machine structures, such as feedforward, feedback control, robust observer-based control, etc., has been continuously developed for reducing the vibrations and improvement of system responses [WST90, LC94, MMT+03, Fen13]. In the chapter 2 of this contribution, the nonlinear modeling considering the higher-order coupling effects of flexible deformations with guided motions in combination with external excitations result in the accurately dynamical behavior of the Bucket-Wheel boom under the real operating conditions. However, it also leads to difficulties in control problems of dynamic systems due to effects of nonlinearities. Normally, nonlinear control approaches are typically limited for special classes of problems. To overcome these difficulties and control them easily, in general, nonlinear dynamical systems are approximated by equivalent linear systems linearized for suitable working points. From this point of view, obviously, control problems of nonlinear dynamical systems can be solved applying linear control theory. A well-known strategy in combination with nonlinear dynamic system control is to use feedback linearization methods in [Isi95, Kha02]. Here nonlinearities are modeled as external disturbances, and can then be linearized by a suitable feedback. However, to apply this approach it is necessary to know nonlinearities as well as the states of system. Normally, direct measurements can be used to determine nonlinearities but, on the other hand, due to some kinds of them as geometric nonlinearities like the couplings of flexible deformations, it becomes impossible to apply these techniques. An indirect measurement to reconstruct nonlinear characteristics along with states of the system using an observer technique is proposed in the work of [SYM95]. This approach is verified by theoretical and validated by experimental results in [MBS94, SYM95, HM97, Söf05, LS14a]. For nonlinear multi-input-multi-output mechanical systems with multi-degree of freedom, as mentioned in this paper, observer-based control approach will be applied to solve the vibration control of complex nonlinear systems.

In the following parts of this chapter, the approach controlling the Bucket-Wheel Boom is presented based on the contributions [SYM95, LS14c]. The system states as well as the time behavior of nonlinearities (as additive effects in relation to the linearized system) in combination with related system states are estimated using a high-gain PI-observer. Then, the PI-Observer-based control approach is realized controlling vibrations of the BWE in combination with the well-known disturbance rejection control approaches.

4.2 State space representation

In the chapter 2, the nonlinear modeling of the Bucket-Wheel boom considering the elasticity of hoisting cables and higher-order couplings between flexible deformations effected by complex motions has been presented. The guided motions including the cutting resistance forces, coupled with states depending in nonlinear matrices can also be seen from the modeling. Here they also appear as part of the nonlinear terms in the nonlinear/state-/motion dependend matrices. The effects of them along with the important dynamic phenomena due to the nonlinearities resulting from geometrical and dynamical couplings of the dynamic behavior of the Bucket-Wheel boom in the sequel have been analyzed in detail.

Based on the previously given results, the total equations of motion of the Bucket-Wheel boom are expressed in matrix form as

$$M\ddot{u}(t) + D\dot{u}(t) + Ku(t) = f(t) + B^*z(t) + N^*n(u,t), \tag{4.1}$$

where $u(t), \dot{u}(t), \ddot{u}(t)$ denote vector of displacements, deflections, and its derivatives of the elastic motion and M, D, K, and $f(t)$ are the global mass, damping, stiffness matricies, and force vectors, respectively. The nonlinearities are denoted as $N^*n(u,t)$.

For control analysis and design purposes, the above mentioned differential equations describing the motion of the Bucket-Wheel boom can be transformed into state-space representation of a mechanical system with constant properties M, D, K as well as nonlinear disturbances Nn as

$$\dot{x}(t) = Ax(t) + Bz(t) + Nn(x(t),t) + e(t), \tag{4.2}$$

$$y(t) = Cx(t), \tag{4.3}$$

with state vector

$$\dot{x}(t) = \begin{bmatrix} u(t) \\ \dot{u}(t) \end{bmatrix}_{n\times 1}, \tag{4.4}$$

system matrix

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}_{n\times n}, \tag{4.5}$$

input matrix

$$B = \begin{bmatrix} 0 \\ -M^{-1}B^* \end{bmatrix}_{n \times l}, \tag{4.6}$$

input matrix of nonlinearities

$$N = \begin{bmatrix} 0 \\ M^{-1}N^* \end{bmatrix}_{n \times n}, \tag{4.7}$$

known input vector (here external forces)

$$e(t) = \begin{bmatrix} 0 \\ -M^{-1}f(t) \end{bmatrix}, \tag{4.8}$$

$y(t)$: vector of measurements of order m,
C : measurement matrix $m \times n$, and
$z(t)$: control input.

4.3 State and disturbance estimation

Taking into account nonlinear effects leads to difficulties in control design tasks. Sloving this problem a typical and often used control approach is based on the linearization of the nonlinear dynamical system in combination with a related linear, working point-based control approach. From this point of view, the well-known results of linear control theory, of course strictly related to the assumed working point and further related restrictions, can be applied to nonlinear problems of dynamical systems. Nonlinearities considered in the modeling of this contribution are geometric nonlinearities resulting from the geometrical couplings of longitudinal and bending deformations and and also from dynamical couplings of higher-order. These nonlinearities are assumed as additive unknown inputs effecting the linearized/linear system. Based on this understanding the detailed nonlinear modeling will be used to describe the complex behavior of the mechanical system. The simplified linear model will be used as the base of the related (robust) control design. The extended linear system including the linear part of the original system and the linear fictitious model replaces the nonlinear system. From this theoretical point of view, PI-observer technique, as shown in Figure. 4.1, has been applied successfully to estimate not only the system states but also unknown inputs as nonlinearities or uncertainties due to modeling errors, external disturbances, etc. in many different areas of robotics, vehicle- and rotor dynamics. In addition, ability of using this technique in a quantitative fault diagnosis and robust control of the system is also introduced. With above mentioned aspects, reviews about development and application of the robust PI-Observer is given detailed in [Söf05, Liu11].

Based on the approach given in [SYM95], nonlinearities (here higher-order couplings between flexible deformations effected by complex motions) can be reconstructed using a high-gain extended state observer. Moreover, the states of the dynamic system can also be estimated.

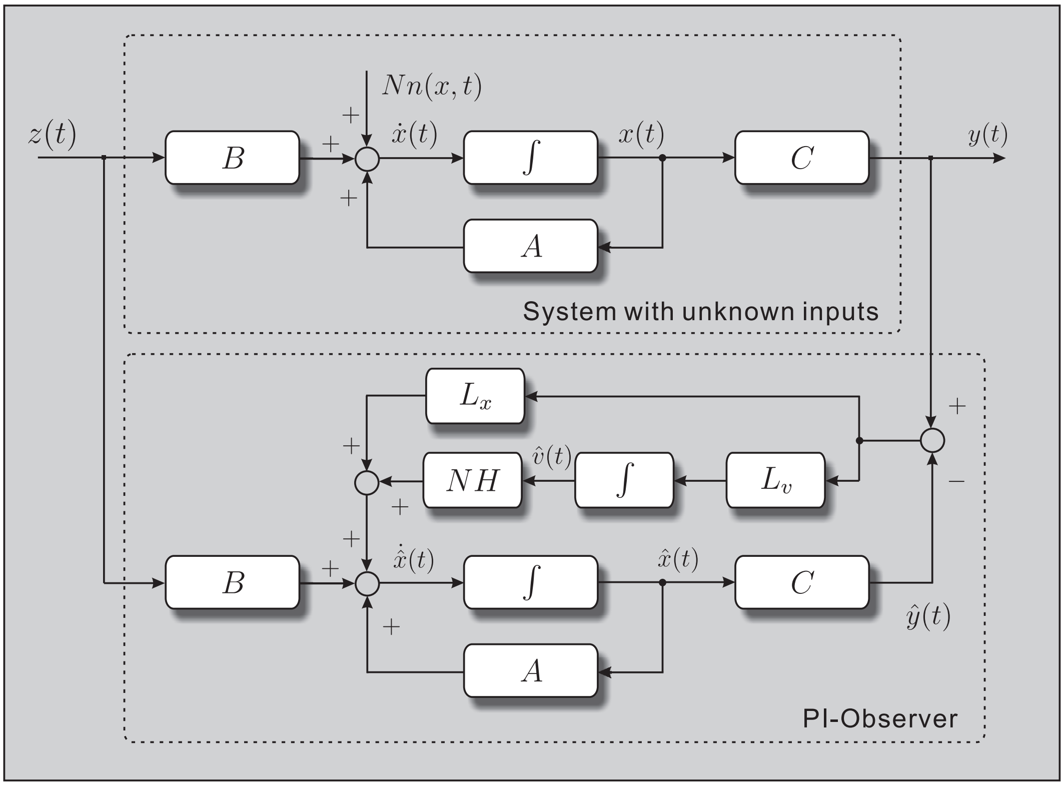

Figure 4.1: Structure of PI-Observer [SYM95]

4.3.1 Fictitious modeling

Figure 4.1 is the structure of PI-observer in which both proportional feedback part and the integral feedback part are used to the estimation error, therefore resulting to a better estimation performance. As shown in [SYM95], nonlinearities $n(x(t),t)$ of the dynamic system are assumed as external disturbances and modeled by a suitable fictitious model F as

$$n(x(t),t) \approx Hv(t), \tag{4.9}$$
$$\dot{v}(t) = Fv(t), \tag{4.10}$$
$$\dim[v] = s, \tag{4.11}$$
$$\text{rank}[N] = r_2, \tag{4.12}$$

where $Hv(t)$ denotes additional variables, which represent fictitious approximations of nonlinearities.

Using the approximation of nonlinearities (Equation 4.9), the nonlinear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bz(t) + Nn(x(t), t) + e(t), && (4.13) \\ y &= Cx(t), && (4.14) \end{aligned}$$

can be replaced by the extended linear system as follows

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & NH \\ 0 & F \end{bmatrix}}_{A_e} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_e} z(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} e(t), \qquad (4.15)$$

$$y(t) = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}, \qquad (4.16)$$

to be used for design of the observer and control gains.

In the applications shown in [SM93, SYM95, Mül00], the model matrices F and H are chosen simply as $H = I_s$ and $F = 0$. Furthermore, a condition for the complete observability of the extended linear system must be guaranteed as follows

$$rank \begin{bmatrix} \lambda I_n - A & -NH \\ 0 & \lambda I_n - F \\ C & 0 \end{bmatrix} = n + s, \text{for all} \quad \lambda \in A_e, \qquad (4.17)$$

with $r \geq s$; this means that the number r of measurements must be equal or larger than the number of nonlinearities s.

4.3.2 Reconstruction of nonlinearities

Using an identity observer, the states $x(t)$ and $v(t)$ of the extended linear system, as given in Equations (4.15), (4.16), can be estimated. The equation of the observer can be given as

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{v}}(t) \end{bmatrix} &= \begin{bmatrix} A & NH \\ 0 & F \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} z(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} e(t) + \begin{bmatrix} L_x \\ L_v \end{bmatrix} (y_e - \hat{y}_e), \\ &= \underbrace{\begin{bmatrix} A - L_x C & NH \\ -L_v C & F \end{bmatrix}}_{\tilde{A}_e} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} z(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} e(t) + \begin{bmatrix} L_x \\ L_v \end{bmatrix} y_e, && (4.18) \\ y_e &= [y^T \ 0]^T, && (4.19) \end{aligned}$$

where L_x, L_v are the observer gain matrices. They are chosen suitably to guarantee the asymptotic stability of the observer using LQR method.

Solving the Riccati equation

$$A_e P + P A_e^T + Q - P C_e^T R^{-1} C_e P = 0, \tag{4.20}$$

with the weighting matrices Q, R chosen as

$$Q = \begin{bmatrix} I_n & O_{n\times s} \\ O_{r\times s} & qI_s \end{bmatrix}, \quad R = I_s, \tag{4.21}$$

where q is a scalar control design parameter ($q > 0$), the observer gain matrices L are calculated as

$$L = [L_x \ \ L_v]^T = P C_e^T R^{-1}. \tag{4.22}$$

The nonlinear effects considered in this contributions (see Chapter 3) result as mentioned from higher-order geometrical and dynamical couplings. They cause strong effects on the dynamic responses of the system undergoing digging forces. Due to the complexity of the dynamics it is impossible to measure the nonlinear dynamical effects directly using the conventional methods. However, the time behavior of the nonlinearities can be reconstructed with the above introduced extended observer approach based on the used equation

$$\hat{n}(\hat{x}(t)) = H\hat{v}(t), \tag{4.23}$$

as $n \approx v$. Details of the approach are given in [SYM95]. Additionally the estimation $\hat{x}$ for x will be used for the control approach.

4.4 Controller design

Based on estimations of the system states as well as of the nonlinearities, the control design can be applied similar to the observer design related approaches. In this section, state feedback control in combination with disturbance rejection control techniques is discussed. From this point of view, nonlinear effects will be dynamically compensated leading to the satisfactory closed-loop control system. As nonlinearities the estimation $\hat{v}$ of the high-gain observer is used in combination with the estimation of the full states of the system $\hat{x}$.

4.4.1 Control law

From the extended linear system (Equations 4.15 and 4.16), the task of a control $z(t)$ in this case is to regulate the system states $x(t) \to 0$ and simultaneously compensate unknown disturbances $n(x(t), t)$ (here nonlinearities due to couplings of flexible deformations effected by complex motions). The state feedback control

$$z(t) = -\begin{bmatrix} K_x & K_v \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \tag{4.24}$$

of the states $x(t)$ of the system (Equation 4.13) and the states $v(t)$ of the disturbance model (Equation 4.9) stabilizes the system (Equations 4.13 and 4.14) and rejects the influence of unknown disturbances asymptotically. When applying the state feedback control Equation (4.24), all states $x(t)$ and $v(t)$ of the extended linear system need to be estimated. Here, the PI-observer (Equations 4.18 and 4.18) can be used for estimating system states. The complete structure of the model-based control approach using the observer technique is illustrated in Figure 4.2. Therefore, in general, the control law designed for the controller using the estimates of the system state variables $\hat{x}(t)$ as well as nonlinearities $\hat{v}(t)$ can be described as follows

$$z(t) = [z_x(t) \;\; z_v(t)]^T = [-K_x\hat{x}(t) \;\; -K_v\hat{v}(t)]^T, \tag{4.25}$$

where the state feedback part of the system $z_x(t) = -K_x\hat{x}(t)$ improves the responses of the system and the compensation part of the nonlinearities $z_v(t) = -K_v\hat{v}(t)$ decouples the destabilizing effects of nonlinearities due to couplings.

4.4.2 Model-based control design using disturbance rejection approaches

As pointed out in chapter 2, uncertainties due to modeling errors, changing environmental and operation conditions, etc. usually lead to difficulties in control or degradation of performance of control systems. Therefore, they are undesirable and need to be rejected in the controller design/analysis to guarantee robust stability and robust performance. This problem has been special attention in control theory with many techniques proposed [Joh71, Din03, Han09, HD09]. In chapter 3, the nonlinear modeling of the dynamical behavior of the Bucket-Wheel boom undergoing guided motions in combination with external excitations is presented including effects of nonlinearities. Based on this modeling, as mentioned in the above part, the PI-observer is applied for estimating states as well as nonlinearities. From this theoretical point of view, controllers can be realized to compensate nonlinearities and improve dynamic responses of the control system. In following part, model-based disturbance rejection approaches are discussed and compared controlling the Bucket-Wheel Excavator.

Static disturbance rejection approach [Joh71]

From the state equation of the extended linear system (Equations 4.15 and 4.16), the control input is divided into

$$z(t) = z_x(t) + z_v(t), \tag{4.26}$$

with the state feedback part of the system $z_x(t) = -K_x\hat{x}(t)$ and the compensation part of the nonlinearities $z_v(t) = -K_v\hat{v}(t)$.

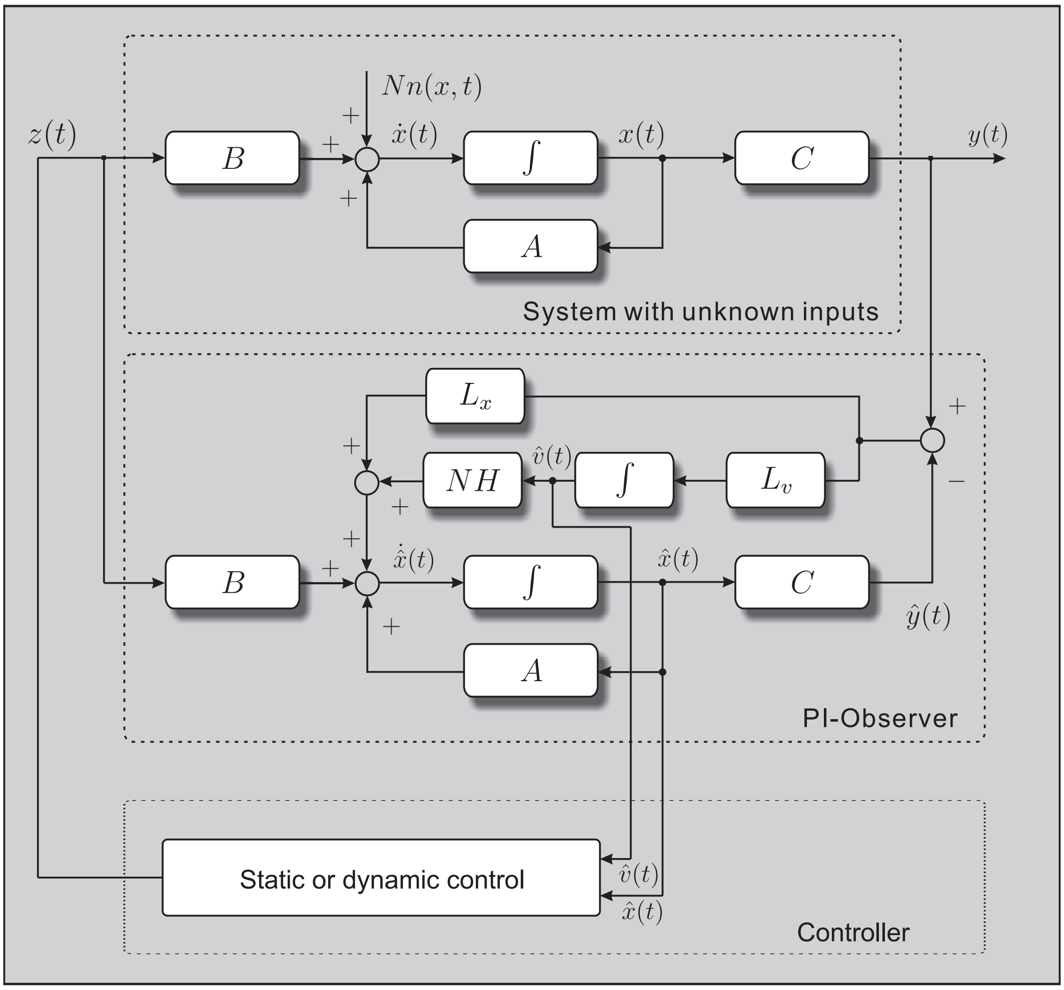

Figure 4.2: Model-based control approach using the observer technique [Söf96]

With this approach, unknown disturbances can be asymptotically compensated when the feedback component $z_v(t)$ is capable of satisfying

$$Bu_v = -NHv(t), \tag{4.27}$$

for all admissible $v(t)$. According to [Joh71], assuming rank $[B|NH]$ = rank $[B]$ results in

$$NH = BK_v, \tag{4.28}$$

for some matrix K_v. Therefore, the matrix K_v of the compensation component u_v can be determined as

$$K_v = (B^T B)' B^T NH. \tag{4.29}$$

As a result, the control law using a feedforward of the states of the disturbance model is expressed as

$$z(t) = -K_x\hat{x}(t) - (B^TB)'B^TNH\hat{v}(t), \tag{4.30}$$

where K_x is determined based on the controlled nominal system (A, B).

Applying the feedback control law determined by the Equation (4.30) to the extended linear system (Equations 4.15 and 4.16), the closed-loop control system with the static input compensation can be synthetized as follow

$$\begin{bmatrix}\dot{x}(t)\\ \dot{v}(t)\end{bmatrix} = \begin{bmatrix}A - BK_x & -BK_v\\ 0 & F\end{bmatrix}\begin{bmatrix}x(t)\\ v(t)\end{bmatrix} + \begin{bmatrix}N\\ 0\end{bmatrix}n(t) + \begin{bmatrix}I\\ 0\end{bmatrix}e(t). \tag{4.31}$$

The effects of nonlinearities using this approach are only compensated asymptotically when the input matrix N of the nonlinearities coincides with the input matrix B. Furthermore, it is difficult to guarantee the robustness of the controlled system using this approach under highly nonlinear effects, uncertainties due to modeling errors, external disturbances, etc.

Davison approach [Dav72]

Considering the system described by Equations (4.13) and (4.14), the approach of Davison introduces the disturbance signal model as

$$\dot{v}(t) = Sv(t) + B_\delta(y(t) - w_p(t)), \tag{4.32}$$

where S, y, and w_p is the system matrix of the disturbance-model, vectors of the measurements, and tracking signals (here $w_p = 0$), respectively.

Applying this disturbance signal model to the system, the augmented system is obtained in the state equations as

$$\begin{bmatrix}\dot{x}(t)\\ \dot{v}(t)\end{bmatrix} = \underbrace{\begin{bmatrix}A & 0\\ B_\delta C & S\end{bmatrix}}_{A_{e,Davison}}\begin{bmatrix}x(t)\\ v(t)\end{bmatrix} + \begin{bmatrix}B\\ 0\end{bmatrix} + \begin{bmatrix}N\\ 0\end{bmatrix}n(t) + \begin{bmatrix}I\\ 0\end{bmatrix}e(t). \tag{4.33}$$

To obtain a stabilizing control for the augmented system (Equation 4.33), a suitable control input is applied as Equation (4.24). Assuming the following assumptions are satisfied.

- A dynamical model S of the disturbance is available.
- The pairs (A, B) and (S, B_δ) are controllable.
- The eigenvalues of S are not transmission-zeroes of the system.

Applying classical approaches as the LQR or pole placement, the feedback matrices

$$K = \begin{bmatrix} K_x \\ K_v \end{bmatrix} \tag{4.34}$$

can be calculated solving the Riccati equation

$$A_{e,Davison}P + PA^T_{e,Davison} + Q - PB^T_{e,Davison}R^{-1}B_{e,Davison}P = 0. \tag{4.35}$$

The state equations of the closed-loop system are expressed in the form

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} A - BK_x & -BK_v \\ B_\delta C & S \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} N \\ 0 \end{bmatrix} n(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} e(t). \tag{4.36}$$

As a result, modeled disturbances can be compensated asymptotically in all controlled outputs using Davison's approach.

Extended approach [Söf96]

As pointed out previously, all system states of the system ($x(t)$) as well as unknown disturbances ($Nn(x,t)$ approximated by $NHv(t)$) are estimated and reconstructed using a high-gain PI-Observer. Therefore, the modification in the sense of Davison's approach for compensating unknown disturbances is present in [Söf96] by introducing a disturbance model

$$\dot{v}(t) = Fv(t) + Qx(t), \tag{4.37}$$

using a suitably chosen matrix Q.

In contrast to Davison's approach, instead of using output channels $(y(t) - w_p)$, this extended approach uses all estimated states for dynamic compensation of unknown disturbances. From Equations (4.15) and (4.37) the extended system can be expressed in the state equations as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & NH \\ Q & F \end{bmatrix}}_{A_{e,Mod}} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} z(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} e(t). \tag{4.38}$$

Controllability of the pairs (A, B) and (F, Q) is also required as assumptions given in Davison's approach. The feedback matrices K_x, K_v are also determined using classical approaches as the LQR or pole placement. Applying state feedback control (Equation 4.24) to the extended system (Equation 4.38), the controlled system is governed as follows

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} A - BK_x & -BK_v \\ Q & F \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} N \\ 0 \end{bmatrix} n(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} e(t). \tag{4.39}$$

Assuming that the estimator reconstructs all states as well as related dynamic effects, as shown in [Söf96], the input matrix Q can be efficiently chosen as

$$Q = (NH)^T. \tag{4.40}$$

4.4.3 General equations of Observer-based controller

Combining Equations (4.13), (4.18), and (4.37), the closed-loop control system can be synthetized by general state equations of motion as follow

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \\ \dot{\hat{v}}(t) \end{bmatrix} = \begin{bmatrix} A & -BK_x & -BK_v \\ L_xC & (A - L_xC - BK_x) & (NH - BK_v) \\ L_vC & -L_vC & F \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} + \\ + \begin{bmatrix} Nn(x,t) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} I \\ I \\ 0 \end{bmatrix} e(t). \quad (4.41)$$

4.5 Summary

The equations of motion describing the dynamic behavior of the Bucket-Wheel boom undergoing guided motions in combination with external excitations is expressed in terms of mass, damping, and stiffness matrices. Using this model it is easy to apply model-based design approaches controlling vibrations of the Bucket-Wheel Excavator. In this chapter, a high-gain PI-Observer is applied efficiently not only to estimate all system states but also to reconstruct unknown disturbances (here the time behavior of nonlinearities due to couplings of flexible deformations). Based on these estimations, model-based control design method is realized controlling vibrations of the system. Disturbance rejection approaches (static and dynamic disturbance rejection), often used for control of nonlinear mechanical systems, are discussed and compared. The efficiency of these approaches for improving responses and for stabilizing the system during the digging process of Bucket-Wheel excavator will be shown in the next chapter of this thesis.

5 Simulation results

As described in chapter 3, the nonlinear modeling of the finite elastic boom of the BWE is presented considering effects of geometric and dynamic couplings of flexile deformations under guided motions. Then, in the chapter 4, observer-based control using disturbance rejection control techniques is applied controlling vibrations of the Bucket-Wheel Excavator. The related results in this chapter are based on [LS13a, LS14b, LS14c]. The dynamical behavior of the Bucket-Wheel boom excited by guided motion is studied through numerical simulations in combination with digging process forces acting simultaneously on the boom-tip according to the three directions (x-, y-, and z-direction) as shown in Figure 5.1.

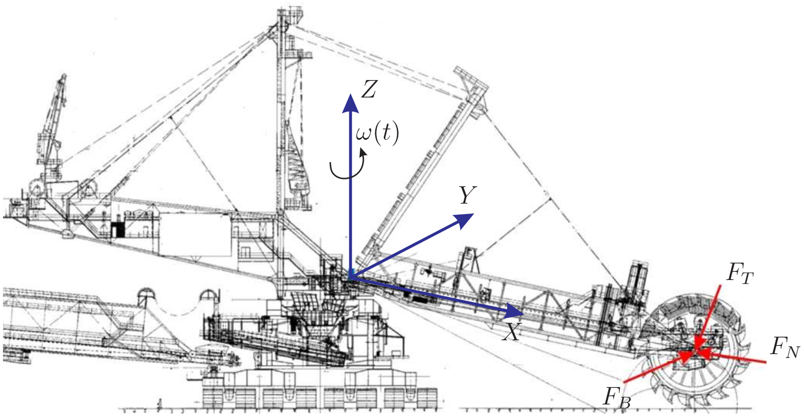

Figure 5.1: Digging forces acting at tip of the Bucket-Wheel boom [Ima12]

Firstly, dynamic responses of the Bucket-Wheel boom in case of uncontrolled motion including higher-order coupling effects between the overall motion and the flexural, longitudinal deformations, effects of resistance forces, hoisting cables, and the angular velocity of the boom are discussed. Next, strategy of using the active control method is applied controlling vibration of the Bucket-Wheel excavator. The simulation results of using PI-Observer to estimate system states and the time behavior of nonlinearities (as additive effects in relation to the linearized system) are included in this section. Finally, the efficient suppression of vibrations as well as stabilization of the system during the digging process of Bucket-Wheel Excavator is illustrated using the PI-Observer-based control approach in combination with disturbance rejection control approaches.

The flexible Bucket-Wheel boom is modeled using Finite Element Method with 5 finite beam elements. Thus, the state-space equation of the control system can be described by

$$\dot{x}(t) = Ax(t) + Bz(t) + Nn(x(t), t) + e(t), \tag{5.1}$$
$$y(t) = Cx(t), \tag{5.2}$$

with the state vector

$$x(t) = [u_1\ u_2\ ...\ u_{30}\ \dot{u}_1\ \dot{u}_2\ ...\ \dot{u}_{30}]^T, \tag{5.3}$$

the system matrix

$$A = \begin{bmatrix} 0_{30\times30} & I_{30\times30} \\ -M^{-1}K & -M^{-1}D \end{bmatrix}_{60\times60}, \tag{5.4}$$

the input matrix

$$N = \begin{bmatrix} 0_{30\times3} \\ M^{-1}N^* \end{bmatrix}_{60\times3}, \text{with } N^* = \begin{bmatrix} 0_{24} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0_{24} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0_{24} & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T_{3\times30}, \tag{5.5}$$

the output matrix

$$C = \begin{bmatrix} 0_{24} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0_{24} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0_{24} & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{5.6}$$

The damping matrix is assumed using Raleigh damping hypothesis with

$$D_g = \alpha K_{li} + \beta M_g. \tag{5.7}$$

For an elastic beam, a proportional damping (α =0.0001, β=0) is used consistently in the following simulation results.

The flexible Bucket-Wheel boom is attached at a rotating base, which is guided by angular velocity about the z-axis as [KRB87]

$$\omega(t) = \begin{cases} \omega_0 \left[t - \frac{t_0}{2\pi} sin\left(\frac{2\pi}{t_0}t\right)\right], & \text{if} \quad t < t_0, \\ \omega_0, & \text{if} \quad t \geq t_0 \end{cases} \tag{5.8}$$

5.1 Modeling: From beam-like structures to the modeling of the elastic boom of BWE

Due to the relatively light weight, high stiffness, and ability for shape control, the truss structures have mostly been used in aerospace and civil engineering applications. With the very large, heavy weight structure of BWE, obviously, the space trusses are applied for the frame-work of the machine such as the Bucket-Wheel boom and the counterpart boom. They can be in lattice frame or girder design. Using the large number of discrete finite elements for modeling such structures usually results to the high cost in computation and may be inaccuracy in analysis of dynamical responses of the whole system. To overcome disadvantages of the above method, the overall elastic behavior of the space truss structure can be assumed as an equivalent continuum beam model, in which the methods based on energy equivalence, stiffness equivalence, etc. are used for calculating the equivalent beam properties and predicting the natural frequencies of vibrations. By doing this, the number of degrees of freedom for modeling the structure is reduced significantly and low cost in computation therefore can be obtained. Moreover, the dynamical behavior of the repetitive structures as well as the related dynamic phenomena can be described and analyzed accurately using the Bernoulli-Euler and Timoshenko beam theory. Many investigations based on substitute continuum approaches are presented [Noo88, BC99, Teu00, SG05, SZ06].

An overview of using the continuum modeling approach for analyzing repetitive lattice structures and determining the equivalent continuum properties of beam-like lattice structures are reviewed detail in [Noo88]. Applications of this approach for solving problems in analysis of beam-like lattice structures such as geometrically nonlinear, free vibration, stability problems, stress analysis, and system identification, control law design problems are also pointed out in this contribution. In [BC99] a general procedure based on the energy equivalence between the continuum modeling and the original structure is pointed out for determining equivalent beam properties. It consists of the following steps: isolating a repeating cell, defining displacement fields in truss, identifying strain parameters of the continuum beam, apply the energy equivalence, calculating equivalent parameters of the continuum beam-like lattice structures. In this contribution, a displacement field in truss is defined using piecewise linear functions based on second order polynomials. Moreover, the continuum stress and strain parameters of the beam-like lattice trusses are determined by their average values at the center of the continuum cell. Compared with classical approaches as mentioned in [Noo88], the numerical simulation results show the effectiveness and the reliability of this approach. This approach is applied not only in the static analysis of planar beam-like lattice trusses but also in dynamic analysis of three-dimensional or planar beam-like lattice trusses. Based on the energy equivalence, a direct energy approach in which the truss forces are expressed in terms of the section forces to define the strain energy in the discrete cell is applied for determining equivalent beam properties [Teu00]. The equivalent stiffness of different types of the plane trusses used commonly in aerospace and civil engineering structures is calculated and systematized in formulas. The results in analysis of the plane trusses then are extended to build up the equivalent continuum properties of the three-dimensional beam-like spatial lattice structures. The accuracy of this approach is verified by comparing with the approach given in [BC99]. By analyzing the eigenvalues of a single cell of planar beam-like structures, the equivalent continuum beam properties can be determined in [SG05, SZ06]. From this theoretical point of view, problems in static and dynamic analysis of such structures including coupling effects of tension-torsion, bending-shear, and determination of natural frequencies of vibration, etc. are therefore solved.

Actually, the steel structure of the Bucket-Wheel excavator is always based on the space truss structures using different common types of the plane trusses such as a triangle truss, a simply supported V-truss, K-truss, N-truss grider, and roof truss, etc. Therefore, applying the formulas given in the work of [Teu00] for determining three-dimensional beam-like lattice structures, the equivalent continuum stiffness parameters of the beam-like spatial trusses of the Bucket-Wheel boom is calculated. Using geometric parameters of the Bucket-Wheel Excavators (SRs8000) obtained from RWE Power AG, the values of equivalent stiffness parameters of the boom of BWE (SRs8000) are calculated and shown in Table 1.

Table 5.1: Physical equivalent stiffness parameters of the boom of BWE SRs8000

Property	Symbol	Value
Beam length	L	63666 mm
Mass per unit volume	ρ	0.8E-05 (kg/mm^3)
Cross-sectional area	A	425160 (mm^2)
Young's modulus	E	210E+3 (N/mm^2)
Beam area moment of inertia	I_y	27.134E+12 (mm^4)
Beam area moment of inertia	I_z	6.8504E+12 (mm^4)

5.2 Uncontrolled system

5.2.1 Effect of geometrical nonlinear terms

Without higher-order couplings of flexible deformations

In this case (the linear modeling), coupling effects of longitudinal and bending deformations are neglected in the modeling. Considering (both separately and together) the rotation motion of the Bucket-Wheel boom at an angular velocity ($\omega = 0.014$ rad/s) and the digging resistance forces, these dynamic excitations causing deformations are shown in Figures 5.2 to 5.4. From the obtained results, it reveals that without the digging resistance forces vibrations of the boom-tip are only observed according to x-and y-direction due to effects of centrifugal and tangential forces.

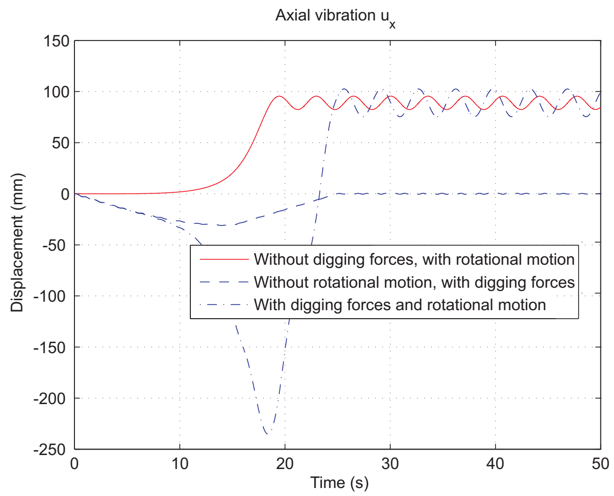

Figure 5.2: Axial deformation at tip of boom with the linear modeling (at $\omega = 0.014$ rad/s) [LS14b]

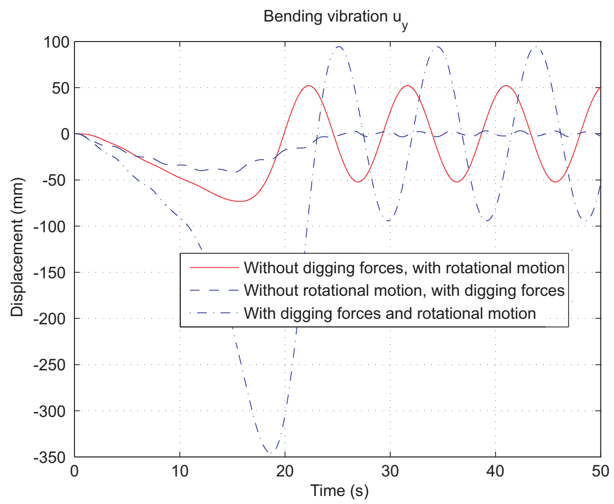

Figure 5.3: Bending deformation at tip of boom with the linear modeling (at $\omega = 0.014$ rad/s) [LS14b]

A negative value of the boom-tip deformation is observed as shown in Figure 5.4 according to z-direction by the effect of the weight of the bucket.

Contemporaneously considering the rotation motion and digging forces using linear modeling, the simulation results reveal that vibration magnitudes of the longitudinal and bending deformations at the tip of the boom increase significantly in comparison to separated cases. As proven in [LL07], these responses can be explained by

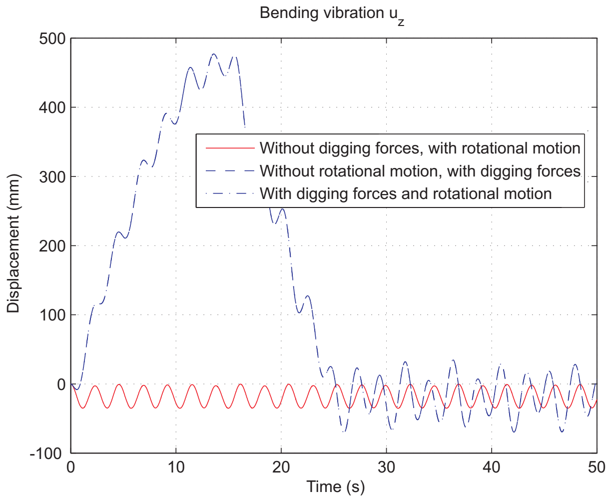

Figure 5.4: Bending deformation at tip of boom with the linear modeling (at $\omega = 0.014$ rad/s) [LS14b]

the coupling effects between the rotation motion and longitudinal and bending deformations caused by the cutting resistance forces.

With higher-order couplings of flexible deformations

Using the nonlinear strain-displacement relationships, from Equations (3.13) to (3.16), and retaining the higher-order terms in these relationships, the higher-order coupling terms involving the longitudinal, torsional, and bending deformations are taken into account in the dynamic modeling. The effects of higher-order couplings on the dynamic responses are studied through simulation examples in case of increasing gradually angular velocity of the Bucket-Wheel excavator ($\omega = 0.012$, 0.0138, and 0.014 rad/s) in combination with the digging resistance forces. Moreover, the results are compared with those resulting from linear models.

During the rotational motion angular velocity ($\omega = 0.012$ rad/s) of the Bucket-Wheel boom, the dynamic excitations cause longitudinal and bending deformations due to the resistance forces of the digging process acting as external inputs with respect to the model. The results for the deflections of the beam-tip (Figures 5.5 to 5.7) show that the magnitudes of the vibrations in case of the nonlinear modeling (with coupling terms) are larger compared with the linear modeling. However, coupling effects on the the dynamical responses of the boom at this angular velocity do not show significant difference with linear/ nonlinear modeling.

When increasing the angular velocity of the boom ($\omega = 0.0138$ rad/s), the simulated results reveal that the maximum magnitudes of the longitudinal and bending

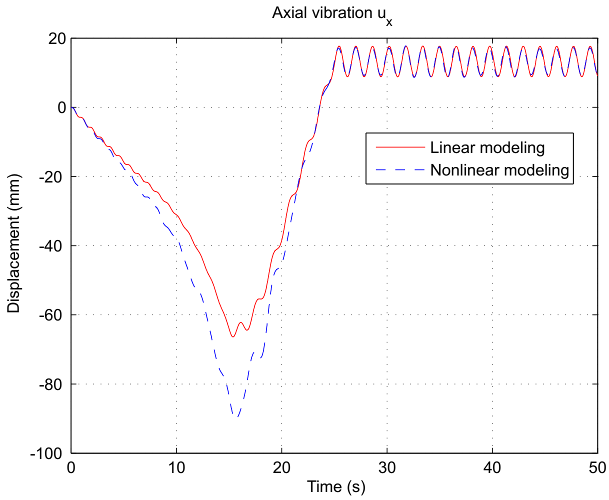

Figure 5.5: Axial deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.012$ rad/s)

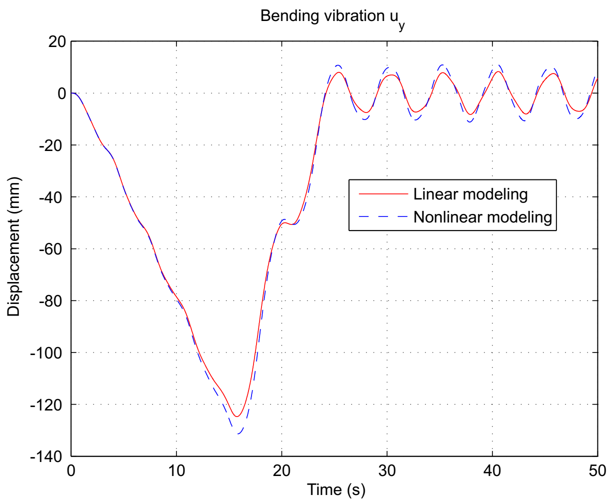

Figure 5.6: Bending deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.012$ rad/s)

deformations at the tip of the boom increase 1.82, 1.89 and 1.1 times, respectively, as shown in Figures 5.8 to 5.10, in comparison with the linear modeling. These responses are explained in [HU94] by the higher-order couplings between the longitudinal and bending deformations in combination with the influence of external forces acting on the beam-tip. Considering fully coupled effects of above mentioned terms at angular velocity ($w = 0.014$ rad/s) an unstable dynamic response of the

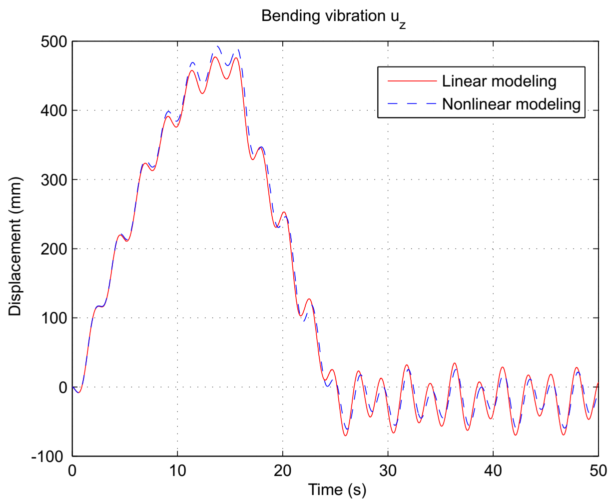

Figure 5.7: Bending deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.012$ rad/s)

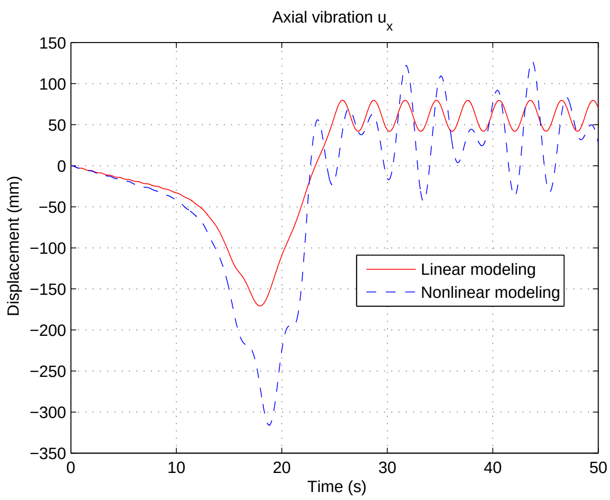

Figure 5.8: Axial deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.0138$ rad/s)

rotating elastic boom is observed from Figures 5.11 to 5.13. This effect results from the strong inertia forces along with the digging forces in combination with the couplings (the bending displacements causing longitudinal deformation) and and it is the main reason leading to very large deformation as well as the instability of the system.

From these above mentioned results, it can be seen that the higher-order model-

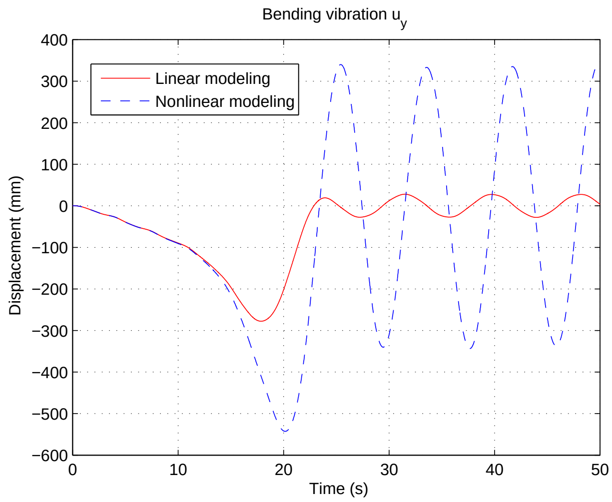

Figure 5.9: Bending deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.0138$ rad/s)

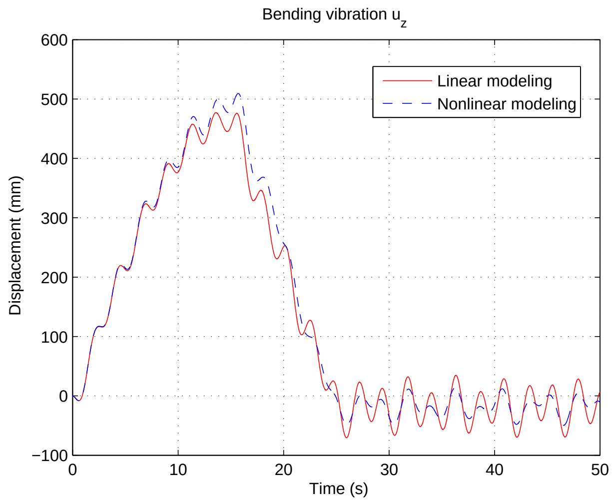

Figure 5.10: Bending deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.0138$ rad/s)

ing including higher-order geometrical couplings as well as the external excitations considered in this investigation is necessary to adequately describe the dynamical behavior of the Bucket-Wheel boom during operation. It gives a good base for the advanced study of the stability of the guided system in combination with external process forces resulting from the digging process of large excavator systems with long and slender booms.

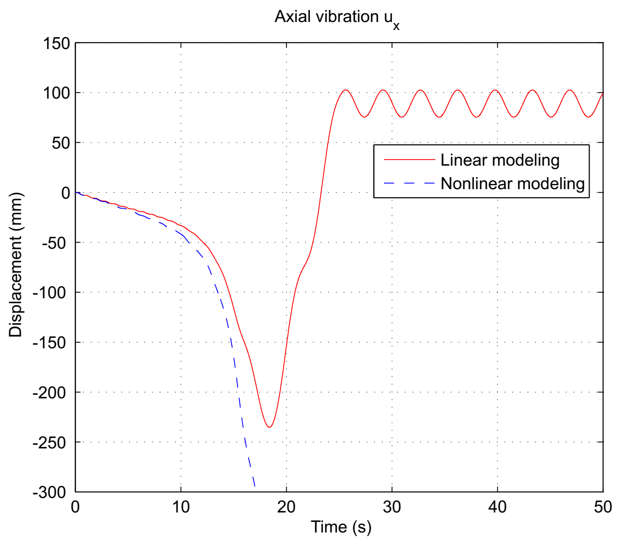

Figure 5.11: Axial deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.014$ rad/s) [LS14b]

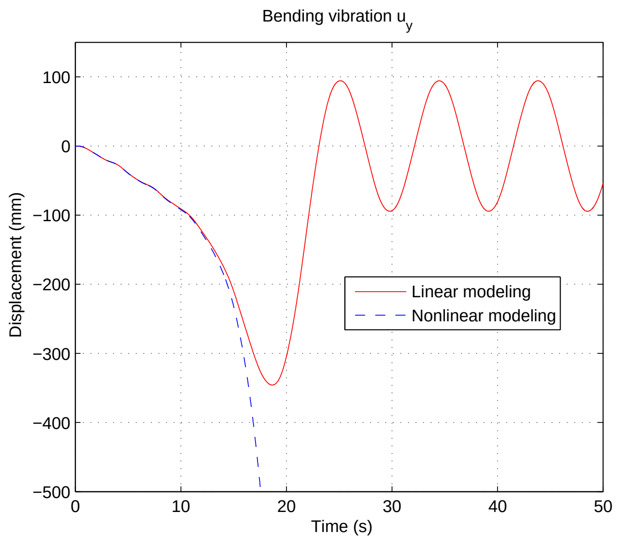

Figure 5.12: Bending deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.014$ rad/s) [LS14b]

5.2.2 Effect of additive hoisting cables

Due to advantages of lightweight and ability for stability and/or shape control, the hoisting cables connected at the tip of the Bucket-Wheel boom will stiffen and

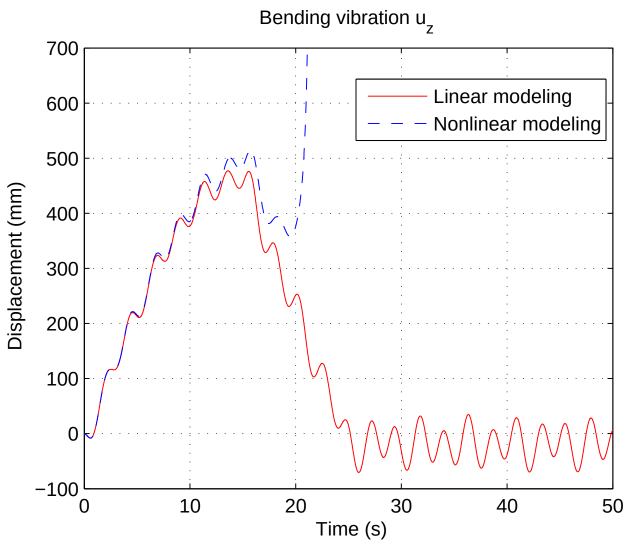

Figure 5.13: Bending deformation at tip of boom with the nonlinear modeling considering digging forces (at $\omega = 0.014$ rad/s) [LS14b]

guarantee equilibrium of the whole system. The effect of the hoisting cables on dynamic responses of the boom is shown in Figures 5.14 and 5.15 in case of with/ without the hoisting cables considering the digging resistance forces at the boom-tip. The obtained results reveal that the stiffness of the boom increases with the hoisting cables. As expected, the magnitudes of the elastic deformations, therefore, become smaller in case of considering the hoisting cable.

In addtion, by using displacement actuators and force sensors the hoisting cables connected at the boom-tip can produce the external forces for damping the vibration energy from the system. By doing this, problems such as vibration attenuation as well as stabilization of the Bucket-Wheel excavator can be easily solved.

5.2.3 Effect of changing digging forces

As pointed out in equation (3.53), the value of the external excitations (here the resistance forces) depends on the specific cutting resistance to digging under diffirent working conditions, the wheel speed of the rotor, and the number of the buckets. Changing the above mentioned factors will obviously affect the deformations as well as the stability of the system. The effects of the different digging work environments (fault zone, transitional zone, and gray clay) on the dynamical response of the boom based on the nonlinear modeling at the angular velocity ($\omega = 0.012$ rad/s) are shown in Figures 5.16 to 5.18. The simulated results reveal that the amplitudes of the longitudinal and bending deformations at the boom-tip increase with the harder materials.

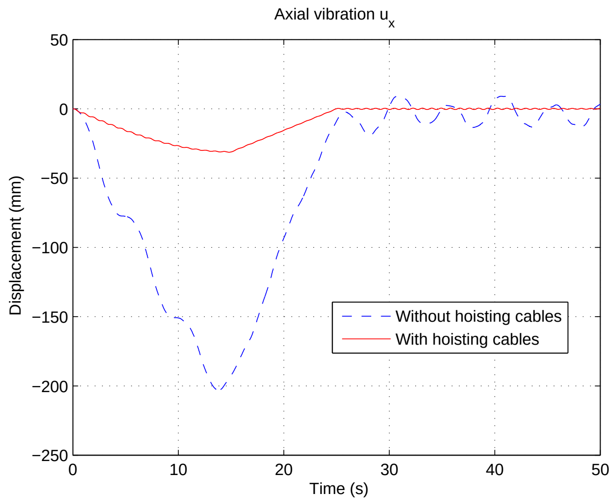

Figure 5.14: Axial deformation at tip of boom with/without hoisting cables considering digging forces (at $\omega = 0.012$ rad/s)

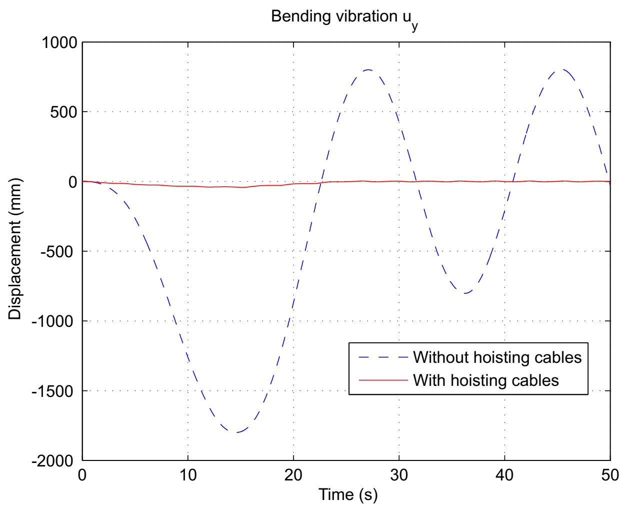

Figure 5.15: Bending deformation at tip of boom with/without hoisting cables considering digging forces (at $\omega = 0.012$ rad/s)

5.3 Controlled system

5.3.1 Estimation of states and nonlinearities

As mentioned in the investigations [SYM95, HM97, Söf05, LS14a], an indirect measuring technique using a state observer allows the estimation of the system states

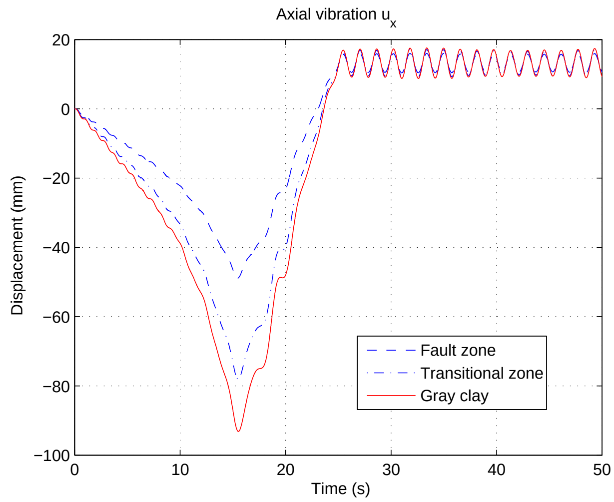

Figure 5.16: Axial deformation at tip of boom with different digging work environments (at $\omega = 0.012$ rad/s)

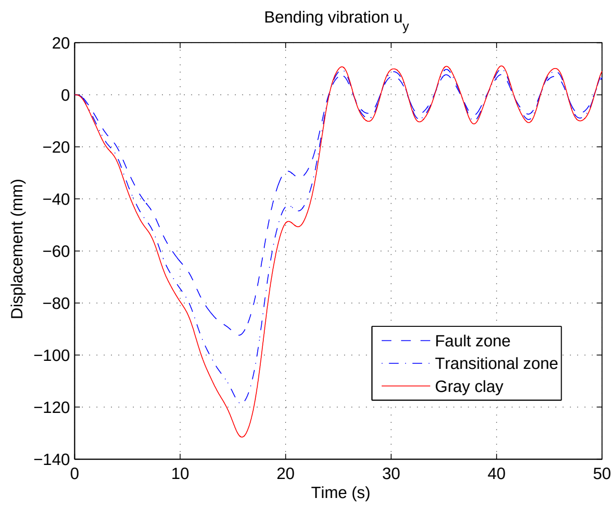

Figure 5.17: Bending deformation at tip of boom with different digging work environments (at $\omega = 0.012$ rad/s)

as well as the unknown nonlinear effects with simple measurements of the vibration system. Simulation and experimental results are verified to prove the successful application of this technique for vibration and control of elastic mechanical structures. In this section, a state observer is applied to estimate system states with the measurement matrix C taken at three tip positions of the boom according to x-, y-, z- direction (Equation 5.6). The input matrix N relating to the state vector of the

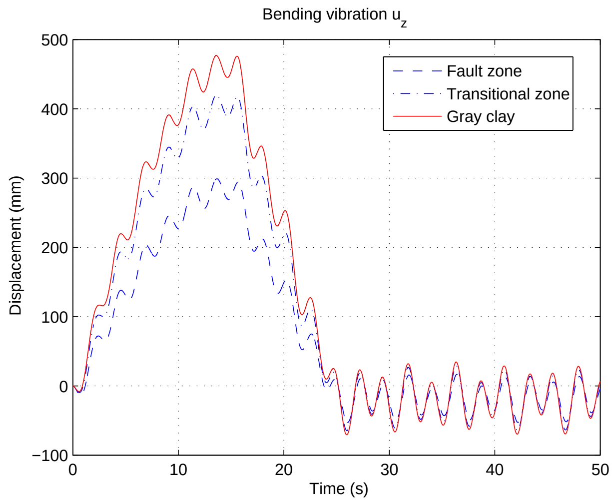

Figure 5.18: Bending deformation at tip of boom with different digging work environments (at $\omega = 0.012$ rad/s)

nonlinearities $Hv(t)$ exactly to the positions to be controlled is chosen as Equation 5.5.

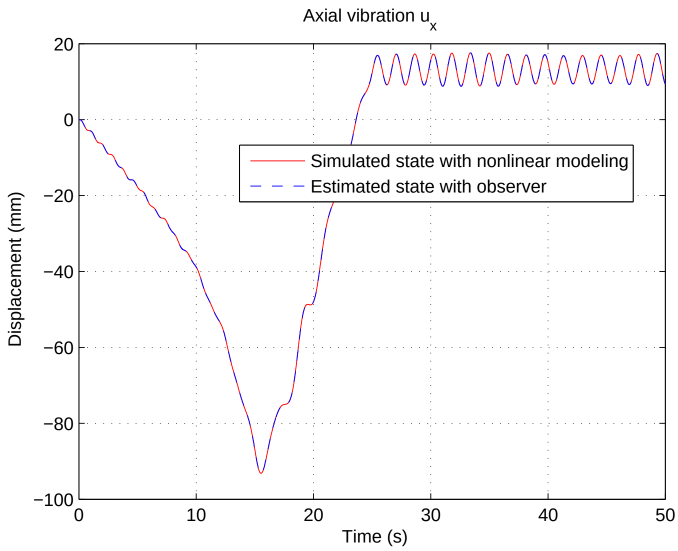

Figure 5.19: Estimated and real (simulated) states of axial deformation at tip of boom considering digging forces (at $\omega = 0.012$ rad/s) [LS14c]

Undergoing the dynamical excitations at the angular velocity ($\omega = 0.012$ rad/s) considering the external digging forces, when the observer is used with the measure-

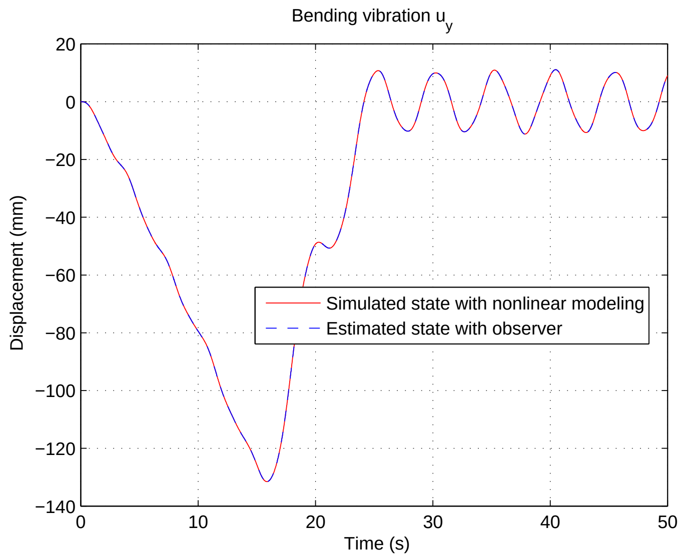

Figure 5.20: Estimated and real (simulated) states of bending deformation at tip of boom considering digging forces (at $\omega = 0.012$ rad/s) [LS14c]

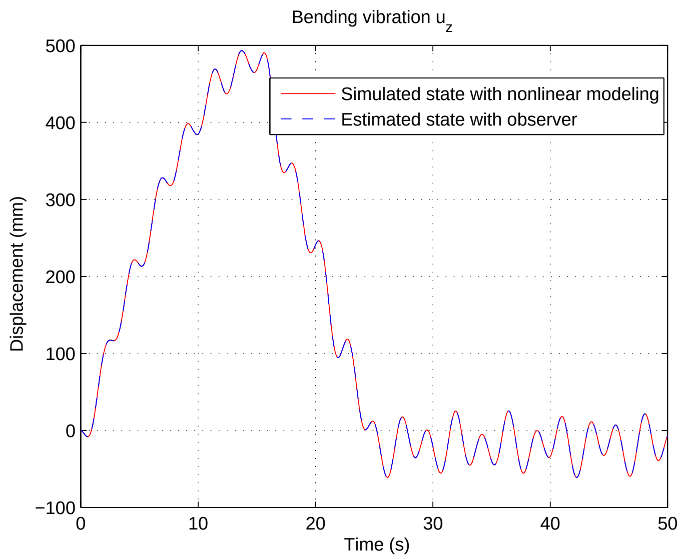

Figure 5.21: Estimated and real (simulated) states of bending deformation at tip of boom considering digging forces (at $\omega = 0.012$ rad/s) [LS14c]

ments defined by matrix C, related simulation results of the beam-tip displacements are shown in Figures 5.19 to 5.21. It can be seen, that the estimated states with the observer are the same with the simulated states using the nonlinear modeling.

In addition, the time behavior of the nonlinearities due to the coupling effects of the flexible deformations is also reconstructed using the state observer. The reconstructed results of the nonlinearities are shown in Figures 5.22 to 5.24 and compared with the real (simulated) nonlinearities.

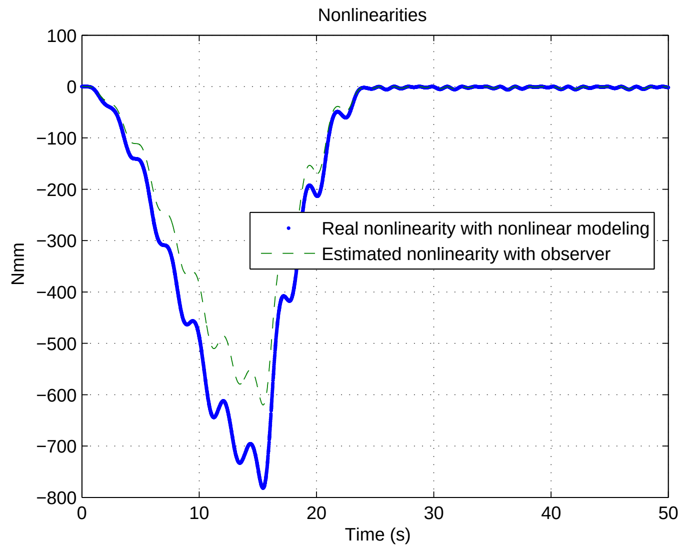

Figure 5.22: Estimated and real (simulated) nonlinear characteristics according to x-direction at tip of boom considering digging forces (at $\omega = 0.012$ rad/s) [LS14c]

As explained in [SYM95, LS12a], the design parameters of the observer approach strongly effecting the quality of the reconstructed nonlinearities will be changed depending on the observer gains. In order to match the characteristics of the real (simulated) and estimated nonlinearities, the observer dynamics must be faster than that of the system. So a good convergence with the real characteristics of the nonlinearities can be observed at high observer gains with constant design parameter $q = 10^{20}$, as shown in Equation (4.21). From the above analyzed results, it can be concluded that the observer works well to obtain the states along with nonlinearities, which are necessary to realize feedback control.

5.3.2 Stability-oriented control

The significant effects of nonlinearities due to the higher-order geometrical and dynamical couplings of flexible deformations leading possibly to larger amplitudes as well as unstable behavior of the Bucket-Wheel boom are analyzed in detail in the first section. Model-based disturbance rejection approaches, often used for control of nonlinear mechanical systems, are discussed in chapter 4 based on a high-gain

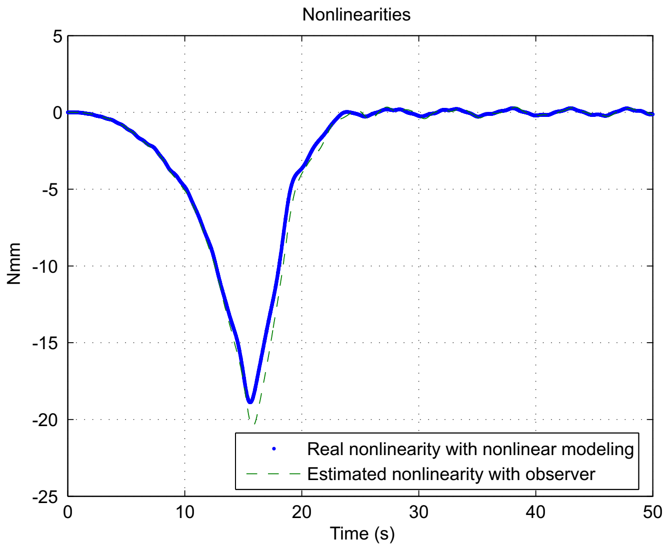

Figure 5.23: Estimated and real (simulated) nonlinear characteristics according to y-direction at tip of boom considering digging forces (at $\omega = 0.012$ rad/s) [LS14c]

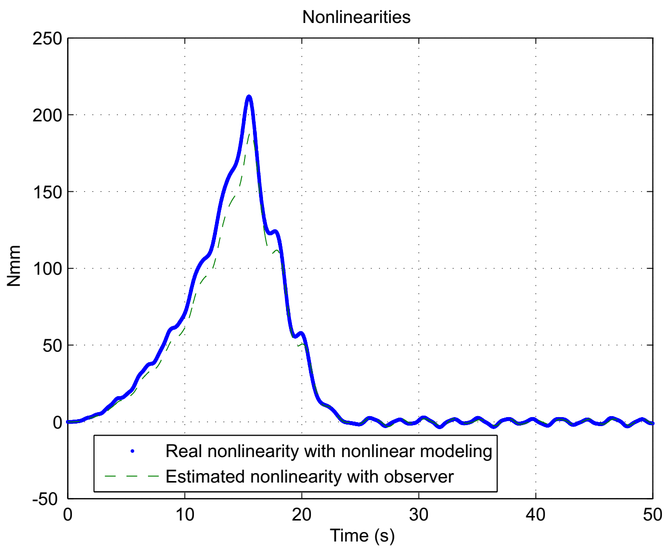

Figure 5.24: Estimated and real (simulated) nonlinear characteristics according to z-direction at tip of boom considering digging forces (at $\omega = 0.012$ rad/s) [LS14c]

PI-observer. In this section, simulation examples are included to illustrate the efficiency of the above mentioned approaches for stabilizing the system during the digging process of Bucket-Wheel excavator. With control input as given in Equation (4.24) using the estimated state $\hat{x}$ of x and $\hat{v}$ of v representing the approximation of the disturbance. The input matrix B, which is taken at the nodal of the beam connected with the hoisting cable, is defined as

$$B = \begin{bmatrix} 0_{30\times3} \\ M^{-1}B^* \end{bmatrix}_{60\times3}, \tag{5.9}$$

$$B^* = \begin{bmatrix} 0_{18} & 1 & 0 & 0 & 0 & 0 & 0 & 0_6 \\ 0_{18} & 0 & 0 & 1 & 0 & 0 & 0 & 0_6 \\ 0_{18} & 0 & 0 & 0 & 0 & 1 & 0 & 0_6 \end{bmatrix}^T_{3\times30}. \tag{5.10}$$

By changing the tension in the hoisting cable, displacements at this nodal can be controlled. The dynamic responses at the boom-tip under the guided motion and digging forces at the angular velocity (ω=0.014 rad/s) considering higher-order geometrical and dynamical couplings are shown in Figures 5.25 to 5.27 with different disturbance rejection approaches (static disturbance rejection approach, Davison approach, and the extended approach of Davison). From the obtained results, it can be seen that the static disturbance rejection approach cannot stabilize the system at an angular velocity ($\omega = 0.014$ rad/s) with a control input determined in Equation (4.30). On the contrary, the destabilizing effects resulting from the geometrical nonlinearities in combination with the digging forces are successfully compensated using Davison approach and the extended approach with the dynamic feedback terms, which are determined from Equations (4.33) and (4.38). Thus, the stable behavior of the Bucket-Wheel boom is observed in comparison with the uncontrolled motion.

Observer-based vibration control using the dynamic disturbance rejection approaches is effectively applied to control (stability control) the system including the ground orientation. Therefore, realization of these approaches (dynamic stability approaches) can replace the static stability approach to improve operating ranges of the BWE.

In order to evaluate the performance of the controller using dynamic disturbance rejection techniques, a criterion based on gains $P_{\mathbf{K}}$ is suggested in [Liu11] as

$$P_K = \left[\int_T^0 u^2 dt, \quad \int_T^0 e^2 dt\right]_K, \tag{5.11}$$

where P_K denotes the trajectory of the control input u and the control error e and T denotes the time window. As shown in Equation 5.11, by changing controller gains K, the trajectory P_K is built up by not only $\int_T^0 u^2 dt$ but also $\int_T^0 e^2 dt$. Based on this trajectory, the performance of a controller can be evaluated. In this criterion, a controller is assumed to be effective if at the same energy the control error is small. Besides, a robust of a controller under uncertainties conditions can be obtained when the change of the trajectory is small.

Applying means of criterion $P_{\mathbf{K}}$, the trajectories P_K are shown in Figure 5.28 with the time window $T = 50$ sec. Thus, control performance of controllers with two

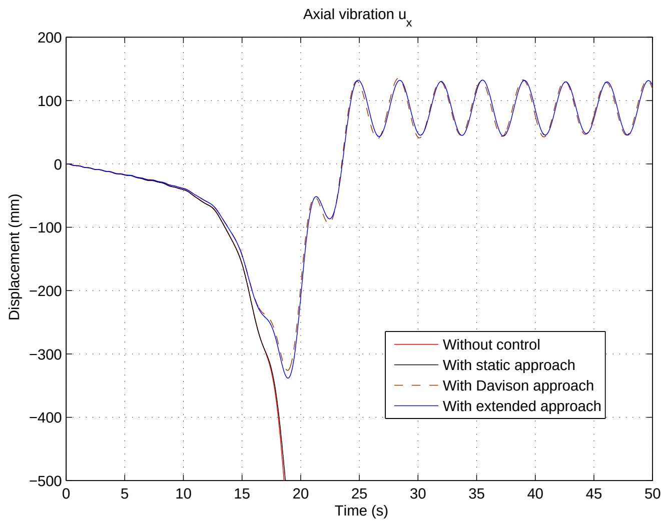

Figure 5.25: Axial deformation at boom-tip considering digging forces ($\omega = 0.014$ rad/s).

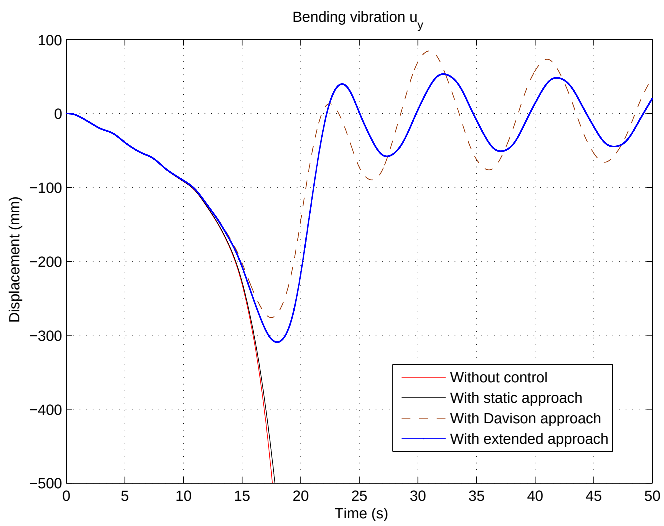

Figure 5.26: Bending deformation at boom-tip considering digging forces ($\omega = 0.014$ rad/s).

dynammic disturbance rejection approaches (Davison approach and extended approach of Davison) can be evaluated. The result in Figure 5.28 shows that at same energy the extended approach leads to small control error and therefore is more efficient compared with Davison approach.

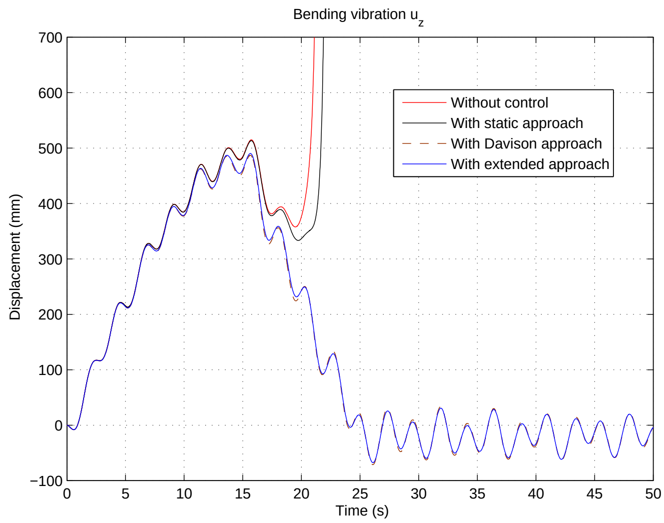

Figure 5.27: Bending deformation at boom-tip considering digging forces ($\omega = 0.014$ rad/s).

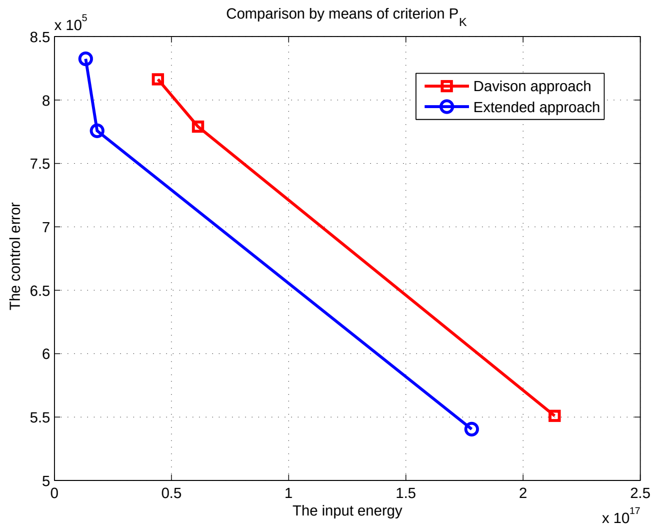

Figure 5.28: Comparison by means of criterion $P_{\mathbf{K}}$.

5.4 Summary

In this chapter, responses of the Bucket-Wheel boom undergoing the rotational motion and digging forces are illustrated considering with/ without the coupling terms due to the flexible deflections. The effects of higher-order geometric and dynamic couplings of flexible deformations are evaluated through simulation results. Considering these effects, the magnitudes of vibrations become larger and can lead to unstable behavior when increasing the angular velocity of the Bucket-Wheel excavator. In addition, the simulation results show the effects of the hoisting cables to the Bucket-Wheel boom resulting to changes of the stiffness of the structure. The dynamic responses of the Bucket-Wheel excavator relating to the digging forces under different exploitation environments are discussed. From the obtained results, it can be concluded that the nonlinear modeling of dynamical behavior of the Bucket-Wheel boom including above mentioned effectsis necessary to adequately predict responses of the system undergoing the rotational motion and digging forces. Based on this nonlinear modeling, control design approaches are synthesized controlling vibrations of the Bucket-Wheel excavator. The simulation results in this chapter illustrate the application of the high-gain PI-Observer to estimate system states as well as unknown disturbances. Using these results, the control method based on PI-Observer is realized controlling vibrations of the Bucket-Wheel excavator with different disturbance rejection approaches. The results are included to illustrate the efficient suppression of vibrations as well as stabilization of the system during the digging process of Bucket-Wheel Excavator. Consequently, application of these approaches can improve operating ranges of the Bucket-Wheel excavator. Therefore, an exploitation productivity of the Bucket-Wheel excavators can be increased.

6 Summary and Outlook

As pointed out in the first part of the thesis, the goals of this thesis are is to model and control complex flexible structures representingthe Bucket-Wheel Excavators. The tasks for solving the above goals are discussed in the next chapters. In this chapter, a summary of the whole thesis is given. The points suggested for future research in this area are recommended.

6.1 Summary

In previous work, the modeling and control of a nonlinear elastic beam representing the length-variable, elastic robot arms were presented [Söf96]. Based on this research, the nonlinear modeling of the dynamical behavior of the Bucket-Wheel boom is developed for the Bucket-Wheel excavator. From this theoretical point of view, the main tasks in this thesis are therefore solved. The contributions of the thesis work are summarized as follows:

- The full nonlinear mathematical model of the boom of the Bucket-Wheel excavator is derived. Here the Bucket-Wheel boom is modeled as a three-dimensional flexible beam using the Euler-Bernoulli beam theory and attached on the excavator turning platform assumed as a rigid body. Retaining higher-order terms in the nonlinear strain-displacement relationship, this allows capturing the higher-order geometrically nonlinear stiffening effects in the modeling of the Bucket-Wheel boom. Therefore, higher-order couplings between the overall motion and the longitudinal and bending deformations are taken into account. Furthermore, the nonlinear modeling of the three-dimensional elastic boom is also considered with the additional elasticity of suspending cables as well as the mass-tip of the Bucket-Wheel. More complex motions, especially the effects of additional guided motions in combination with digging resistance forces coupled with state as well as state dependences in the nonlinear matrices, are discussed and considered. So far, the elastic effects of the boom along with the interaction between the cutting head and the face material is taken into account. The effects of higher-order couplings between flexible deformations, the hoisting cables, and resistance forces on the dynamical responses of the Bucket-Wheel boom are illustrated by simulation examples in case of considering with linear/ nonlinear modeling. The dynamic phenomena resulting from the nonlinear modeling including higher-order geometrical couplings as well as the digging forces on the dynamic behavior of the Bucket-Wheel boom are analyzed in detail. Considering these effects, the magnitudes of vibrations become larger and can lead to unstable behavior when increasing the angular velocity of the Bucket-Wheel excavator. The results in case of considering the additional hosting cables to the boom show the change of the stiffness of the structure. Additionally, dynamic responses of the Bucket-Wheel boom undergoing different digging conditions are given. From the obtained results, it can be concluded that the nonlinear modeling of dynamical behavior of the Bucket-Wheel boom including above mentioned effects is necessary to adequately predict responses of the system undergoing the rotational motion and digging forces. Thus, the developed model and the related dynamic systems representation give a good base for the advanced study of the stability of the guided system in combination with external process forces.

- For control analysis and design purposes, the nonlinear dynamical system of the Bucket-Wheel boom is approximated by an extended linear system, in which nonlinearities of the dynamic system (here the higher-order geometrically nonlinear couplings of the flexible deformations) are assumed as external disturbances and modeled by a suitable fictitious model. Based on this extended linear system, the high-gain PI-Observer is applied not only to estimate all states of the system but also to reconstruct the time behavior of the nonlinearities. From this point of view, the high-gain PI-Observer-based state feedback control in combination with disturbance rejection control approaches is realized. Three disturbance rejection control approaches, the static disturbance rejection approach, Davison approach, and the extended approach of Davison, are discussed for compensating nonlinearities. From the simulation results, it can be concluded that the static disturbance rejection approach cannot stabilize the system at the high angular velocity. On the contrary, the destabilizing effects resulting from the geometrical nonlinearities in combination with the digging forces are successfully compensated using Davison approach and the extended approach with the dynamic feedback terms. The control results are included to illustrate the efficient suppression of vibrations as well as stabilization of the system during the digging process of Bucket-Wheel Excavator. Then, a criterion based on gains $P_{\mathbf{K}}$ is used to evaluate the performance of the controller using dynamic disturbance rejection approaches. The efficiency of using the extended approach of Davison is illustrated by the simulation result. Consequently, realization of these approaches (dynamic stability approaches) can improve operating ranges of the BWE.

6.2 Outlook

For further studies and future works, some points are recommended as follows.

- The verification of the above mentioned results can be included using the experimental data analysis or the test rig.
- The bounds/ regions of dynamic stability of the Bucket-Wheel boom can be determined from operating parameters.
- For a detail modeling of the Bucket-Wheel excavator, this modeling can be developed and extended with the counterbalance arm.
- Different advanced control approaches controlling vibrations of the Bucket-Wheel excavator can be applied and compared.

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The contents and results of this thesis are sumarized based on the following previous publications:

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